GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

GALACTIC COSMIC RAYS

ACCELERATION AND TRANSPORT

Pasquale Blasi

Gran Sasso Science Institute

ISAPP School 2018, CERN, October 29, 2018

OUTLINE OF THE LECTURE - 1

PARTICLE ACCELERATION

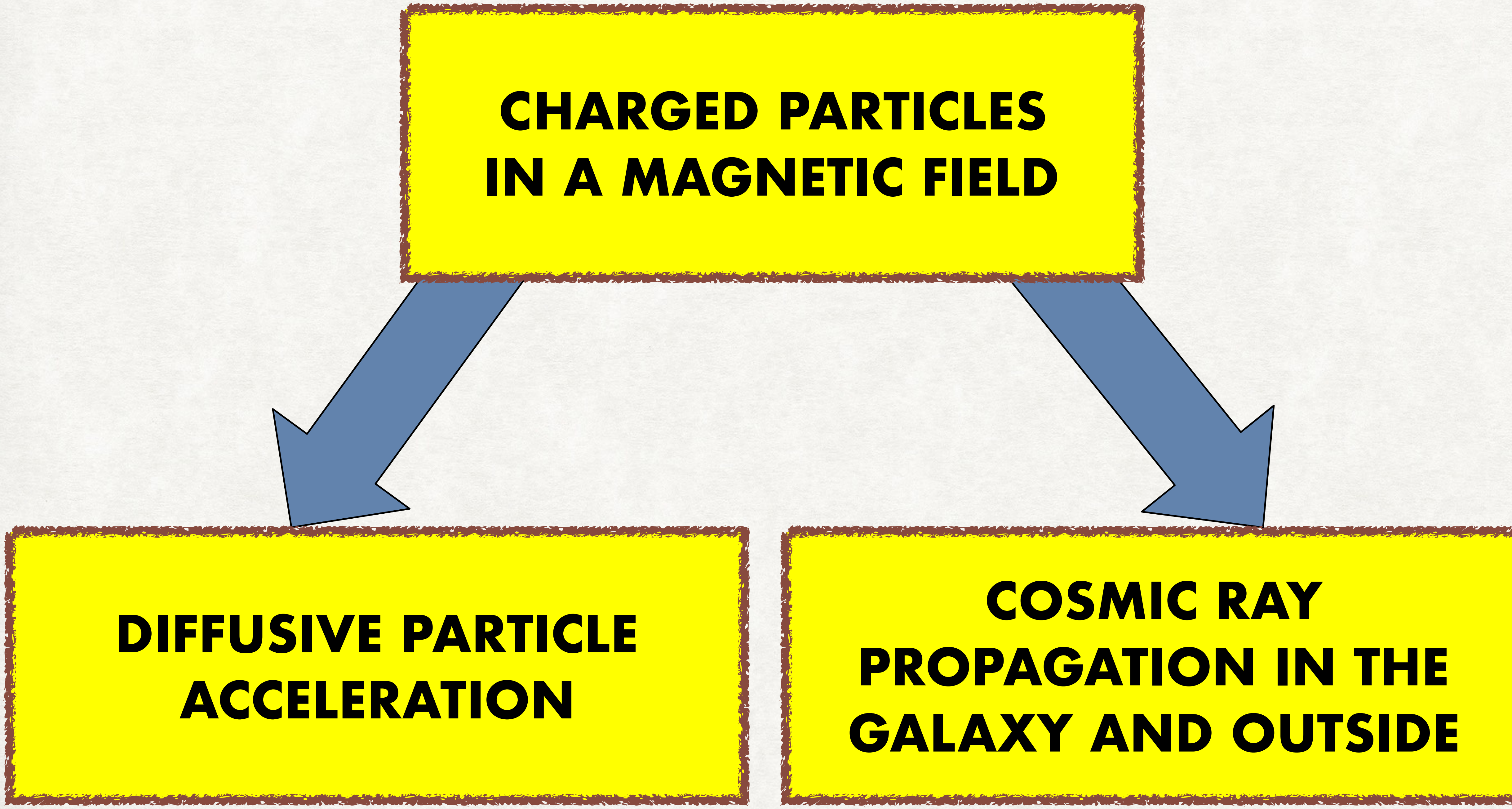
- *Principles of CR transport*
- *Second Order Fermi Acceleration*
- *Diffusive Shock Acceleration (DSA): test particle theory*
- *Diffusive Shock Re-acceleration*
- *Non linear theory of DSA*
    - ✓ *non linear dynamical reaction of accelerated particles*
    - ✓ *non linear B-field amplification due to accelerated particles*
    - ✓ *phenomenology of non linear DSA*
- *The problem of Emax in different types of sources*

OUTLINE OF THE LECTURE - 2

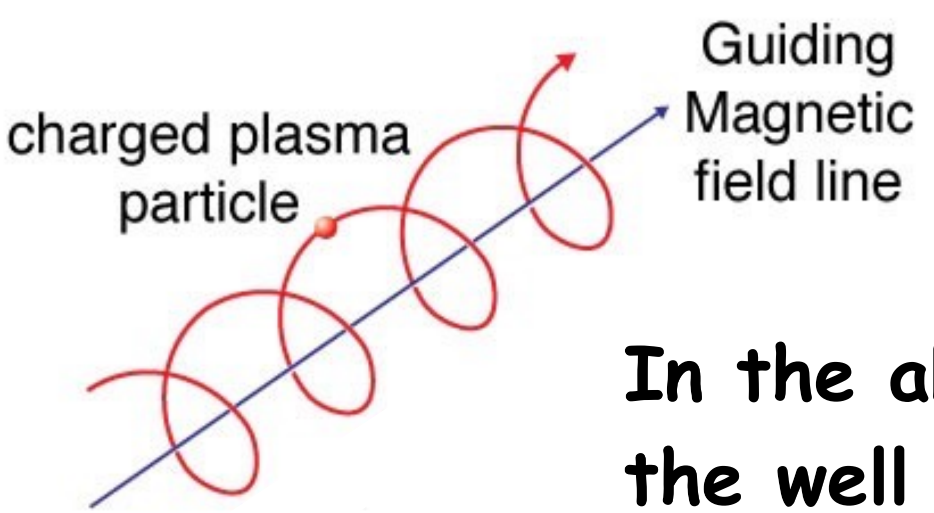
COSMIC RAY PROPAGATION IN THE GALAXY

- *A toy model of the Galaxy: leaky box*
- *CR diffusion, advection and losses for nuclei - Test Particle Theory*
- *CR transport of electrons*
- *Secondary/Primary ratios*
- *Non linear theories of CR transport in the Galaxy*
- *Some advanced topics*

COSMIC RAY TRANSPORT



CHARGED PARTICLES IN A REGULAR B FIELD



$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

In the absence of an electric field one obtains the well known solution:

$$p_z = \text{Constant}$$

$$v_x = V_0 \cos[\Omega\, t]$$

$$v_y = V_0 \sin[\Omega\, t]$$

LARMOR FREQUENCY

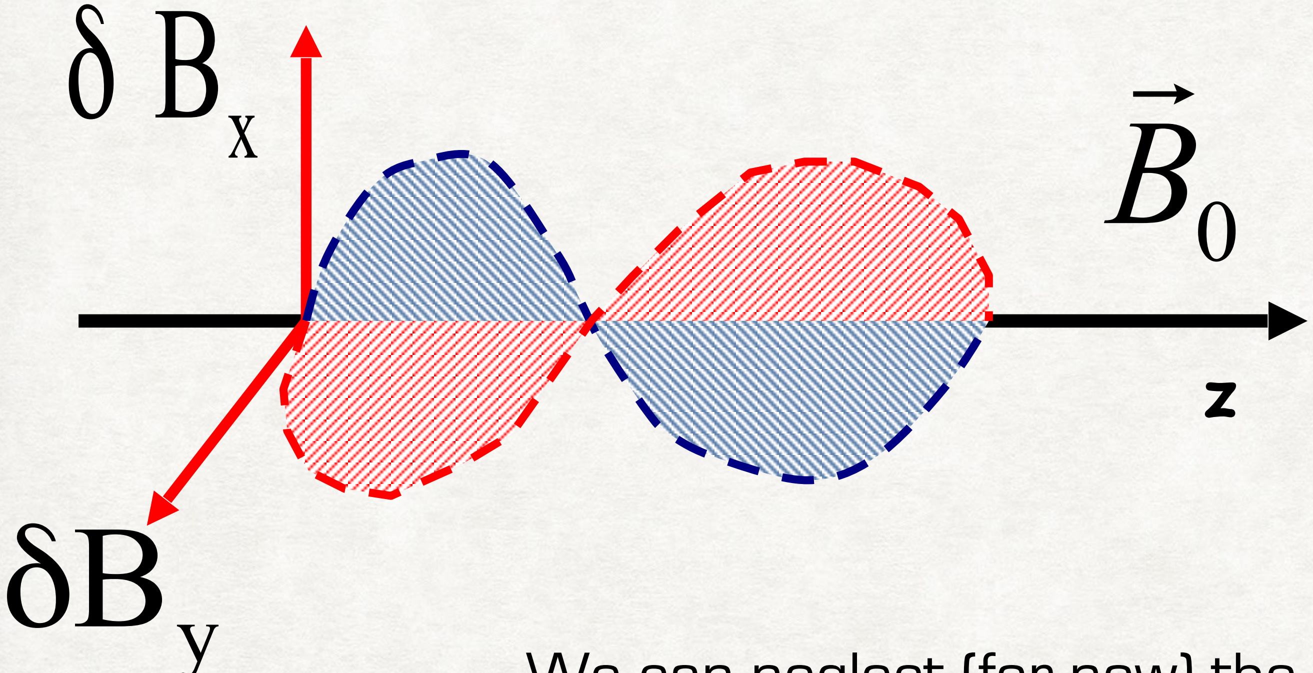
$$\Omega = \frac{q\, B_0}{m\, c\, \gamma}$$

A FEW THINGS TO KEEP IN MIND

- THE MAGNETIC FIELD DOES NOT CHANGE PARTICLE ENERGY -> NO ACCELERATION BY B FIELDS

- A RELATIVISTIC PARTICLE MOVES IN THE Z DIRECTION ON AVERAGE AT C/3

MOTION OF A PARTICLE IN A WAVY FIELD



Let us consider an Alfven wave propagating in the z direction:

$$\delta B << B_0 \qquad \delta\vec{B} \perp \vec{B}_0$$

We can neglect (for now) the electric field associated with the wave, or in other words we can sit in the reference frame of the wave:

$$\frac{d\vec{p}}{dt} = q\,\frac{\vec{v}}{c} \times (\vec{B}_0 + \delta\vec{B})$$

THIS CHANGES ONLY THE X AND Y COMPONENTS OF THE MOMENTUM

THIS TERM CHANGES ONLY THE DIRECTION OF $P_Z = P\mu$

Remember that the wave typically moves with the Alfven speed:

$$v_a = \frac{B}{(4\pi\rho)^{1/2}} = 2 \times 10^6 B_\mu n_1^{-1/2} \ cm/s$$

Alfven waves have frequencies << ion gyration frequency $\Omega_p = qB/m_p c$

It is therefore clear that for a relativistic particle these waves, in first approximation, look like static waves.

The equation of motion can be written as:

$$\frac{d\vec{p}}{dt} = \frac{q}{c}\vec{v} \times (\vec{B}_0 + \vec{\delta}B)$$

If to split the momentum in parallel and perpendicular, the perpendicular component cannot change in modulus, while the parallel momentum is described by

$$\frac{dp \parallel}{dt} = \frac{q}{c}|\vec{v}_\perp \times \delta\vec{B}| \qquad p_{||} = p \ \mu$$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos(\Omega t - kx + \psi)$$

Wave form of the magnetic field with
a random phase and frequency

$$\Omega = qB_0/mc\gamma$$

Larmor frequency

In the frame in which the wave is at rest we can write $x = v\mu t$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos\left[(\Omega - kv\mu)t + \psi\right]$$

It is clear that the mean value of the pitch angle variation over a long enough time vanishes

$$\langle \Delta\mu \rangle_t = 0$$

We want to see now what happens to $\langle \Delta\mu \Delta\mu \rangle$

Let us first average upon the random phase of the waves:

$$\langle \Delta\mu(t')\Delta\mu(t")\rangle_\psi = \frac{q^2 v^2 (1-\mu^2)\delta B^2}{2c^2p^2} cos\left[(\Omega - kv\mu)(t' - t")\right]$$

And integrating over time:

$$\langle \Delta\mu\Delta\mu\rangle_t = \frac{q^2 v^2 (1-\mu^2)\delta B^2}{2c^2p^2} \int dt' \int dt" cos\left[(\Omega - kv\mu)(t' - t")\right]$$

$$= \frac{q^2 v (1-\mu^2)\delta B^2}{c^2p^2\mu} \delta(k - \Omega/v\mu)\Delta t$$

RESONANCE

Many waves

IN GENERAL ONE DOES NOT HAVE A SINGLE WAVE BUT RATHER A POWER SPECTRUM:

$$P(k) = B_k^2/4\pi$$

THEREFORE INTEGRATING OVER ALL OF THEM:

$$\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \rangle = \frac{q^2(1-\mu^2)\pi}{m^2c^2\gamma^2}\frac{1}{v\mu}4\pi\int dk \frac{\delta B(k)^2}{4\pi}\delta(k-\Omega/v\mu)$$

OR IN A MORE IMMEDIATE FORMALISM:

$$\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \frac{\pi}{2}\Omega(1-\mu^2)k_{res}F(k_{res})$$

$$k_{res} = \frac{\Omega}{v\mu}$$

RESONANCE!!!

DIFFUSION COEFFICIENT

THE RANDOM CHANGE OF THE PITCH ANGLE IS DESCRIBED BY A DIFFUSION COEFFICIENT

$$D_{\mu\mu} = \left\langle \frac{\Delta\theta\Delta\theta}{\Delta t} \right\rangle = \frac{\pi}{4}\Omega k_{res} F(k_{res})$$

FRACTIONAL POWER $(\delta B/B_0)^2$ $=G(k_{res})$

THE DEFLECTION ANGLE CHANGES BY ORDER UNITY IN A TIME:

$$\tau \approx \frac{1}{\Omega\, G(k_{res})}$$

PATHLENGTH FOR DIFFUSION ~ VT

$$\left\langle \frac{\Delta z\Delta z}{\Delta t} \right\rangle \approx v^2\ \tau = \frac{v^2}{\Omega\, G(k_{res})}$$

SPATIAL DIFFUSION COEFF.

SPATIAL DIFFUSION COEFFICIENT

THESE FEW STEPS ARE AT THE VERY BASIS OF QUASI-LINEAR THEORY, WHICH LEADS TO A GENERAL EXPRESSION FOR THE DIFFUSION COEFFICIENT IN SPACE

$$D(p) = \frac{1}{2}\langle\frac{\Delta z \Delta z}{\Delta t}\rangle = \frac{1}{3} r_L(p) v(p) \frac{1}{\mathcal{F}(k_{res})}; \qquad k_{res} = \frac{1}{r_L(p)}$$

- THIS EXPRESSION REFERS TO PARALLEL DIFFUSION
- IT CONNECTS THE BEHAVIOUR OF PARTICLES WITH THE SPECTRUM OF TURBULENCE
- IT STRESSES THE CRUCIAL ROLE OF RESONANCES FOR THE OCCURRENCE OF DIFFUSION

PARTICLE SCATTERING

- EACH TIME THAT A RESONANCE OCCURS THE PARTICLE CHANGES PITCH ANGLE BY $\Delta\theta \sim \delta B/B$ WITH A RANDOM SIGN

- THE RESONANCE OCCURS ONLY FOR RIGHT HAND POLARIZED WAVES IF THE PARTICLES MOVES TO THE RIGHT (AND VICEVERSA)

- THE RESONANCE CONDITION TELLS US THAT 1) IF $k \ll 1/r_L$ PARTICLES SURF ADIABATICALLY AND 2) IF $k \gg 1/r_L$ PARTICLES HARDLY FEEL THE WAVES

ACCELERATION OF NONTHERMAL PARTICLES

The presence of non-thermal particles is ubiquitous in the Universe (solar wind, Active galaxies, supernova remnants, gamma ray bursts, Pulsars, micro-quasars)

WHEREVER THERE ARE MAGNETIZED PLASMAS THERE ARE NON-THERMAL PARTICLES

↓

PARTICLE ACCELERATION

BUT THERMAL PARTICLES ARE USUALLY DOMINANT, SO WHAT DETERMINES THE DISCRIMINATION BETWEEN THERMAL AND ACCELERATED PARTICLES?

INJECTION

ALL ACCELERATION MECHANISMS ARE ELECTROMAGNETIC IN NATURE

MAGNETIC FIELD CANNOT MAKE WORK ON CHARGED PARTICLES THEREFORE ELECTRIC FIELDS ARE NEEDED FOR ACCELERATION TO OCCUR

REGULAR ACCELERATION

THE ELECTRIC FIELD IS LARGE SCALE:

$$\langle \vec{E} \rangle \neq 0$$

STOCHASTIC ACCELERATION

THE ELECTRIC FIELD IS SMALL SCALE:

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

STOCHASTIC ACCELERATION

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

Most acceleration mechanisms that are operational in astrophysical environments are of this type. We have seen that the action of random magnetic fluctuations is that of scattering particles when the resonance is achieved. In other words, the particle distribution is isotropized in the reference frame of the wave.

Although in the reference frame of the waves the momentum is conserved (B does not make work) in the lab frame the particle momentum changes by

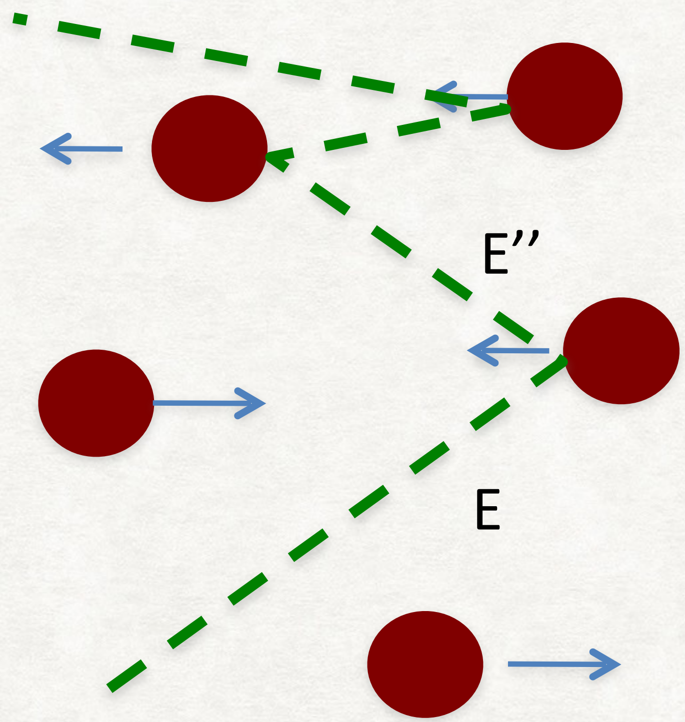
$$\Delta p \sim p \frac{v_A}{c}$$

In a time T which is the diffusion time as found in the last lecture. It follows that

$$D_{pp} = \langle \frac{\Delta P \Delta p}{\Delta t} \rangle \sim p^2 \frac{1}{T} \left(\frac{v_A}{c} \right)^2 \rightarrow \tau_{pp} = \frac{p^2}{D_{pp}} T \left(\frac{c}{v_A} \right)^2 \gg T$$

THE MOMENTUM CHANGE IS A SECOND ORDER PHENOMENON !!!

SECOND ORDER FERMI ACCELERATION



We inject a particle with energy E. In the reference frame of a cloud moving with speed β the particle energy is:

$$E' = \gamma E + \beta\gamma p\mu$$

and the momentum along x is:

$$p'_x = \beta\gamma E + \gamma p\mu$$

Assuming that the cloud is very massive compared with the particle, we can assume that the cloud is unaffected by the scattering, therefore the particle energy in the cloud frame does not change and the momentum along x is simply inverted, so that after 'scattering' $p'_x \rightarrow -p'_x$. The final energy in the Lab frame is therefore:

$$E'' = \gamma E' + \beta\gamma p'_x =$$

$$\gamma^2 E \left(1 + \beta^2 + 2\beta\mu\frac{p}{E}\right)$$

$$\frac{p}{E} = \frac{mv\gamma}{m\gamma} = v$$

Where v is now the dimensionless particle velocity

It follows that: $$E'' = \gamma^2 E \left(1 + \beta^2 + 2\beta\mu v\right)$$

and:

$$\frac{E'' - E}{E} = \gamma^2 \left(1 + 2\beta v\mu + \beta^2\right) - 1$$

and finally, taking the limit of non-relativistic clouds $\gamma \rightarrow 1$:

$$\frac{E'' - E}{E} \approx 2\beta^2 + 2\beta v\mu$$

We can see that the fractional energy change can be both positive or negative, which means that particles can either gain or lose energy, depending on whether the particle-cloud scattering is head-on or tail-on.

We need to calculate the probability that a scattering occurs head-on or Tail-on. The scattering probability along direction μ is proportional to the Relative velocity in that direction:

$$P(\mu) = Av_{rel} = A\frac{\beta\mu + v}{1 + v\beta\mu} \rightarrow_{v\rightarrow 1} \approx A(1 + \beta\mu)$$

The condition of normalization to unity:

$$\int_{-1}^{1} P(\mu) d\mu = 1$$

leads to A=1/2. It follows that the mean fractional energy change is:

$$\langle \frac{\Delta E}{E} \rangle = \int_{-1}^{1} d\mu P(\mu) \left(2\beta^2 + 2\beta\mu\right) = \frac{8}{3}\beta^2$$

NOTE THAT IF WE DID NOT ASSUME RIGID REFLECTION AT EACH CLOUD BUT RATHER ISOTROPIZATION OF THE PITCH ANGLE IN EACH CLOUD, THEN WE WOULD HAVE OBTAINED (4/3) β^2 INSTEAD OF (8/3) β^2

THE FRACTIONAL CHANGE IS A SECOND ORDER QUANTITY IN $\beta<<1$. This is the reason for the name SECOND ORDER FERMI ACCELERATION

The acceleration process can in fact be shown to become more important in the relativistic regime where $\beta \rightarrow 1$

THE PHYSICAL ESSENCE CONTAINED IN THIS SECOND ORDERDEPENDENCE IS THAT IN EACH PARTICLE-CLOUD SCATTERING THE ENERGY OF THE PARTICLE CAN EITHER INCREASE OR DECREASE → WE ARE LOOKING AT A PROCESS OF DIFFUSION IN MOMENTUM SPACE

THE REASON WHY ON AVERAGE THE MEAN ENERGY INCREASES IS THAT HEAD-ON COLLISIONS ARE MORE PROBABLE THAN TAIL-ON COLLISIONS

WHAT IS DOING THE WORK?

We just found that particles propagating in a magnetic field can change their momentum (in modulus and direction)…

BUT MAGNETIC FIELDS CANNOT CHANGE THE MOMENTUM MODULUS… ONLY ELECTRIC FIELDS CAN

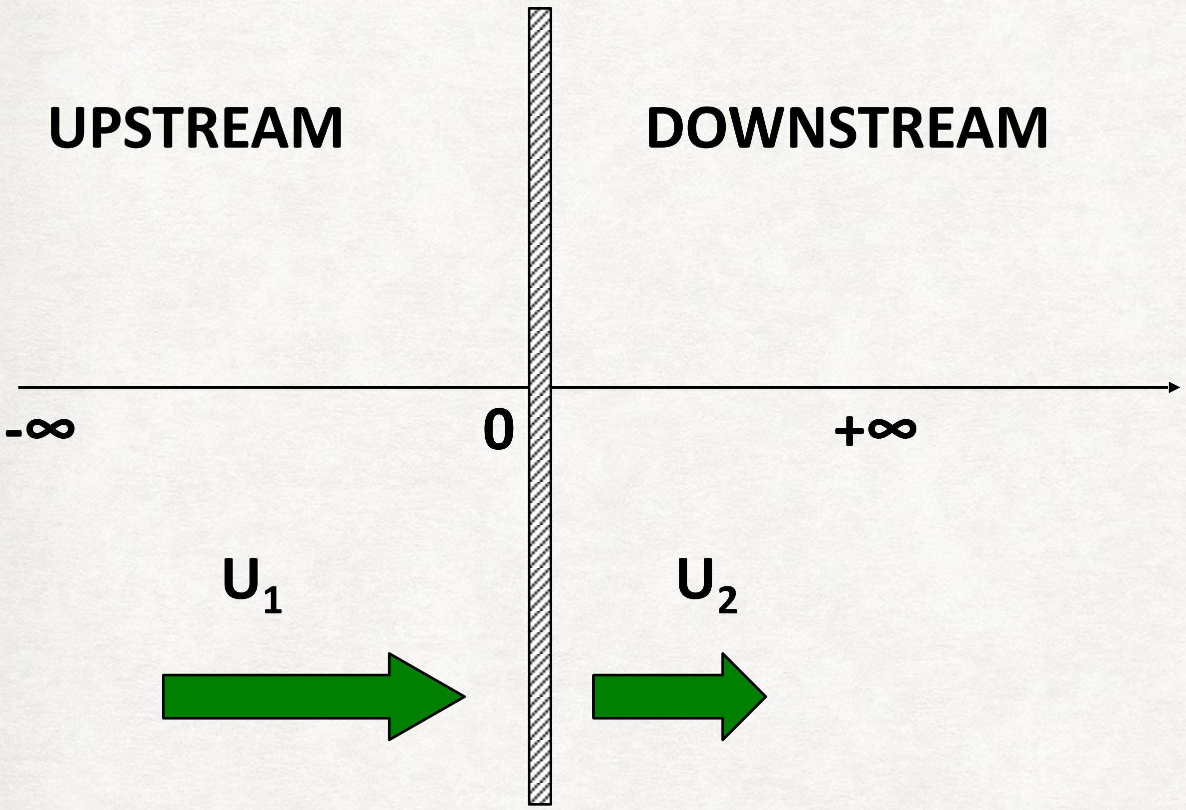
WHAT IS THE SOURCE OF THE ELECTRIC FIELDS???
Moving Magnetic Fields

The induced electric field is responsible for this first instance of particle acceleration

The scattering leads to momentum transfer, but to WHAT?

Recall that particles isotropize in the reference frame of the waves…

SHOCK SOLUTIONS



Let us sit in the reference frame in which the shock is at rest and look for stationary solutions

$$\frac{\partial}{\partial x}(\rho u) = 0$$

$$\frac{\partial}{\partial x}(\rho u^2 + P) = 0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma - 1}uP\right) = 0$$

It is easy to show that aside from the trivial solution in which all quantities remain spatially constant, there is a discontinuous solution:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}$$

M_1 is the upstream Fluid Mach number

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - \gamma(\gamma-1)][(\gamma-1)M_1^2 + 2]}{(\gamma+1)^2 M_1^2}$$

STRONG SHOCKS $M_1 >> 1$

In the limit of strong shock fronts these expressions get substantially simpler and one has:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_1^2, \quad T_2 = 2\frac{\gamma-1}{(\gamma+1)^2} m u_1^2$$

ONE CAN SEE THAT SHOCKS BEHAVE AS VERY EFFICIENT HEATING MACHINES IN THAT A LARGE FRACTION OF THE INCOMING RAM PRESSURE IS CONVERTED TO INTERNAL ENERGY OF THE GAS BEHIND THE SHOCK FRONT...

COLLISIONLESS SHOCKS

While shocks in the terrestrial environment are mediated by particle-particle collisions, astrophysical shocks are almost always of a different nature. The pathlength for ionized plasmas is of the order of:

$$\lambda \simeq \frac{1}{n\sigma} = 3.2 Mpc\ n_1^{-1} \left(\frac{\sigma}{10^{-25} cm^2}\right)^{-1}$$

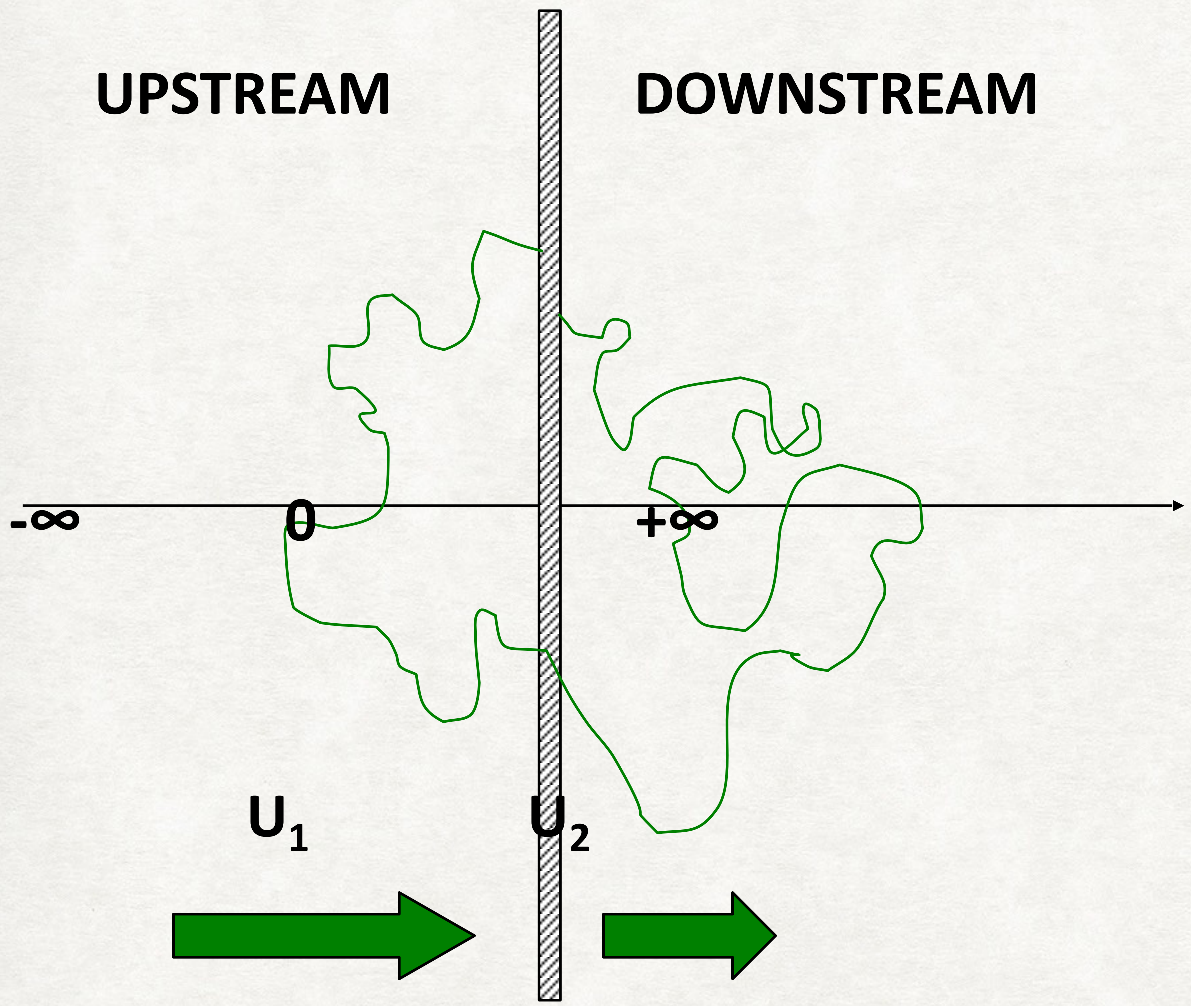
Absurdly large compared with any reasonable length scale. It follows that astrophysical shocks can hardly form because of particle-particle scattering but REQUIRE the mediation of magnetic fields. In the downstream gas the Larmor radius of particles is:

$$r_{L,th} \approx 10^{10} B_\mu T_8^{1/2}\ cm$$

The slowing down of the incoming flow and its isotropization (thermalization) is due to the action of magnetic fields in the shock region (COLLISIONLESS SHOCKS)

DIFFUSIVE SHOCK ACCELERATION
OR
FIRST ORDER FERMI ACCELERATION

BOUNCING BETWEEN APPROACHING MAGNETIC MIRRORS



Let us take a relativistic particle with energy E~p upstream of the shock. In the downstream frame:

$$E_d = \gamma E(1+\beta\mu) \quad 0 \le \mu \le 1$$

where $\beta = u_1 - u_2 > 0$. In the downstream frame the direction of motion of the particle is isotropized and reapproaches the shock with the same energy but pitch angle μ'

$$E_u = \gamma E_d - \beta E_d \gamma \mu' = \gamma^2 E(1+\beta\mu)(1-\beta\mu')$$

$$-1 \le \mu' \le 0$$

In the non-relativistic case the particle distribution is, at zeroth order, isotropic
Therefore:

TOTAL FLUX

$$J = \int_0^1 d\Omega \frac{N}{4\pi} v\mu = \frac{Nv}{4} \quad \Rightarrow \quad P(\mu)d\mu = \frac{ANv\mu}{\frac{Nv}{4}} d\mu = 2\mu d\mu$$

The mean value of the energy change is therefore:

$$\langle \frac{E_u - E}{E} \rangle = -\int_0^1 d\mu 2\mu \int_{-1}^0 d\mu' 2\mu' \left[\gamma^2 (1+\beta\mu)(1-\beta\mu') - 1\right] \approx \frac{4}{3}\beta = \frac{4}{3}(u_1 - u_2)$$

A FEW IMPORTANT POINTS:

I. There are no configurations that lead to losses

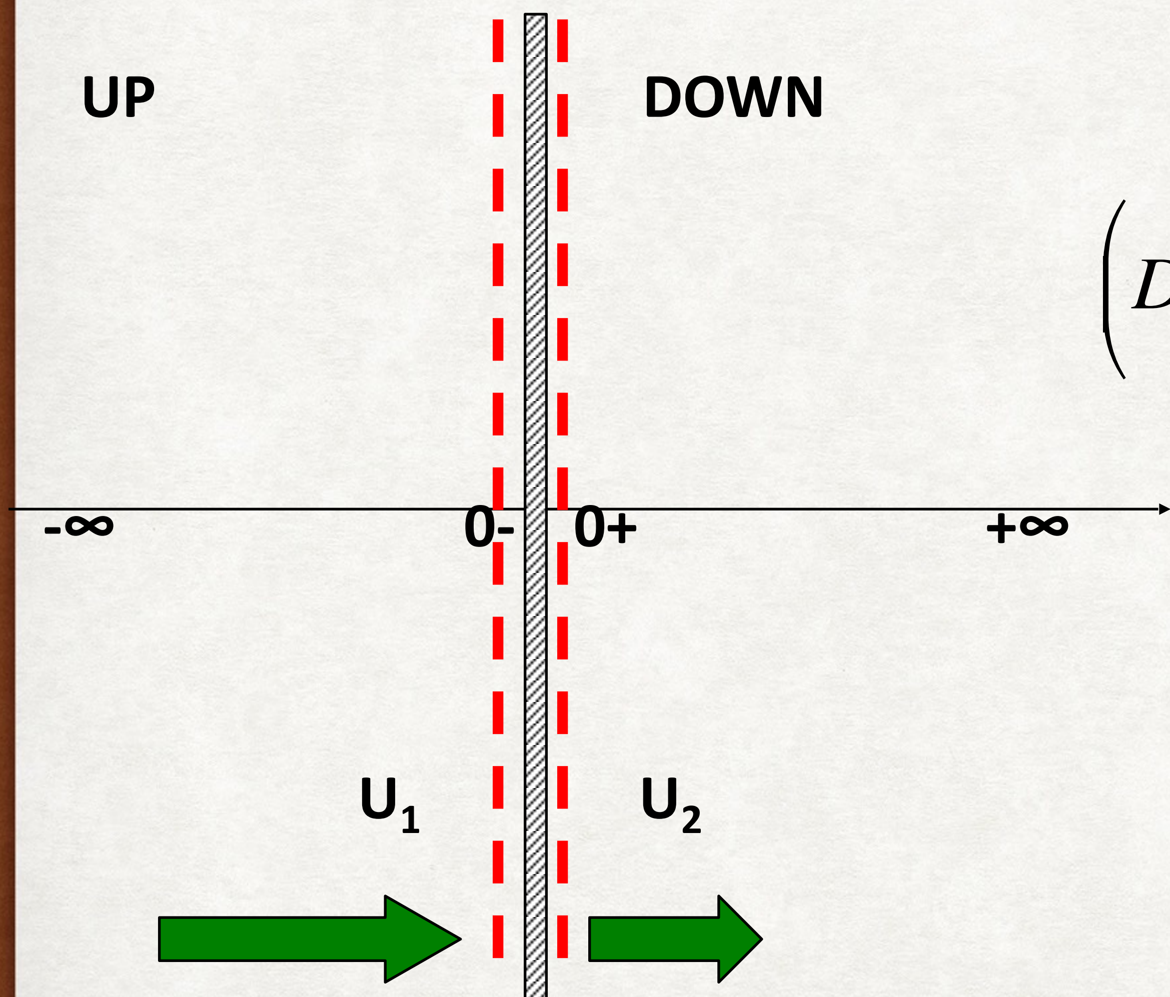
II. The mean energy gain is now first order in β

III. The energy gain is basically independent of any detail on how particles scatter back and forth!

THE TRANSPORT EQUATION APPROACH

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p,t)$$

DIFFUSION ADVECTION COMPRESSION INJECTION



Integrating around the shock:

$$\left(D\frac{\partial f}{\partial x}\right)_2 - \left(D\frac{\partial f}{\partial x}\right)_1 + \frac{1}{3}(u_2 - u_1)p\frac{df_0(p)}{dp} + Q_0(p) = 0$$

Integrating from upstr. infinity to 0-:

$$\left(D\frac{\partial f}{\partial x}\right)_1 = u_1 f_0$$

and requiring homogeneity downstream:

$$p\frac{df_0}{dp} = \frac{3}{u_2 - u_1}(u_1 f_0 - Q_0)$$

THE TRANSPORT EQUATION APPROACH

INTEGRATION OF THIS SIMPLE EQUATION GIVES:

$$f_0(p) = \frac{3u_1}{u_1 - u_2} \frac{N_{inj}}{4\pi p_{inj}^2} \left(\frac{p}{p_{inj}} \right)^{\frac{-3u_1}{u_1 - u_2}}$$

DEFINE THE COMPRESSION FACTOR $r = u_1/u_2 \rightarrow 4$ (strong shock)

THE SLOPE OF THE SPECTRUM IS

$$\frac{3u_1}{u_1 - u_2} = \frac{3}{1 - 1/r} \rightarrow 4 \quad \text{if r} \rightarrow 4$$

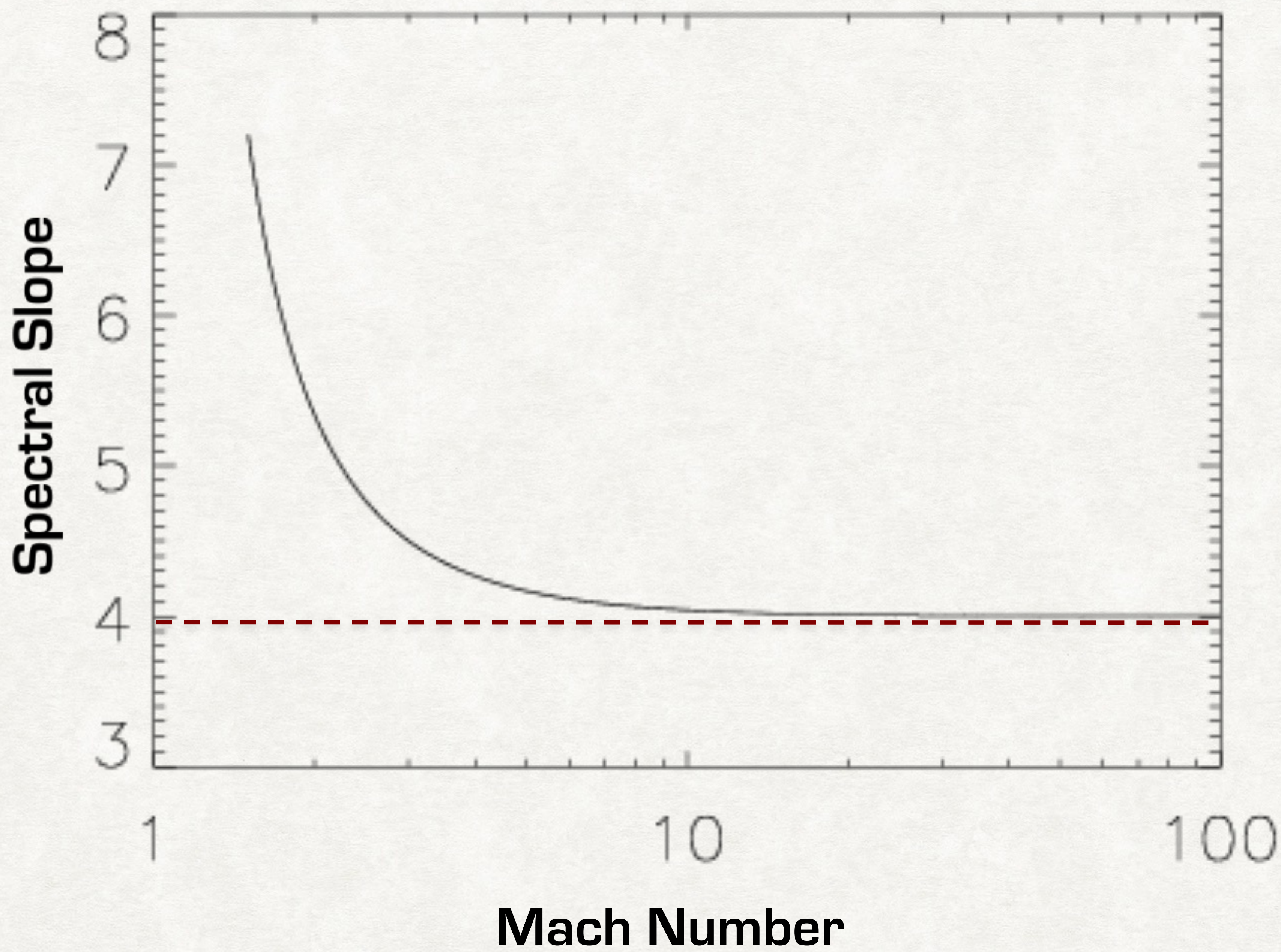
NOTICE THAT: $N(p)dp = 4\pi p^2 f(p)dp \rightarrow N(p) \propto p^{-2}$

1. **THE SPECTRUM OF ACCELERATED PARTICLES IS A POWER LAW IN MOMENTUM EXTENDING TO INFINITE MOMENTA**

2. **THE SLOPE DEPENDS UNIQUELY ON THE COMPRESSION FACTOR AND IS INDEPENDENT OF THE DIFFUSION PROPERTIES**

3. **NO DEPENDENCE UPON DIFFUSION (MICRO-PHYSICS) —- BUT E_{MAX}**

TEST PARTICLE SPECTRUM



REACCELERATION VS ACCELERATION

SHOCKS ARE BLIND TO THE NATURE OF THE CHARGED PARTICLES

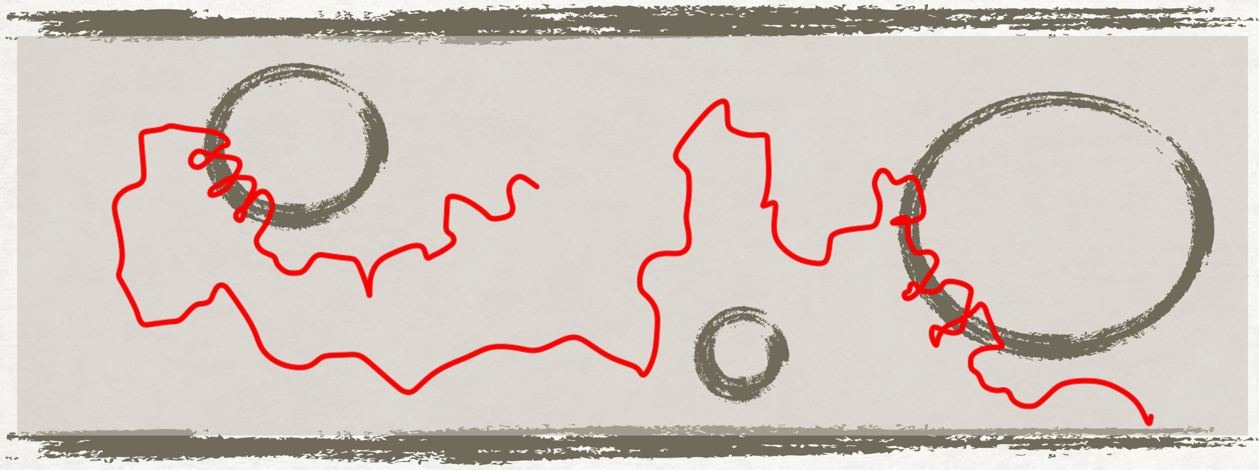
SEED CR ARE ACCELERATED

IF THEIR SPECTRUM IS STEEPER THAN THE ONE THAT IS ASSOCIATED WITH THE SHOCK MACH NUMBER —> THEIR SPECTRUM GETS HARDER

IF THEIR SPECTRUM IS HARDER THAN THE ONE THAT IS ASSOCIATED WITH THE SHOCK MACH NUMBER —> THEIR SPECTRUM REMAINS THE SAME

IN BOTH CASES ENERGY IS ADDED BUT THE TOTAL NUMBER OF PARTICLES IS CONSERVED

SHOCK ACCELERATION OF SECONDARY NUCLEI



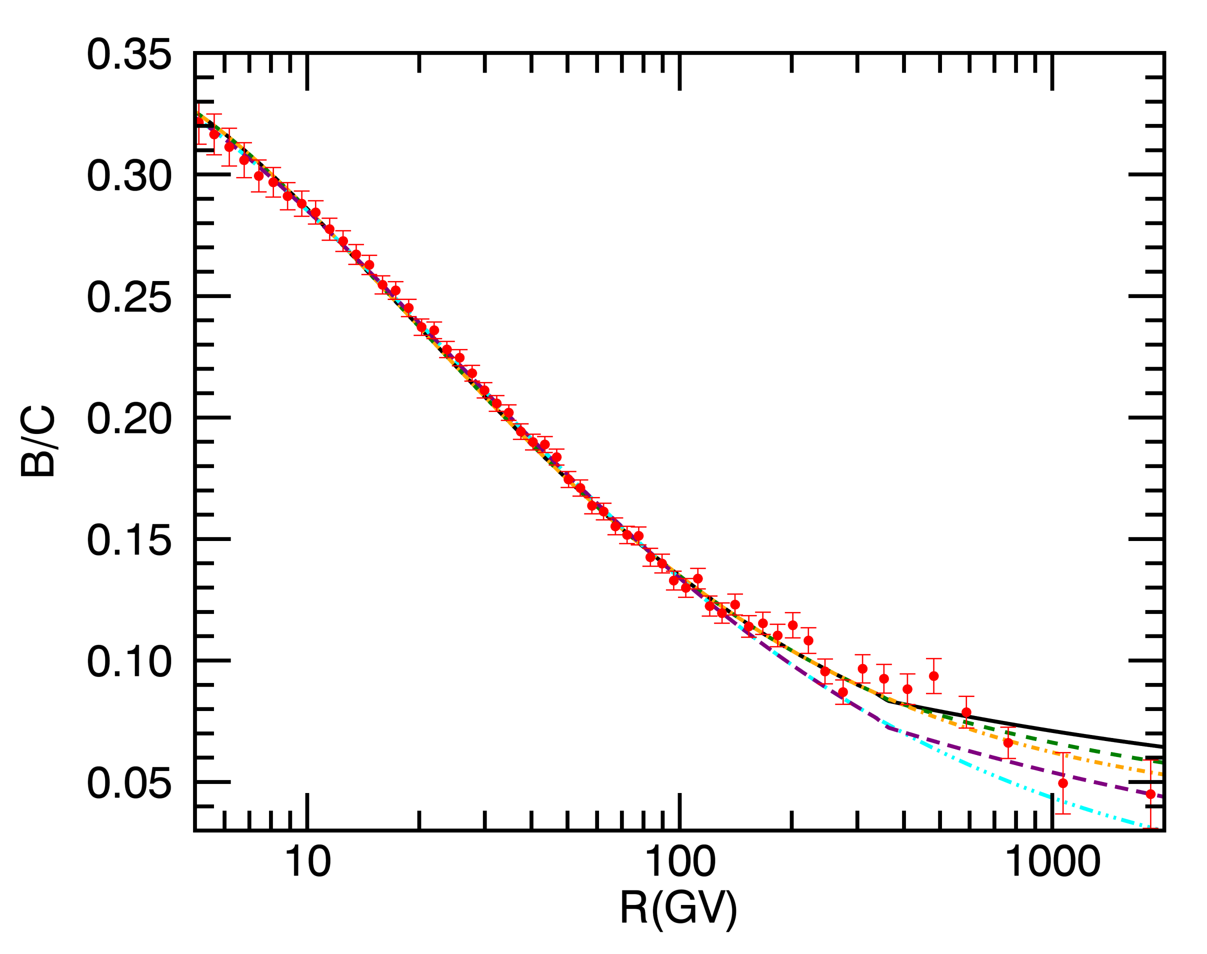
SECONDARY NUCLEI (AS WELL AS PRIMARY) OCCASIONALLY ENCOUNTER A SN SHOCK AND GET ACCELERATED AT IT — SHOCK IS BLIND TO THE NATURE OF PARTICLES

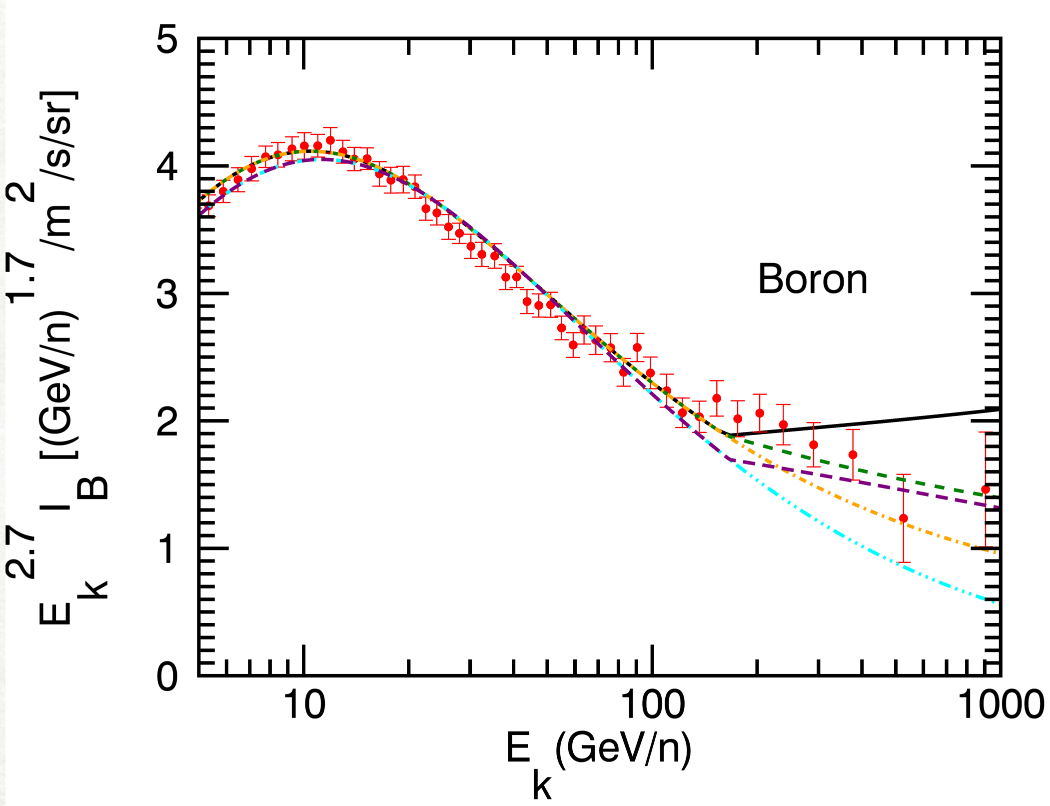
PRIMARY NUCLEI	SECONDARY NUCLEI
thermal seeds —> $E^{-\gamma}$	$E^{-\gamma-\delta}$ —> $E^{-\gamma}$

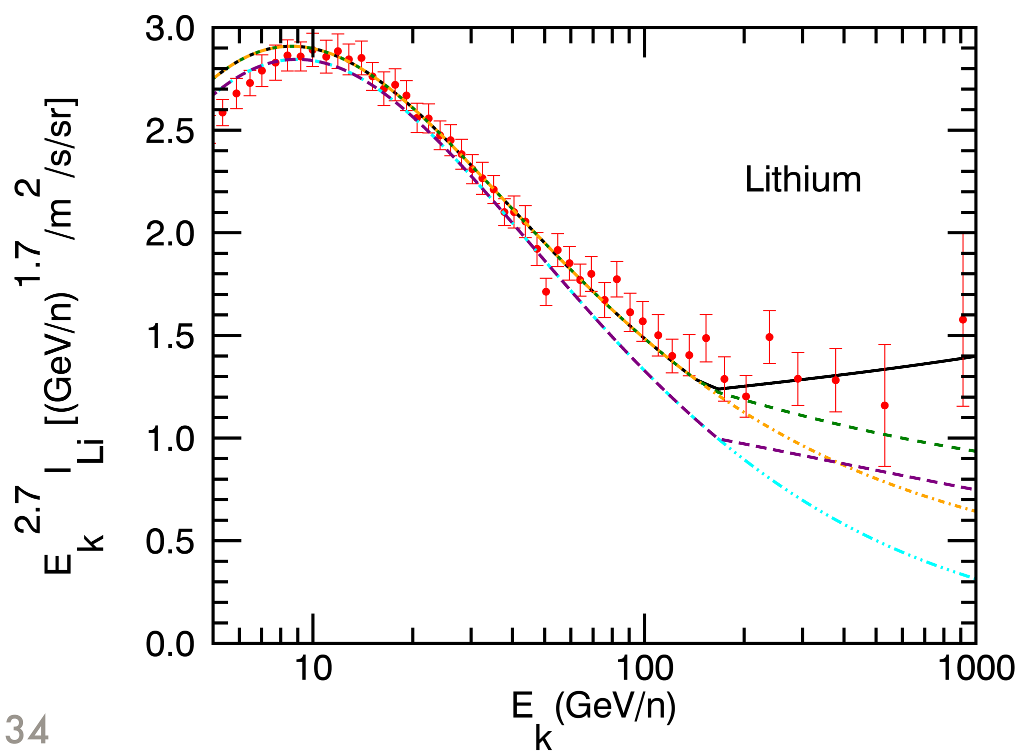
IT IS CLEAR THAT THE OCCASIONAL ACCELERATION OF SECONDARY NUCLEI MUST BE THE MAIN CONTRIBUTION AT SUFFICIENTLY HIGH E, TYPICALLY ABOVE TeV (PB 2017)

SHOCK ACCELERATION OF SECONDARY NUCLEI

PB 2017







CLEARLY NOT AN ATTEMPT TO MAKE A DETAILED PREDICTION, BUT RATHER DISCUSSION OF A NEW EFFECT

SOME IMPORTANT COMMENTS

- THE STATIONARY PROBLEM DOES NOT ALLOW TO HAVE A MAX MOMENTUM WITH BOUNDARY CONDITION AT INFINITY!
- THE NORMALIZATION IS ARBITRARY THEREFORE THERE IS NO CONTROL ON THE AMOUNT OF ENERGY IN CR
- AND YET IT HAS BEEN OBTAINED IN THE TEST PARTICLE APPROXIMATION
- THE SOLUTION DOES NOT DEPEND ON WHAT IS THE MECHANISM THAT CAUSES PARTICLES TO BOUNCE BACK AND FORTH
- FOR STRONG SHOCKS THE SPECTRUM IS UNIVERSAL AND CLOSE TO E^{-2}
- IT HAS BEEN IMPLICITELY ASSUMED THAT WHATEVER SCATTERS THE PARTICLES IS AT REST (OR SLOW) IN THE FLUID FRAME

MAXIMUM ENERGY

The maximum energy in an accelerator is determined by either the age of the accelerator compared with the acceleration time or the size of the system compared with the diffusion length D(E)/u. The hardest condition is the one that dominates.

Using the diffusion coefficient in the ISM derived from the B/C ratio:

$$D(E) \approx 3 \times 10^{28} E_{GeV}^{1/3} cm^2/s$$

and the velocity of a SNR shock as u=5000 km/s one sees that:

$$t_{acc} \sim D(E)/u^2 \sim 4 \times 10^3 E_{GeV}^{1/3} \ years$$

Too long for any useful acceleration → NEED FOR ADDITIONAL TURBULENCE

$$t_{acc}(p) = \langle t \rangle = \frac{3}{u_1 - u_2} \int_{p_0}^{p} \frac{dp'}{p'} \left[\frac{D_1(p')}{u_1} + \frac{D_2(p')}{u_2} \right]$$

Drury 1983

ENERGY LOSSES AND ELECTRONS

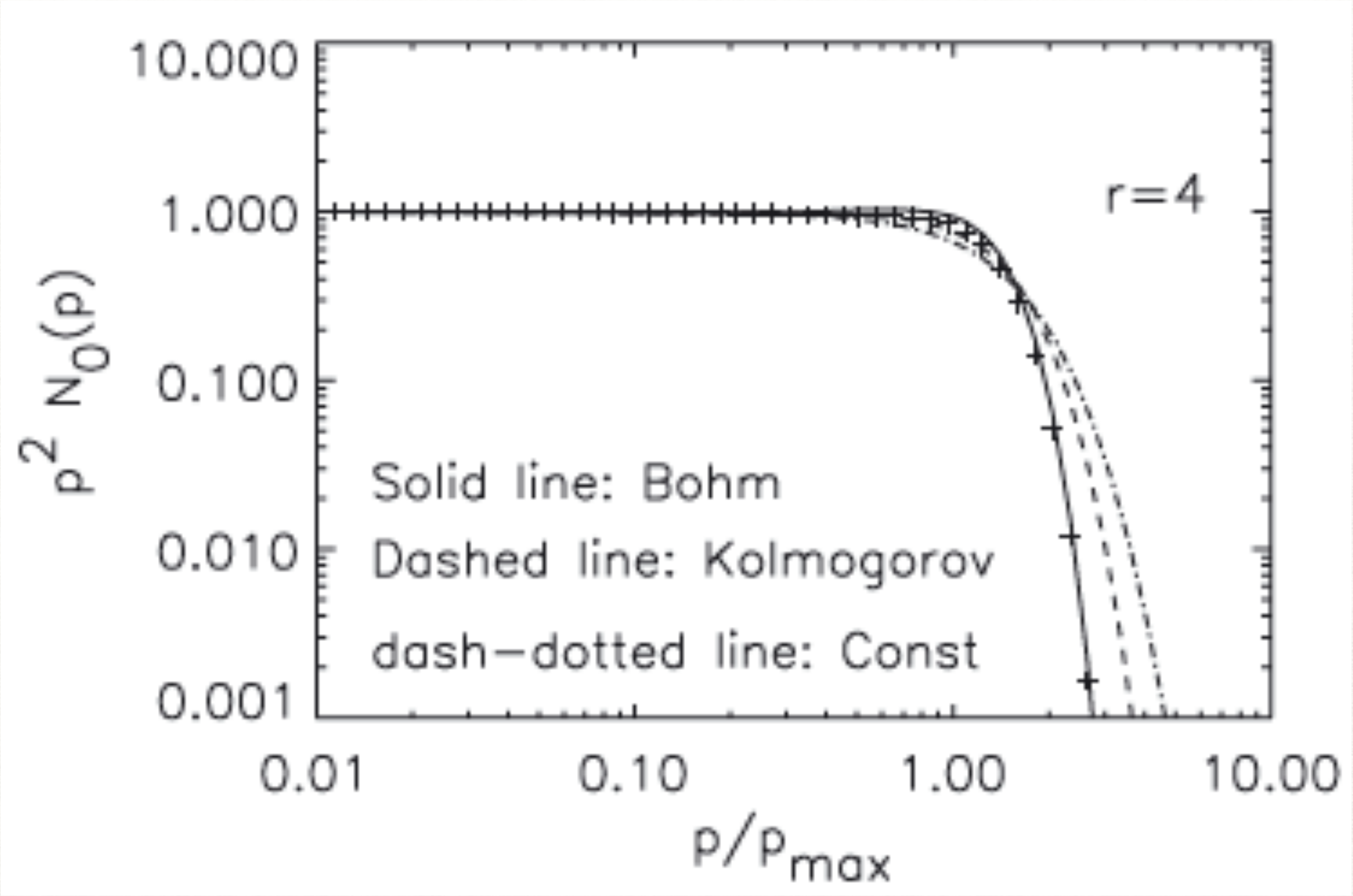
For electrons, energy losses make acceleration even harder.

The maximum energy of electrons is determined by the condition:

$$t_{acc} \leq Min\left[Age, \tau_{loss}\right]$$

Where the losses are mainly due to synchrotron and inverse Compton Scattering.

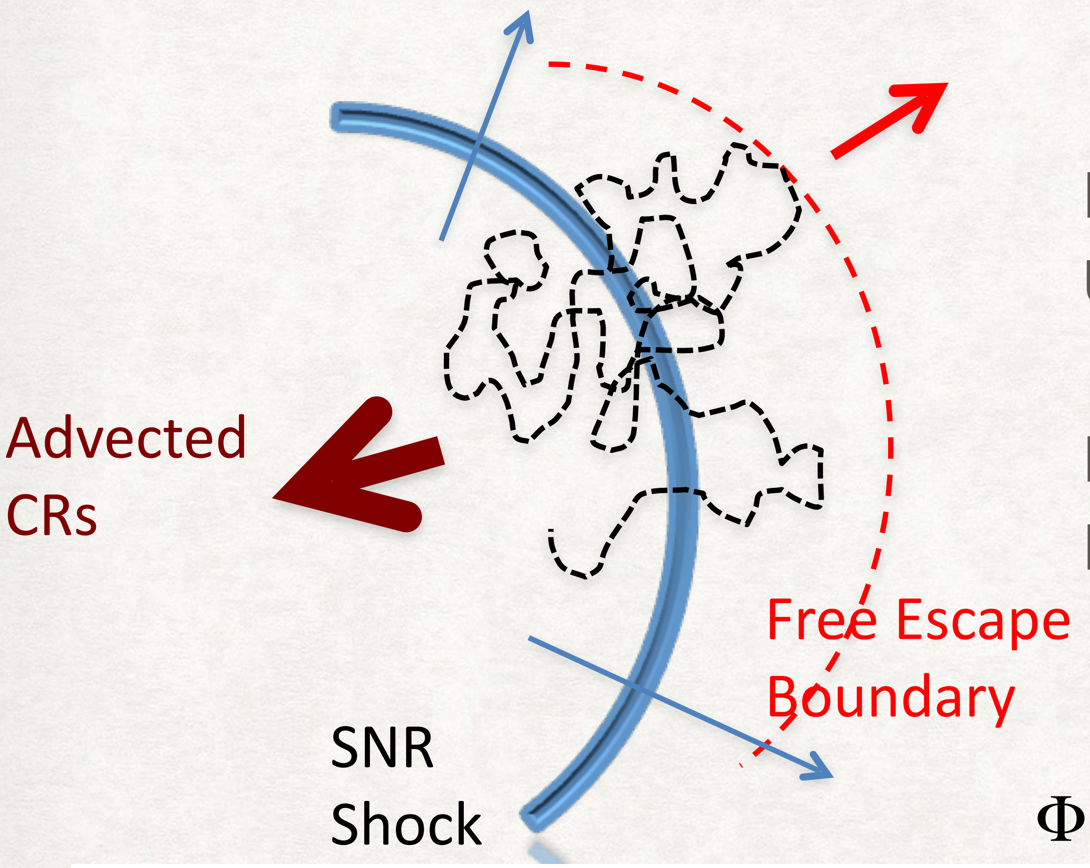
ELECTRONS IN ONE SLIDE



PB 2010

Zirakashvili&Aharonian 2007

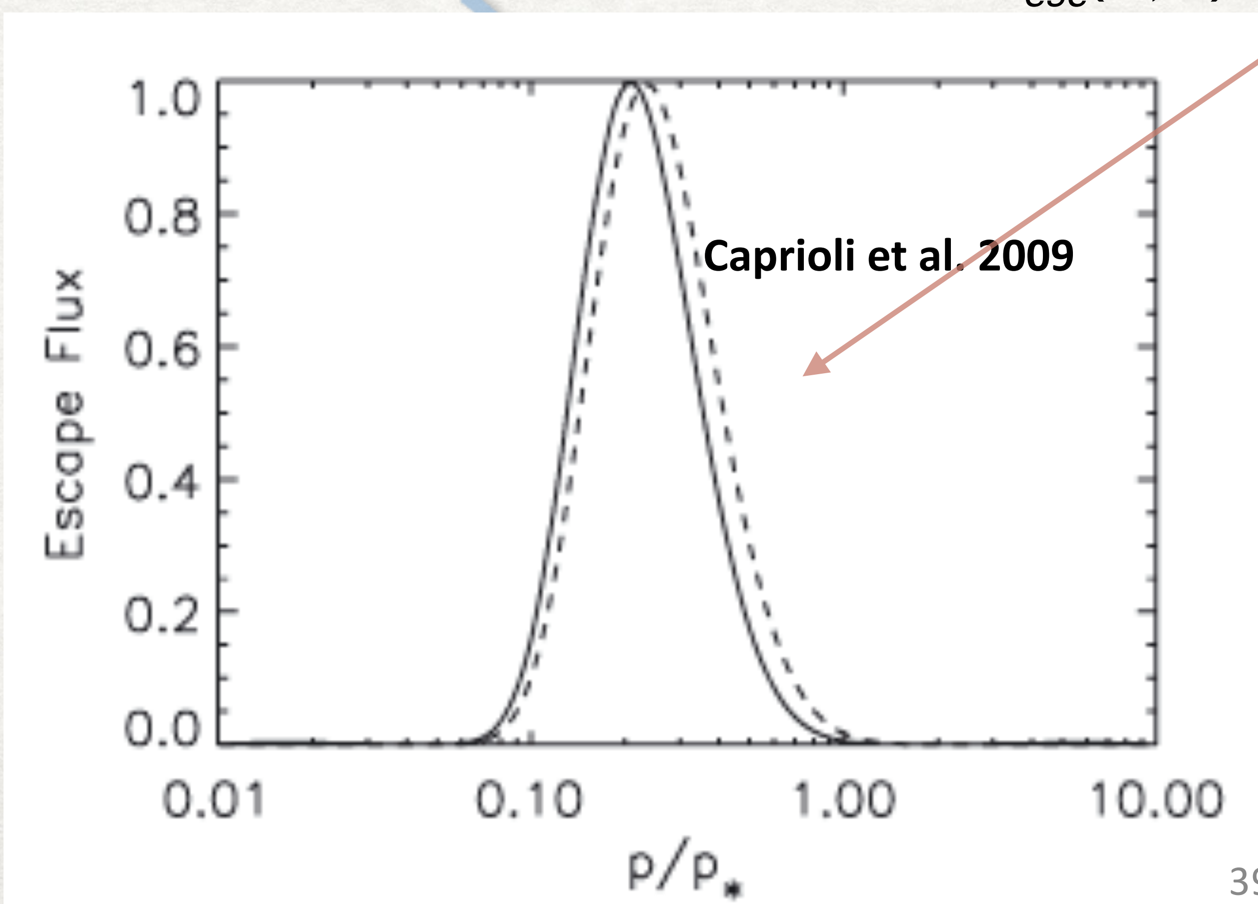
THE PROBLEM OF ESCAPE FROM THE ACCELERATOR

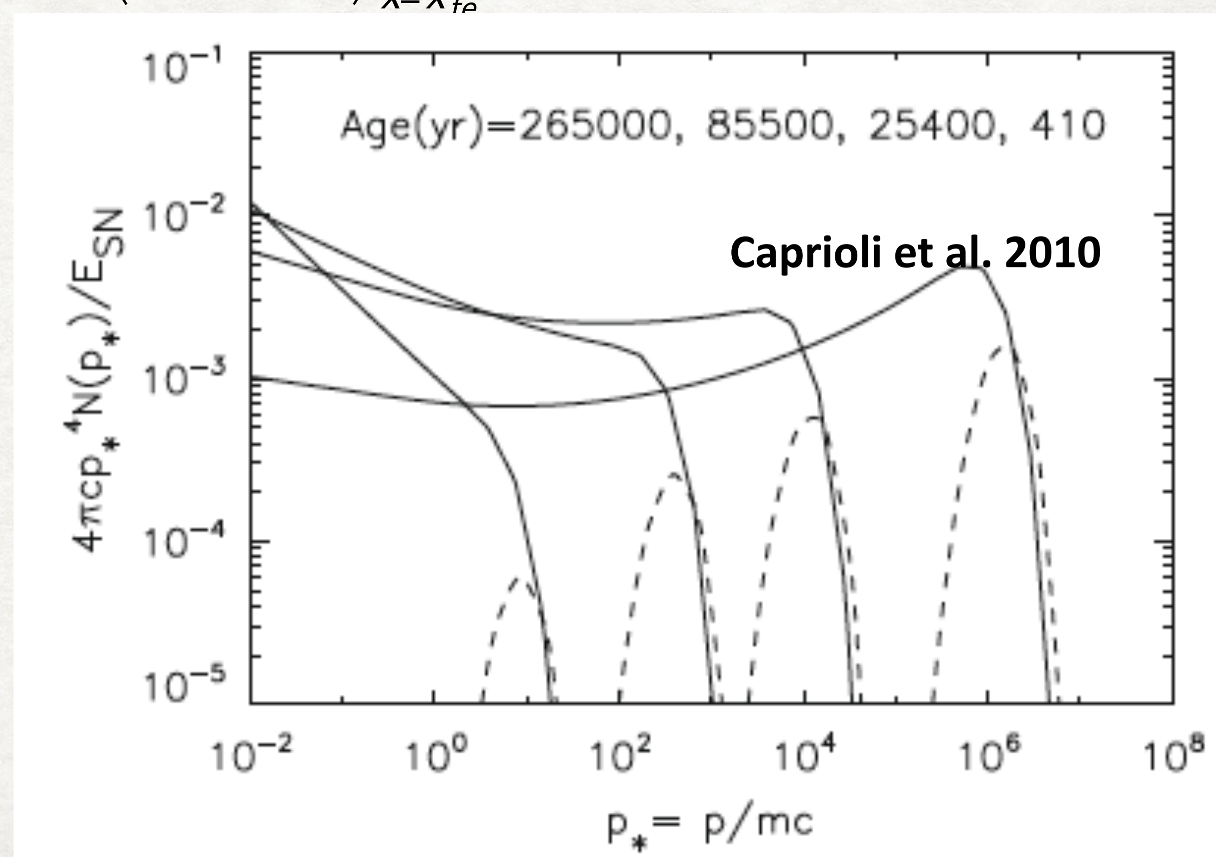


IN STANDARD DSA THERE IS NO ESCAPE FROM UPSTREAM

ESCAPE CAN BE FORCED BY A IMPOSING A FREE ESCAPE BOUNDARY CONDITION

$$\Phi_{esc}(E,x) = D(E)\left(\frac{\partial f(E,x)}{\partial x}\right)_{x=x_{fe}}$$





NON LINEAR THEORY OF DSA

WHY DO WE NEED A NON LINEAR THEORY?

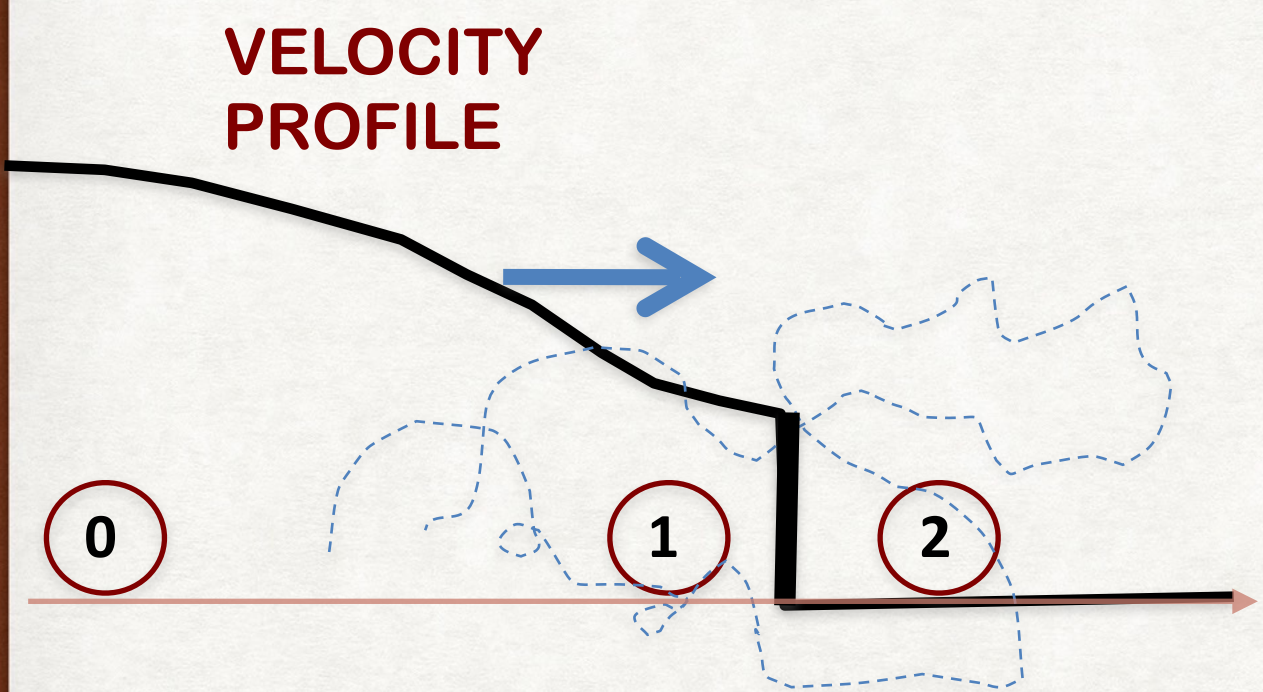
TEST PARTICLE THEORY PREDICTS ENERGY DIVERGENT SPECTRA

THE TYPICAL EFFICIENCY EXPECTED OF A SNR (~10%) IS SUCH THAT TEST PARTICLE THEORY IS A BAD APPROXIMATION

THE MAX MOMENTUM CAN ONLY BE INTRODUCED BY HAND IN TEST PARTICLE THEORY

SIMPLE ESTIMATES SHOW THAT EMAX IS VERY LOW UNLESS CR TAKE PART IN THE ACCELERATION PROCESS, BY AFFECTING THEIR OWN SCATTERING

DYNAMICAL REACTION OF ACCELERATED PARTICLES



Particle transport is described by using the usual transport equation including diffusion and advection

But now dynamics is important too:

$$\rho_0 u_0 = \rho_1 u_1$$

Conservation of Mass

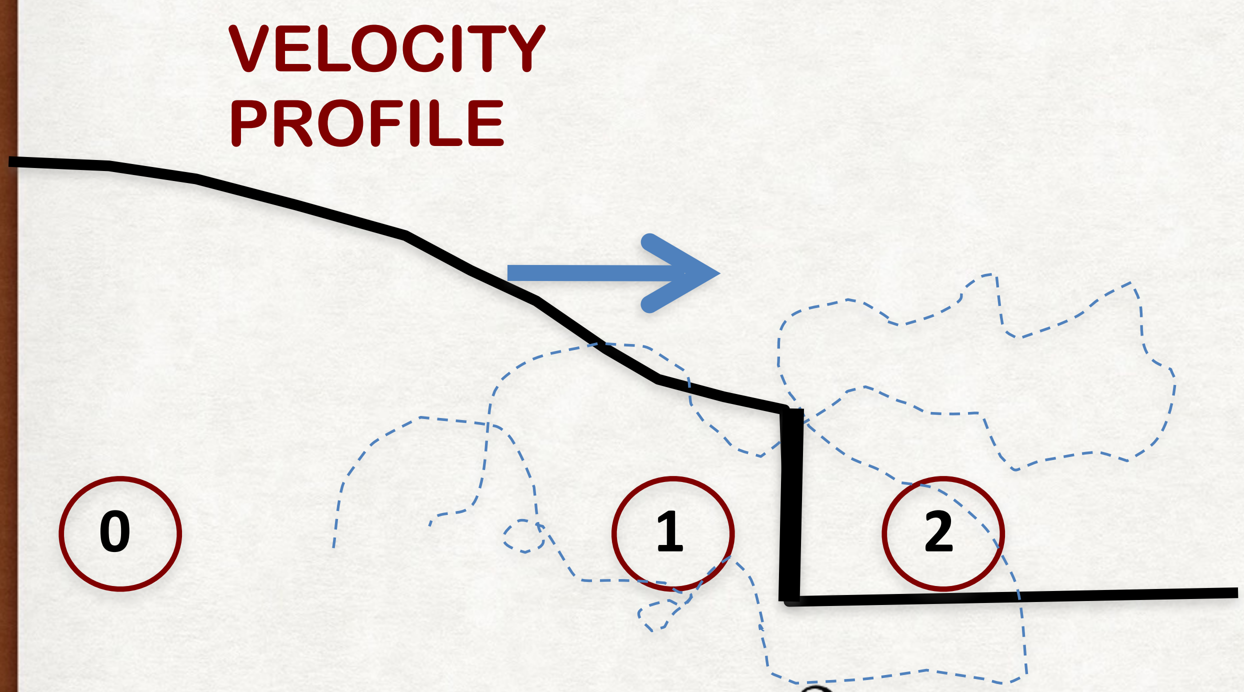
$$\rho_0 u_0^2 + P_{g,0} = \rho_1 u_1^2 + P_{g,1} + P_{c,1}$$

Conservation of Momentum

Conservation of Energy

$$\frac{1}{2}\rho_0 u_0^3 + \frac{P_{g,0} u_0 \gamma_g}{\gamma_g - 1} - F_{esc} = \frac{1}{2}\rho_1 u_1^3 + \frac{P_{g,1} u_1 \gamma_g}{\gamma_g - 1} + \frac{P_{c,1} u_1 \gamma_c}{\gamma_c - 1}$$

FORMATION OF A PRECURSOR - SIMPLIFIED



$$\frac{\partial}{\partial x}[\rho u] = 0 \rightarrow \rho(x)u(x) = \rho_0 u_0$$

$$\frac{\partial}{\partial x}\left[P_g + \rho u^2 + P_{CR}\right] = 0$$

$$P_g(x) + \rho u^2 + P_{CR} = P_{g,0} + \rho_0 u_0^2$$

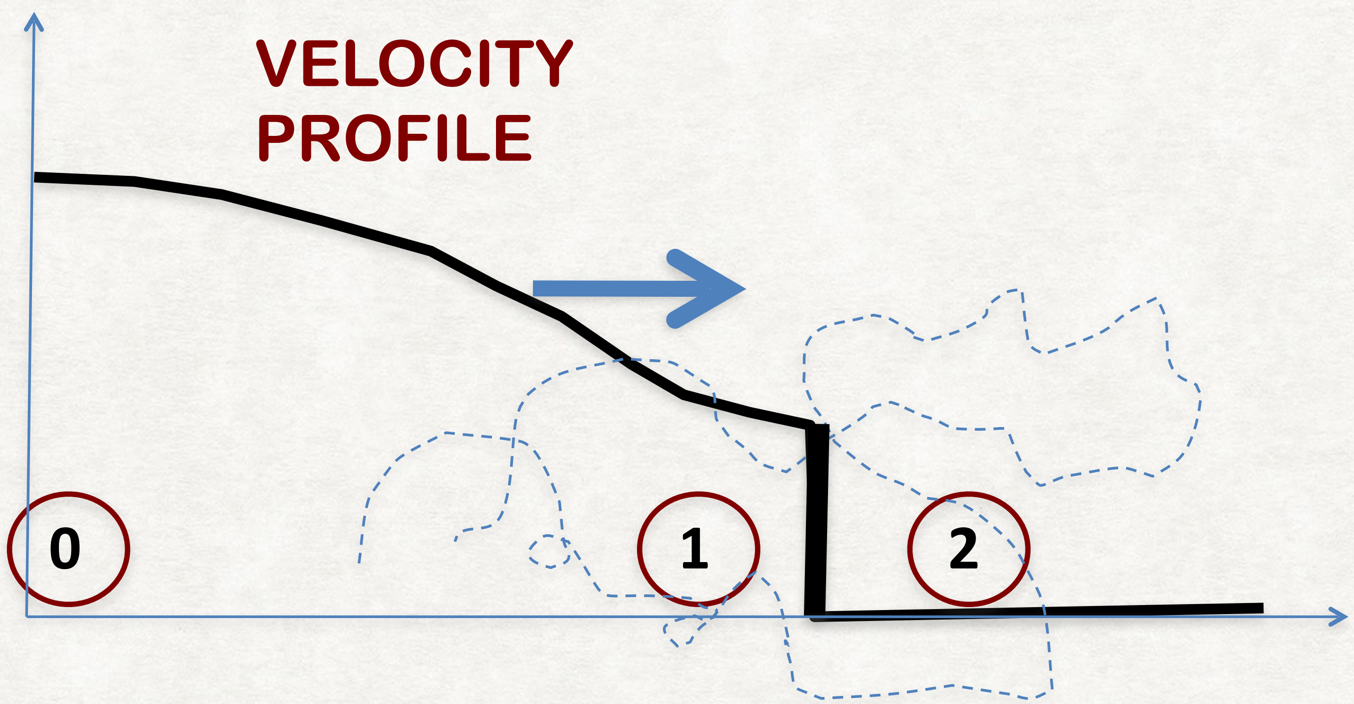
AND DIVIDING BY THE RAM PRESSURE AT UPSTREAM INFINITY:

$$\frac{P_g}{\rho_0 u_0^2} + \frac{u}{u_0} + \frac{P_{CR}}{\rho_0 u_0^2} = \frac{P_{g,0}}{\rho_0 u_0^2} + 1 \Longrightarrow \frac{u}{u_0} \approx 1 - \xi_{CR}(x)$$

WHERE WE NEGLECTED TERMS OF ORDER $1/M^2$

$$\xi_{CR}(x) = \frac{P_{CR}(x)}{\rho_0 u_0^2}$$

BASIC PREDICTIONS OF NON LINEAR THEORY



COMPRESSION FACTOR BECOMES FUNCTION OF ENERGY

SPECTRA ARE NOT PERFECT POWER LAWS (CONCAVE)

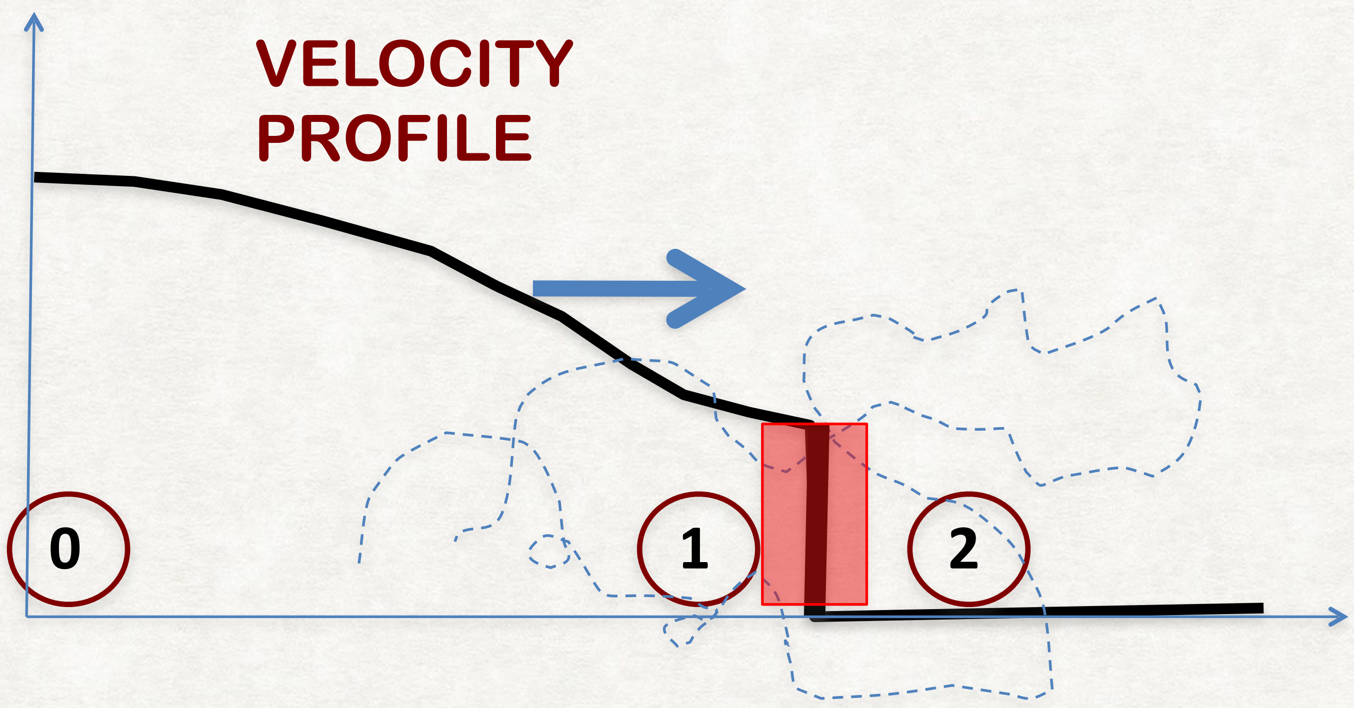
GAS BEHIND THE SHOCK IS COOLER FOR EFFICIENT SHOCK ACCELERATION

SYSTEM SELF REGULATED

EFFICIENT GROWTH OF B-FIELD IF ACCELERATION EFFICIENT

PB+2010

BASIC PREDICTIONS OF NON LINEAR THEORY



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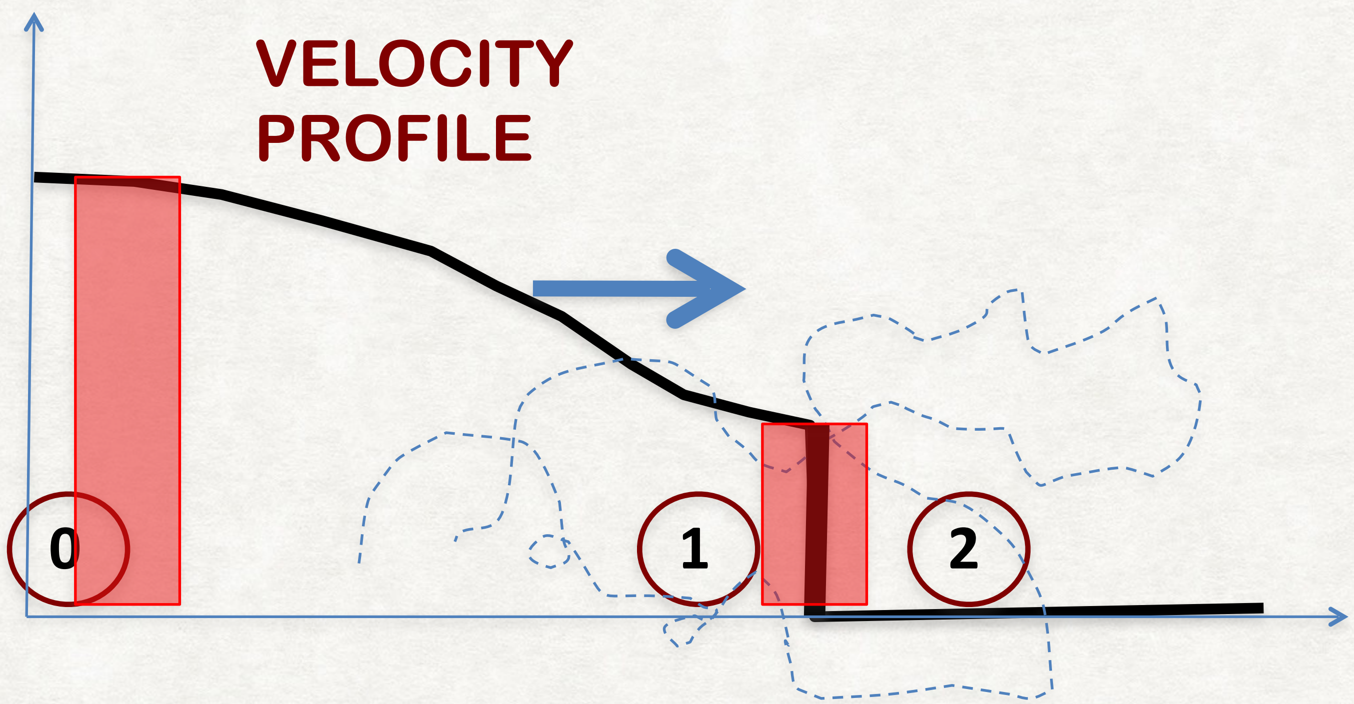

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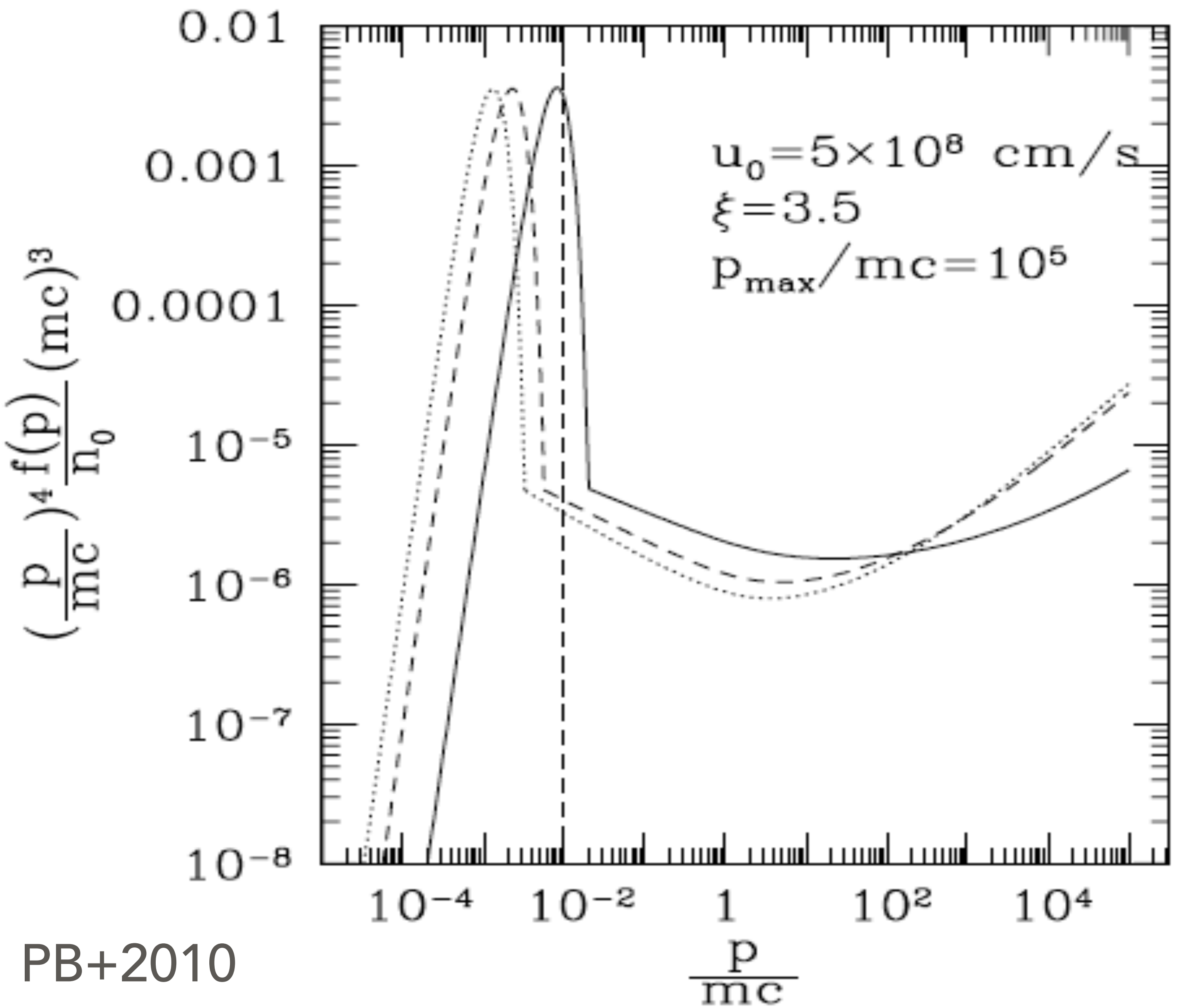

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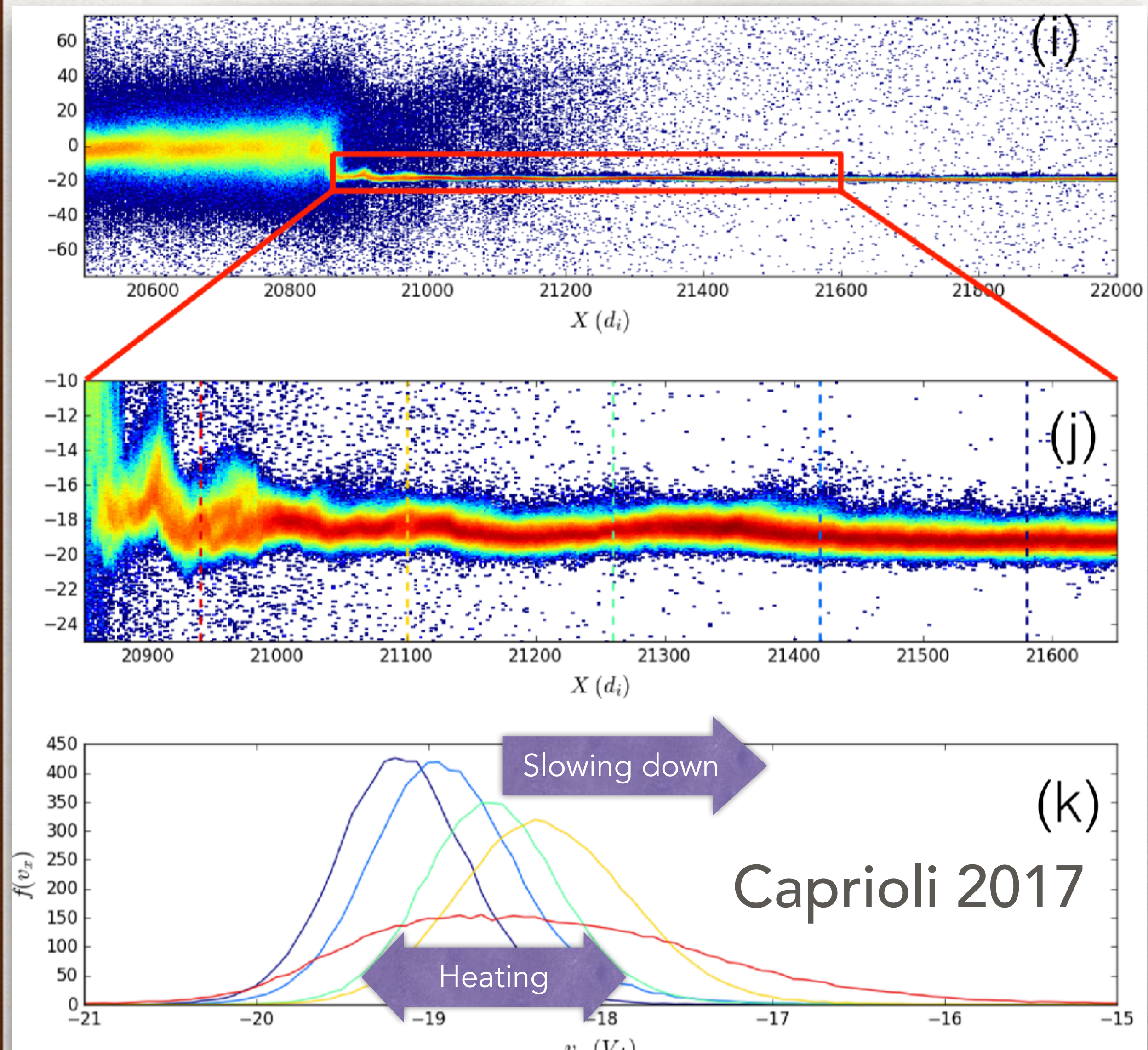
EFFECT OF TURBULENT DAMPING

AT LEAST A FRACTION OF THE ENERGY OF CR UPSTREAM IS TRANSFERRED TO THE THERMAL ENERGY OF THE BACKGROUND PLASMA

THIS PROCESS (TURBULENT HEATING) LEADS TO A REDUCTION OF THE MACH NUMBER IN THE PRECURSOR —> SMOOTHER PRECURSOR —> **SPECTRA AGAIN CLOSE TO E^{-2}**

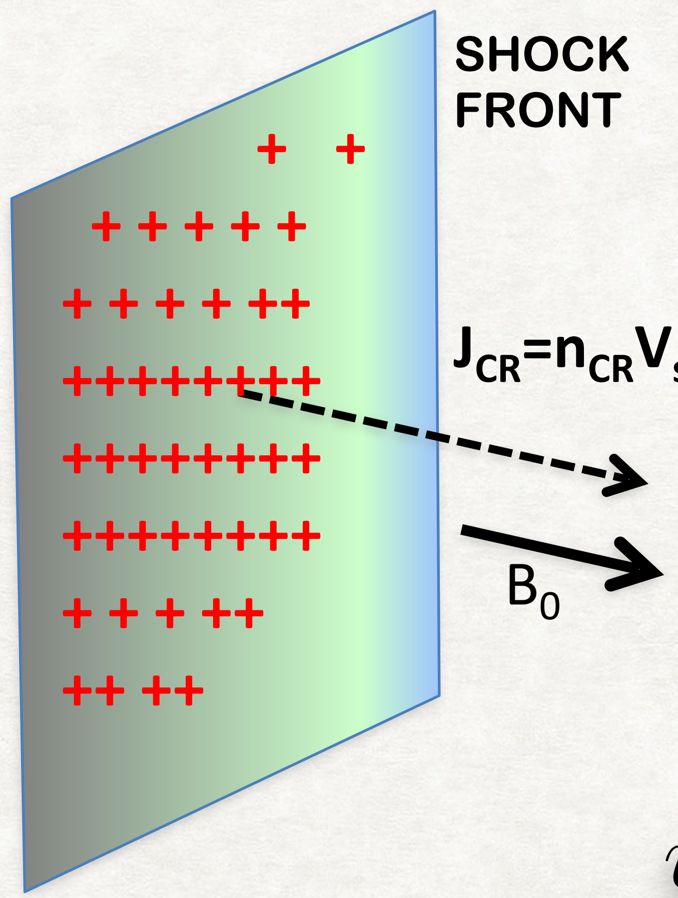


HYBRID SIMS SHOW THIS EFFECT IN THE FORM OF A SLOWING DOWN OF THE PLASMA AND HEATING

YET NO APPRECIABLE DEVIATION FROM E^{-2}

HOWEVER THESE SIMULATIONS ARE NON RELATIVISTIC

BASICS OF CR STREAMING INSTABILITY



THE UPSTREAM PLASMA REACTS TO THE UPCOMING CR CURRENT BY CREATING A RETURN CURRENT TO COMPENSATE THE POSITIVE CR CHARGE

THE SMALL INDUCED PERTURBATIONS MAY BE UNSTABLE (ACHTERBERG 1983, ZWEIBEL 1978, BELL 1978, BELL 2004, AMATO & PB 2009)

$$n_p + n_{CR} = n_e$$

$$n_{CR} v_{shock} = n_e v_e$$

$$v_e = \frac{n_{CR}}{n_{CR}+n_p} v_{shock} \approx v_{shock} \frac{n_{CR}}{n_p}$$

CR MOVE WITH THE SHOCK SPEED (>> V_A). THIS UNSTABLE SITUATION LEADS THE PLASMA TO REACT IN ORDER TO SLOW DOWN CR TO <V_A BY SCATTERING PARTICLES IN THE PERP DIRECTION (B-FIELD GROWTH)

STREAMING INSTABILITY - THE SIMPLE VIEW

CR streaming with the shock leads to growth of waves. The general idea is simple to explain:

$$n_{CR} m v_D \rightarrow n_{CR} m V_A \Rightarrow \frac{dP_{CR}}{dt} = \frac{n_{CR} m (v_D - V_A)}{\tau}$$

$$\frac{dP_w}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{V_A}$$

and assuming equilibrium:

$$\gamma_W = \sqrt{2} \frac{n_{CR}}{n_{gas}} \frac{v_D - V_A}{V_A} \Omega_{cyc}$$

And for parameters typical of SNR shocks:

$$\gamma_W \simeq \sqrt{2}\ \xi_{CR} \left(\frac{V_s}{c}\right)^2 \frac{V_s}{V_A} \Omega_{cyc} \sim \mathcal{O}(10^{-4}\ \ seconds^{-1})$$

BRANCHES OF THE CR INDUCED STREAMING INSTABILITY

A CAREFUL ANALYSIS OF THE INSTABILITY REVEALS THAT THERE ARE TWO BRANCHES

RESONANT	NON RESONANT
MAX GROWTH AT K=1/LARMOR	MAX GROWTH AT K>>1/LARMOR

THE MAX GROWTH CAN ALWAYS BE WRITTEN IN THE FORM

$$\gamma_{max} = k_{max} v_A$$

WHERE THE WAVENUMBER IS DETERMINED BY THE TENSION CONDITION:

$$k_{max} B_0 \approx \frac{4\pi}{c} J_{CR} \rightarrow k_{max} \approx \frac{4\pi}{c B_0} J_{CR}$$

THE SEPARATION BETWEEN THE TWO REGIMES IS AT $k_{MAX}\ r_L=1$

IF WE WRITE THE CR CURRENT AS $J_{CR} = n_{CR}(>E)ev_D$

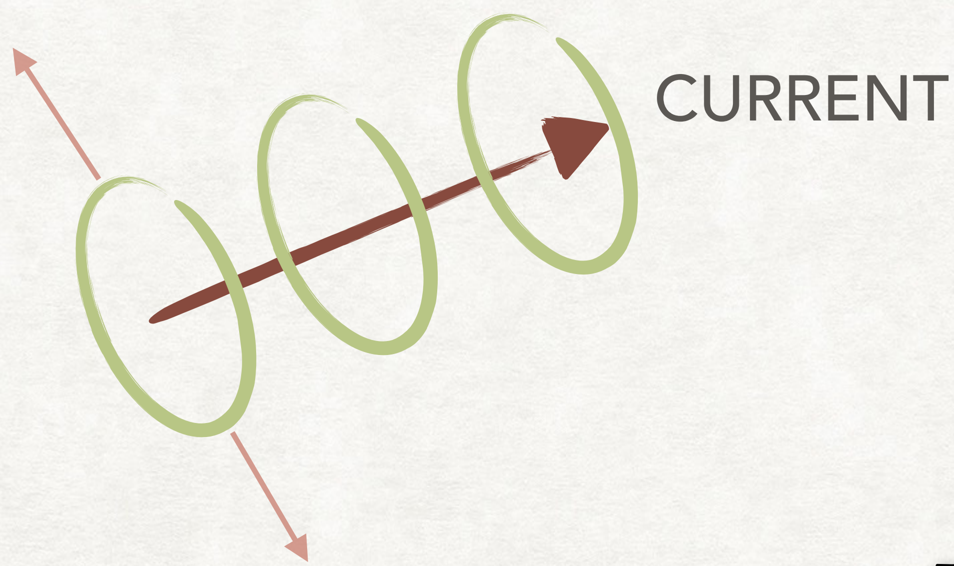
WHERE E IS THE ENERGY OF THE PARTICLES DOMINATING THE CR CURRENT, WE CAN WRITE THE CONDITION ABOVE AS

$$\boxed{\frac{U_{CR}}{U_B} = \frac{c}{v_D}} \qquad U_{CR} = n_{CR}(>E)E \qquad U_B = \frac{B^2}{4\pi}$$

IN CASE OF SHOCKS **V_D=SHOCK VELOCITY** AND THE CONDITION SAYS THAT THE NON-RESONANT MODES DOMINATED WHEN THE SHOCK IS VERY FAST AND ACCELERATION IS EFFICIENT —- *FOR TYPICAL CASES THIS IS ALWAYS THE CASE*

BUT RECALL! THE WAVES THAT GROW HAVE K MUCH LARGER THAN THE LARMOR RADIUS OF THE PARTICLES IN THE CURRENT —> NO SCATTERING BECAUSE EFFICIENT SCATTERING REQUIRES RESONANCE!!!

THE EASY WAY TO SATURATION OF GROWTH



The current exerts a force of the background plasma

$$\rho\frac{dv}{dt} \sim \frac{1}{c}J_{CR}\delta B$$

which translates into a plasma displacement:

$$\Delta x \sim \frac{J_{CR}}{c\rho}\frac{\delta B(0)}{\gamma_{max}^2}exp(\gamma_{max}t)$$

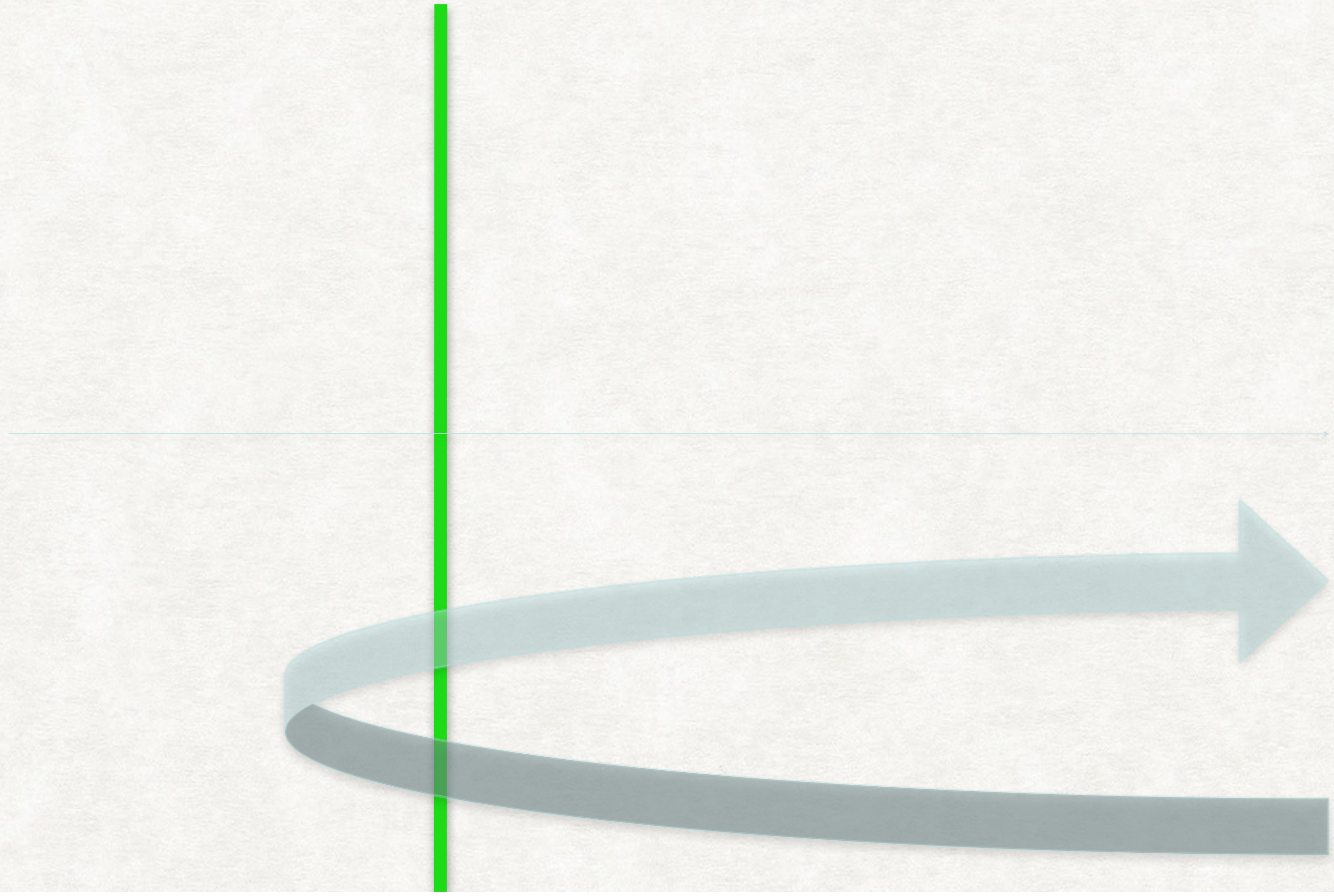
which stretches the magnetic field line by the same amount…

The saturation takes place when the displacement equals the Larmor radius of the particles in the field **δ**B … imposing this condition leads to:

$$\frac{\delta B^2}{4\pi} = \frac{\xi_{CR}}{\Lambda}\rho v_s^2\frac{v_s}{c} \qquad \Lambda = \ln(E_{max}/E_{min})$$

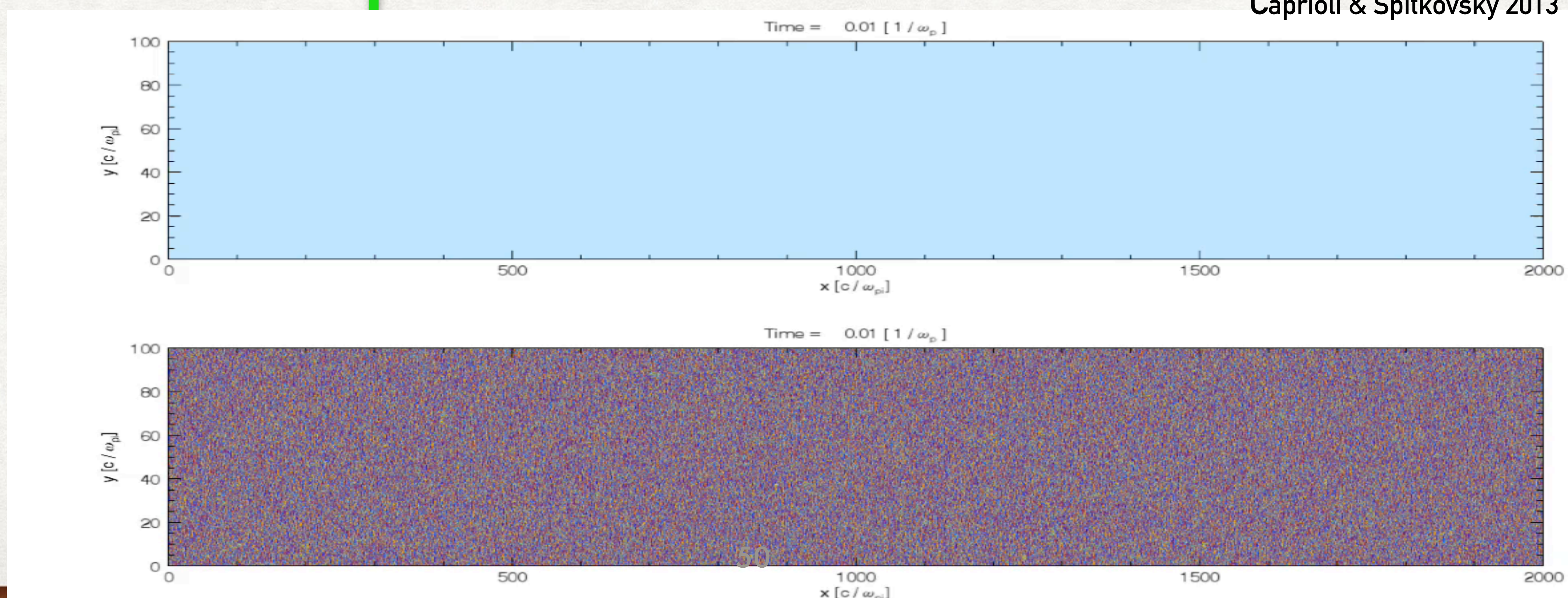
specialized to a strong shock and a spectrum E^{-2}

A QUALITATIVE PICTURE OF ACCELERATION



Bell & Schure 2013
Cardillo, Amato & PB 2015

Caprioli & Spitkovsky 2013



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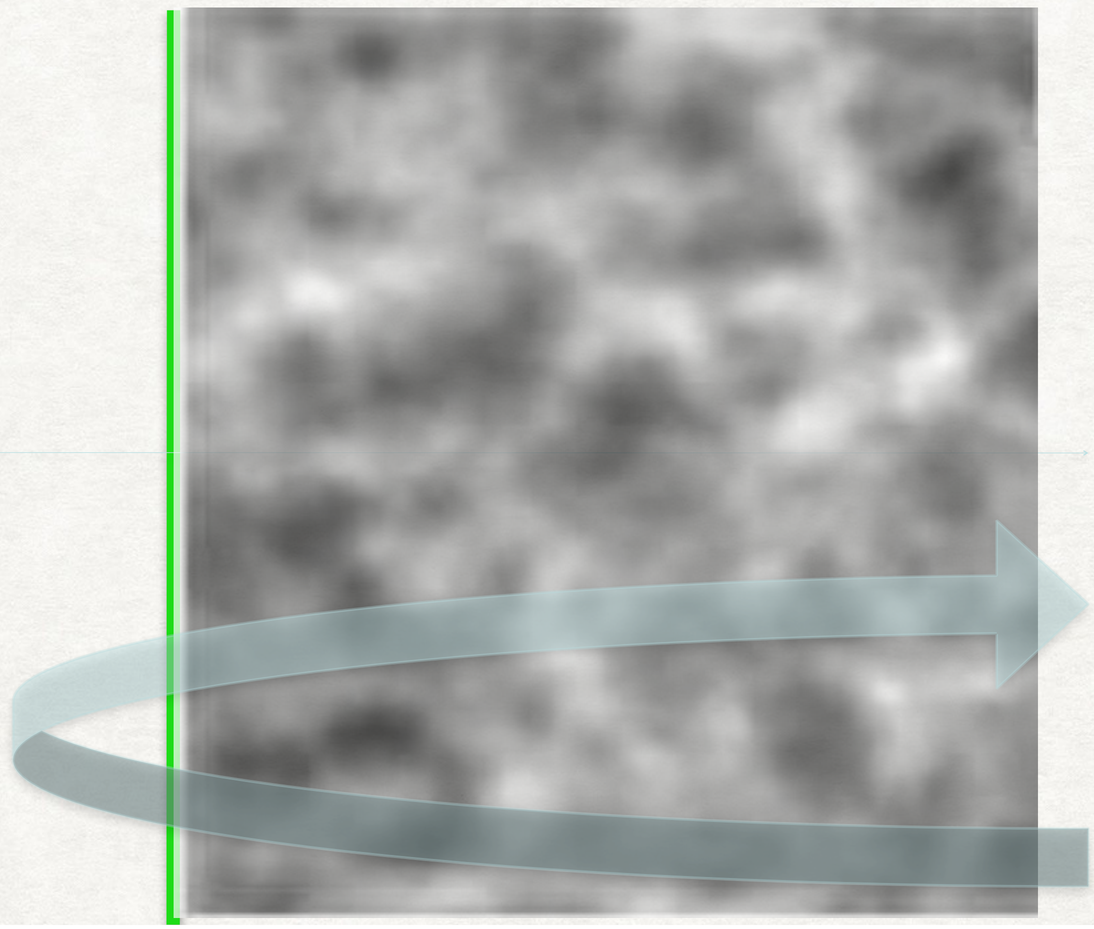

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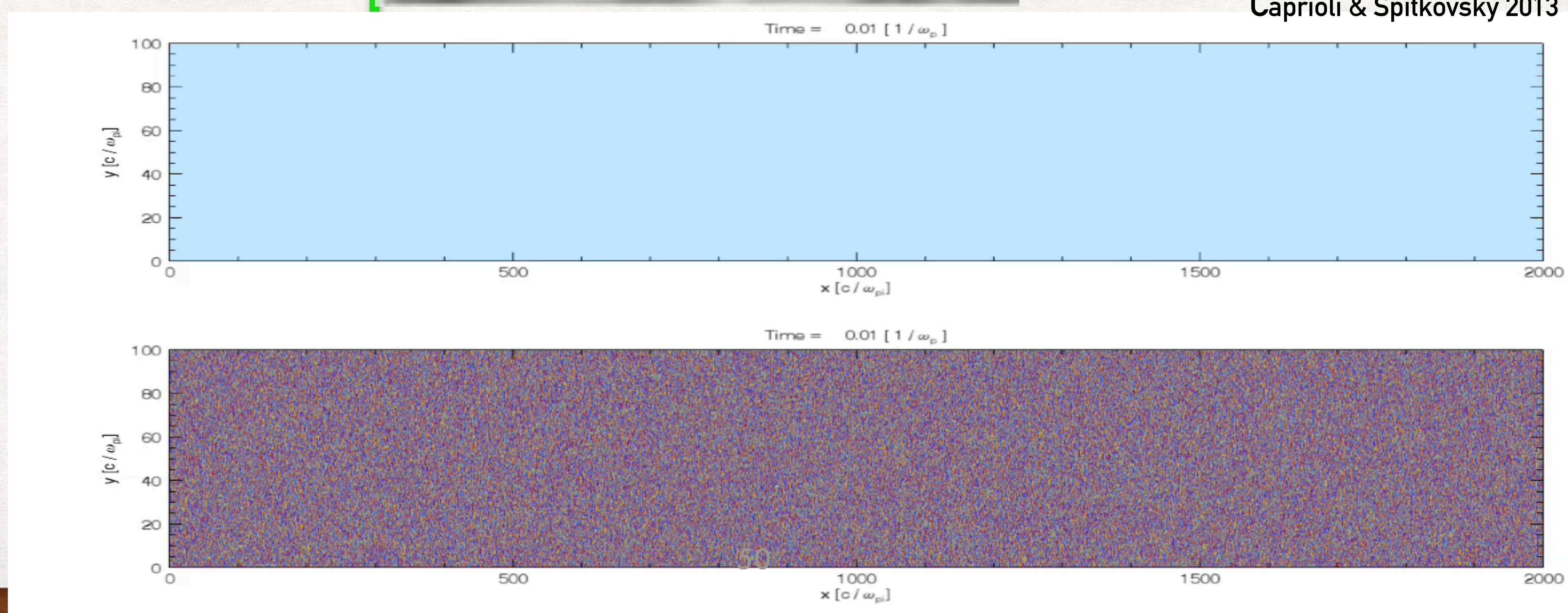

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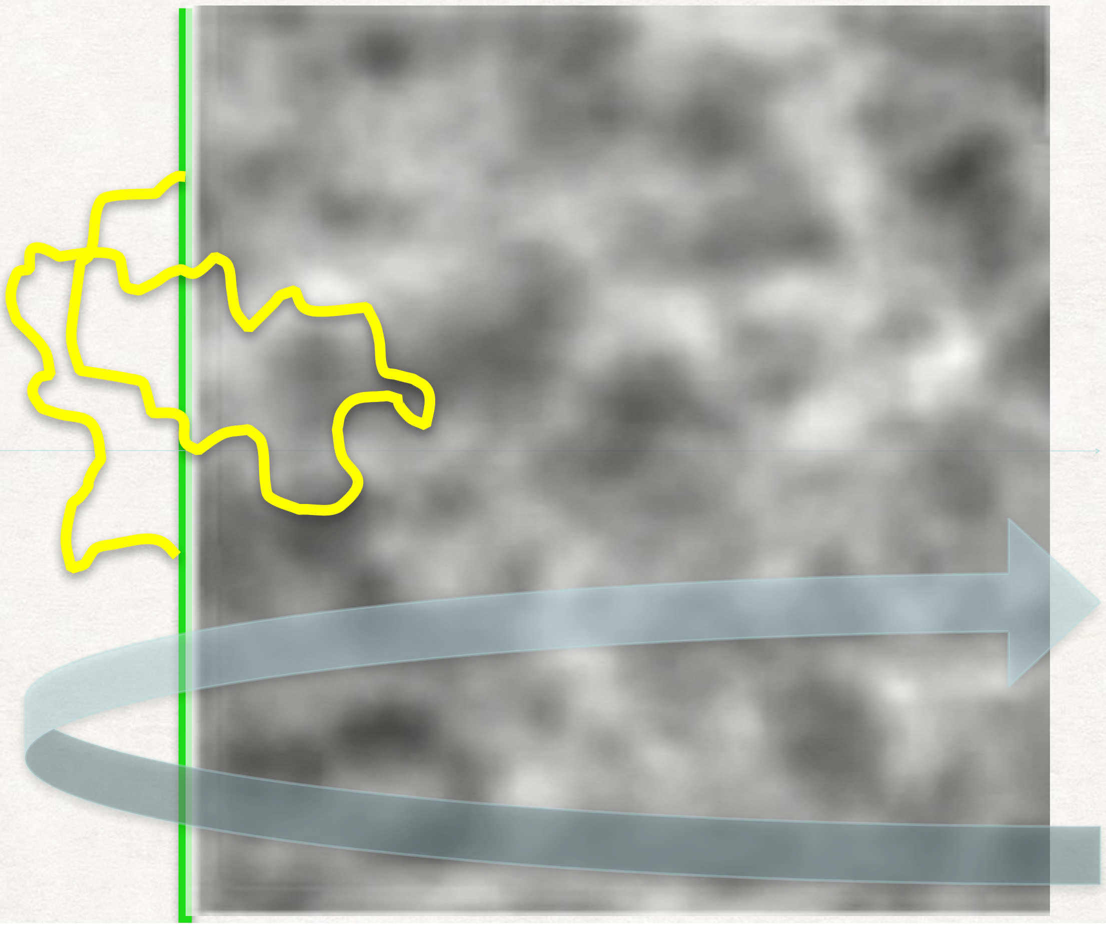

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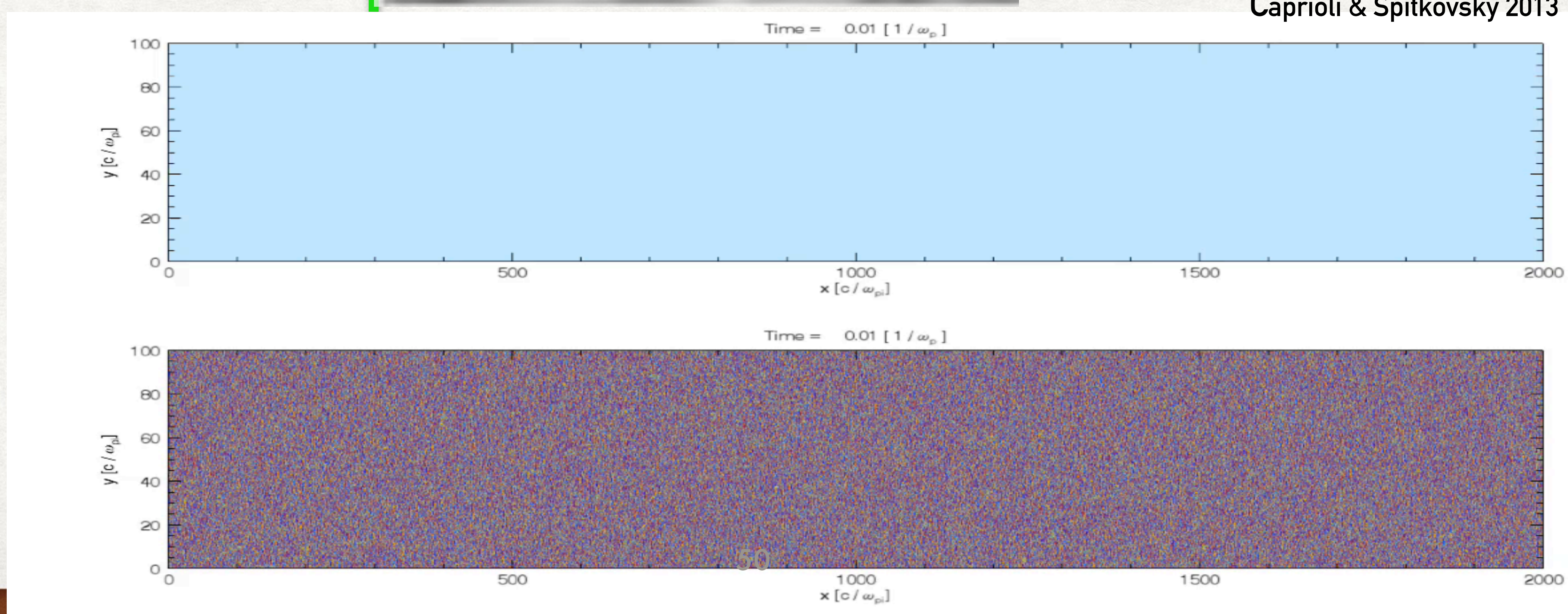

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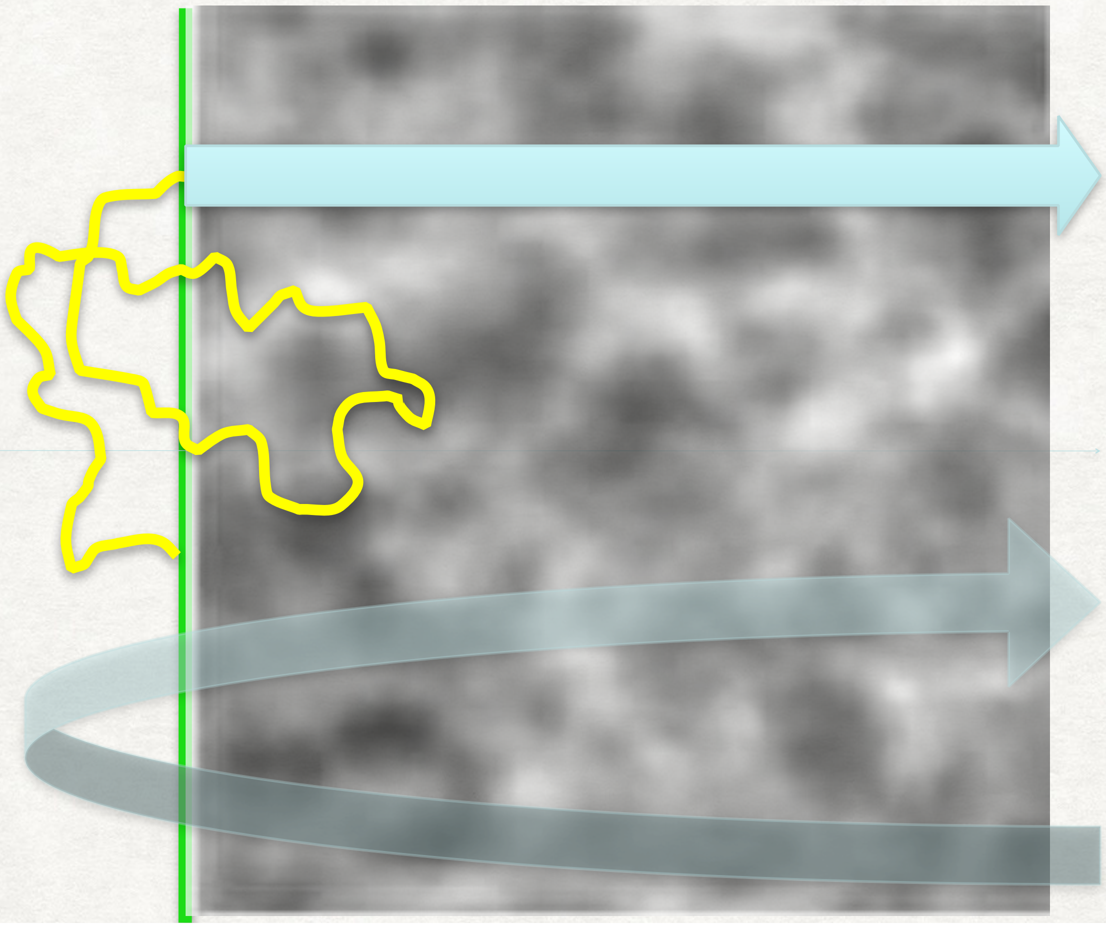

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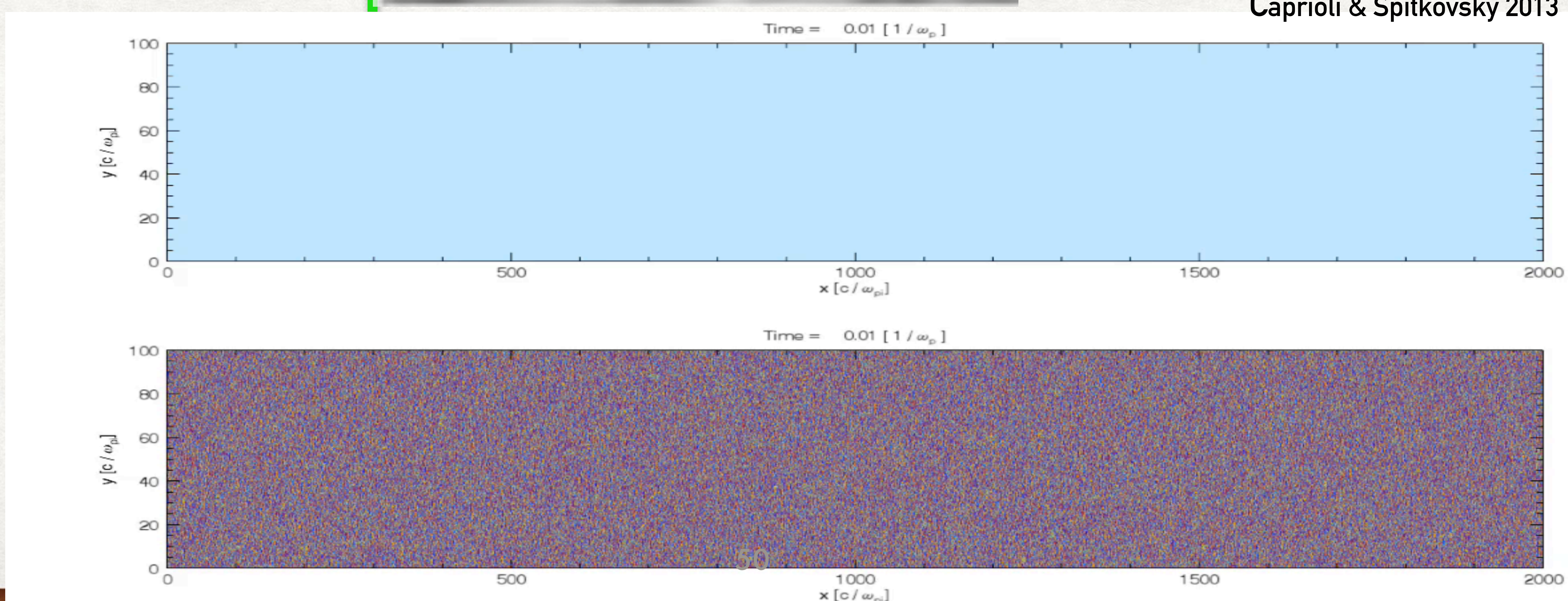

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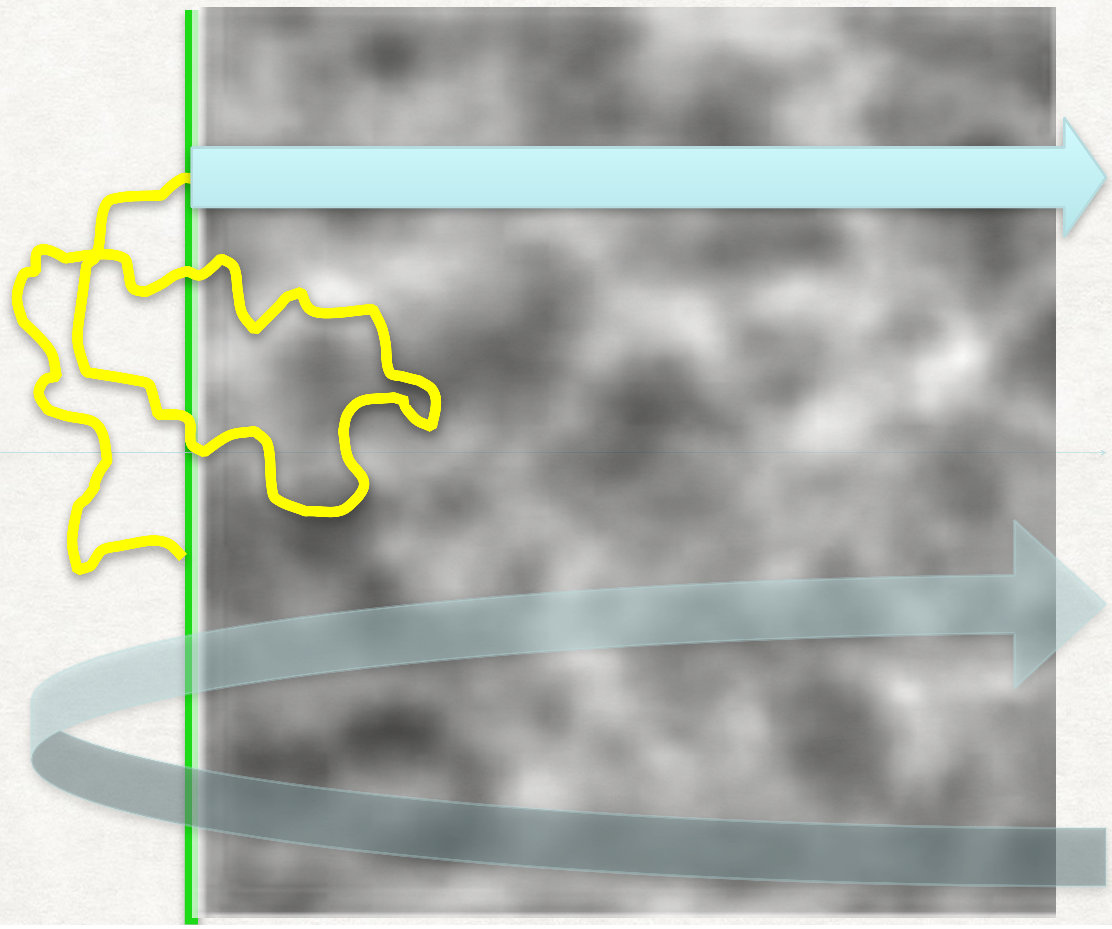

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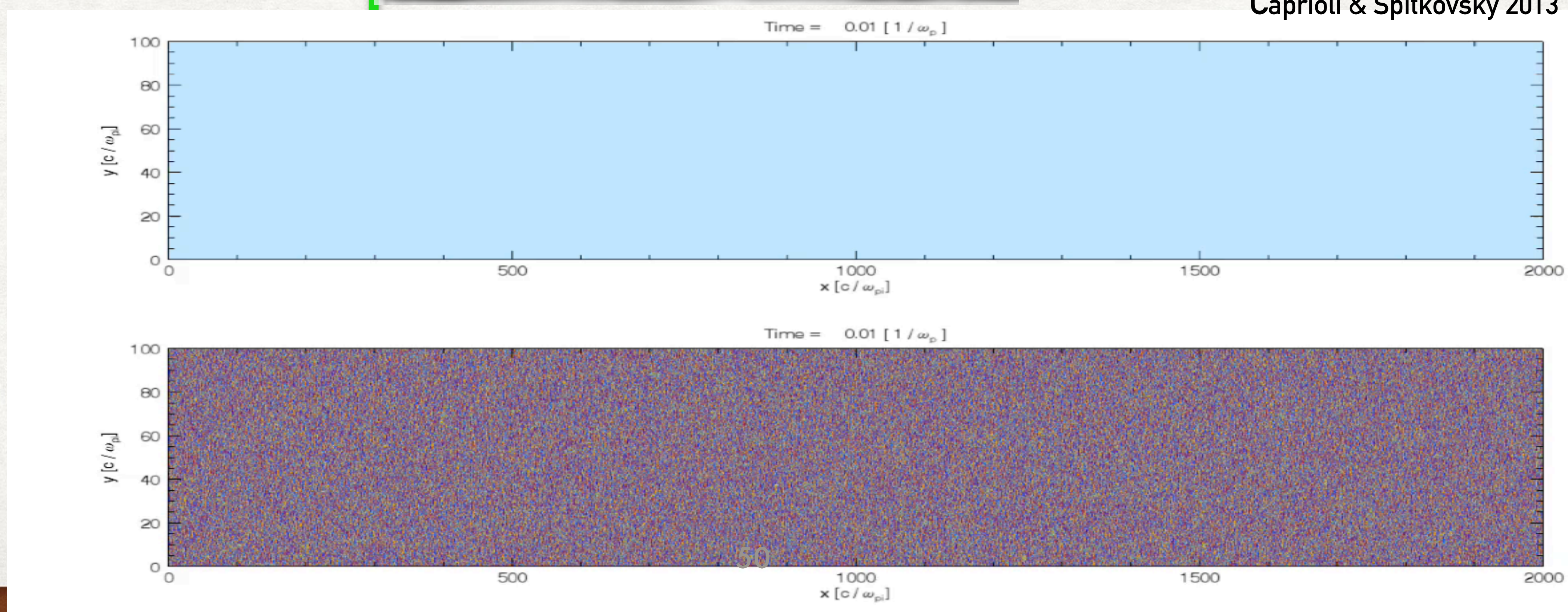
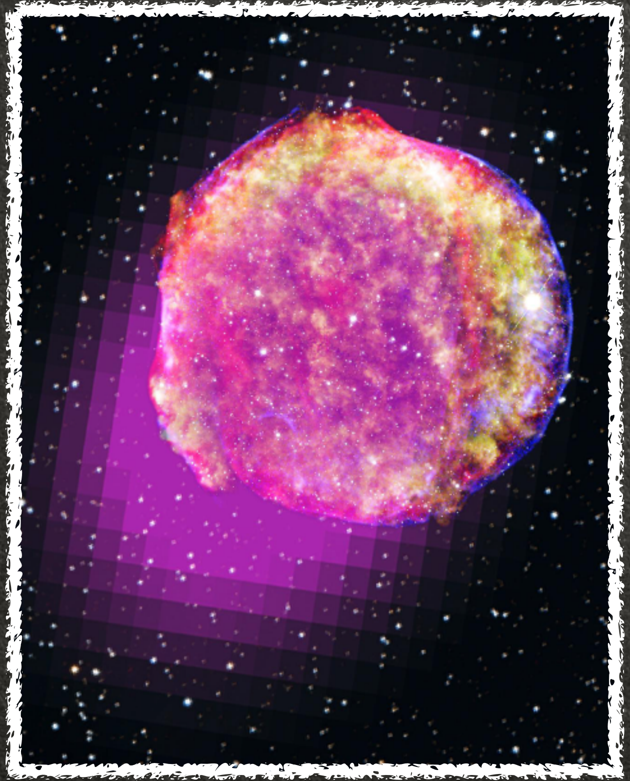


IMPLICATIONS FOR MAXIMUM ENERGY

Supernovae of type Ia

Explosion takes place in the ISM with spatially constant density

$$E_{max} \approx 130\ TeV \left(\frac{\xi_{CR}}{0.1}\right)\left(\frac{M_{ej}}{M_\odot}\right)^{-2/3}\left(\frac{E_{SN}}{10^{51} erg}\right)\left(\frac{n_{ISM}}{cm^{-3}}\right)^{1/6}$$



Supernovae of type II

IMPLICATIONS FOR MAXIMUM ENERGY

Supernovae of type Ia

Explosion takes place in the ISM with spatially constant density

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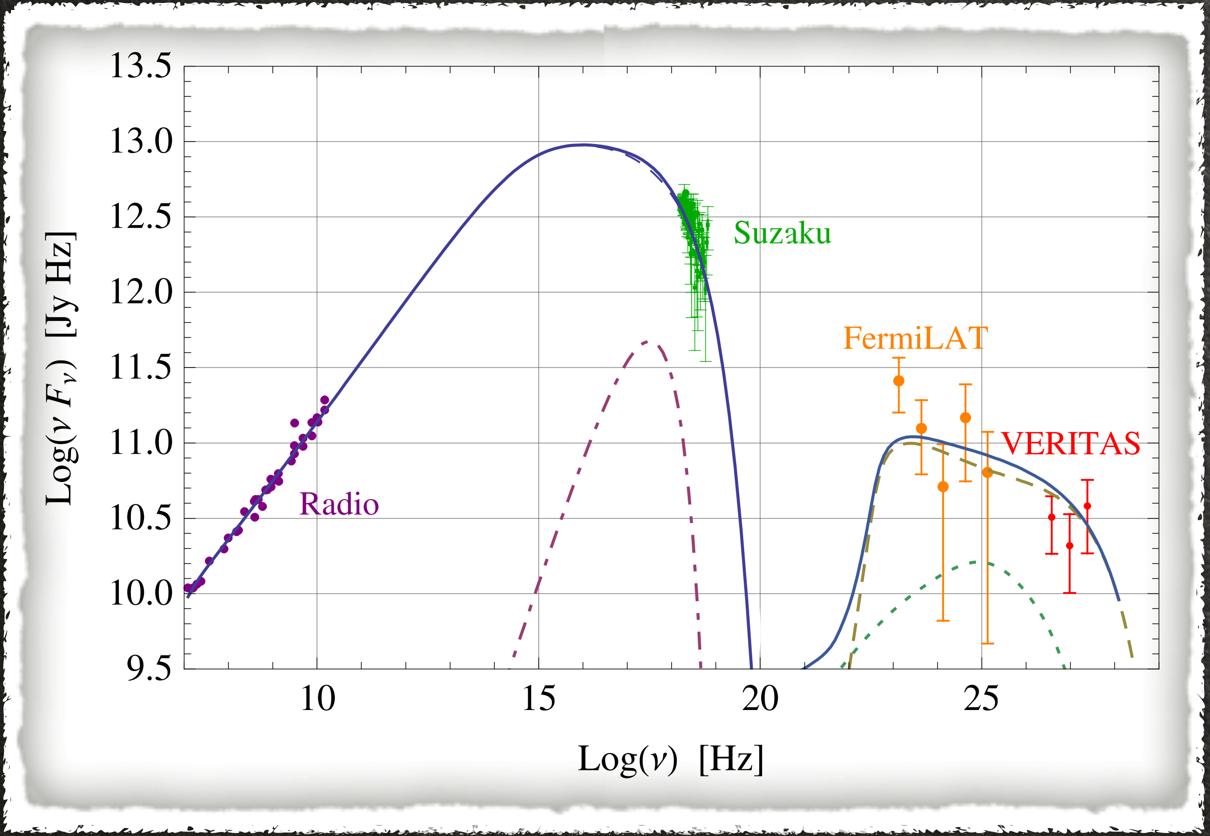

Supernovae of type II

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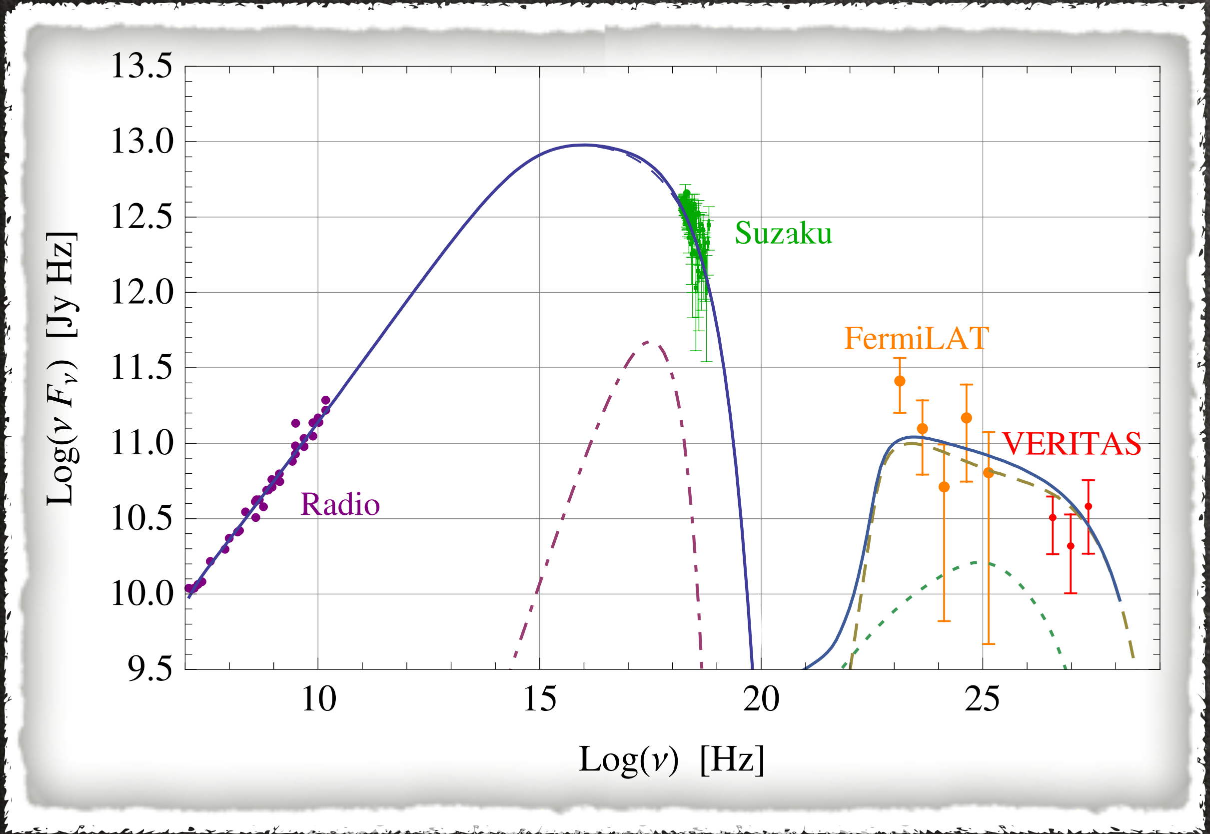


Supernovae of type II

In most cases the explosion takes place in the dense wind of the red super-giant progenitor

$$\rho(r) = \frac{\dot{M}}{4\pi r^2 v_W}$$

The Sedov phase reached while the shock expands inside the wind

$$R = M_{ej} v_W / \dot{M}$$

This corresponds to typical times of few tens of years after the SN explosion !!!

$$E_{max} \approx 1\ PeV \left(\frac{\xi_{CR}}{0.1}\right) \left(\frac{M_{ej}}{M_\odot}\right)^{-1} \left(\frac{E_{SN}}{10^{51} erg}\right) \times \left(\frac{\dot{M}}{10^{-5} M_\odot yr^{-1}}\right)^{1/2} \left(\frac{v_{wind}}{10 km/s}\right)^{-1/2}$$

X-ray rims and B-field amplification

TYPICAL THICKNESS OF FILAMENTS: ~ 10^{-2} pc

The synchrotron limited thickness is:

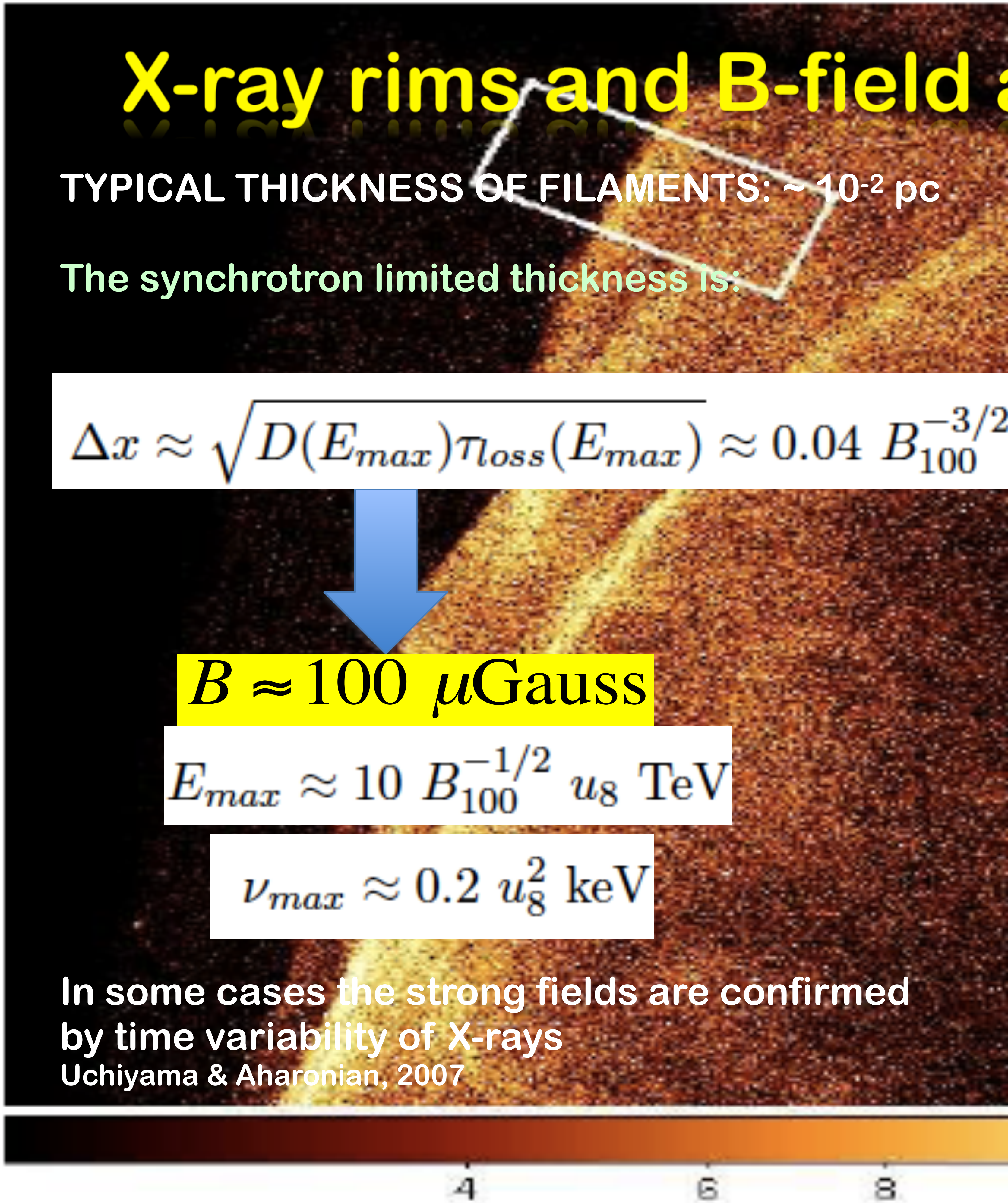
$$\Delta x \approx \sqrt{D(E_{max})\tau_{loss}(E_{max})} \approx 0.04\ B_{100}^{-3/2}\ \mathrm{pc}$$

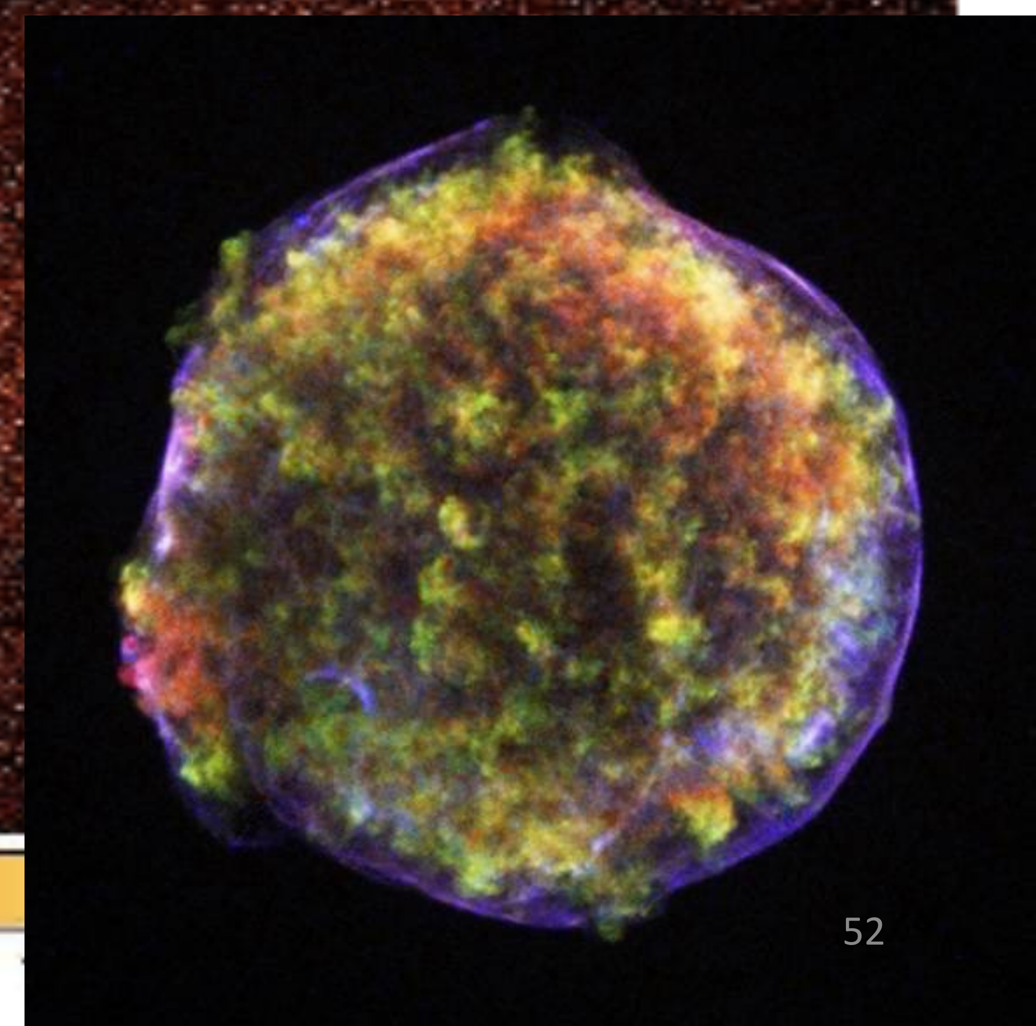
$$B \approx 100\ \mu\mathrm{Gauss}$$

$$E_{max} \approx 10\ B_{100}^{-1/2}\ u_8\ \mathrm{TeV}$$

$$\nu_{max} \approx 0.2\ u_8^2\ \mathrm{keV}$$

In some cases the strong fields are confirmed by time variability of X-rays
Uchiyama & Aharonian, 2007

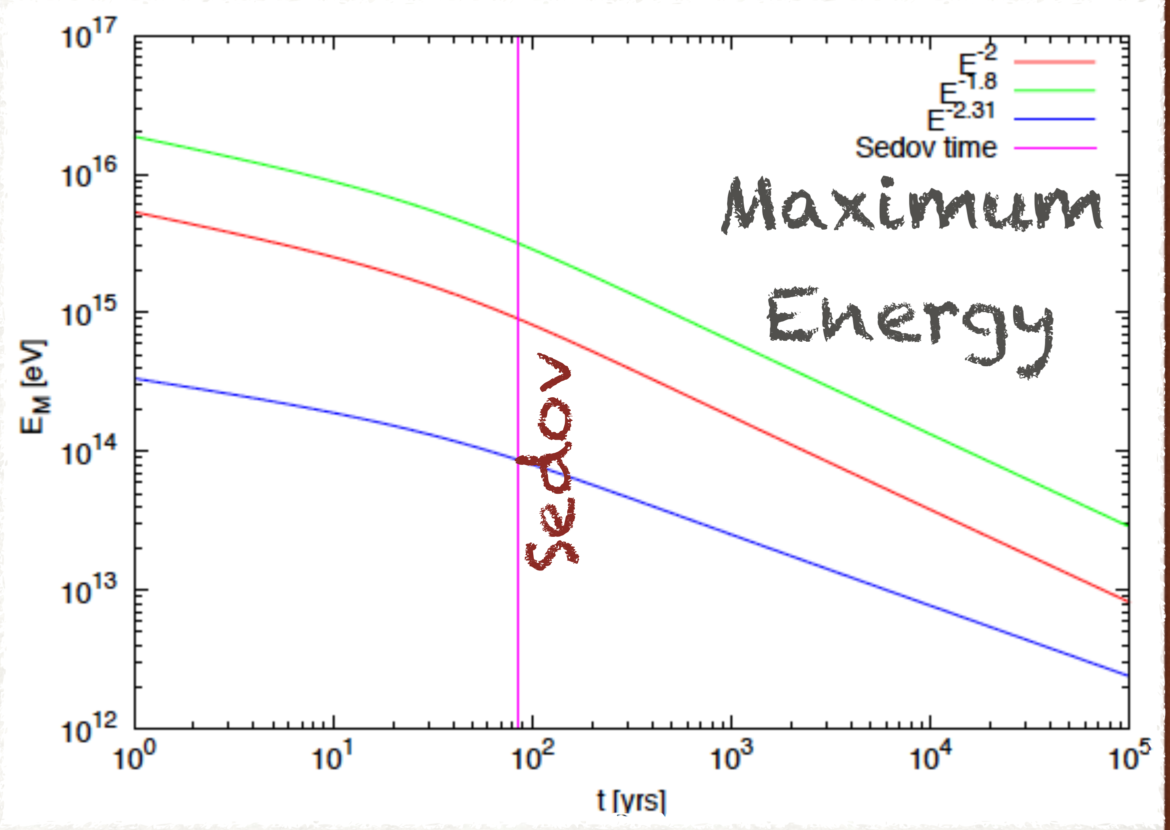




SUCCESS AND PROBLEMS

- Effective max energy at the beginning of Sedov phase (~30 years...) - not easy to catch Pevatrons with gamma rays...

Cardillo, Amato & PB 2015



- No exponential cutoff at E_{max} (broken power law)

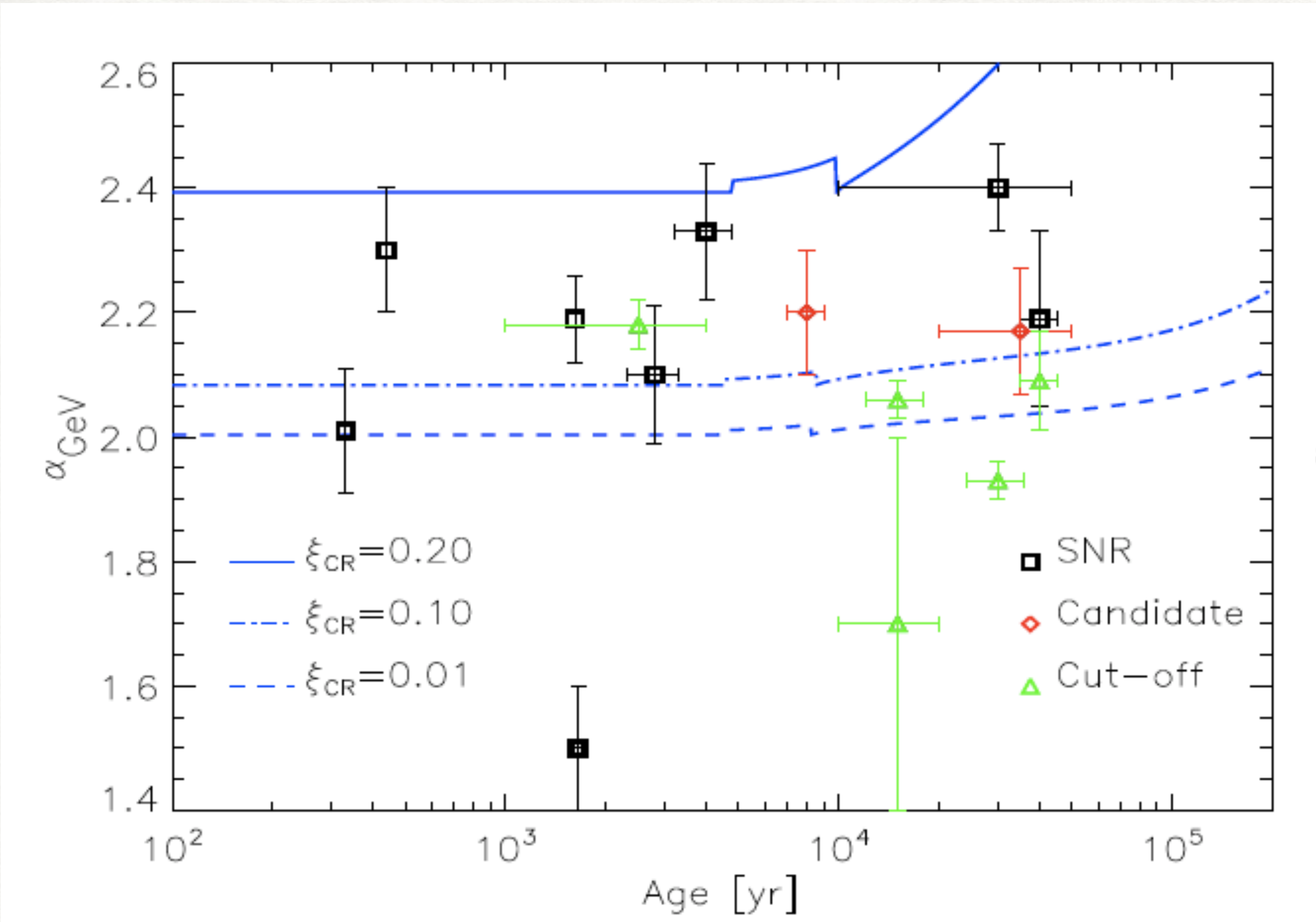
- SPECTRUM OF ACCELERATED PARTICLES IS VERY CLOSE TO E^{-2}

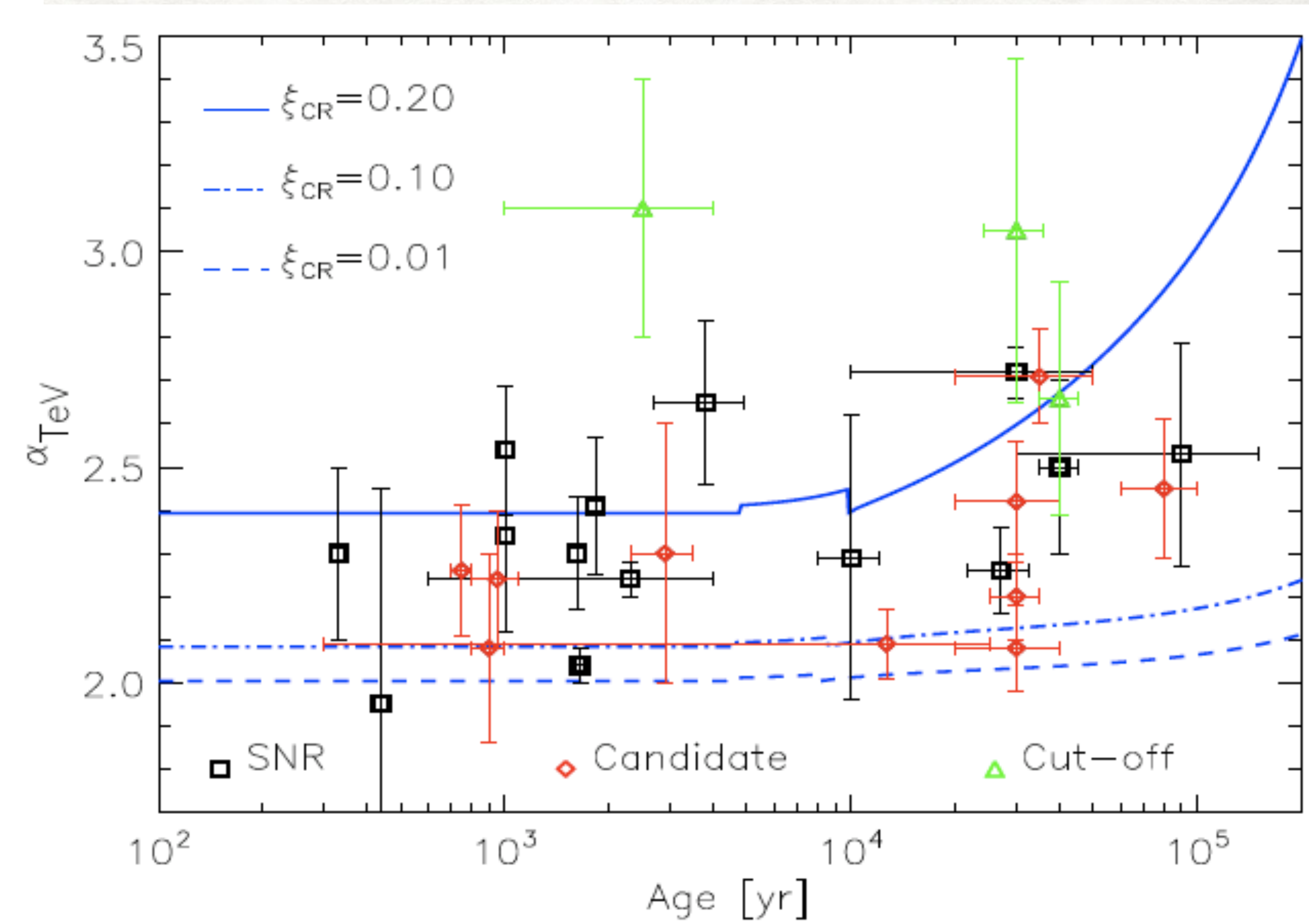
- SUCH HARD SPECTRUM NEEDED FOR GROWTH OF MODES

- BUT NOT OBSERVED IN ANY SNR SO FAR!

- AND CERTAINLY NOT THE ONE REQUIRED BY TRANSPORT THEORY

ISSUES WITH SPECTRA INSIDE SNR

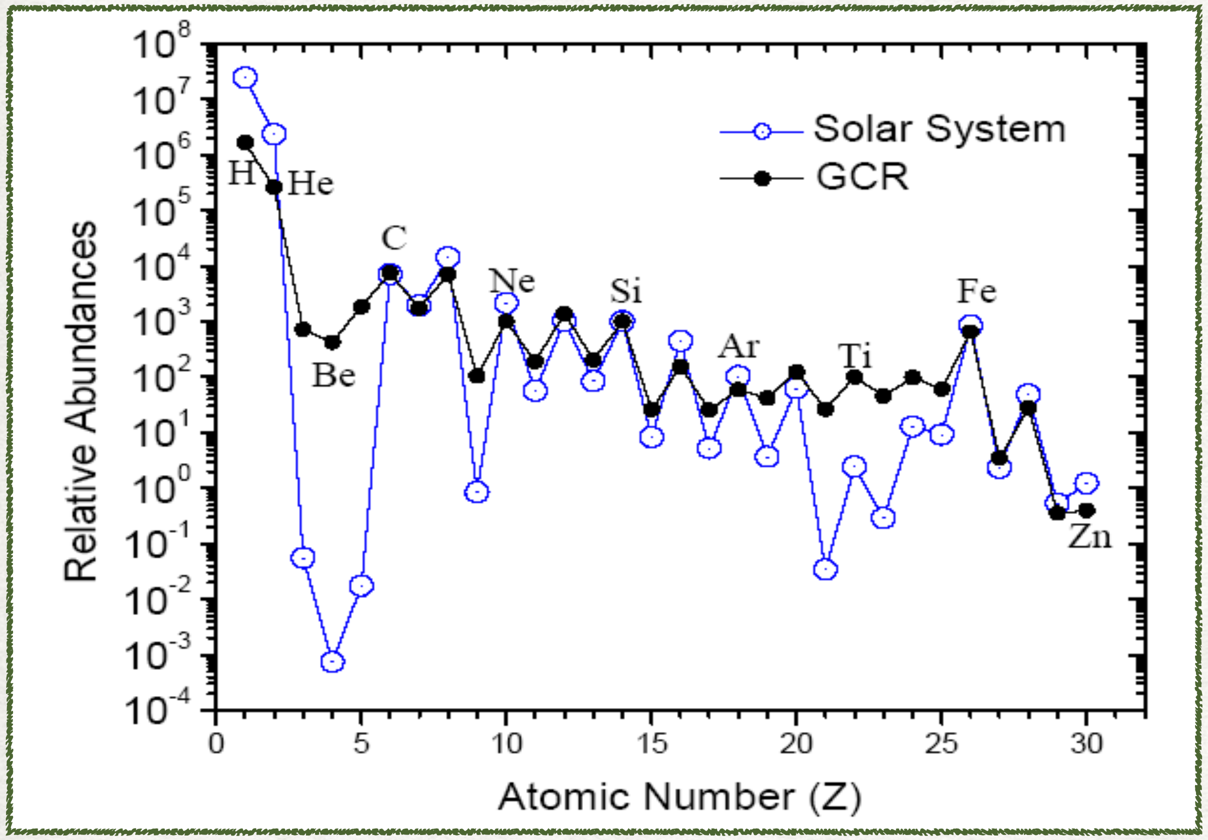




Caprioli 2011

CR TRANSPORT IN THE GALAXY

BASIC INDICATORS OF DIFFUSIVE TRANSPORT

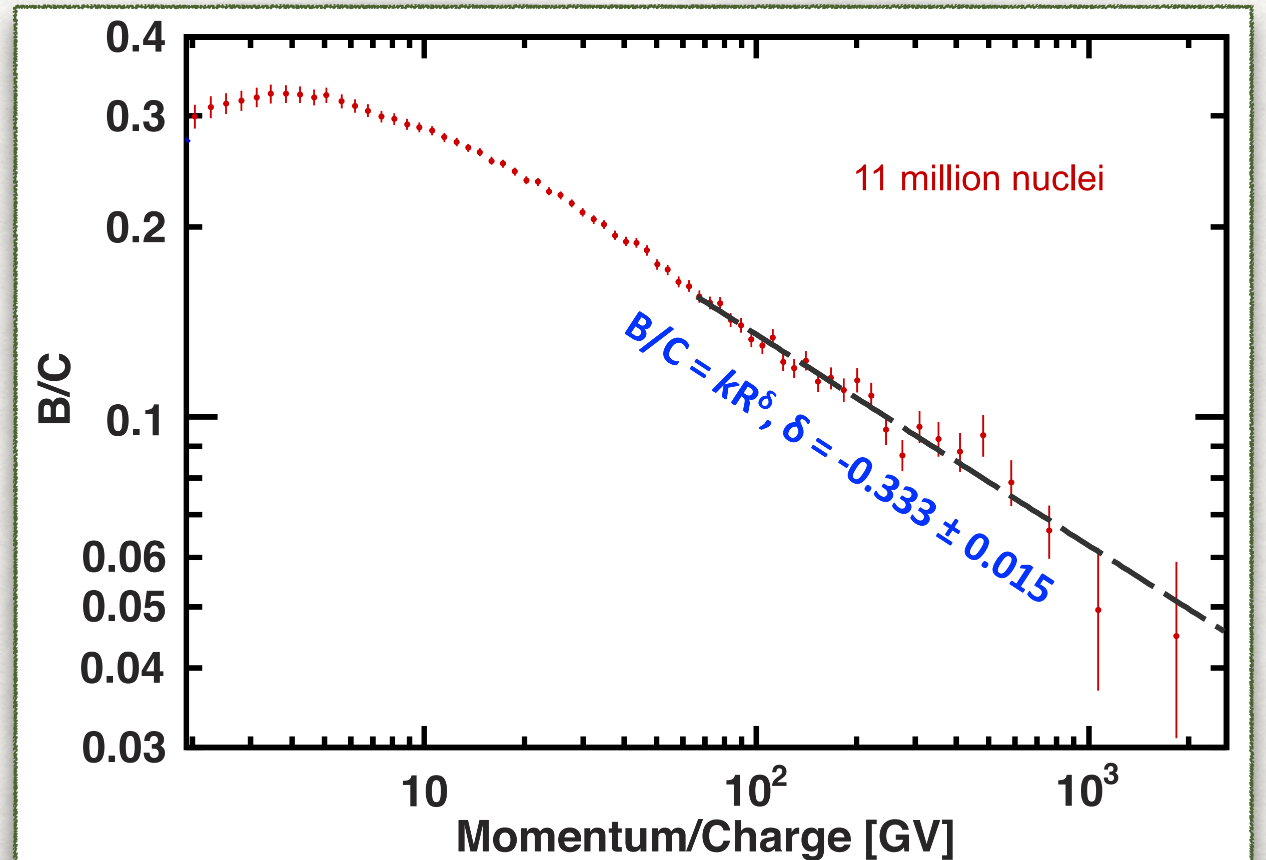


Measurements of the Boron and sub-iron elements in CRs show that CR live for tens of million years in the Galaxy

DIFFUSIVE TRANSPORT

$$\sigma_{sp}(A) \approx 45A^{0.7}\ \mathrm{mb}$$

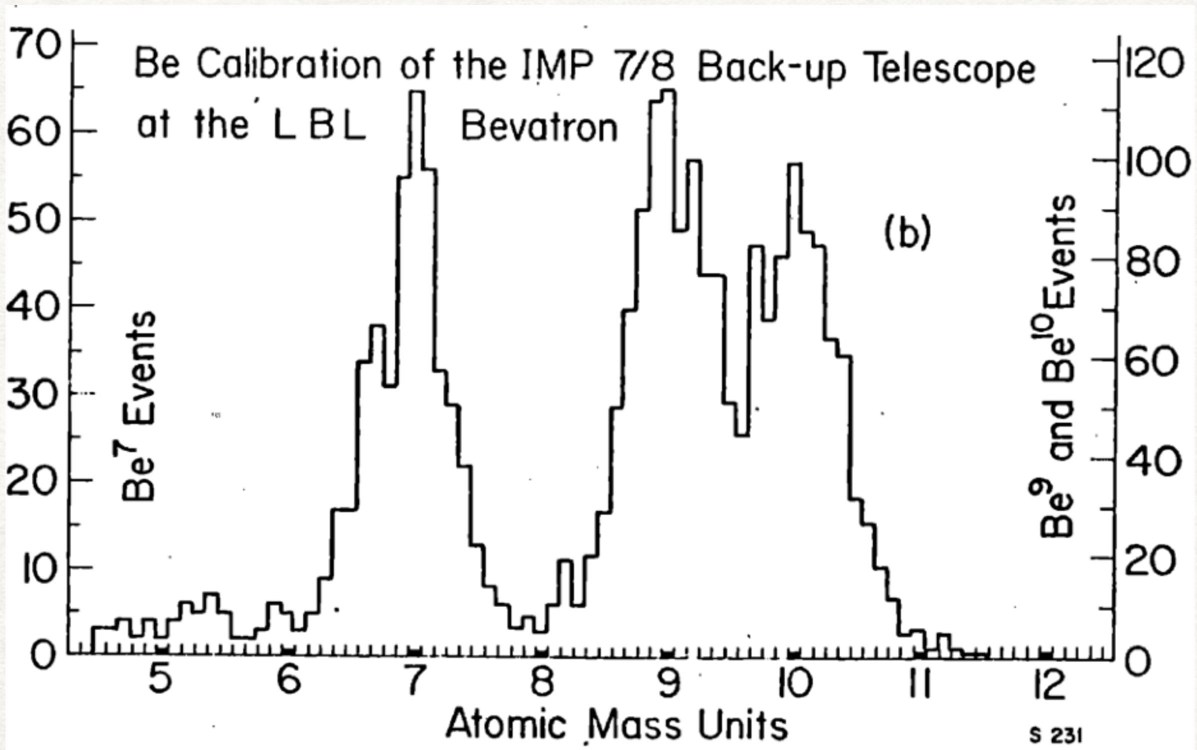
$$\tau_{sp} \approx [n_d(h/H)c\sigma_{sp}]^{-1} \approx 80 H_4 A_{12}^{-0.7}\ \mathrm{Myr}$$

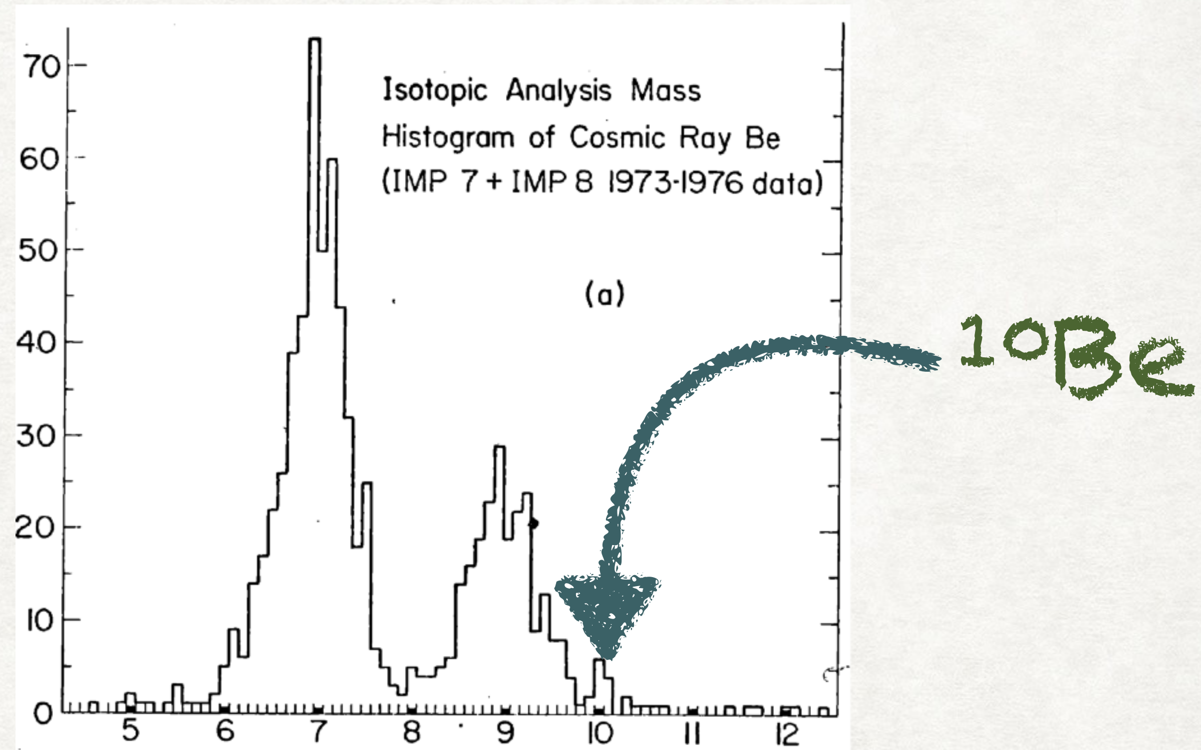


M. Aguilar *et al.*, Phys. Rev. Lett. **117**, 231101 (2016)

BASIC INDICATORS OF DIFFUSIVE TRANSPORT

Garcia-Munoz et al. 1977

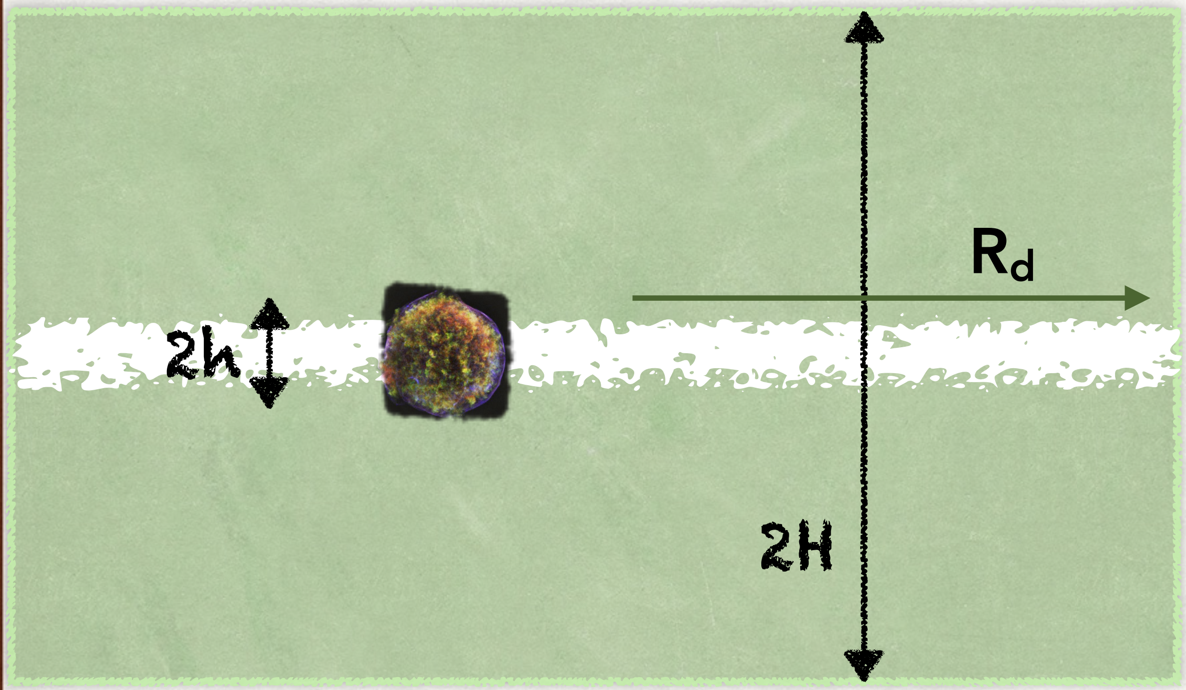




Be comes in three isotopes and ^{10}Be is unstable with a decay time of 15 Myr.

While in the Lab the three isotopes are roughly equally produced, in the CR we see the peak of ^{10}Be being much smaller —> information of decay vs production vs confinement

A TOY MODEL FOR OUR GALAXY



HALO ~ several kpc

DISC ~ 300 pc

Assumptions of the model:

1. *CR are injected in an infinitely thin disc*
2. *CR diffuse in the whole volume*
3. *CR freely escape from a boundary*

1 $$Q(p,z) = \frac{Q_0(p)}{\pi R_d^2}\delta(z)$$

2 $$-\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] = Q(p,z)$$

3 $$f(z=H,p) = 0$$

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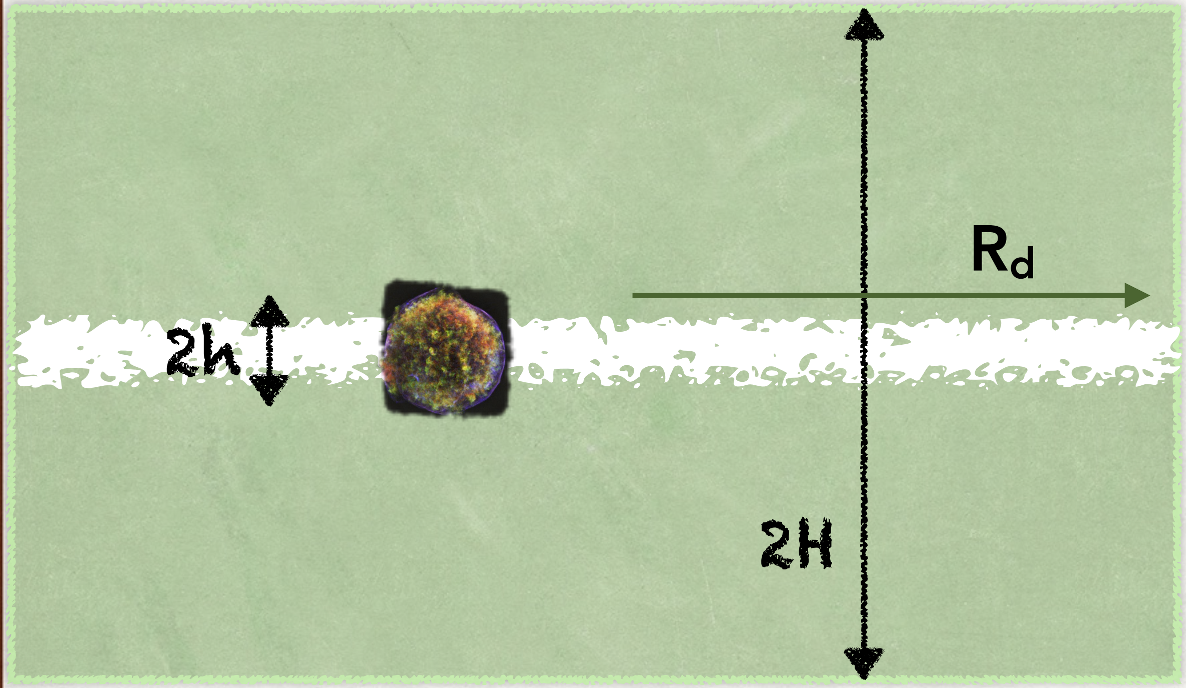

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For z≠0:

$$D\frac{\partial f}{\partial z} = Constant \rightarrow f(z) = f_0\left(1 - \frac{z}{H}\right)$$

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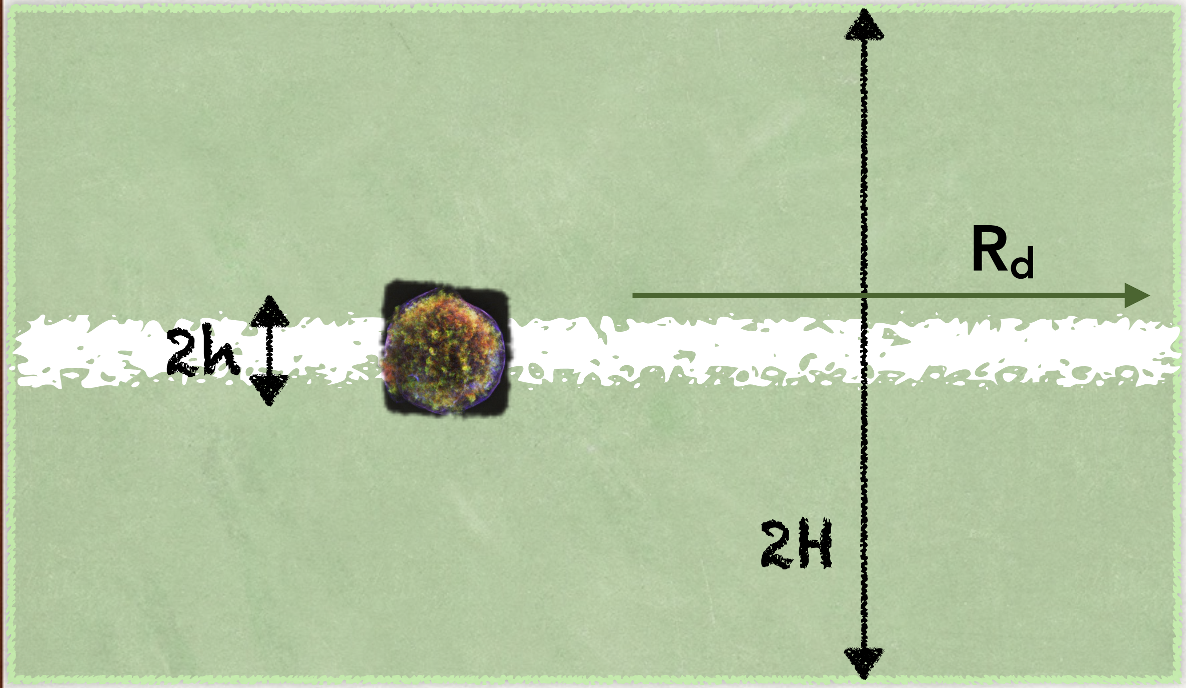

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For z≠0:

$$D\frac{\partial f}{\partial z} = Constant \rightarrow f(z) = f_0\left(1-\frac{z}{H}\right)$$

$$D\frac{\partial f}{\partial z}|_{z=0^+} = -\frac{f_0}{H}$$

A TOY MODEL FOR OUR GALAXY

Let us now integrate the diffusion equation around z=0

$$-\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] = \frac{Q_0(p)}{\pi R_d^2}\delta(z) \quad \Rightarrow \quad -2D\frac{\partial f}{\partial z}|_{z=0^+} = \frac{Q_0(p)}{\pi R_d^2}$$

and recalling that

Diffusion Time

$$D\frac{\partial f}{\partial z}|_{z=0^+} = -D\frac{f_0}{H} \quad \Rightarrow \quad f_0(p) = \frac{Q_0(p)}{2\pi R_d^2}\frac{H}{D} = \frac{Q_0(p)}{2\pi R_d^2 H}\quad\frac{H^2}{D}$$

Rate of injection per unit volume

Since $Q_0(p)\sim p^{-\gamma}$ and $D(p)\sim p^{\delta}$
$f_0(p) \sim p^{-\gamma-\delta}$

A TOY MODEL FOR OUR GALAXY: ESCAPE FLUX

WHICH CR FLUX WOULD BE MEASURED BY AN OBSERVER OUTSIDE OUR GALAXY?

WE ALREADY ESTABLISHED THAT

$$D\frac{\partial f}{\partial z} = constant$$

BUT THIS IS EXACTLY THE FLUX ACROSS A SURFACE IN DIFFUSIVE REGIME:

$$\Phi_{esc}(p) = D\frac{\partial f}{\partial z}|_{z=H} = D\frac{\partial f}{\partial z}|_{z=0^+} = \frac{Q_0(p)}{2\pi R_d^2}$$

THE SPECTRUM OF COSMIC RAYS OBSERVED BY AN OBSERVER OUTSIDE OUR GALAXY IS THE SAME AS INJECTED BY SOURCES, NOT THE SAME AS WE MEASURE AT THE EARTH!

MEANING OF FREE ESCAPE BOUNDARY?

The physics of CR transport is as much regulated by diffusion as it is by boundary conditions (this is true for toy models as well as it is for GALPROP)

What does "free escape" mean? $f(z=H,p)=0$

Conservation of flux at the boundary implies:

$$D\frac{\partial f}{\partial z}|_{z=H}=\frac{c}{3}f_{out}$$

$$D\frac{f_0}{H}=\frac{c}{3}f_{out}\to f_{out}=\frac{3D}{cH}f_0\approx\frac{\lambda(p)}{H}f_0\ll f_0$$

Beware that despite the great importance of this assumption we do not have any handle on what determines the halo size or weather the halo size depends on energy

A SIMPLE DESCRIPTION OF TRANSPORT OF NUCLEI

At high enough energy (typically above few GeV/n) CR nuclei only suffer **spallation** reactions while interacting with the ISM

Spallation reaction are such that a nucleus of type (A,Z) is transformed to a nucleus (A-1,Z')+N

Spallation reactions **conserve the energy per nucleon**, namely the Lorentz factor

We shall assume that the gas is concentrated in a thin Galactic disc

A SIMPLE DESCRIPTION OF TRANSPORT OF NUCLEI

For nuclei of mass A, it is customary to introduce the flux as a function of the kinetic energy per nucleon E_k: $I_\alpha(E_k)dE_k = p^2 F_\alpha(p)\, v(p)\, dp$ which implies: $I_\alpha(E_k) = Ap^2 F_\alpha(p)$

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_\alpha(E_k)}{\partial z}\right] + 2h_d n_d v(E_k)\sigma_\alpha \delta(z) I_\alpha(E_k) =$$

$$= 2Ap^2 h_d q_{0,\alpha}(p)\delta(z) + \sum_{\alpha' > \alpha} 2h_d n_d\, v(E_k)\sigma_{\alpha' \to \alpha}\delta(z) I_{\alpha'}(E_k)$$

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FOR SIMPLICITY THIS EQUATION DOES NOT CONTAIN SOME LOSS TERMS (IONIZATION), ADVECTION AND SECOND ORDER FERMI ACCELERATION IN ISM

ALL THESE EFFECTS MAY BECOME IMPORTANT AT E<10 GeV/nucleon

PRIMARY NUCLEI

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_\alpha(E_k,z)}{\partial z}\right] + 2h_d n_d v(E_k)\sigma_\alpha \delta(z) I_\alpha(E_k) = \ 2Ap^2 h_d q_{0,\alpha}(p)\delta(z)$$

Formally similar to the equation for protons but with **spallation** taken into account

Technically the equation is solved in the same way:

1) *consider z>0 (or z<0) and then*
2) *integrate around z=0 between 0^- and 0^+*

1 $$D_\alpha \frac{\partial I_\alpha}{\partial z} = constant \rightarrow I_\alpha = I_{0,\alpha}\left(1 - \frac{z}{H}\right)$$ with free escape boundary condition

2 $$-D_\alpha \frac{\partial I_\alpha}{\partial z}|_{z=0} = -h_d n_d v \sigma_\alpha I_{0,\alpha} + Ap^2 h_d Q_{0,\alpha}(p)$$

PRIMARY NUCLEI

Injection x escape time

$$I_{0,\alpha}(E_k) = \frac{\frac{N_{inj,\alpha} R_{SN}}{2\pi R_{disc}^2 H} \frac{H^2}{D_\alpha}}{1 + \frac{X(E_k)}{X_\alpha}}$$

Flux of nuclei of type a

$$X(E_k) = n_d \left(\frac{h}{H}\right) m_p v \frac{H^2}{D_\alpha}$$

Grammage traversed by nuclei of type a

$$X_\alpha = \frac{m_p}{\sigma_\alpha}$$

Critical grammage for nuclei of type a

For $X \ll X_\alpha$ the equilibrium spectrum is the standard $E_k^{-\gamma-\delta}$

For $X \gg X_\alpha$ the equilibrium spectrum reproduces the injection spectrum $E_k^{-\gamma}$

SECONDARY NUCLEI - CASE OF B/C

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_B}{\partial z}\right] + 2h_d n_d v \sigma_B \delta(z) I_B = 2h_d n_d \,\sigma_{CB} v I_C \delta(z) + 2h_d n_d \,\sigma_{OxB} v I_{Ox} \delta(z)$$

Destruction of B

Production of B from carbon spallation

Production of B from oxygen spallation

Following the same strategy as in the previous cases one obtains easily:

$$I_{B,0}(E_k) = \frac{I_{C,0}(E_k)\frac{X(E_k)}{X_{cr,CB}}}{1+\frac{X(E_k)}{X_{cr,B}}} + \frac{I_{Ox,0}(E_k)\frac{X(E_k)}{X_{cr,OxB}}}{1+\frac{X(E_k)}{X_{cr,B}}}$$

which reflects in the following B/C ratio:

$$\boxed{\frac{I_{B.0}(E_k)}{I_{C,0}(E_k)} = \frac{\frac{X(E_k)}{X_{cr,CB}}}{1+\frac{X(E_k)}{X_{cr,B}}} + \frac{I_{Ox,0}(E_k)}{I_{C,0}(E_k)}\frac{\frac{X(E_k)}{X_{cr,OxB}}}{1+\frac{X(E_k)}{X_{cr,B}}}}$$

AT E>50-100 GeV THE B/C RATIO SCALES AS $X(E_K)$ NAMELY AS $1/D_\alpha$ NAMELY WE CAN MEASURE THE SLOPE OF D(E) FROM THE ENERGY DEPENDENCE OF B/C

EXERCISES

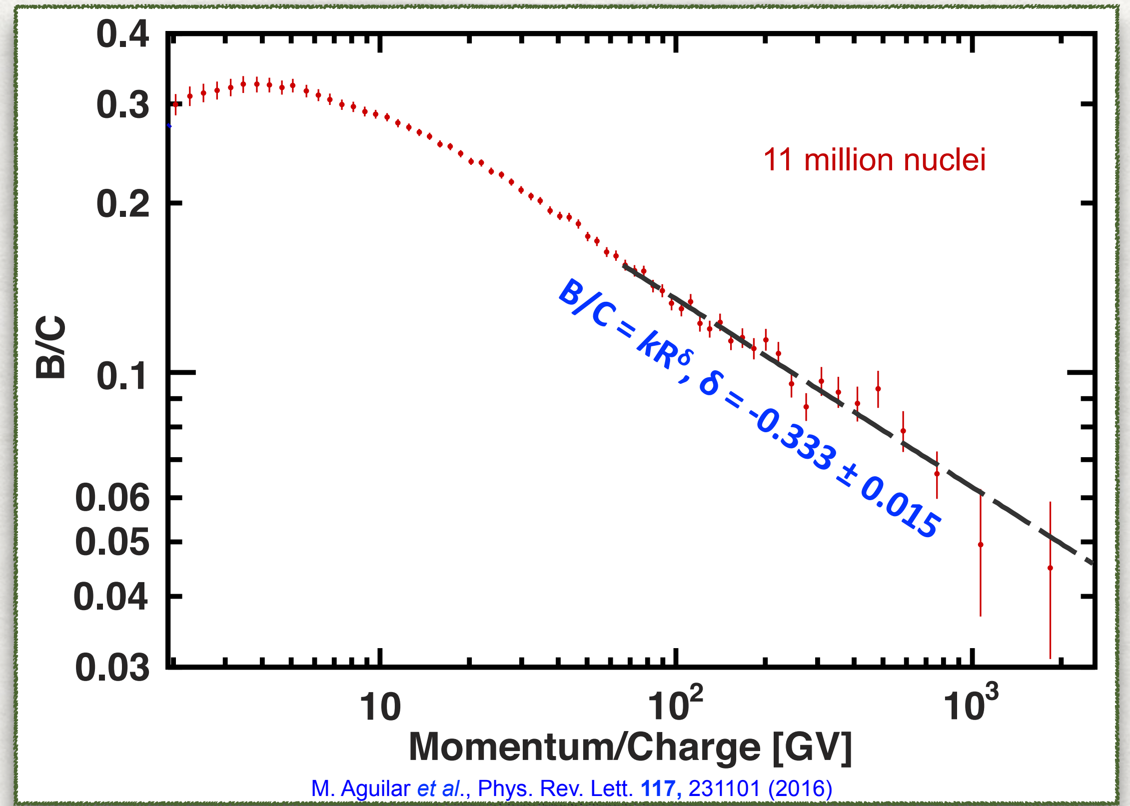
1. Solve the transport equation in the case in which there is an advection term

2. Solve the transport equation in the case of a bi-halo, namely a halo made of two zones with different diffusion coefficients. Show that in this case the spectrum of CRs is no longer a power law

3. Calculate the B/C ratio in case (2)

SUMMARY: SECONDARY/PRIMARY: B/C

Evidence for CR diffusive transport



primary equilibrium

$$n_{pr}(E/n) \propto Q(E/n)\tau_{diff}(E/n)$$

secondary injection

$$q_{sec}(E/n) \approx n_{pr}(E/n)\sigma v n_{gas}$$

secondary equilibrium

$$n_{sec}(E/n) \approx q_{sec}(E/n)\tau_{diff}(E/n)$$

$$\frac{n_{sec}}{n_{pr}} \approx \frac{\sigma}{m_p}\left[v n_{gas} m_p \tau_{diff}\right]$$

GRAMMAGE: $X(E/n) \propto \tau_{diff}(E/n) \sim 1/D(E/n)$

ELECTRONS AND POSITRONS

FOR TYPICAL PARAMETERS OF CR PROPAGATION, FOR ELECTRONS ENERGY LOSSES KICK IN ABOVE ~10 GeV

equilibrium primary electrons:

$$n_e(E) \sim \frac{Q(E)\tau_{loss}(E)}{2\pi R_d^2 \sqrt{D(E)\tau_{loss}(E)}} \sim E^{-\gamma-\frac{1}{2}-\frac{\delta}{2}}$$

injection secondary e⁻e⁺:

$$q_{sec}(E)dE \sim n_p(E')dE'\sigma_{pp}n_{gas}c \sim E^{-\gamma-\delta'}$$

equilibrium secondary pairs:

$$n_{sec}(E) \sim \frac{q_{sec}(E)\tau_{loss}(E)}{2\pi R_d^2 \sqrt{D(E)\tau_{loss}(E)}} \sim E^{-\gamma-\delta'-\frac{1}{2}-\frac{\delta}{2}}$$

RATIO:

$$\frac{n_{e^+}(E)}{n_e(E)} \sim E^{-\delta'}$$

It reflects the slope of the proton spectrum at E'~20E

ANTIPROTONS

injection pbar: $q_{\bar{p}}(E)dE \sim n_p(E')dE'\sigma_{pp\to\bar{p}}(E')n_{gas}c \sim E^{-\gamma-\delta'+s}$

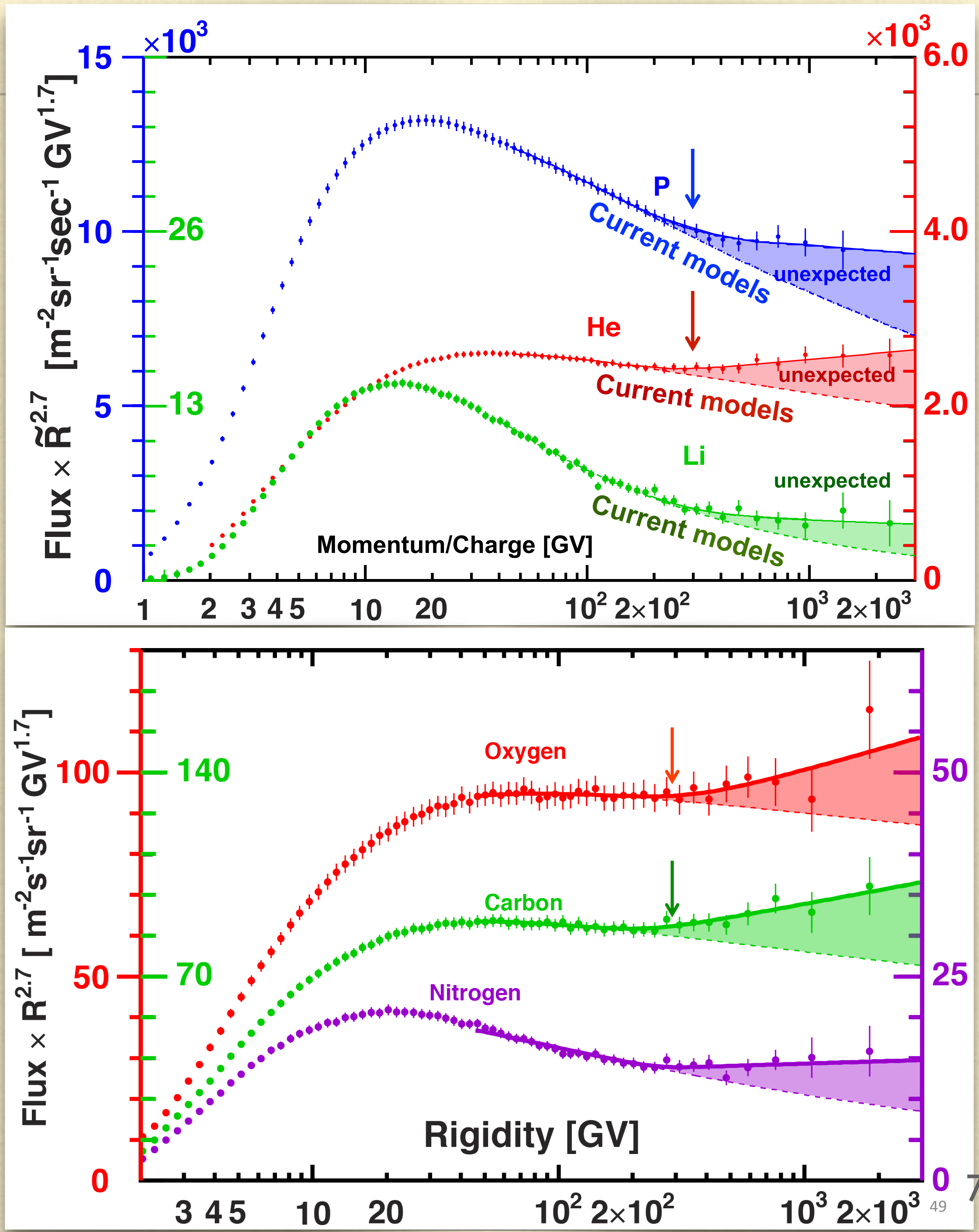
equilibrium pbar: $n_{\bar{p}}(E) \sim q_{\bar{p}}(E)\tau_{diff}(E) \sim E^{-\gamma-\delta'+s-\delta}$

RATIO 1: $\dfrac{n_{\bar{p}}}{n_p} \sim E^{-\delta'+s}$

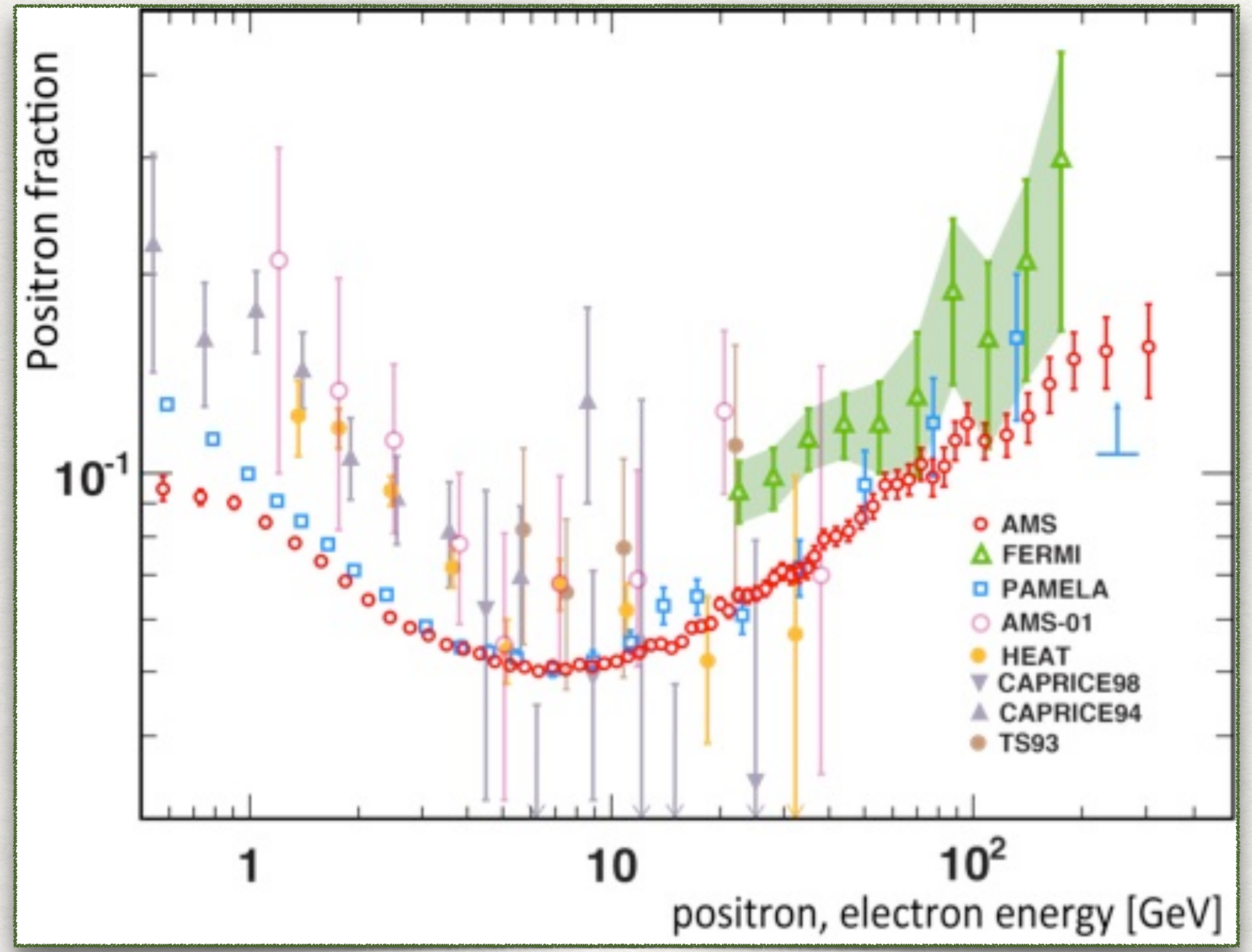
RATIO 2: $\dfrac{n_{\bar{p}}}{n_{e^+}} \sim E^{s-\frac{\delta}{2}+\frac{1}{2}}$

SURPRISES: SPECTRA OF PROTONS, HELIUM AND HEAVIER PRIMARY NUCLEI

- **Both protons and helium spectra show a break @~200-300 GV (PAMELA and AMS-02) - *Some Physics kicking in?***
- **The He spectrum is slightly harder than that of protons - *Acceleration or propagation?***
- **There is some indication that a similar break exists for heavier nuclei (CREAM)**



SECONDARY/PRIMARY: POSITRON FRACTION



AMS-02 Coll. 2013

Reacceleration of secondary Pairs in old SNRs

PB 2009, PB & Serpico 2009; Mertsch & Sarkar 2009

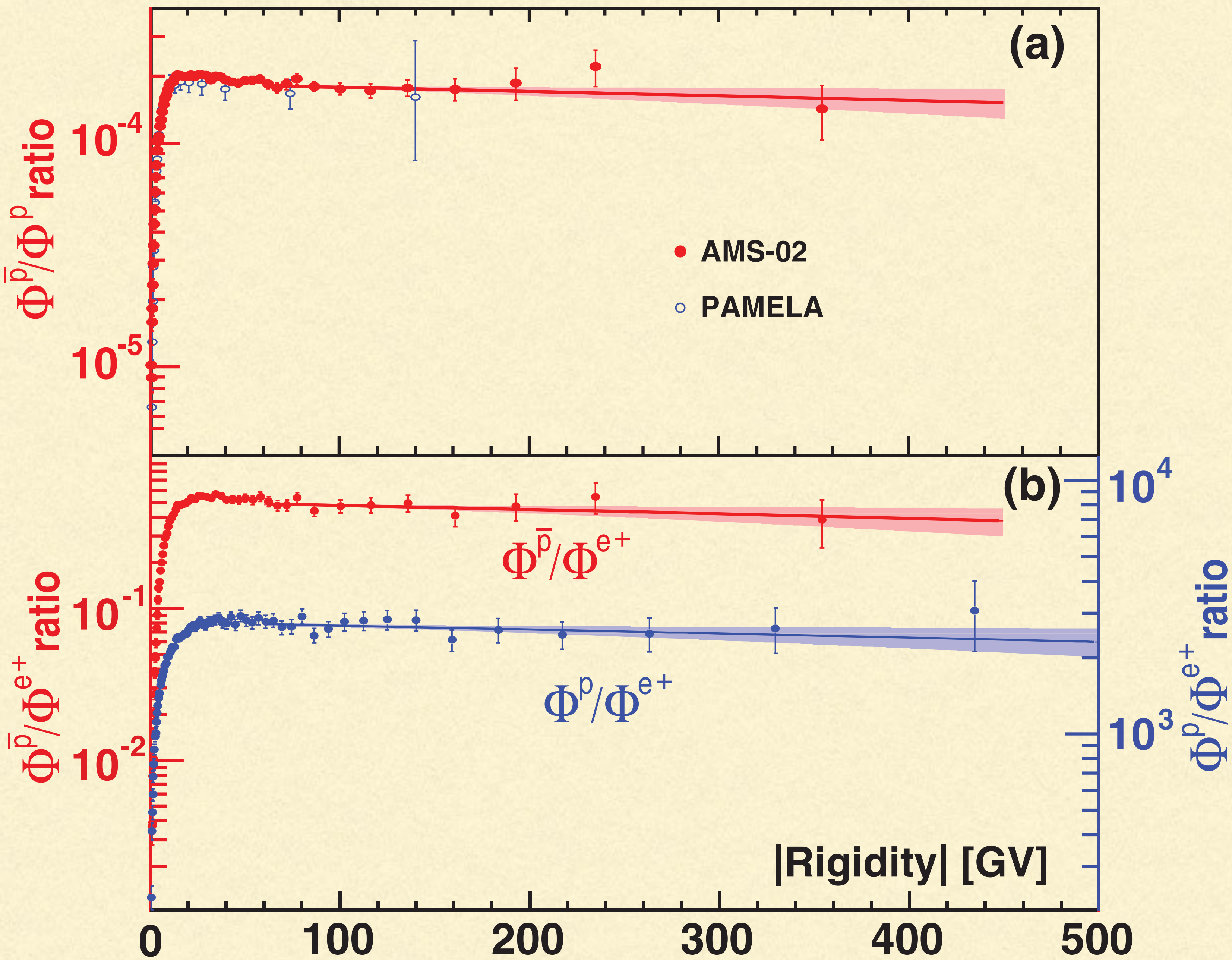
Pulsar Wind Nebulae

Hooper, PB & Serpico (2009); PB & Amato 2010

Dark Matter Annihilation

Difficult: high annihilation, Cross section, leptophilia, Boosting factor [Serpico (2012)]

A GLOBAL PICTURE - AMS02



NON LINEAR COSMIC RAY TRANSPORT IN THE GALAXY

A tale of how cosmic rays tell the environment what to do … while the environment tells cosmic rays how to behave

STREAMING INSTABILITY - THE SIMPLE VIEW

We have already obtained that the CR density has a gradient of about f_0/H, which translates to a flux Df_0/H. This streaming leads to the excitation of an instability. The general idea is simple to explain **(see the case of shocks discussed earlier)**

$$n_{CR} m v_D \to n_{CR} m V_A \Rightarrow \frac{dP_{CR}}{dt} = \frac{n_{CR} m (v_D - V_A)}{\tau} \qquad \frac{dP_w}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{V_A}$$

and assuming equilibrium:

$$\gamma_W = \sqrt{2}\, \frac{n_{CR}}{n_{gas}} \frac{v_D - V_A}{V_A} \Omega_{cyc}$$

The effective drift velocity is of order $\mathbf{v_D \sim D(df/dz)/f \sim D(E)/H}$

The instability is excited only if the CR streaming is super-Alfvenic. For typical parameters of the Galaxy this happens for E>1 GeV and the instability grows on time scales of order a few hundred years << confinement time

GROWTH AND DAMPING

The saturation of the instability is established by the balance between growth and damping

Different damping mechanisms operate. Let's see what happens with non linear damping (in the ionized halo)

$$\Gamma_{NL} \approx v_{sound} k \mathcal{F}$$

The growth rate, written in the correct way is:

$$\Gamma_{CR}(k) = \frac{16\pi^2}{3} \frac{v_A}{B^2 \mathcal{F}} \left[p^4 v(p) \frac{\partial f}{\partial z} \right]_{k=k_{res}} \approx \frac{P_{CR}(>p)}{U_B} \frac{v_A}{H} \frac{1}{\mathcal{F}} \approx \left(\frac{r_L}{r_0} \right)^{-0.7} \frac{v_A}{H} \frac{1}{\mathcal{F}}$$

ROUGHLY IN EQUIPARTITION AT p_0 ~ 1 GeV

Equating growth and damping rates:

$$\mathcal{F}^2(k) = \left(\frac{r_L}{r_0} \right)^{0.3} \frac{v_A}{v_{sound}} \frac{r_0}{H} \rightarrow D(p) = \frac{1}{3} \frac{r_L v}{\mathcal{F}} \approx 3 \times 10^{28} \beta \left(\frac{E}{10GeV} \right)^{0.85} \left(\frac{H}{3kpc} \right)^{0.5} cm^2/s$$

Tantalisingly close to that inferred from B/C @ low E

SOME ADVANCED TOPICS:

ALTERNATIVE TRANSPORT MODELS

SELF-CONFINEMENT AROUND SOURCES

IN NORMAL CONDITIONS THE ISM IS INSUFFICIENT TO GUARANTEE ANY DECENT NEAR-SOURCE GRAMMAGE

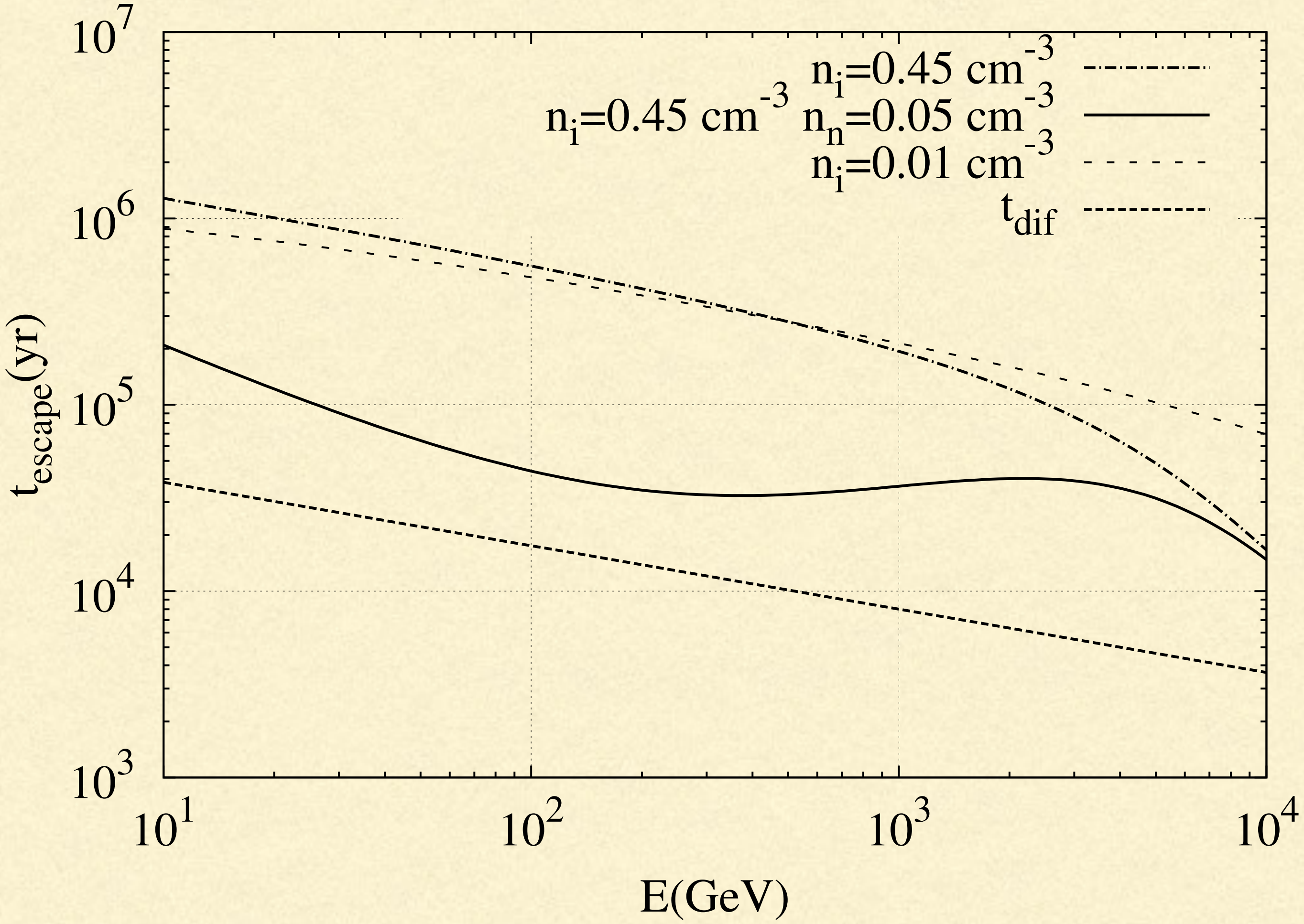
BUT CR TRANSPORT NEAR SOURCES IS STRONGLY NON-LINEAR (large CR density and density gradients) WHICH MAKES CONFINEMENT TIME LONGER (COCOON?)

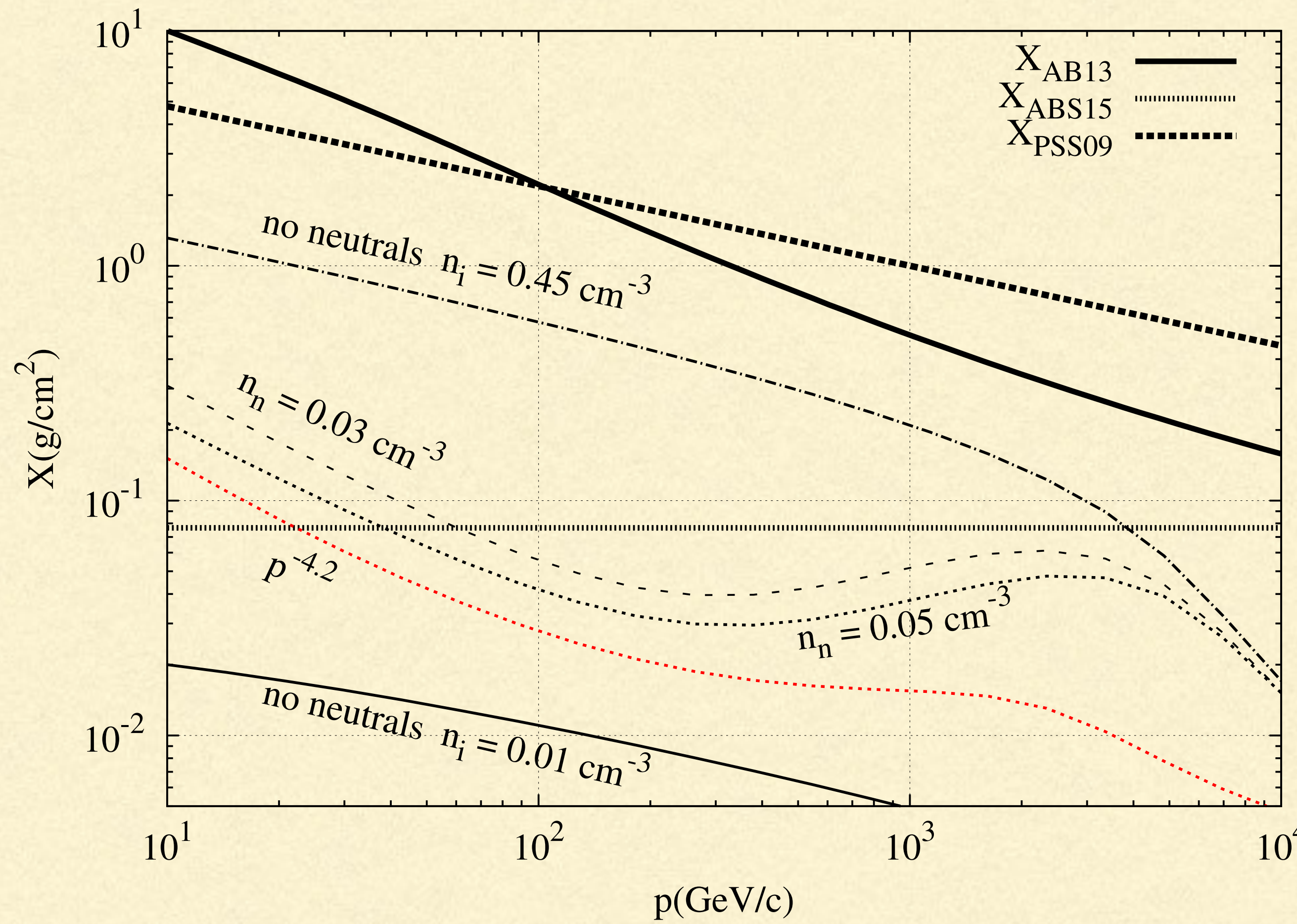
FOR A STANDARD SN THIS MAY IN FACT BE THE CASE (**D'Angelo+,Malkov+, Nava+**)

FOR SOME TIME AFTER SN EXPLOSION THE CR DENSITY IS >> MEAN AND ITS GRADIENT IS VERY LARGE

SELF-CONFINEMENT AROUND SOURCES

D'ANGELO, PB & AMATO 2017)



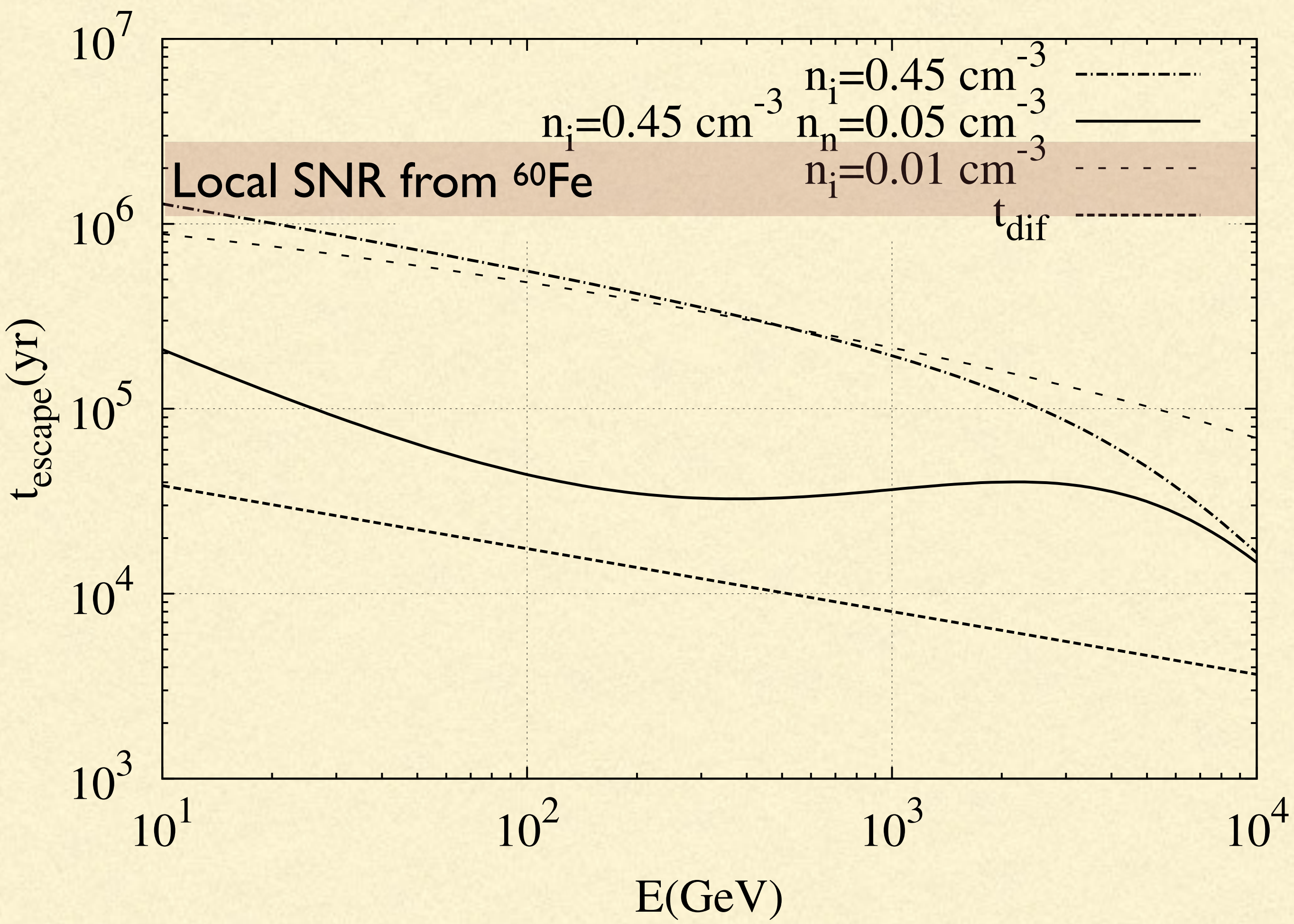


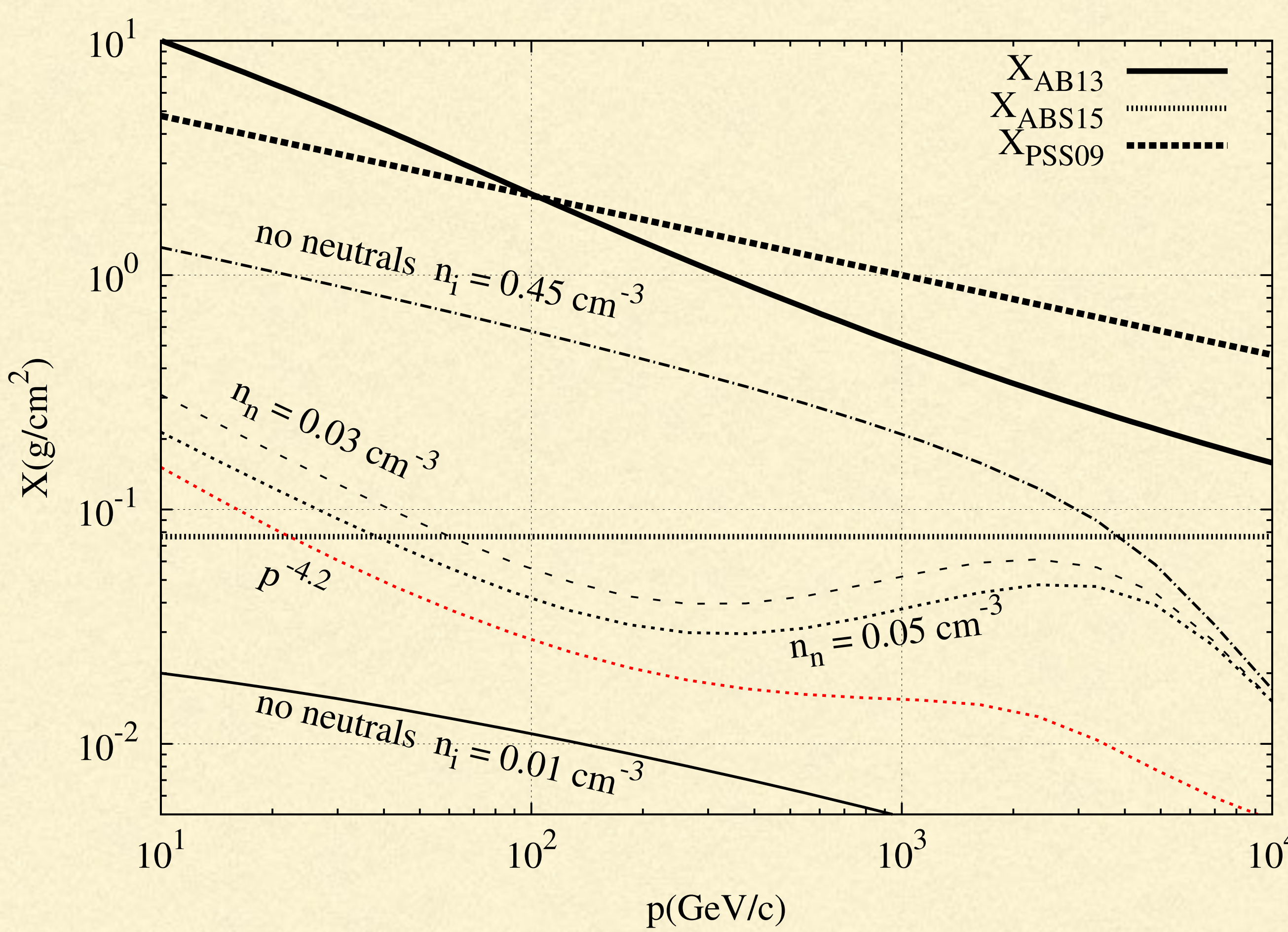
A naive estimate of when the cocoon grammage becomes comparable to the Galactic grammage

$$\frac{L_c^2}{D_{nl}} n_d = \frac{H^2}{D_{gal}} n_d \frac{h}{H} \rightarrow \frac{D_{nl}}{D_{gal}} = \frac{L_c^2}{H h} \sim \frac{1}{40}$$

SELF-CONFINEMENT AROUND SOURCES

D'ANGELO, PB & AMATO 2017)





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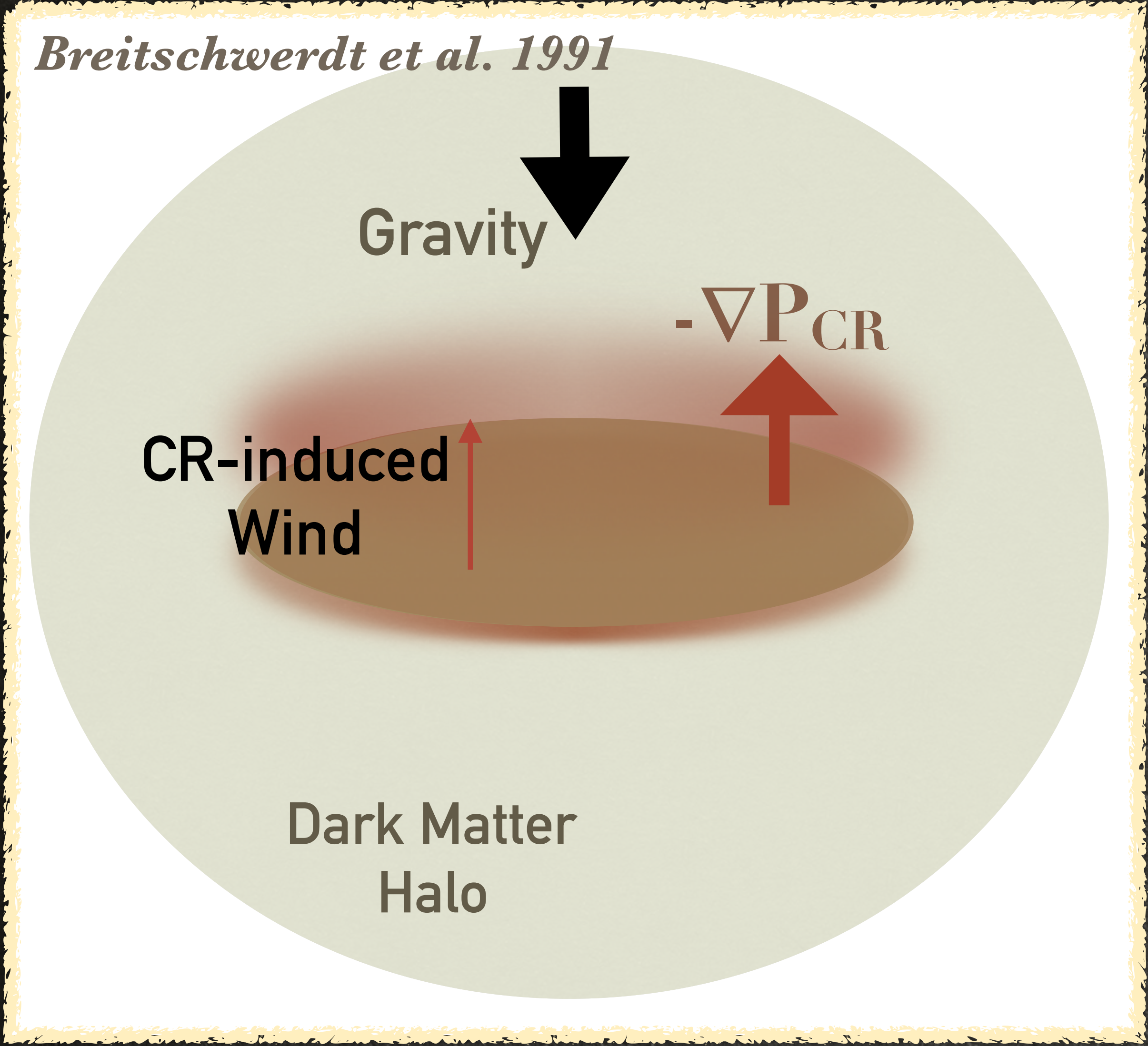
SELF-CONFINEMENT AROUND SOURCES

- THE FEASIBILITY IF THIS SCENARIO DEPENDS STRONGLY ON THE AMOUNT OF NEUTRAL GAS (ion-neutral damping)
- THE PROXIMITY OF A CLOUD INCREASES THE GRAMMAGE IN THE NEAR SOURCE REGION
- THE EMISSIVITY INTEGRATED ALONG A LINE OF SIGHT IS SENSITIVE TO WHETHER THIS PHENOMENON IS TAKING PLACE (*MORLINO ET AL. 2017*)
- WHEN PRESENT, IT STOPS BEING IMPORTANT FOR E>1 TeV, AS WOULD BE REQUIRED TO BE A "COCOON"
- **SEVERAL INDICATIONS ALREADY THAT THE DIFFUSION COEFFICIENT CLOSE TO SOURCES IS MUCH SMALLER THAN THE AVERAGE (FERMI OBSERVATIONS OF MOLECULAR CLOUDS, HAWC EVIDENCE FOR DIFFUSE EMISSION, …)**

SOME ADVANCED TOPICS:

COSMIC RAY INDUCED GALACTIC WINDS

Cosmic Rays vs Gravity: Cosmic Ray Induced Galactic Winds



The force exerted by CR may win over gravity and a wind may be launched

$$\vec{\nabla}\cdot(\rho\vec{u}) = 0,$$

$$\rho(\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla}(P_g + P_c) - \rho\vec{\nabla}\Phi,$$

$$\vec{u}\cdot\vec{\nabla}P_g = \frac{\gamma_g P_g}{\rho}\vec{u}\cdot\vec{\nabla}\rho - (\gamma_g - 1)\vec{v}_A\cdot\vec{\nabla}P_c,$$

$$\vec{\nabla}\cdot\left[\rho\vec{u}\left(\frac{u^2}{2} + \frac{\gamma_g}{\gamma_g - 1}\frac{P_g}{\rho} + \Phi\right)\right] = -(\vec{u} + \vec{v}_A)\cdot\vec{\nabla}P_c,$$

$$\vec{\nabla}\cdot\left[(\vec{u} + \vec{v}_A)\frac{\gamma_c P_c}{\gamma_c - 1} - \frac{\overline{D}\vec{\nabla}P_c}{\gamma_c - 1}\right] = (\vec{u} + \vec{v}_A)\cdot\vec{\nabla}P_c,$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\cdot\left[D\vec{\nabla}f\right] - (\vec{u} + \vec{v}_A)\cdot\vec{\nabla}f + \vec{\nabla}\cdot(\vec{u} + \vec{v}_A)\frac{1}{3}\frac{\partial f}{\partial \ln p} + Q = 0$$

Diffusion determined by self-generation at CR gradients balanced by local damping of the same waves

No pre-established diffusion coefficient and no pre-fixed halo size

Cosmic Rays vs Gravity: CR driven winds

Aside from math, the Physics of the problem can be understood easily, though it turns out to be unrealistic: There is a critical distance above (and below) the disc (which depends on particle energy) where diffusion turns into advection:

$$\frac{z^2}{D(p)} \simeq \frac{z}{u(z)} \to z_*(p) \propto p^{\delta/2} \qquad D(p) \sim p^{\delta}$$

Ptuskin et al. 1997

No effective halo size H

$$f_0(p) = \frac{Q(p)}{2A_{disc}} \frac{H}{D(p)} \sim E^{-\gamma-\delta} \qquad f_0(p) = \frac{Q(p)}{2A_{disc}} \frac{z_*(p)}{D(p)} \sim E^{-\gamma-\delta/2}$$

STANDARD CASE — **CR-INDUCED WIND WITH SELF-GENERATION**

At high energy, the critical scale becomes larger than the location where the geometry to he wind becomes spherical, and a steepening of the spectrum may be expected