Power supply ripples & 6D Frequency Map Analysis

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Acknowledgments: H. Bartosik, R. De Maria, M. Fitterer, G. Iadarola, D. Pellegrini, N.Triantafyllou

Introduction

- Previous studies: DA studies with power supply ripples for the HL-LHC triplet showed a sensitivity at 300 & 600 Hz *.

- The combination of non-linear resonances and modulation effects: degradation of dynamic aperture and beam lifetime.

- Transverse tune modulation: **additional resonance sidebands**, which can reach the footprint and cause particle diffusion.

- **Frequency Map Analysis** in the presence of:
  I. **Power supply ripples**
  II. **Synchrotron motion**

* “BEAM DYNAMICS REQUIREMENTS FOR THE POWERING SCHEME OF THE HL-LHC TRIPLET”, M. Fitterer, R. De Maria, S. Fartoukh and M. Giovannozzi

Modulation of the betatron tunes (I)

- **Power supply ripples:**

Modulation in the **current of a quadrupole's power supply** → Change in magnetic field → Modulation of the **normalized quadrupolar strength**

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$$\Delta Q = \frac{1}{4\pi} \oint \beta(s) \Delta k(s) ds$$

Instantaneous tune

$$Q_{inst} = Q_0 + \frac{1}{4\pi} \overline{\beta K_{depth}} \sin(2\pi Q_p n)$$

Modulation depth

$$x(n) = x_0 \cos(2\pi Q_0 n + \underbrace{\frac{\overline{\beta K_{depth}}}{4\pi Q_p}}_{\text{Modulation index}} \sin(2\pi Q_p n))$$

Modulation of the betatron tunes (II)

- **<u>Modulation from synchrotron motion</u>:**
- Chromaticity, synchrotron tune & Δp/p define the value of the **modulation index**.
- Large modulation index $\Rightarrow$ production of **higher order harmonics**.

Instantaneous tune

$$Q_{inst} = Q_0 + \frac{Q'_x \Delta p}{p} \sin(2\pi Q_s n)$$

Transverse motion- Linear approximation

$$x(n) = x_0 \cos\left(2\pi Q_0 n + \underbrace{\frac{Q'_x}{Q_s} \cdot \frac{\Delta p_{max}}{p}}_{\textit{Modulation index}} \sin(2\pi Q_s n)\right)$$

Simplified model with FM – Frequency domain

- Linear map, octupolar kick, modulating quadrupole
- **Fourier integral:**

$$\boxed{|\varphi(\omega)| = |< x(n), e^{-i\omega n} >| = |\int e^{-i2\pi fn} \cdot \overline{x_h(n)}\, dn|}$$

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- **How easy is it to compute the tune of a modulated signal?**

Simplified model with FM – Frequency domain

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- **Fourier integral:**

$$|\varphi(\omega)| = |<\mathbf{x(n)}, e^{-i\omega n}>| = \left|\int e^{-i2\pi f n} \cdot \overline{\mathbf{x_h(n)}}\, dn\right|$$

- **Bessel functions of the first kind:**

$$\mathbf{x(n)} = x_0 \cos(2\pi Q_x n + \beta \sin(2\pi Q_m n)) = x_0 \sum_{m=-\infty}^{\infty} J_m(\beta)\cos(2\pi(Q_x + mQ_m)n)$$

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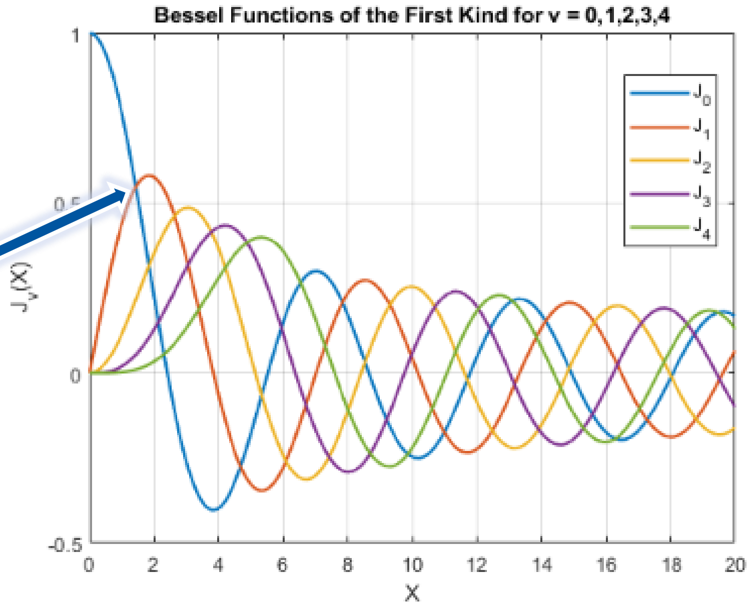
- **Bessel functions of the first kind:**

β > 1.5



Amplitude of the Bessel function of the m order

Modulation index

Simplified model with FM – Frequency domain

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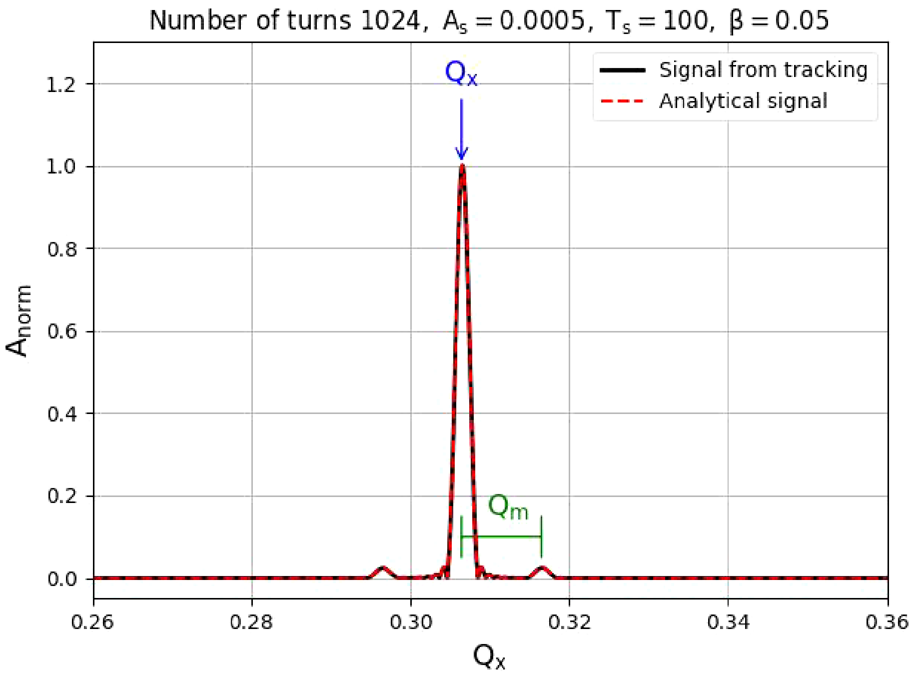

- **Modulation frequency is constant**
- **Increase of modulation depth**
- **<u>Modulation index increases</u>**

- **Red line: analytical signal**
- **Black line: signal from tracking**

Simplified model with FM – Frequency domain

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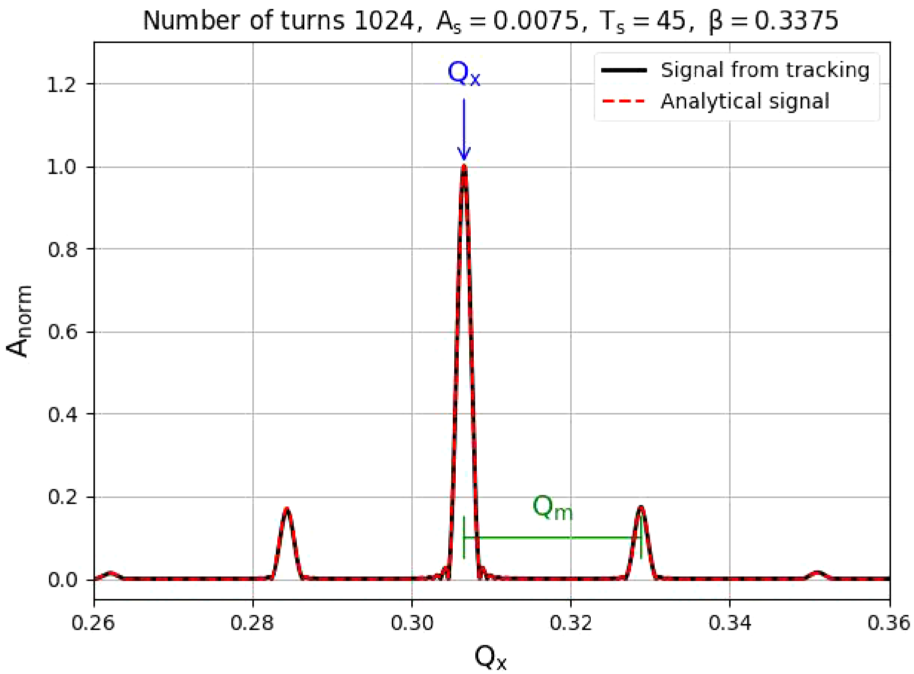
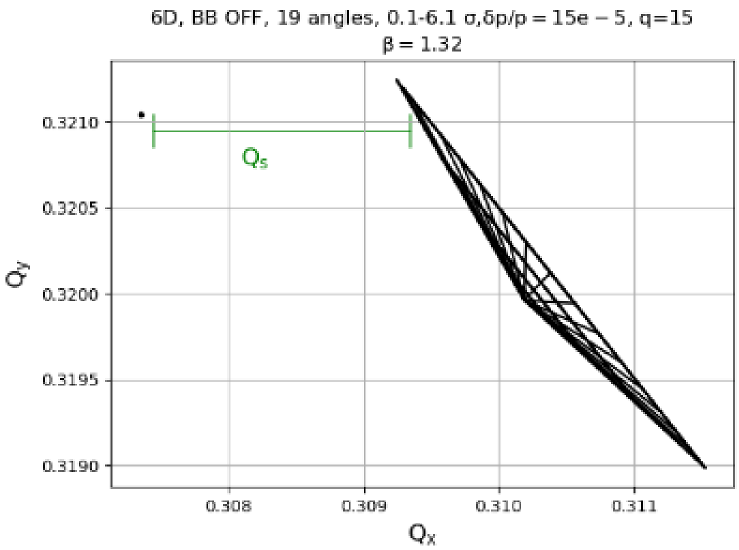


LHC: Application of correction with Bessel functions

β = 1.32



Tools

- Sixtrack
- NAFF

Parameters

- ❑ 6D footprint
- ❑ 19 angles
- ❑ 0.1-6.1 σ
- ❑ **δp/p = 15e-5**
- ❑ Q ‘ = 15
- ❑ β* = 40 cm
- ❑ (Qx, Qy) = (62.31,60.32)
- ❑ xing = 150 μrad

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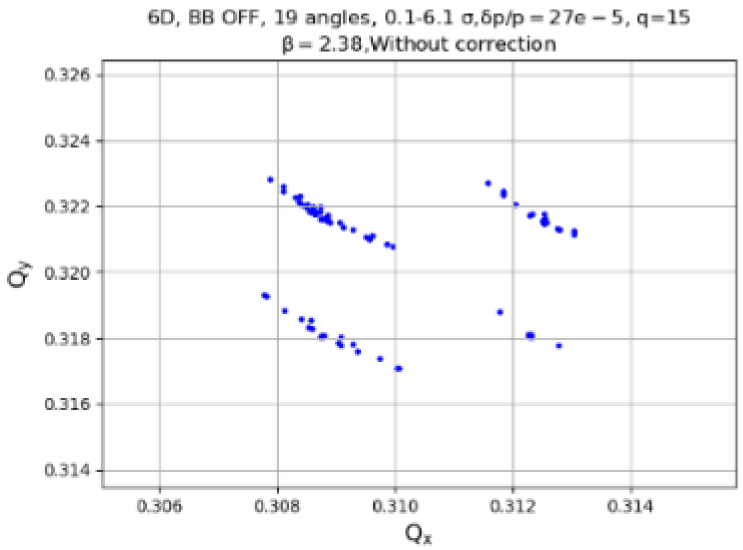
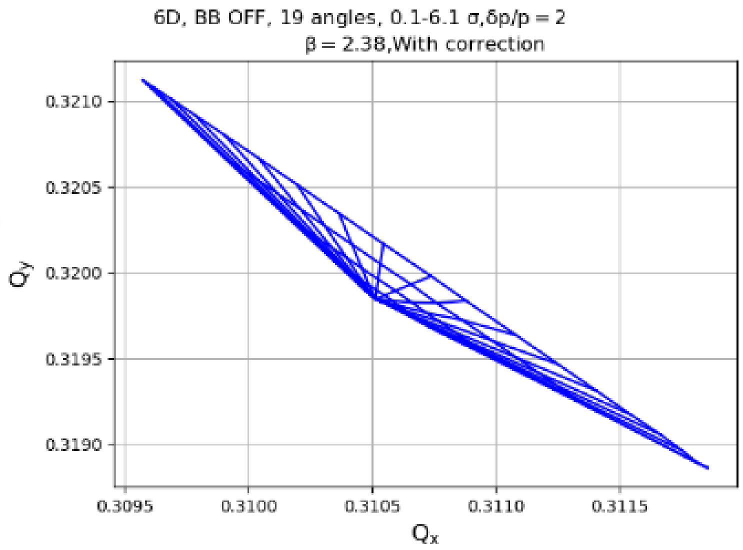
- ❑ xing = 150 μrad

β = 2.38



Corrected

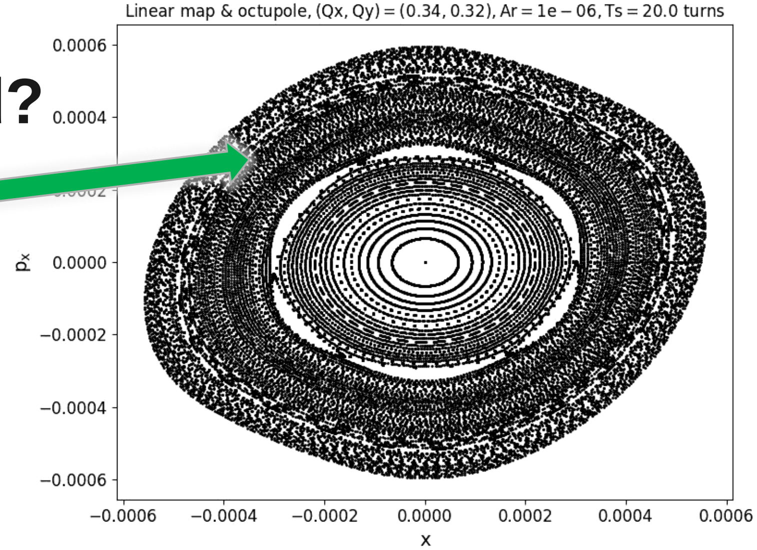


Simplified model with FM: Phase space

- **6th order** resonance excited from octupole
- Introduction of modulated quadrupole

Sideband?

↑ Modulation depth,
Modulation frequency = ct



- For a *<u>small modulation depth</u>* the **1st sideband** of the **6th order** resonance starts to appear
- For a *<u>larger modulation depth</u>* more particles are trapped inside the 1st sideband

Simplified model with FM: Phase space

- **6^{th} order** resonance excited from octupole
- Introduction of modulated quadrupole

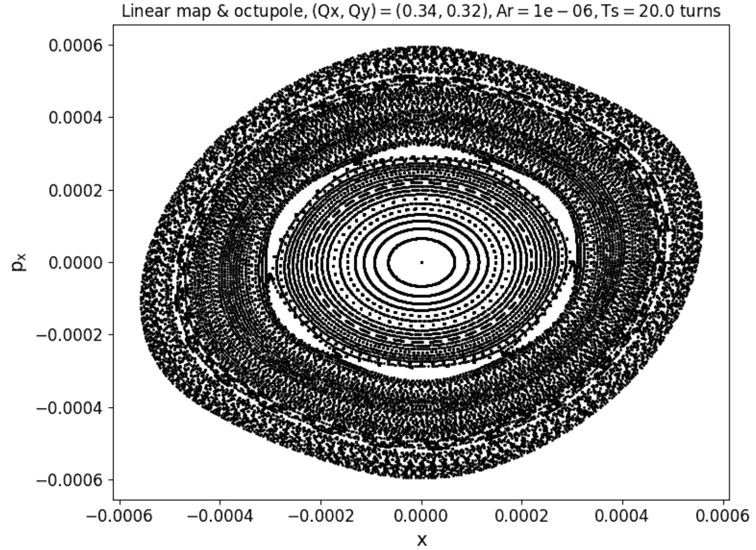
Sideband?

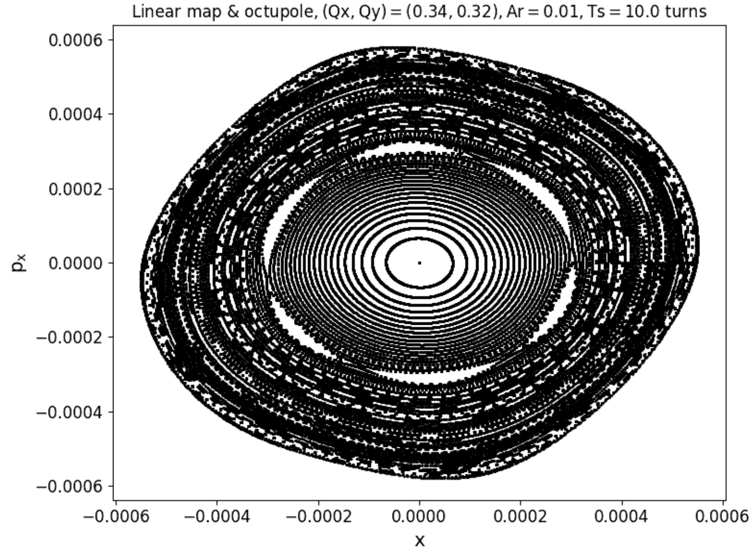
↑ **Modulation depth, Modulation frequency = ct**

- For a *small modulation depth* the **1^{st} sideband** of the **6^{th} order** resonance starts to appear
- For a *larger modulation depth* more particles are trapped inside the 1^{st} sideband

↓ **Modulation frequency, Modulation depth = ct**

- For a *smaller modulation frequency* sidebands approach the main resonance & more sidebands start to appear





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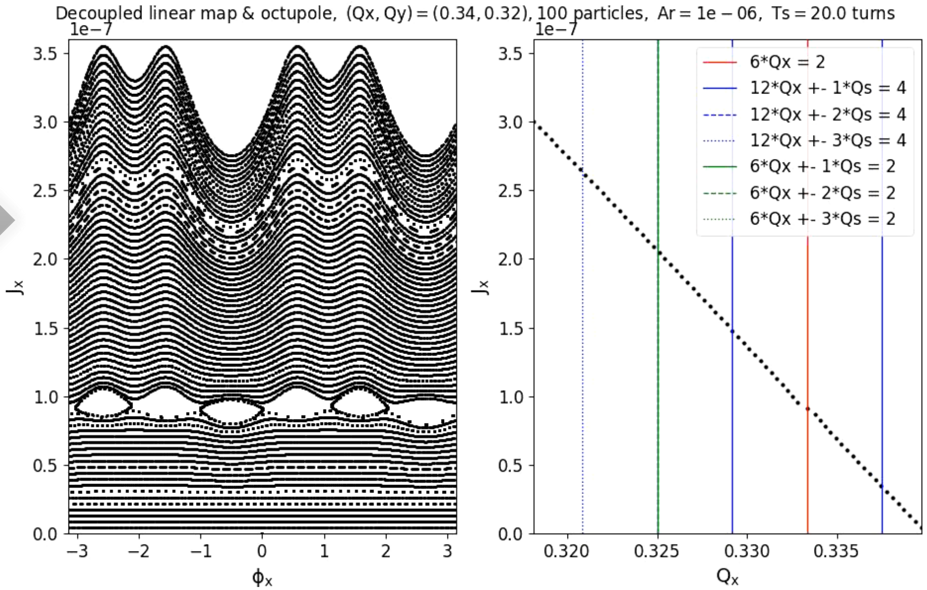

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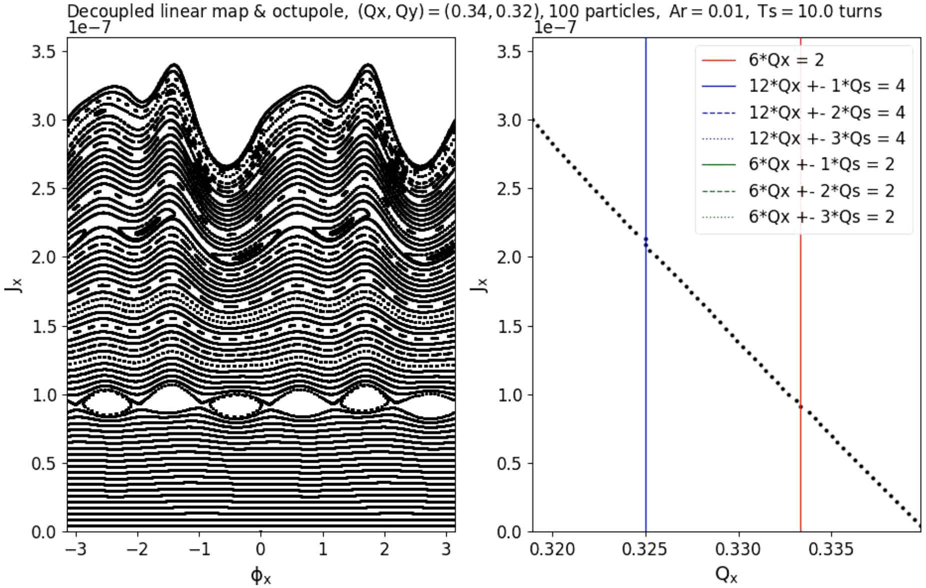


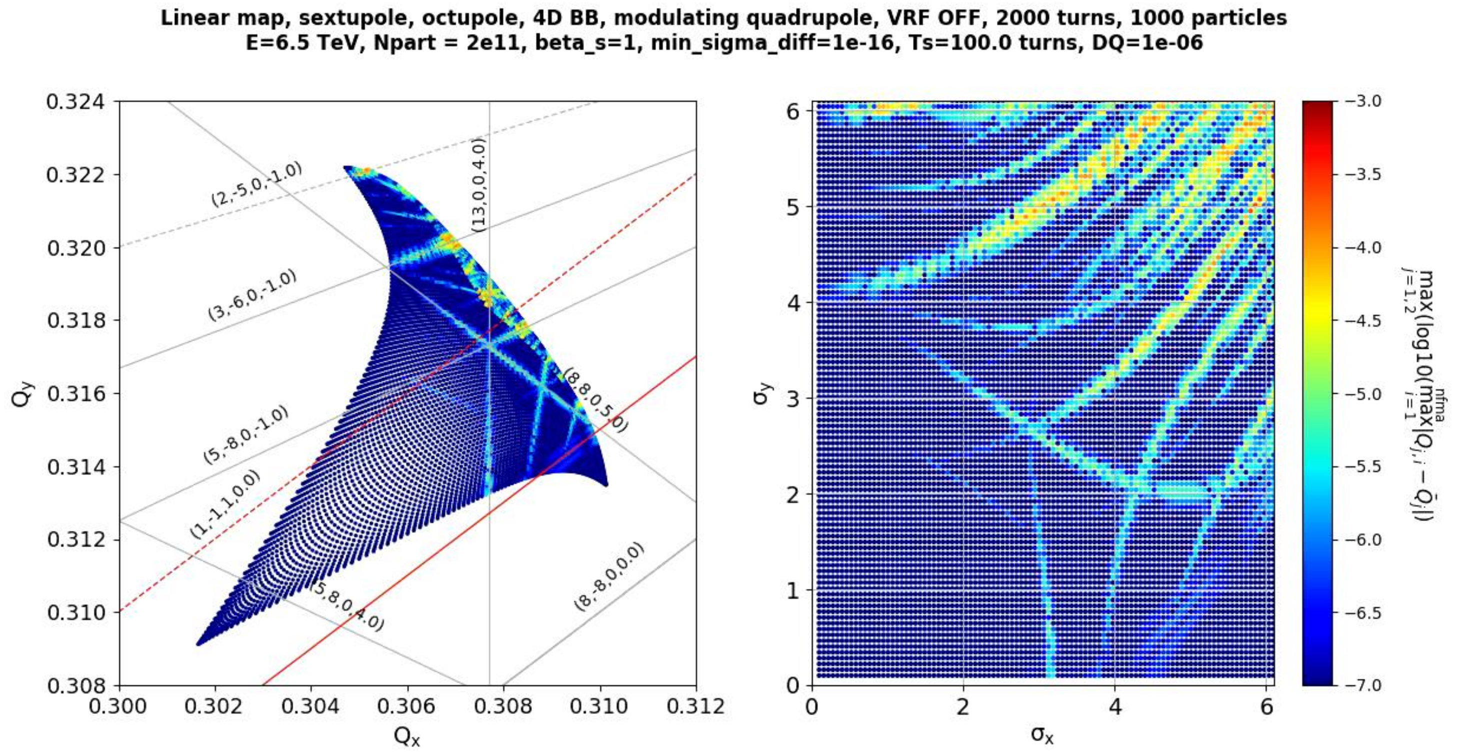
Distance between 1st sideband and main resonance is $\frac{2 \pm Q_s}{6}$

Simplified model with FM & 4D Beam-beam

- Resonance conditions:

$$\mathrm{aQ_x + bQ_y + cQ_m = k\ for\ a, b, c, k\ integers}$$

4D BB, modulation depth scan from ΔQ=1e-6 to ΔQ=5e-4

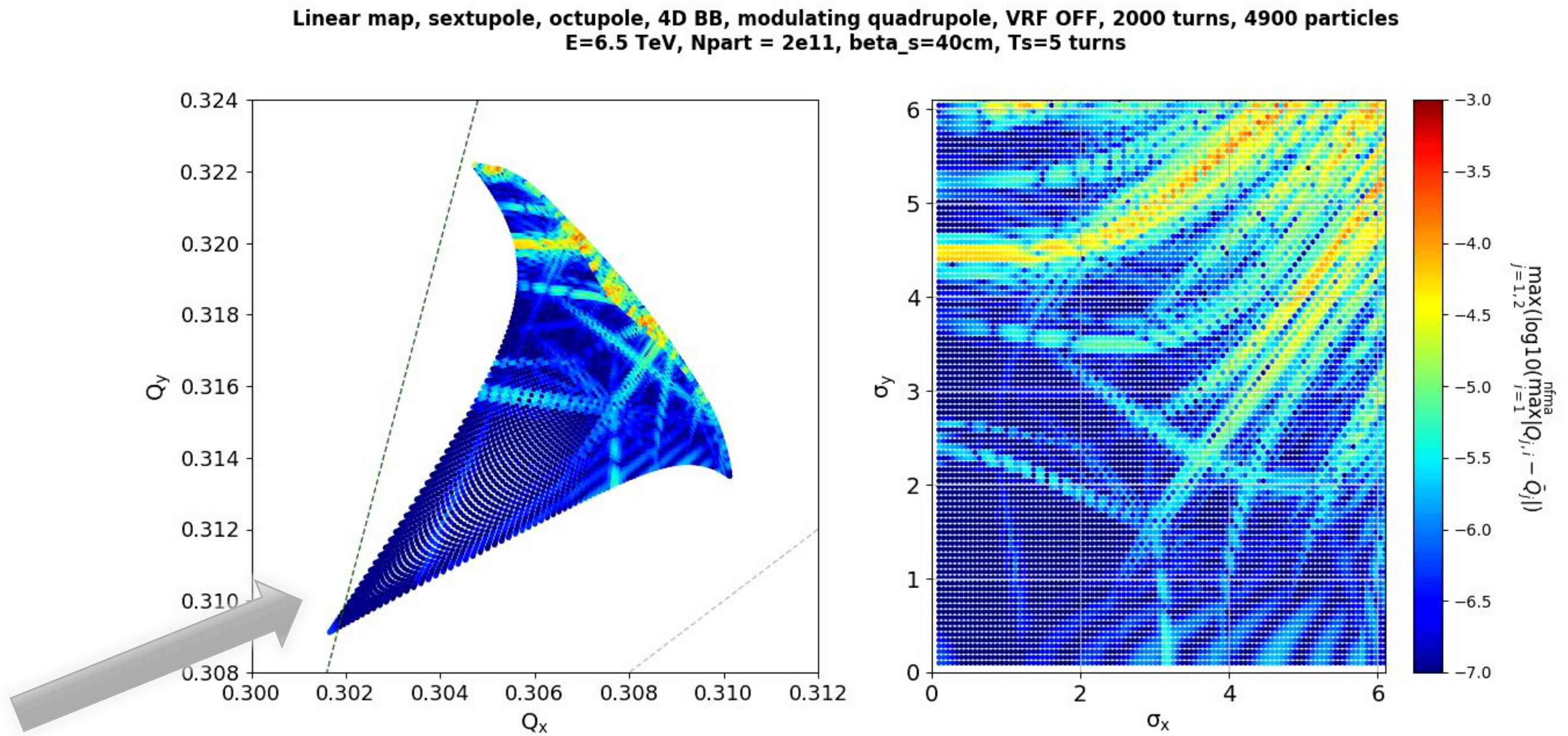


Simplified model with FM & 4D Beam-beam

- Resonance conditions:

$$aQ_x + bQ_y + cQ_m = k \text{ for } a, b, c, k \text{ integers}$$

4D BB, modulation frequency scan from Qm = 0.2 to Qm=0.05



1st sideband of 3rd order resonance

LHC: Power supply ripples

- Quadrupoles of the **inner triplet** right and left **of IP1 and IP5**, **large beta-functions** increase the sensitivity to non-linear effects
- **Resonance conditions**:

$$\mathrm{a}Q_x + \mathrm{b}Q_y + \mathrm{c}\frac{f_{\mathrm{modulation}}}{f_{\mathrm{revolution}}} = \mathrm{k} \text{ for a, b, c, k integers}$$

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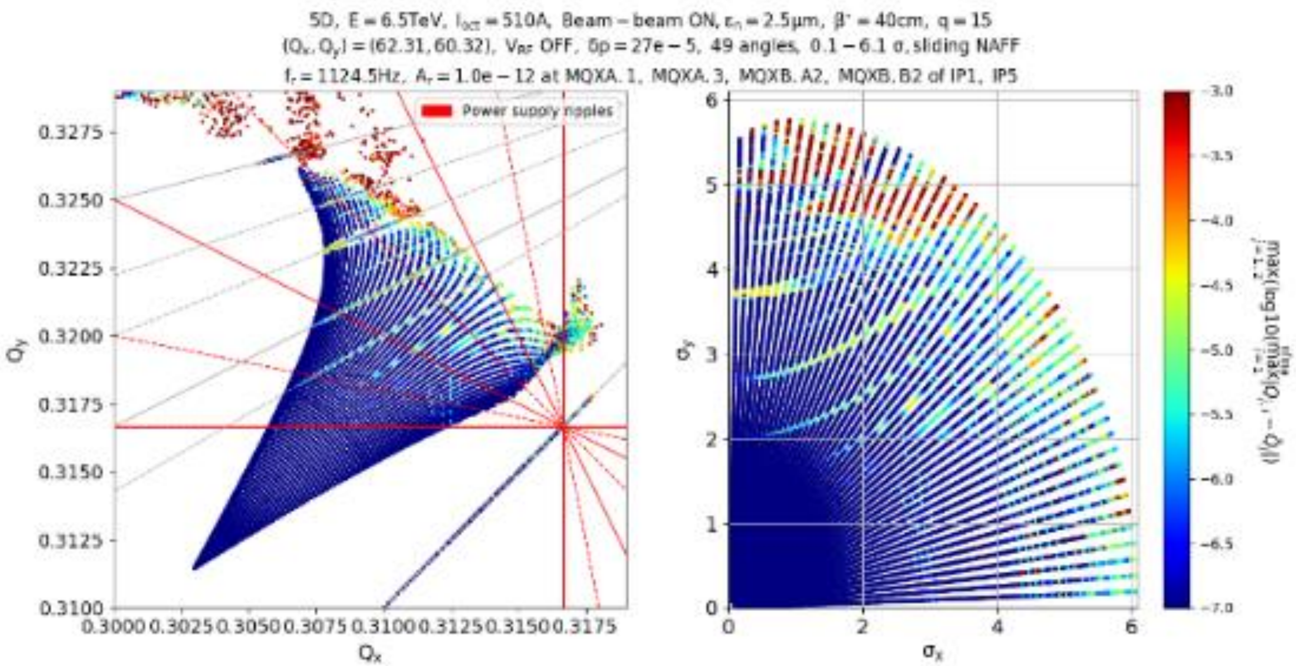

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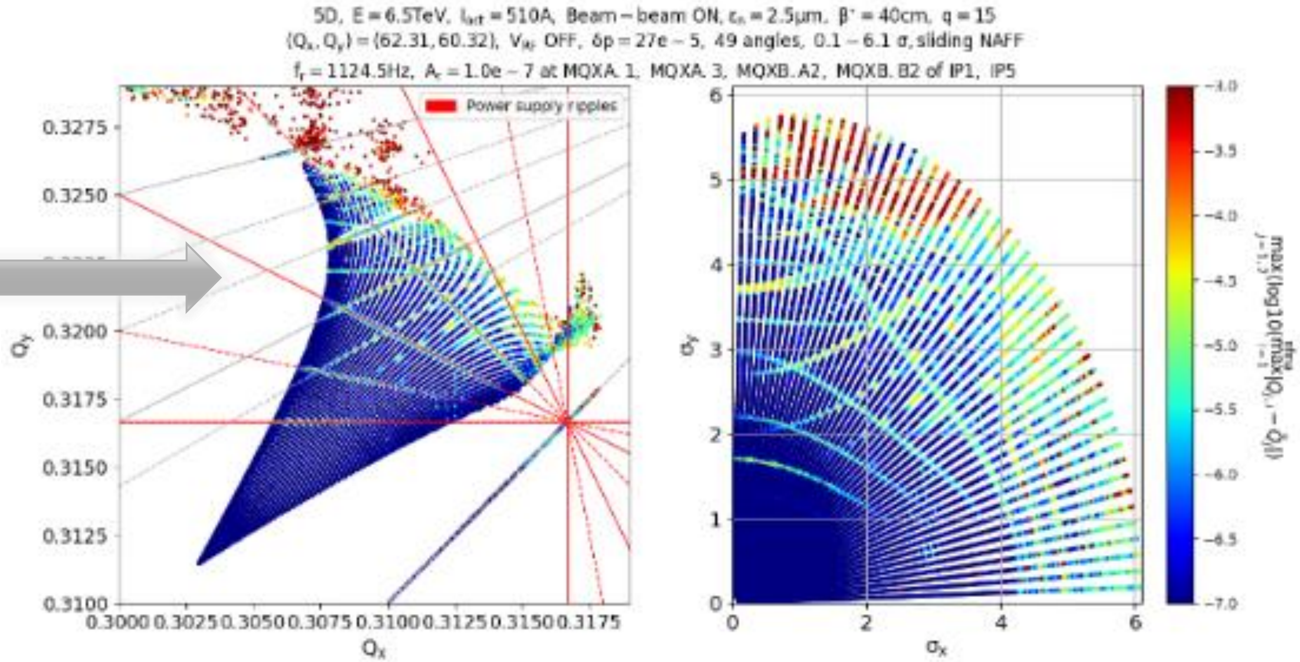
$$aQ_x + bQ_y + c\frac{f_{modulation}}{f_{revolution}} = k \text{ for } a, b, c, k \text{ integers}$$

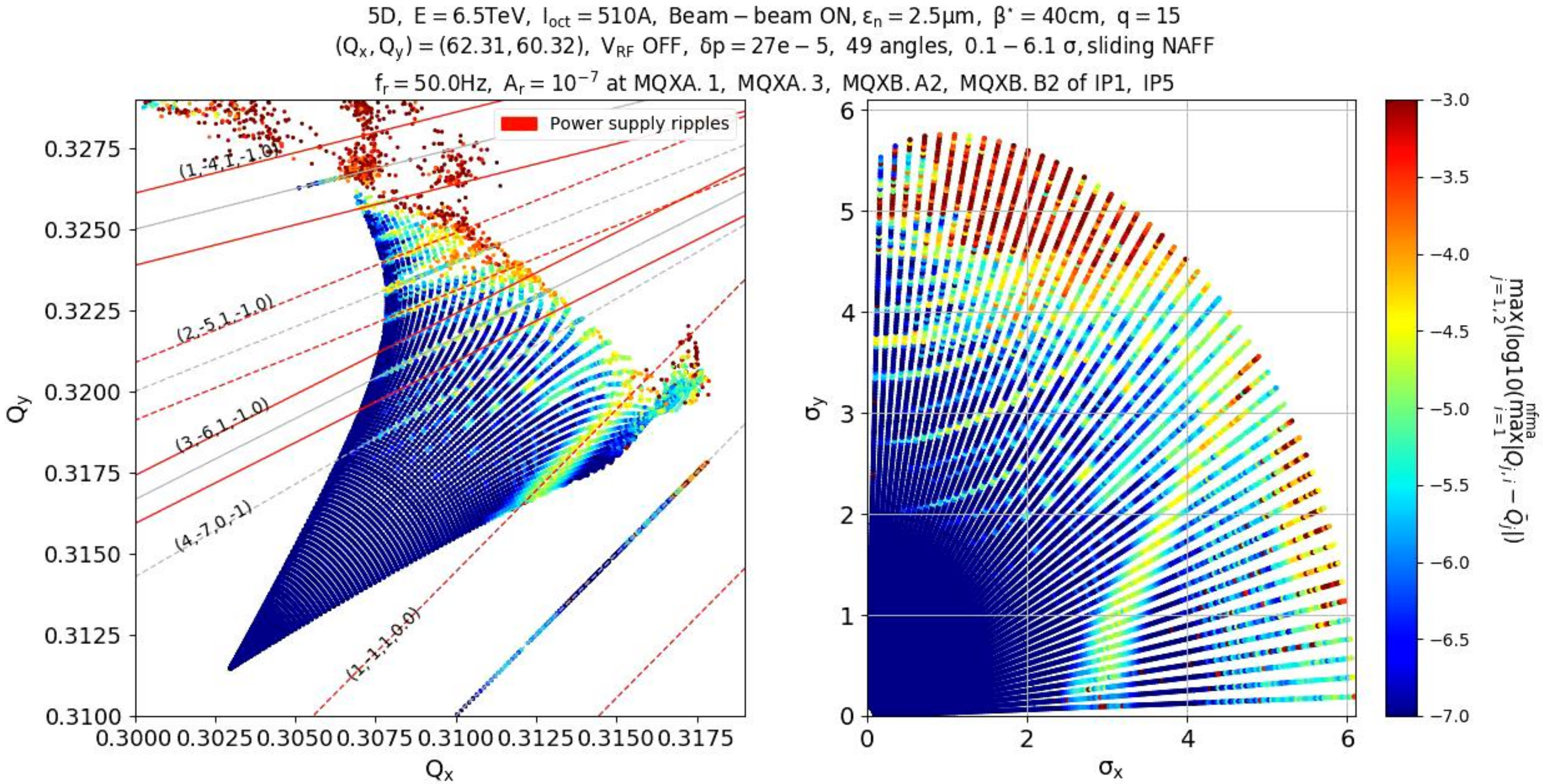
-**By increasing the modulation depth, sidebands start to appear in the footprints**

ΔQ=1e-4



LHC: Power supply ripples

- Scan of different ripple frequencies **(50-900 Hz)**



6D studies

Goal

Full dynamics of the beam **along with synchrotron motion**

Difference

Coherent motion of the footprint due to synchro-betatron coupling

Tools

I. Sixtrack for single particle tracking

II. NAFF

Methods

Long term tracking: average picture of this motion in frequency space

Instantaneous picture in the frequency domain→ small number of turns & high sampling rate (accuracy)

Purpose of the study

✓Frequency maps with a **30-turn window length** and reasonable number of BPMs

✓**Long term frequency maps** with 1BPM per turn & power supply ripples

Setup & parameters

Parameters

- E = 6.5 TeV
- **6D Beam beam**
- $Q_s = 0.0017$
- $\mathbf{I_{oct} = 510\ A}$
- $\varepsilon_N = 2.5\ \mu m$
- **δ = 27e-5**
- $\mathbf{Q_x = 62.31}$
- $\mathbf{Q_y = 60.32}$
- **dq = 15**

Setup for 6D FMAs

1 BPM analysis

at IP3

Multiple BPM analysis

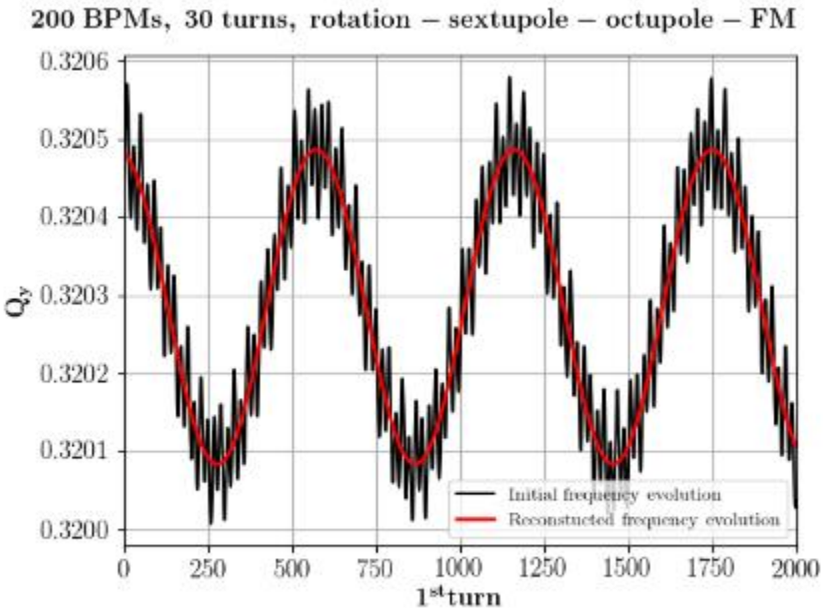
- **200 virtual** BPMs
- Adjusted phase between BPMs around the ring to have cleaner spectra (discrepancies for particles at different amplitudes and energies)
- Tracking data are collected **"turn wise":**

$$\mathbf{x} = x_{\mathrm{BPM1,turn1}}\, , x_{\mathrm{BPM2},turn1}\, , \ldots , x_{\mathrm{BPM200},turn1}\, , \ldots , x_{\mathrm{BPM200},turn30}$$

Multiple BPM analysis

- Window length of **30 turns**

- Before building the instantaneous footprints, the leakage from the resonance driving terms (high frequency modulation) needs to be removed.



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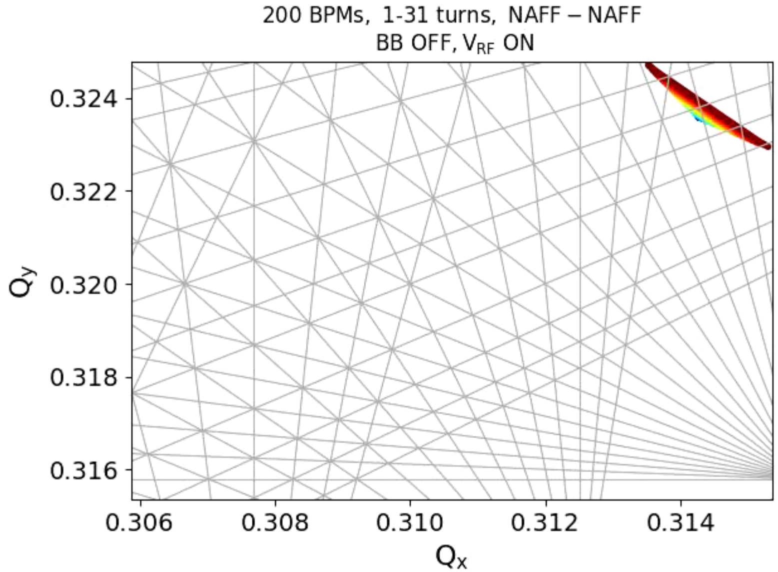

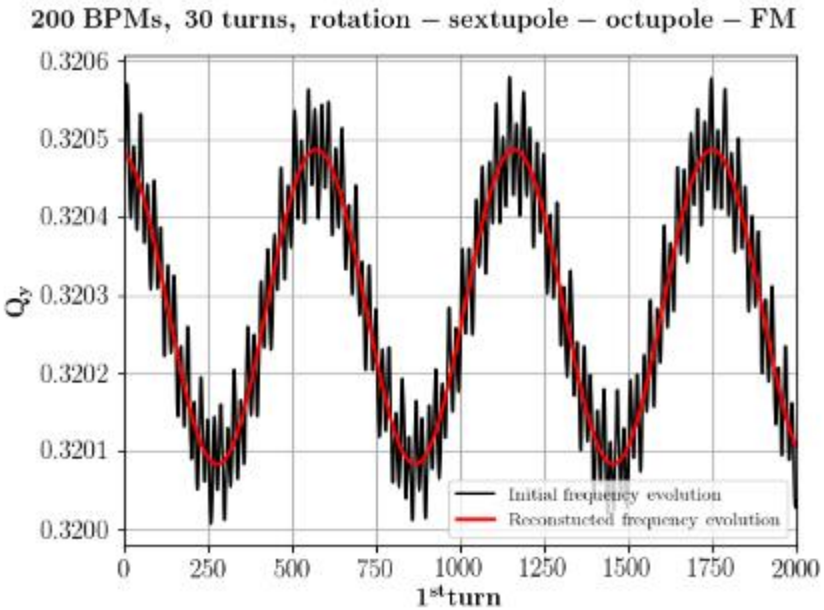

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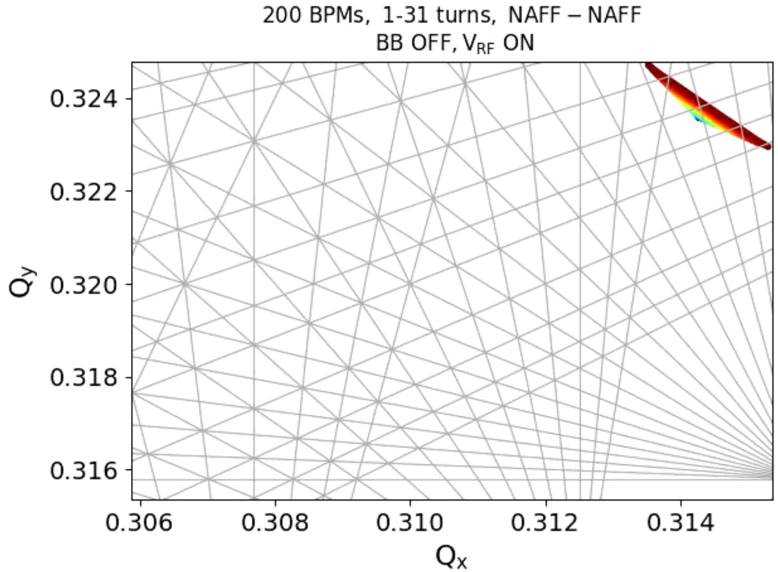
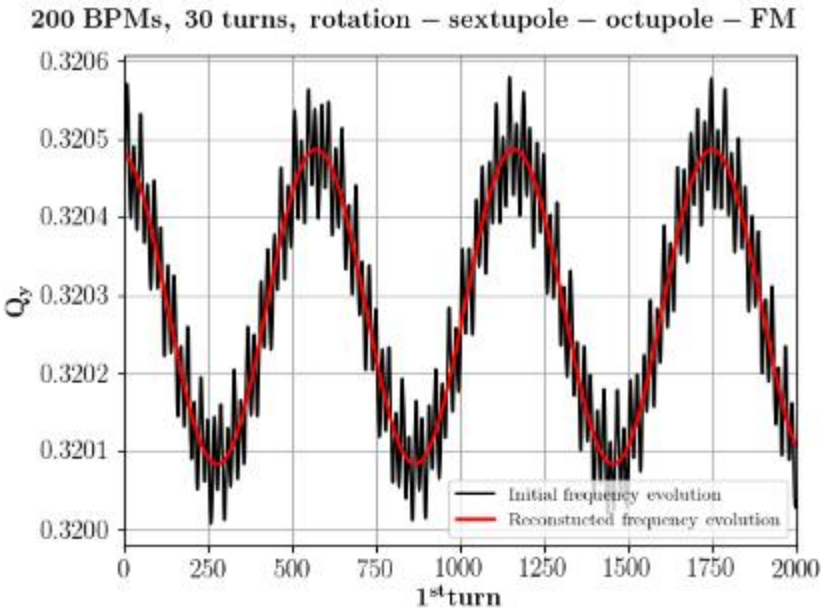
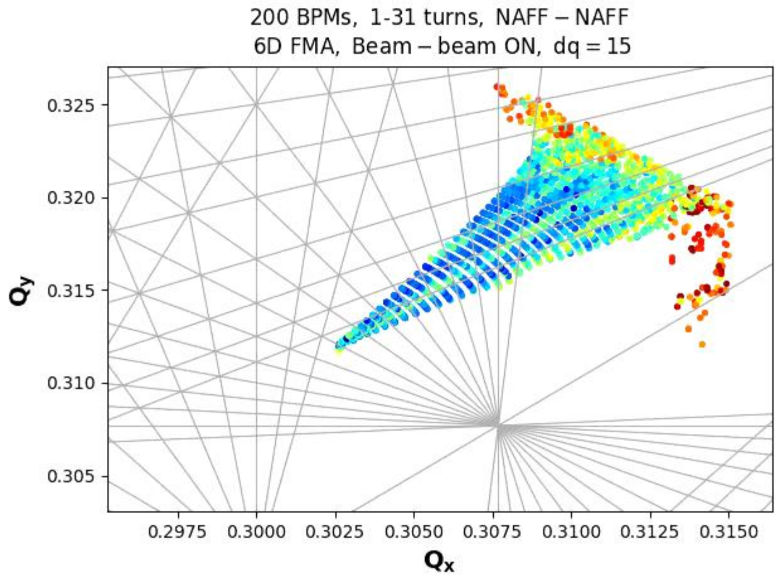
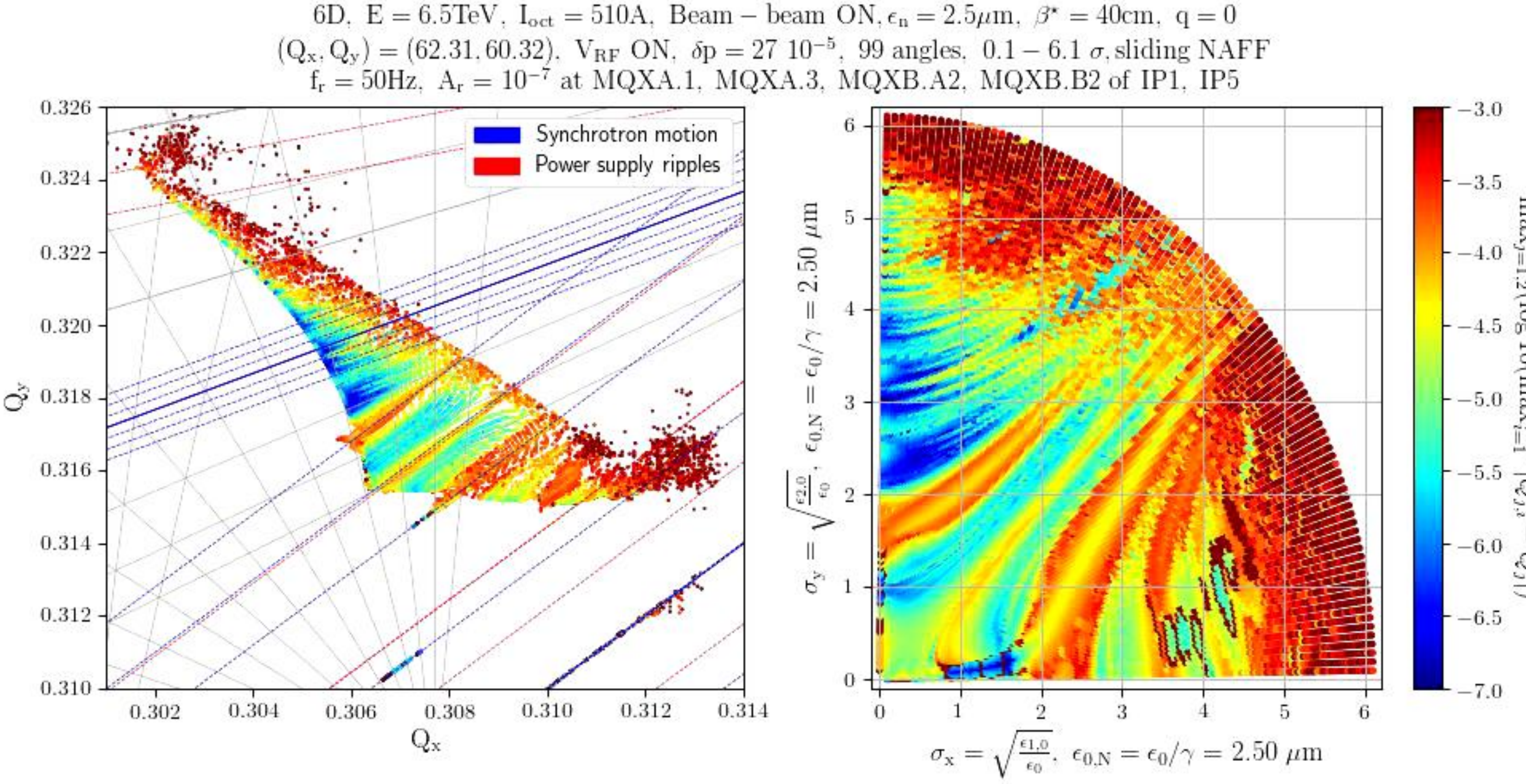






6D FMAs with power supply ripples

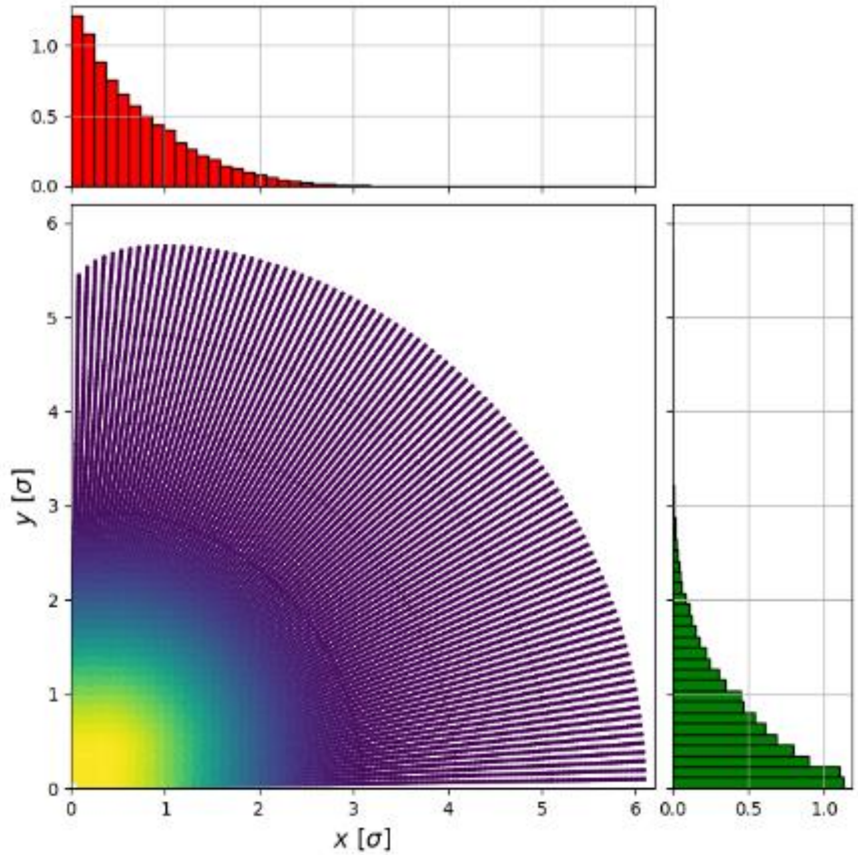


Weights in the distribution

- Each particle contributes its associated weight towards the bin count (instead of 1)

- $P(a < x < b) = P(x < b) - P(x < a)$ according to the initial position in configuration space

- From Uniform to Gaussian: Particles at the core are more important than the ones in the tails



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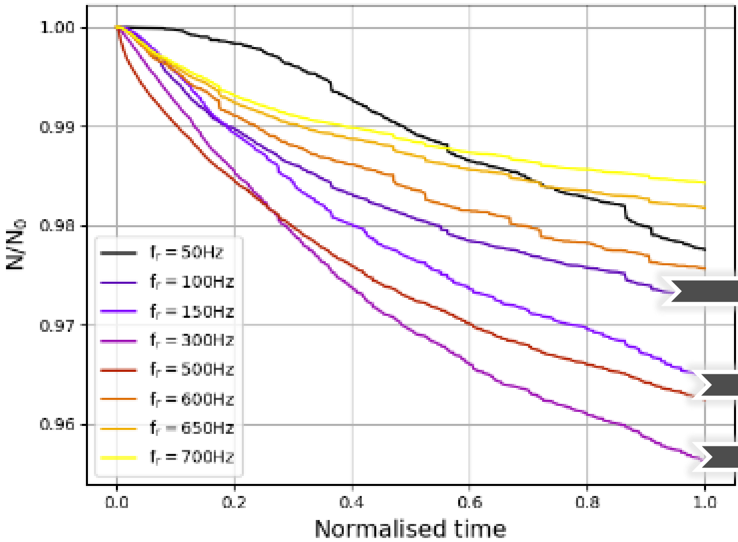
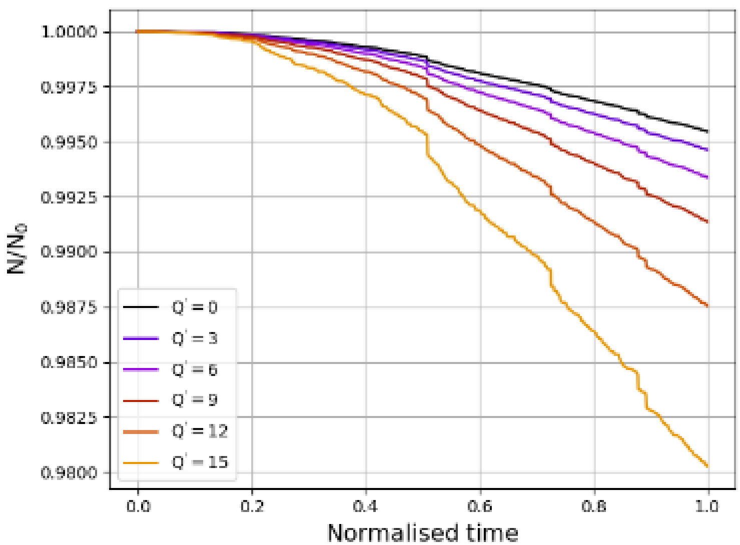


5D, power supply ripple, frequency scan

1st sideband of the diagonal

1st sideband of the 3rd order

2nd sideband of the 3rd order



6D, chromaticity scan

Conclusions & future steps

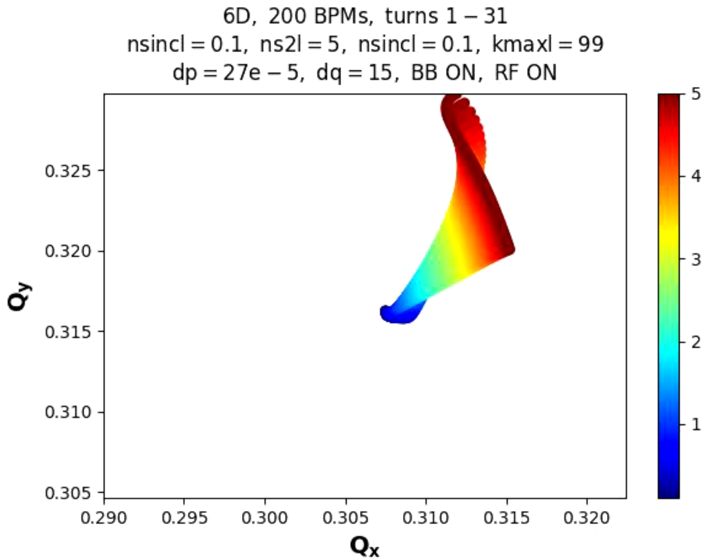
Conclusions

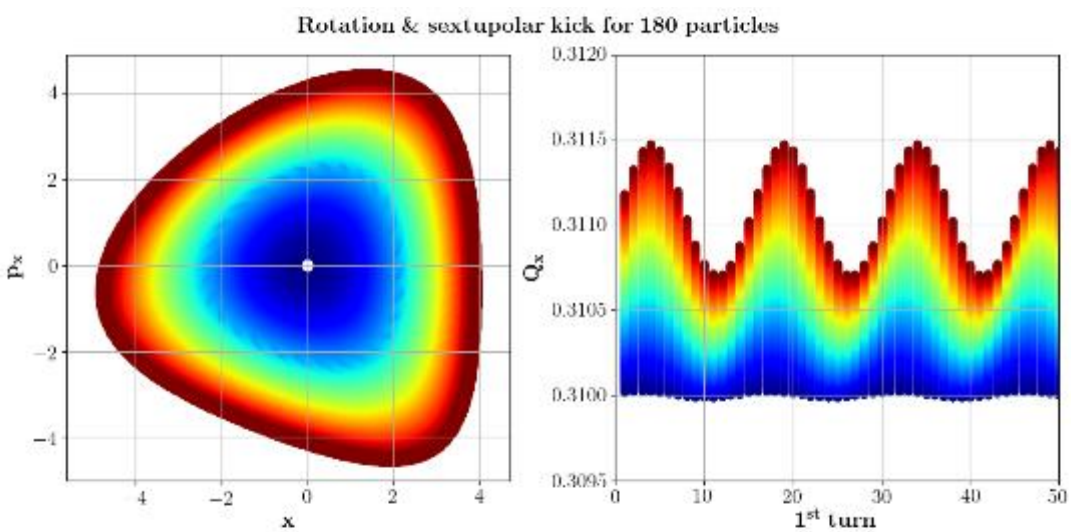
- The additional resonance lines from power supply ripples and chromatic tune modulation have been identified with FMAs.
- We are able to show the coherent motion of the 6D footprint in the presence of chromaticity with the multiple-BPM instantaneous tune determination method.
- We showed the impact of different amplitudes and frequencies of the modulation from power supply ripples, from synchrotron motion and from the combination of both effects

Future steps

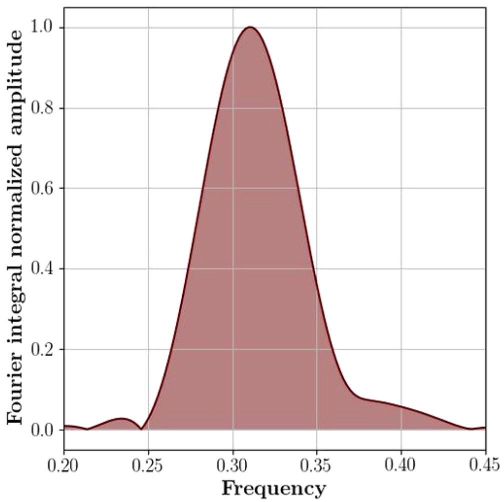
- Investigate what is the impact of 6D BB in the modulation of the synchrotron motion
- Simulations with distributions, in order to identify the impact of these effects in emittance growth, transportation of particles in the tails of the distribution and losses.
- Real LHC spectrum from power supply ripples.

Backup

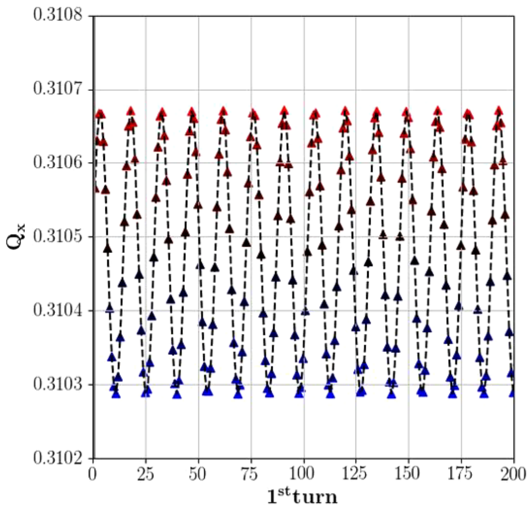

6D, 200 BPMs, turns 1 – 31
nsincl = 0.1, ns2l = 5, nsincl = 0.1, kmaxl = 99
dp = 27e – 5, dq = 15, BB ON, RF ON
Qy
Qx


Rotation & sextupolar kick for 180 particles
px
x
Qx
1st turn

Backup



Fourier integral normalized amplitude
0.0
0.2
0.4
0.6
0.8
1.0
0.20
0.25
0.30
0.35
0.40
0.45
Frequency

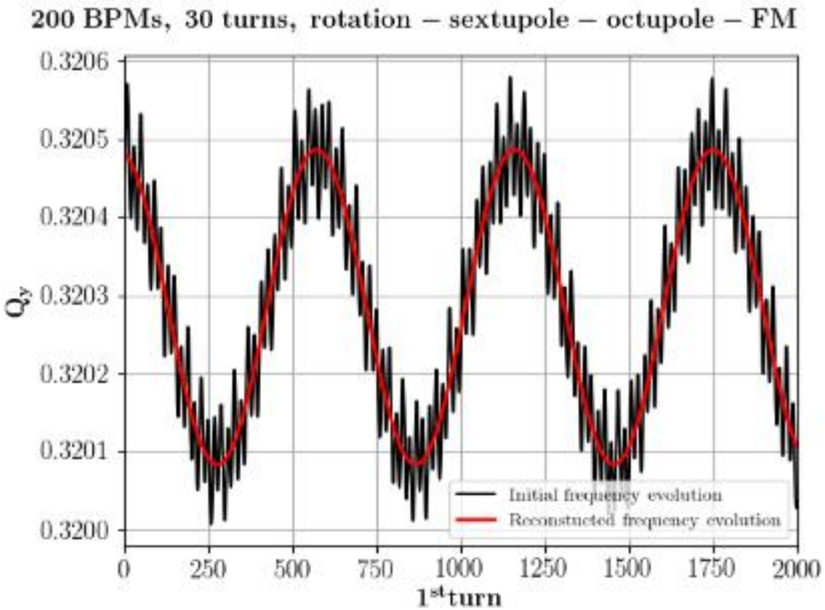


Q_x
0.3102
0.3103
0.3104
0.3105
0.3106
0.3107
0.3108
0
25
50
75
100
125
150
175
200
1^{st}turn

Simplified model: NAFF - NAFF

- With a sliding window of 30 turns the frequency components can be identified
- By frequency analyzing the NAFF results the three components can be identified and separated:

1) **DC component**, the un-modulated betatron tune with **A0**
2) **fs**, the 1st sideband of the betatron tune with **A1** amplitude and **φ1** phase
3) **fN**, the frequency from the non-linear elements



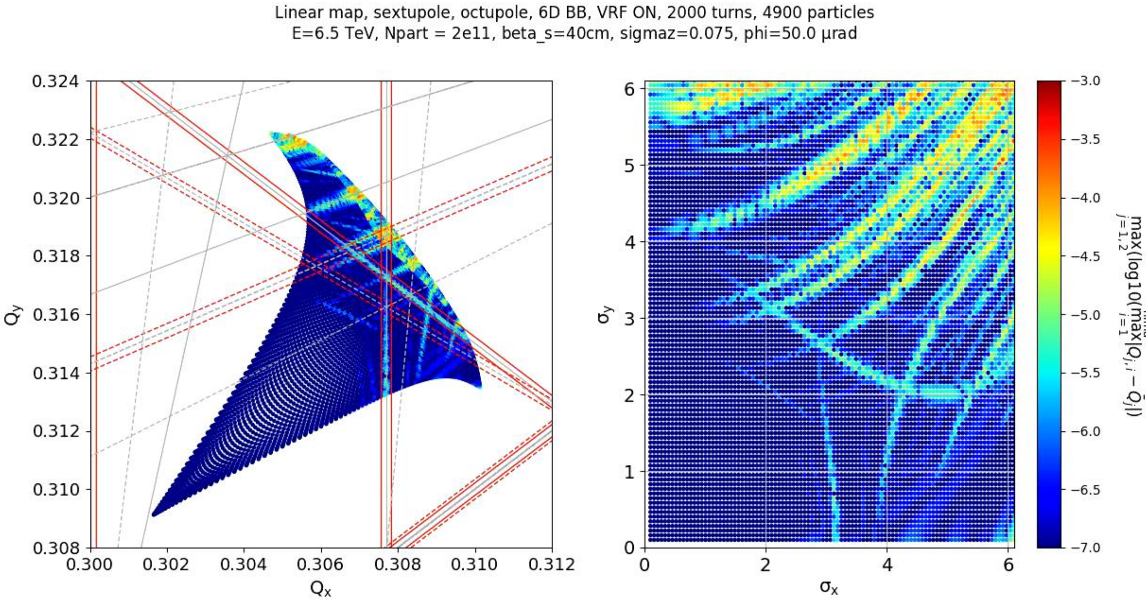
$$Q_{reconstructed} = \sum_{t=1}^{t=L_{SW}} \mathbf{A_0} + \mathbf{A_1} e^{i(2\pi \mathbf{f_1} t + \boldsymbol{\varphi_1})}$$

Simplified model with FM & 6D Beam-beam

- Resonance conditions:

$$\mathbf{aQ_x + bQ_y + c}{\color{red}\mathbf{Q_s}} = \mathbf{k\ for\ a, b, c, k\ integers}$$

6D BB, crossing angle scan



Sixtrack LHC