

Power supply ripples & 6D Frequency Map Analysis

Sofia Kostoglou, Yannis Papaphilippou

Acknowledgments: H. Bartosik, R. De Maria, M. Fitterer, G. Iadarola, D. Pellegrini, N. Triantafyllou

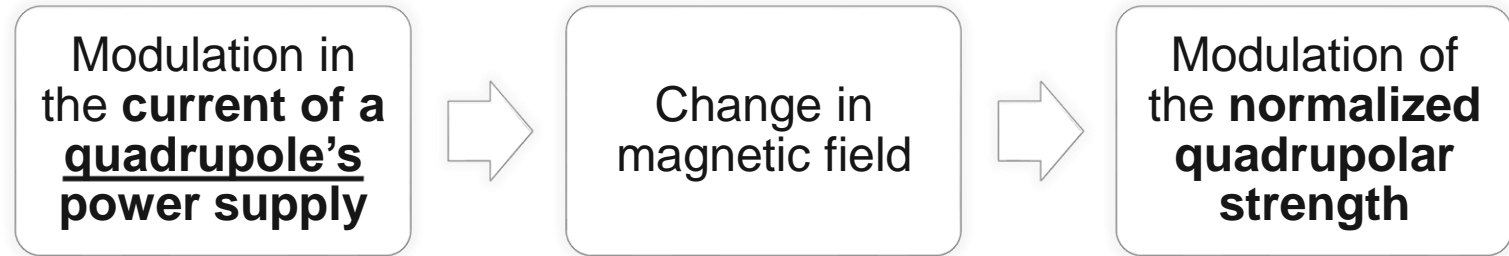
Introduction

- Previous studies: DA studies with power supply ripples for the HL-LHC triplet showed a sensitivity at 300 & 600 Hz *.
- The combination of non-linear resonances and modulation effects: degradation of dynamic aperture and beam lifetime.
- Transverse tune modulation: **additional resonance sidebands**, which can reach the footprint and cause particle diffusion.
- **Frequency Map Analysis** in the presence of:
 - I. **Power supply ripples**
 - II. **Synchrotron motion**

* “BEAM DYNAMICS REQUIREMENTS FOR THE POWERING SCHEME OF THE HL-LHC TRIPLET”, M. Fitterer, R. De Maria, S. Fartoukh and M. Giovannozzi

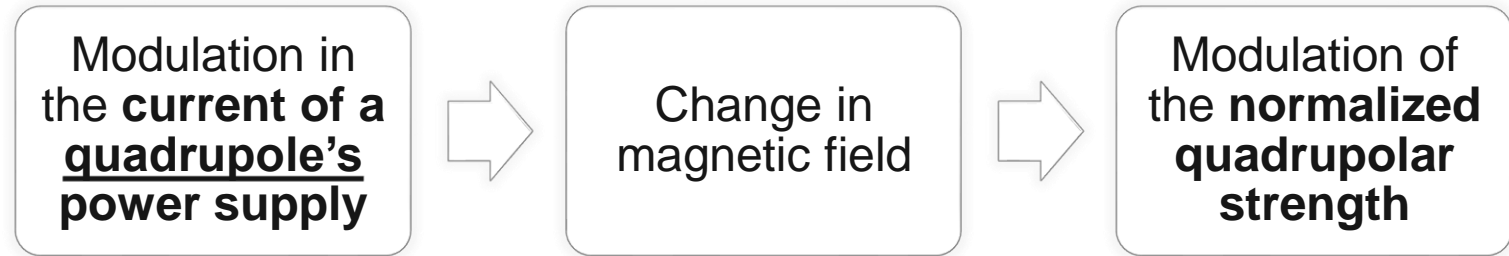
Modulation of the betatron tunes (I)

➤ Power supply ripples:



Modulation of the betatron tunes (I)

➤ Power supply ripples:



$$\Delta Q = \frac{1}{4\pi} \oint \beta(s) \Delta k(s) ds$$

Instantaneous tune

$$Q_{\text{inst}} = Q_0 + \frac{1}{4\pi} \overline{\beta K_{\text{depth}}} \sin(2\pi Q_p n)$$


Modulation depth

$$x(n) = x_0 \cos(2\pi Q_0 n + \underbrace{\frac{\overline{\beta K_{\text{depth}}}}{4\pi Q_p}}_{\text{Modulation index}} \sin(2\pi Q_p n))$$

Modulation index

Modulation of the betatron tunes (II)

➤ Modulation from synchrotron motion:

- Chromaticity, synchrotron tune & $\Delta p/p$ define the value of the **modulation index**.
- Large modulation index  production of **higher order harmonics**.

Instantaneous tune

$$Q_{\text{inst}} = Q_0 + \frac{Q'_x \Delta p}{p} \sin(2\pi Q_s n)$$

Transverse motion- Linear approximation

$$x(n) = x_0 \cos \left(2\pi Q_0 n + \underbrace{\frac{Q'_x}{Q_s} \cdot \frac{\Delta p_{\text{max}}}{p}}_{\text{Modulation index}} \sin(2\pi Q_s n) \right)$$

Modulation index

Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
- **Fourier integral:**

$$|\varphi(\omega)| = | \langle x(n), e^{-i\omega n} \rangle | = \left| \int e^{-i2\pi f n} \cdot \overline{x_h(n)} \, dn \right|$$

Simplified model with FM – Frequency domain

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- **How easy is it to compute the tune of a modulated signal?**

Simplified model with FM – Frequency domain

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- **Bessel functions of the first kind:**

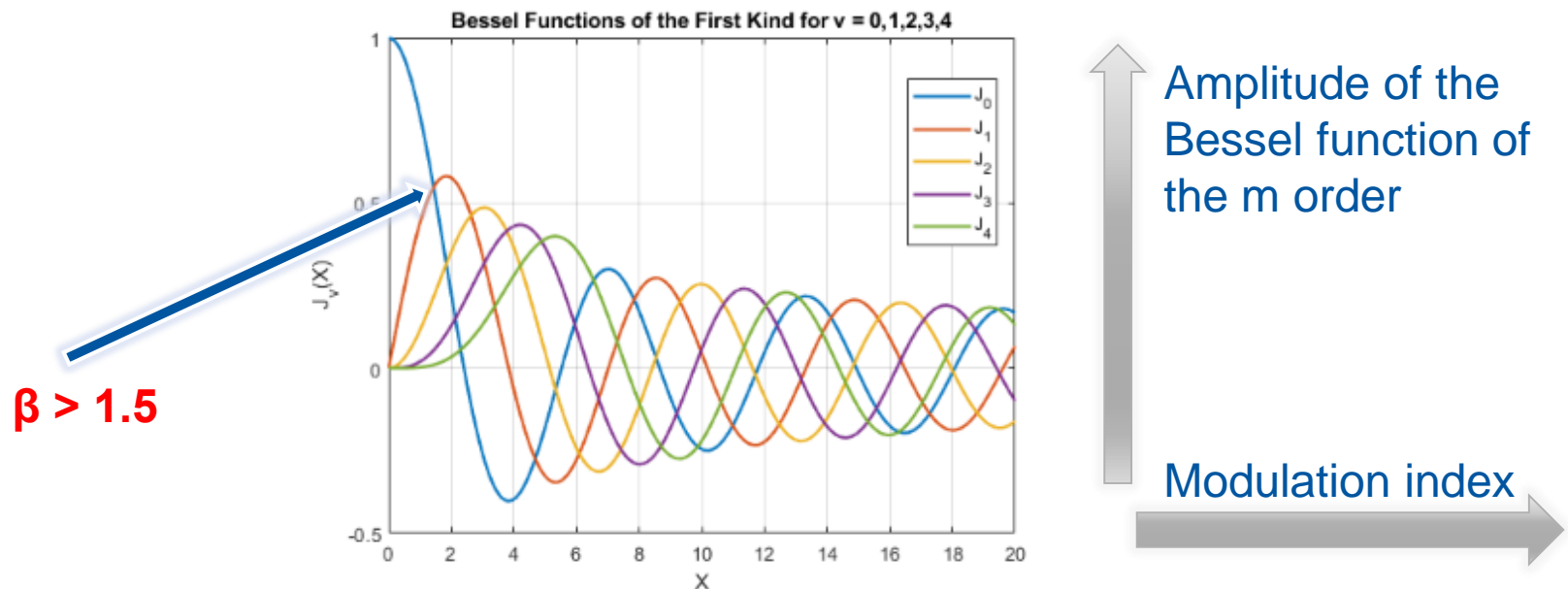
$$\mathbf{x}(\mathbf{n}) = x_0 \cos(2\pi Q_x \mathbf{n} + \beta \sin(2\pi Q_m \mathbf{n})) = x_0 \sum_{m=-\infty}^{\infty} J_m(\beta) \cos(2\pi(Q_x + mQ_m)\mathbf{n})$$

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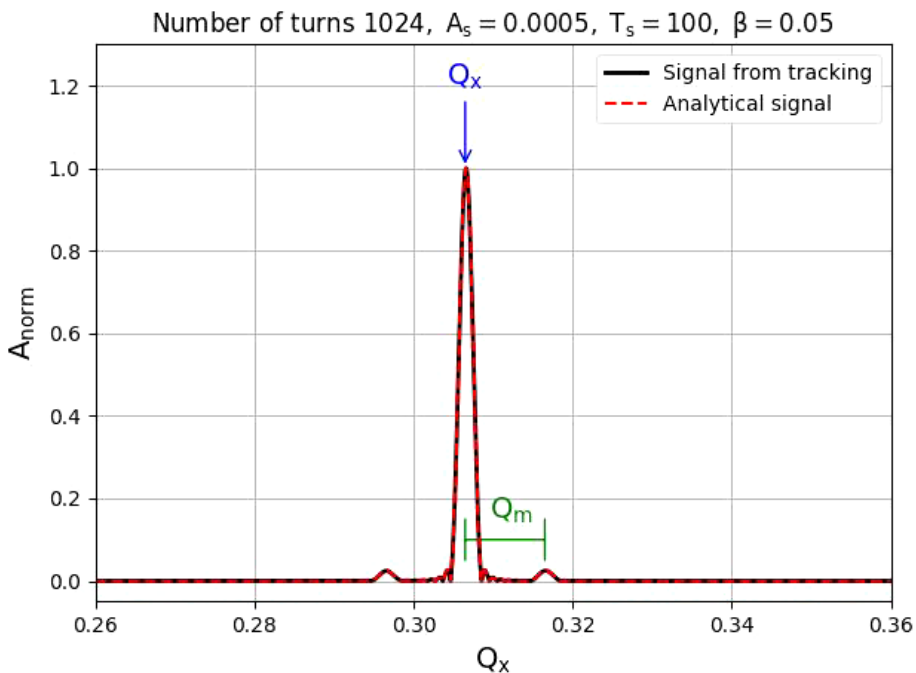


Simplified model with FM – Frequency domain

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- **Bessel functions of the first kind:**



- **Modulation frequency is constant**
- **Increase of modulation depth**
- **Modulation index increases**

- **Red line: analytical signal**
- **Black line: signal from tracking**

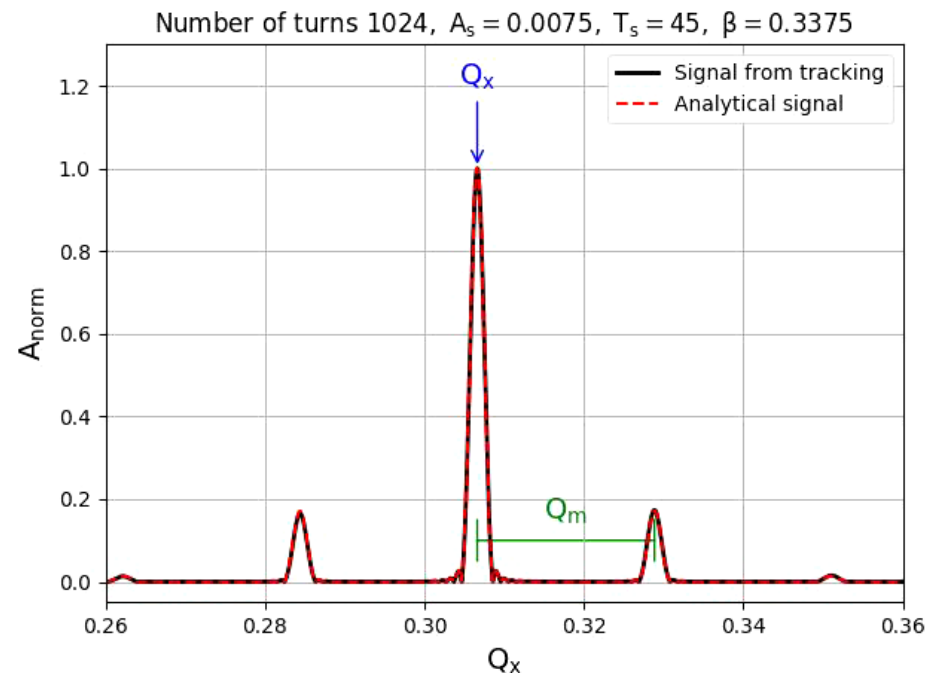
Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
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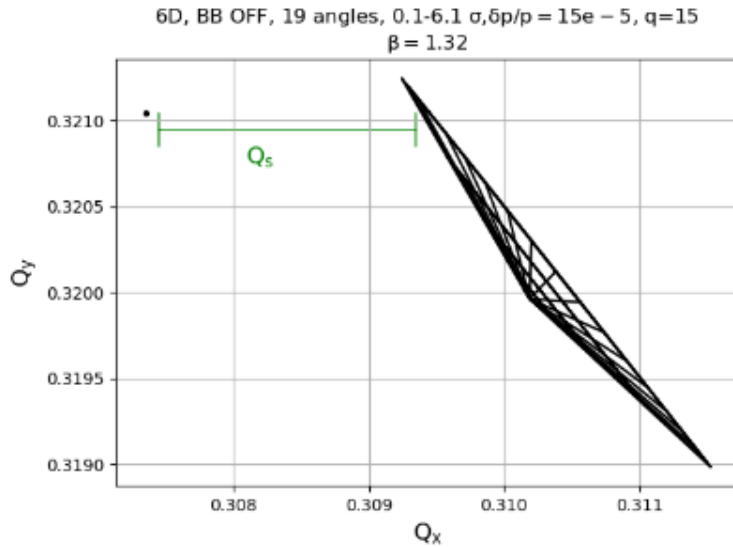
- **Bessel functions of the first kind:**

- **Modulation frequency decreases**
- **Modulation depth is constant**
- **Modulation index increases**



LHC: Application of correction with Bessel functions

$$\beta = 1.32$$



Tools

- Sixtrack
- NAFF

Parameters

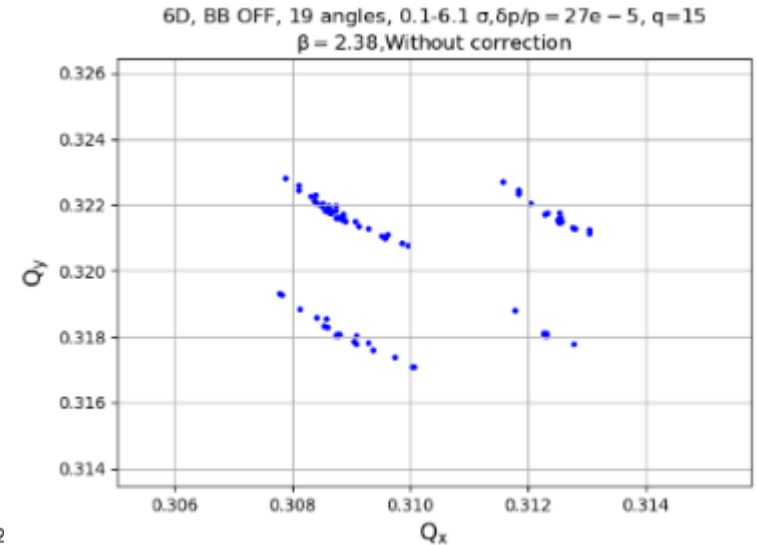
- 6D footprint
- 19 angles
- 0.1-6.1 σ
- $\delta p/p = 15e-5$
- $Q' = 15$
- $\beta^* = 40$ cm
- $(Q_x, Q_y) = (62.31, 60.32)$
- $x_{ing} = 150$ μ rad

LHC: Application of correction with Bessel functions

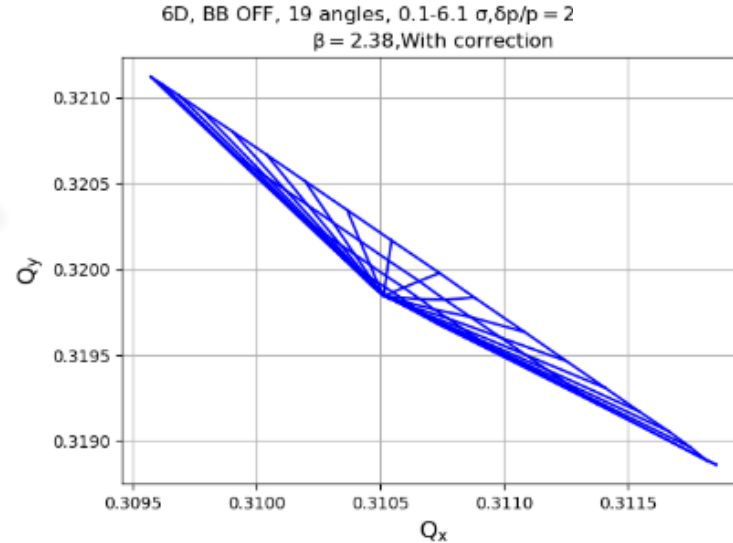
Parameters

- ❑ 6D footprint
- ❑ 19 angles
- ❑ 0.1-6.1 σ
- ❑ $\delta p/p = 27e-5$
- ❑ $Q' = 15$
- ❑ $\beta^* = 40$ cm
- ❑ $(Q_x, Q_y) = (62.31, 60.32)$
- ❑ $x_{ing} = 150$ μ rad

$\beta = 2.38$



Corrected \rightarrow



Simplified model with FM: Phase space

- ❑ **6th order** resonance excited from octupole

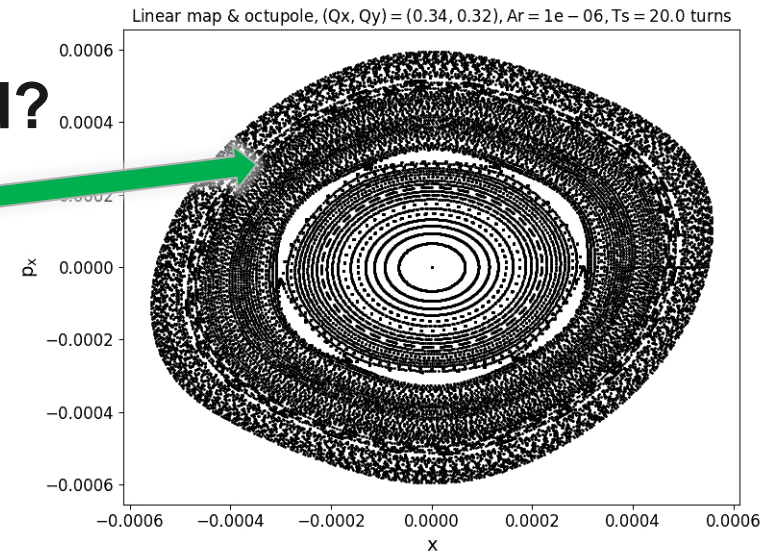
Sideband?

- ❑ Introduction of modulated quadrupole

↑ Modulation depth,
Modulation frequency = ct

- ❑ For a small modulation depth the **1st sideband** of the **6th order** resonance starts to appear

- ❑ For a larger modulation depth more particles are trapped inside the **1st sideband**



Simplified model with FM: Phase space

- ❑ **6th order** resonance excited from octupole **Sideband?**

- ❑ Introduction of modulated quadrupole

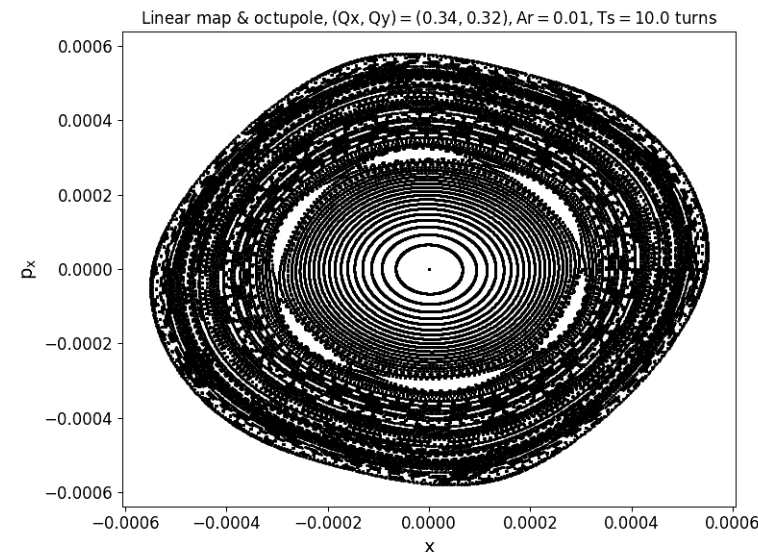
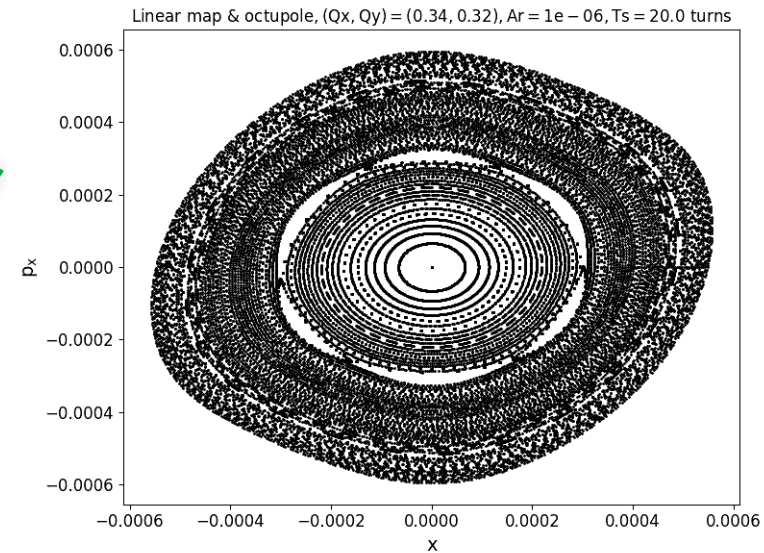
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- ❑ For a larger modulation depth more particles are trapped inside the **1st sideband**

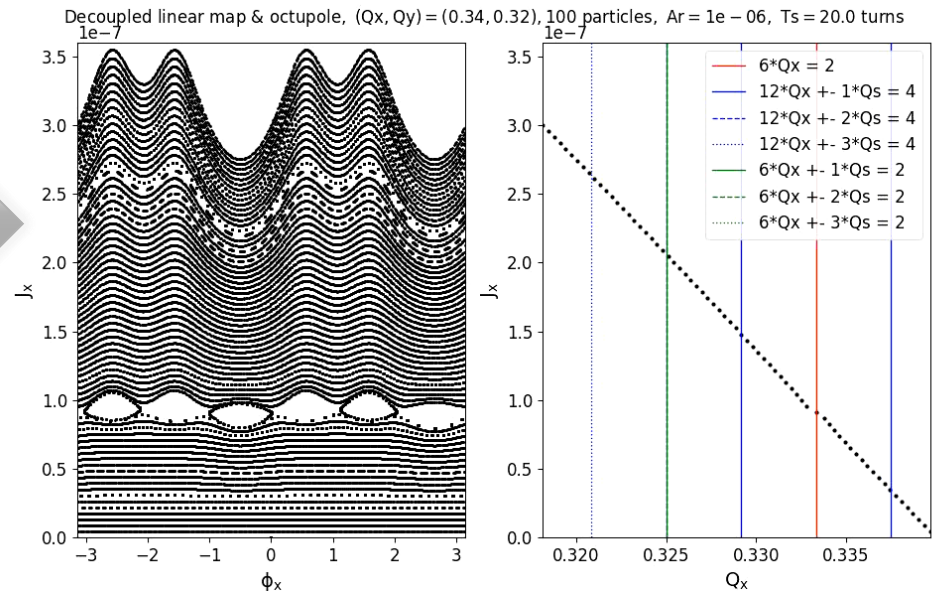
↓ **Modulation frequency,**
Modulation depth = ct

- ❑ For a smaller modulation frequency sidebands approach the main resonance & more sidebands start to appear

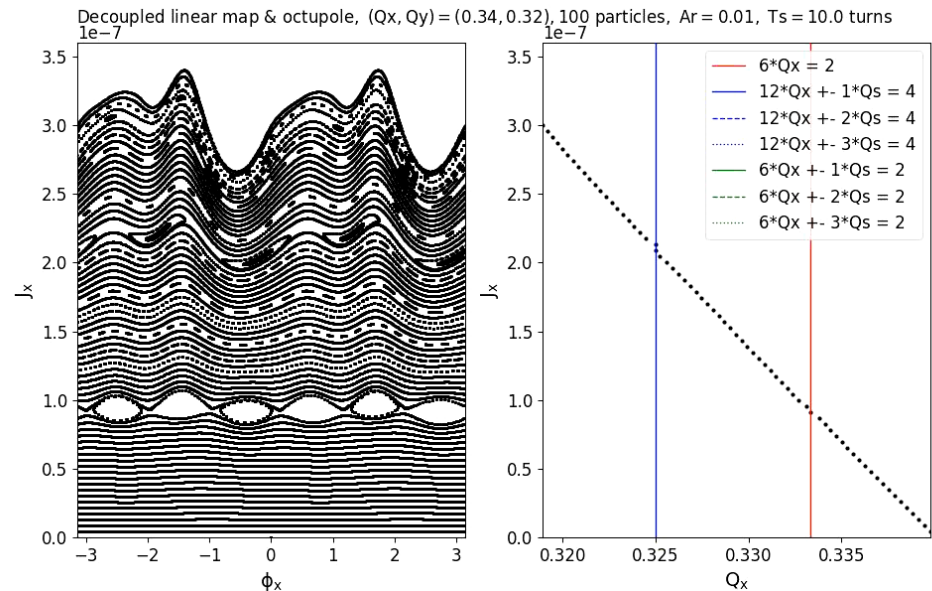


Simplified model with FM: Phase space

↑ Modulation depth,
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↓ Modulation frequency,
Modulation depth = ct



Distance between 1st sideband and main resonance is $\frac{2 \pm Q_s}{6}$

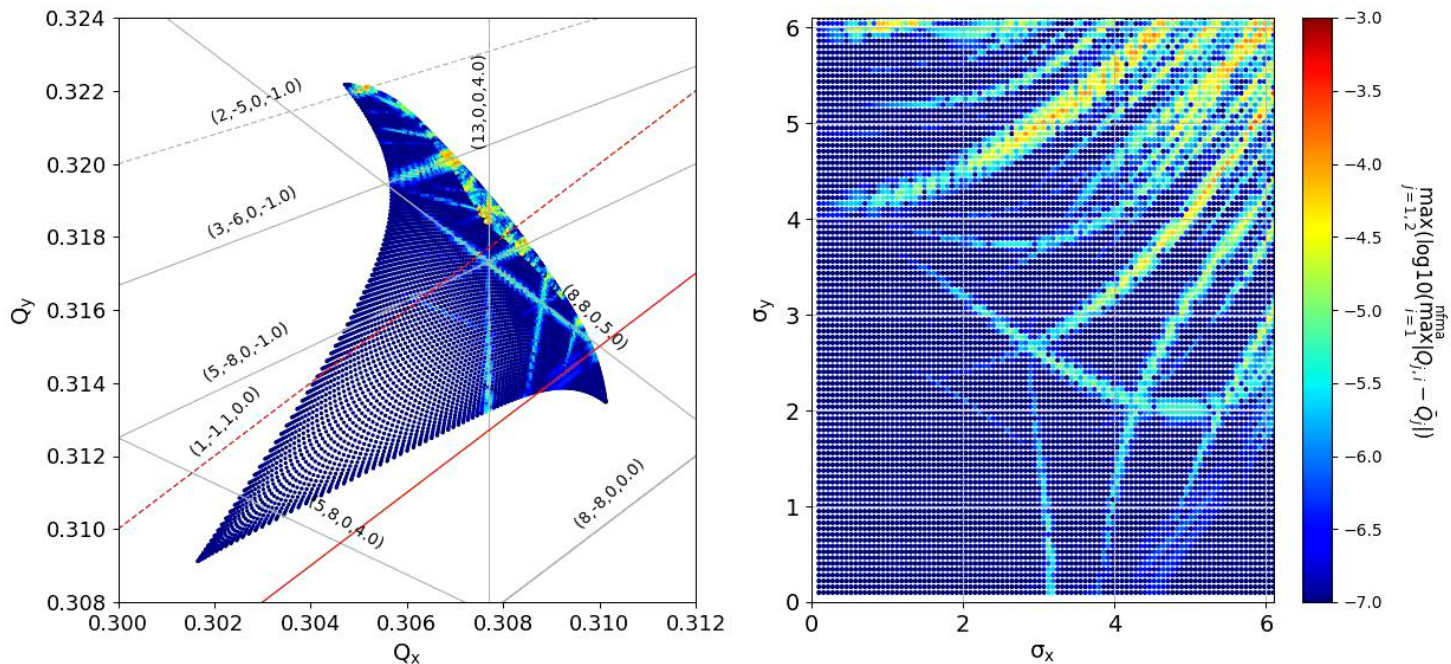
Simplified model with FM & 4D Beam-beam

- Resonance conditions:

$$aQ_x + bQ_y + cQ_m = k \text{ for } a, b, c, k \text{ integers}$$

4D BB, modulation depth scan from $\Delta Q=1e-6$ to $\Delta Q=5e-4$

Linear map, sextupole, octupole, 4D BB, modulating quadrupole, VRF OFF, 2000 turns, 1000 particles
E=6.5 TeV, Npart = 2e11, beta_s=1, min_sigma_diff=1e-16, Ts=100.0 turns, DQ=1e-06



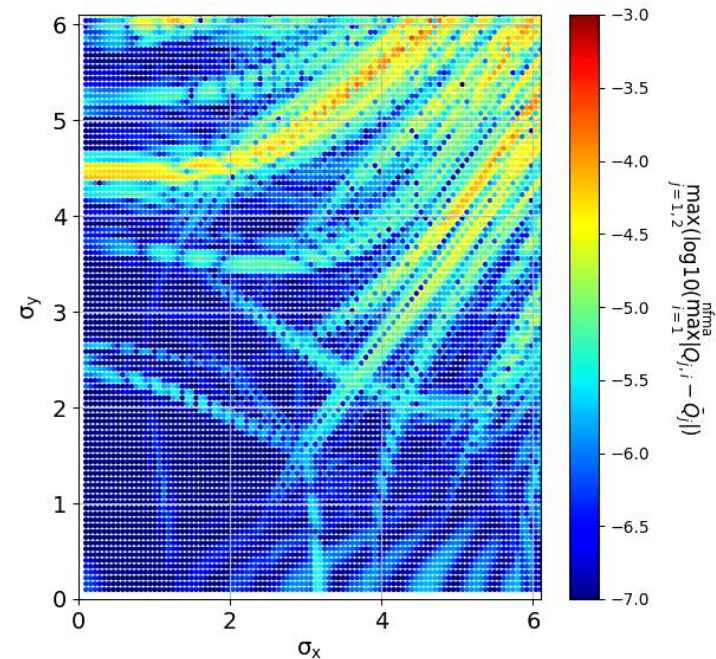
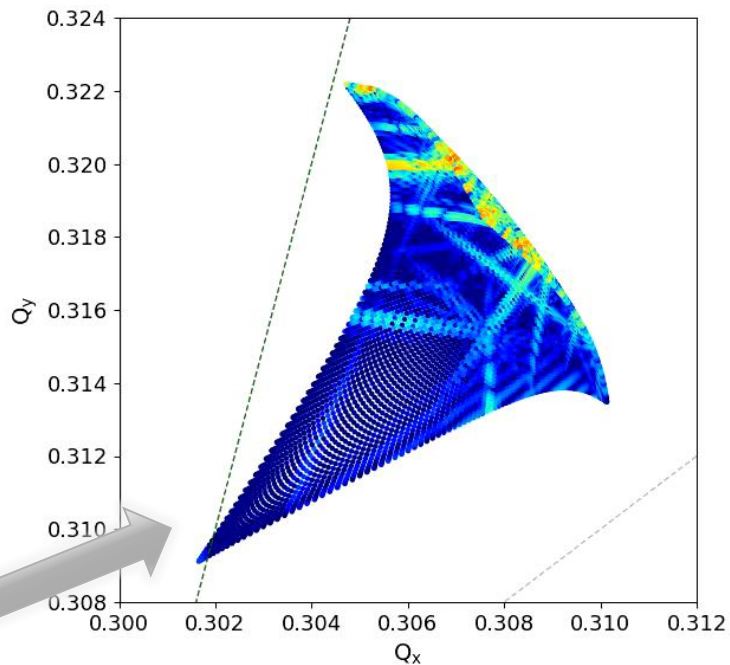
Simplified model with FM & 4D Beam-beam

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4D BB, modulation frequency scan from $Q_m = 0.2$ to $Q_m=0.05$

Linear map, sextupole, octupole, 4D BB, modulating quadrupole, VRF OFF, 2000 turns, 4900 particles
E=6.5 TeV, Npart = 2e11, beta_s=40cm, Ts=5 turns



1st sideband of 3rd order resonance

LHC: Power supply ripples

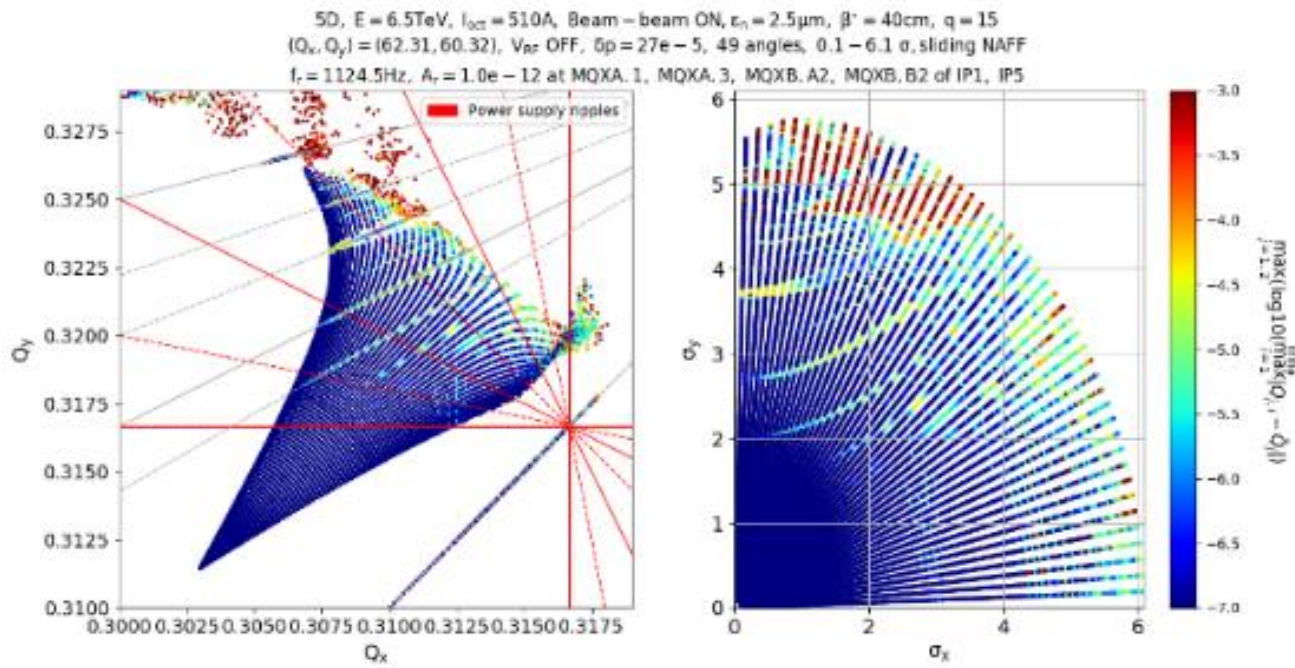
- Quadrupoles of the **inner triplet** right and left of **IP1 and IP5**, **large beta-functions** increase the sensitivity to non-linear effects
- **Resonance conditions:**

$$aQ_x + bQ_y + c \frac{f_{\text{modulation}}}{f_{\text{revolution}}} = k \text{ for } a, b, c, k \text{ integers}$$

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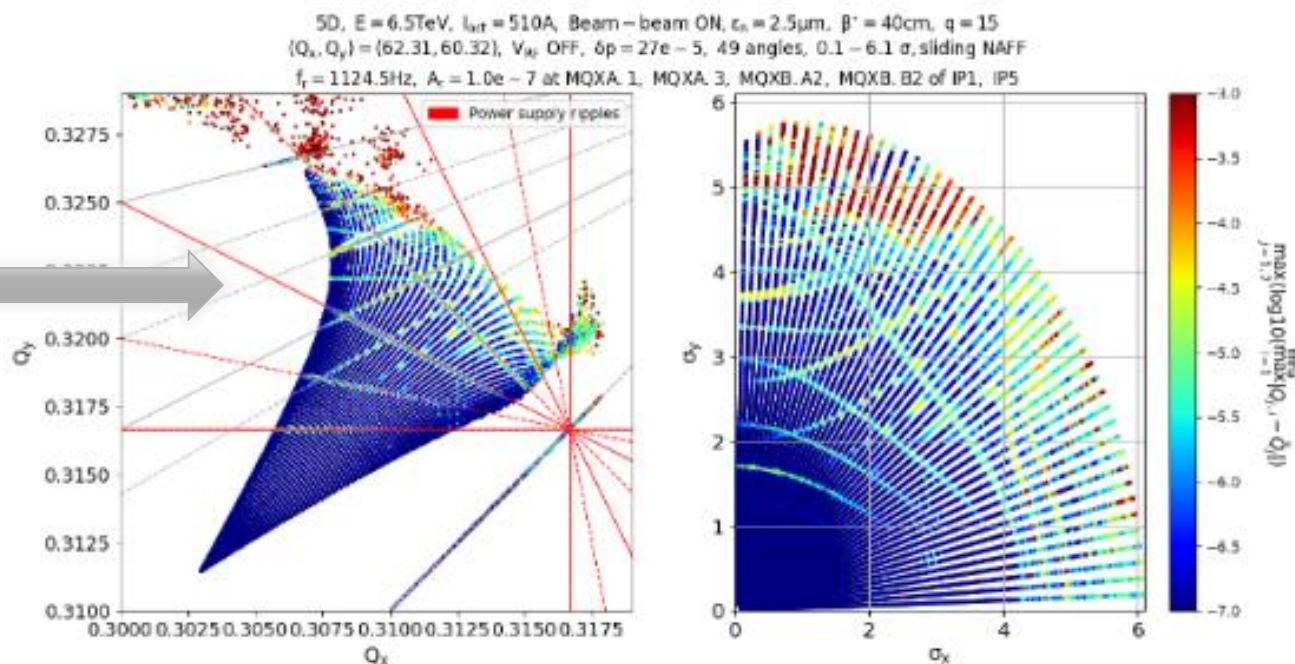
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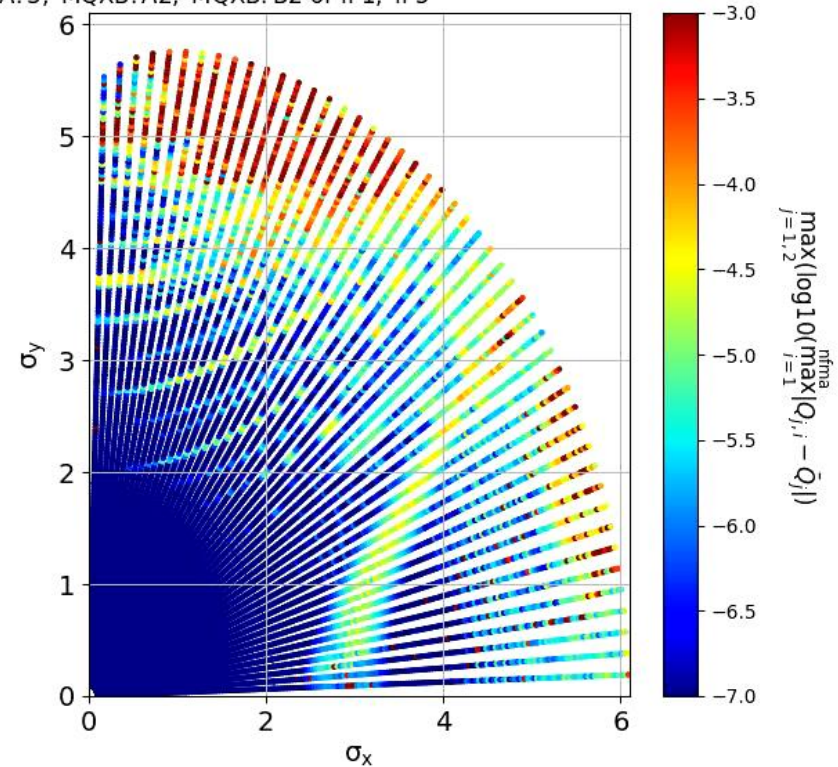
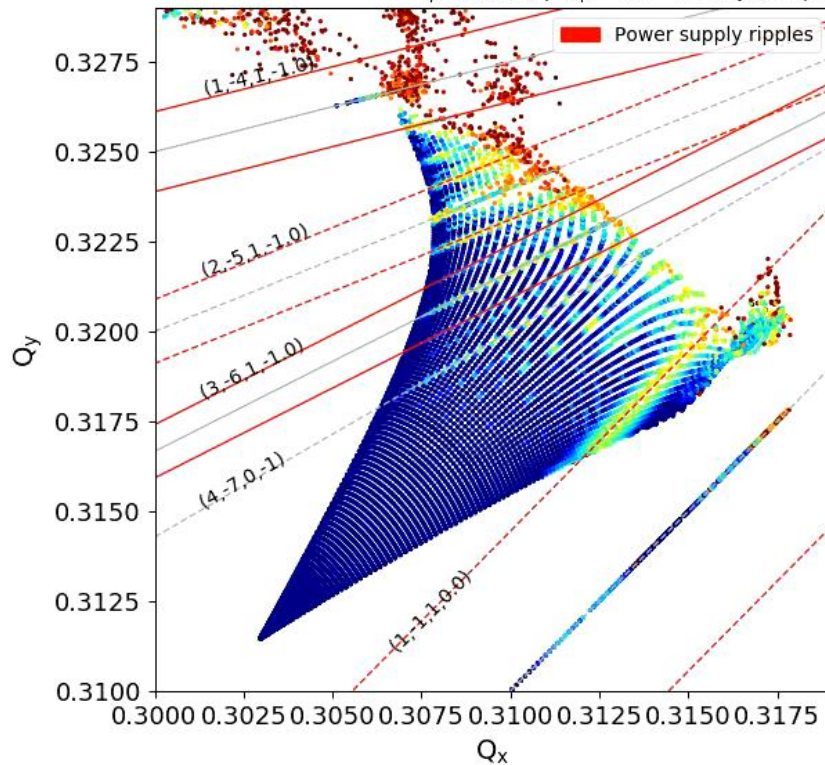
-By increasing the modulation depth, sidebands start to appear in the footprints



LHC: Power supply ripples

□ Scan of different ripple frequencies (50-900 Hz)

5D, $E = 6.5\text{TeV}$, $I_{\text{oc}} = 510\text{A}$, Beam - beam ON, $\epsilon_n = 2.5\mu\text{m}$, $\beta^* = 40\text{cm}$, $q = 15$
 $(Q_x, Q_y) = (62.31, 60.32)$, V_{RF} OFF, $\delta p = 27e-5$, 49 angles, $0.1 - 6.1 \sigma$, sliding NAFF
 $f_r = 50.0\text{Hz}$, $A_r = 10^{-7}$ at MQXA. 1, MQXA. 3, MQXB. A2, MQXB. B2 of IP1, IP5



6D studies

➤ Goal

- Full dynamics of the beam **along with synchrotron motion**

➤ Difference

- **Coherent motion of the footprint** due to synchro-betatron coupling

Tools

- i. Sixtrack for single particle tracking
- ii. NAFF

➤ Methods

- **Long term tracking**: average picture of this motion in frequency space
- **Instantaneous picture** in the frequency domain → small number of turns & high sampling rate (accuracy)

✓ Purpose of the study

- ✓ Frequency maps with a **30-turn window length** and reasonable number of BPMs
- ✓ **Long term frequency maps** with 1BPM per turn & power supply ripples

Setup & parameters

Parameters

- $E = 6.5 \text{ TeV}$
- **6D Beam beam**
- $Q_s = 0.0017$
- $I_{\text{oct}} = 510 \text{ A}$
- $\varepsilon_N = 2.5 \mu\text{m}$
- $\delta = 27\text{e-}5$
- $Q_x = 62.31$
- $Q_y = 60.32$
- $dq = 15$

Setup for 6D FMAs

1 BPM analysis

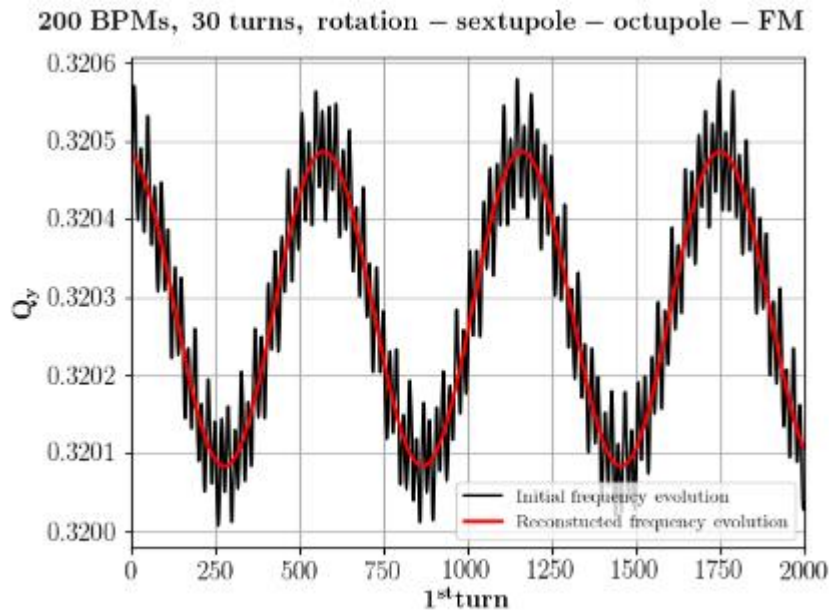
at IP3

Multiple BPM analysis

- **200 virtual BPMs**
- Adjusted phase between BPMs around the ring to have cleaner spectra (discrepancies for particles at different amplitudes and energies)
- Tracking data are collected **“turn wise”**:
 $\mathbf{x} = \mathbf{x}_{\text{BPM1,turn1}}, \mathbf{x}_{\text{BPM2,turn1}}, \dots, \mathbf{x}_{\text{BPM200,turn1}}, \dots, \mathbf{x}_{\text{BPM200,turn30}}$

Multiple BPM analysis

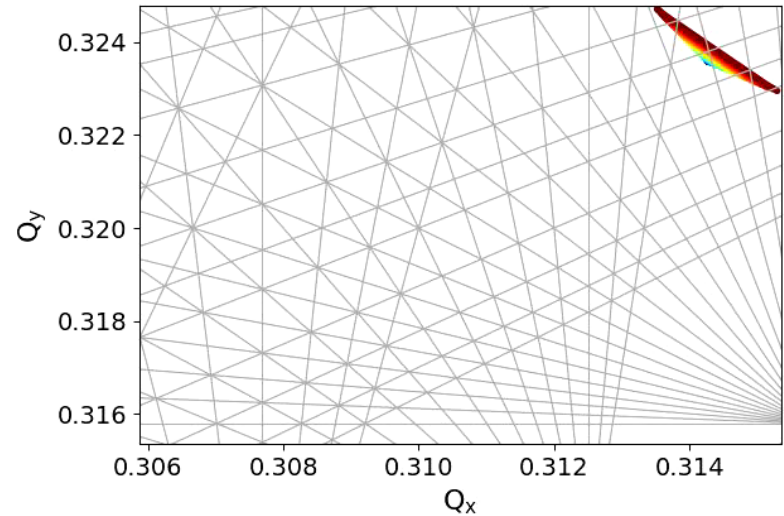
- Window length of **30 turns**
- Before building the instantaneous footprints, the leakage from the resonance driving terms (high frequency modulation) needs to be removed.



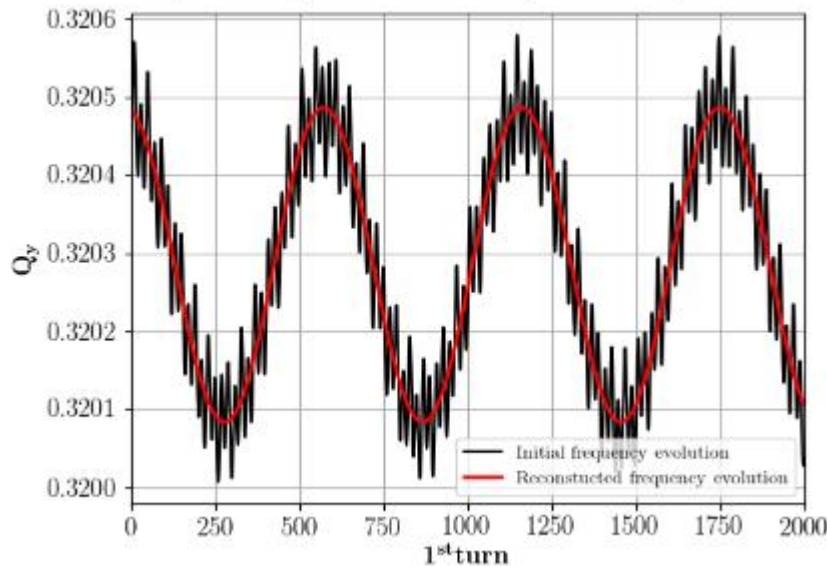
Multiple BPM analysis

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200 BPMs, 1-31 turns, NAFF – NAFF
BB OFF, V_{RF} ON



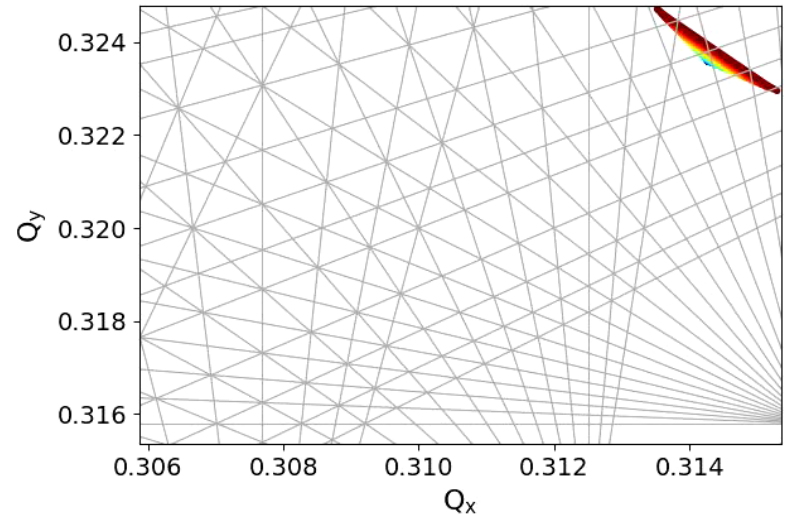
200 BPMs, 30 turns, rotation – sextupole – octupole – FM



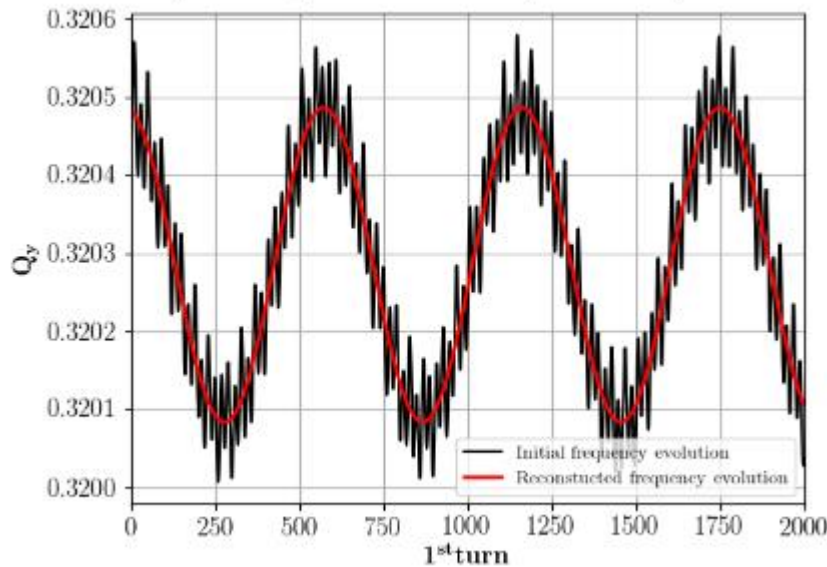
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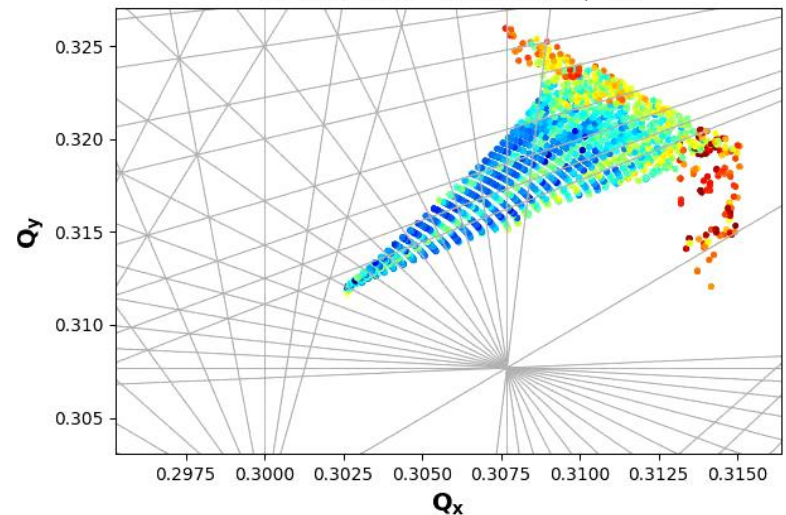
200 BPMs, 1-31 turns, NAFF – NAFF
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200 BPMs, 30 turns, rotation – sextupole – octupole – FM

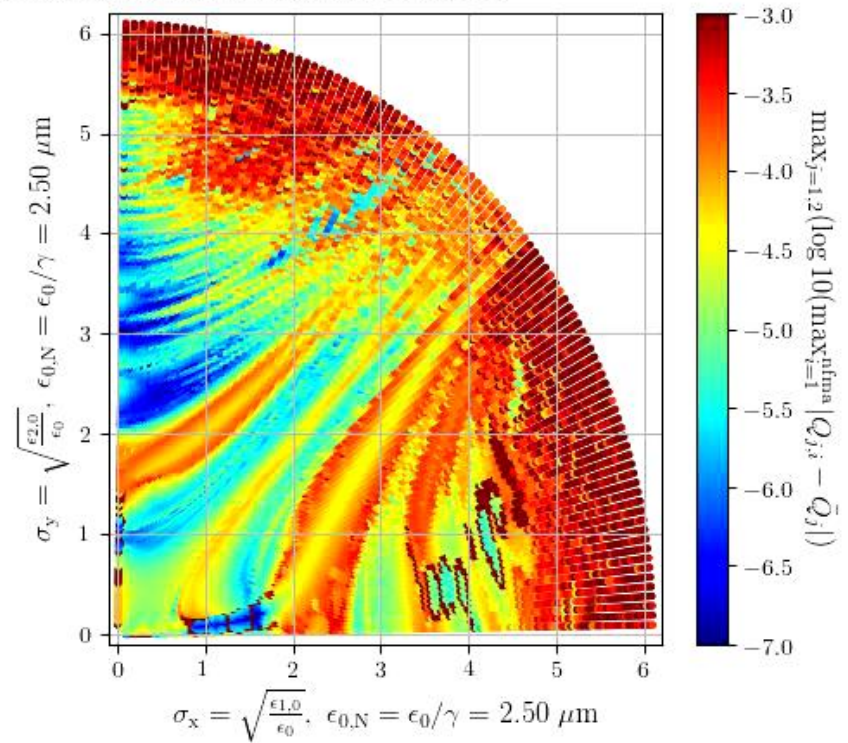
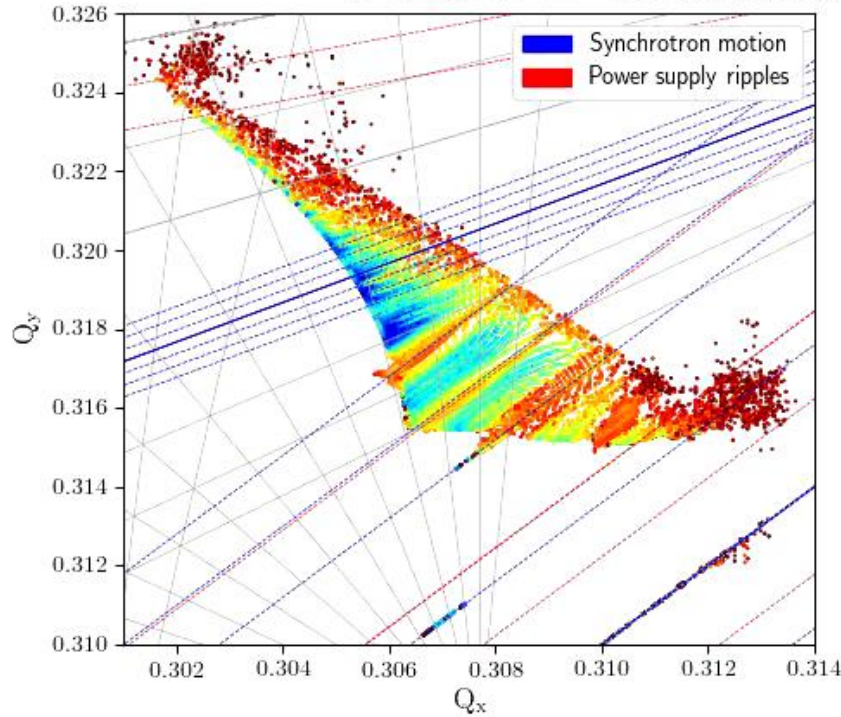


200 BPMs, 1-31 turns, NAFF – NAFF
6D FMA, Beam – beam ON, $dq = 15$



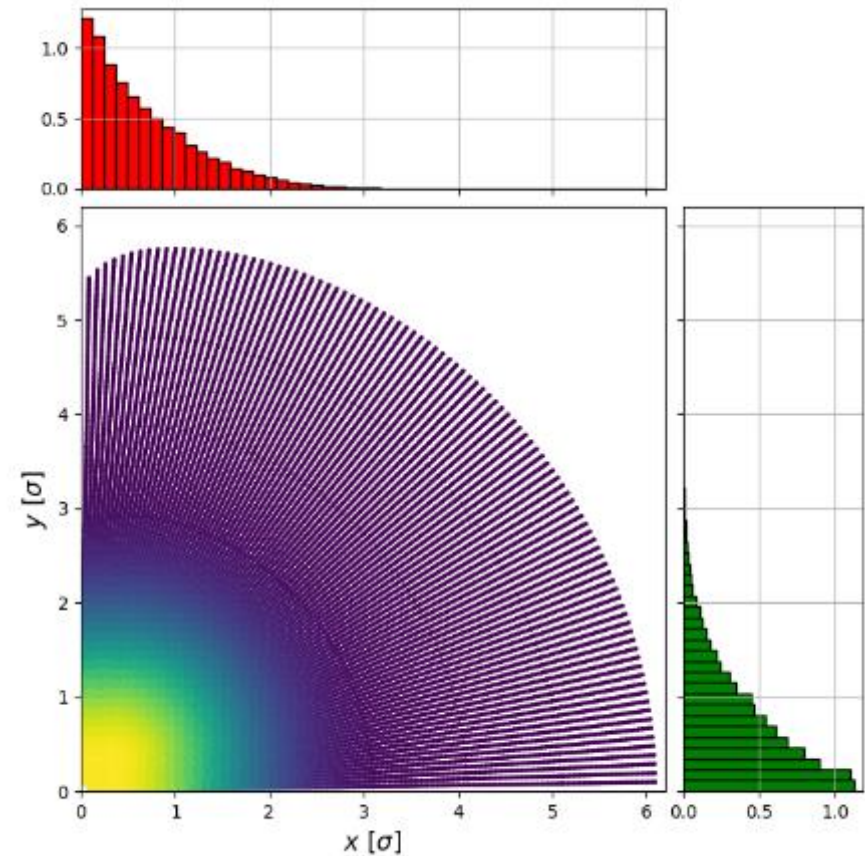
6D FMAs with power supply ripples

6D, $E = 6.5\text{TeV}$, $I_{\text{oct}} = 510\text{A}$, Beam - beam ON, $\epsilon_n = 2.5\mu\text{m}$, $\beta^* = 40\text{cm}$, $q = 0$
 $(Q_x, Q_y) = (62.31, 60.32)$, V_{RF} ON, $\delta p = 27 \cdot 10^{-5}$, 99 angles, $0.1 - 6.1 \sigma$, sliding NAFF
 $f_r = 50\text{Hz}$, $A_r = 10^{-7}$ at MQXA.1, MQXA.3, MQXB.A2, MQXB.B2 of IP1, IP5

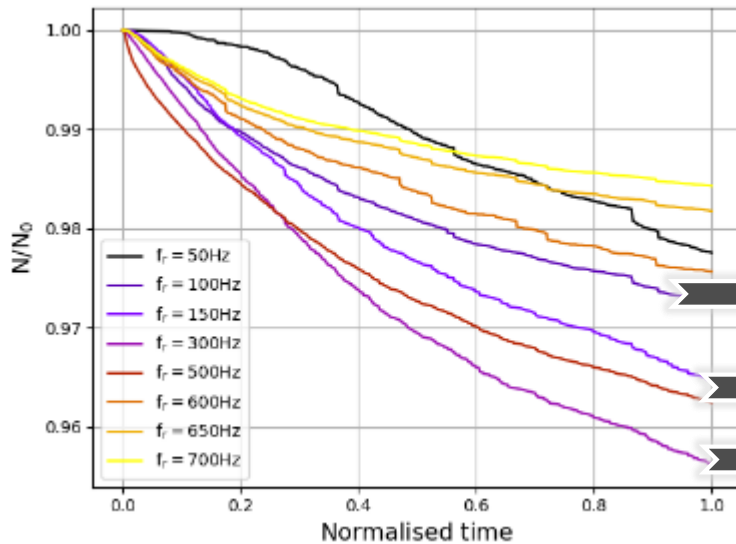


Weights in the distribution

- ❑ Each particle contributes its associated weight towards the bin count (instead of 1)
- ❑ $P(a < x < b) = P(x < b) - P(x < a)$ according to the initial position in configuration space
- ❑ From Uniform to Gaussian: Particles at the core are more important than the ones in the tails



Weights in the distribution

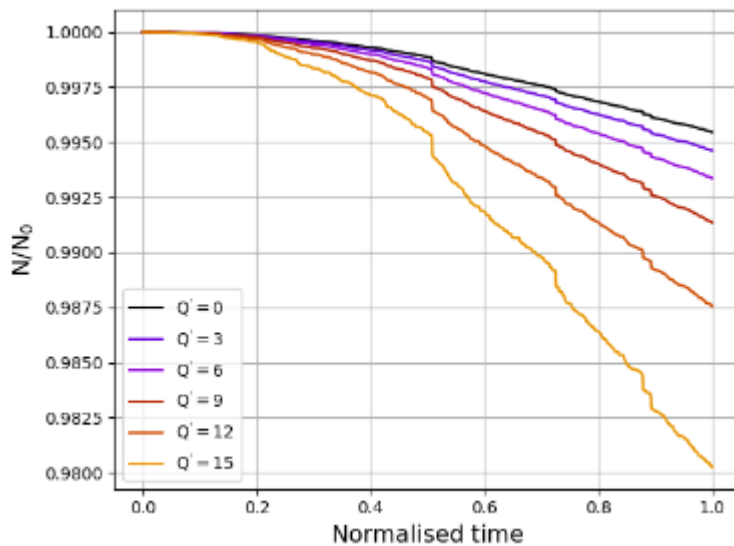


5D, power supply ripple, frequency scan

1st sideband of the diagonal

1st sideband of the 3rd order

2nd sideband of the 3rd order



6D, chromaticity scan

Conclusions & future steps

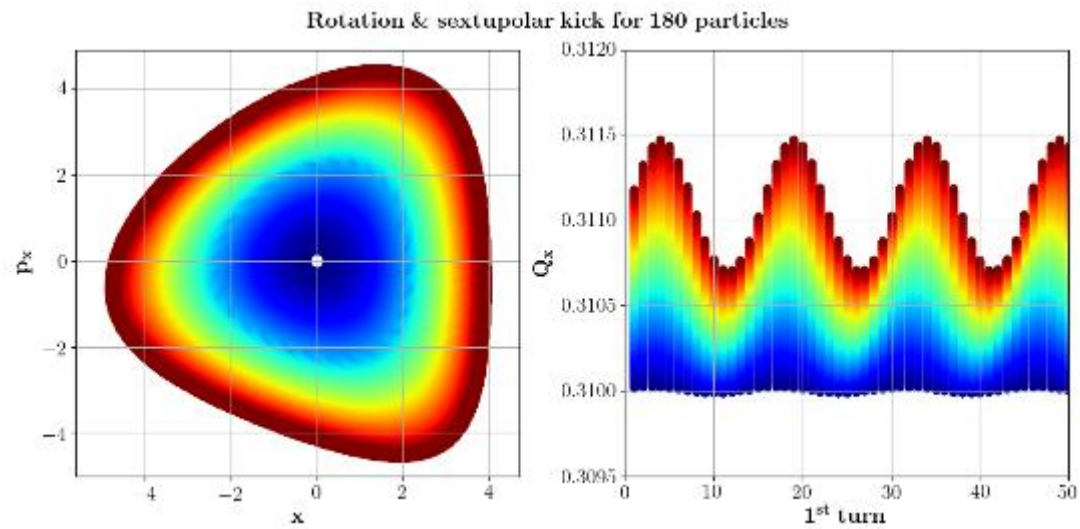
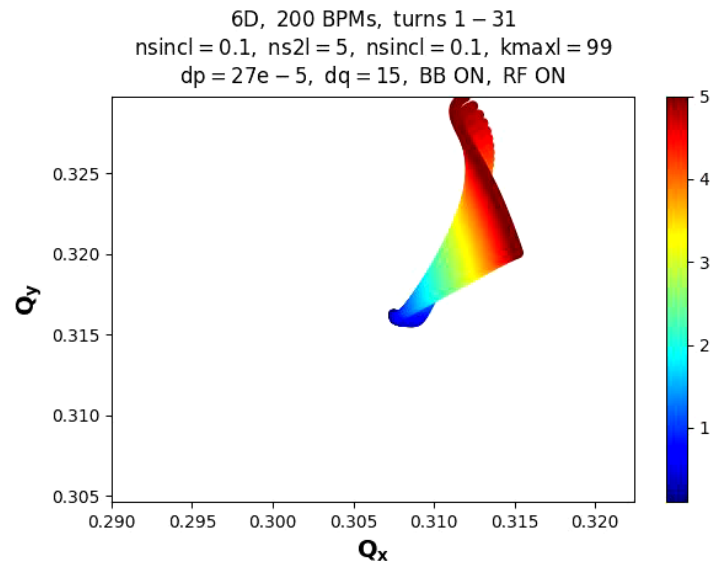
Conclusions

- ❑ The additional resonance lines from power supply ripples and chromatic tune modulation have been identified with FMAs.
- ❑ We are able to show the coherent motion of the 6D footprint in the presence of chromaticity with the multiple-BPM instantaneous tune determination method.
- ❑ We showed the impact of different amplitudes and frequencies of the modulation from power supply ripples, from synchrotron motion and from the combination of both effects

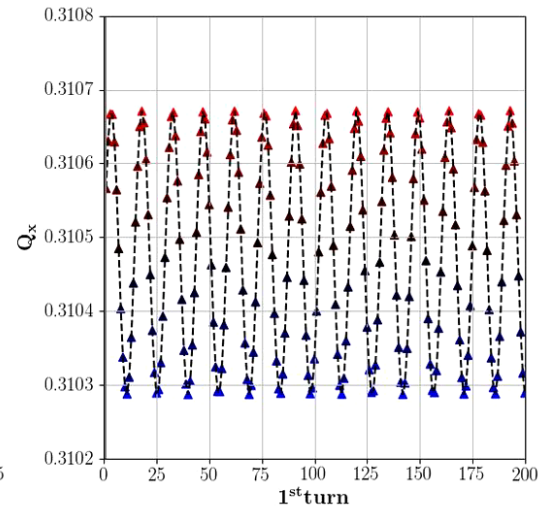
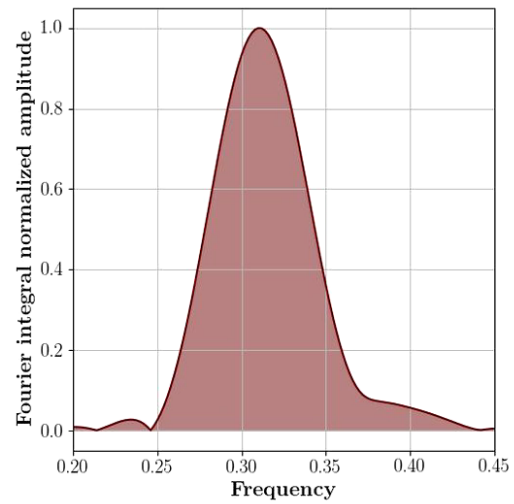
Future steps

- ❑ Investigate what is the impact of 6D BB in the modulation of the synchrotron motion
- ❑ Simulations with distributions, in order to identify the impact of these effects in emittance growth, transportation of particles in the tails of the distribution and losses.
- ❑ Real LHC spectrum from power supply ripples.

Backup

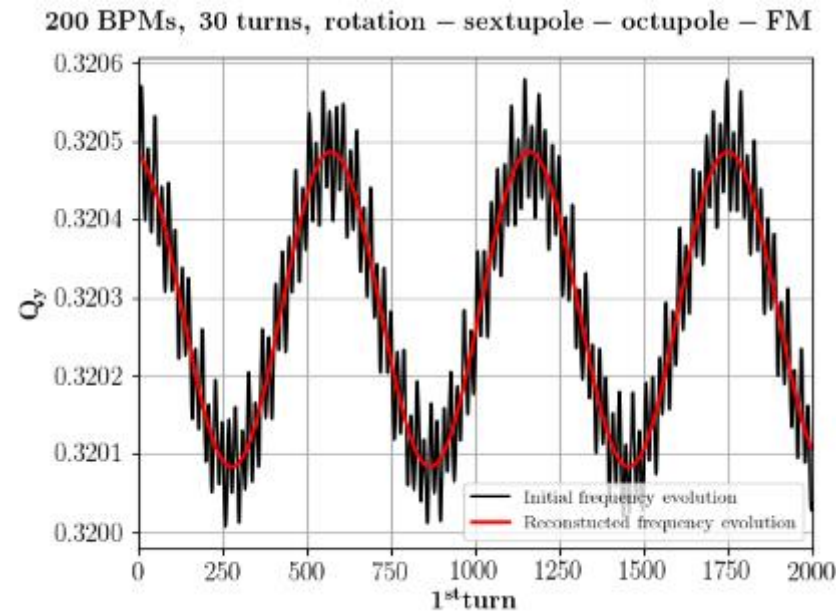


Backup



Simplified model: NAFF - NAFF

- With a sliding window of 30 turns the frequency components can be identified
- By frequency analyzing the NAFF results the three components can be identified and separated:
 - 1) **DC component**, the un-modulated betatron tune with **A0**
 - 2) **fs**, the 1st sideband of the betatron tune with **A1** amplitude and **φ1** phase
 - 3) **fN**, the frequency from the non-linear elements



$$Q_{\text{reconstructed}} = \sum_{t=1}^{t=L_{\text{SW}}} \mathbf{A}_0 + \mathbf{A}_1 e^{i(2\pi f_1 t + \varphi_1)}$$

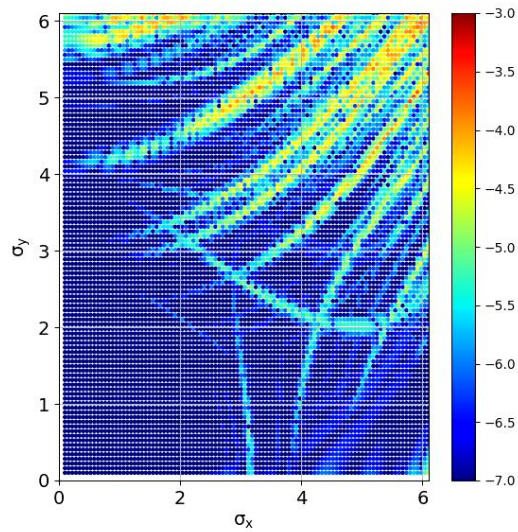
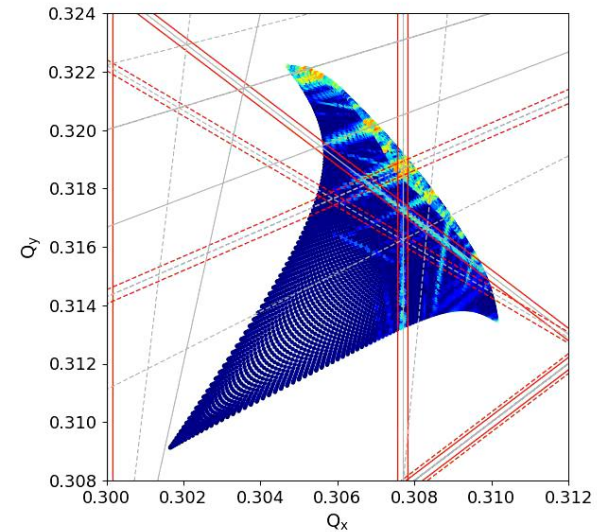
Simplified model with FM & 6D Beam-beam

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6D BB, crossing angle scan

Linear map, sextupole, octupole, 6D BB, VRF ON, 2000 turns, 4900 particles
E=6.5 TeV, Npart = 2e11, beta_s=40cm, sigma_z=0.075, phi=50.0 μrad



Sixtrack LHC

