Beyond the Standard Model

CERN summer student lectures 2018

Lecture 2/4

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Outline

Monday
- General introduction
- What kind of physics can be probed at colliders?
- Higgs physics as a door to BSM

Tuesday
- Naturalness
- Supersymmetry
- Grand unification, proton decay

Wednesday
- Composite Higgs
- Extra dimensions
- Quantum gravity

Thursday
- Cosmological relaxation
- Beyond colliders searches for new physics
Higgs Mechanism

Symmetry of the Lagrangian

\[ SU(2)_L \times U(1)_Y \]

Higgs Doublet

\[ H = \left( \begin{array}{c} h^+ \\ h^0 \end{array} \right) \]

Symmetry of the Vacuum

\[ U(1)_{e.m.} \]

Vacuum Expectation Value

\[ \langle H \rangle = \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right) \text{ with } v \approx 246 \text{ GeV} \]

\[ D_\mu H = \partial_\mu H - \frac{i}{2} \left( \begin{array}{cc} gW^3_\mu + g'B_\mu & \sqrt{2}gW^+_\mu \\ \sqrt{2}gW^-_\mu & -gW^3_\mu + g'B_\mu \end{array} \right) H \text{ with } W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \pm W^2_\mu) \]

\[ |D_\mu H|^2 = \frac{1}{4} g^2 v^2 W^+ \mu W^- \mu + \frac{1}{8} (W^3_\mu B_\mu) \left( \begin{array}{cc} g^2 v^2 & -gg'v^2 \\ -gg'v^2 & g'^2 v^2 \end{array} \right) \left( \begin{array}{c} W^3_\mu \\ B_\mu \end{array} \right) \]

Gauge boson spectrum

- electrically charged bosons
  \[ M^2_W = \frac{1}{4} g^2 v^2 \]

- electrically neutral bosons
  \[ Z_\mu = cW^3_\mu - sB_\mu \]
  \[ \gamma_\mu = sW^3_\mu + cB_\mu \]
  \[ c = \frac{g}{\sqrt{g^2 + g'^2}} \]
  \[ s = \frac{g'}{\sqrt{g^2 + g'^2}} \]
  \[ M^2_Z = \frac{1}{4} (g^2 + g'^2) v^2 \]
  \[ M_\gamma = 0 \]

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Higgs and EW vacuum Stability

\[ V(h) = -\frac{1}{2} \mu^2 h^2 + \frac{1}{4} \lambda h^4 \]

vev: \( v^2 = \frac{\mu^2}{\lambda} \) \hspace{1cm} mass: \( m_H^2 = 2\lambda v^2 \)

the vacuum is not empty even classically \((\bar{h} \to 0)\)

How is Quantum Mechanics changing the picture?

16\pi^2 \frac{d\lambda}{d\ln Q} = 24\lambda^2 - (3g'^2 + 9g^2 - 12y_t^2)\lambda + \frac{3}{8} g'^4 + \frac{3}{4} g'^2 g^2 + \frac{9}{8} g^4 - 6y_t^4 + \text{Higher loops}

Small Yukawa
Higgs and EW vacuum Stability

Small mass ($y_t$ dominated RGE)

$$\lambda(Q) = \lambda_0 - \frac{3}{8\pi^2} y_0^4 \ln \frac{Q}{Q_0} \left( 1 - \frac{9}{16\pi^2} y_0^2 \ln \frac{Q}{Q_0} \right)$$

$\lambda < 0 \Rightarrow$ potential unbounded from below

$$\Lambda \leq v e^{4\pi^2 m_H^2 / 3 y_t^2 v^2}$$

New physics should appear before that point to restore stability

Linde '76, '80
Weinberg '76
Maini et al '78, '79
Politzer, Wolfram '79
Lindner '86
+...
Higgs and EW vacuum Stability

Small mass ($y_t$ dominated RGE)

$$\lambda(Q) = \lambda_0 - \frac{3}{8\pi^2} y_0^4 \frac{\ln \frac{Q}{Q_0}}{1 - \frac{9}{16\pi^2} y_0^2 \ln \frac{Q}{Q_0}}$$

Buttazzo et al '13

Linde '76, '80
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+...

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Quantum Instability of the Higgs Mass

so far we looked only at the RG evolution of the Higgs quartic coupling (dimensionless parameter). The Higgs mass has a totally different behavior: it is highly dependent on the UV physics, which leads to the so called hierarchy problem

\[ \delta m_H^2 = \left( 2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2 \right) \frac{3G_F \Lambda^2}{8\sqrt{2\pi^2}} \]

\[ m_H^2 \sim m_0^2 - (115 \text{ GeV})^2 \left( \frac{\Lambda}{700 \text{ GeV}} \right)^2 \]
Naturalness principle @ work

Following the arguments of Wilson, 't Hooft (and others):
only small numbers associated to the breaking of a symmetry survive quantum corrections

Introduce new degrees of freedom to regulate the high-energy behavior

Beautiful examples of naturalness to understand the need of “new” physics

see for instance Giudice ’13 (and refs. therein) for an account

- the need of the positron to screen the electron self-energy: \( \Lambda < m_e/\alpha_{em} \)

- the rho meson to cutoff the EM contribution to the charged pion mass: \( \Lambda < \delta m_\pi^2/\alpha_{em} \)

- the kaon mass difference regulated by the charm quark: \( \Lambda^2 < \frac{\delta m_K}{m_K} \frac{6\pi^2}{G_F^2 f_K^2 \sin^2 \theta_C} \)

- the light Higgs boson to screen the EW corrections to gauge bosons self-energies

- ...

- new physics at the weak scale to cancel the UV sensitivity of the Higgs mass?
How to Stabilize the Higgs Potential

The spin trick

2s+1 polarization states
...with the only exception of a particle moving at the speed of light
... fewer polarization states

a particle of spin s:

Spin 1  Gauge invariance  ➔  no longitudinal polarization  ➔  m=0
Spin 1/2  Chiral symmetry  ➔  only one helicity

If the symmetries are broken, the radiative mass will be set by the scale of symmetry breaking, not the UV/Planck scale

... but the Higgs is a spin 0 particle
Symmetries to Stabilize a Scalar Potential

Supersymmetry

fermion ~ boson

Higher Dimensional Lorentz invariance

\[ A_\mu \sim A_5 \]

4D spin 1

4D spin 0

gauge-Higgs unification models

\[ [\text{Manton '79, Fairlie 79, Hosotani '83 +...}] \]

These symmetries cannot be exact symmetry of the Nature. They have to be broken. We want to look for a soft breaking in order to preserve the stabilization of the weak scale.
Other approaches to the hierarchy problem

the hierarchy problem can be reformulated as:
why the weak scale so much smaller than the Planck scale of quantum gravity?

\[ M_{\text{Pl}} = \sqrt{\frac{\hbar c}{G_N}} \sim 10^{19} \text{ GeV}/c^2 \]

\* large extra dimensions (~1mm): dilute gravitational interactions into large volume not accessible to other forces. Scale of quantum gravity around 1TeV. Black holes could be produced at the LHC.

\* many different species: \( M^* = M_{\text{Pl}}/\sqrt{N} \). \( M^* \sim 1\text{TeV} \) if \( N \sim 10^{32} \)

\* composite Higgs: above the scale of compositeness, the Higgs boson dissolves into its fundamental constituents. Momentum-dependent form factors cut off the divergent integrals

\* break EW symmetry without a Higgs boson, aka technicolor models. Ruled out by the Higgs boson discovery
Could the EW scale accidentally small?

The Sun and the Moon have the same angular size seen from Earth. Why?

- Dynamical explanation?
- Accident?
- Multiverse... there exist many (exo)planets with moons!
- Anthropic selection (probably not for the Moon, but maybe for the Higgs)
SUSY 1.0.1

fermion ⇔ boson

\[ \mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + i \bar{\psi} \gamma^\mu \partial_\mu \psi \]

**susy transformations:**

\[ \delta \phi = \bar{\epsilon} \psi \]
\[ \delta \psi = -i (\gamma^\mu \partial_\mu \phi) \epsilon \]

**susy algebra:**

\[ [\delta_{\epsilon_1}, \delta_{\epsilon_2}] \begin{pmatrix} \phi \\ \psi \end{pmatrix} = -i (\bar{\epsilon}_2 \gamma^\mu \epsilon_1) \partial_\mu \begin{pmatrix} \phi \\ \psi \end{pmatrix} \]

susy² = 4D translation

How to introduce interactions?
SUSY: a quantum space-time

Lorentz transformations

Supersymmetry transformations

quantum dimensions
\[ \theta_1 \theta_2 = -\theta_2 \theta_1 \]

3D space

4D space-time

space-time

superparticle

boson
(integer spin)

fermion
(half-integer spin)

particle

time

antiparticle

3D space

4D space-time

(G. Giudice HCPSS'09)
SUSY: a quantum space-time

SUSY is the most general extension of Lorentz/Poincaré invariance

(L. Giudice HCPSS'09)

Quantum dimensions
θ₁θ₂ = -θ₂θ₁

3D space

Lorentz transformations

Time

4D space-time

Supersymmetry

Particle

Fermion (half-integer spin)

Particle

3D space

4D space-time

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SUSY Interactions

heuristic rule:
replace bosons with fermions in the interaction

Scalar potential is not arbitrary any longer:
dictated by gauge and Yukawa interactions.
One important consequence: upper bound on Higgs mass in simplest models

SUSY predictions
many new particles
many new interactions
SUSY and the (big) hierarchy problem

how to dynamically generate soft breaking terms compatible with exp constraints?

SUSY biggest pb:

\[ \delta m^2_H \propto (y_t^2 - \tilde{y}_t^2) \Lambda^2 + (m_t^2 - \tilde{m}_t^2) \log \Lambda \]

\[ y_t \neq \tilde{y}_t \quad \Lambda^2 \text{ dv} \quad \text{hard susy breaking} \]

\[ m_t \neq \tilde{m}_t \quad \log \Lambda \text{ dv} \quad \text{soft susy breaking} \]
### Minimal Supersymmetric SM - Matter Content

<table>
<thead>
<tr>
<th>Particles</th>
<th>Sparticles</th>
</tr>
</thead>
<tbody>
<tr>
<td>quarks $\begin{pmatrix} u_L \ d_L \end{pmatrix}$ $u_R$ $d_R$</td>
<td>squarks $\begin{pmatrix} \tilde{u}_L \ \tilde{d}_L \end{pmatrix}$ $\tilde{u}_R$ $\tilde{d}_R$</td>
</tr>
<tr>
<td>leptons $\begin{pmatrix} e_L \ \nu_L \end{pmatrix}$ $e_R$</td>
<td>sleptons $\begin{pmatrix} \tilde{e}_L \ \tilde{\nu}_L \end{pmatrix}$ $\tilde{e}_R$</td>
</tr>
<tr>
<td>Higgs doublets $H_1$ (hypercharge $= -1$)</td>
<td>Higgsinos $\tilde{H}_1$ $\tilde{H}_2$</td>
</tr>
<tr>
<td>$H_2$ (hypercharge $= +1$)</td>
<td></td>
</tr>
<tr>
<td>$W^\pm$, $W^3$</td>
<td>winos $\tilde{\omega}^\pm$, $\tilde{\omega}^3$</td>
</tr>
<tr>
<td>$B_\mu$</td>
<td>bino $\tilde{b}$</td>
</tr>
<tr>
<td>$G^{A}_\mu$ $A = 1, \ldots, 8$</td>
<td>gluinos $\tilde{g}^A$</td>
</tr>
</tbody>
</table>

(\text{G. Giudice HCPSS’09})

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**WHERE IS SUSY?**
- Higgsinos, stops, and gluinos.
- Fermions
  - Leptons
  - Quarks
- Bosons
  - Sleptons
  - Squarks
- Gluino
- Neutralinos and charginos
  - $H^0$
  - $Z^0$
  - $W^\pm$
  - Higgs

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SUSY searches

gluinos and squarks are produced by QCD interactions

LSP (lightest supersymmetric particle) is stable \( \approx \) Missing Energy
LSP (lightest supersymmetric particle) is stable \(\approx\) Missing Energy
The MSSM Higgs mass and stop searches

Figure 5: Allowed values of the OS stop mass reproducing $m_h = 125 \text{ GeV}$ as a function of the stop mixing, with $\tan \beta = 20$, $\mu = 300 \text{ GeV}$ and all the other sparticles at 2 TeV. The band reproduce the theoretical uncertainties while the dashed line the $2\sigma$ experimental uncertainty from the top mass. The wiggle around the positive maximal mixing point is due to the physical threshold when $m_{\tilde{t}}$ crosses $M_3 + M_1$.

Current and future bounds on stop mass

LHC (2018)

One needs heavy stop(s) to obtain a 125GeV Higgs (within the MSSM)

Pardo Vega, Villadoro '15 + many others
The MSSM Higgs mass and stop searches

Figure 5: Allowed values of the OS stop mass reproducing $m_h = 125$ GeV with $\tan \beta = 20$, $\mu = 300$ GeV and all the other sparticles at 2 TeV. The band $\tau$ while the dashed line the 2σ experimental uncertainty from the top mass. The mixing point is due to the physical threshold when $m_{\tilde{t}}$ crosses $M_3 + m_t$.

One needs heavy stop(s) to obtain a 125GeV Higgs (within the MSSM)

Current and future bounds on stop mass

HL-LHC (2030)

Pardo Vega, Villadoro ’15 + many others

Direct stop searches (ATLAS Snowmass doc)

Direct gluino searches

EWino searches

5σ @ 300/fb

95% excl @ 3000/fb

95% excl @ 300/fb

5σ @ 300/fb

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One needs heavy stop(s) to obtain a 125GeV Higgs (within the MSSM)

Current and future bounds on stop mass

FCC-hh @ 100TeV (2060)
Saving SUSY

SUSY is Natural but not plain vanilla

- CMSSM
- pMSSM
- NMSSM
- colorless stops ("folded susy")
- Hide SUSY, e.g. smaller phase space
  - reduce production (e.g. split families) Mahbubani et al
  - reduce MET (e.g. R-parity, compressed spectrum) Csaki et al
  - dilute MET (decay to invisible particles with more invisible particles)
  - soften MET (stealth susy, stop - top degeneracy) Fan et al

LHC\(_{300(0)fb^{-1}}\) will tell!

Good coverage of hidden natural susy

- mono-top searches (DM, flavored naturalness - mixing among different squark flavors-, stop-higgsino mixings)
- mono-jet searches with ISR recoil (compressed spectra)
- precise tt inclusive measurement + spin correlations (stop \(\rightarrow\) top + soft neutralino)
- multi-hard-jets (RPV, hidden valleys, long decay chains)
Grand Unified Theory
Evolution of coupling constants

Classical physics: the forces depend on distances

Quantum physics: the charges depend on distances

QED
virtual particles screen the electric charge: $\alpha \downarrow$ when $d \uparrow$

QCD
virtual particles (quarks and *gluons*) screen the strong charge: $\alpha_s \uparrow$ when $d \uparrow$

'asymptotic freedom'

$$\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi} \left( -\frac{11N_c}{6} + \frac{N_f}{3} \right)$$
Grand Unified Theories

A single form of matter
A single fundamental interaction
SU(5) GUT: Gauge Group Structure

SU(3)_c x SU(2)_L x U(1)_Y: SM Matter Content

\[ Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3, 2)_{1/6}, \quad u_R^c = (\bar{3}, 1)_{-2/3}, \quad d_R^c = (\bar{3}, 1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1, 2)_{-1/2}, \quad e_R^c = (1, 1)_1 \]

How can you ever remember all these numbers?

SU(3)_c x SU(2)_L x U(1)_Y \subset SU(5)

SU(5) Adjoint rep. \( \begin{pmatrix} SU(2) \\ SU(3) \end{pmatrix} \)

\[ \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab} \]

\[ \bar{5} = (1, 2)_{-1/2} \sqrt{3/5} + (3, 1)_{1/3} \sqrt{3/5} \]

\[ 5 = L + d_R^c \]

\[ 10 = (5 \times 5)_A = (3, 1)_{-2/3} \sqrt{3/5} + (3, 2)_{1/6} \sqrt{3/5} + (1, 1) \sqrt{3/5} \]

\[ 10 = u_R^c + Q_L + e_R^c \]

additional U(1) factor that commutes with SU(3) x SU(2)

\[ T^{12} = \begin{pmatrix} 1/2 \\ 1/2 \\ -1/3 \end{pmatrix} \]

\[ T^{12} = \sqrt{3/5} Y \]

\[ g_5 T^{12} = g' Y \]

\[ g_5 \sqrt{3/5} = g' \quad g_5 = g = g_s \]

\[ \sin^2 \theta_W = \frac{3}{8} @ M_{\text{GUT}} \]
SU(5) GUT: Gauge Group Structure

SU(3)_c \times SU(2)_L \times U(1)_Y: SM Matter Content

\[ Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3, 2)_{1/6}, \quad u_R^c = (\bar{3}, 1)_{-2/3}, \quad d_R^c = (\bar{3}, 1)_{1/3}, \quad L \sim (\nu_L) = (1, 1)_1 \]

How can you even remember all these numbers?

the SM matter fits nicely into representations of SU(5), even more nicely into SO(10)

unification baryon-lepton

\[ T_{12}^\dagger = \begin{pmatrix} \frac{1}{2} \sqrt{\frac{3}{5}} \\ \frac{1}{3} \sqrt{\frac{3}{5}} \end{pmatrix} \]

\[ g_5 \sqrt{\frac{3}{5}} = g' \]

\[ g_5 = g = g_s \]

\[ \sin^2 \theta_W = \frac{3}{8} \quad @ M_{\text{GUT}} \]

\[ 10 = (5 \times 5)_A = (3, 1)_{-\frac{2}{3}} \sqrt{\frac{3}{5}} + (3, 2)_{\frac{1}{2}} \sqrt{\frac{3}{5}} + (1, 1)_{\frac{3}{5}} \]

\[ 10 = u_R^c + Q_L + e_R^c \]

\[ 5 = L + d_R^c \]

\[ T_{12} = \sqrt{\frac{3}{5}} Y \]

\[ g_5 T_{12} = g' Y \]
SU(5) GUT: low energy consistency condition

\[
\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2}, \quad i = SU(3), SU(2), U(1)
\]

\[\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z) \quad \text{experimental inputs}\]
\[b_3, b_2, b_1 \quad \text{predicted by the matter content}\]

3 equations & 2 unknowns \((\alpha_{GUT}, M_{GUT})\)

one consistency relation on low energy parameters

\[
\epsilon_{ijk} \frac{b_j - b_k}{\alpha_i(M_Z)} = 0
\]

\[
\sin^2 \theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1} \frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)}
\]

\[
\alpha_{em}(M_Z) \approx \frac{1}{128} \quad \alpha_s(M_Z) \approx 0.1184 \pm 0.0007
\]

\[
\sin^2 \theta_W \approx 0.207 \quad \text{not bad... (observed value: 0.23)}
\]

Even better in MSSM
SU(5) GUT: low energy consistency condition

\[
\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \quad i = SU(3), SU(2), U(1)
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\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z) \quad \text{experimental inputs}
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\[
b_3, b_2, b_1 \quad \text{predicted by the matter content}
\]

3 equations & 2 unknowns \( (\alpha_{GUT}, M_{GUT}) \)

one consistency relation on low energy parameters

\[
M_{GUT} = M_Z \exp \left( 2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \right) \approx 7 \times 10^{14} \text{ GeV}
\]

\[
\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 41.5
\]

self-consistent computation:
- \( M_{GUT} < M_{Pl} \) safe to neglect quantum gravity effects
- \( \alpha_{GUT} \ll 1 \) perturbative computation
SU(5) GUT: SM $\beta$ fcts

$g$, $g'$ and $g_s$ are different but it is a low energy artifact!

$$\beta = \frac{dg}{d\log \mu} = -\frac{1}{16\pi^2}bg^3 + \ldots$$

$$\frac{1}{g^2(Q)} = \frac{1}{g^2(Q_0)} + \frac{b}{16\pi^2} \ln \frac{Q^2}{Q_0^2}$$

$\beta = \frac{11}{3} T_2(\text{spin-1}) - \frac{2}{3} T_2(\text{chiral spin-1/2}) - \frac{1}{3} T_2(\text{complex spin-0})$

$$\text{Tr}(T^a(R)T^b(R)) = T_2(R)\delta^{ab} \quad T_2(\text{fund}) = \frac{1}{2} \quad T_2(\text{adj}) = N$$

$$g = \frac{11}{3} \times 3 - \frac{2}{3} \left( \frac{1}{2} \times 2 \times 3 + \frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 1 \times 3 \right) = 7$$

$$b_{SU(3)} = \frac{11}{3} \times 2 - \frac{2}{3} \left( \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 1 \times 3 \right) - \frac{1}{3} \times \frac{1}{2} = \frac{19}{6}$$

$$b_Y = -\frac{2}{3} \left( \left( \frac{1}{6} \right)^2 \times 2 \times 3 \times 3 \right) + \left( -\frac{2}{3} \right)^2 \times 3 \times 3 + \left( \frac{1}{3} \right)^2 \times 3 \times 3 + \left( -\frac{1}{2} \right)^2 \times 2 \times 3 \times (1)^2 \times 3 - \frac{1}{3} \left( \frac{1}{2} \right)^2 \times 2 = \frac{-41}{6}$$

$$b_{T12} = \frac{-41}{10}$$
SU(5) GUT: SM vs MSSM $\beta$ fcts

chiral superfield
complex spin-0
Weyl spin-1/2
in same representation of gauge group

vector superfield
Weyl spin-1/2
real spin-1
in same representation of gauge group

$$b = \frac{11}{3} T_2(\text{vector}) - \frac{2}{3} T_2(\text{vector}) - \frac{2}{3} T_2(\text{chiral}) - \frac{1}{3} T_2(\text{chiral}) = 3 T_2(\text{vector}) - T_2(\text{chiral})$$

MSSM Chiral Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad U = (\bar{3},1)_{-2/3}, \quad D = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad E = (1,1)_1, \quad H_u = (1,2)_{1/2}, \quad H_d = (1,2)_{-1/2}$$

$$b_{SU(3)} = 3 \times 3 - \left( \frac{1}{2} \times 2 \times 3 + \frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 1 \times 3 \right) = 3$$

$$b_{SU(2)} = 3 \times 2 - \left( \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 1 \times 3 \right) - \frac{H_u}{2} - \frac{H_d}{2} = -1$$

$$b_Y = - \left( \left( \frac{1}{6} \right)^2 3 \times 2 \times 3 + \left( \frac{2}{3} \right)^2 3 \times 3 + \left( \frac{1}{3} \right)^2 3 \times 3 + \left( -\frac{1}{2} \right)^2 2 \times 3 + (1)^2 \times 3 \right) - \left( \frac{1}{2} \right)^2 2 - \left( \frac{1}{2} \right)^2 2 = -11$$

$$b_{T12} = -\frac{33}{5}$$
SU(5) GUT: MSSM GUT

$b_3 = 3, \ b_2 = -1, \ b_1 = -33/5$

low-energy consistency relation for unification

$$\sin^2 \theta_W = \frac{3(b_3 - b_2)}{8b_3 - 3b_2 - 5b_1} + \frac{5(b_2 - b_1)}{8b_3 - 3b_2 - 5b_1} \frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)} \approx 0.23$$

squarks and sleptons form complete SU(5) reps → they don’t improve unification!

gauginos and higgsinos are improving the unification of gauge couplings

GUT scale predictions

$$M_{GUT} = M_Z \exp \left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \right) \approx 2 \times 10^{16} \text{ GeV}$$

$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 24.3$$
Proton Decay

in GUT, matter is unstable
decay of proton mediated by new SU(5)/SO(10) gauge bosons

GUT: \( \tau_p(p \rightarrow e^+\pi^0) = \left( \frac{M_X}{10^{15} \text{ GeV}} \right)^4 \cdot 10^{31-32} \text{ yr} \)

Exp: \( \tau_p(p \rightarrow e^+\pi^0) > 8.2 \times 10^{33} \text{ yr} \)

(Age of the Universe: \( 10^{10} \) years)
# Proton Decay

<table>
<thead>
<tr>
<th>Mode</th>
<th>Partial mean life (10^{30} years)</th>
<th>Confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \rightarrow e^+ K^0$</td>
<td>&gt; 2000 (n), &gt; 8200 (p)</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow e^+ \pi$</td>
<td>&gt; 228 (n), &gt; 160 (p)</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow e^+ \pi$</td>
<td>&gt; 1000 (n), &gt; 6600 (p)</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow \nu \pi$</td>
<td>&gt; 1100 (n), &gt; 390 (p)</td>
<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow e^+ \eta$</td>
<td>&gt; 4200</td>
<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+ \eta$</td>
<td>&gt; 1300</td>
<td>90%</td>
</tr>
<tr>
<td>$n \rightarrow \nu \eta$</td>
<td>&gt; 158</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow e^+ \rho$</td>
<td>&gt; 217 (n), &gt; 710 (p)</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow \mu^+ \rho$</td>
<td>&gt; 228 (n), &gt; 160 (p)</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow \nu \rho$</td>
<td>&gt; 19 (n), &gt; 162 (p)</td>
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</tr>
<tr>
<td>$p \rightarrow e^+ \omega$</td>
<td>&gt; 320</td>
<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow \mu^+ \omega$</td>
<td>&gt; 780</td>
<td>90%</td>
</tr>
<tr>
<td>$n \rightarrow \nu \omega$</td>
<td>&gt; 108</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow e^+ K^0$</td>
<td>&gt; 17 (n), &gt; 1000 (p)</td>
<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow e^+ K^0_L$</td>
<td>&gt; 26 (n), &gt; 1600 (p)</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow \mu^+ K^0$</td>
<td>&gt; 86 (n), &gt; 5900 (p)</td>
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<tr>
<td>$n \rightarrow \nu K^0_L$</td>
<td>&gt; 260</td>
<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow e^+ K^*(892)^0$</td>
<td>&gt; 84</td>
<td>90%</td>
</tr>
<tr>
<td>$N \rightarrow \nu K^*(892)$</td>
<td>&gt; 78 (n), &gt; 51 (p)</td>
<td>90%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>Partial mean life (10^{30} years)</th>
<th>Confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n \rightarrow e^- \pi^+$</td>
<td>&gt; 65</td>
<td>90%</td>
</tr>
<tr>
<td>$n \rightarrow \mu^- \pi^+$</td>
<td>&gt; 49</td>
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</tr>
<tr>
<td>$n \rightarrow e^- \rho^+$</td>
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<td>90%</td>
</tr>
<tr>
<td>$n \rightarrow \mu^- \rho^+$</td>
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</tr>
<tr>
<td>$n \rightarrow e^- K^+$</td>
<td>&gt; 32</td>
<td>90%</td>
</tr>
<tr>
<td>$n \rightarrow \mu^- K^+$</td>
<td>&gt; 57</td>
<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow e^- \pi^+ \pi^+$</td>
<td>&gt; 30</td>
<td>90%</td>
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<tr>
<td>$n \rightarrow e^- \pi^+ \pi^0$</td>
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<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow \mu^- \pi^+ \pi^+ $</td>
<td>&gt; 17</td>
<td>90%</td>
</tr>
<tr>
<td>$n \rightarrow \mu^- \pi^+ \pi^0$</td>
<td>&gt; 34</td>
<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow e^- \pi^+ K^+$</td>
<td>&gt; 75</td>
<td>90%</td>
</tr>
<tr>
<td>$p \rightarrow \mu^- \pi^+ K^+$</td>
<td>&gt; 245</td>
<td>90%</td>
</tr>
</tbody>
</table>

## ΔB=ΔL=1 decay bounds

**Christophe Grojean**

**BSM**

**CERN, July 2018**