

# What is String Theory?

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# What makes up the Universe?



The visible part of the Universe is made of particles (electrons, quarks, ...)

**But what makes up, say, an electron?**

# What makes up an electron?

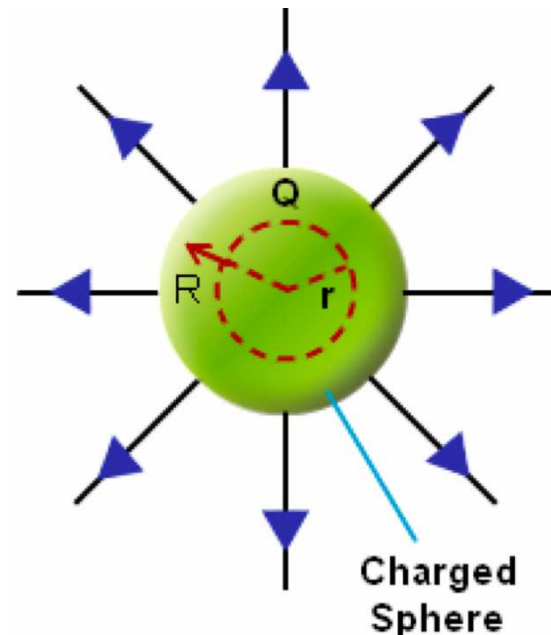
Working hypothesis:

Think of electrons are pointlike objects.

As it stands, this leads to problems.

## 1) Classical Electrodynamics:

- If the electron were literally a point, its mass should be infinite.
- Model electron as a ball of radius  $r_e$ .

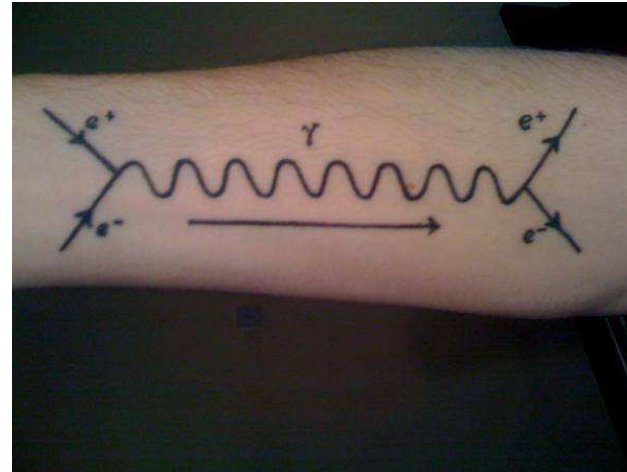


$$m_e c^2 = \text{energy} = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e} \rightarrow \infty \quad \text{as} \quad r_e \rightarrow 0.$$

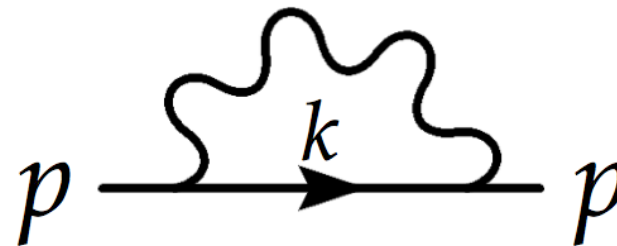
# Ultraviolet Divergences - I

## 2) Quantum Electrodynamics (QED)

- describes electromagnetism at subatomic scales
- Interaction mediated by photons =  $U(1)$  gauge bosons = spin 1 particle .



This same ultraviolet (UV) divergence remains.



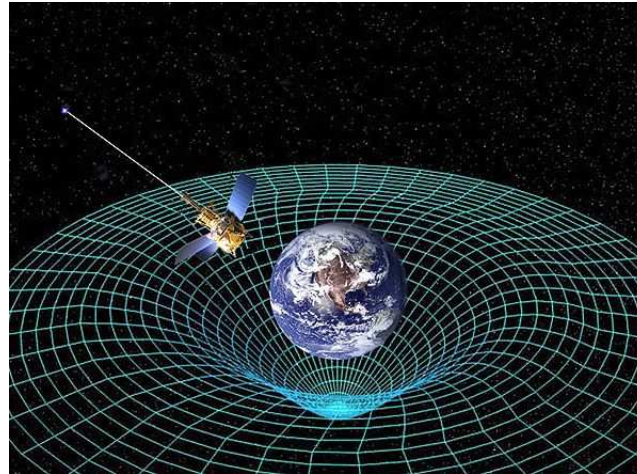
### Wilsonian renormalisation:

- Powerful techniques hide this infinity for all practical purposes.
- QED is a perfectly working effective theory.
- Something's not right at the fundamental level.

# Ultraviolet Divergences - II

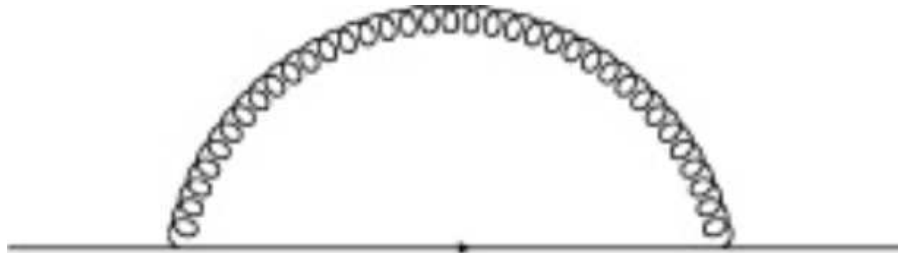
## 3) General Relativity

- Gravitational physics at astronomical scales
- Classical gravity mediated by curvature of spacetime.

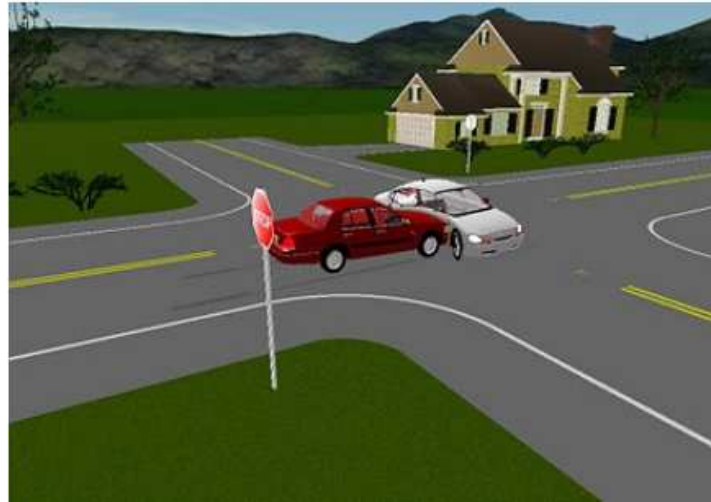


Similar issues arise in **perturbative quantum treatment of gravity**:

- Carrier of force is **graviton = spin 2 particle**.
- The divergences are worse and the same **(perturbative) techniques fail**.



# Physics at a crossroads



**Two alternative conclusions are possible:**

- 1) Stick to gravity and pointlike particles, but change quantization.  
(Loop Quantum Gravity, Asymptotic Safety, . . .)
- 2) Stick to usual quantisation, but rethink pointlike nature of particles.  
(That's what we do in string theory.)

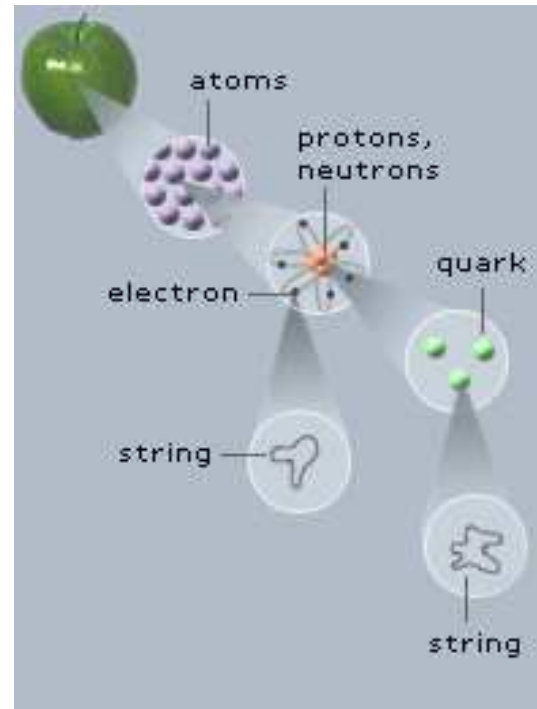
**”Kinder, schafft Neues!”**



# If it's not a point - what is it?

The simplest object with substructure is 1-dimensional → **String theory**

- **Dynamical input:** The fundamental objects in Nature are not pointlike, but 1-dimensional strings
- **Kinematical input:** Describe these strings via the familiar rules of quantum theory and general covariance



**Claim:** This next simplest option inevitably leads to

- a consistent theory of gauge interactions and gravity
- free of UV divergences (order by order in pert. theory)
- with no free dimensionless parameters.



# A symphony from 1 string

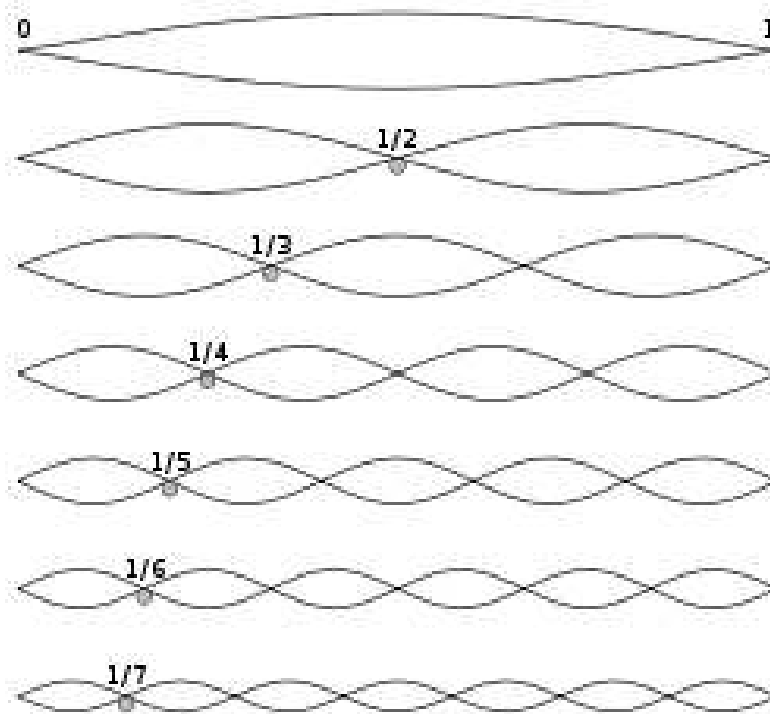
- ✓ A string can vibrate just like the string of a violin does.
- ✓ The different oscillation modes correspond to different particles.

⇒ **Maximal unification:**

- There is only one kind of "stuff" - the string.
- All physics is captured by its excitations.

**Analogy:**

- Suppose your favorite violin has only one string.
- Many different oscillations still allow for a full symphony of different tones.



# Roadmap

## Part I: Basic Principles of String Theory

- 1) Classical strings in pics and formulae
- 2) Quantisation and spectrum

## Part II: String Compactification

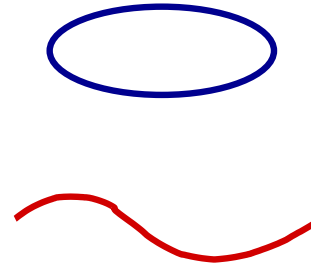
- 3) The concept of compactification
- 4) Brane Worlds
- 5) The String Landscape

# Classical Strings

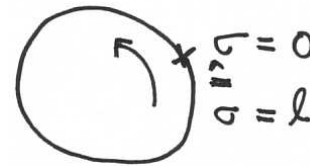
2 types of strings:

closed strings

open strings



- Position along string parametrized by  $0 \leq \sigma < \ell$



- String coordinates in spacetime:  $X^\mu(\tau, \sigma)$ ,  $\mu = 0, \dots, d - 1$

## Equations of motion

- free point particle:  $(\frac{\partial}{\partial \tau})^2 X^\mu(\tau) = 0$
- free string:  $\left( \left( \frac{\partial}{\partial \tau} \right)^2 - \left( \frac{\partial}{\partial \sigma} \right)^2 \right) \mathbf{X}^\mu(\tau, \sigma) = \mathbf{0} \leftrightarrow$  **2D wave equation**

# Classical Strings - Dynamics I

## Strings carry energy

- c.o.m. momentum
- oscillations along string

## Strings carry spin

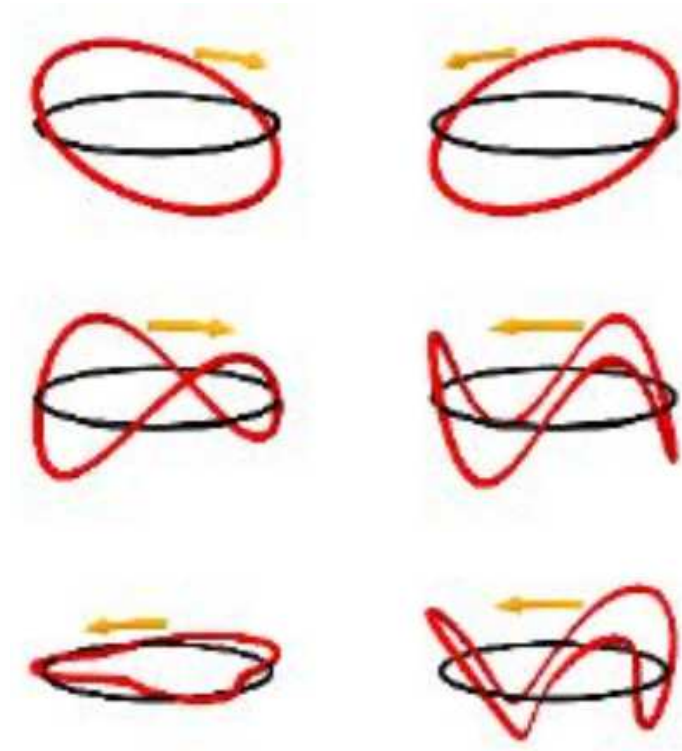
↔ polarisation of oscillation

Energy scale set by **string length**

$$\ell_s \equiv 2\pi\sqrt{\alpha'}$$

Typical **string scale**  $M_s = \ell_s^{-1}$  is the only 'free parameter' of the theory:

$$3.5\text{TeV} \leq M_s \leq M_{\text{Pl.}} \simeq 10^{18}\text{GeV}$$



# Classical Strings - Dynamics II

- **2D wave equation:** 
$$\left( \left( \frac{\partial}{\partial \tau} \right)^2 - \left( \frac{\partial}{\partial \sigma} \right)^2 \right) X^\mu(\tau, \sigma) = 0$$

- **Ansatz:** 
$$X^\mu(\tau, \sigma) = \underbrace{X_R^\mu(\tau - \sigma)}_{\text{right-moving wave}} + \underbrace{X_L^\mu(\tau + \sigma)}_{\text{left-moving wave}}$$

- **Boundary conditions** for **closed string**

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \ell) \quad \ell : \text{circumference of string}$$

**Most general solution:** Fourier expansion

$$X_{R/L}^\mu = \frac{1}{2} x^\mu + \frac{\pi \alpha'}{\ell} p^\mu (\tau \pm \sigma) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z} \neq 0} \frac{1}{n} \alpha_n^\mu / \tilde{\alpha}_n^\mu e^{-i \frac{2\pi}{\ell} n (\tau \pm \sigma)}$$

- **Frequencies:**  $\frac{2\pi}{\ell} n$       **Amplitudes:**  $\alpha_n^\mu / n$  (Right)       $\tilde{\alpha}_n^\mu / n$  (Left)
- c.o.m momentum  $p^\mu$  and position  $x^\mu$

# String Quantisation - I

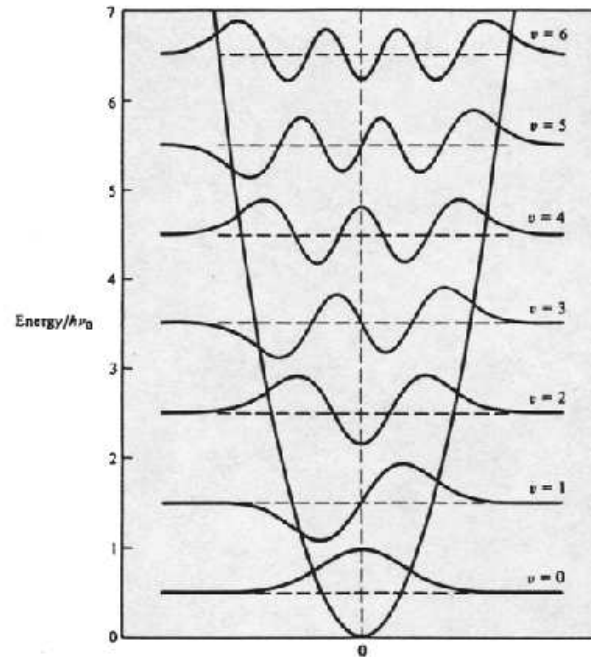
= quantisation of waves along the string

Each excitation mode

$\alpha_m^\mu, \tilde{\alpha}_m^\mu$  represents a  
**harmonic oscillator**

**States:**

- c.o.m. momentum  $p$ :  $|0, p\rangle$
- **Excite each left/right oscillation**  
frequency  $\frac{2\pi}{\ell}n$  arbitrarily often:



$$\prod_{m>0, \mu} (\alpha_{-m}^\mu)^{n_{m, \mu}} \prod_{m>0, \mu} (\tilde{\alpha}_{-m}^\mu)^{\tilde{n}_{m, \mu}} |0; p\rangle$$

(Special technicality here: equal number of left/rightmoving quanta)

# String Quantisation - II

**Tower of string excitations** - characterized by oscillation number  $N_L = N_R$

- $N_L = 0 = \tilde{N}_R$ :  $|0, p\rangle$ : momentum eigenstate with zero oscillations
- $N_L = 1 = \tilde{N}_R$ :  $\zeta_{\mu\nu} \alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0; p\rangle$ : first mode excited
- ...

**Mass of string excitations:** (for bosonic string)

$$M^2 = 4M_s^2 \times (N - a) \quad a = 1 \quad N = N_L = N_R$$

$M_s \simeq \ell_s^{-1}$ : **string scale**  $\leftrightarrow$  sets **scale** of oscillations

$N_L = 0 = N_R$ : tachyon - removed in superstring theory

**$N_L = 1 = N_R$ : massless excitations**

$N = 2, 3, \dots$ : massive states of mass-squared set by  $M_s$

**Each oscillation = object with mass and spin = particle.**

# Gravitons from closed strings

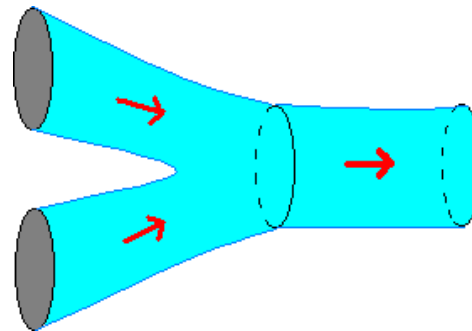
**Low-energy regime** ( $E \ll M_s$ ): only **massless modes** relevant

Closed massless :  $\zeta_{\mu\nu} \alpha_{-1}^{\mu} \tilde{\alpha}_{-1}^{\nu} |0; p\rangle$ ,  $\zeta_{\mu\nu}$  : polarisation tensor

- This object contains a **spin-2 mode** = 2-index symmetric tensor.
- **This must be the graviton**  $h_{\mu\nu}$ .  
 $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ : fluctuation around background

Direct check:

- Compute interactions in string perturbation theory
- Find same interactions as for perturbative graviton



**High energy regime** ( $E \geq M_s$ ):

Characteristic tower of massive, higher spin excitations visible

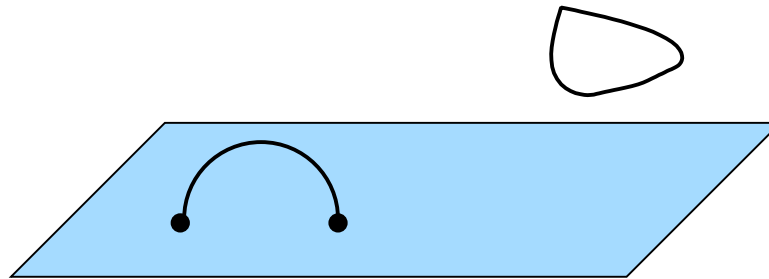
$$M^2 \simeq N M_s^2 \quad J \simeq N$$



# Photons from open strings

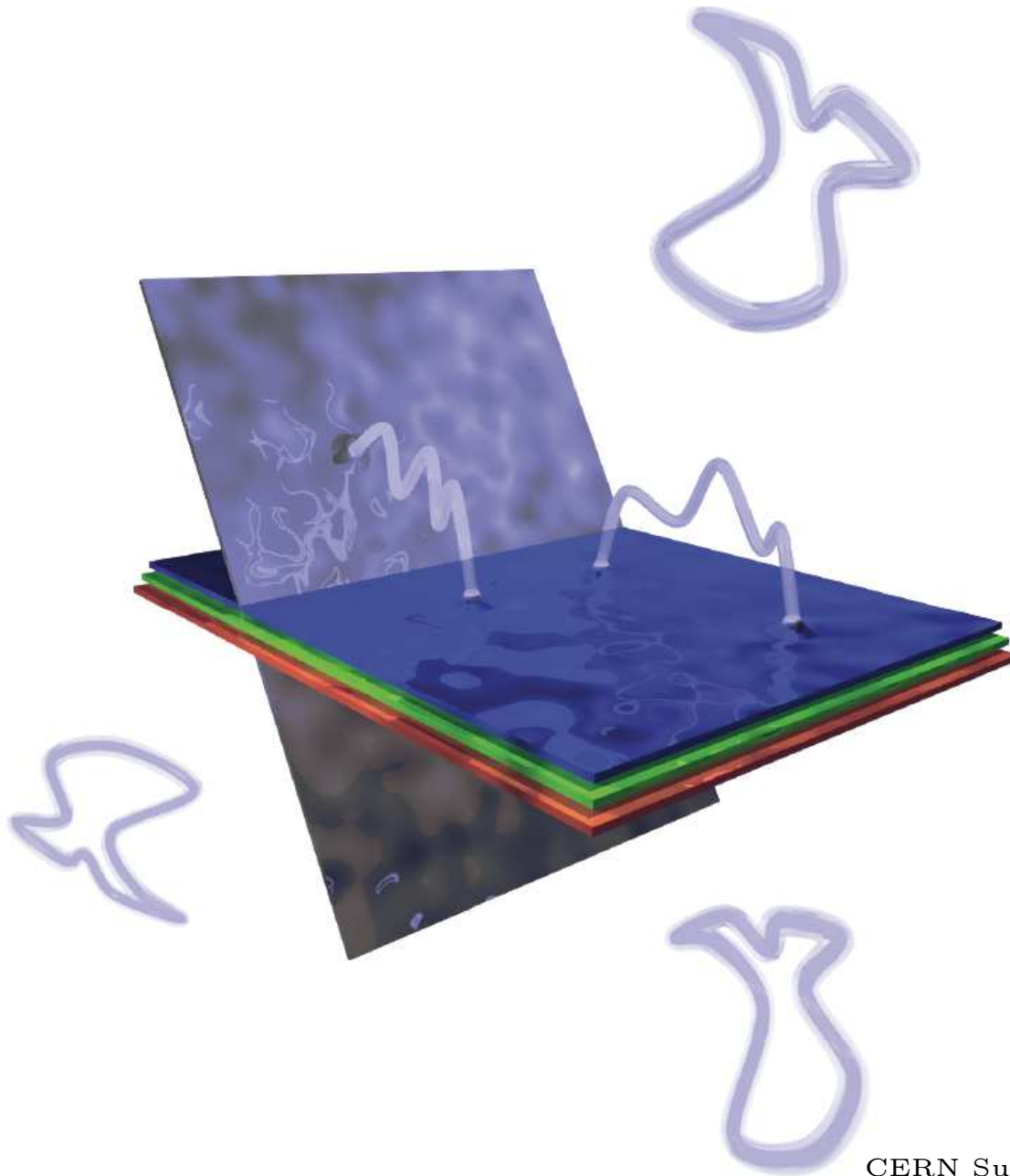
- An open string has **two endpoints** at  $\sigma = 0$  and  $\sigma = \ell$
- Repeat program of classical solutions and quantisation with **suitable boundary conditions**
- **Result:** String endpoints can move freely along an object called a **Dp-brane** =  $(p+1)$ -dimensional hypersurface of spacetime

Polchinski 1996



- Boundary conditions relate left/rightmoving waves
- **Massless level:**  $\zeta_\mu \alpha_{-1}^\mu |0; p\rangle$ : spin-1 particle
- Interpretation as **vector boson** responsible for a **U(1) gauge theory**

# Gravity in bulk - EM on brane

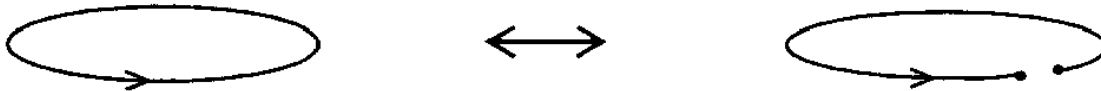


# A crucial consistency check

In string theory, gauge theory implies gravity.

Reason:

- Strings interact by joining and splitting.



- Open string endpoints can join to form a stable closed string.  
(The converse is not always true)

✓ Behaviour consistent with universality of gravity:

photons  $\implies$  energy  $\implies$  gravity

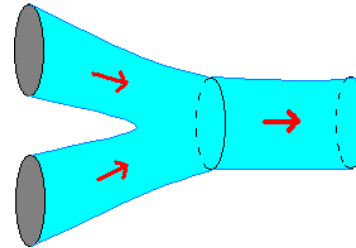
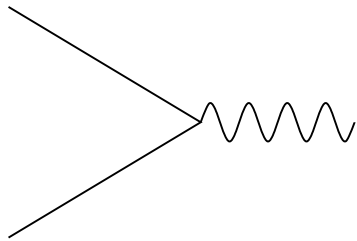
- ✓ In string theory, gauge interactions and gravity are not independent.  
They are linked by the internal consistency of the theory.

**String theory is the only known theory with this property.**

# UV finiteness

**General picture:** String as intrinsic UV regulator

- High energy scattering probes string length  $\leftrightarrow$  non-local behaviour
- Point-like interaction vertex is smoothed out.



This can be made very precise quantitatively, and finiteness of loop diagrams in perturbation theory can be checked.

# Why strings are special

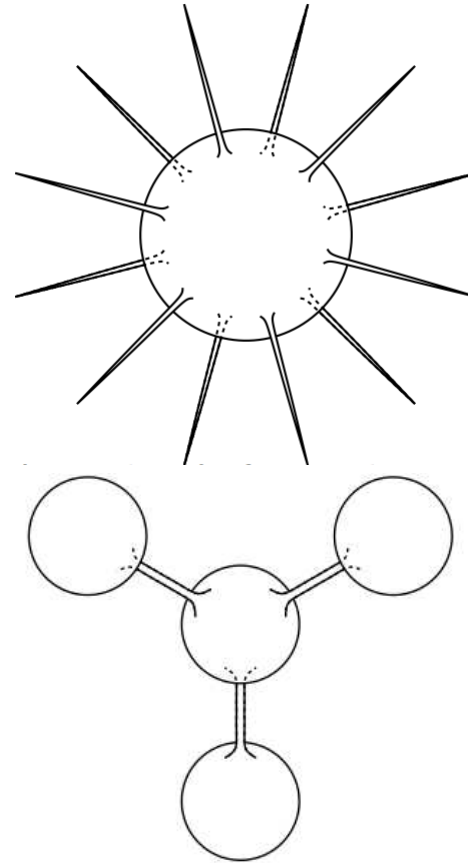
Can a particle have even higher-dimensional substructure?

Model particle as a **membrane** -  
2 spatial dimensions

Tubes of length  $L$  and radius  $R$   
have spatial volume  $\simeq L \times R$ .

**Quantum fluctuations:**

- Long, thin tubes can form without energy cost.
- **Membranes automatically describe multi-particle states.**



**No first quantisation of higher-branes à la strings possible.**

# Consequences of string theory

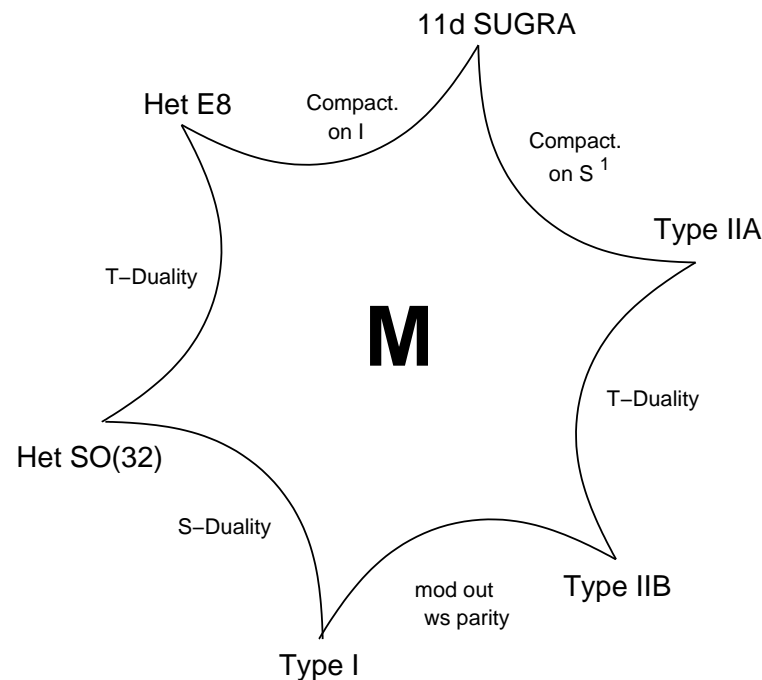
Internal consistency conditions make further predictions:

- Spacetime is not 4-dimensional, but 10-dimensional.

- In 10 dimensions there is only one unique type of string theory.

It has many equivalent formulations which are dual to each other.

[Witten 1995, ...]



- The 10-dim. theory is supersymmetric:  
Every boson has a fermionic superpartner.

**This does NOT imply that supersymmetry must be found at LHC.**

# Summary so far

Superstring theory is well-defined and unique (up to dualities) in 10d.

## 1) Low energy regime $E \ll M_s$

Theory **predicts** Einstein gravity and gauge theory.

- **Within** the full 10d **bulk** a **graviton** propagates.
- Along **lower dimensional D-branes** a **gauge boson** propagates.

## 2) High energy regime $E \geq M_s$

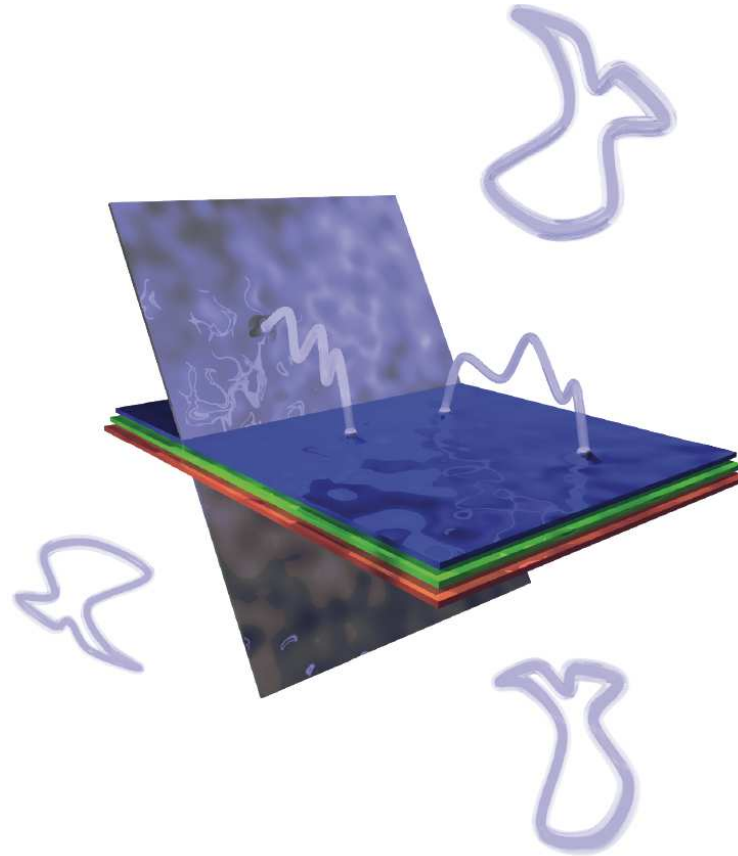
- **Characteristic tower of massive string excitations**  
→ measurable (in principle) as resonances!
- Energy dependence of interactions differs from field theory.
- Scattering amplitudes are **ultra-violet finite** without the need for renormalisation.  
⇒ **truly fundamental (as opposed to effective) theory**

# Part II: String Compactification

3) Compactification

4) Brane Worlds

5) The String Landscape





# Extra dimensions

Superstring Theory is well-defined only if spacetime is 10 dimensional.

It is thus an example of a theory of extra dimensions:

- Such theories are considered also in point particle framework.
- Extra dimensions are compact and very small.

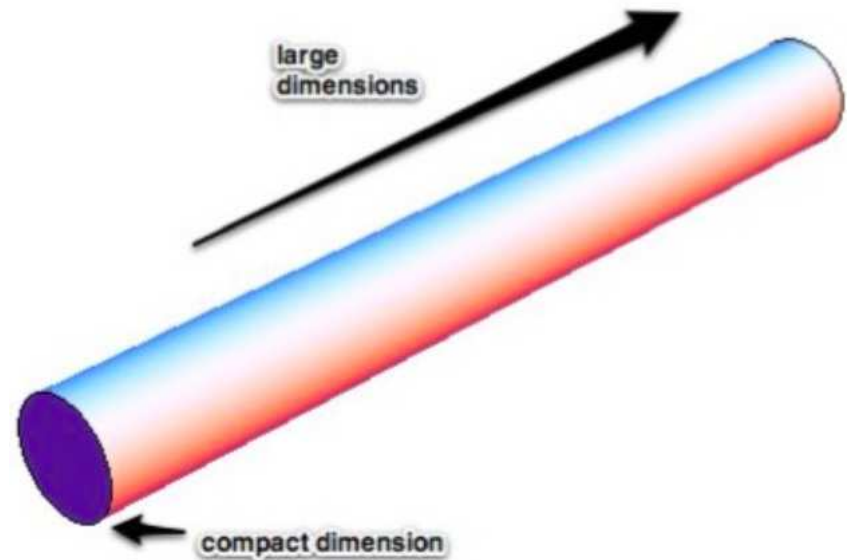
Toy example:

- Consider theory in 5 dimensions:

$$x^M, M = 0, 1, \underbrace{\dots, 3, 4}_{\mu}$$

- Compactify direction  $x^4$  along circle  $S^1$  of radius  $R$

- As radius  $R \rightarrow 0$  this becomes an effectively 4D theory



# String compactification

Back to strings:

To arrive at 4 large extra dimensions we need to **compactify 6 dimensions**.

- **Simplest solution:**

Each dimension is a circle  $S^1$ , i.e.

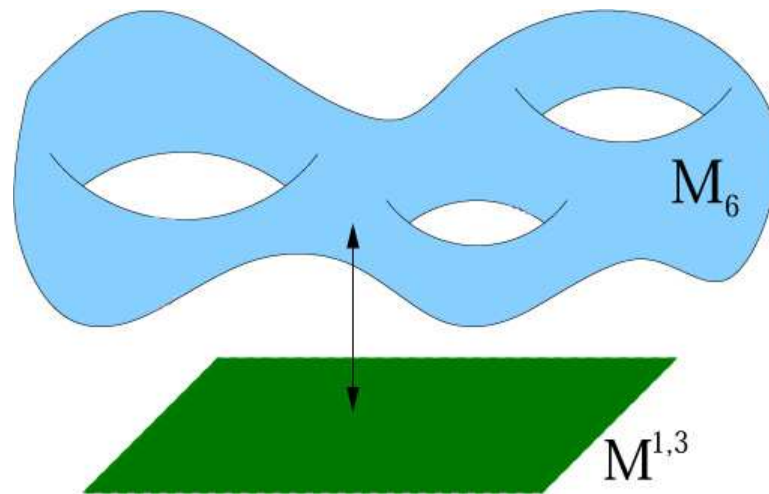
internal space is a six-dimensional torus  $T^6 = S^1 \times \dots \times S^1$

$x_0, x_1, x_2, x_3$ : macroscopic

$x_4, x_5, \dots, x_9$ : rolled up on  $T^6$

- More general 6-dimensional spaces allowed.

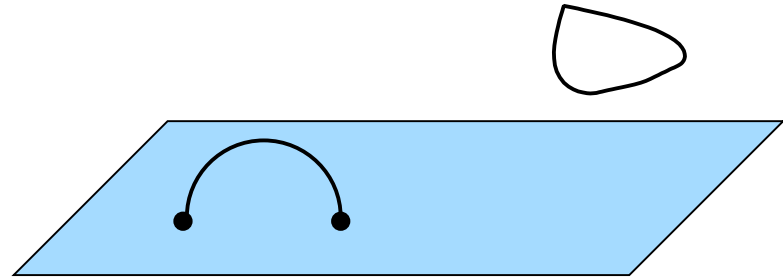
- **Each consistent compactification yields a solution to string equation of motions with specific physics in 4D.**



# Intersecting Brane Worlds - I

String excitations along 1 Dp-brane:

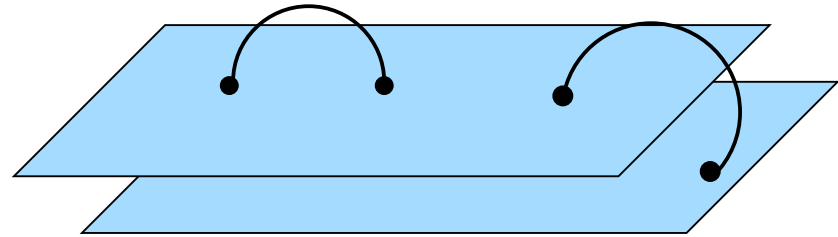
$U(1)$  gauge  $A^i, i = 0, \dots, p$



N coincident Dp-branes:

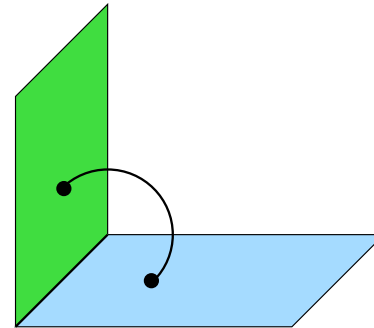
$U(N)$  gauge symmetry

$N \times N$  gauge bosons



Dp-branes at intersection:

Matter fields (chiral fermions) in  
bifundamental representation  $(\bar{N}_a, N_b)$



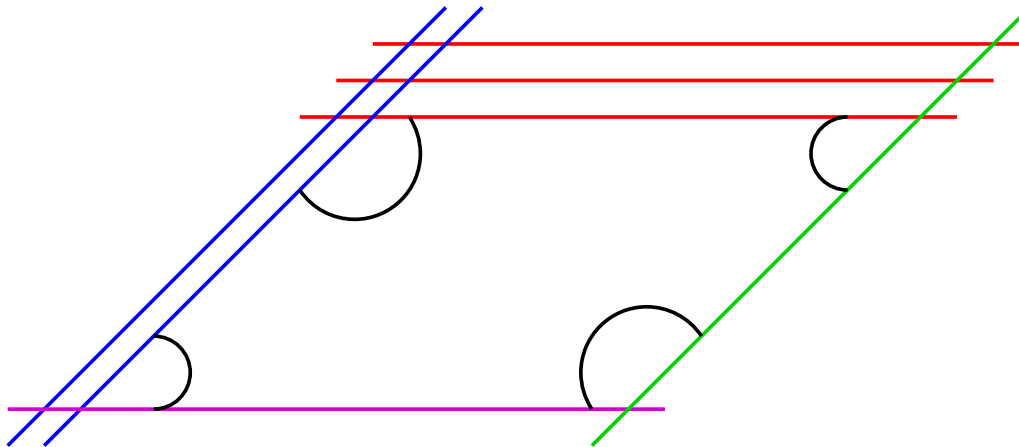
# Intersecting Brane Models

- Simple realisation of gauge groups of the type  $\prod_i U(N_i)$  with chiral matter in bifundamental representations

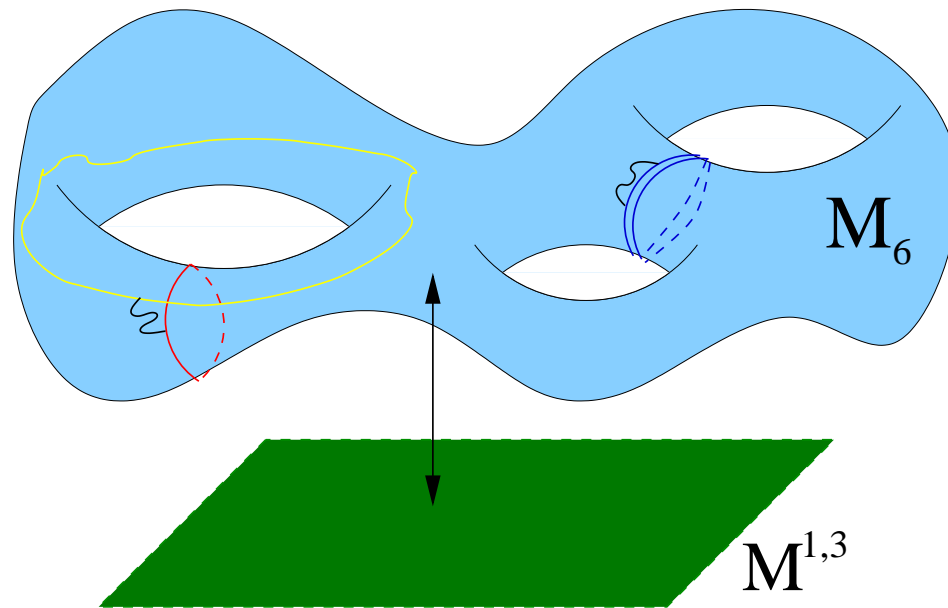
- Basic ingredients of the Standard Model

$$SU(3) \times SU(2) \times U(1)_Y$$

Direct implementations of Standard Model gauge interactions and matter via "Intersecting Brane Worlds"



# Intersecting Brane Models



- Configuration of multiple branes  $\leftrightarrow$  gauge groups
- Intersection pattern  $\leftrightarrow$  charged matter
- Specifics of geometry  $\leftrightarrow$  interactions (computable!)

## String phenomenology:

Explore **interplay of string geometry and physics in 4 dimensions**

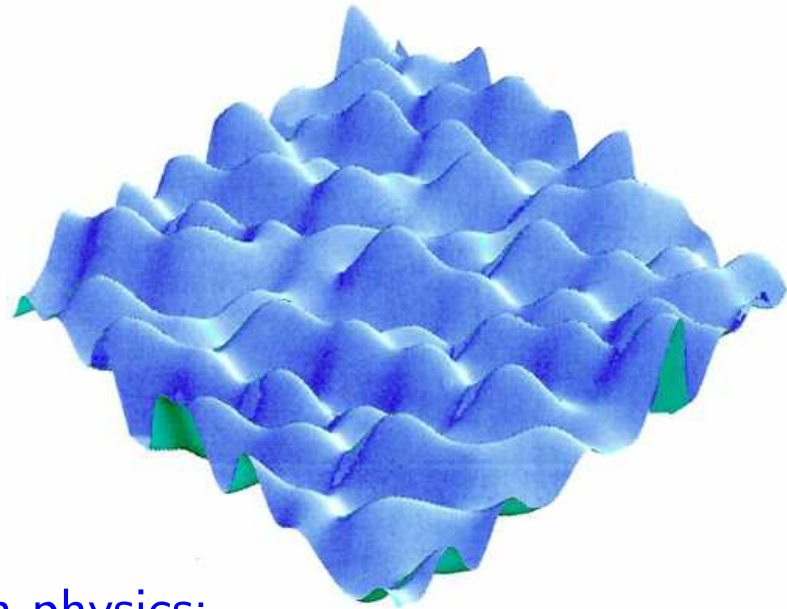
# The landscape of string vacua

Each consistent compactification is a solution to string equ. of motion.

Each 4d solution is called a **4d string vacuum**.

In 10d: All interactions uniquely determined

In 4d: Plethora of consistent solutions exists - the **landscape of string vacua**



Existence of many solutions is typical in physics:

Einstein gravity is one theory with many solutions!

**Pressing question:** Consequences for physics in 4D?

Solution to fine-tuning problems (Higgs, Cosmological Constant)?

# String Geometry

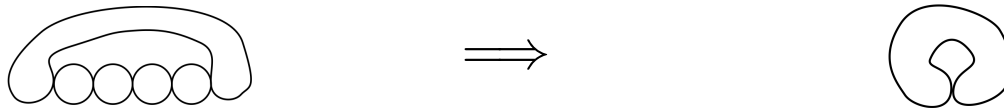
Geometry of compactification space  $\leftrightarrow$  Physics in 4d (or higher)

- Strings as extended objects probe geometry differently than points
- Opens door for fascinating interplay between mathematics and physics

$\Rightarrow$  New physics ways to think about geometry by translating into physics

Ex.: Classification of singularities in geometry

- Singularities occur when submanifolds shrink to zero size



- Branes can wrap these vanishing cycles and give rise to massless particles in effective theory
- Gives interpretation for classification of singularities in mathematics and guidelines for new situations unknown to mathematicians

Many more, beautiful examples of this type

# Summary

String theory is a **maximally economic quantum theory of gravity, gauge interactions and matter**.

Assumption of stringlike nature of particles leads to **calculable theory without UV divergences**.

Challenge for **String Phenomenology**: understanding the **vacuum of this theory**

This talk has focused on **String Theory as a fundamental theory**.

What we haven't discussed at all:

- **String Theory as modern mathematical physics**:  
deep interplay with sophisticated mathematics [Witten,Douglas,...]  
(Mirror symmetry, D-brane categories,...)
- **String Theory as a tool**: [Maldacena'97,Witten'98,...]  
Holographic principle - AdS/CFT: Insights and applications

**String Theory is a framework for modern physics.**