Measuring the Rate of Isotropization of Quark-Gluon Plasma Using Rapidity Correlations

Krakow, Poland, 8 January 2019

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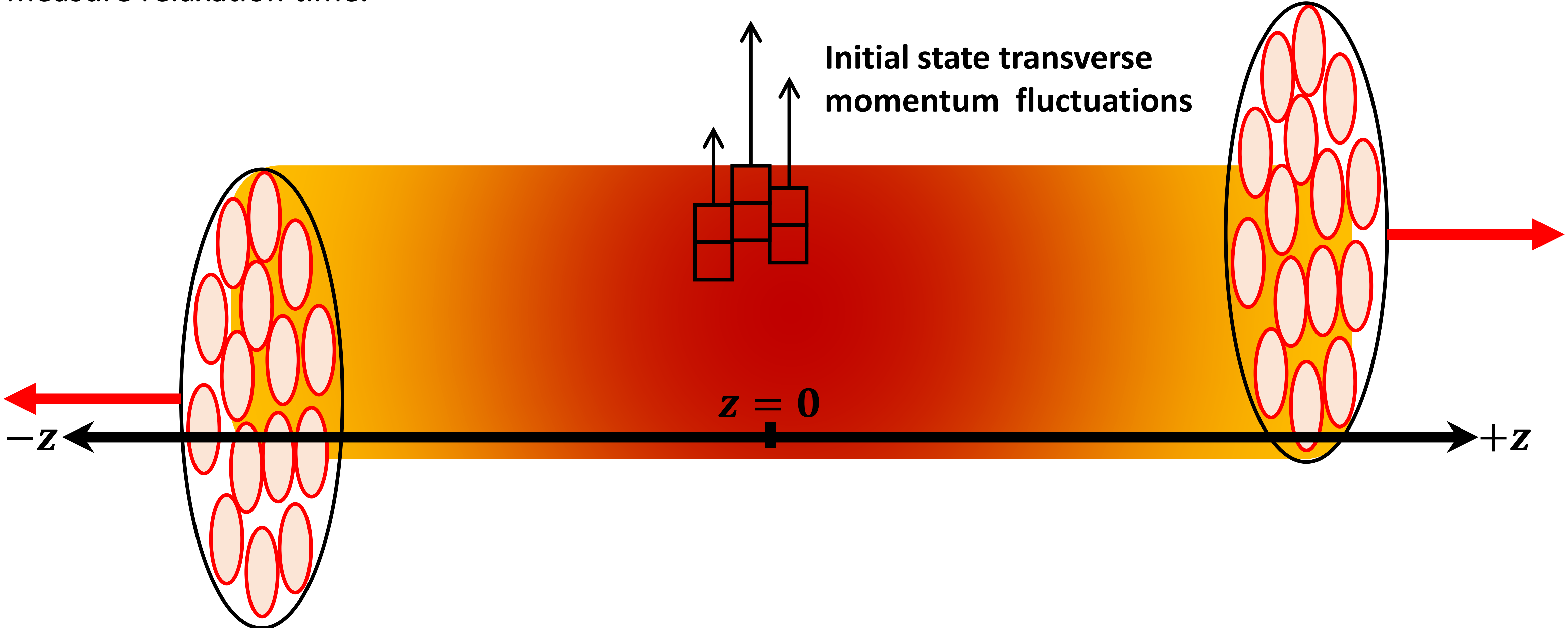
Fluctuations in Nuclear Collisions

Damping of transverse flow fluctuations can be used to measure viscosity.

Gavin & Abdel-Aziz, Phys. Rev. Lett. 97 (2006) 162302

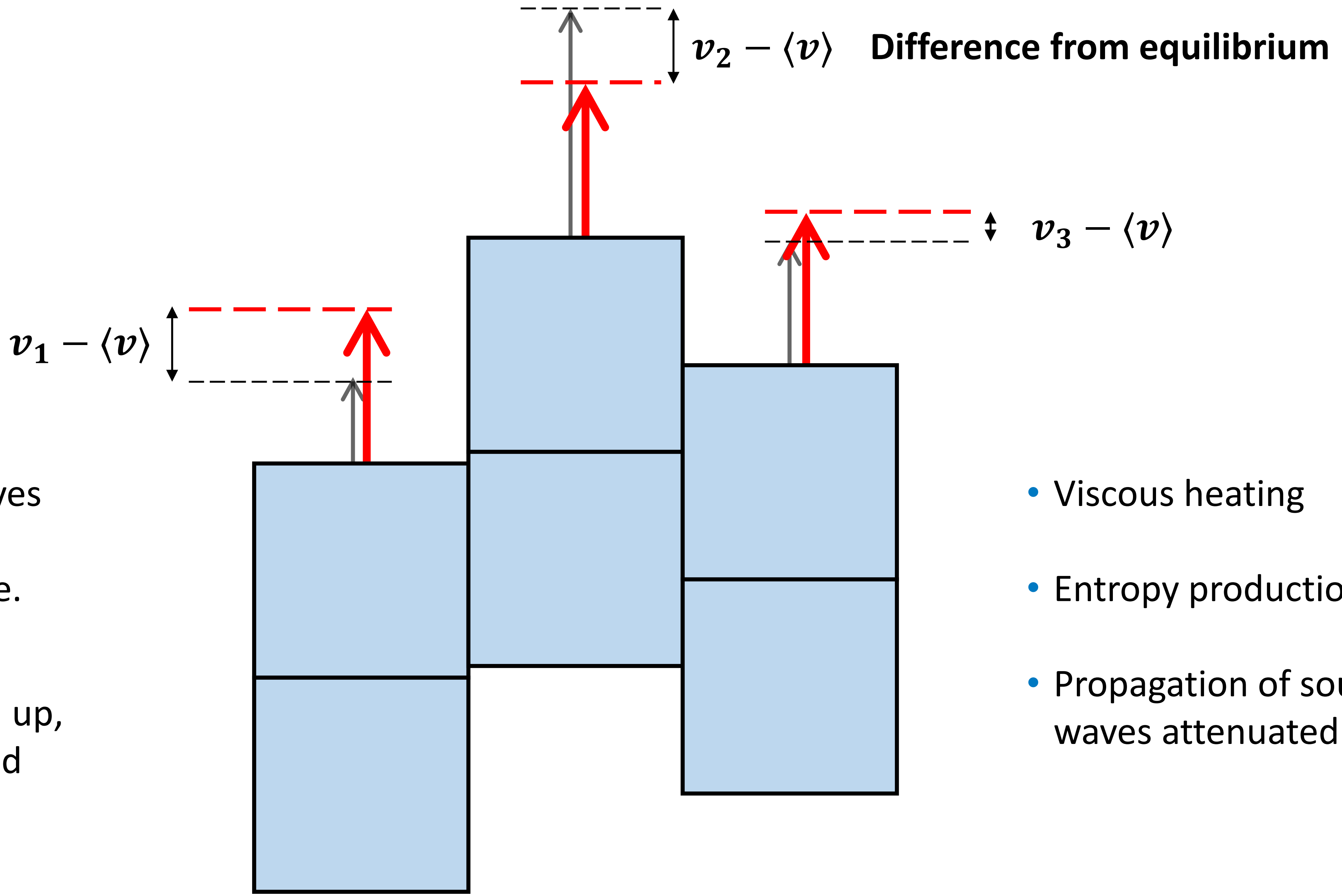
Rapidity dependence of flow fluctuations can measure relaxation time.

Gavin, GM, Zin, Phys. Rev. C94 (2016) no.2, 024921



Fluctuations from Equilibrium

$$T_{zr} = -\eta \frac{\partial v_r}{\partial z}$$



Shear viscosity drives the fluid velocities toward the average.

Slow cells are sped up, fast cells are slowed down.

- Viscous heating
- Entropy production
- Propagation of sound waves attenuated

Momentum in Fluctuating Hydrodynamics

- **Momentum current** – small fluctuations $M_i \equiv T_{0i} - \langle T_{0i} \rangle$
- **Momentum conservation** linearized Navier-Stokes $$\frac{\partial M_i}{\partial t} + \nabla_i p = \frac{\eta/3 + \zeta}{sT} \nabla_i (\vec{\nabla} \cdot \vec{M}) + \frac{\eta}{sT} \nabla^2 M_i$$
- **Helmholtz decomposition** $\vec{M} = \vec{g} + \vec{h}$
- **“longitudinal” mode** (curl free part) $\vec{\nabla} \times \vec{h} = 0$
- **“transverse” mode** (divergence free part) $\vec{\nabla} \cdot \vec{g} = 0$

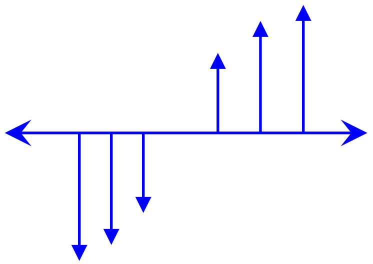
The Shear Mode and Noise

- **Momentum conservation**
  linearized Navier-Stokes

$$\partial_i M_i + \nabla_i p = \frac{\eta/3 + \zeta}{sT} \nabla_i(\vec{\nabla} \cdot \vec{M}) + \frac{\eta}{sT} \nabla^2 M_i \qquad \bigstar$$

- **Viscous diffusion of divergence free modes**

$$\vec{\nabla} \times (\star) \rightarrow \qquad \boldsymbol{\partial_t \vec{g} = \nu \nabla^2 \vec{g}}$$



- kinematic viscosity $\nu = \eta/Ts$

- **Dissipative parts modified by noise**

$$\boldsymbol{T_{ji}^{diss} \approx -\nu \nabla_j g_i + \mathbf{noise}}$$

$$\boldsymbol{\partial_t g_i = \nu \nabla^2 (g_i + noise)}$$

People are working on this problem in general. See:
Akamatsu, Mazeliauskas, Teaney,
Ohnishi, Kitazawa, Asakawa,
Stephanov, Kapusta, Mueller,
Plumberg, Pratt, Young, Schlichting

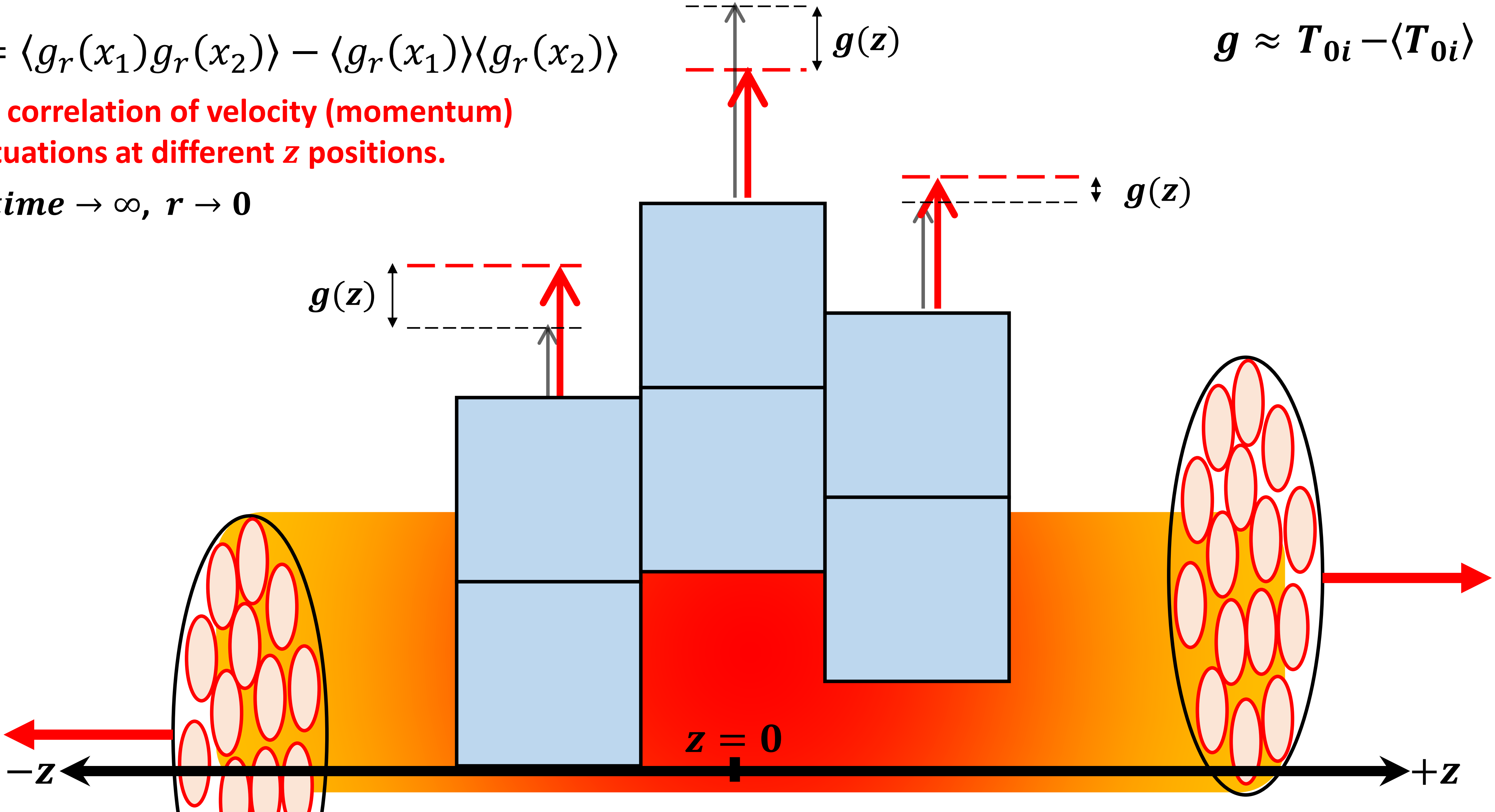
Comparing Changes in Fluctuations with Position

$$r = \langle g_r(x_1) g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle$$

$\boldsymbol{r}$ = correlation of velocity (momentum) fluctuations at different z positions.

As $\boldsymbol{time} \to \infty$, $\boldsymbol{r} \to 0$

$$\boldsymbol{g} \approx \boldsymbol{T_{0i}} - \langle \boldsymbol{T_{0i}} \rangle$$

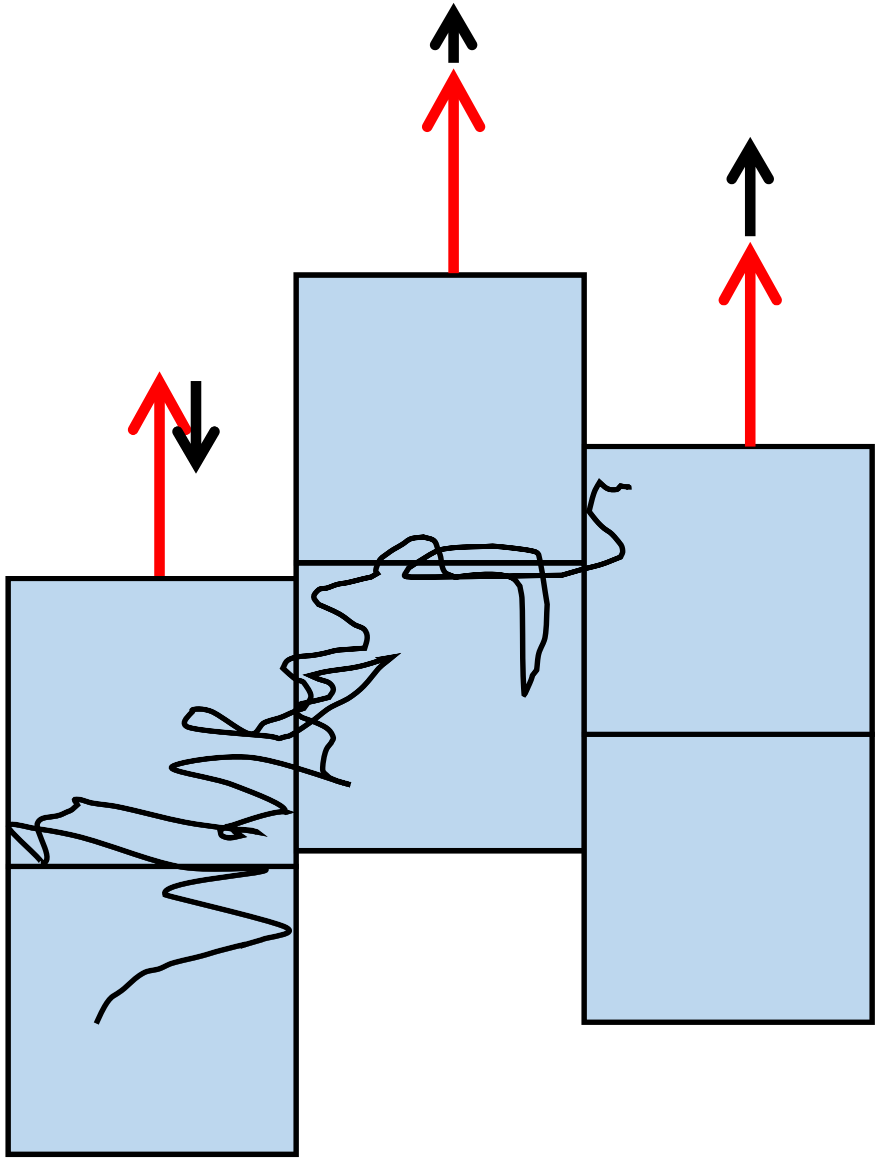


Fluctuations from Noise

- **No noise:**
  as $time \to \infty$, $r \to 0$

- **Including noise:**
  as $time \to \infty$, $r \to r_{eq}$

- **The difference**
  $\Delta r = r - r_{eq}$
  still satisfies a diffusion equation



Diffusion of Momentum Correlations

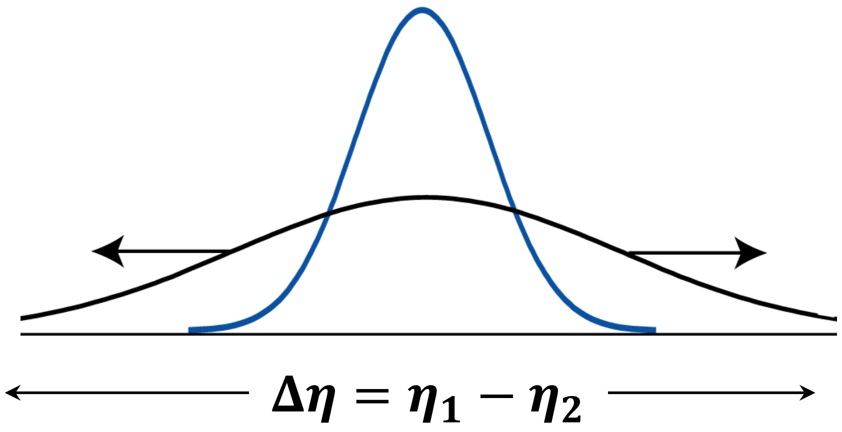
- **Momentum flux density correlation function** $$r = \langle g_r(x_1) g_r(x_2)\rangle - \langle g_r(x_1)\rangle\langle g_r(x_2)\rangle$$

- **The difference $\boldsymbol{\Delta r = r - r_{eq}}$ still satisfies a diffusion equation** $$\left(\frac{\partial}{\partial\tau} - \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial\eta_1^2} + \frac{\partial^2}{\partial\eta_2^2}\right)\right)\Delta r = 0$$

Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

- **fluctuations diffuse through volume, driving $\boldsymbol{r \rightarrow r_{eq}}$**

width in relative spatial rapidity grows from initial value σ_0



Measuring Correlations

- **Momentum flux density correlation function** $r = \langle g_r(x_1) g_r(x_2)\rangle - \langle g_r(x_1)\rangle\langle g_r(x_2)\rangle$

- **The difference $\boldsymbol{\Delta r = r - r_{eq}}$ still satisfies a diffusion equation**

$$\left(\frac{\partial}{\partial\tau} - \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial\eta_1^2} + \frac{\partial^2}{\partial\eta_2^2}\right)\right)\Delta r = 0$$

Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

- **observable:** $C = \frac{1}{\langle N\rangle^2}\left\langle \sum_{pairs} p_{ti} p_{tj} \right\rangle - \langle p_t\rangle^2 = \frac{1}{\langle N\rangle^2}\int \Delta r \, dx_1 \, dx_2$

- **assumptions:**

1) only “transverse” shear modes
2) proper-time freeze out

Abdel-Aziz & Gavin., PRL 97 (2006) 162302; PR C70 (2004) 034905
Pratt, Schlichting, Gavin, Phys. Rev. C 84 (2011) 024909

Measuring Correlations

STAR Phys.Lett. B704 (2011) 467-473 arXiv:1106.4334

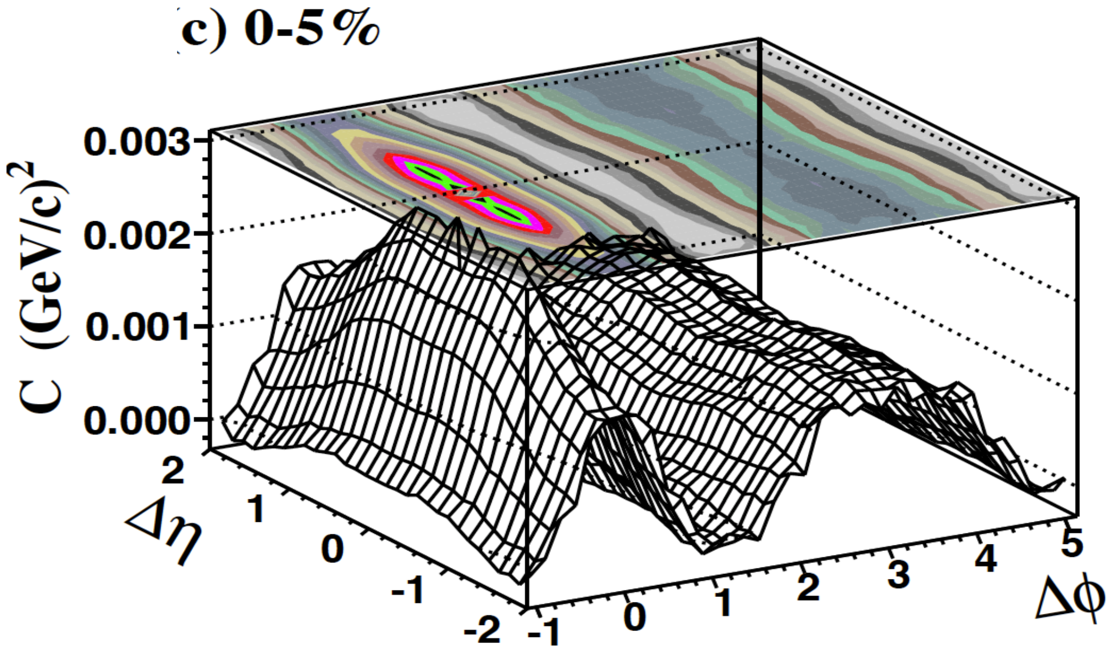
measured: rapidity width of near side peak

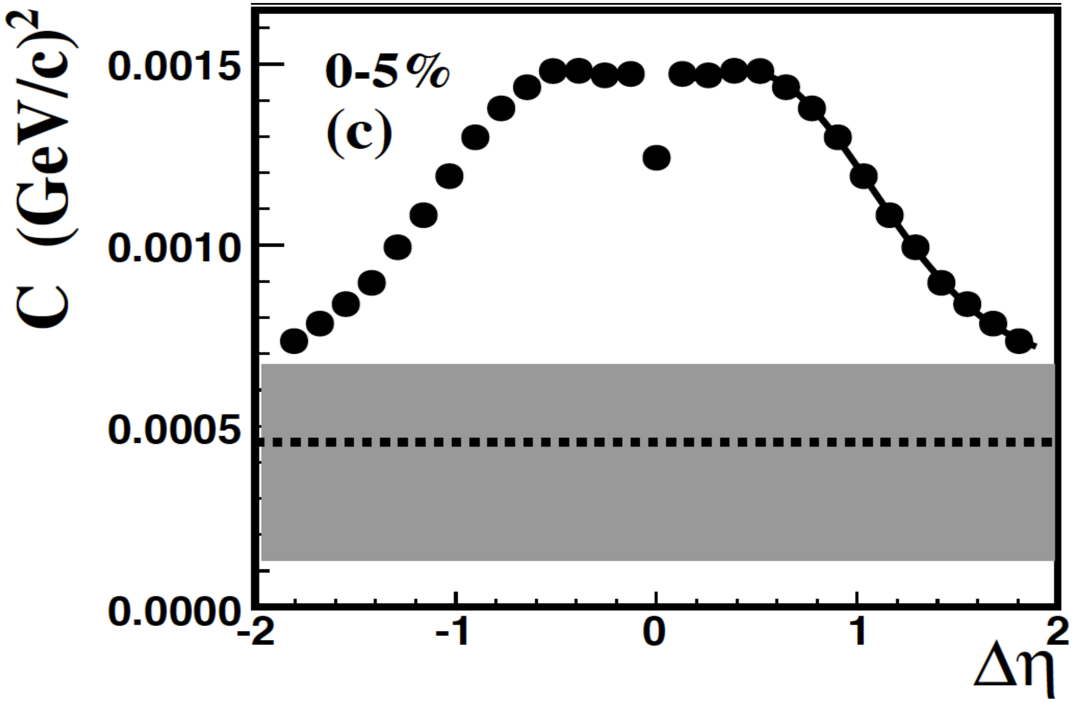
- fit peak + constant offset
- offset is ridge, i.e., long range rapidity correlations
- report RMS width of the peak

- **find:** width increases in central collisions

$$\sigma_{central} = 1.0 \pm 0.2$$

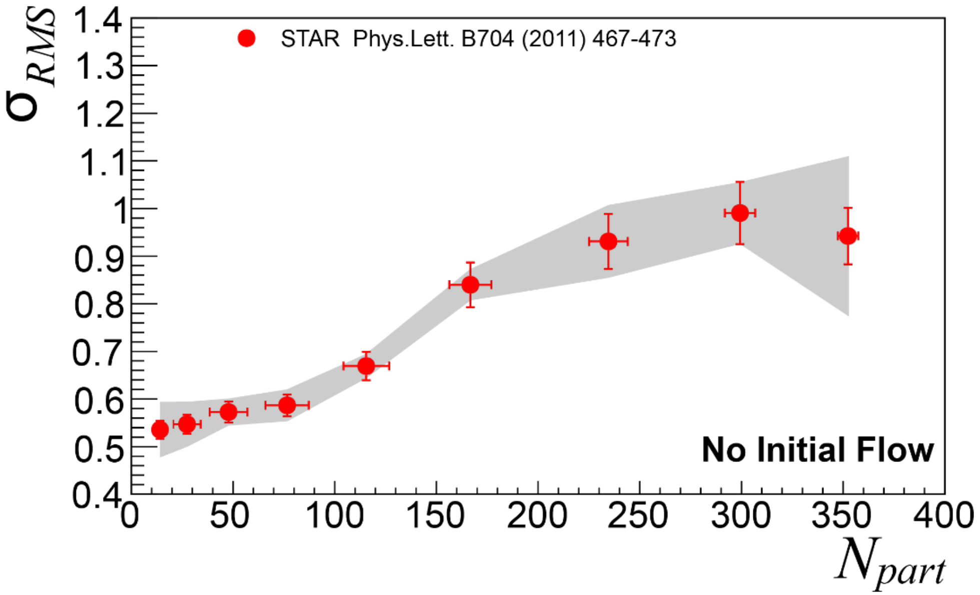
$$\sigma_{peripheral} = 0.54 \pm 0.02$$

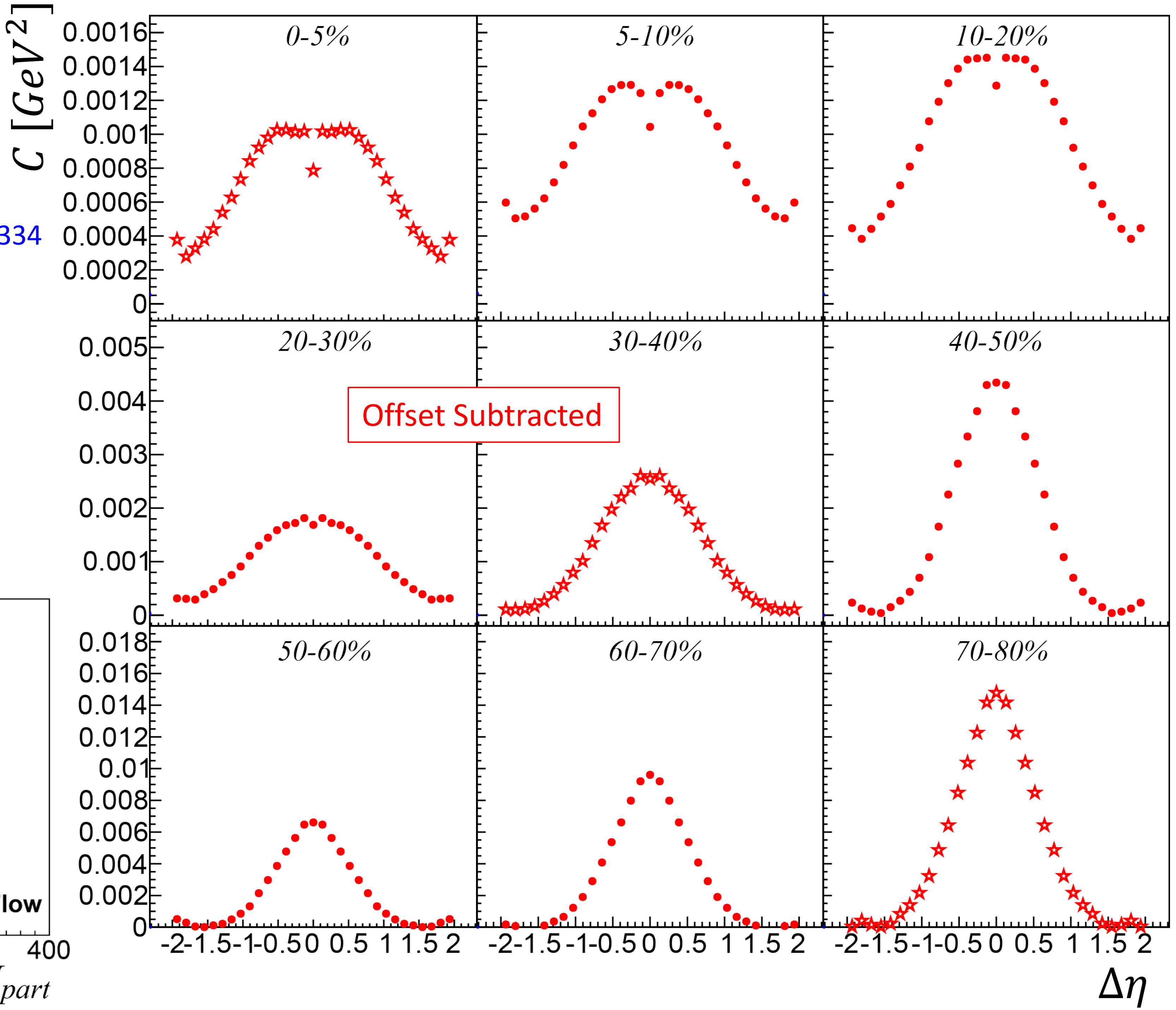




Longitudinal p_T Fluctuations

STAR Phys.Lett. B704 (2011) 467-473 arXiv:1106.4334





First Order Diffusion

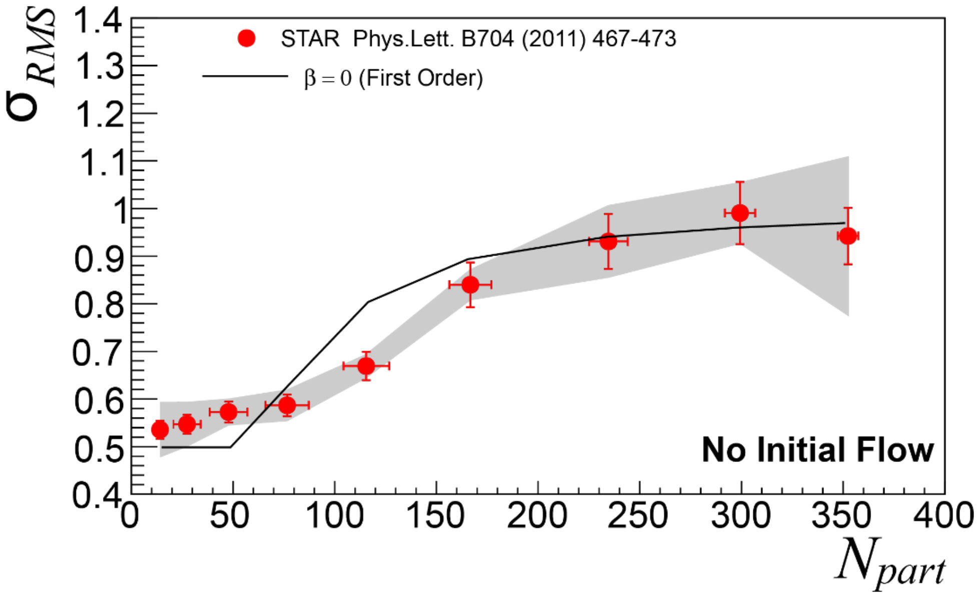
$\eta/s = 1/4\pi$

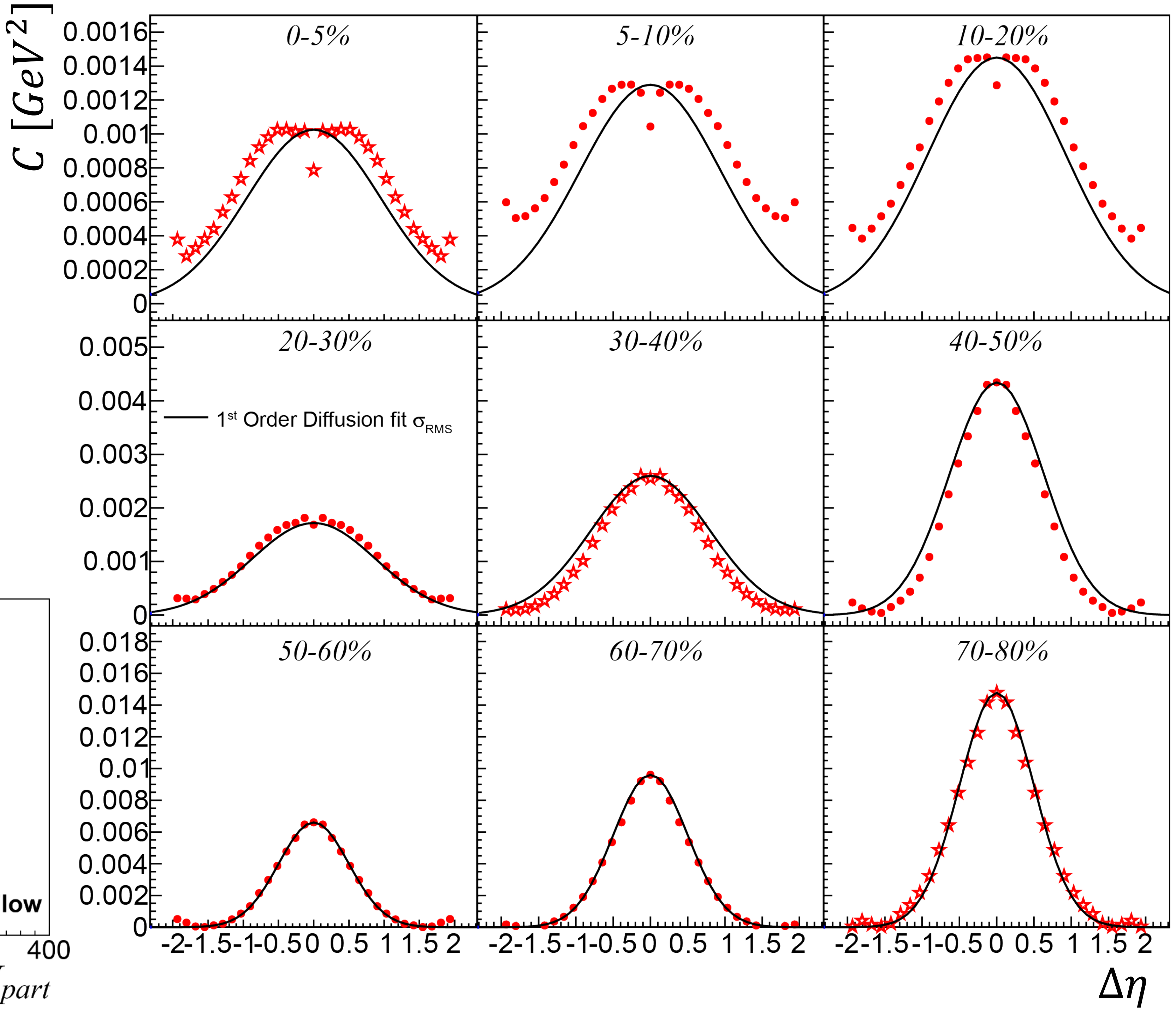
$\nu = \frac{\eta}{T_F s} = constant$

$T_F = 143\ MeV$

$\tau_0 = 0.6\ fm$

$\tau_{F,central} = 10\ fm$





Second Order Diffusion Equation for Correlations

$$\left[\frac{\tau_\pi}{2}\frac{\partial^2}{\partial\tau^2} + \left(1 + \frac{\kappa\tau_\pi}{\tau}\right)\frac{\partial}{\partial\tau} - \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial\eta_1^2} + \frac{\partial^2}{\partial\eta_2^2}\right)\right]\Delta r = 0$$

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Relaxation Time
$\boldsymbol{\tau_\pi = \beta\nu}$

Heating from gradients in velocities

Kinematic Viscosity
$\boldsymbol{\nu = \eta/Ts}$

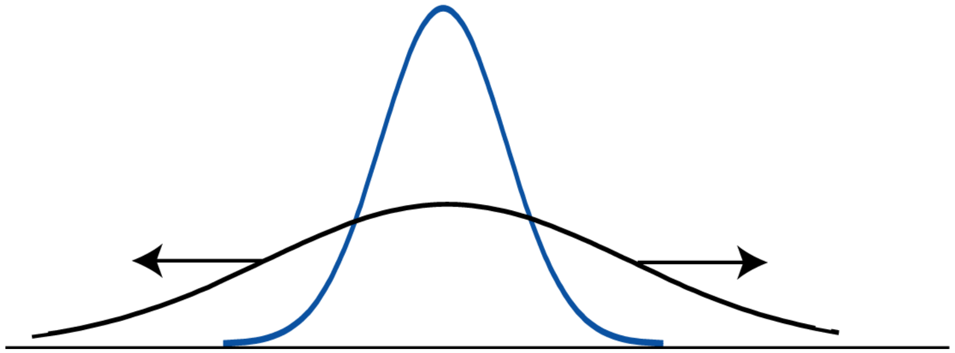
Second Order Diffusion Equation for Correlations

$$\left[\frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2}\right)\right] \Delta r = 0$$

Diffusion Equation

Diffusion (1st Order)

- Gaussian peak spreads
- tails violate causality



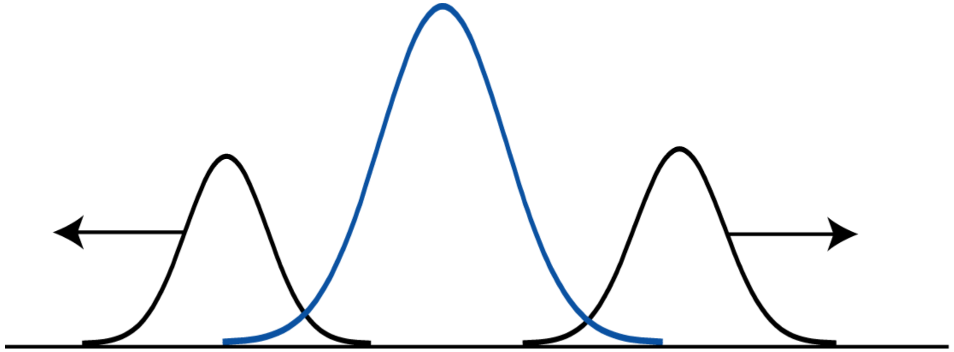
Second Order Diffusion Equation for Correlations

$$\left[\frac{\tau_\pi}{2}\frac{\partial^2}{\partial\tau^2} \qquad - \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial\eta_1^2}+\frac{\partial^2}{\partial\eta_2^2}\right)\right]\Delta r = 0$$

Wave Equation

Wave propagation

- peak splits into left and right traveling pulses
- propagation speed c_s

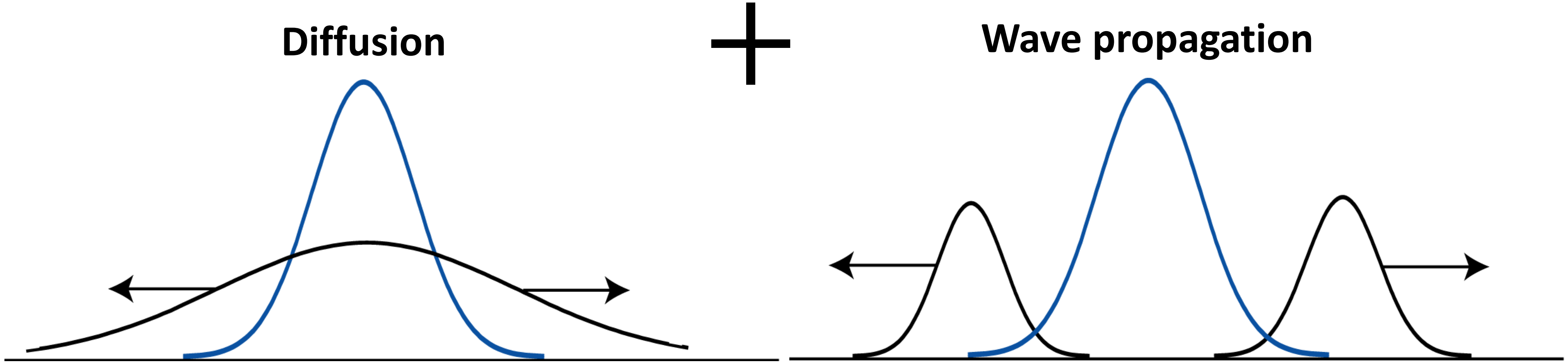


Second Order Diffusion Equation for Correlations

$$\left[\frac{\tau_\pi}{2}\frac{\partial^2}{\partial\tau^2} + \left(1 + \frac{\kappa\tau_\pi}{\tau}\right)\frac{\partial}{\partial\tau} - \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial\eta_1^2} + \frac{\partial^2}{\partial\eta_2^2}\right)\right]\Delta r = 0$$

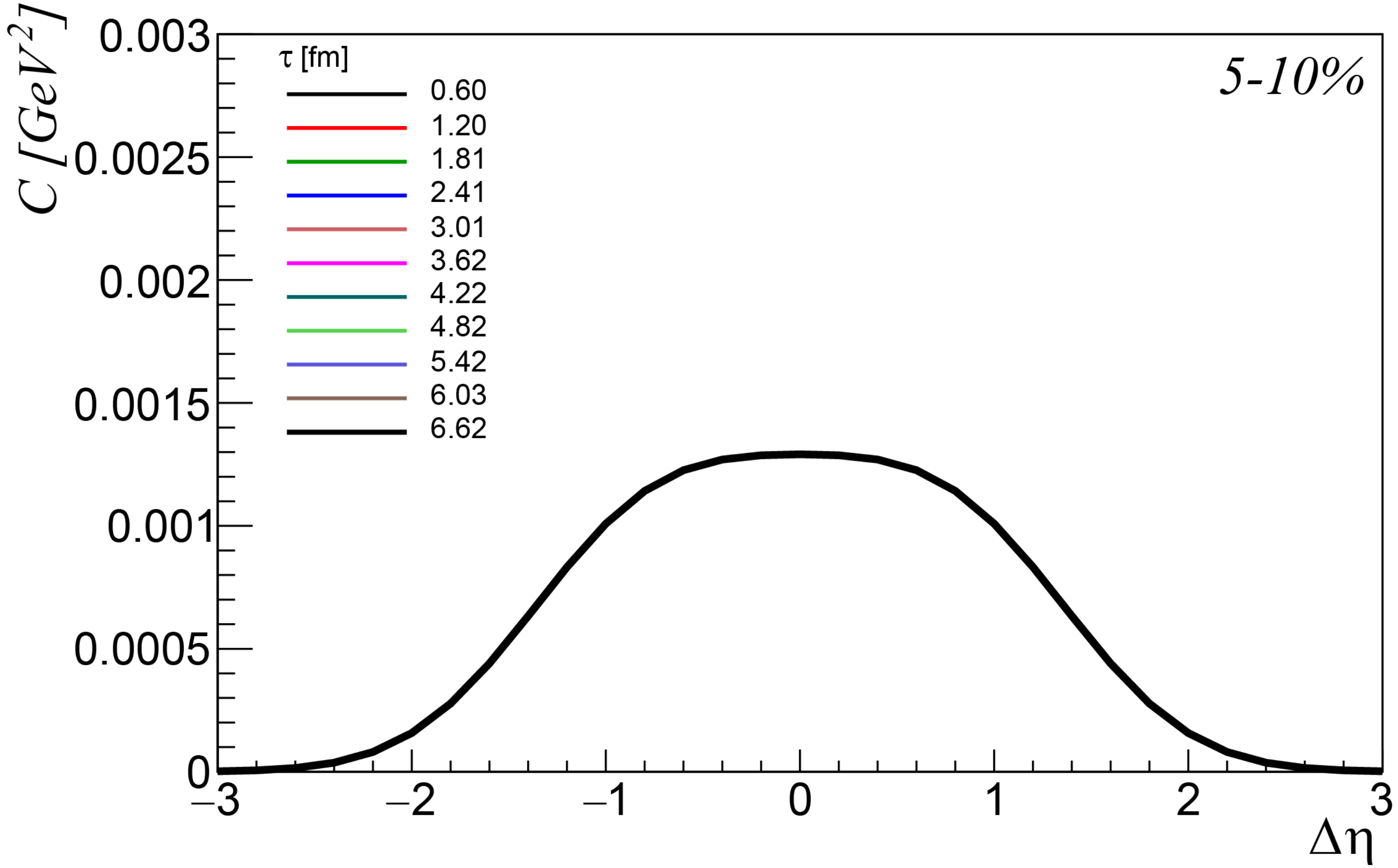
Relaxation Time $\boldsymbol{\tau_\pi = \beta\nu}$

- $\beta = 0$, diffusion only, waves move at infinite speed
- $\beta = 5$, predicted by kinetic theory of gas of massless particles
- Larger β means slower wave → slower changes in fluctuation correlations



The Effect of τ_π

$$\left[\frac{\tau_\pi}{2}\frac{\partial^2}{\partial\tau^2} + \left(1 + \frac{\kappa\tau_\pi}{\tau}\right)\frac{\partial}{\partial\tau} - \frac{\nu}{\tau^2}\left(2\frac{\partial^2}{\partial\Delta\eta^2} + \frac{1}{2}\frac{\partial^2}{\partial\eta_a^2}\right)\right]\Delta r = 0$$



$\kappa = 0$

$\beta = 10$

$\eta/s = 1/4\pi$

$\nu = \frac{\eta}{T_F s} = constant$

$T_F = 143\ MeV$

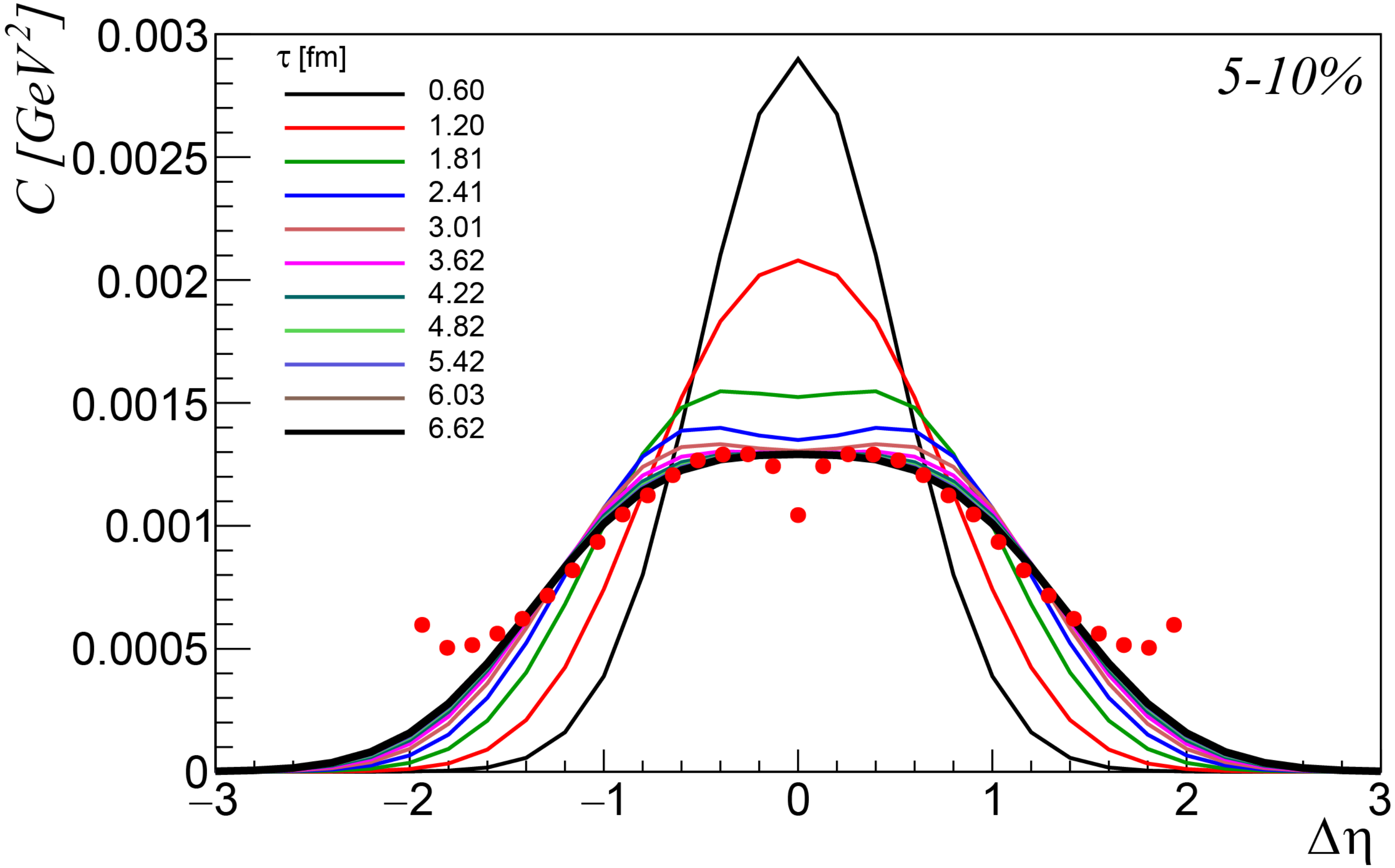
$\tau_0 = 0.6\ fm$

$\tau_{F,central} = 10\ fm$

No initial flow

Comparison to Experiment

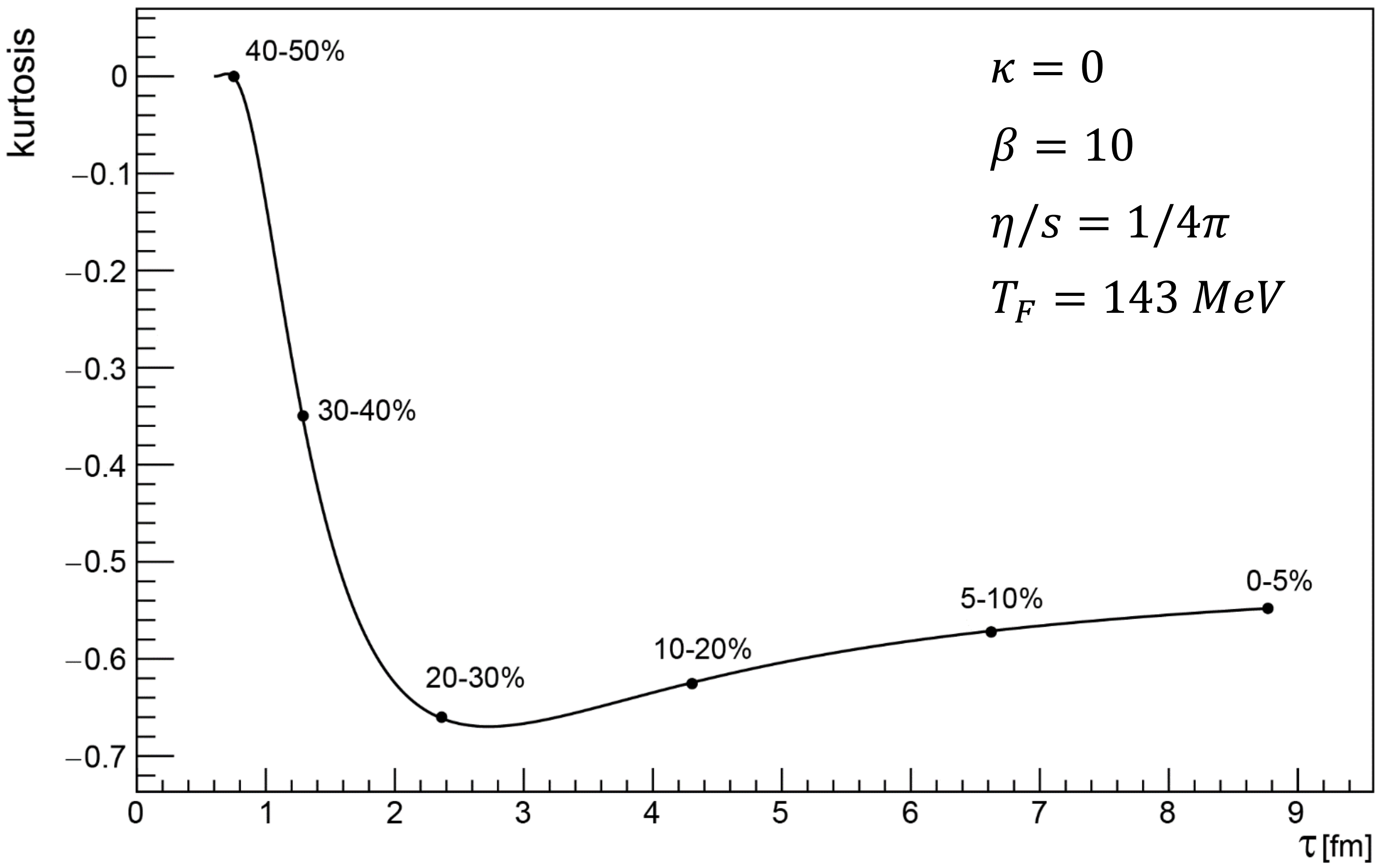
$$\left[\frac{\tau_\pi}{2}\frac{\partial^2}{\partial\tau^2} + \left(1 + \frac{\kappa\tau_\pi}{\tau}\right)\frac{\partial}{\partial\tau} - \frac{\nu}{\tau^2}\left(2\frac{\partial^2}{\partial\Delta\eta^2} + \frac{1}{2}\frac{\partial^2}{\partial\eta_a^2}\right)\right]\Delta r = 0$$



Data from the STAR Collaboration, Phys.Lett. B704 (2011) 467

Wave and Diffusion Competition

- For constant viscosity and $\kappa = 0$ every centrality follows the same time evolution
- Wave effects happen early and rapidly, diffusion influences show up later
- For this specific set of calculation conditions 20-30% collisions freeze out near the maximum (visible) effect of the wave behavior



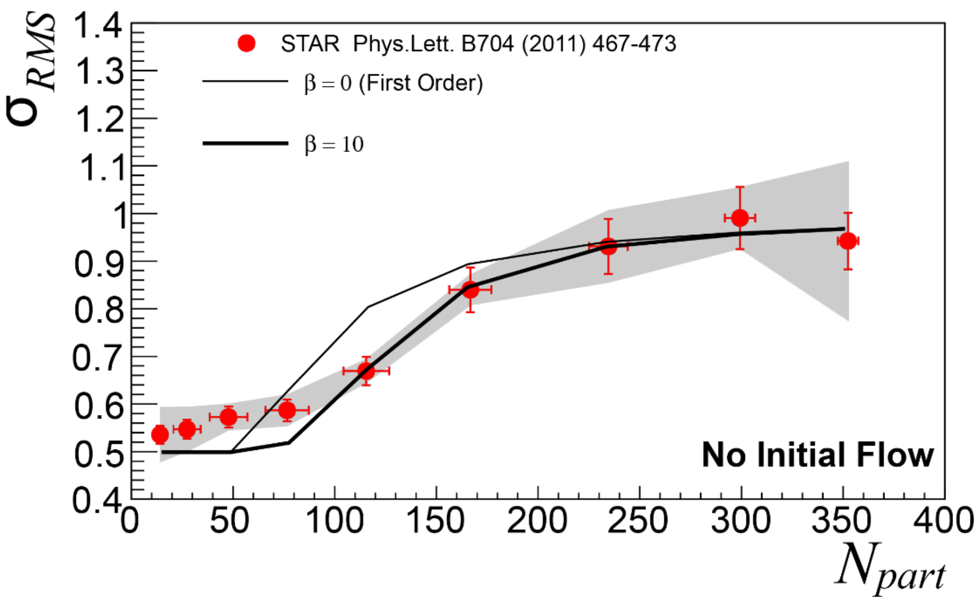
Second Order Diffusion

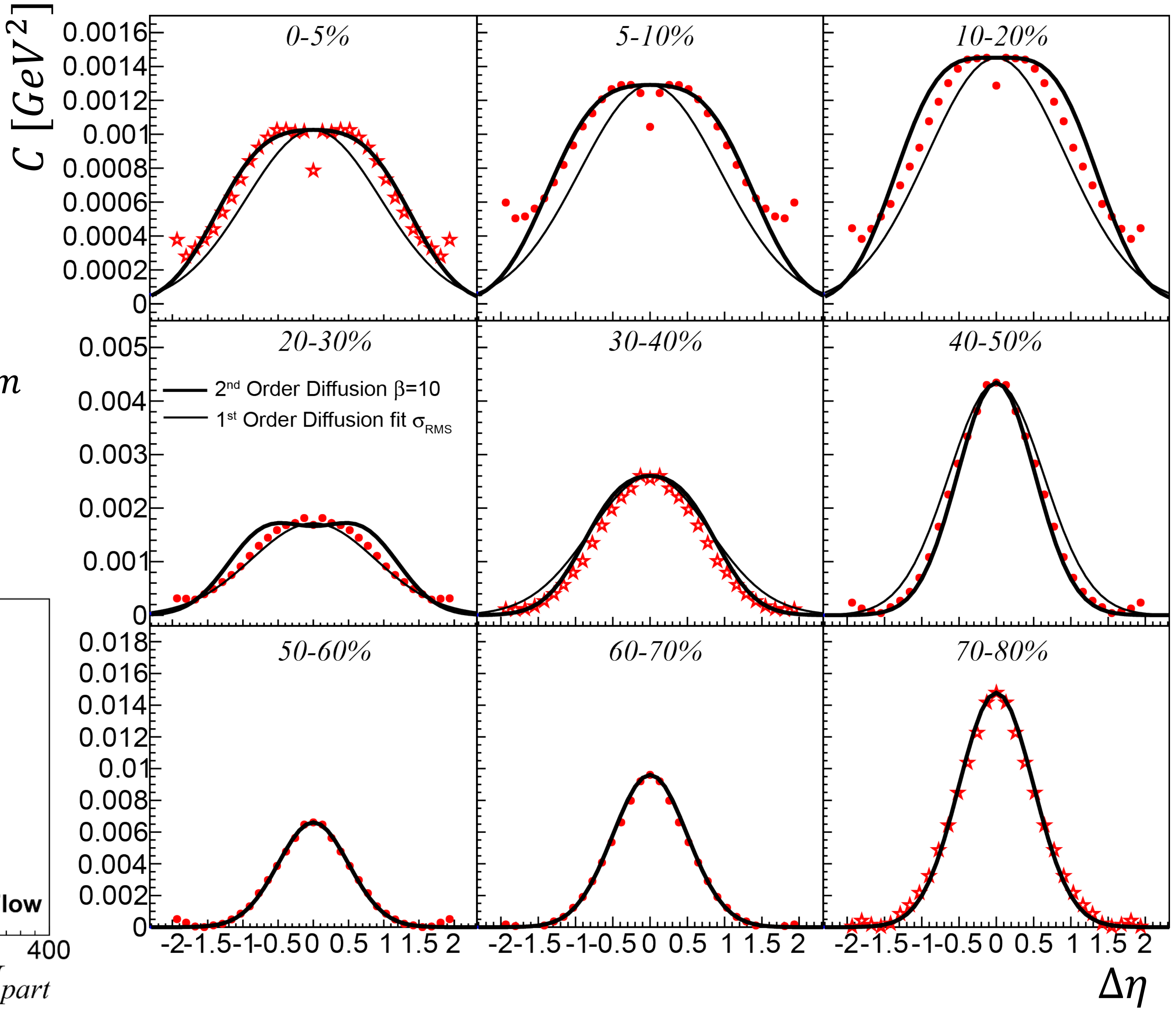
$\kappa = 0$ $T_F = 143\ MeV$

$\beta = 10$ $\tau_0 = 0.6\ fm$

$\eta/s = 1/4\pi$ $\tau_{F,central} = 10\ fm$

$$\nu = \frac{\eta}{T_F s} = constant$$





Evolving Parameters

- Entropy production due to viscous heating and longitudinal expansion.

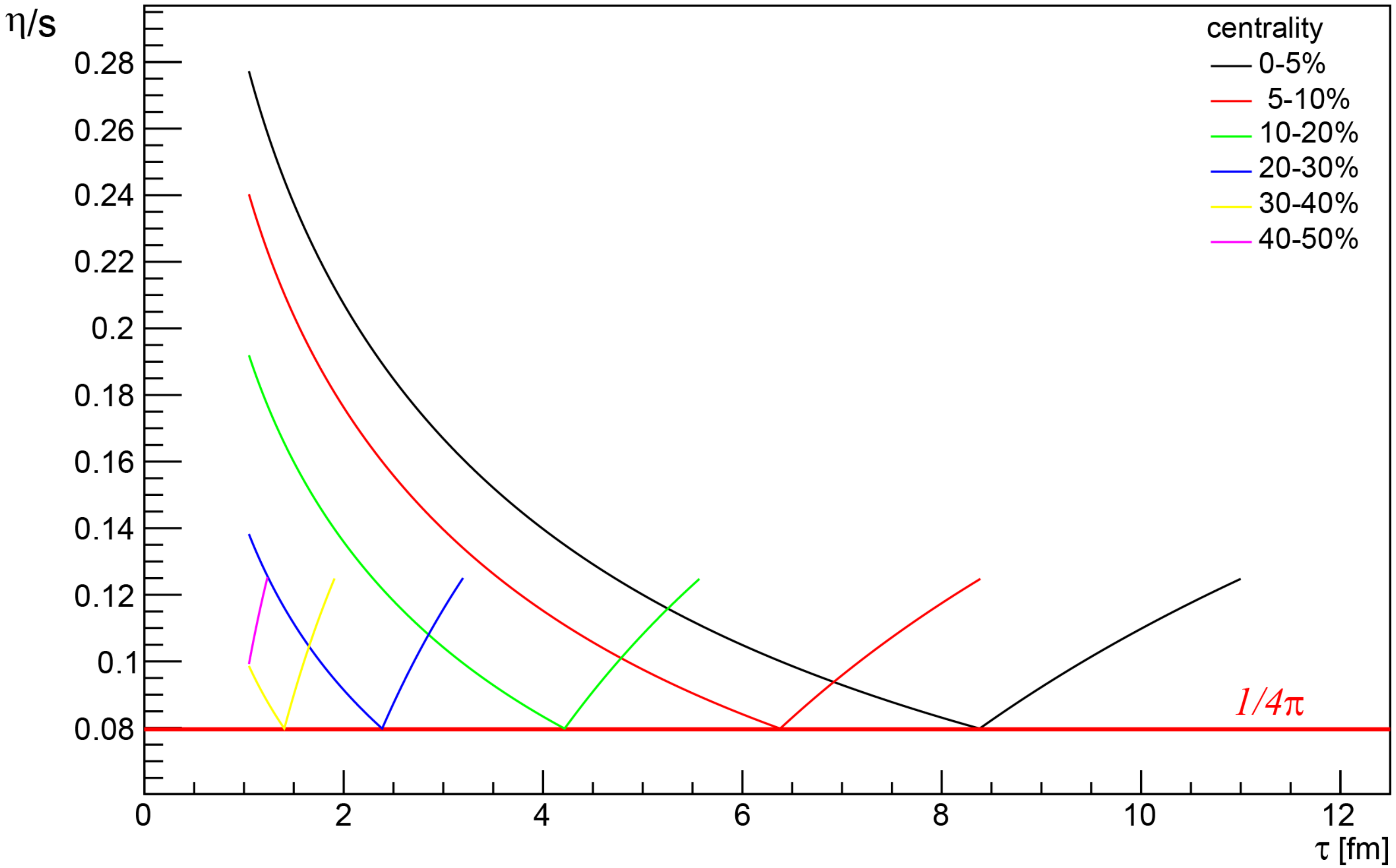
$$\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{\mathrm{T}\tau}$$

- Relaxation equation. Causality delays heating.

$$\frac{d\Phi}{d\tau} = -\frac{1}{\tau_\pi}\left(\Phi - \frac{4\eta}{3\tau}\right) - \frac{\kappa}{\tau}\Phi$$

- Coefficient associated with the gradient of speeds of fluid cells

$$\kappa = \frac{1}{2}\left\{1 + \frac{d\ln(\tau_\pi/\eta T)}{d\ln\tau}\right\}$$



η/s parameterization follows Phys. Rev. C86 (2012) 014909

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$$\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{T\tau}$$

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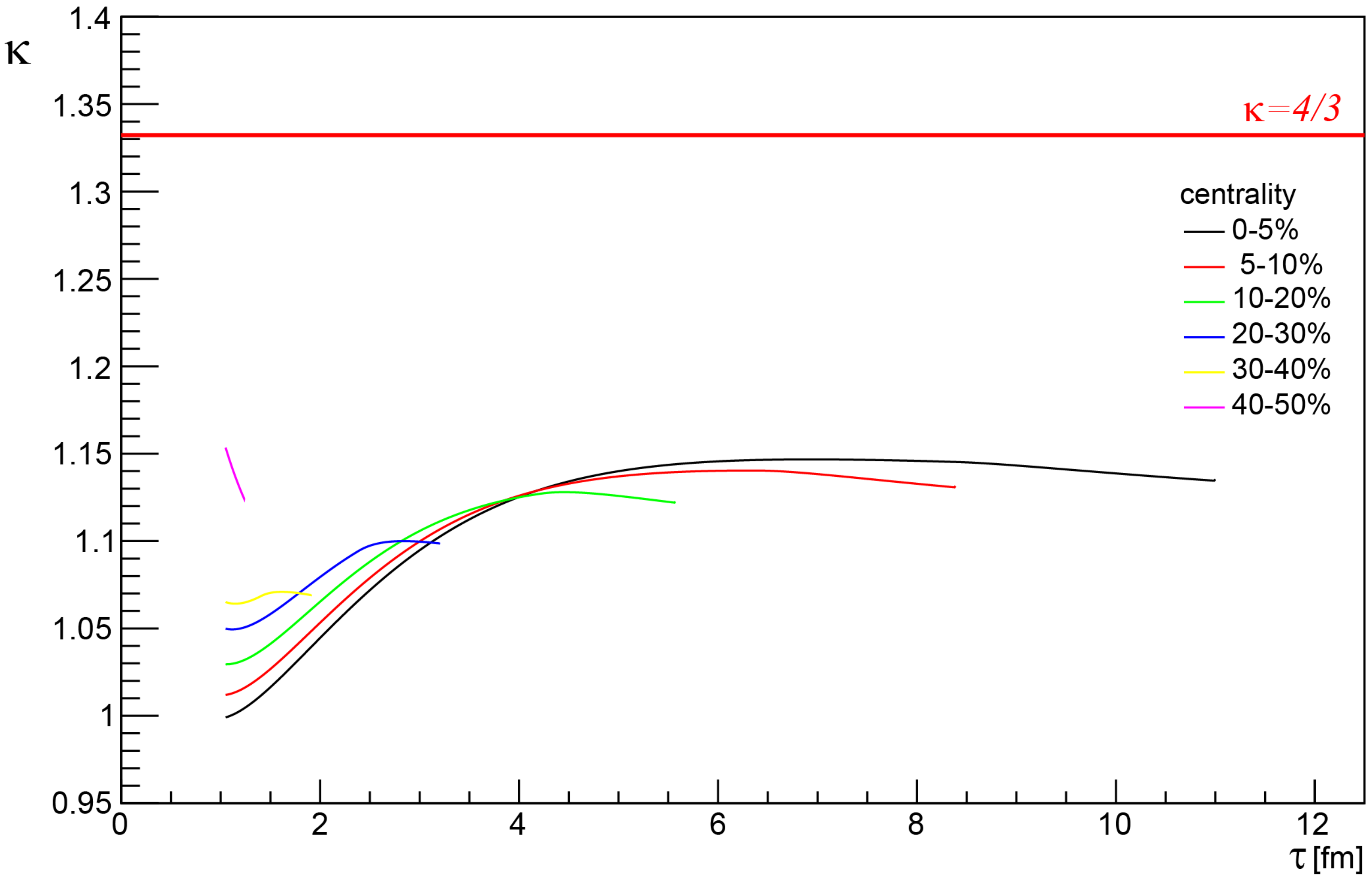


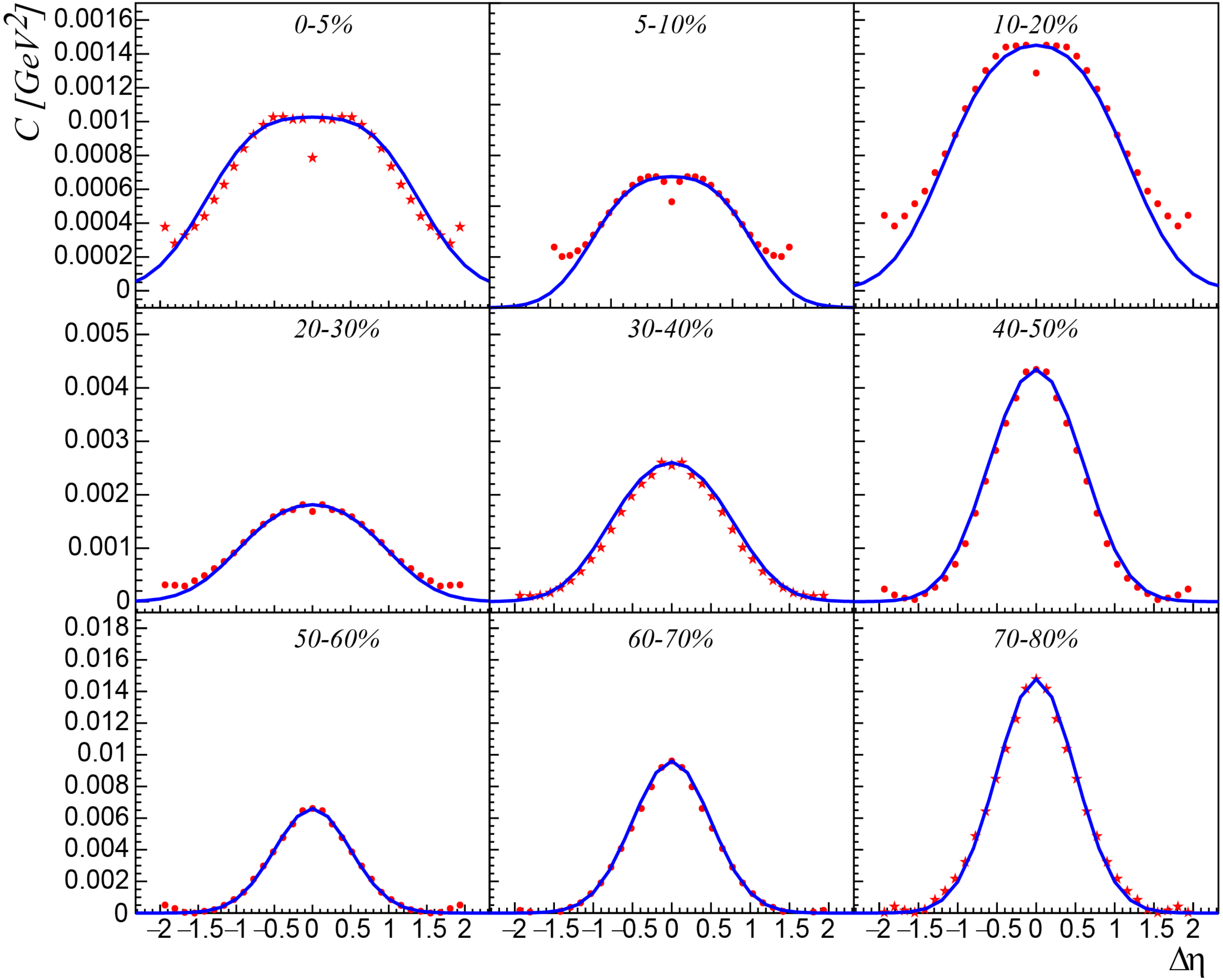
Evolving Parameters

$\beta = 5.5$

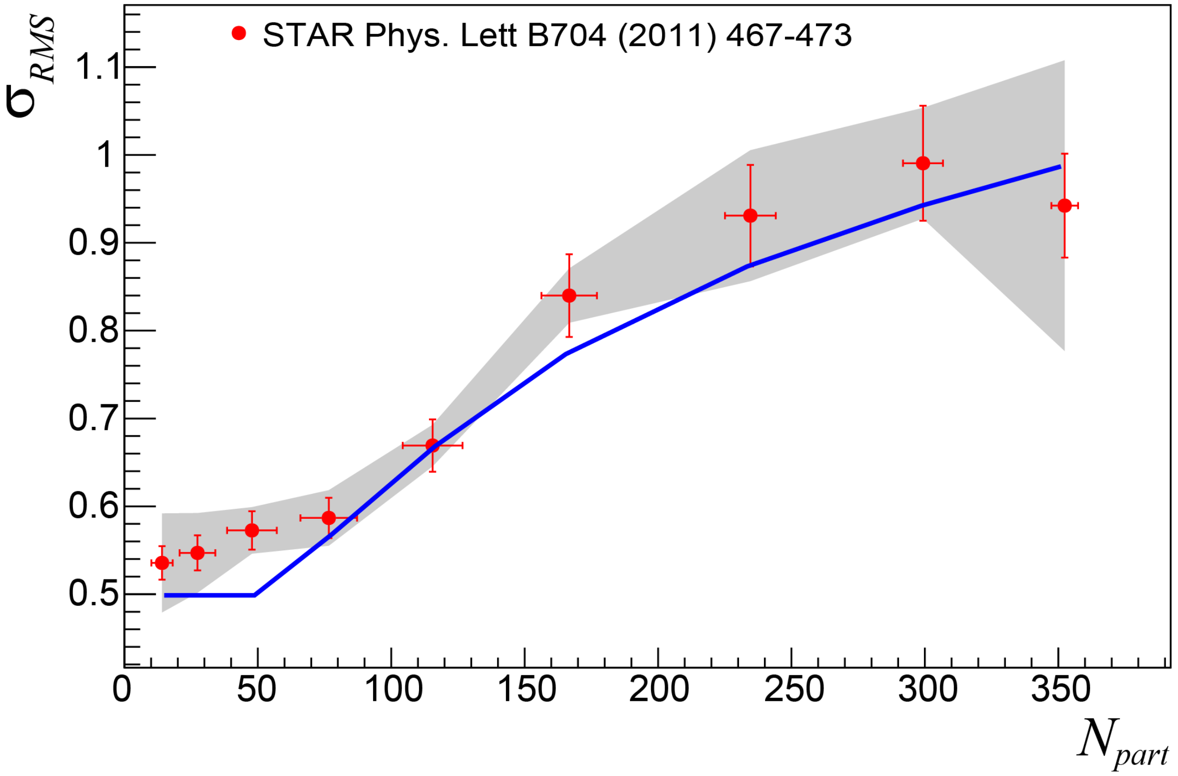
$T_F = 150\ MeV$

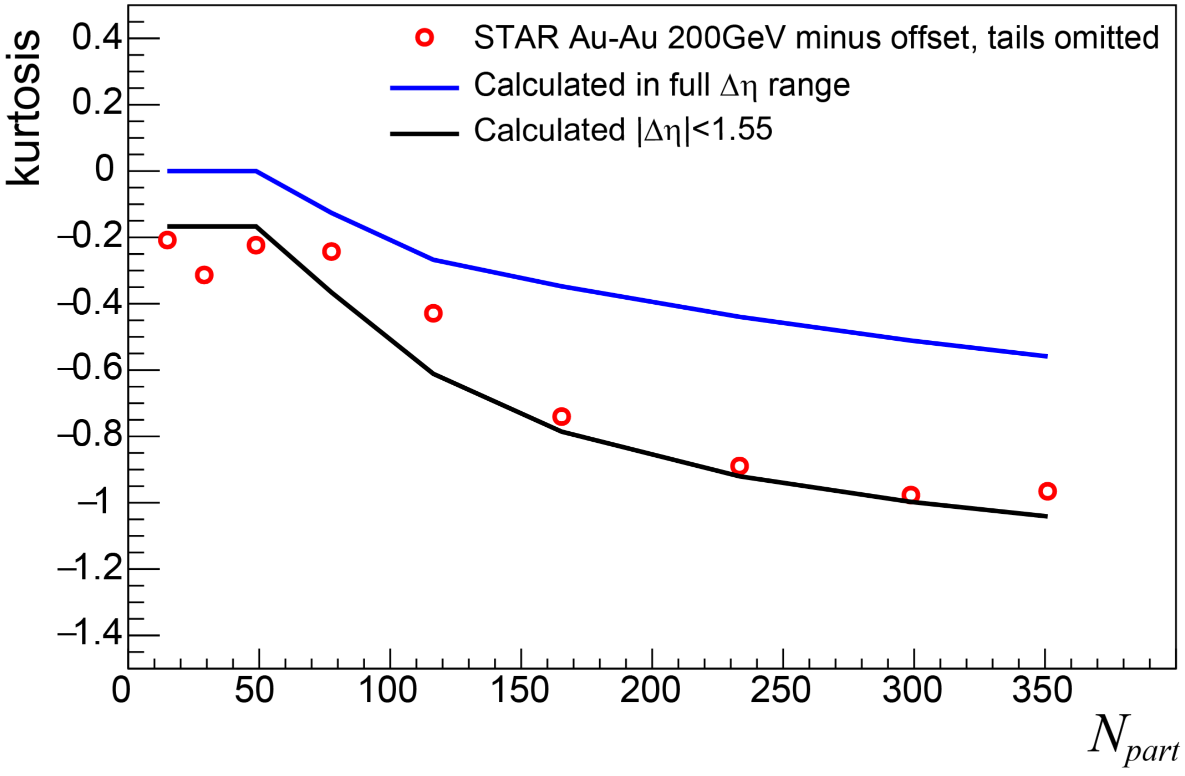
$\tau_0 = 1.05\ fm$

$\tau_{F,central} = 12.5\ fm$



Width and Kurtosis





Realistic Limits on $\beta = \tau_\pi/\nu$

$T_F = 150\ MeV$

$\tau_0 = 1.05\ fm$

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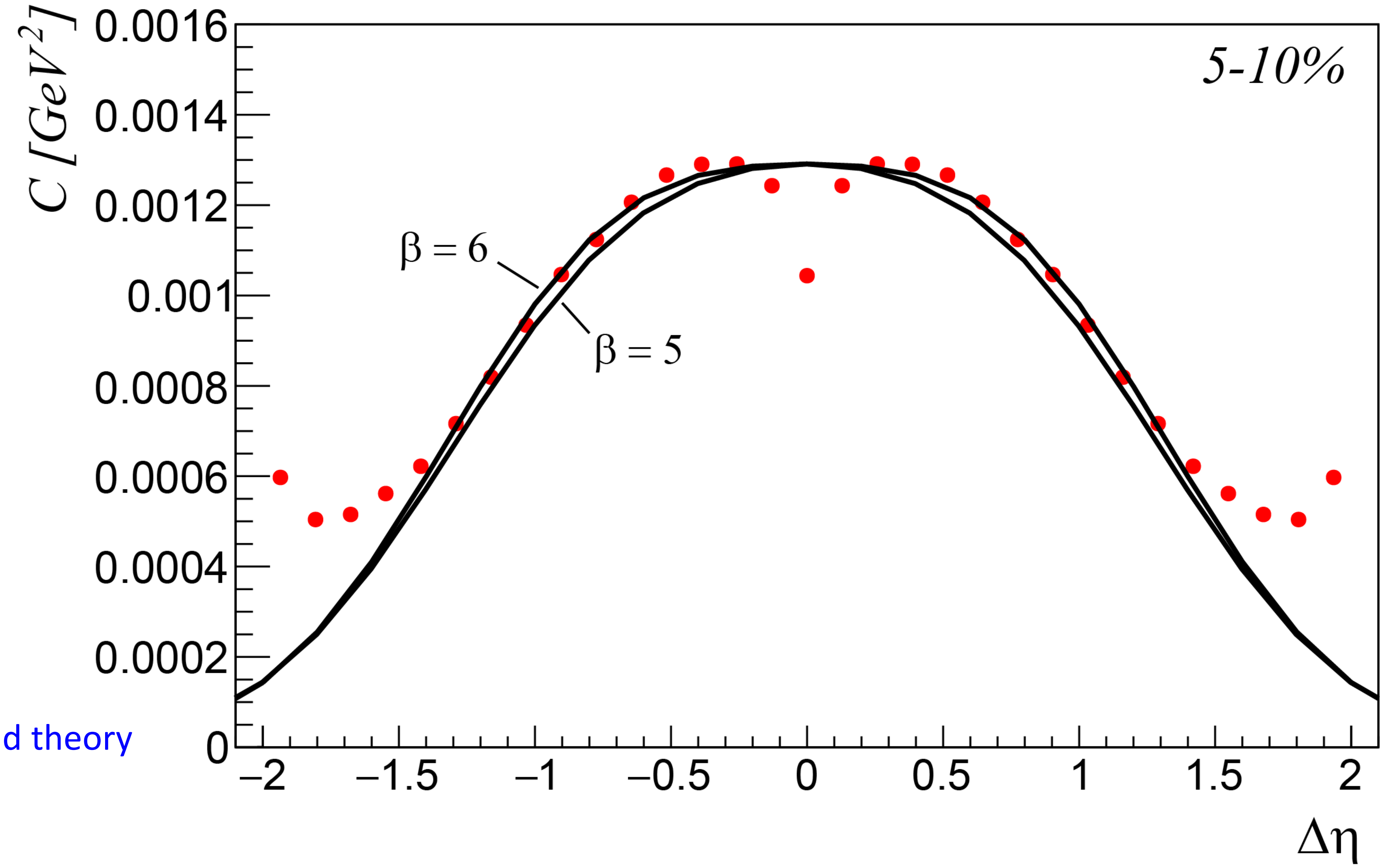
With initial flow



Jeon & Czajka massless scalar field theory

$\tau_\pi/\nu = 5 - 7$

Phys.Rev. C95 (2017) no.6, 064906

Summary

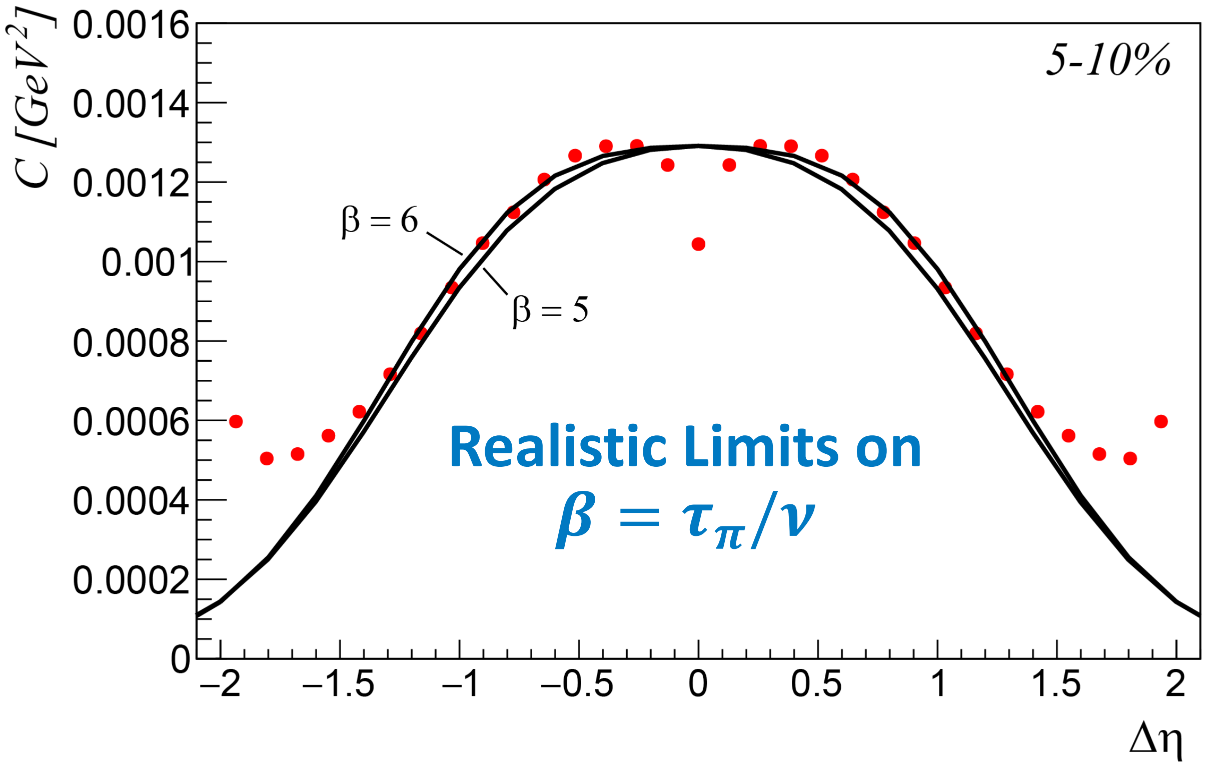
Hydro formulation: only transverse modes effect these correlations

- Wavelike propagation of fluctuations important at early times
- Diffusive effects show up as waves attenuate and separate

Causality shapes the rapidity dependence of correlations

$\tau_\pi = \beta\nu \rightarrow$ **double humps**

$\nu = \frac{\eta}{Ts} \rightarrow$ **width**



Open Questions

- Influence of sound and heat modes on observables
- Charge balancing, resonances, jets, HBT

Kurtosis with Evolving Parameters

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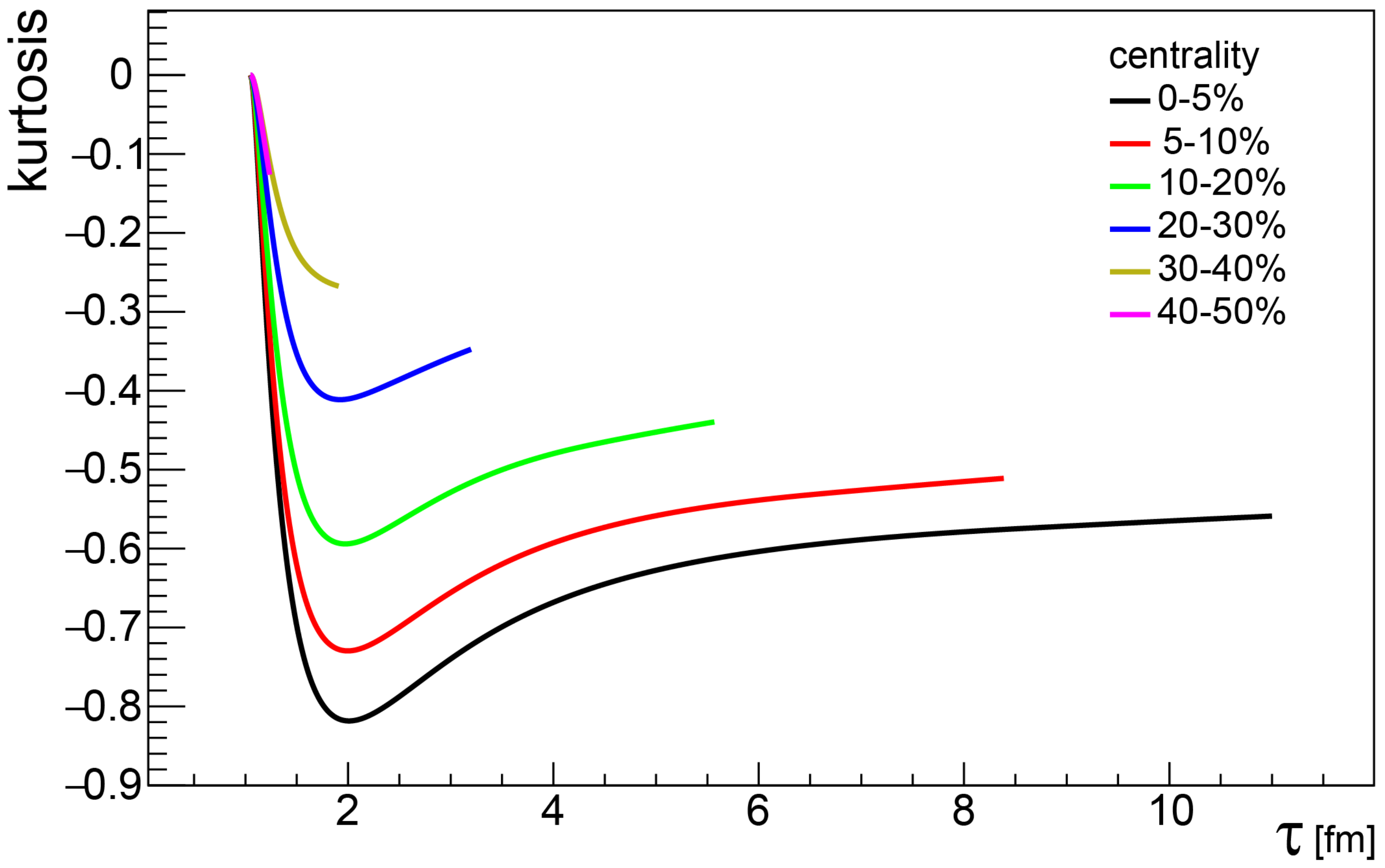
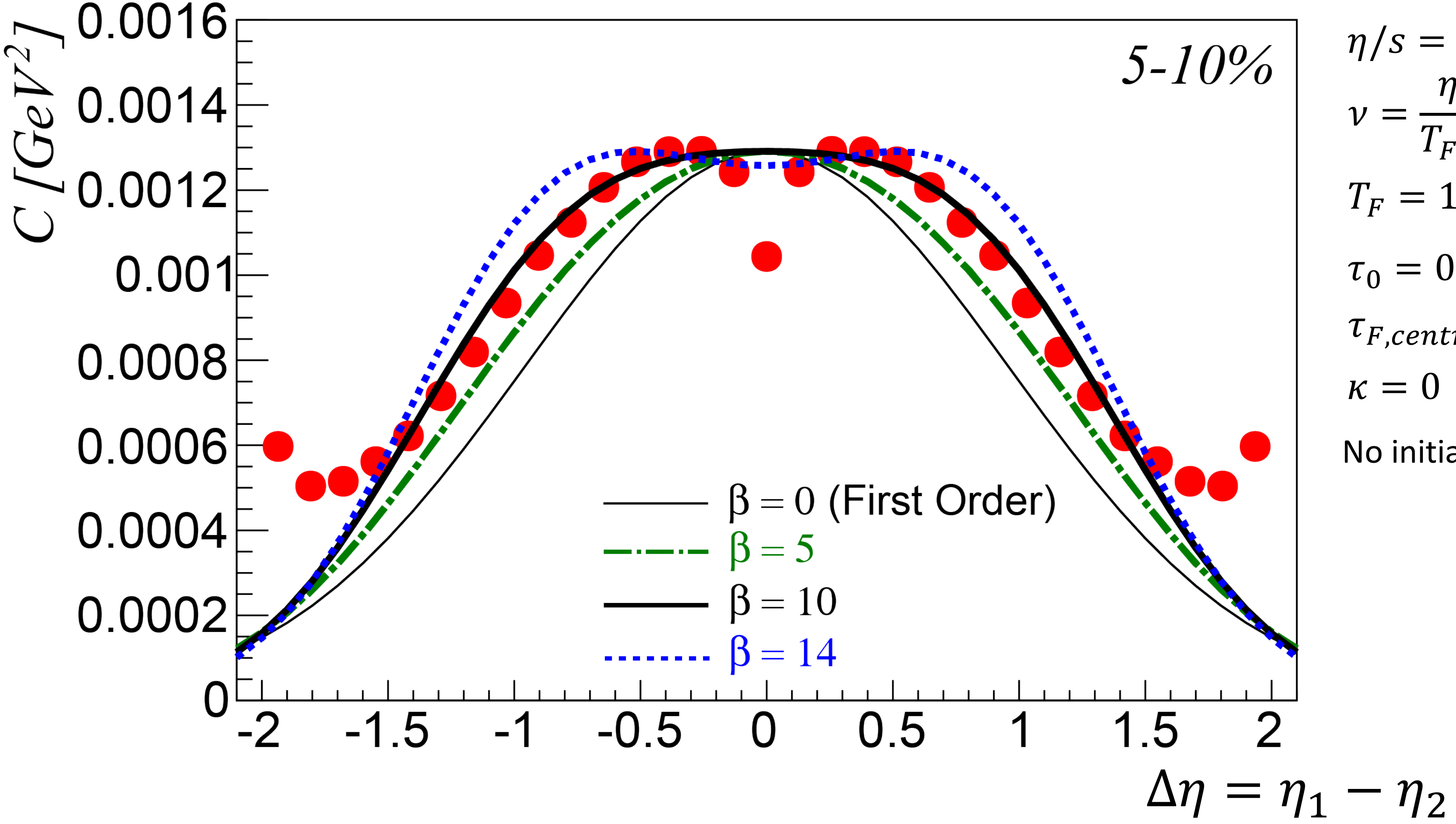
With initial flow



What the Experiment Can Show Us

Data from the STAR Collaboration, Phys.Lett. B704 (2011) 467



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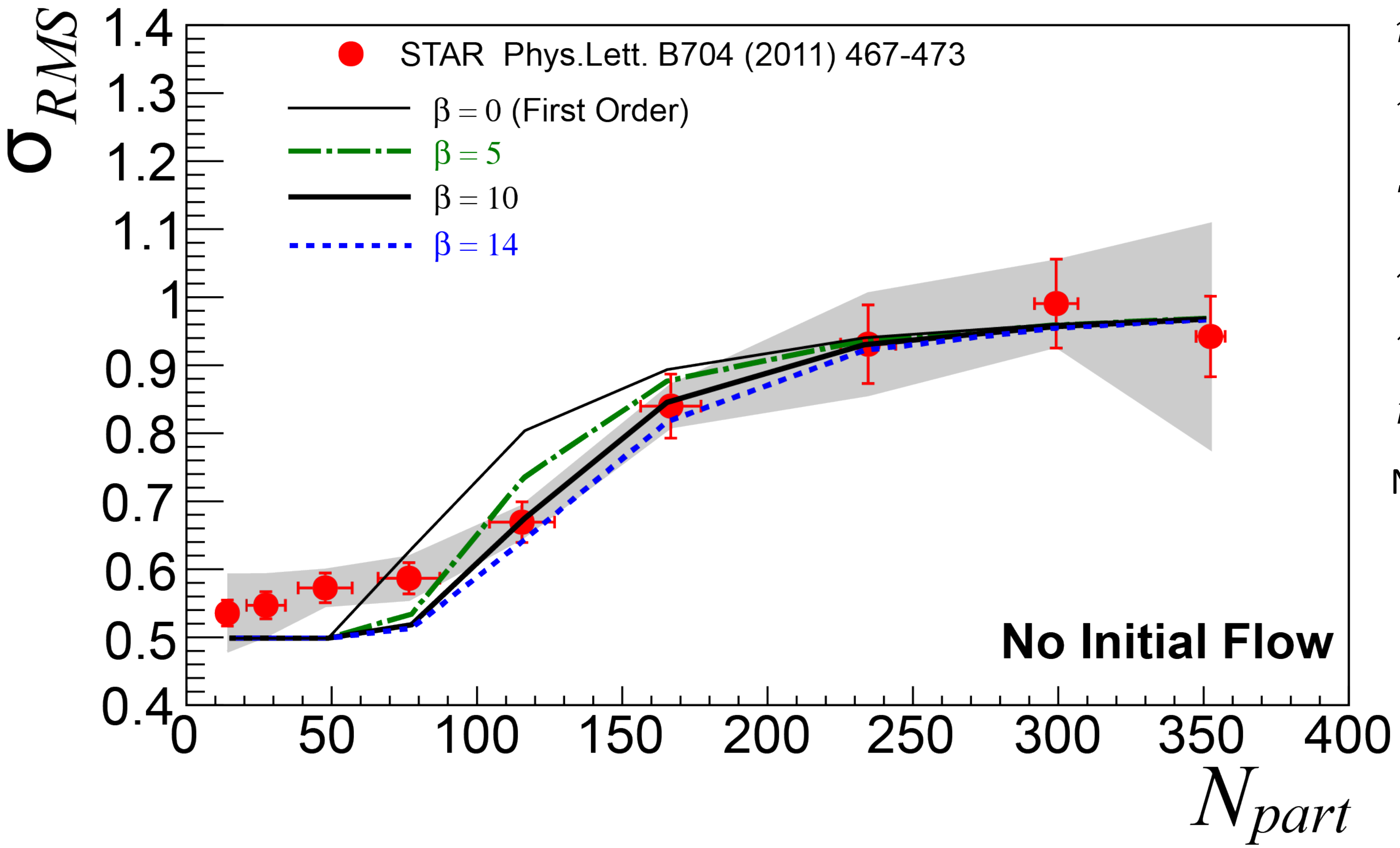

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No initial flow

The stress energy tensor relaxes due to shear viscous forces. In second order hydrodynamics (we will keep only shear contributions)

$$D = u^{\mu}\partial_{\mu}$$

$$\nabla_{\mu} = \partial_{\mu} - u_{\mu}u^{\nu}\partial_{\nu}$$

$$\boldsymbol{\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}D\Pi^{\alpha\beta} = -\frac{1}{\tau_{\pi}}(\Pi^{\mu\nu} - S^{\mu\nu}) - \kappa\nabla_{\alpha}u^{\alpha}\Pi^{\mu\nu}}$$

$$S^{\mu\nu} = \eta\left(\nabla^{\mu}u^{\nu} + \nabla^{\nu}u^{\mu} - \frac{2}{3}\Delta^{\mu\nu}\nabla_{\alpha}u^{\alpha}\right)$$

Here we require $\partial_{\mu}\left(T^{\mu i}_{ideal} + \Pi^{\mu i}\right) = 0$ leading to Israel-Stewart equations.

Schematically $\quad \frac{\partial}{\partial t}\Pi^{\mu i} \sim -\frac{1}{\tau_{\pi}}(\Pi^{\mu\nu} - S^{\mu\nu})$