Ab-initio approach to anisotropic flow in nucleus-nucleus collisions

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Work in progress with Giuliano Giacalone, Matt Luzum, Cyrille Marquet, Pablo Guerrero Rodriguez

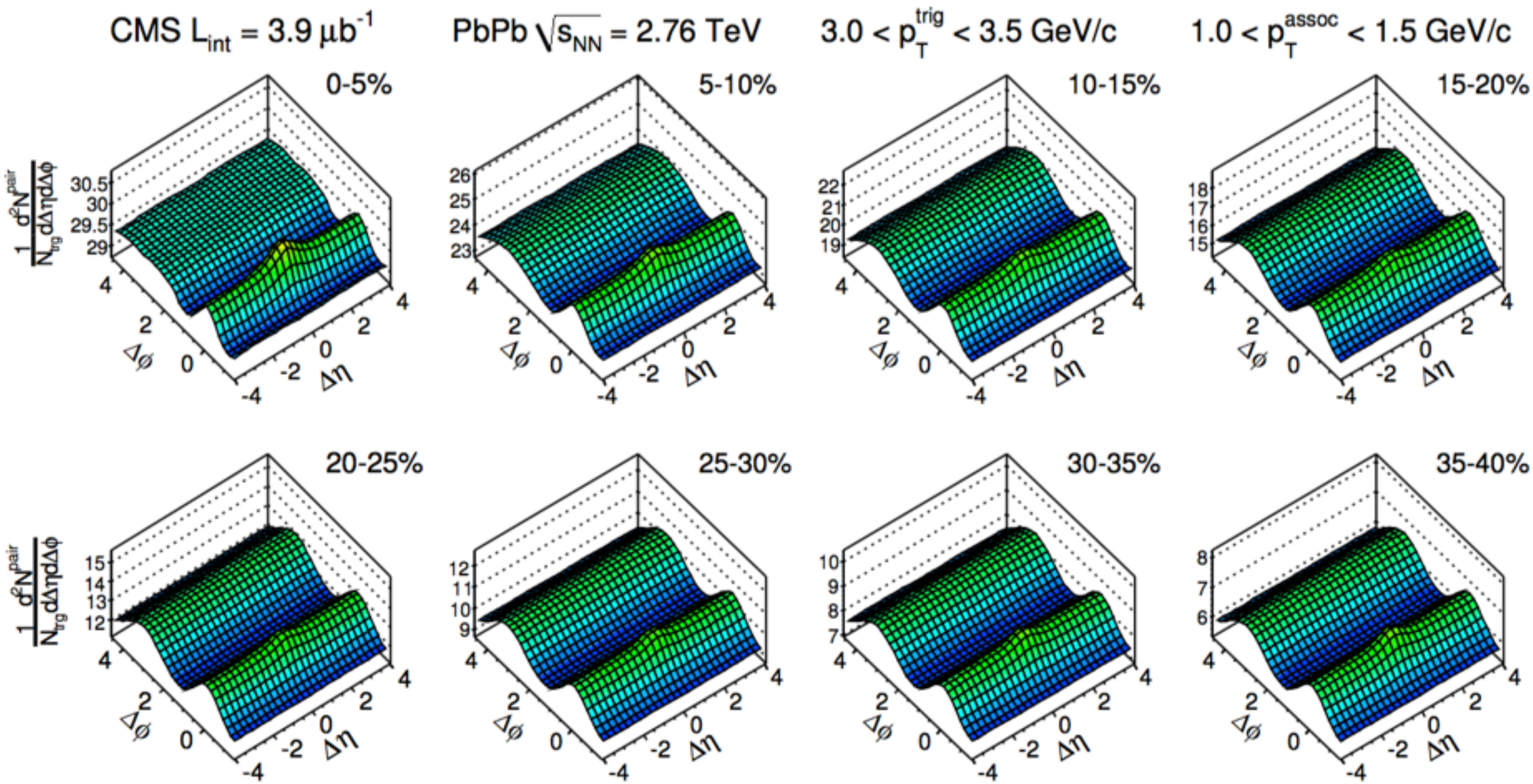
Outline

- Hydrodynamics is successful in describing anisotropic flow
- The main limitation is the poor knowledge of the initial density profile
- We use recent results from high-density QCD calculations to determine the profile and its fluctuations

Albacete, Guerrero-Rodríguez, Marquet, 1808.00795

- These new initial conditions describe elliptic flow and triangular flow at RHIC and LHC.

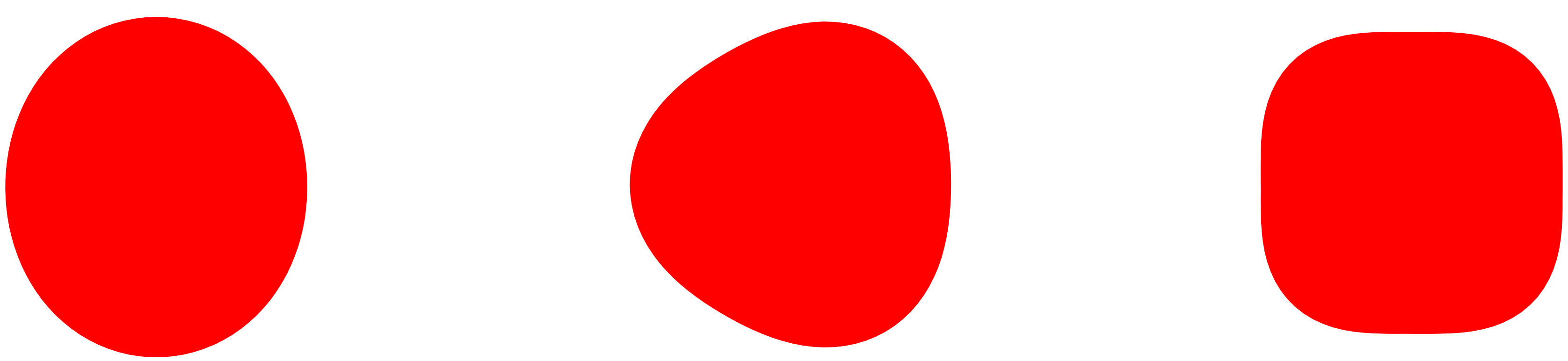
Anisotropic flow: what we see



A wave-like pattern of the distribution of particle pairs versus relative azimuthal angle Δφ, independent of relative rapidity Δη

How we understand this pattern

- The single-particle distribution is essentially independent of rapidity η but depends on azimuthal angle, φ in each event
- Fourier decomposition : $f(\varphi)=\sum_n V_n\, e^{-in\varphi}$
- $v_n \equiv |V_n|$=anisotropic flow fluctuates event to event



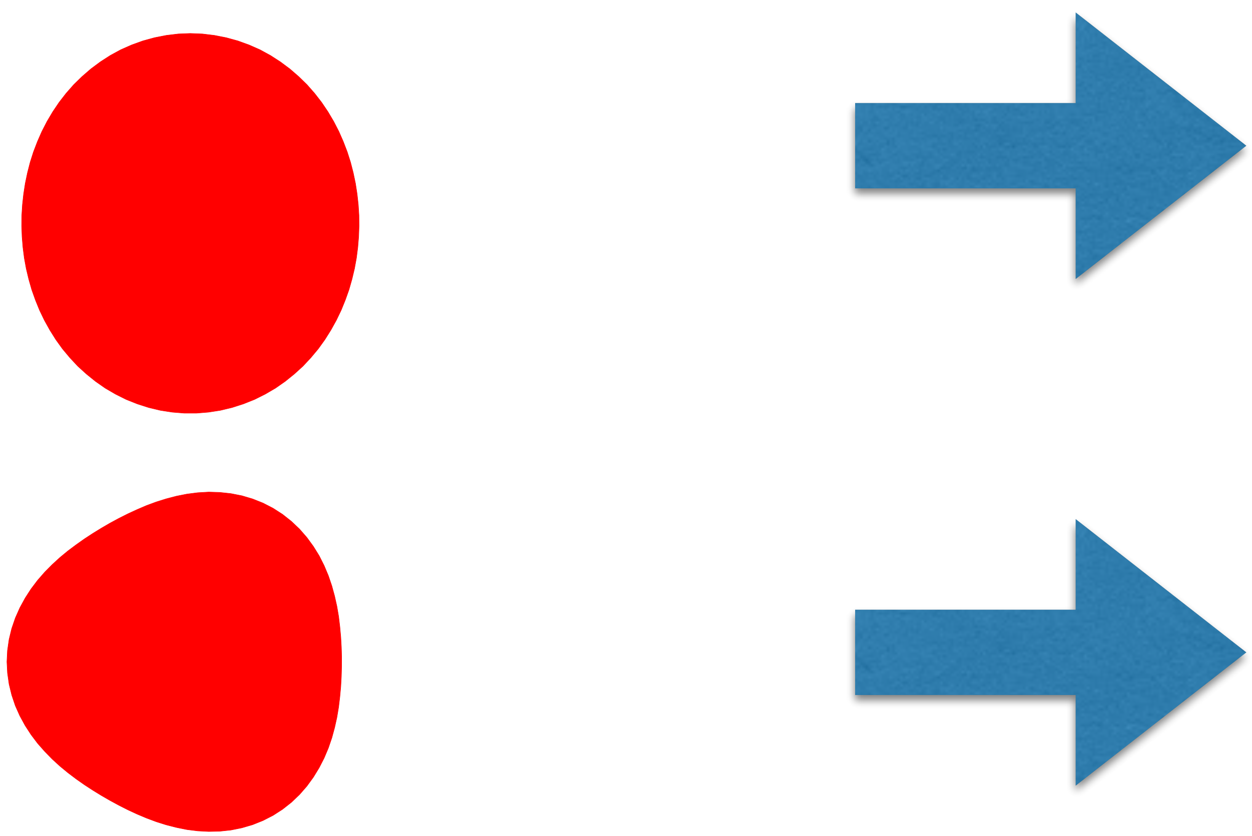
v_2 v_3 v_4

Anisotropic flow and hydrodynamics

Initial transverse density profile

Expansion

Final distribution

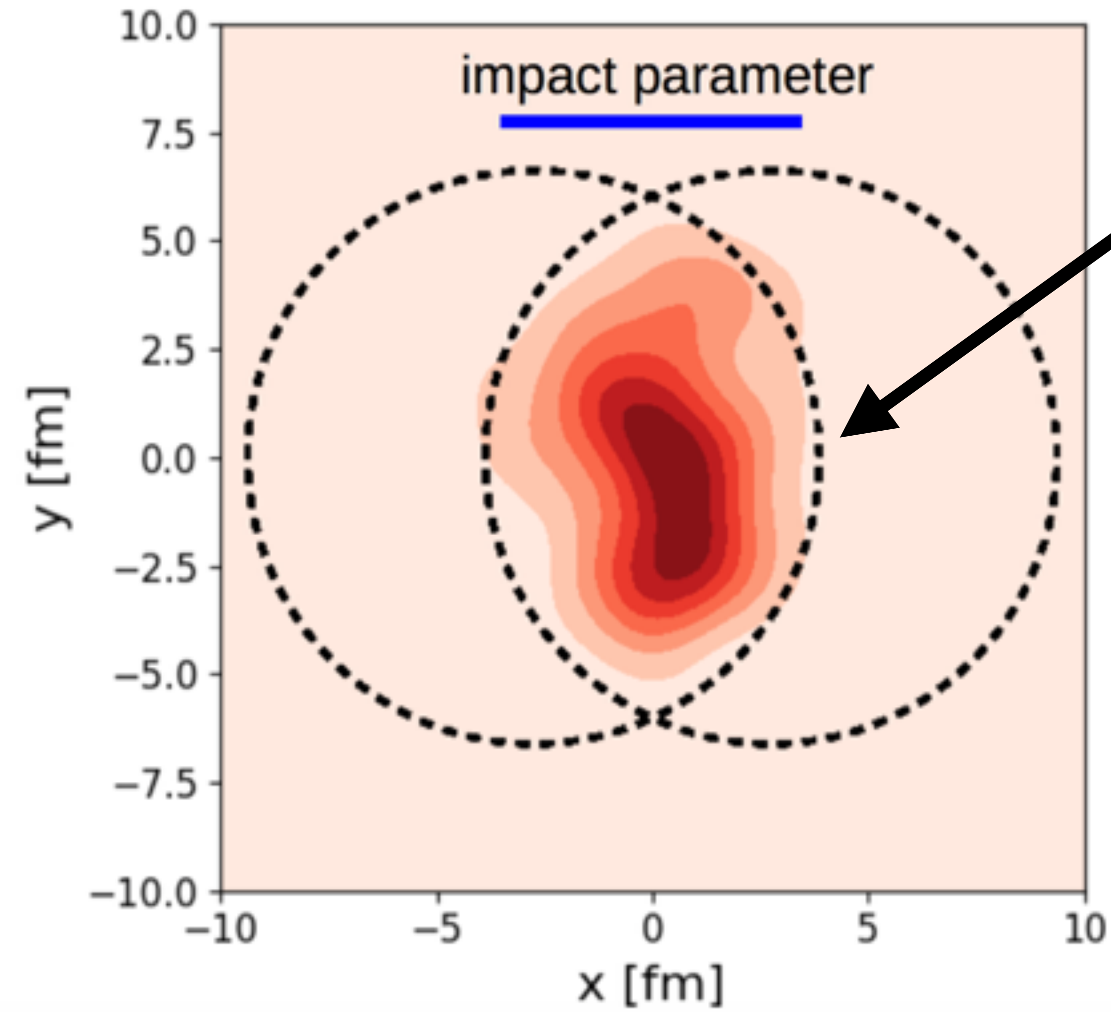


Elliptic flow v_2

Triangular flow v_3

In hydrodynamics, anisotropic flow is a response to the anisotropy of the initial density profile.

Initial density as a fluctuating field



Typical initial transverse density profile ρ(x,y) in a given event

The initial anisotropy ε_n is defined by

$$\varepsilon_n \equiv \frac{\int_z z^n \rho(z)}{\int_z |z|^n \rho(z)}$$

with $z \equiv x+iy = r\, e^{i\varphi}$

$z^n = r^n\, e^{in\varphi}$ → n^{th} harmonic

$v_2 = \kappa_2\, \varepsilon_2$ and $v_3 = \kappa_3\, \varepsilon_3$ where κ_2, κ_3 : hydro response coefficients

Event-averaged quantities

- v_2 and v_3 are not measured event by event.
- The simplest measures are rms averages, sqrt($\langle v_n^2 \rangle$), denoted by $v_2\{2\}$ and $v_3\{2\}$.
- Less simple measures use higher-order cumulants, such as $v_2\{4\}=(2\langle v_2^2\rangle^2-\langle v_2^4\rangle)^{1/4}$.
- In hydro, all are determined by initial anisotropies: $v_2\{2\}=\kappa_2\varepsilon_2\{2\}$, $v_3\{2\}=\kappa_3\varepsilon_3\{2\}$, $v_2\{4\}=\kappa_2\varepsilon_2\{4\}$

Event-averaged anisotropies in terms of density field

- One can express these quantities in terms of the mean density, $\langle\rho(z)\rangle$, and of the two-point function

$$S(z_1, z_2) \equiv \langle \delta\rho(z_1)\delta\rho(z_2)\rangle = \langle \rho(z_1)\rho(z_2)\rangle - \langle \rho(z_1)\rangle\langle \rho(z_2)\rangle$$

Blaizot, Broniowski, JYO, 1405.3572

- $\varepsilon_2\{4\}$ is just the anisotropy of the mean density profile $\langle\rho(z)\rangle$, to a good approximation.

Voloshin, Poskanzer, Aihong Tang, Gang Wang 0708.0800

- $\varepsilon_3\{2\}^2$ and $\varepsilon_2\{2\}^2 - \varepsilon_2\{4\}^2$ = $\dfrac{\int_{z_1 z_2} z_1^n \, \bar{z}_2^n \, S(z_1, z_2)}{\left(\int_z |z|^n \langle\rho(z)\rangle\right)^2}$

New ab-initio approach

arXiv.org > hep-ph > arXiv:1808.00795

Initial correlations of the Glasma energy-momentum tensor

Javier L. Albacete, Pablo Guerrero-Rodríguez, Cyrille Marquet

(Submitted on 2 Aug 2018)

We present an analytical calculation of the covariance of the energy-momentum tensor associated to the gluon field produced in ultra-relativistic heavy ion collisions at early times, the Glasma. This object involves the two-point and single-point correlators of the energy-momentum tensor ($\langle T^{\mu\nu}(x_\perp)T^{\sigma\rho}(y_\perp)\rangle$ and $\langle T^{\mu\nu}(x_\perp)\rangle$, respectively) at proper time $\tau=0^+$.

- A first ab-initio analytic calculation of one- and two point functions of the initial density field in the Glasma, or CGC, framework.
- Exactly the ingredients we need to evaluate $\varepsilon_2\{2\}$, $\varepsilon_3\{2\}$, $\varepsilon_2\{4\}$.

A technical *tour de force*

$$\mathrm{Cov}[\epsilon_0](x_\perp, y_\perp) = \langle \epsilon_0(x_\perp)\epsilon_0(y_\perp)\rangle - \langle \epsilon_0(x_\perp)\rangle\langle \epsilon_0(y_\perp)\rangle$$

$$\begin{aligned}
\mathrm{Cov}[\epsilon](\tau = 0^+; x_\perp, y_\perp) = {} & \frac{\partial_x^i\Gamma\partial_y^i\Gamma(N_c^2-1)A(4A^2-B^2)}{16N_c^2\Gamma^5 g^4}(p_1q_2+p_2q_1) \\
& + \frac{(N_c^2-1)(16A^4+B^4)}{2N_c^2\Gamma^4 g^4}p_1p_2 + \frac{(\partial_x^i\Gamma\partial_y^i\Gamma)^2(N_c^2-1)A^2}{64N_c^2\Gamma^6 g^4}q_1q_2 \\
& + \frac{(4A^2+B^2)^2r^2}{N_c^2\Gamma^4 g^4}\left(\left[\frac{1}{2}Q_{s1}^2Q_{s2}^2r^2 + 4Q_{s2}^2e^{-\frac{Q_{s1}^2r^2}{4}} - 4Q_{s1}^2\right] + [1\leftrightarrow 2]\right) \\
& + \frac{(N_c^2-1)(4A^2+B^2)}{2N_c^2\Gamma^2 g^4}\left(4\pi\,\partial^2L(0_\perp)\right)^2\left(\left[\bar{Q}_{s1}^4(Q_{s2}^2r^2 - 4 + 4e^{-\frac{Q_{s2}^2r^2}{4}})\right] + [1\leftrightarrow 2]\right) \\
& + \frac{(4A^2+B^2)^2}{\Gamma^4 g^4 N_c^2(N_c^2-1)^2(N_c^2-4)^2}\left(\left[-4(N_c^2-1)(N_c^2-4)(N_c^6-3N_c^4-26N_c^2+16)e^{-\frac{Q_{s1}^2r^2}{4}}\right.\right. \\
& + (N_c-3)(N_c+1)^3(N_c+2)^2N_c^3\left((N_c-2)e^{\frac{Q_{s1}^2r^2}{4}} - 2(N_c-1)\right)e^{-\frac{r^2\left(N_cQ_{s1}^2+2(N_c-1)Q_{s2}^2\right)}{4N_c}} \\
& + (N_c+3)(N_c-1)^3(N_c-2)^2N_c^3\left((N_c+2)e^{\frac{Q_{s1}^2r^2}{4}} - 2(N_c+1)\right)e^{-\frac{r^2\left(N_cQ_{s1}^2+2(N_c+1)Q_{s2}^2\right)}{4N_c}} \\
& + 4(N_c^2-8)(N_c^2-1)^3(N_c^2+4)e^{-\frac{1}{4}r^2(Q_{s1}^2+Q_{s2}^2)} \\
& + \frac{1}{2}(N_c-2)^2(N_c-1)^3(N_c+3)N_c^4e^{-\frac{(N_c+1)r^2\left(Q_{s1}^2+Q_{s2}^2\right)}{2N_c}} \\
& \left. + \frac{1}{2}(N_c-3)(N_c+1)^3(N_c+2)^2N_c^4e^{-\frac{(N_c-1)r^2\left(Q_{s1}^2+Q_{s2}^2\right)}{2N_c}}\right] + [1\leftrightarrow 2] \\
& \left. + 2\left(N_c^2-4\right)^2(N_c^6+2N_c^4-19N_c^2+8)\right)
\end{aligned}$$

notation $\varepsilon_0(x_\perp)$ for initial density, instead of $\rho(z)$ in previous slides

With: $p_{1,2} \equiv e^{-\frac{Q_{s1,2}^2r^2}{4}}(Q_{s1,2}^2r^2+4) - 4$

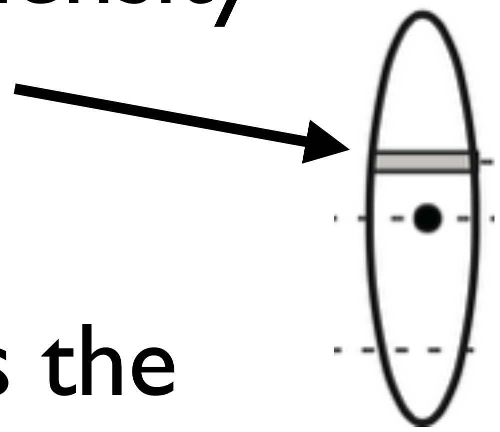
$q_{1,2} \equiv e^{-\frac{Q_{s1,2}^2r^2}{4}}(Q_{s1,2}^4r^4 + 8Q_{s1,2}^2r^2 + 32) - 32.$

and the saturation scale: $\frac{r^2Q_s^2}{4} = g^2\frac{N_c}{2}\Gamma(r_\perp)\bar{\lambda}(b_\perp)$

borrowed from Pablo Guerrero Rodríguez

But simple results eventually

- The only dimensionful quantities are the saturation scales (momenta) of the two nuclei Q_{sA} and Q_{sB} at a given point in the transverse plane
- Q_{sA}^2 is in turn proportional to the nuclear density integrated over the longitudinal coordinate: thickness function T_A of the Glauber model.
- The only free parameter in the calculation is the proportionality coefficient.



One- and two-point functions in CGC

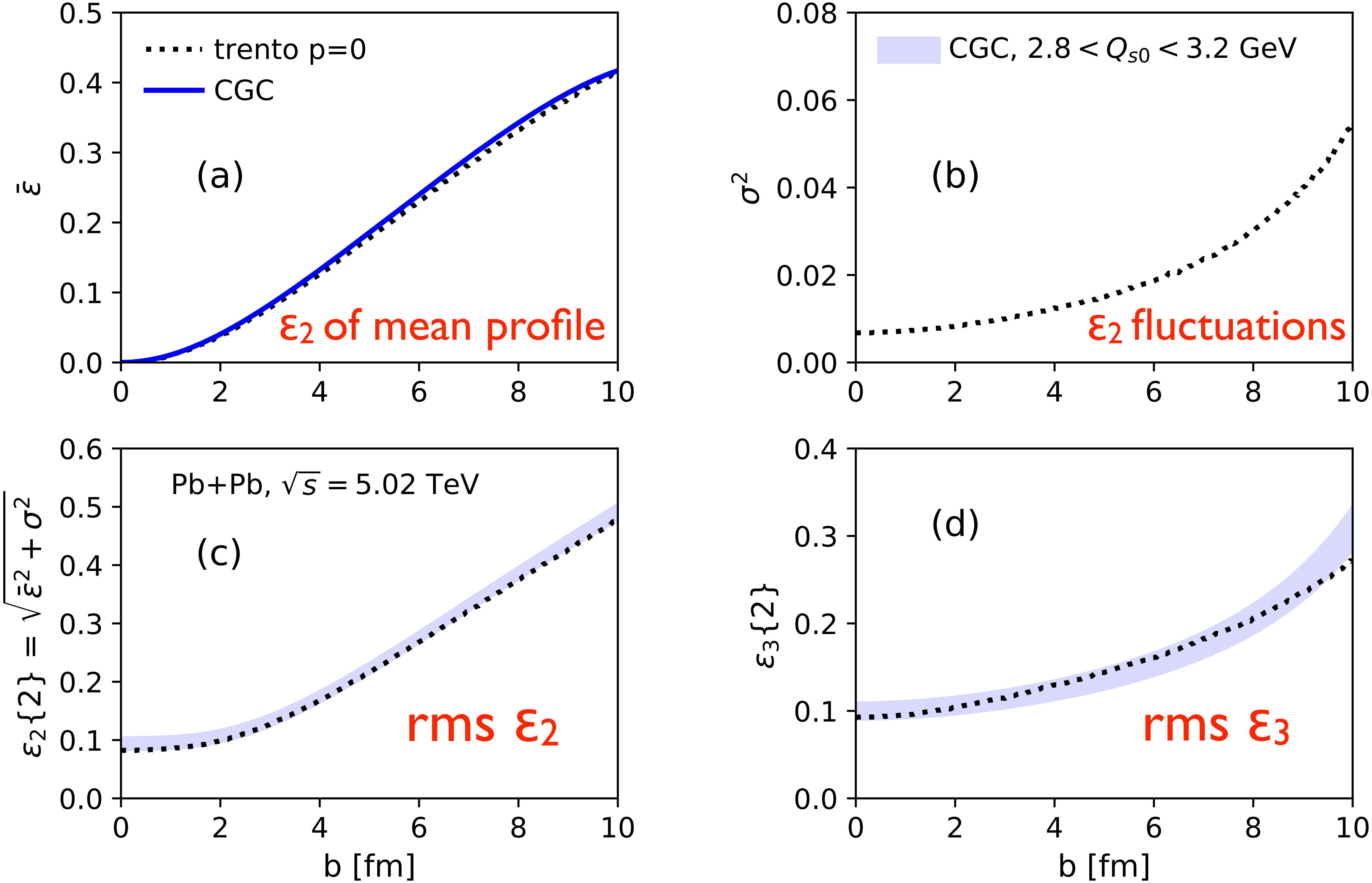
- $\langle \rho(z) \rangle = Q_{sA}^2(z) Q_{sB}^2(z)$,
  (up to a dimensionless proportionality constant)

Tuomas Lappi hep-ph/0606207

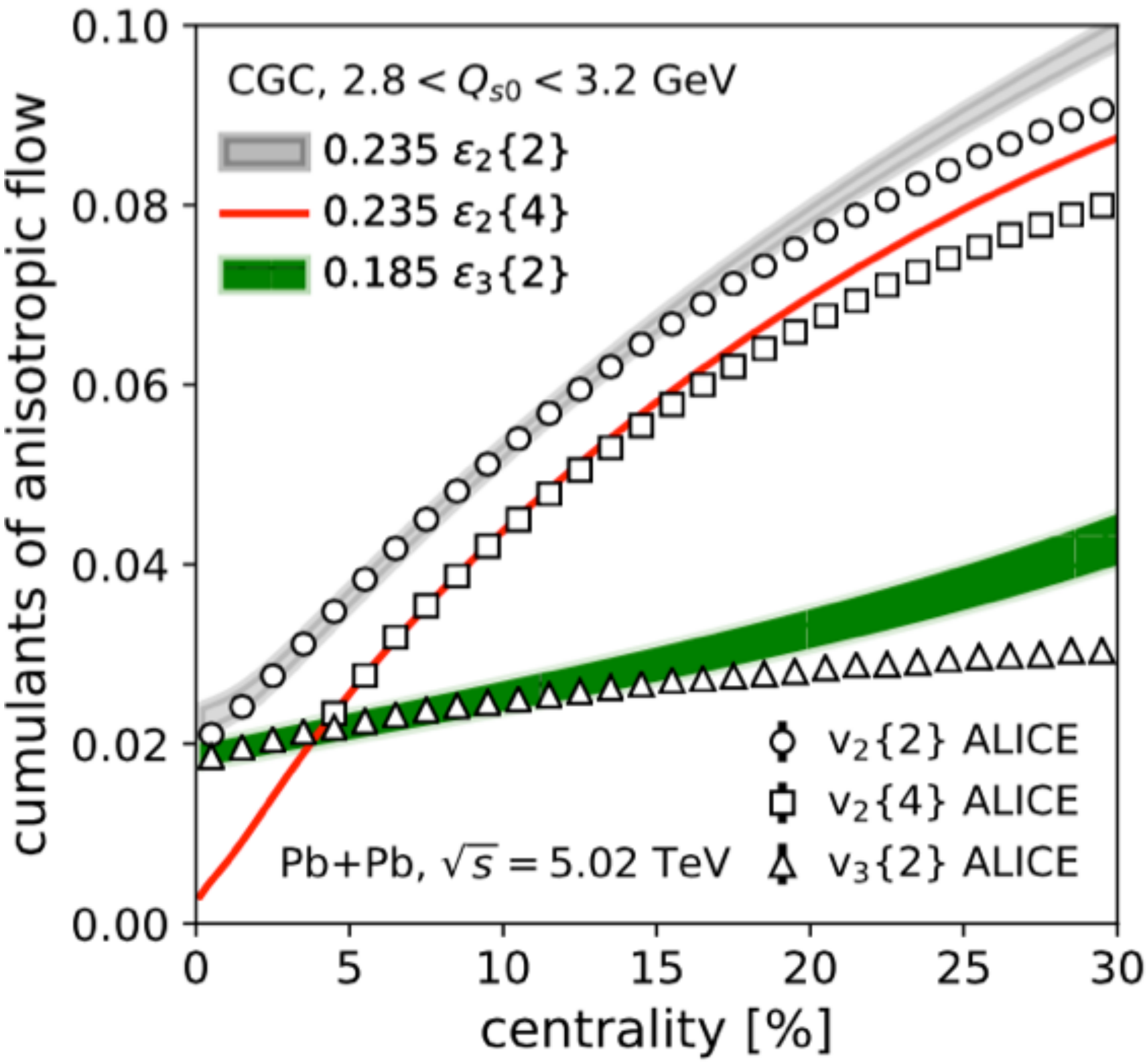
- The full structure of the two-point function is not needed. Correlation is short range, and we only need its integral over the relative coordinate $r=z_1-z_2$:
  $\int S(z+r/2, z-r/2)\, dr = Q_{sA}^2(z) Q_{sB}^2(z) (Q_{sA}^2(z) + Q_{sB}^2(z))$
  (up to a dimensionless proportionality constant)

- Inserting into the general expressions of Blaizot & Broniowski, one obtains initial eccentricities.

Results for eccentricities



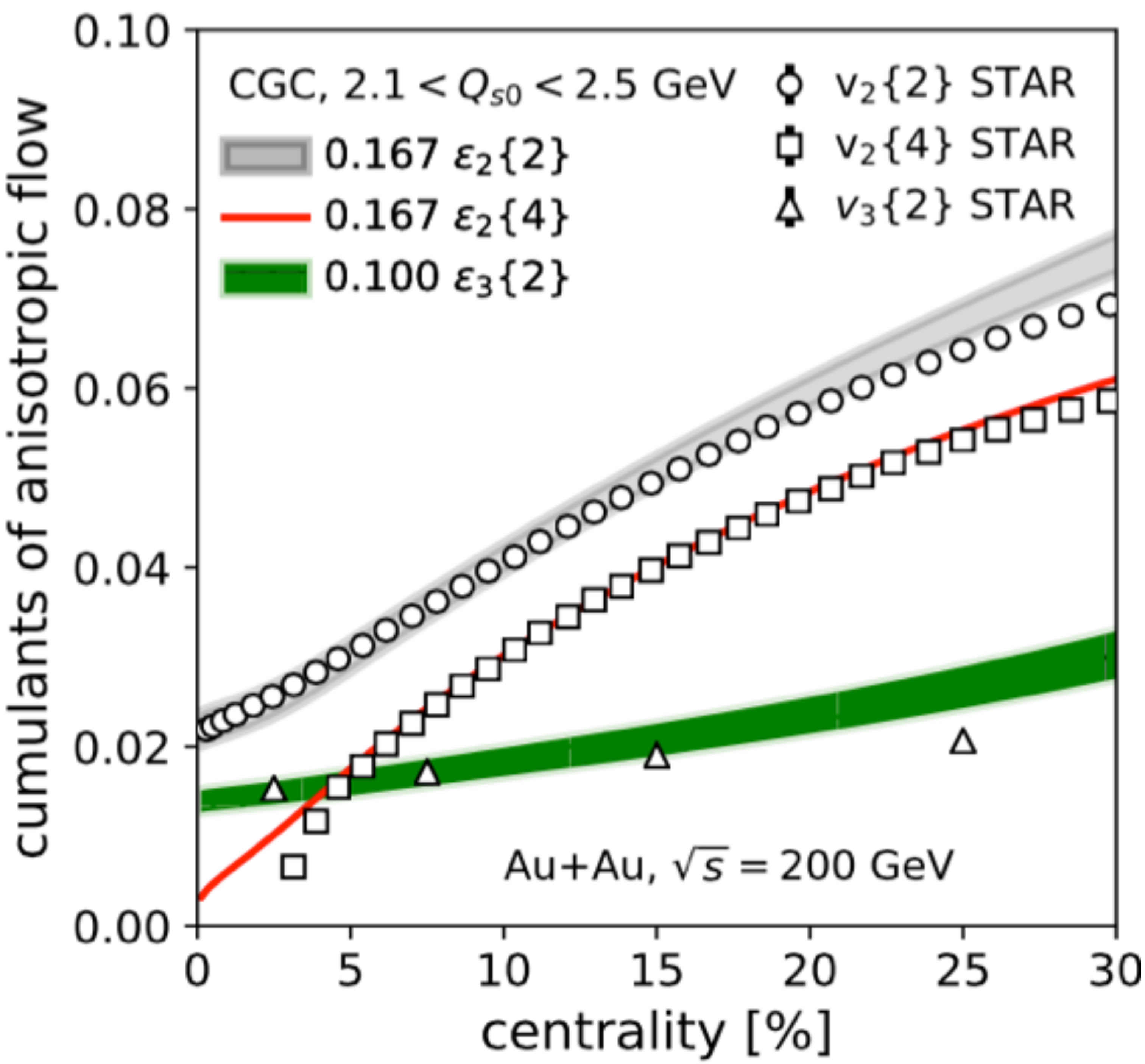
Application to LHC data



3 parameters:
1 saturation scale
and 2 response coeff.
all values in ballpark
of expectations

CGC initial conditions agree with data. Viscous damping likely to explain small discrepancies at large centralities.

Application to RHIC data



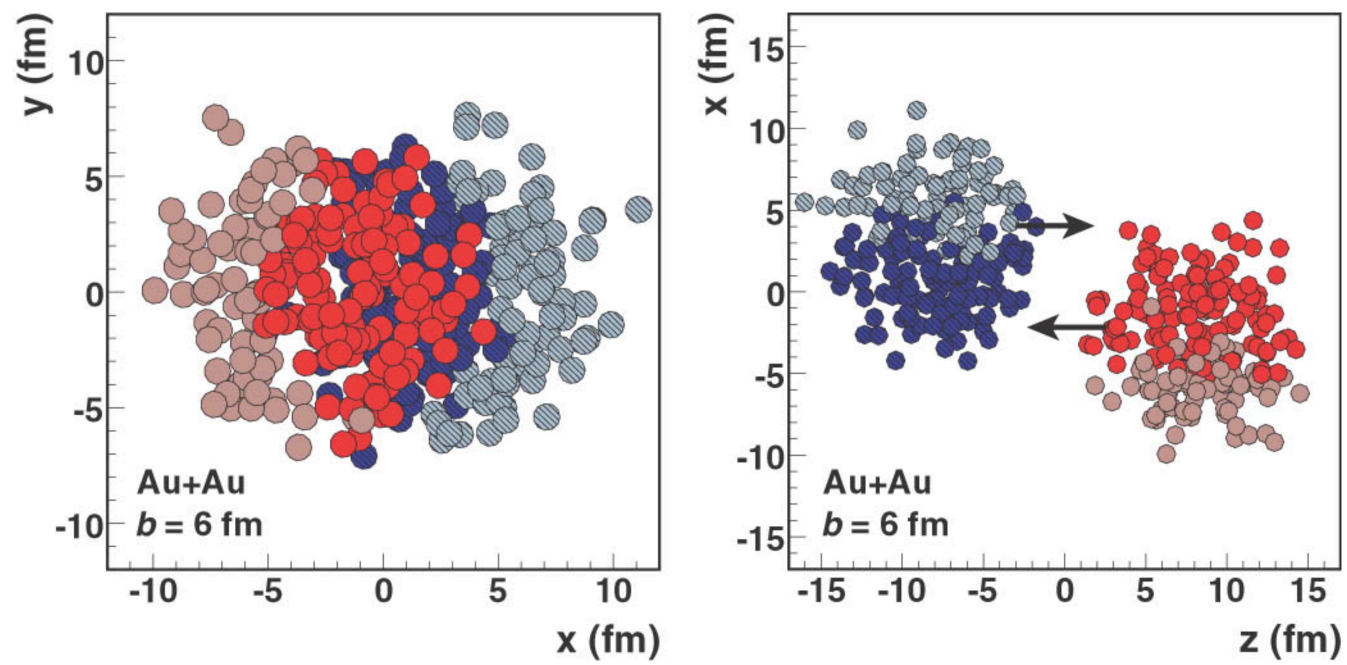
RHIC data also well described.

v_2 fluctuations larger than at LHC: naturally explained by smaller saturation scale

Response coefficients smaller than at LHC as expected from hydro.

New paradigm for fluctuations

- It is traditionally assumed that event-by-event fluctuations of the density originate from the position of the nucleons within nuclei, *à la* Glauber



- These fluctuations are not taken into account in the Glasma calculation.

New paradigm for fluctuations

- In this calculation, fluctuations are solely due to small-x QCD dynamics.
- The smaller the saturation scale, the larger these fluctuations.
- The values of the saturation scale returned by our fits to data are already fairly large.
- This leaves little room for « traditional » fluctuations coming from nucleon positions.

Conclusions

- Recent ab-initio calculations of the fluctuations of the initial energy density explain the magnitude and centrality dependence of v_2 and v_3.
- QCD evolution naturally explains why elliptic flow fluctuations are smaller at LHC than at RHIC.
- A new paradigm for event-to-event fluctuations, which no longer involves nucleon positions.