Lecture 5

Neutrinos

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How can they cross the earth or the sun without being stopped?

How can we get to know their masses and mixings?

How can they change their nature by travelling in empty space?

Can we count how many there are?

What's peculiar about them?

1. The only matter particles without strong or em interactions

Consider neutrinos from the sun

$$l^\nu = \frac{1}{n\sigma^\nu(E_\nu \approx 1MeV)} \qquad \sigma^\nu \approx 10^{-43} cm^2 \frac{E_\nu}{MeV} \qquad n \approx 100g/cm^3$$

$$\Rightarrow \quad l^\nu \approx 10^{18} cm \approx 1 \; light \; year \qquad \Phi^\nu \approx \frac{10^{11}}{cm^2 \cdot sec}$$

!! Seen in experiment from ${}^{37}Cl + \nu_e \to {}^{37}Ar + e^-$, by counting a few ${}^{37}Ar$ atoms in a 4000-liter tank of CCl_4 !!

2. The only matter particles without a mass

$$\lambda_e \bar{L} h^+ e_R = \lambda_e(\bar{\nu}_L (h^-)^* + \bar{e}_L (h^0)^*) e_R \to \lambda_e v \bar{e}_L e_R$$

What about the Lorentz symmetry?

Under Lorentz: J=0, 1/2, 1 for $h, \ \Psi, \ A_\mu$

However for J=1/2 $\Psi(x) = \Psi_L(x) + \Psi_R(x)$ "L-R chirality"

In general, 3 different cases possible:

1. massive (charged or neutral) = "Dirac" $\Psi = \Psi_L + \Psi_R, \ \bar{\Psi} = \bar{\Psi}_L + \bar{\Psi}_R$

$\Psi(\Uparrow\Downarrow) \neq \bar{\Psi}(\Uparrow\Downarrow)$

2. massless (neutral) = "Weyl" $\nu = \nu_L, \quad \bar{\nu} = \bar{\nu}_R$ (chirality = "helicity")

$\nu(\Leftarrow) \neq \bar{\nu}(\Rightarrow)$

3. massive (neutral) = "Majorana" $\nu_M = \nu_L + (\nu_L)^c \equiv \bar{\nu}_M$

$\nu(\Uparrow\Downarrow) = \bar{\nu}(\Uparrow\Downarrow)$

Ψ_L and Ψ_R can transform differently under the "internal" symmetry group

They do not in QCD and QED (hence P and C conserved, see below)

Two ways to give the neutrinos a mass

1. Add 3 ν_R's ($(\mathbf{1},\mathbf{1})_0$ under $SU(3)\times SU(2)\times U(1)_Y$)

$\Delta\mathcal{L}_Y = \lambda^\nu \bar{L}_L h \nu_R \rightarrow \lambda^\nu v\ \bar{\nu}_L \nu_R$ "Dirac neutrinos"

$m_\nu = \lambda^\nu v$ very similar to any charged fermion mass

Lepton Number still conserved $L(\nu_R) = L(\nu_L)$

2. Introduce a dimension 5 interaction

(the only one allowed of dim=5)

$$\Delta\mathcal{L} = \frac{\lambda^\nu}{M}(L_L h^+)(L_L h^+) \rightarrow \frac{\lambda^\nu v^2}{M}\nu_L\nu_L$$ "Majorana neutrinos"

$m_\nu = \frac{\lambda^\nu v^2}{M}$ Lepton Number "broken" whenever m_ν relevant

(m_ν small $\leftrightarrow M >> v$)

Neutrino mixing

As soon as neutrinos pick up a mass (remember the quark case):

$$\frac{g}{\sqrt{2}} W_\mu \bar{e}_{Li} \gamma_\mu \nu_{Li} \;\Rightarrow\; \frac{g}{\sqrt{2}} W_\mu \bar{e}_{Li} \gamma_\mu V_{ij}^{PMNS} \nu_{Lj}$$

where only on the r.h.s. ν_{Li} are true (physical) states of definite mass

V^{PMNS} relevant in any observable with mass/energy scales not too much larger than the neutrino mass itself

By going through analogous considerations as in the quark case:

$V^{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}; \delta^l)$ in the Dirac case

$V^{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}; \delta^l : \phi_1, \phi_2)$ in the Majorana case

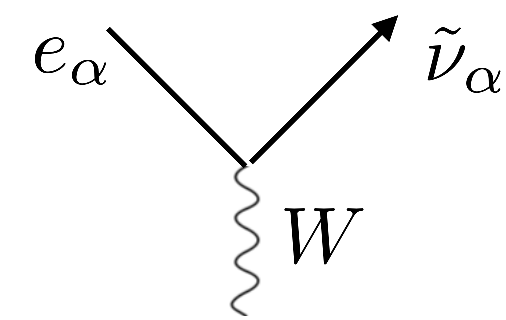
Neutrino oscillations 1

Simplified (naive) picture

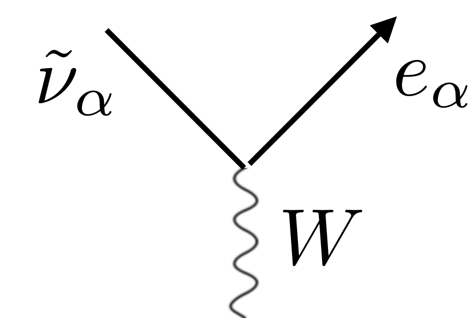
Consider:

$$e_\alpha = e, \mu, \tau$$

(1) A neutrino produced "at x=0" by the interaction with a corresponding charged lepton



(2) A neutrino detected "at x=R" by the appearance of a corresponding charged lepton



Remember that

$$\boxed{\tilde{\nu}_\alpha = V_{\alpha i}^{PMNS} \nu_i}$$

and that

$$i\frac{d}{dt}|\nu_i, t> = E_i|\nu_i, t>; \qquad |\nu_i, t> = e^{iE_i t}|\nu_i, 0>$$

$$E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p}$$

Neutrino oscillations 2

$|\tilde{\nu}_\alpha >$ $\quad t=0 \qquad R = ct \qquad t \quad$ $|\tilde{\nu}_\beta >$

$\alpha, \beta = e, \nu, \tau$

Question: What is the probability that $\tilde{\nu}_\alpha$ at time t=0 has turned to $\tilde{\nu}_\beta$ at time t?

$$|\tilde{\nu}_\alpha, t >= \Sigma_\beta \Psi_\beta(t) |\tilde{\nu}_\beta >$$

$$P(\tilde{\nu}_\alpha, 0; \tilde{\nu}_\beta, t) = |<\tilde{\nu}_\beta|\tilde{\nu}_\alpha, t>|^2$$

$$<\tilde{\nu}_\beta|\tilde{\nu}_\alpha, t >= \Sigma_{i=1,2,3} V^*_{i\beta} e^{iE_i t} V_{i\alpha} \approx e^{ipt} \Sigma_i V^*_{i\beta} V_{i\alpha} e^{i\frac{m_i^2 t}{2p}}$$

The overall phase does not count in P

$$\Downarrow$$

From $i, j = 1, 2, 3$ there are 2 independent $\Delta m^2_{ij} = m^2_i - m^2_j$

Oscillations do not depend on the absolute scale of neutrino masses

Neutrino oscillations 3

Apparent paradoxes of the simplified picture

1. Why is the neutrino state given a definite p rather than E?
2. If p defined $(\Delta p = 0)$, $\Delta x = \infty$. R = ? Why $t = R/c$?
3. If p defined, how about coherence? $v_1 = \frac{p}{E_1} \neq v_2 = \frac{p}{E_2}$

Sketch of the resolution

The neutrino is actually a wave packet

$$|\nu_\alpha^{phys}(x,t)> = \int dE f(E) \Sigma_i V_{\alpha i} e^{-i(p_i x - Et)} |\nu_i> \qquad p_i(E) = \sqrt{E^2 - m_i^2}$$

so that $< \nu_\beta | \nu_\alpha^{phys}(x,t) > = \int dE f(E) \Sigma_i V_{\alpha i} e^{-i(p_i x - Et)} V_{\beta i}^*$

In realistic oscillation experiments, due to the properties of the source and/or of the detector:

The naive P

$$| < \nu_\beta | \nu_\alpha^{phys}(R,t) > |^2 \approx \int dE |f(E)|^2 |\Sigma_i V_{\alpha i} e^{-i p_i(E) R} V_{\beta i}^*|^2$$

The 2x2 case

$$V_{(2\times 2)} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$

$$P(\tilde{\nu}_1, 0; \tilde{\nu}_2, R) = sin^2 2\theta \; sin^2 \frac{R\Delta m^2_{12}}{4E}$$

$$= sin^2 2\theta \; sin^2 \left(1.27 \frac{R(km)\Delta m^2_{12}(eV^2)}{4E(GeV)} \right)$$

$$P(\tilde{\nu}_1, 0; \tilde{\nu}_1, R) = 1 - P(\tilde{\nu}_1, 0; \tilde{\nu}_2, R)$$

2 ways NOT to see any oscillation:

1. $\frac{E(GeV)}{R(km)} >> \Delta m^2_{12}(eV^2) \;\; \Rightarrow$ (no effect) $\quad P_{12} \approx 0$

2. $\frac{E(GeV)}{R(km)} << \Delta m^2_{12}(eV^2) \;\; \Rightarrow$ (an average effect) $\quad P_{12} \approx \frac{1}{2} sin^2 2\theta$

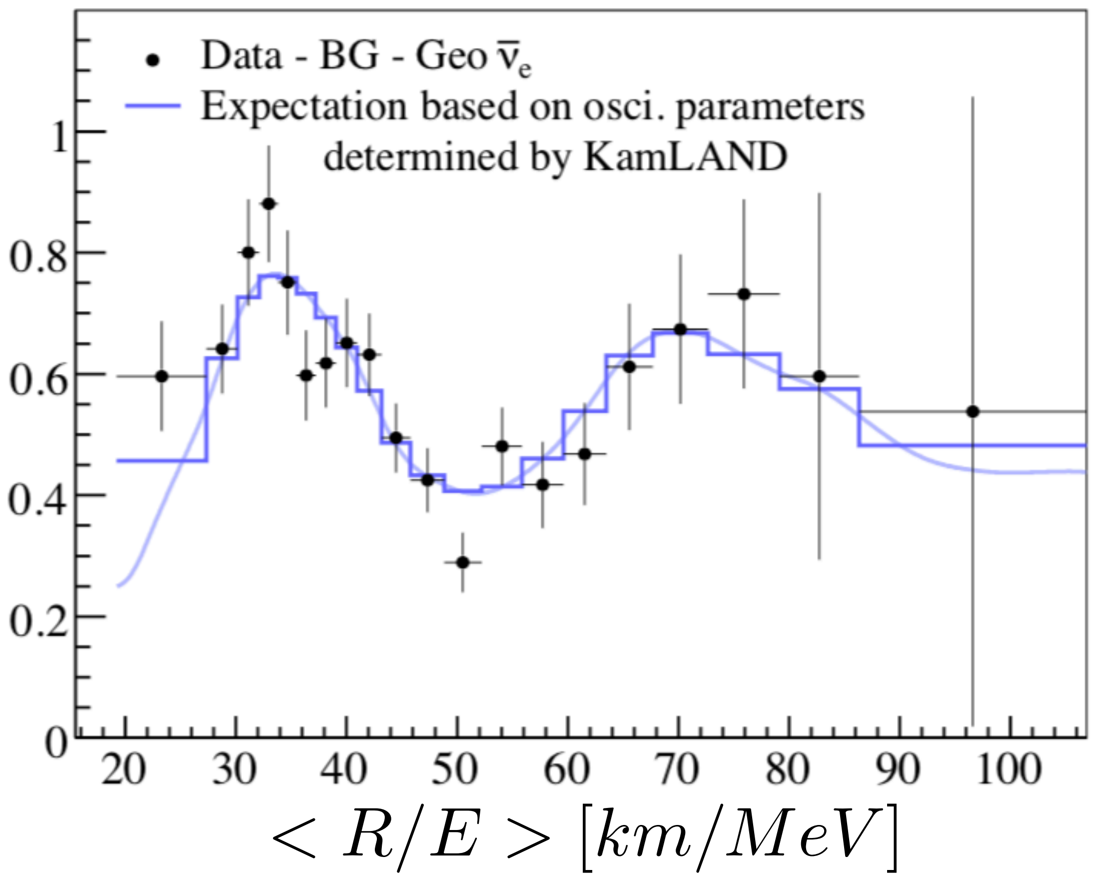
Two remarkable examples

(both using reactor neutrinos)

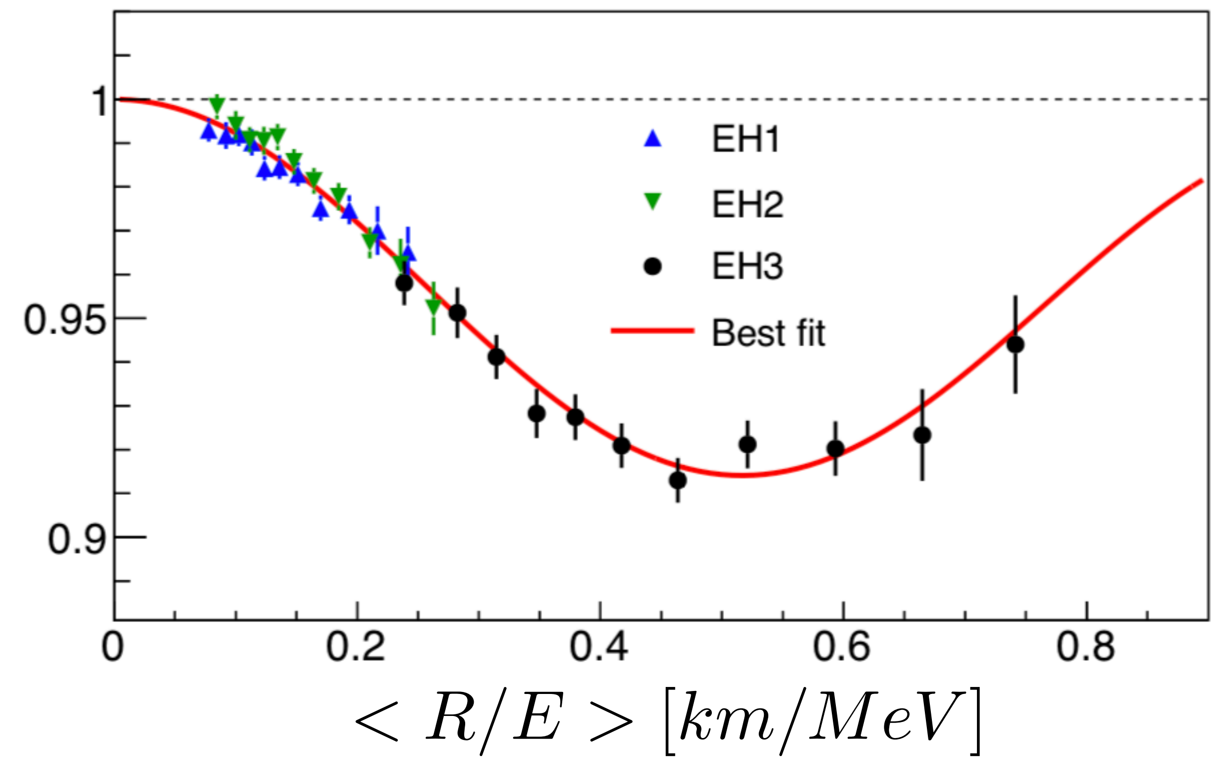
$$n \to p + e^- + \bar{\nu}_e$$

$$P(\bar{\nu}_e \to \bar{\nu}_e)$$

Kamland 2008



DayaBay 2016



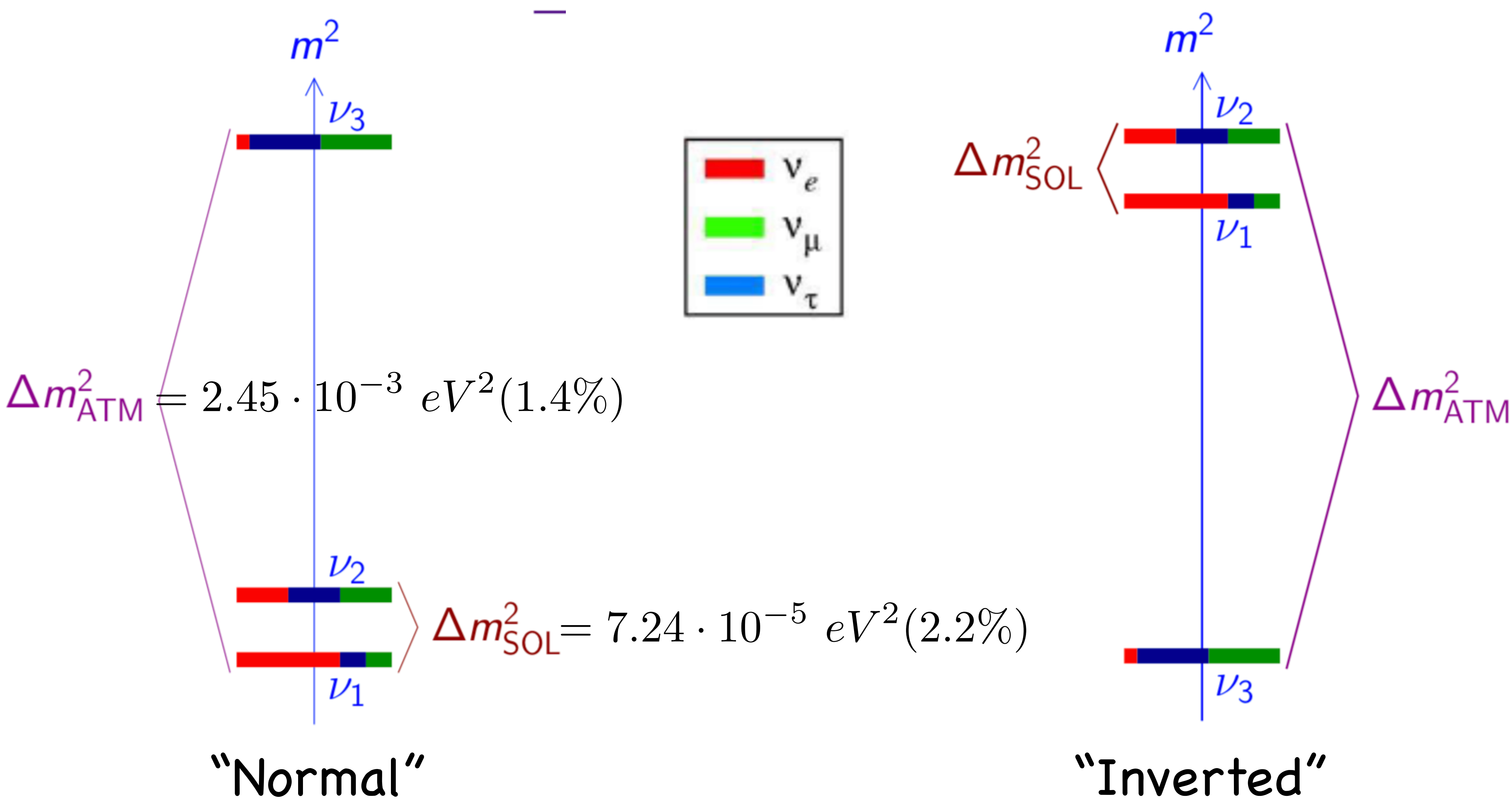
$$\Delta m^2_{SOL} = (7.58 \pm 0.14 \pm 0.14)10^{-5}\ eV^2$$

$$tan^2\theta_{12} = 0.56 \pm 0.10 \pm 0.10$$

$$\Delta m^2_{ATM} = (2.50 \pm 0.06 \pm 0.06)10^{-3}\ eV^2$$

$$sin^2 2\theta_{13} = 0.0841 \pm 0.0027 \pm 0.0019$$

Current knowledge



Different colours correspond to the different compositions of ν_e ν_μ ν_τ

The absolute scale unknown, but $m_\nu^{max} \geq \sqrt{\Delta m^2_{ATM}} \approx 0.05\ eV$

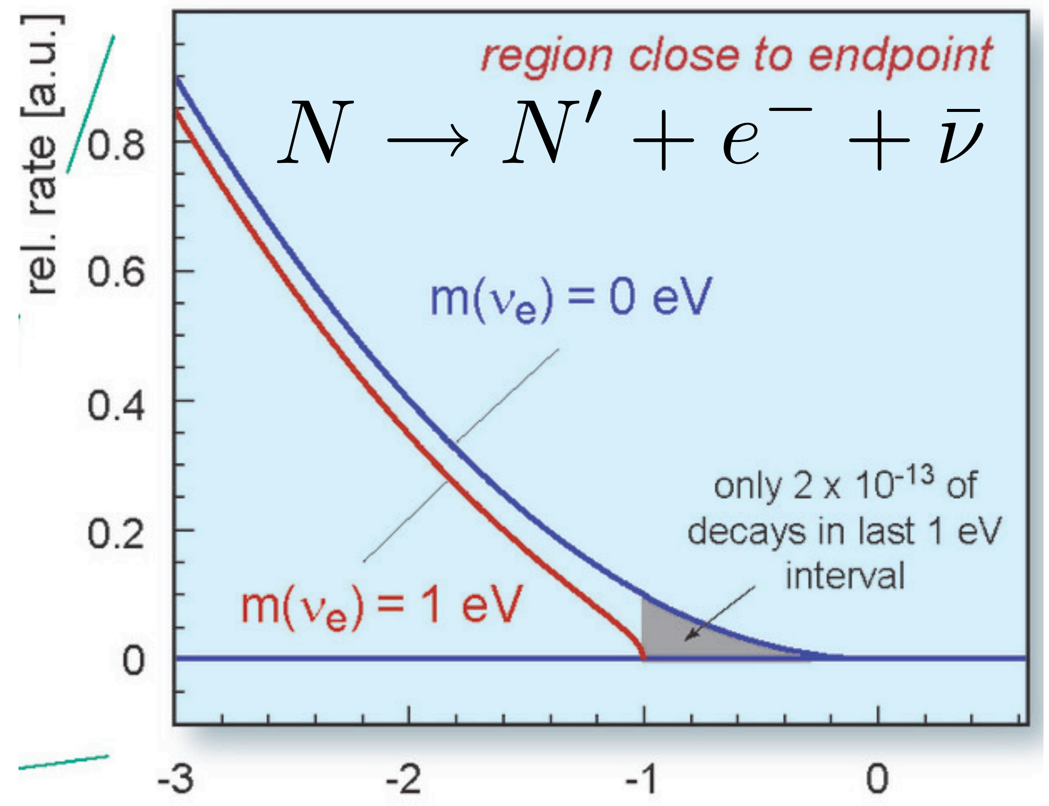
3 ways to be sensitive to the ν-mass scale

1- beta-decay endpoint

$$(Z,A) \to (Z+1,A) + e + \bar{\nu}_e$$

$$m_\beta = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

proposed by Fermi in his 1934 work

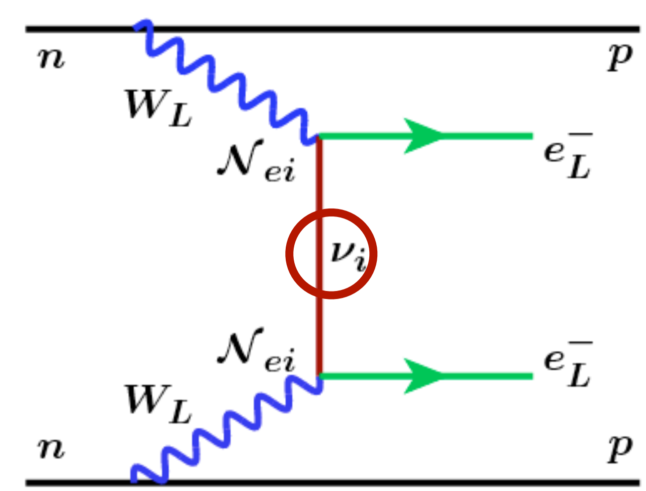


2- neutrino-less ββ-decay

$$(Z,A) \to (Z+2,A) + 2e$$

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

Lepton number broken by 2 units



3 - cosmology (large scale structures)

$$\Sigma = m_1 + m_2 + m_3$$

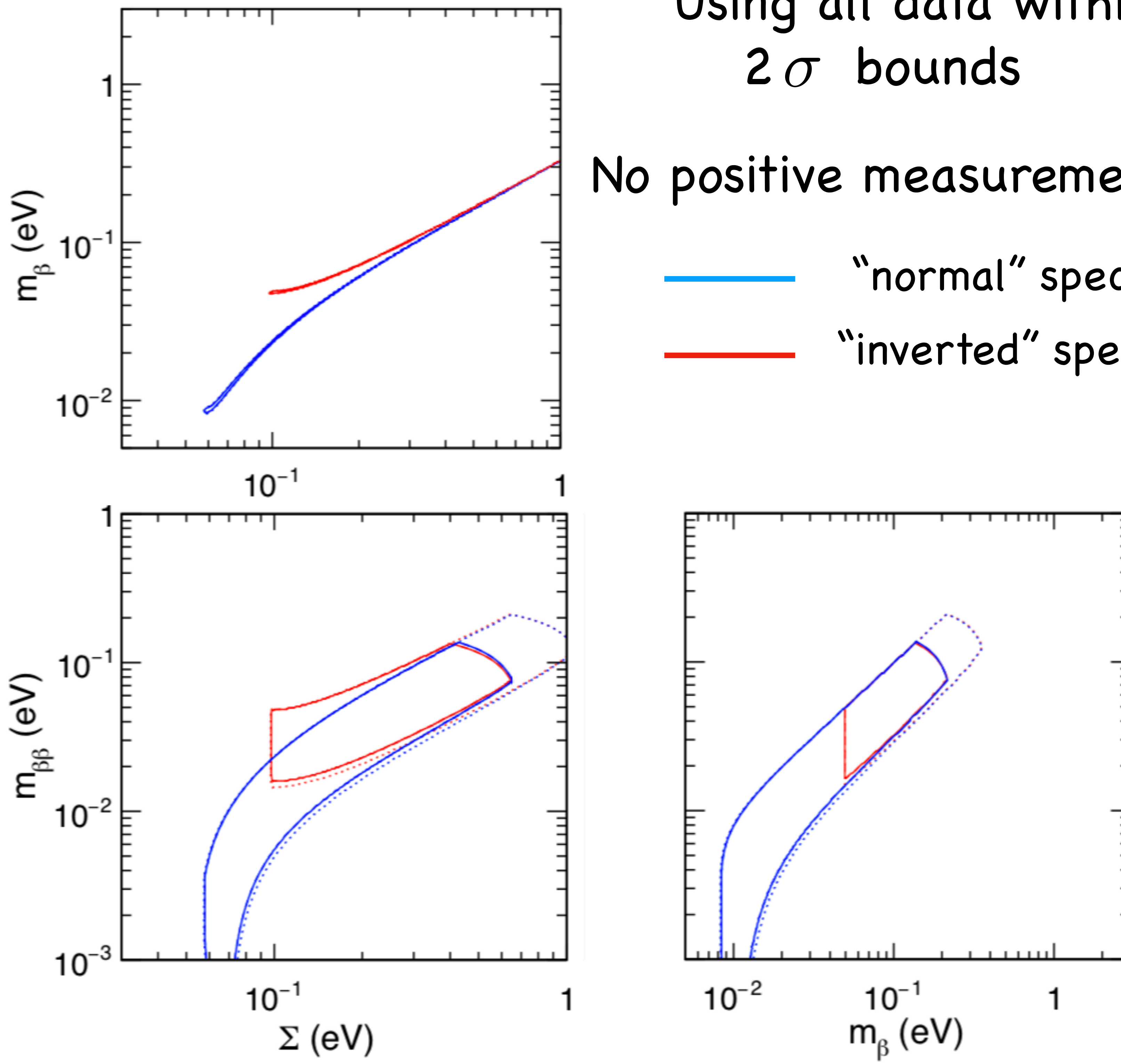
Cosmological ν's affect the average size of "objects" in the sky

Constraints from current knowledge

m_β
beta-decay endpoint

$m_{\beta\beta}$
neutrino-less $\beta\beta$ decay

$\Sigma = m_1 + m_2 + m_3$
large scale structures



Using all data within 2σ bounds

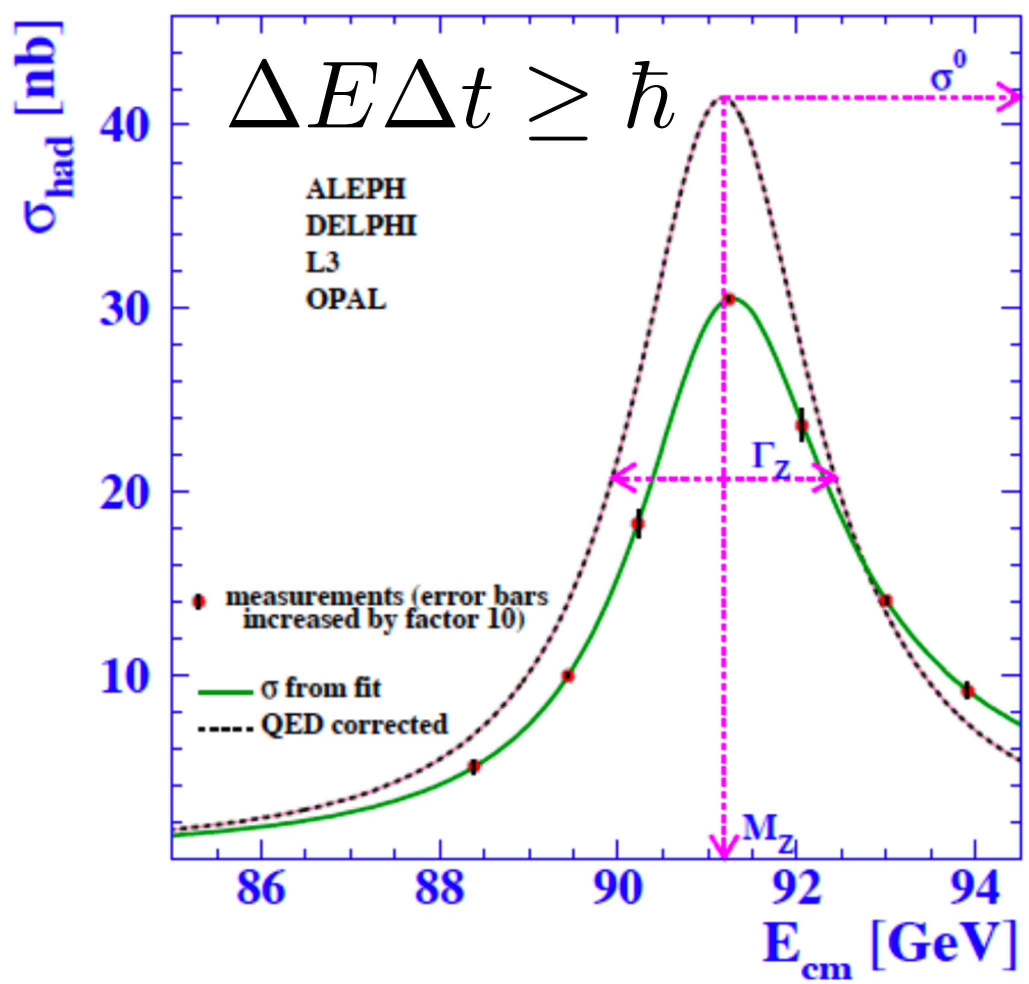
No positive measurement, YET

"normal" spectrum

"inverted" spectrum

Lisi et at, 2018

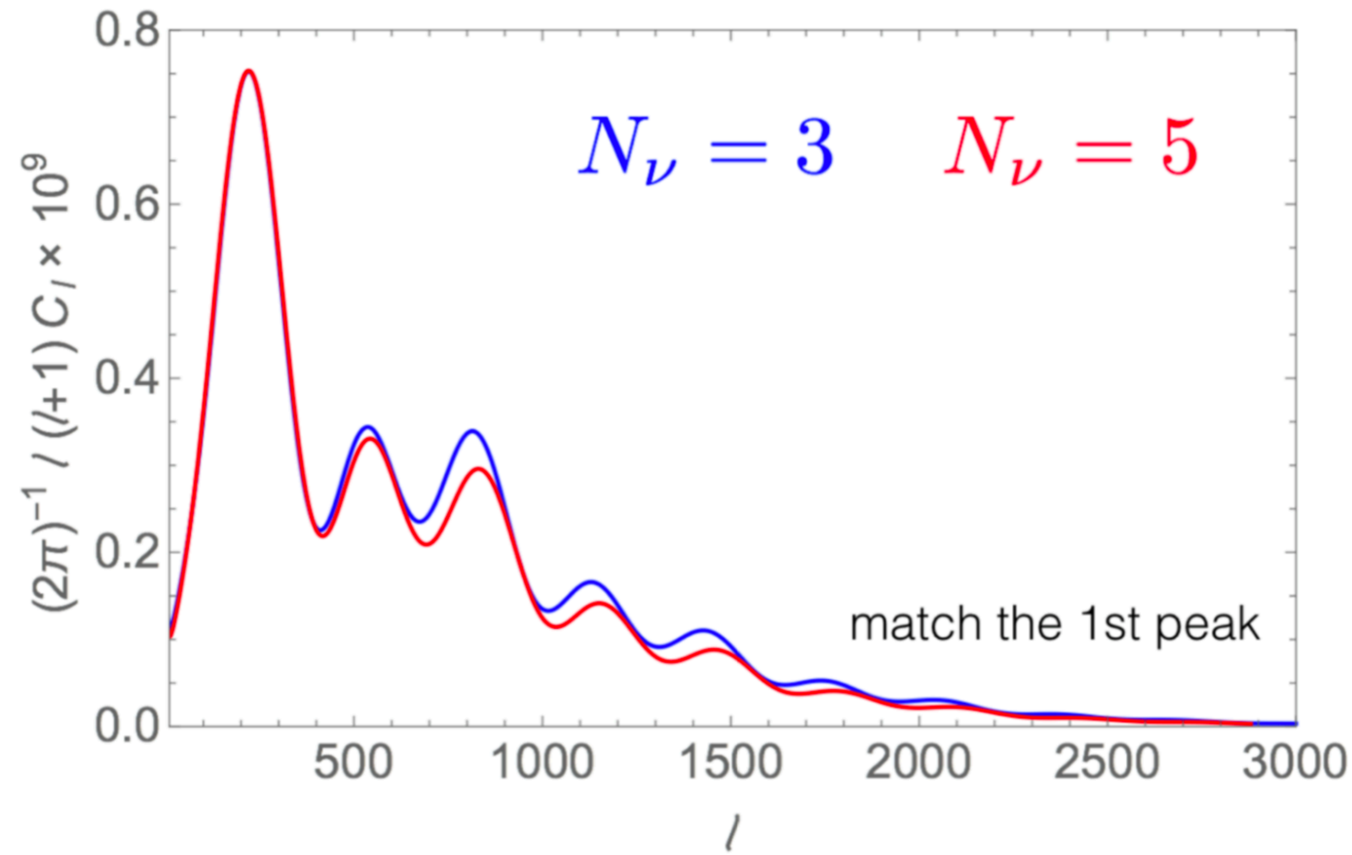
How many neutrinos?



From precision measurements of the Z-"lineshape" and of all visible decays

$$\frac{1}{\tau_Z} \equiv \Gamma_Z^{tot} = \Gamma_Z^{vis} + N_\nu \Gamma_{Z\to\nu\bar{\nu}}$$

$$N_\nu = 2.992 \pm 0.007$$



From precision measurements of the temperature fluctuations of the Cosmic Microwave Background

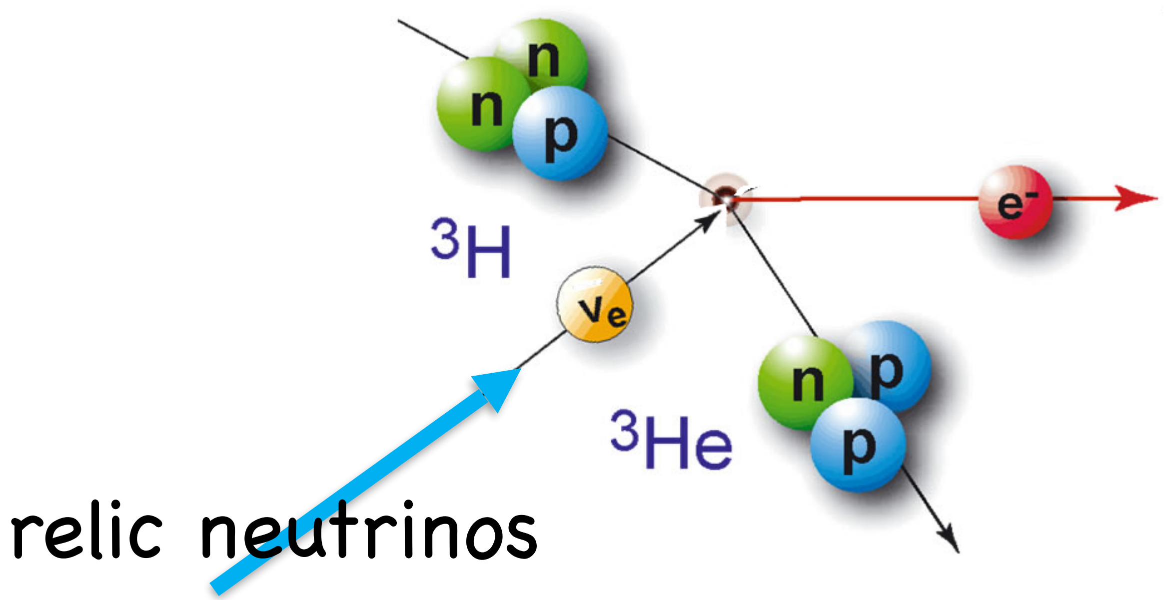
(of broader significance)

$$N_\nu = 3.04 \pm 0.33$$

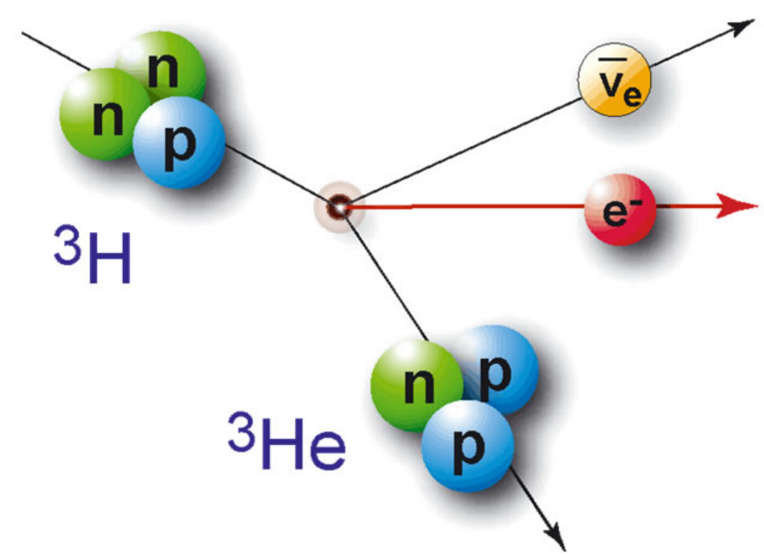
The ultimate challenge

Can we ever detect the cosmological neutrinos?

We think to know that they are released after about 1 second from the big bang in about the same number as the relic photons, released after about 100000 years and detected with great accuracy in the CMB



The process to observe



The background to avoid

semi/final slide of the entire course

(as promised)

How many of you have I convinced

that the SM is useful to understand

the world (a relevant quadrant of it)?

Please, frank answers only!

Problems of (questions for) the SM

0. Which rationale for matter quantum numbers?

$$|Q_p + Q_e| < 10^{-21} e$$

1. Phenomena unaccounted for

neutrino masses
Dark matter

matter-antimatter asymmetry
inflation?

2. Why $\theta \lesssim 10^{-10}$? $\theta G_{\mu\nu} \tilde{G}^{\mu\nu}$

Axions

3. $\mathcal{O}_i : d(\mathcal{O}_i) \leq 4$ only?

neutrino masses
Gravity

Are the protons forever?

4. Lack of calculability (a euphemism)

$\Rightarrow$ the hierarchy problem
the flavour paradox $\Leftarrow$