Exclusive diffractive processes in high energy *eA* collisions

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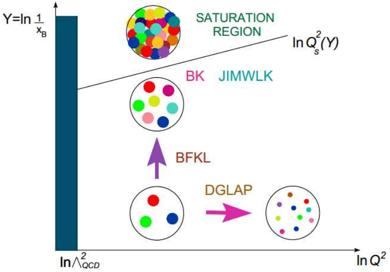
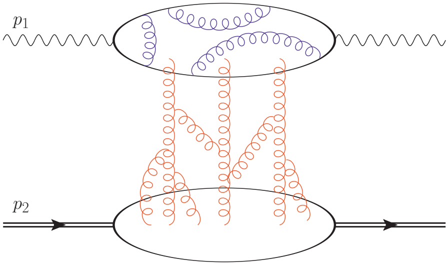
The shockwave description of the Color Glass Condensate



Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

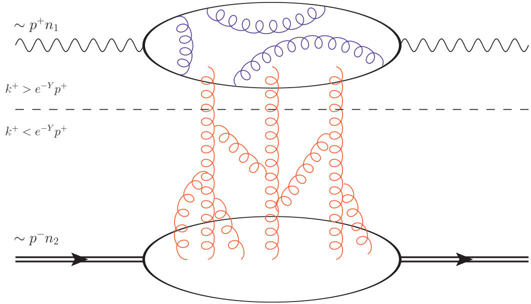
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \to (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation

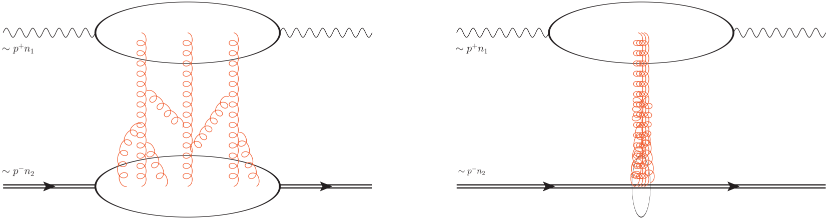


Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) + b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k})$$

$e^\eta = e^{-Y} \ll 1$

Large longitudinal boost to the projectile frame

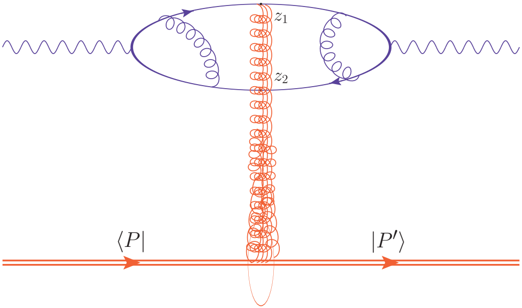


Large longitudinal boost $\Lambda \propto \sqrt{s}\; b^{\mu}(x) \to b^{-}(x)\, n_2^{\mu} \simeq \delta(x^{+})\, \mathbf{B}(\vec{x})\, n_2^{\mu}$
(*Shockwave* approximation)

Multiple interactions with the target can be resummed into path-ordered Wilson lines attached to each parton crossing lightcone time 0:

$$\tilde{U}^{\eta}(\vec{p}) = \int d^{D-2}\vec{z}\; e^{-i(\vec{p}\cdot\vec{z})} U^{\eta}_{\vec{z}}, \quad U^{\eta}_{i} = U^{\eta}_{\vec{z}_i} = P e^{ig\int b^{-}_{\eta}(z_i^{+},\vec{z}_i)\, dz_i^{+}}$$

Factorized picture



Factorized amplitude

$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^\eta(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Evolution for the dipole operator

B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\,\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta}\,\mathcal{U}_{32}^{\eta}\right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta}\,\mathcal{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

Mean field approximation (large N_C)

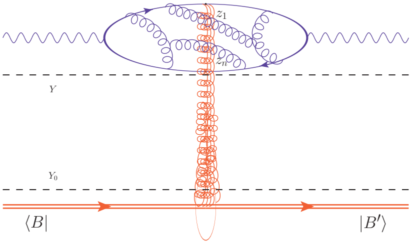
$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\,\vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle\ \langle \mathcal{U}_{32}^{\eta} \rangle\right]$$

Non-linear term : saturation

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor



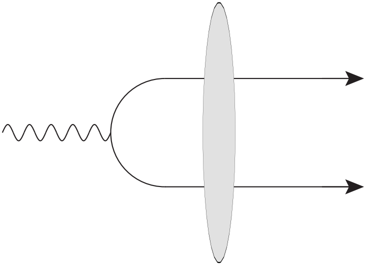
$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

Exclusive diffraction probes the $b_\perp$-dependent, off-diagonal part of the non-perturbative scattering amplitude

Exclusive diffractive dijet production

Exclusive diffractive dijet production

LO diagram for diffractive dijet production



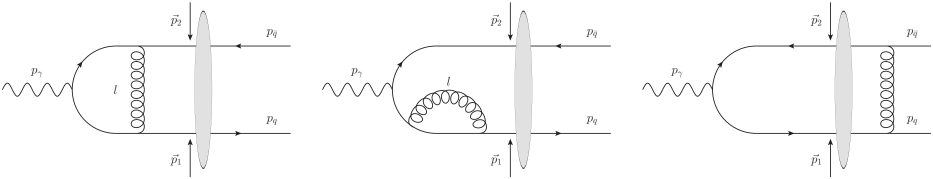
$$\mathcal{A} = \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_0(\vec{p}_1, \vec{p}_2) \times \left\langle P' \left| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \right| P \right\rangle$$

$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d\vec{z}_1 d^d\vec{z}_2\, e^{-i(\vec{p}_1\cdot\vec{z}_1)-i(\vec{p}_2\cdot\vec{z}_2)} [\frac{1}{N_c}\mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1]$$

Probes the Dipole Wigner distribution [Hatta, Xiao, Yuan]

First kind of virtual corrections

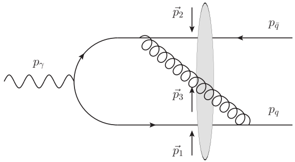
NLO dipole diagrams

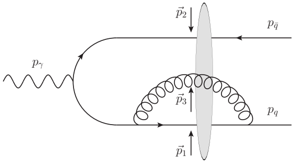


$$\mathcal{A}_{NLO}^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_{V1}(\vec{p}_1, \vec{p}_2)$$
$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

Second kind of virtual corrections

NLO double dipole corrections





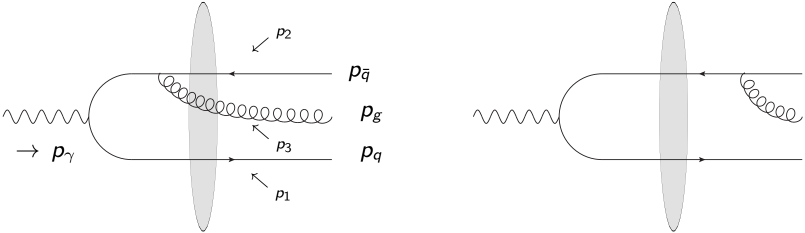
$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3\, \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

$$\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle (2\pi)^d \delta(\vec{p}_3)$$

$$+ \Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle]$$

Real corrections

Real dipole and double dipole corrections



$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle (2\pi)^d \delta(\vec{p}_3)$$
$$+ \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \langle P' | \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) | P \rangle]$$

$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$
$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle$$

Divergences

Divergences

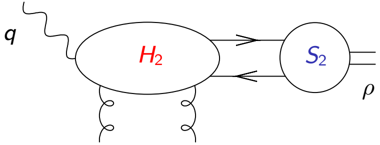
- Rapidity divergence $p_g^+ \to 0$ (spurious gauge pole in axial gauge)
  - Removed via JIMWLK evolution
- UV, soft divergence, collinear divergence
  - Cancels between real and virtual corrections, along with renormalization
- Soft and collinear divergence
  - Removed via a jet algorithm

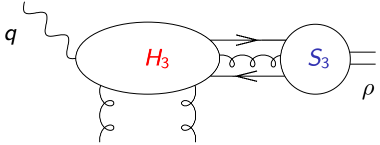
We thus built a finite NLO exclusive diffractive cross section with saturation effects

Exclusive diffractive light vector meson production

Collinear factorization: basic principle

The impact factor is the convolution of a hard part and the vacuum-to-meson matrix element of an operator





$$\int_x (H_2(x))_{ij}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^{\alpha}(x)\psi_j^{\beta}(0) \right| 0 \right\rangle \qquad \int_{x_1,x_2} (H_3^{\mu}(x_1,x_2))_{ij,c}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^{\alpha}(x_1) A_{\mu}^{c}(x_2)\psi_j^{\beta}(0) \right| 0 \right\rangle$$

H and S are by convolution and by summation over spinor and color indices

Once factorization in the t channel is done, now factorize in the s channel with collinear factorization: expand the impact factor in powers of the hard scale

Twist 2

Collinear factorization at twist 2

- Leading twist DA for a longitudinally polarized light vector meson

$$\left\langle \rho \left| \bar{\psi}(z)\gamma^{\mu}\psi(0) \right| 0 \right\rangle \rightarrow p^{\mu} f_{\rho} \int_{0}^{1} dx e^{ix(p\cdot z)} \varphi_{1}(x)$$

- Leading twist DA for a transversely polarized light vector meson

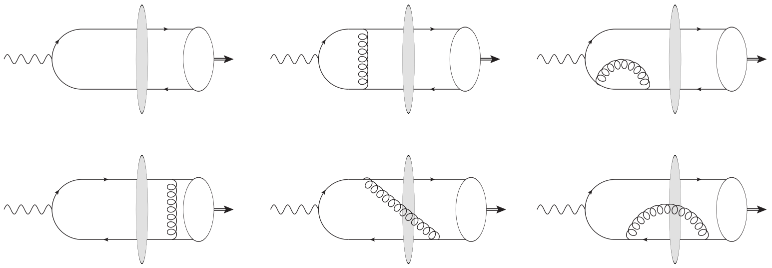
$$\left\langle \rho \left| \bar{\psi}(z)\sigma^{\mu\nu}\psi(0) \right| 0 \right\rangle \rightarrow i(p^{\mu}\varepsilon_{\rho}^{\nu} - p^{\nu}\varepsilon_{\rho}^{\mu}) f_{\rho}^{T} \int_{0}^{1} dx e^{ix(p\cdot z)} \varphi_{\perp}(x)$$

The twist 2 DA for a transverse meson is chiral odd, thus $\gamma^{*}A \rightarrow \rho_{T}A$ starts at twist 3

Exclusive diffractive ρ_L production:

NLO corrections to a twist 2 process

Exclusive diffractive production of a light neutral vector meson



$$\mathcal{A} = -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_\parallel(x) \int \frac{d^d\vec{p}_1}{(2\pi)^d}\frac{d^d\vec{p}_2}{(2\pi)^d}\frac{d^d\vec{p}_3}{(2\pi)^d}$$
$$\times (2\pi)^{d+1}\,\delta\left(p_V^+ - p_\gamma^+\right)\delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3\right)$$
$$\times \left[\left(\Phi_0^\beta(x,\,\vec{p}_1,\,\vec{p}_2) + C_F\Phi_{V1}^\beta(x,\,\vec{p}_1,\,\vec{p}_2)\right)\tilde{\mathcal{U}}_{12}^\eta\,(2\pi)^d\delta(\vec{p}_3)\right.$$
$$\left. + \,\Phi_{V2}^\beta(x,\vec{p}_1,\vec{p}_2,\vec{p}_3)\,\tilde{\mathcal{W}}_{123}^\eta\right]$$

Probes gluon GPDs at low x, as well as twist 2 DAs

Divergences

Divergences

- Rapidity divergence $p_g^+ \to 0$ (spurious gauge pole in axial gauge)
  - Removed via JIMWLK evolution
- UV, soft divergence, collinear divergence
  - Mostly cancel each other, but requires renormalization of the operator in the vacuum-to-meson matrix element $\to$ ERBL evolution equation for the DA

We thus built a finite NLO exclusive diffractive amplitude with saturation effects

Theoretical issues

Two theoretical questions

- How to get to the BFKL limit at NLL?
- What about end-point singularities for the power-suppressed $\gamma_T \to \rho_L$ contribution?

Comparison with previous results:JIMWLK/BFKL equivalence

In the forward $t = 0$ limit and in the linear BFKL limit, the $\gamma_L \to \rho_L$ impact factor was computed at NLO [Ivanov, Kotsky, Papa].

JIMWLK convolution

$$\int d^d p_1 d^d p_2 \Phi_{CGC}(p_1, p_2) \tilde{\mathcal{U}}(p_1, p_2)$$

$\tilde{\mathcal{U}}(p_1, p_2)$ dipole scattering operator

BFKL convolution

$$\int d^d q_1 d^d q_2 \Phi_{BFKL}(q_1, q_2)\, R(q_1) R(q_2)$$

$R(q)$ Reggeon field

Defining the Reggeon field in the CGC as the logarithm of a Wilson line [Caron-Huot]

$$R^a(x) \equiv \frac{f^{abc}}{g C_A} \left(\ln U_x^{adj} \right)^{bc}$$

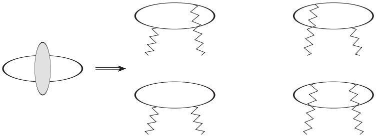
$$U_x = 1 + i g t^a R^a(x) - \frac{g^2}{2} t^a t^b R^a(x) R^b(x) + O(g^3)$$

Such fields are Reggeized by the JIMWLK Hamiltonian, satisfy the BFKL equation and satisfy bootstrap equations.

JIMWLK/BFKL equivalence

Linear limit of diffractive CGC impact factors

$$\int d^2p_1 d^2p_2\, \varphi(p_1,p_2)\, \tilde{\mathcal{U}}(p_1,p_2)$$
$$= \frac{g^2}{4N_c}\int d^2q_1 d^2q_2 R^a(q_1)\, R^a(q_2)\left[2\varphi(q_1,q_2) - \varphi(q_1+q_2,0) - \varphi(0,q_1+q_2)\right]$$



This matches our result to the leading order BFKL result.

At NLL accuracy, things are interestingly worse due to the ambiguity of distribution of radiative corrections between impact factors and kernels.

Equivalence with BFKL at NLL accuracy

Linear limit: usual k_t-factorization (BFKL framework)

s-channel discontinuity of $A + B \to A' + B'$ scattering amplitudes

$$\delta(p_{A'} + p_{B'} - p_A - p_B) Disc_s \mathcal{A}_{AB}^{A'B'} \propto \Phi(A', A) \otimes \mathcal{K} \otimes \Phi(B', B)$$

For any non-singular operator $\mathcal{O}$ this discontinuity is invariant under

$$\Phi(A', A) \to \Phi(A', A)\, \mathcal{O}, \quad \mathcal{K} \to \mathcal{O}^{-1} \mathcal{K} \mathcal{O}, \quad \Phi(B', B) \to \mathcal{O}^{-1} \Phi(B', B)$$

i.e. there is an ambiguity of distribution of corrections between the impact factors and the kernel. In the linear approximation of BK there exists an operator $\mathcal{O}$ such that

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi_{BK} = (\Phi_{BFKL} \otimes \mathcal{O}) \otimes (\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL} \otimes \mathcal{O}) \otimes (\mathcal{O}^{-1} \otimes \Phi_{BFKL})$$

The expression for $\mathcal{O}$ to make the kernels explicitly equivalent at NLO accuracy under such a change of variables is known [Fadin, Fiore, Grabovsky, Papa]
Comparing our NLL CGC impact factor with the NLL BFKL impact factor should confirm this expression.

End point singularities and factorization

End point singularities?

Leading order impact factor for, respectively, $\gamma_L^* \to V_L$ and $\gamma_T^* \to V_L$ transitions:

$$\Phi_L^{(0)} = \frac{2x\bar{x}p_V^+ Q}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2},$$

$$\Phi_T^{(0)} = -\frac{(x-\bar{x})p_V^+(\bar{x}\vec{p}_{1\perp} - x\vec{p}_{2\perp}) \cdot \vec{\varepsilon}_{\gamma_T}}{(\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2}$$

No end point singularity, even for a transverse photon and even in the photoproduction limit and even at NLO.
With null transverse momenta in the t channel, one could encounter $x \in \{0, 1\}$ end point singularities as $\frac{1}{x\bar{x}Q^2}$ thus breaking collinear factorization.

Exclusive diffractive ρ_T production:

LO but twist 3 process

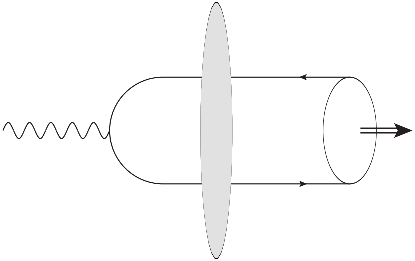
Previous study

Previous works [Anikin, Besse, Ivanov, Pire, Szymanowski, Wallon]

- Full $\gamma_T \to \rho_T$ impact factor, but
  - Linear BFKL regime only
  - Forward $t = 0$ case only
  - Hence No $\gamma_L^* \to \rho_T$ transition allowed
- Proved the equivalence between two major schemes for collinear factorization at twist 3, but in a process-dependent way
- Required interesting algebra to restore QCD gauge invariance, but no deep understanding for the origin of invariance breaking in the first place

2-body diagrams

2-body contribution

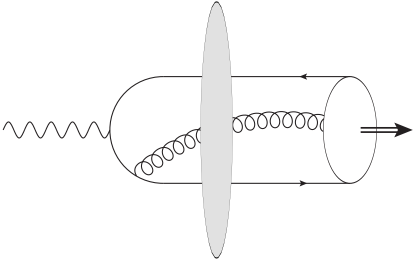


$$\int d^2\vec{z}_1 d^2\vec{z}_2\ \Phi^{2b}_{q\bar{q}}(\vec{z}_1,\vec{z}_2)\,\mathrm{Tr}(U_1 U_2^\dagger)\,\langle\rho\,|\bar{\psi}\psi|\,0\rangle$$

Note that this is not the whole story. This nice and simple contribution only arises once we cancel all contributions which break QCD gauge invariance up to twist 4 corrections.

3-body contribution

Natural 3-body CGC diagram



$$\int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \;\Phi^{3b}_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right) \mathrm{Tr}[U_1 t^b U_2^\dagger t^a] U_3^{ab} \left\langle \rho \left| \bar{\psi} g A \psi \right| 0 \right\rangle$$

Double-dipole term even at tree level $\Rightarrow$ Great sensitivity to saturation

Note that this is not the whole story. This nice and simple contribution only arises once we cancel all contributions which break QCD gauge invariance up to twist 4 corrections.

Divergences

Divergences and issues?

- No divergence. No end point singularity which would break factorization in a pure collinear framework. The mixed CGC/collinear framework gets rid of *s*-channel factorization breaking.
- QCD gauge invariance is restored up to twist 4 terms
- Presence of a double dipole term at LO: enhanced saturation effects?
- In the Wandzura-Wilczek approximation, it will be easy get the NLO corrections to this twist 3 process and no end point singularity is to be expected

We thus built a finite twist 3 exclusive diffractive amplitude with saturation effects

Conclusion

- We provided the full computation of the $\gamma^{(*)} \to JetJet$ and $\gamma^*_{L,T} \to \rho_L$ impact factors at NLO accuracy, and the twist 3 impact factors for $\gamma^*_{L,T} \to \rho_T$ in the shockwave framework.
- Our results are perfectly finite, even for photoproduction (at large t for ρ)
- The computation can be adapted for twist 3 NLO production in the Wandzura-Wilczek approximation, removing factorization breaking end-point singularities even at NLO for a process which would not factorize in a full collinear factorization scheme
- Exclusive diffractive processes are perfectly suited for precision saturation physics and gluon tomography with $b_\perp$ dependence at the EIC. Dijet production probes the dipole Wigner distribution, ρ meson production probes gluon GPDs at small x.

Effective CGC Feynman rules for fields

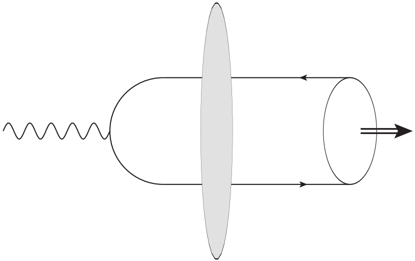
The recursion to exponentiate slow gluon scatterings into a Wilson line only starts at order g_s

$$A^{\mu}_{eff}\left(z_0\right)|_{z_0^+<0} = A^{\mu}\left(z_0\right) - 2i\int d^D z_3\,\delta\left(z_3^+\right) G^{\mu}_{\sigma_\perp}\left(z_{30}\right)\left(U^{ba}_{\vec{z}_3} - \delta^{ba}\right) F^{+\sigma_\perp}\left(z_3\right)$$

$$\overline{\psi}_{eff}\left(z_0\right)|_{z_0^+<0} = \bar{\psi}\left(z_0\right) + \int d^D z_1\,\delta\left(z_1^+\right)\overline{\psi}\left(z_1\right)\left(U_{\vec{z}_1} - 1\right)\gamma^+ G\left(z_{10}\right)$$

$$\psi_{eff}\left(z_0\right)|_{z_0^+<0} = \psi\left(z_0\right) - \int d^D z_2\delta\left(z_2^+\right) G\left(z_{02}\right)\gamma^+\psi\left(z_2\right)\left(U^{\dagger}_{\vec{z}_2} - 1\right)$$

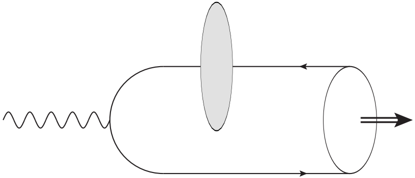
Natural 2-body CGC diagram



$$\int d^2\vec{z}_1 d^2\vec{z}_2\ \Phi^{2b}_{q\bar{q}}(\vec{z}_1,\vec{z}_2)\,\mathrm{Tr}[(U_1-1)(U_2^\dagger-1)]\,\langle\rho\,|\bar{\psi}\psi|\,0\rangle$$

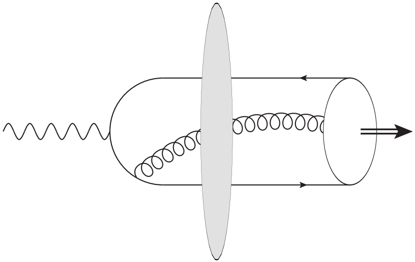
Contains monopole contributions

Antiquark monopole 2-body diagram



$$\int d^2\vec{z}_2\ \Phi^{2b}_{\bar{q}}(\vec{z}_2)\,\mathrm{Tr}[(U_2^\dagger - 1)]\,\langle\rho\,|\bar{\psi}\psi|\,0\rangle$$

Natural 3-body CGC diagram

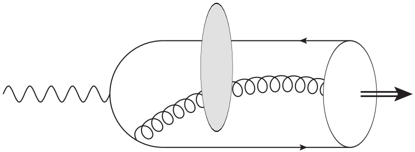


$$\int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\ \Phi^{3b}_{q\bar{q}g}(\vec{z}_1,\vec{z}_2,\vec{z}_3)\,\mathrm{Tr}[(U_1-1)t^b(U_2^\dagger-1)t^a](U_3^{ab}-\delta^{ab})\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle$$

Contains dipole and monopole contributions

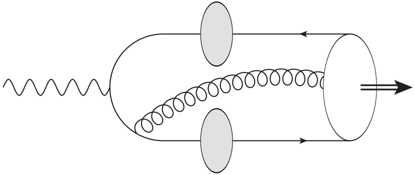
Double-dipole term even at tree level $\Rightarrow$ Great sensitivity to saturation

3-body ($\bar{q}g$)-dipole diagram



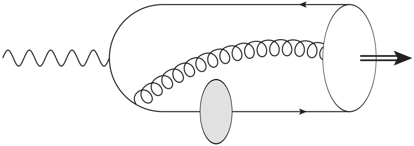
$$\mathcal{A}^{3b}_{\bar{q}g} = \int d^2\vec{z}_2 d^2\vec{z}_3 \; \Phi^{3b}_{\bar{q}g}\left(\vec{z}_2, \vec{z}_3\right) \mathrm{Tr}[t^b(U_2^{\dagger} - 1)t^a](U_3^{ab} - \delta^{ab}) \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

3-body $(q\bar{q})$-dipole diagram



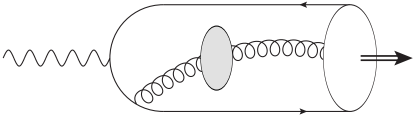
$$\mathcal{A}_{q\bar{q}}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 \;\Phi_{q\bar{q}}^{3b}(\vec{z}_1,\vec{z}_2)\,\mathrm{Tr}[(U_1-1)t^b(U_2^\dagger-1)t^a]\delta^{ab}\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle$$

3-body (q)-monopole diagram



$$\mathcal{A}_q^{3b} = \int d^2\vec{z}_1 \Phi_q^{3b}(\vec{z}_1)\,\mathrm{Tr}[(U_1 - 1)t^b t^a]\delta^{ab}\left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

3-body (g)-monopole diagram



$$\mathcal{A}_g^{3b} = \int d^2\vec{z}_3\, \Phi_g^{3b}(\vec{z}_3)\, \mathrm{Tr}[t^b t^a](U_3^{ab} - \delta^{ab}) \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

Cancelling the 2-body monopoles

Antiquark monopole part of the natural CGC diagram

- Monopole part of the quark line

$$\overline{\psi}_{eff}\left(z_0\right)|_{z_0^+<0} = \bar{\psi}\left(z_0\right) + \int d^D z_1\, \delta\left(z_1^+\right) \overline{\psi}\left(z_1\right)\left(U_{\vec{z}_1} - 1\right)\gamma^+ G\left(z_{10}\right)$$

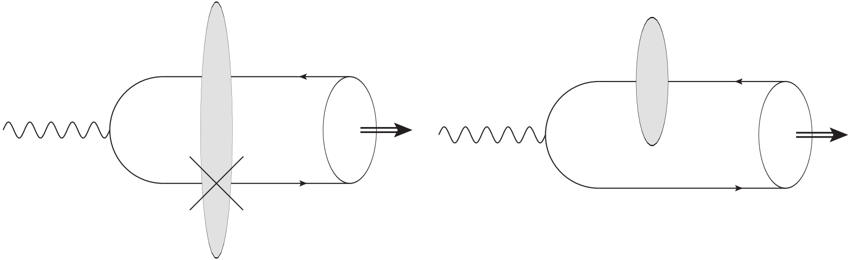
- Simple algebra allows one to get

$$\int d^D z_1 \int \frac{d^D q}{(2\pi)^D} \delta\left(z_1^+\right)\left(\frac{-i\bar{\psi}\left(z_1\right)}{\left(q^- - \frac{\vec{q}^2 - i0}{2q^+}\right)} + \frac{\bar{\psi}\left(z_1\right)\overleftarrow{\partial}\gamma^\mu\gamma^+}{2q^+\left(q^- - \frac{\vec{q}^2 - i0}{2q^+}\right)}\right) e^{-i(q\cdot z_{10})}$$

- Thus one term contributes to a 2-body monopole contribution, and (Dirac equation) the other term contributes to a 3-body monopole contribution.

Sum of the 2-body antiquark monopoles

Using the fact that the non-perturbative collinear matrix elements do not depend on z^+ variables at twist 3 accuracy ...[censored technicalities]... we get the sum between the natural 2-body antiquark monopole diagram and the 2-body antiquark monopole part of the natural CGC diagram



$$\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} \quad - \quad \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - 0}$$

Sum of the 2-body antiquark monopoles

Using the fact that the non-perturbative collinear matrix elements are at most linear in $z_\perp$, the sum cancels iff

$$\left.\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} - \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)}}\right|_{\vec{q}=\vec{0}} = 0$$

$$\left.\frac{\partial}{\partial q_\perp^\mu}\left(\frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)} - \frac{\vec{q}^2}{2q^+}} - \frac{1}{p_\gamma^- - \frac{\left(\vec{p}_\gamma - \vec{q}\right)^2}{2\left(p_\gamma^+ - q^+\right)}}\right)\right|_{\vec{q}=\vec{0}} = 0$$

Cancelling the 3-body unnatural dipoles, and monopoles

"Unnatural" 3-body diagrams

$$\Phi_{qg}\left(\vec{z}_1,\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle=\int d^2\vec{z}_2\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle+\mathrm{Twist}\,4$$

$$\Phi_{\bar{q}g}\left(\vec{z}_2,\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle=\int d^2\vec{z}_1\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle+\mathrm{Twist}\,4$$

$$\Phi_{q\bar{q}}\left(\vec{z}_1,\vec{z}_2\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle=\int d^2\vec{z}_3\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle+\mathrm{Twist}\,4$$

$$\Phi_{g}\left(\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle=\int d^2\vec{z}_1\,d^2\vec{z}_2\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle+\mathrm{Twist}\,4$$

$$\Phi_{q}\left(\vec{z}_1\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle=\int d^2\vec{z}_2\,d^2\vec{z}_3\,\Phi_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle+\mathrm{Twist}\,4$$

Hence the 3-body total from 3-body diagrams

$$\mathcal{A}_3^{3b}=\int d^2\vec{z}_1\,d^2\vec{z}_2\,d^2\vec{z}_3\,\Phi^{3b}_{q\bar{q}g}\left(\vec{z}_1,\vec{z}_2,\vec{z}_3\right)\left\langle\rho\left|\bar{\psi}A\psi\right|0\right\rangle$$
$$\times\left[\mathrm{Tr}(U_1 t^b U_2^\dagger t^a)U_3^{ab}-\mathrm{Tr}(t^b U_2^\dagger t^a \delta^{ab})\right]$$

3-body diagrams

Total from 3-body diagrams

$$\mathcal{A}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\, \Phi^{3b}_{q\bar{q}g}\left(\vec{z}_1, \vec{z}_2, \vec{z}_3\right) \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$
$$\times \left[\mathrm{Tr}(U_1 t^b U_2^\dagger t^a) U_3^{ab} - \mathrm{Tr}(t^b U_2^\dagger t^a \delta^{ab})\right]$$

"3-body" antiquark monopole from the natural 2-body diagram

$$\Phi_2^{3b}\left(\vec{z}_2\right) = \int d^2\vec{z}_1 d^2\vec{z}_3\, \Phi_{q\bar{q}g}\left(\vec{z}_1, \vec{z}_2, \vec{z}_3\right) + \mathrm{Twist}\ 4$$

Sums up to a gauge invariant amplitude

$$\mathcal{A}^{3b} = \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\, \Phi^{3b}_{q\bar{q}g}\left(\vec{z}_1, \vec{z}_2, \vec{z}_3\right)$$
$$\times \left[\mathrm{Tr}(U_1 t^b U_2^\dagger t^a) U_3^{ab} - C_F\right] \left\langle \rho \left| \bar{\psi} A \psi \right| 0 \right\rangle$$

Final amplitude

$$\mathcal{A} = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi^{2b}_{q\bar{q}}(\vec{z}_1, \vec{z}_2) \left[\mathrm{Tr}\left(U_1 U_2^\dagger\right) - N_c \right] + \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3\, \Phi^{3b}_{q\bar{q}g}(\vec{z}_1, \vec{z}_2, \vec{z}_3) \left[\mathrm{Tr}\left(U_1 t^b U_2^\dagger t^a\right) U_3^{ab} - C_F \right]$$

Expansion in Reggeons in the dilute limit: (Reggeon momenta q_1, q_2)

$$\Phi_{BFKL} = \int d^2\vec{z}_1 d^2\vec{z}_2\, \Phi^{2b}_{q\bar{q}}(\vec{z}_1, \vec{z}_2) \left(e^{i(\vec{q}_1\cdot\vec{z}_2)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_1)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right)$$
$$- \int d^2\vec{z}_1 d^2\vec{z}_2 d^2\vec{z}_3 \Phi^{3b}_{q\bar{q}g}(\vec{z}_1, \vec{z}_2, \vec{z}_3) \left[N_c \left(e^{i(\vec{q}_1\cdot\vec{z}_3)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_3)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right) - \left(\frac{N_c^2 - 1}{2N_c}\right)\left(e^{i(\vec{q}_1\cdot\vec{z}_2)} - e^{i(\vec{q}_1\cdot\vec{z}_1)}\right)\left(e^{i(\vec{q}_2\cdot\vec{z}_1)} - e^{i(\vec{q}_2\cdot\vec{z}_2)}\right)\right]$$

Obviously gauge invariant in the BFKL sense: $\Phi_{BFKL} = 0$ for $q_1 = 0$ or $q_2 = 0$.

In the dilute, forward limit, our result matches the previous BFKL results