# HH production at NNLO including $M_t$ effects

**Javier Mazzitelli**

In collaboration with M. Grazzini, G. Heinrich, S. Jones,
S. Kallweit, M. Kerner, J. Lindert

Universität Zürich[UZH]

# HH at NNLO with $M_t$ effects

[arXiv:1803.02463]

- We combined **full NLO** with **HTL NNLO**, fully differential predictions
- We studied different **reweightings** to account for **finite $M_t$ effects** at NNLO
- Our best prediction: **$\text{NNLO}_{\text{FTapprox}}$**

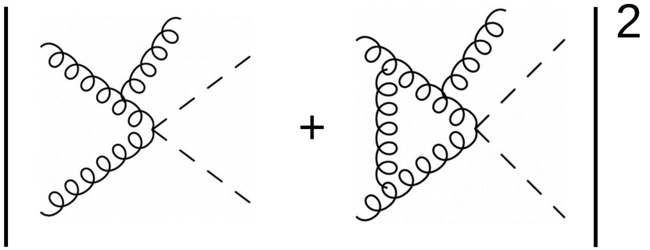
We perform a subprocess-wise reweighting:
for each n-loop squared amplitude

$$\mathcal{A}^{(n)}_{\text{HEFT}}(ij \to HH + X)$$

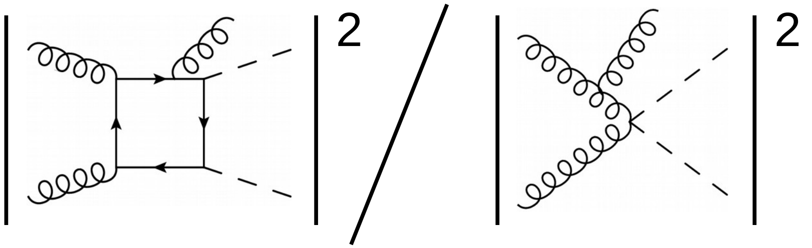
we apply the reweighting

$$\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}^{\text{Born}}_{\text{Full}}(ij \to HH + X)}{\mathcal{A}^{(0)}_{\text{HEFT}}(ij \to HH + X)}$$

E.g. the squared amplitude:




is reweighted by:




- Amplitudes that are tree-level in the HTL are treated exactly (**full double-reals**)
- Great performance at NLO (4% difference with full NLO)

# Total cross sections

| $\sqrt{s}$ | 13 TeV | 14 TeV | 27 TeV | 100 TeV |
|---|---|---|---|---|
| NLO [fb] | $27.78^{+13.8\%}_{-12.8\%}$ | $32.88^{+13.5\%}_{-12.5\%}$ | $127.7^{+11.5\%}_{-10.4\%}$ | $1147^{+10.7\%}_{-9.9\%}$ |
| $\mathrm{NLO_{FTapprox}}$ [fb] | $28.91^{+15.0\%}_{-13.4\%}$ | $34.25^{+14.7\%}_{-13.2\%}$ | $134.1^{+12.7\%}_{-11.1\%}$ | $1220^{+11.9\%}_{-10.6\%}$ |
| $\mathrm{NNLO_{NLO-i}}$ [fb] | $32.69^{+5.3\%}_{-7.7\%}$ | $38.66^{+5.3\%}_{-7.7\%}$ | $149.3^{+4.8\%}_{-6.7\%}$ | $1337^{+4.1\%}_{-5.4\%}$ |
| $\mathrm{NNLO_{B-proj}}$ [fb] | $33.42^{+1.5\%}_{-4.8\%}$ | $39.58^{+1.4\%}_{-4.7\%}$ | $154.2^{+0.7\%}_{-3.8\%}$ | $1406^{+0.5\%}_{-2.8\%}$ |
| $\mathrm{NNLO_{FTapprox}}$ [fb] | $31.05^{+2.2\%}_{-5.0\%}$ | $36.69^{+2.1\%}_{-4.9\%}$ | $139.9^{+1.3\%}_{-3.9\%}$ | $1224^{+0.9\%}_{-3.2\%}$ |
| $M_t$ unc. $\mathrm{NNLO_{FTapprox}}$ | ±2.6% | ±2.7% | ±3.4% | ±4.6% |
| $\mathrm{NNLO_{FTapprox}}/\mathrm{NLO}$ | 1.118 | 1.116 | 1.096 | 1.067 |

- Increase with respect to NLO at 14TeV: ~12%
- About 8% smaller than YR4 recommendation
- Smaller scale and $M_t$ uncertainties

- Proposal: **update** current **total XS** and **$M_t$ uncertainties** recommendation to the **$\mathrm{NNLO_{FTapprox}}$**
- For distributions **rescale NLO+PS by $\mathrm{NNLO_{FTapprox}}$ total XS**

# $M_t$ scheme uncertainty

- Question raised in the HXSWG general meeting: $M_t$ scheme dependence is not included in the previous uncertainties, and in principle can be large

- For our predictions we renormalize the top quark mass in the on-shell scheme

- Using the $\overline{\text{MS}}$ scheme is not possible at the moment (NLO two-loop virtuals available only for fixed $M_t$ = 173GeV)

$$m_t(\mu) = M_t \left[ 1 - \frac{\alpha_S(\mu)}{\pi} \left( \frac{4}{3} + \log \frac{\mu^2}{M_t^2} \right) + \mathcal{O}(\alpha_S^2) \right]$$

$\overline{\text{MS}}$ mass (↓ $m_t(\mu)$)   OS mass (↓ $M_t$)

- As a first estimation we can replace the OS mass by the $\overline{\text{MS}}$ mass using the above relation for the LO cross section

↓

Total cross section reduced by about
- 11% for $\mu=M_{hh}/2$
- 5% for $\mu=M_t$

- Full NLO is expected to reduce this dependence

# $M_t$ scheme uncertainty

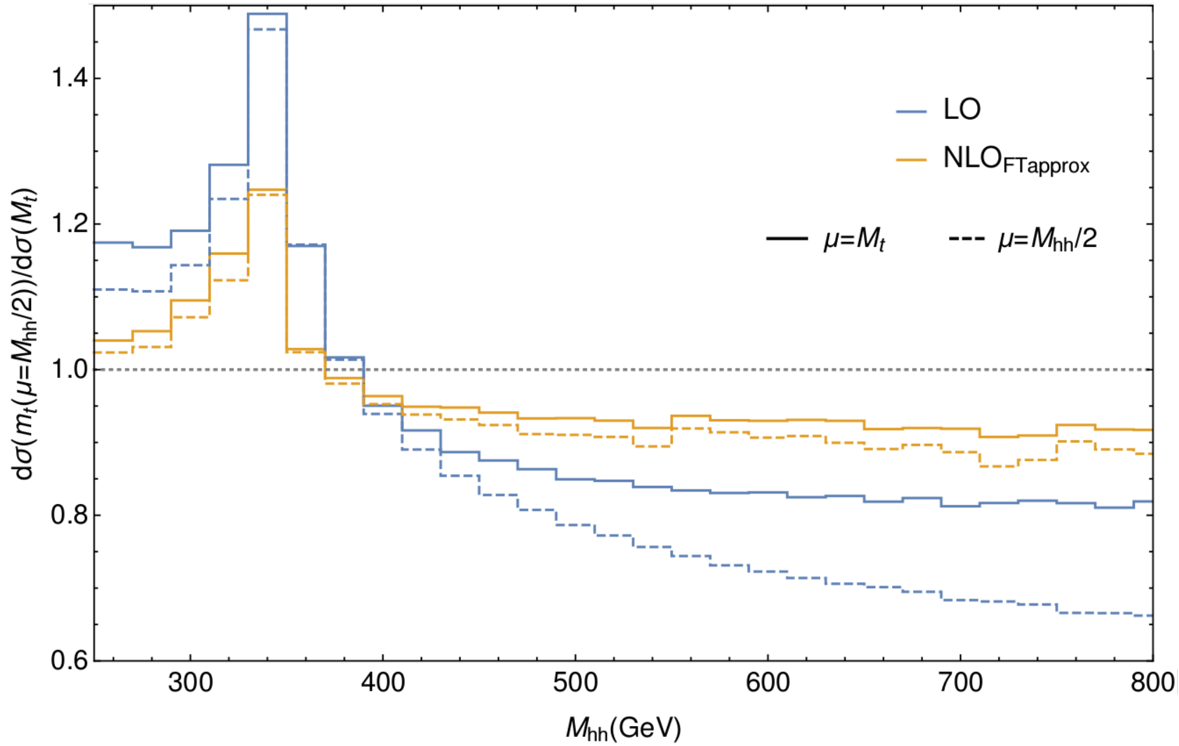
- We can use the FTapprox in order to estimate the scheme dependence at NLO

- Even more: even though we cannot compute the two-loop virtuals in the $\overline{\text{MS}}$ scheme, we can replace the OS counterterm by the $\overline{\text{MS}}$ one

- $\text{NLO}_{\text{FTapprox}}$ total cross section in the $\overline{\text{MS}}$ scheme:

  2.7% (4.5%) smaller than the OS one for $\mu=M_t$ ($\mu=M_{hh}/2$)

- Scheme dependence reduced by about factor of 2 w.r.t. LO




- This effect should probably be smaller using full NLO

  → Use this as an upper limit for scheme dependence

# Some questions

- Is there a preferred choice for the $\overline{\text{MS}}$ top quark mass scale?

- Assuming a 3% (5%) scheme dependence at NLO, what is the $\text{NNLO}_{\text{FTapprox}}$ scheme uncertainty?

  Approximation to full NNLO in the OS scheme

  Difference w.r.t. $\overline{\text{MS}}$ expected to be further reduced at NNLO

- Should these uncertainties (scale, scheme and $M_t$ uncertainties) be combined linearly?

**Thanks!**

# Backup slides

# NLO-improved approximation – $\mathrm{NNLO}_{\mathrm{NLO\text{-}i}}$

Done originally in Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk and Zirke, arXiv:1608.04798 [hep-ph]

Simplest approach: for **each bin** of each histogram we do

$$\mathrm{NNLO}_{\mathrm{NLO\text{-}i}} = \mathrm{NLO} \times \left( \frac{\mathrm{NNLO}}{\mathrm{NLO}} \right)_{\mathrm{HEFT}}$$

- Observable level reweighting, technically simple
- Finite $M_t$ effects in the NNLO piece enter via the full NLO
- Has to be repeated for each observable and binning (bin size dependent!)
- We compute the total cross section based on the $M_{hh}$ distribution

# Born-projected approximation – $\mathrm{NNLO}_{\text{B-proj}}$

Reweight each NNLO event by the ratio of
the full and HEFT Born squared amplitudes

Different multiplicities (double real and real-virtual corrections)

↓

Projection to Born kinematics needed

We make use of the $q_T$-recoil procedure:

Catani, de Florian, Ferrera and Grazzini, arXiv:1507.06937 [hep-ph]

- Momenta of the Higgs bosons remain unchanged
- The new initial state partons momenta absorb the $q_T$ due to the additional radiation
- Initial state momenta remain massless, and their transverse component goes to zero when $q_T$ goes to zero (and then $q_T$-cancellation is not spoiled)

Finite $M_t$ effects entering only via the Born amplitude: no information about real radiation

# Full-theory approximation – $\text{NNLO}_{\text{FTapprox}}$

- Double real corrections can be computed in the full theory (one-loop amplitudes)
- Idea: construct an approximation in which they are treated in an exact way

We perform a subprocess-wise reweighting: for each n-loop squared amplitude

$$\mathcal{A}^{(n)}_{\rm HEFT}(ij \to HH + X)$$

we apply the reweighting

$$\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}^{\rm Born}_{\rm Full}(ij \to HH + X)}{\mathcal{A}^{(0)}_{\rm HEFT}(ij \to HH + X)}$$

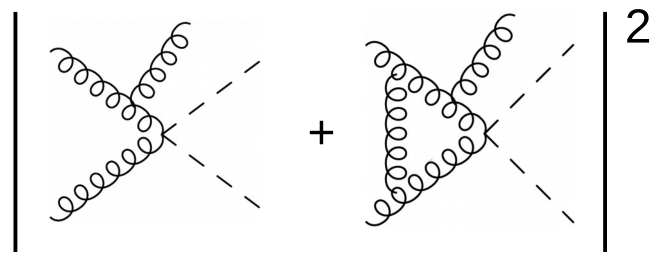
- Same partonic subprocess used for reweighting: no need for a projection
- Amplitudes that are tree-level in the HEFT are treated exactly
- At NLO this agrees with the FTapprox in Maltoni, Vryonidou and Zaro, arXiv:1408.6542 [hep-ph]
- Great performance at NLO (4% difference with full NLO) + full $M_t$ dependence in double reals

**Our best NNLO prediction**

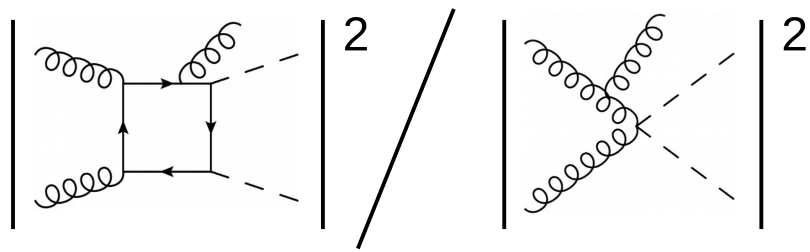
# Full-theory approximation – $NNLO_{FTapprox}$

- Double real corrections can be computed in the full theory (one-loop amplitudes)
- Idea: construct an approximation in which they are treated in an exact way

E.g. the squared amplitude:



is reweighted by:



- Same partonic subprocess used for reweighting: no need for a projection
- Amplitudes that are tree-level in the HEFT are treated exactly
- At NLO this agrees with the FTapprox in Maltoni, Vryonidou and Zaro, arXiv:1408.6542 [hep-ph]
- Great performance at NLO (4% difference with full NLO) + full $M_t$ dependence in double reals

**Our best NNLO prediction**

# Top quark mass uncertainties

| $\sqrt{s}$ | 13 TeV | 14 TeV | 27 TeV | 100 TeV |
|---|---|---|---|---|
| NLO [fb] | $27.78^{+13.8\%}_{-12.8\%}$ | $32.88^{+13.5\%}_{-12.5\%}$ | $127.7^{+11.5\%}_{-10.4\%}$ | $1147^{+10.7\%}_{-9.9\%}$ |
| $\mathrm{NLO_{FTapprox}}$ [fb] | $28.91^{+15.0\%}_{-13.4\%}$ | $34.25^{+14.7\%}_{-13.2\%}$ | $134.1^{+12.7\%}_{-11.1\%}$ | $1220^{+11.9\%}_{-10.6\%}$ |
| $\mathrm{NNLO_{NLO-i}}$ [fb] | $32.69^{+5.3\%}_{-7.7\%}$ | $38.66^{+5.3\%}_{-7.7\%}$ | $149.3^{+4.8\%}_{-6.7\%}$ | $1337^{+4.1\%}_{-5.4\%}$ |
| $\mathrm{NNLO_{B-proj}}$ [fb] | $33.42^{+1.5\%}_{-4.8\%}$ | $39.58^{+1.4\%}_{-4.7\%}$ | $154.2^{+0.7\%}_{-3.8\%}$ | $1406^{+0.5\%}_{-2.8\%}$ |
| $\mathrm{NNLO_{FTapprox}}$ [fb] | $31.05^{+2.2\%}_{-5.0\%}$ | $36.69^{+2.1\%}_{-4.9\%}$ | $139.9^{+1.3\%}_{-3.9\%}$ | $1224^{+0.9\%}_{-3.2\%}$ |

- At NLO the FTapprox overestimates full NLO by 4% → 11% for the pure NLO contribution
- Assuming a ±11% uncertainty for the pure NNLO piece → ±1.2% uncertainty at NNLO
- Multiply by a factor of 2 to be more conservative (14TeV)

# Top quark mass uncertainties

| $\sqrt{s}$ | 13 TeV | 14 TeV | 27 TeV | 100 TeV |
|---|---|---|---|---|
| NLO [fb] | $27.78^{+13.8\%}_{-12.8\%}$ | $32.88^{+13.5\%}_{-12.5\%}$ | $127.7^{+11.5\%}_{-10.4\%}$ | $1147^{+10.7\%}_{-9.9\%}$ |
| $\mathrm{NLO_{FTapprox}}$ [fb] | $28.91^{+15.0\%}_{-13.4\%}$ | $34.25^{+14.7\%}_{-13.2\%}$ | $134.1^{+12.7\%}_{-11.1\%}$ | $1220^{+11.9\%}_{-10.6\%}$ |
| $\mathrm{NNLO_{NLO-i}}$ [fb] | $32.69^{+5.3\%}_{-7.7\%}$ | $38.66^{+5.3\%}_{-7.7\%}$ | $149.3^{+4.8\%}_{-6.7\%}$ | $1337^{+4.1\%}_{-5.4\%}$ |
| $\mathrm{NNLO_{B-proj}}$ [fb] | $33.42^{+1.5\%}_{-4.8\%}$ | $39.58^{+1.4\%}_{-4.7\%}$ | $154.2^{+0.7\%}_{-3.8\%}$ | $1406^{+0.5\%}_{-2.8\%}$ |
| $\mathrm{NNLO_{FTapprox}}$ [fb] | $31.05^{+2.2\%}_{-5.0\%}$ | $36.69^{+2.1\%}_{-4.9\%}$ | $139.9^{+1.3\%}_{-3.9\%}$ | $1224^{+0.9\%}_{-3.2\%}$ |
| $M_t$ unc. $\mathrm{NNLO_{FTapprox}}$ | $\pm 2.3\%$ | $\pm 2.4\%$ | $\pm 2.7\%$ | $\pm 3.1\%$ |

- At NLO the FTapprox overestimates full NLO by 4% → 11% for the pure NLO contribution
- Assuming a ±11% uncertainty for the pure NNLO piece → ±1.2% uncertainty at NNLO
- Multiply by a factor of 2 to be more conservative (14TeV)

# Top quark mass uncertainties

| $\sqrt{s}$ | 13 TeV | 14 TeV | 27 TeV | 100 TeV |
|---|---|---|---|---|
| NLO [fb] | $27.78^{+13.8\%}_{-12.8\%}$ | $32.88^{+13.5\%}_{-12.5\%}$ | $127.7^{+11.5\%}_{-10.4\%}$ | $1147^{+10.7\%}_{-9.9\%}$ |
| $\mathrm{NLO}_{\mathrm{FTapprox}}$ [fb] | $28.91^{+15.0\%}_{-13.4\%}$ | $34.25^{+14.7\%}_{-13.2\%}$ | $134.1^{+12.7\%}_{-11.1\%}$ | $1220^{+11.9\%}_{-10.6\%}$ |
| $\mathrm{NNLO}_{\mathrm{NLO-i}}$ [fb] | $32.69^{+5.3\%}_{-7.7\%}$ | $38.66^{+5.3\%}_{-7.7\%}$ | $149.3^{+4.8\%}_{-6.7\%}$ | $1337^{+4.1\%}_{-5.4\%}$ |
| $\mathrm{NNLO}_{\mathrm{B-proj}}$ [fb] | $33.42^{+1.5\%}_{-4.8\%}$ | $39.58^{+1.4\%}_{-4.7\%}$ | $154.2^{+0.7\%}_{-3.8\%}$ | $1406^{+0.5\%}_{-2.8\%}$ |
| $\mathrm{NNLO}_{\mathrm{FTapprox}}$ [fb] | $31.05^{+2.2\%}_{-5.0\%}$ | $36.69^{+2.1\%}_{-4.9\%}$ | $139.9^{+1.3\%}_{-3.9\%}$ | $1224^{+0.9\%}_{-3.2\%}$ |
| $M_t$ unc. $\mathrm{NNLO}_{\mathrm{FTapprox}}$ | $\pm 2.3\%$ | $\pm 2.4\%$ | $\pm 2.7\%$ | $\pm 3.1\%$ |
| $M_t$ unc. $\mathrm{NNLO}_{\mathrm{B-proj}}$ | $\pm 14\%$ | $\pm 15\%$ | $\pm 20\%$ | $\pm 36\%$ |

- At NLO the FTapprox overestimates full NLO by 4% → 11% for the pure NLO contribution
- Assuming a ±11% uncertainty for the pure NNLO piece → ±1.2% uncertainty at NNLO
- Multiply by a factor of 2 to be more conservative (14TeV)

We can repeat the procedure for the Born-projected approximation

→ Compatible results even without the factor of 2

# Top quark mass uncertainties

| $\sqrt{s}$ | 13 TeV | 14 TeV | 27 TeV | 100 TeV |
|---|---|---|---|---|
| NLO [fb] | $27.78^{+13.8\%}_{-12.8\%}$ | $32.88^{+13.5\%}_{-12.5\%}$ | $127.7^{+11.5\%}_{-10.4\%}$ | $1147^{+10.7\%}_{-9.9\%}$ |
| $\mathrm{NLO}_{\mathrm{FTapprox}}$ [fb] | $28.91^{+15.0\%}_{-13.4\%}$ | $34.25^{+14.7\%}_{-13.2\%}$ | $134.1^{+12.7\%}_{-11.1\%}$ | $1220^{+11.9\%}_{-10.6\%}$ |
| $\mathrm{NNLO}_{\mathrm{NLO-i}}$ [fb] | $32.69^{+5.3\%}_{-7.7\%}$ | $38.66^{+5.3\%}_{-7.7\%}$ | $149.3^{+4.8\%}_{-6.7\%}$ | $1337^{+4.1\%}_{-5.4\%}$ |
| $\mathrm{NNLO}_{\mathrm{B-proj}}$ [fb] | $33.42^{+1.5\%}_{-4.8\%}$ | $39.58^{+1.4\%}_{-4.7\%}$ | $154.2^{+0.7\%}_{-3.8\%}$ | $1406^{+0.5\%}_{-2.8\%}$ |
| $\mathrm{NNLO}_{\mathrm{FTapprox}}$ [fb] | $31.05^{+2.2\%}_{-5.0\%}$ | $36.69^{+2.1\%}_{-4.9\%}$ | $139.9^{+1.3\%}_{-3.9\%}$ | $1224^{+0.9\%}_{-3.2\%}$ |
| $M_t$ unc. $\mathrm{NNLO}_{\mathrm{FTapprox}}$ | $\pm 2.3\%$ | $\pm 2.4\%$ | $\pm 2.7\%$ | $\pm 3.1\%$ |
| $M_t$ unc. $\mathrm{NNLO}_{\mathrm{B-proj}}$ | $\pm 14\%$ | $\pm 15\%$ | $\pm 20\%$ | $\pm 36\%$ |

- But the difference between FTapprox and NLO-i increases with the collider energy faster than this uncertainty estimate
- To be more conservative, take half the difference between FTapprox and NLO-i

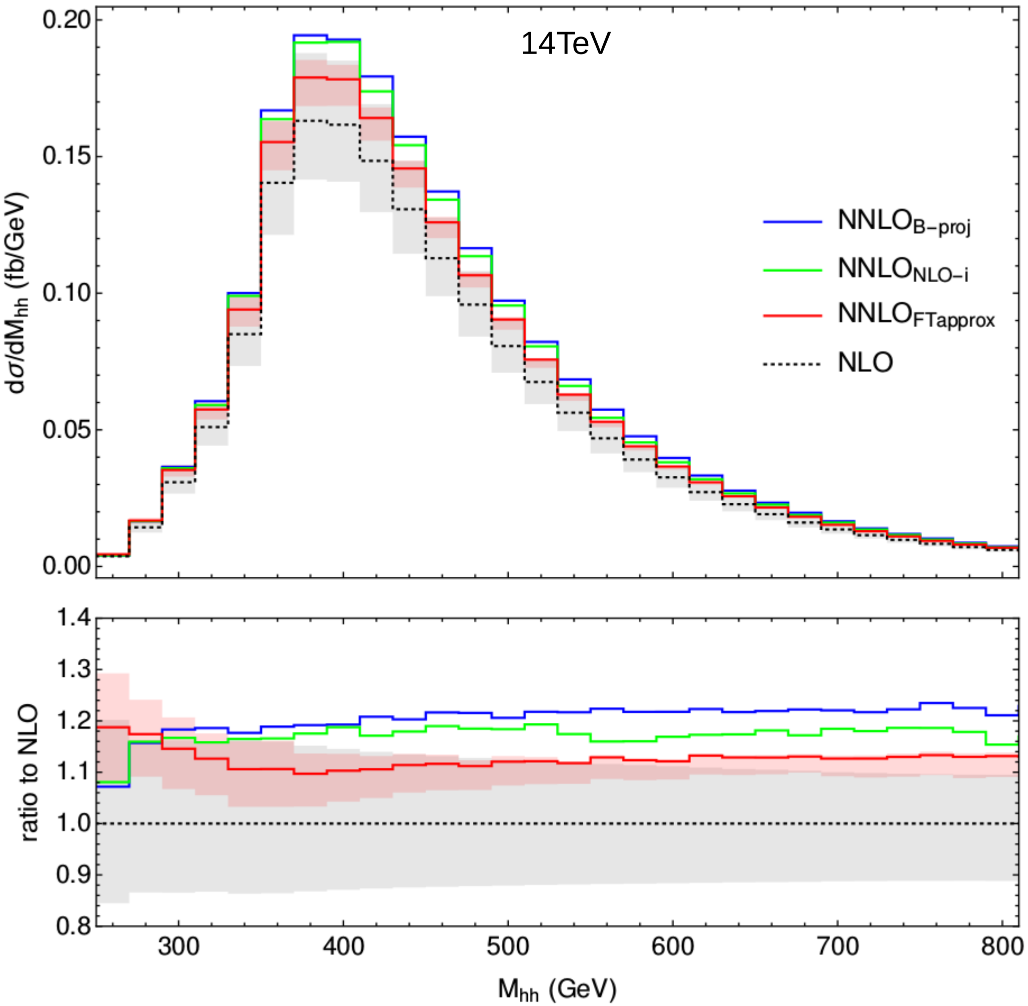
# Top quark mass uncertainties

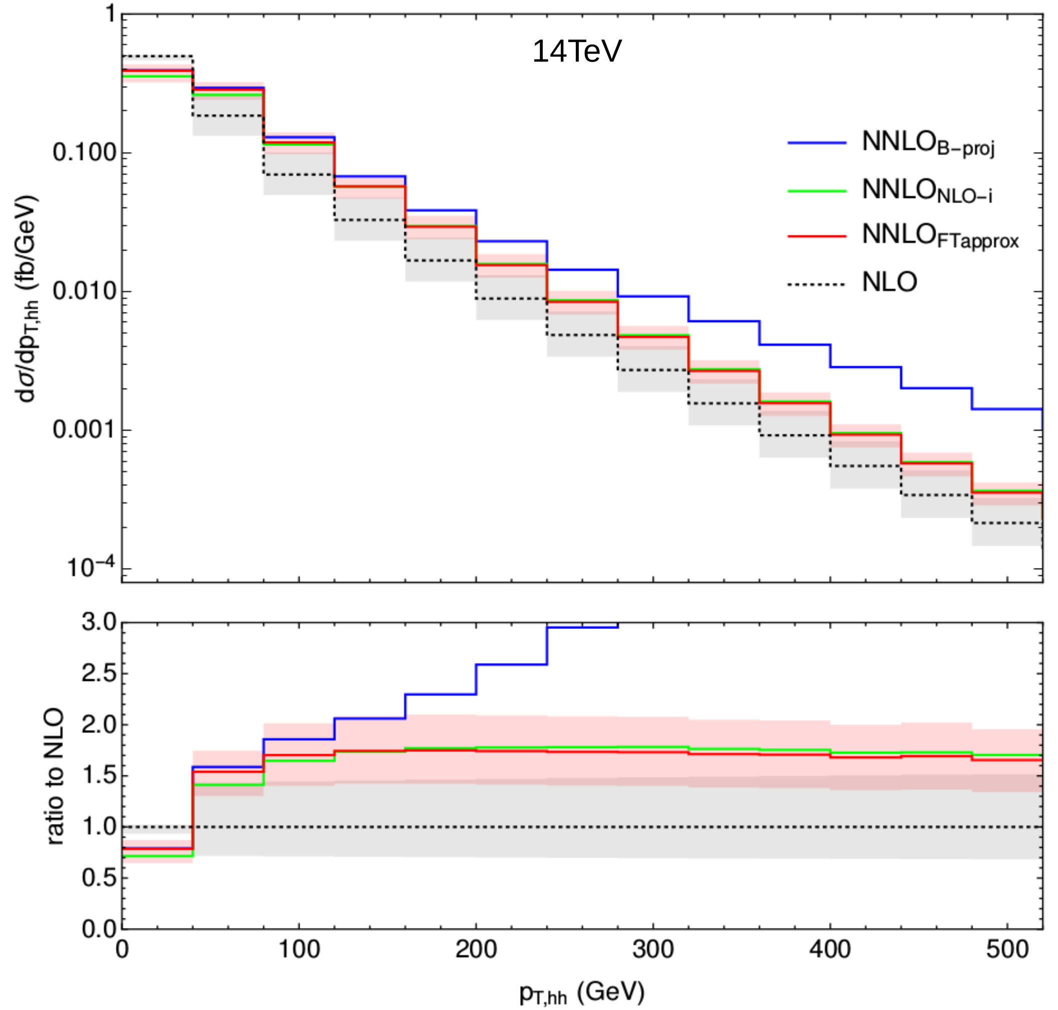
| $\sqrt{s}$ | 13 TeV | 14 TeV | 27 TeV | 100 TeV |
|---|---|---|---|---|
| NLO [fb] | $27.78^{+13.8\%}_{-12.8\%}$ | $32.88^{+13.5\%}_{-12.5\%}$ | $127.7^{+11.5\%}_{-10.4\%}$ | $1147^{+10.7\%}_{-9.9\%}$ |
| $\mathrm{NLO}_{\mathrm{FTapprox}}$ [fb] | $28.91^{+15.0\%}_{-13.4\%}$ | $34.25^{+14.7\%}_{-13.2\%}$ | $134.1^{+12.7\%}_{-11.1\%}$ | $1220^{+11.9\%}_{-10.6\%}$ |
| $\mathrm{NNLO}_{\mathrm{NLO-i}}$ [fb] | $32.69^{+5.3\%}_{-7.7\%}$ | $38.66^{+5.3\%}_{-7.7\%}$ | $149.3^{+4.8\%}_{-6.7\%}$ | $1337^{+4.1\%}_{-5.4\%}$ |
| $\mathrm{NNLO}_{\mathrm{B-proj}}$ [fb] | $33.42^{+1.5\%}_{-4.8\%}$ | $39.58^{+1.4\%}_{-4.7\%}$ | $154.2^{+0.7\%}_{-3.8\%}$ | $1406^{+0.5\%}_{-2.8\%}$ |
| $\mathrm{NNLO}_{\mathrm{FTapprox}}$ [fb] | $31.05^{+2.2\%}_{-5.0\%}$ | $36.69^{+2.1\%}_{-4.9\%}$ | $139.9^{+1.3\%}_{-3.9\%}$ | $1224^{+0.9\%}_{-3.2\%}$ |
| $M_t$ unc. $\mathrm{NNLO}_{\mathrm{FTapprox}}$ | ±2.3% | ±2.4% | ±2.7% | ±3.1% |
| $M_t$ unc. $\mathrm{NNLO}_{\mathrm{B-proj}}$ | ±14% | ±15% | ±20% | ±36% |
| $M_t$ unc. $\mathrm{NNLO}_{\mathrm{FTapprox}}$ | ±2.6% | ±2.7% | ±3.4% | ±4.6% |

- But the difference between FTapprox and NLO-i increases with the collider energy faster than this uncertainty estimate
- To be more conservative, take half the difference between FTapprox and NLO-i

Small difference for LHC, more conservative for larger energies

# Some differential distributions




- B-proj and NLO-i have similar behaviors
- FTapprox presents larger corrections at threshold, minimum corrections at $M_{hh}$ ~ 400GeV, slow increase towards the tail
- Scale uncertainties are substantially reduced
- Overlap with the NLO band

- NNLO$_{B-proj}$ has wrong scaling in the tail
  No information about lowest order for $p_{T,hh}$
- NNLO$_{FTapprox}$ agrees with NNLO$_{B-proj}$ for low $p_{T,hh}$, and with NNLO$_{NLO-i}$ in the tail
- Distribution trivial at LO: NNLO is effectively NLO
  Large corrections and sizeable scale uncertainties