Power supply ripples & 6D Frequency Map Analysis

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Introduction

- Previous studies: DA studies with power supply ripples for the HL-LHC triplet showed a sensitivity at 300 & 600 Hz *

- The combination of non-linear resonances and modulation effects: degradation of dynamic aperture and beam lifetime.

- Transverse tune modulation: additional resonance sidebands, which can reach the footprint and cause particle diffusion.

- **Frequency Map Analysis** in the presence of:
  I. Power supply ripples
  II. Synchrotron motion

* "BEAM DYNAMICS REQUIREMENTS FOR THE POWERING SCHEME OF THE HL-LHC TRIPLET", M. Fitterer, R. De Maria, S. Fartoukh and M. Giovannozzi
Modulation of the betatron tunes (I)

- **Power supply ripples:**
  - Modulation in the *current of a quadrupole’s power supply*
  - Change in magnetic field
  - Modulation of the *normalized quadrupolar strength*
Modulation of the betatron tunes (I)

- **Power supply ripples:**

  - Modulation in the current of a quadrupole’s power supply
  - Change in magnetic field
  - Modulation of the normalized quadrupolar strength

### Instantaneous tune

\[
\Delta Q = \frac{1}{4\pi} \int \beta(s) \Delta k(s) \, ds
\]

\[
Q_{\text{inst}} = Q_0 + \frac{1}{4\pi} \beta K_{\text{depth}} \sin(2\pi Q_p n)
\]

\[
x(n) = x_0 \cos(2\pi Q_0 n + \frac{\beta K_{\text{depth}}}{4\pi Q_p} \sin(2\pi Q_p n))
\]

- **Modulation depth**
- **Modulation index**
Modulation of the betatron tunes (II)

- **Modulation from synchrotron motion:**
  - Chromaticity, synchrotron tune & $\Delta p/p$ define the value of the modulation index.
  - Large modulation index production of **higher order harmonics**.

\[
Q_{\text{inst}} = Q_0 + \frac{Q'_x \Delta p}{p} \sin(2\pi Q_s n)
\]

Transverse motion - Linear approximation

\[
x(n) = x_0 \cos \left( 2\pi Q_0 n + \frac{Q'_x \Delta p_{\text{max}}}{Q_s \frac{p}{p}} \sin(2\pi Q_s n) \right)
\]
Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
- **Fourier integral:**

\[
|\varphi(\omega)| = | < x(n), e^{-i\omega n} > | = \left| \int e^{-i2\pi f n} \cdot x_h(n) \, dn \right|
\]
Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
- Fourier integral:

\[ |\phi(\omega)| = |\langle x(n), e^{-i\omega n} \rangle| = |\int e^{-i2\pi fn} \cdot x_h(n) \, dn| \]

- How easy is it to compute the tune of a modulated signal?
Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
- **Fourier integral:**
  \[
  |\varphi(\omega)| = |< x(n), e^{-i\omega n} >| = |\int e^{-i2\pi fn} \cdot x_h(n) \, dn |
  \]

- **Bessel functions of the first kind:**
  \[
  x(n) = x_0 \cos(2\pi Q_x n + \beta \sin(2\pi Q_m n)) = x_0 \sum_{m=-\infty}^{\infty} J_m(\beta) \cos(2\pi (Q_x + mQ_m)n)
  \]
Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
- Fourier integral:

\[ |\varphi(\omega)| = |< x(n), e^{-i\omega n} >| = |\int e^{-i2\pi fn} \cdot x_h(n) \, dn | \]

- Bessel functions of the first kind:

\[ \beta > 1.5 \]

Amplitude of the Bessel function of the m order

Modulation index
Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
- Fourier integral:

\[ |\varphi(\omega)| = | < x(n), e^{-i\omega n} > | = | \int e^{-i2\pi fn} \cdot x_h(n) \, dn | \]

- Bessel functions of the first kind:

Number of turns 1024, \( A_5 = 0.0005 \), \( T_s = 100 \), \( \beta = 0.05 \)

- Modulation frequency is constant
- Increase of modulation depth
- **Modulation index increases**
- Red line: analytical signal
- Black line: signal from tracking
Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
- Fourier integral:
  \[ |\varphi(\omega)| = | \langle x(n), e^{-i\omega n} \rangle | = | \int e^{-i2\pi fn} \cdot x_h(n) \, dn | \]

- Bessel functions of the first kind:
  - Modulation frequency decreases
  - Modulation depth is constant
  - Modulation index increases

Number of turns 1024, \( A_s = 0.0075 \), \( T_s = 45 \), \( \beta = 0.3375 \)

Diagram showing Qx and Qm with normalized amplitude (Anorm) vs. Qx.
LHC: Application of correction with Bessel functions

\[ \beta = 1.32 \]

**Tools**
- Sixtrack
- NAFF

**Parameters**
- 6D footprint
- 19 angles
- 0.1-6.1 \( \sigma \)
- \( \delta p/p = 15e-5 \)
- \( Q' = 15 \)
- \( \beta^* = 40 \text{ cm} \)
- \( (Q_x, Q_y) = (62.31, 60.32) \)
- \( \text{xing} = 150 \mu \text{rad} \)
LHC: Application of correction with Bessel functions

**Parameters**
- 6D footprint
- 19 angles
- 0.1-6.1 $\sigma$
- $\delta p/p = 27e-5$
- $Q' = 15$
- $\beta^* = 40$ cm
- $(Q_x, Q_y) = (62.31, 60.32)$
- xing = 150 $\mu$rad

$\beta = 2.38$
- **6th order** resonance excited from octupole
- Introduction of modulated quadrupole

For a **small modulation depth** the 1st sideband of the 6th order resonance starts to appear

For a **larger modulation depth** more particles are trapped inside the 1st sideband

**Simplified model with FM: Phase space**

↑ Modulation depth, Modulation frequency = ct

Linear map & octupole, \((Qx, Qy) = (0.34, 0.32)\), \(Ar = 1e-06\), \(Ts = 20.0\) turns
- **6th order** resonance excited from octupole
- Introduction of modulated quadrupole

  - For a *small modulation depth* the 1st sideband of the 6th order resonance starts to appear
  - For a *larger modulation depth* more particles are trapped inside the 1st sideband
  - For a *smaller modulation frequency* sidebands approach the main resonance & more sidebands start to appear

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**Simplified model with FM: Phase space**

![Graphs](https://via.placeholder.com/150)

- Modulation depth, Modulation frequency = ct
- Modulation frequency, Modulation depth = ct
Simplified model with FM: Phase space

\[ \uparrow \text{Modulation depth,} \quad \text{Modulation frequency } = ct \]

\[ \downarrow \text{Modulation frequency,} \quad \text{Modulation depth } = ct \]

Distance between 1\textsuperscript{st} sideband and main resonance is \( \frac{2 \pm Q_s}{6} \)
Simplified model with FM & 4D Beam-beam

- Resonance conditions:

\[ aQ_x + bQ_y + cQ_m = k \text{ for } a, b, c, k \text{ integers} \]

4D BB, modulation depth scan from \( \Delta Q=1e-6 \) to \( \Delta Q=5e-4 \)

Linear map, sextupole, octupole, 4D BB, modulating quadrupole, VRF OFF, 2000 turns, 1000 particles
E=6.5 TeV, Npart = 2e11, beta_s=1, min_sigma_diff=1e-16, Ts=100.0 turns, DQ=1e-06
Simplified model with FM & 4D Beam-beam

- Resonance conditions:

\[ aQ_x + bQ_y + cQ_m = k \text{ for } a, b, c, k \text{ integers} \]

4D BB, modulation frequency scan from \( Q_m = 0.2 \) to \( Q_m = 0.05 \)

Linear map, sextupole, octupole, 4D BB, modulating quadrupole, VRF OFF, 2000 turns, 4900 particles

\( E = 6.5 \text{ TeV}, N_{\text{part}} = 2 \times 10^4, \beta_s = 40 \text{ cm}, T_s = 5 \text{ turns} \)

1\textsuperscript{st} sideband of 3\textsuperscript{rd} order resonance
LHC: Power supply ripples

- Quadrupoles of the **inner triplet** right and left of IP1 and IP5, large beta-functions increase the sensitivity to non-linear effects

- Resonance conditions:

\[ aQ_x + bQ_y + c \frac{f_{\text{modulation}}}{f_{\text{revolution}}} = k \text{ for } a, b, c, k \text{ integers} \]
LHC: Power supply ripples

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LHC: Power supply ripples

- Quadrupoles of the **inner triplet right and left of IP1 and IP5, large beta-functions** increase the sensitivity to non-linear effects.

- **Resonance conditions:**
  
  \[ aQ_x + bQ_y + c \frac{f_{\text{modulation}}}{f_{\text{revolution}}} = k \text{ for } a, b, c, k \text{ integers} \]

- By increasing the modulation depth, sidebands start to appear in the footprints.

\[ \Delta Q = 1 \times 10^{-4} \]
LHC: Power supply ripples

- Scan of different ripple frequencies (50-900 Hz)

5D, E = 6.5 TeV, \( I_{\text{coll}} = 510 \text{A} \), Beam – beam ON, \( \varepsilon_n = 2.5 \mu\text{m} \), \( \beta^* = 40 \text{cm} \), \( q = 15 \)

\((Q_x, Q_y) = (62.31, 60.32)\), \( V_{RF} \) OFF, \( \delta p = 27 \times 10^{-5} \), 49 angles, 0.1 – 6.1 \( \sigma \), sliding NAFF

\( f_r = 50.0 \text{Hz, } A_r = 10^{-7} \) at MQXA.1, MQXA.3, MQXB.A2, MQXB.B2 of IP1, IP5
6D studies

- **Goal**
  - Full dynamics of the beam **along with synchrotron motion**

- **Difference**
  - Coherent motion of the footprint due to synchro-betatron coupling

- **Tools**
  1. Sixtrack for single particle tracking
  2. NAFF

- **Methods**
  - **Long term tracking**: average picture of this motion in frequency space
  - **Instantaneous picture** in the frequency domain → small number of turns & high sampling rate (accuracy)

- **Purpose of the study**
  - Frequency maps with a **30-turn window length** and reasonable number of BPMs
  - **Long term frequency maps** with 1BPM per turn & power supply ripples
Setup & parameters

### Parameters

- \( E = 6.5 \text{ TeV} \)
- \( 6D \text{ Beam beam} \)
- \( Q_s = 0.0017 \)
- \( I_{\text{oct}} = 510 \text{ A} \)
- \( \varepsilon_N = 2.5 \mu\text{m} \)
- \( \delta = 27\times10^{-5} \)
- \( Q_x = 62.31 \)
- \( Q_y = 60.32 \)
- \( dq = 15 \)

Setup for 6D FMAs

**Multiple BPM analysis**

- 200 virtual BPMs
- Adjusted phase between BPMs around the ring to have cleaner spectra (discrepancies for particles at different amplitudes and energies)
- Tracking data are collected "turn wise":
  \[ x = x_{\text{BPM1,turn1}}, x_{\text{BPM2,turn1}}, \ldots, x_{\text{BPM200,turn1}}, \ldots, x_{\text{BPM200,turn30}} \]
Multiple BPM analysis

- Window length of **30 turns**

- Before building the instantaneous footprints, the leakage from the resonance driving terms (high frequency modulation) needs to be removed.
Multiple BPM analysis

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Multiple BPM analysis

- Window length of 30 turns
- Before building the instantaneous footprints, the leakage from the resonance driving terms (high frequency modulation) needs to be removed.
6D FMAs with power supply ripples

6D, $E = 6.5 \text{TeV}$, $I_{\text{sect}} = 510 \text{A}$. Beam-beam ON, $\epsilon_n = 2.5 \mu\text{m}$, $\beta^* = 40 \text{cm}$, $q = 0$

$(Q_x, Q_y) = (62.31, 60.32)$, $V_{\text{RF}}$ ON, $\delta p = 27 \times 10^{-5}$, 99 angles, $0.1 - 6.1 \sigma$, sliding NAFF

$f_c = 50 \text{Hz}$, $A_c = 10^{-7}$ at MQXA.1, MQXA.3, MQXB.A2, MQXB.B2 of IP1, IP5

\[
\sigma_x = \sqrt{\frac{\epsilon_{10}}{\epsilon_0}} \quad \epsilon_{0,N} = \epsilon_0 / \gamma = 2.50 \mu\text{m}
\]
Weights in the distribution

- Each particle contributes its associated weight towards the bin count (instead of 1)

- \[ P(a < x < b) = P(x < b) - P(x < a) \] according to the initial position in configuration space

- From Uniform to Gaussian: Particles at the core are more important than the ones in the tails
Weights in the distribution

5D, power supply ripple, frequency scan

1\textsuperscript{st} sideband of the diagonal

1\textsuperscript{st} sideband of the 3\textsuperscript{rd} order

2\textsuperscript{nd} sideband of the 3\textsuperscript{rd} order

6D, chromaticity scan
Conclusions & future steps

Conclusions

- The additional resonance lines from power supply ripples and chromatic tune modulation have been identified with FMAs.
- We are able to show the coherent motion of the 6D footprint in the presence of chromaticity with the multiple-BPM instantaneous tune determination method.
- We showed the impact of different amplitudes and frequencies of the modulation from power supply ripples, from synchrotron motion and from the combination of both effects.

Future steps

- Investigate what is the impact of 6D BB in the modulation of the synchrotron motion.
- Simulations with distributions, in order to identify the impact of these effects in emittance growth, transportation of particles in the tails of the distribution and losses.
- Real LHC spectrum from power supply ripples.
Backup

6D, 200 BPMs, turns 1 – 31
nsincl = 0.1, ns2l = 5, nsincl = 0.1, kmaxl = 99
dp = 27e-5, dq = 15, BB ON, RF ON

Rotation & sextupolar kick for 180 particles
Simplified model: NAFF - NAFF

- With a sliding window of 30 turns the frequency components can be identified.
- By frequency analyzing the NAFF results the three components can be identified and separated:
  1) **DC component**, the un-modulated betatron tune with $A_0$
  2) $f_s$, the 1\textsuperscript{st} sideband of the betatron tune with $A_1$ amplitude and $\varphi_1$ phase
  3) $f_N$, the frequency from the non-linear elements

\[
Q_{\text{reconstructed}} = \sum_{t=1}^{t=L_{SW}} A_0 + A_1 e^{i(2\pi f_1 t + \varphi_1)}
\]
Simplified model with FM & 6D Beam-beam

- Resonance conditions:

\[ aQ_x + bQ_y + cQ_s = k \text{ for } a, b, c, k \text{ integers} \]

6D BB, crossing angle scan

- Linear map, sextupole, octupole, 6D BB, VRF ON, 2000 turns, 4900 particles
  - \( E = 6.5 \text{ TeV} \), \( N_{\text{part}} = 2 \times 10^4 \), beta_5 = 40 cm, sigma_m = 0.075, phi = 30.0 \mu\text{rad}

Sixtrack LHC

- Frequency plot with \( Q' = 0.0 \)