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EPOS

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Klaus Werner

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1 Parton evolution and Pomerons in EPOS

1.1 Parton evolution

A fast moving proton



emits successively partons (mainly gluons), quasi-real (large gamma factors) 4

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... which can be probed by a virtual photon (emitted from an electron)



What precisely the photon "sees" depends on two kinematic variables,

the **virtuality**

$$Q^2 = -k^2$$

and the Bjorken variable

$$x = \frac{Q^2}{2pk}$$

which probes partons with momentum fraction *x*. It determines also the **approximation scheme** to compute the parton cloud.



BFKL (Balitsky, Fadin, Kuraev, and Lipatov):

$$\frac{\partial \varphi(x, \boldsymbol{q})}{\partial \ln \frac{1}{x}} \frac{\alpha_s N_c}{\pi^2} \int d^2 k \, K(\boldsymbol{q}, \boldsymbol{k}) \varphi(x, \boldsymbol{k})$$

with
$$xg(x, Q^2) = \int_0^{Q^2} \frac{d^2k}{k^2} \varphi(x, k),$$

DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli and Parisi):

$$\frac{\partial g(x,Q^2)}{\partial \ln q^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) g(\frac{x}{z},Q^2)$$

Very large $\ln 1/x$: Saturation domain



Non-linear effects 9

Gluon from one cascade is absorbed by another one



Same evolution as in proton-photon (causality)

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Different way of plotting the same reaction



inelastic scattering diagram

Corresponding cut diagram



referred to as "cut parton ladder" = amplitude squared of the inelastic diagram

Corresponding elastic diagram



referred to as "(uncut) parton ladder"

1.3 Soft domain

Very small $\ln Q^2$: No perturbative treatment!

But one may use the hypothesis of **Lorentz invariance** and **analyticity** of the T-matrix. One starts with a partial wave expansion of the T-matrix (Watson-Sommerfeld transform) :

$$T(t,s) = \sum_{j=0}^{\infty} (2j+1)\mathcal{T}(j,s)P_j(z)$$

with $t \propto z - 1$, $z = \cos \vartheta$, P_j : Legendre polynomials.

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With $\alpha(s)$ being the rightmost pole of $\mathcal{T}(j,s)$ one gets for $t \to \infty$:

$$T(t,s) \propto t^{\alpha(s)}$$



and assuming crossing symmetry one gets the famous asymptotic result

$$T(s,t) \propto s^{\alpha(t)}$$

with the "Regge pole"
 $lpha(t) = lpha(0) + lpha' t$



Formulas (see Phys.Rept. 350 (2001) 93-289):

$$T_{\text{soft}}(\hat{s}, t) = 8\pi s_0 i \gamma_{\text{Pom-parton}}^2 \left(\frac{\hat{s}}{s_0}\right)^{\alpha_{\text{soft}}(0)} \times \exp(\lambda_{\text{soft}} t)$$

with $\lambda_{\text{soft}} = 2R_{\text{Pom-parton}}^2 + \alpha'_{\text{soft}} \ln \frac{\hat{s}}{s_0}.$

Interaction cross section,

$$\sigma_{\text{soft}}(\hat{s}) = \frac{1}{2\hat{s}} 2\text{Im} T_{\text{soft}}(\hat{s}, 0) ,$$

$$= 8\pi \gamma_{\rm part}^2 \left(\frac{\hat{s}}{s_0}\right)^{\alpha_{\rm soft}(0)-1}$$

which grows too fast





Space-time picture of semihard Pomeron



Hard cross section and amplitude (see Phys.Rept. 350 (2001) 93-289):

(

$$\begin{aligned} \sigma_{\text{hard}}^{jk}(\hat{s},Q_0^2) &= \frac{1}{2\hat{s}} 2\text{Im} \, T_{\text{hard}}^{jk}(\hat{s},t=0) \\ &= K \sum_{m} \int dx_B^+ dx_B^- dp_\perp^2 \frac{d\sigma_{\text{Born}}^{ml}}{dp_\perp^2} (x_B^+ x_B^- \hat{s},p_\perp^2) \\ &\times E_{\text{QCD}}^{jm}(x_B^+,Q_0^2,M_F^2) \, E_{\text{QCD}}^{kl}(x_B^-,Q_0^2,M_F^2) \theta \left(M_F^2 - Q_0^2\right), \end{aligned}$$

One knows (Lipativ, 86): amplitude is imaginary, and nearly independent on $t \Rightarrow$ (with $R_{hard}^2 \simeq 0$):

$$T_{\text{hard}}^{jk}(\hat{s},t) = i\hat{s}\,\sigma_{\text{hard}}^{jk}(\hat{s},Q_0^2)\,\exp\left(R_{\text{hard}}^2\,t\right)$$

Semihard amplitude :

$$iT_{\text{semihard}}(\hat{s}, t) = \sum_{jk} \int_0^1 \frac{dz^+}{z^+} \frac{dz^-}{z^-}$$
$$\times \text{Im } T^j_{\text{soft}}\left(\frac{s_0}{z^+}, t\right) \text{ Im } T^k_{\text{soft}}\left(\frac{s_0}{z^-}, t\right) iT^{jk}_{\text{hard}}(z^+z^-\hat{s}, t)$$

(valid for $s \rightarrow \infty$ and small parton virtualities except for the ones in the ladder)

2 Multiple scattering in EPOS

in collaboration with T. Pierog and B. Guiot

Parton based Gribov-Regge theory. By H.J. Drescher, M. Hladik, S. Ostapchenko, T. Pierog, K. Werner. hep-ph/0007198. Published in Phys.Rept. 350 (2001) 93-289.

2.1 Single scattering (single Pomeron)



somewhat simplified



Be *T* the elastic (pp,pA,AA) scattering T-matrix =>

$$2s\,\sigma_{\rm tot}=\frac{1}{\rm i}{\rm disc}\,T$$

Basic assumption : Multiple "Pomerons"

$$iT = \sum_{k} \frac{1}{k!} \{ iT_{\text{Pom}} \times ... \times iT_{\text{Pom}} \}$$

Example: 2 "Pomerons"



Evaluate

$$\frac{1}{i} \operatorname{disc} \left\{ i T_{\operatorname{Pom}} \times ... \times i T_{\operatorname{Pom}} \right\}$$

using "cutting rules" :

A "cut" multi-Pomeron diagram amounts to the sum of all possible cuts Example of two Pomerons



Using "Pomeron = parton ladder + soft", we have (first diagram)



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Using a simplified notation for "cut" and "uncut" Pomeron



one gets ...

2.3 Complete result (strict energy conservation)

(Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)



$$\begin{split} \sigma^{\text{tot}} &= \int d^2 b \int \prod_{i=1}^A d^2 b_i^A \, dz_i^A \, \rho_A(\sqrt{(b_i^A)^2 + (z_i^A)^2}) \\ &\prod_{j=1}^B d^2 b_j^B \, dz_j^B \, \rho_B(\sqrt{(b_j^B)^2 + (z_j^B)^2}) \\ &\sum_{m_1 l_1} \dots \sum_{m_{AB} l_{AB}} (1 - \delta_{0\Sigma m_k}) \int \prod_{k=1}^{AB} \left(\prod_{\mu=1}^{m_k} dx_{k,\mu}^+ dx_{k,\mu}^- \prod_{\lambda=1}^l d\tilde{x}_{k,\lambda}^+ d\tilde{x}_{k,\lambda}^- \right) \left\{ \\ &\prod_{k=1}^{AB} \left(\frac{1}{m_k!} \frac{1}{l_k!} \prod_{\mu=1}^{m_k} G(x_{k,\mu'}^+ x_{k,\mu'}^- s, |\vec{b} + \vec{b}_{\pi(k)}^A - \vec{b}_{\pi(k)}^B|) \right) \\ &\prod_{\lambda=1}^l - G(\tilde{x}_{k,\lambda'}^+ \tilde{x}_{k,\lambda'}^- s, |\vec{b} + \vec{b}_{\pi(k)}^A - \vec{b}_{\pi(k)}^B|) \right) \\ &\prod_{i=1}^A \left(1 - \sum_{\pi(k)=i} x_{k,\mu_i}^+ - \sum_{\pi(k)=i} \tilde{x}_{k,\lambda}^+ \right)^{\alpha} \prod_{j=1}^B \left(1 - \sum_{\tau(k)=j} x_{k,\mu}^- - \sum_{\tau(k)=j} \tilde{x}_{k,\lambda}^- \right)^{\alpha} \right) \end{split}$$

Complicated due to strict energy conservation => 10,000,000-dimensional intergrals, not separable

but doable:

 \Box Parameterizations for $G(x^+, x^-, s, b)$

□ Analytical integrations

□ Employing Markov chain techniques to generate configurations *K* according to multidimensional probalility distributions $f(K) = \sigma_K / \sigma_{tot}$ σ_K :



Dotted lines : Cut Pomerons Full lines : Uncut Pomerons LHC meets Cosmic Rays 29 October 2018 # Klaus Werner # Subatech, Nantes

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2.4 Configurations via Markov chains

the heart of EPOS, see Phys. Rept. 350, 2001 or https://ejc2018.sciencesconf.org/data/pages/joliot.20.pdf



like a physical process reaching eqilibrium

The law changes step by step ($f_t \rightarrow f_{t+1}$) :

$$f_{t+1}(K) = \sum_{K'} f_t(K') p(K' \to K).$$

The transition probability *p* has to be chosen properly to assure convergence towards *f*

Sufficient condition: detailed balance $f(K') p(K' \rightarrow K) = f(K) p(K \rightarrow K')$,
Metropolis:

One can prove that a $p(K \rightarrow K')$ of the form

$$w(K \to K') \times \min\left(1, \frac{f(K')}{f(K)} \frac{w(K' \to K)}{w(K \to K')}\right)$$

with any choice of *w* fulfills detailed balance!!

But still *w* needs to be chosen in an intelligent way ... even then long iterations,

but the method allows to generate very complex configurations according to very complex laws

2.5 Parton saturation

Computing the expressions G for single Pomerons: A cutoff Q_0 is needed (for the DGLAP integrals).

Taking *Q*₀ constant leads to a power law increase of cross sections vs energy (=> wrong)

because non-linear effects like gluon fusion are not taken into account



Solution: Instead of a constant *Q*₀, use a dynamical saturation scale for each Pomeron:

$$Q_s = Q_s(N_{\mathbf{I}}, s_{\mathbf{I}})$$

with

 $N_{\mathbb{IP}}$ = number of Pomerons connected to a given Pomeron (whose probability distribution depends on Q_s)

 $s_{\mathbb{IP}}$ = energy of considered Pomeron



Parton distributions



=> Increase of $\langle p_t \rangle$ with multiplicity

2.6 "Outside to inside" parton production

How to reconstruct the Pomerons? (knowing the properties of the Pomeron ends)

Having the end partons i, j, how to get the intermediate ones (like m, k etc)? We iterate from outside to inside!



Actually the diagram *k* to *j* correponds to $\sigma_{hard}^{kj}(\hat{s}, Q_1^2, Q_2^2)$, already used to generate multiple scattering configurations



Probability of single emission $m \rightarrow k$:

$$prob(\xi, Q^2) = d\xi \frac{dQ^2}{Q^2} \Delta^m(Q_1^2, Q^2) \frac{\alpha_s}{2\pi} P_m^k(\xi) \sigma_{\text{hard}}^{kj}(\xi \hat{s}, Q^2, Q_2^2)$$

with a given parton *j* on the other end.

Attention: emission on one side depends on existing parton the the other end!

=> precalculation of cross sections, tabulations, interpolation

Recent: Heavy quark (Q) production in EPOS framework





Electron-positron annihilation



Color field between two color charges => relativistic string

B. Andersson, G. Gustafson, G. Ingelman, and T. Sjostrand, Phys. Rep. 97 (83) 31 X. Artru, Phys. Rep. 97 (83) 147

High pt gluon emission in e⁺e⁻



Kinky relativistic string

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Two kinky relativistic strings (at least)

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String evolution



Mapping partons => string initial conditions



pp: Parton ladder = color flux tubes = **2 kinky strings**



(here no IS radiation, only hard process producing two gluons)



String segment = hadron. Close to "kink": jets

Unless we have many color flux tubes



=> core + corona

3 Collectivity in EPOS

□ Core-corona separation (pp,pA,AA)

□ **EPOS 3**:

- Hydrodynamic expansion of core
- Statistical decay of fluid (Grand canonical, big systems)

□ EPOS LHC:

- Effective flow, droplet decay

(like resonance decay, small systems)

□ "Unification": Microcanonical decay (small and big)



□ No need to match dynamical part of hydro evolution

Energy and flavor conservation for small systems

□ Needed to "unify" EPOSLHC and EPOS3

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Grand canonical decay, T = 130 MeV

V=50 fm³; V=1000 fm³



Microcanonic decay

of given volume in its CMS into *n* hadrons



Different from decay rate of a massive particle (using LIPS), where asymptotic states are defined over an infinitely large volume (see Becattini et al, EPJC35:243-258,2004). But $E_i = \sqrt{p_i^2 + m_i^2}$

Microcanonical decay

$$dP \propto d\Phi_{\text{NRPS}} = \delta(M - \Sigma E_i) \,\delta(\Sigma \vec{p}_i) \prod_{i=1}^n d^3 p_i$$

- \Box Hagedorn 1958 methods to compute Φ_{NRPS}
- □ Lorentz invariant phase space (LIPS) (James 1968)
- □ Hagedorn methods used for decaying QGP droplets (Werner, Aichelin, 1994, Becattini 2003)
- \Box 2012 (Bignamini,Becattini,Piccinini) compute Φ_{NRPS} via the Lorentz invariant phase space (LIPS)

Hagedorn integral method can be made very efficient at large n (new), but it is VERY time consuming at small n

□ **LIPS method very fast for small n**, gets time consuming at large n

□ around $n \approx 30 - 40$ both methods work (=> checks)

Hagedorn integral method

The phase-space integral:

$$\phi_{\text{NRPS}}(M, m_1, \dots, m_n) = (4\pi)^n \int \prod_{i=1}^n p_i^2 \,\delta(E - \sum_{i=1}^n E_i) \,W(p_1, \dots, p_n) \prod_{i=1}^n dp_i,$$

with the "random walk function" W (angular integral)

$$W(p_1,\ldots,p_n):=\frac{1}{(4\pi)^n}\int \delta\big(\sum_{i=1}^n p_i\times\vec{u}_i\big)\prod_{i=1}^n d\Omega_i$$

We obtain (Werner, Aichelin 94)

$$\phi(M, m_1, \ldots, m_n) = \int_0^1 dr_1 \ldots \int_0^1 dr_{n-1} \psi(r_1, \ldots, r_{n-1})$$

$$\psi = \frac{(4\pi)^n T^{n-1}}{(n-1)!} \prod_{i=1}^n p_i E_i W(p_1, \ldots, p_n),$$

with
$$z_i = r_i^{1/i}$$
, $x_i = z_i x_{i+1}$, $s_i = x_i T$, $t_i = s_i - s_{i-1}$,
 $E_i = t_i + m_i$, $T = M - \sum_{i=1}^n m_i$

Suitable for MC provided W is known

The random walk function may be written as

$$W(p_1,\ldots,p_n)=\frac{1}{(4\pi)^n}\frac{1}{(2\pi)^3}\int\int e^{-i\vec{\lambda}\Sigma p_j\hat{p}_j}\prod_{j=1}^n d\Omega_j\,d^3\lambda,$$

which gives $W = \int_0^\infty F(\lambda) \, d\lambda$ with

$$F(\lambda) = \frac{1}{2\pi^2} \lambda^2 \prod_{j=1}^n \frac{\sin p_j \lambda}{p_j \lambda}.$$

New: using the fact that for large *n* $\prod_{j=1}^{n} \frac{\sin p_{j}\lambda}{p_{j}\lambda} \approx \exp\left(-P^{2}\lambda^{2}\right), \quad P = \sqrt{\frac{1}{6}\sum_{j=1}^{n}p_{j}^{2}}$

With
$$F_0(\lambda) = F(\lambda) \times \exp(P^2 \lambda^2)$$
:
 $W = \int_0^\infty F(\lambda) \, d\lambda = \frac{1}{P} \int_0^\infty F_0\left(\frac{x}{P}\right) \times \exp\left(-x^2\right) \, dx$

with F_0 being a slowly varying function of x, which allows to use the Gauss-Hermite formula

$$W pprox rac{1}{P} \sum\limits_{k=1}^{K} w_j^{GH} F_0\left(rac{x_j^{GH}}{P}
ight)$$
 ,

with nodes and weights x_j^{GH} and w_j^{GH} found in text books.

With only six nodes we get excellent results.

Sampling hadron configurations $K = \{h_1, ..., h_n; \vec{p}_1, ... \vec{p}_n\}$ via Markov chains

We construct sequences of random configurations $K_1, K_2, K_3, ...K_t, ...$ such that $f_t(K_t)$ converges towards f(K) for $t \to \infty$

with *f* = microcanonical probability distribution

The law changes step by step ($f_t \rightarrow f_{t+1}$) :

$$f_{t+1}(K) = \sum_{K'} f_t(K') p(K' \to K).$$

with $p(K \rightarrow K')$ of the form

$$w(K \to K') \times \min\left(1, \frac{f(K')}{f(K)} \frac{w(K' \to K)}{w(K \to K')}\right)$$

3.2 Grand canonical limit

For very large *M* we should recover the "grand canonical limit" for single particle spectra:

$$f_k = rac{g_k V}{(2\pi\hbar)^3} \exp\left(-rac{E_k}{T}
ight),$$

The average energy is

$$\bar{E} = \frac{g_k V}{(2\pi\hbar)^3} \sum_k \int_0^\infty E_k \exp\left(-\frac{E_k}{T}\right) 4\pi p^2 dp$$

Changing variables via $E_k dE_k = pdp$, and using $K_1(z) = z \int_1^\infty \exp(-zx)\sqrt{x^2 - 1}dx$, and $3K_2(z) = z^2 \int_1^\infty \exp(-zx)\sqrt{x^2 - 1}^3 dx$,

$$=> \qquad \bar{E} = \frac{4\pi g_k V}{(2\pi\hbar)^3} m^2 T \left(3T K_2(\frac{m}{T}) + m K_1(\frac{m}{T}) \right).$$

The microcanonical decay of an object of mass M and volume V should converge (for $M \rightarrow \infty$) to the GC single particle spectra

with *T* obtained from $M = \overline{E}$.

3.3 Comparing GC et MiC decay

We consider a complete (?) set of hadrons (\approx 400, PDG list)

We check the effect of

- □ energy conservation
- □ flavor conservation














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circles = pp (7TeV)

squares = pPb (5TeV)

stars = PbPb (2.76TeV)

Data partly collected by A. G. Knospe Refs:

<dNch/deta> in Pb+Pb: Phys. Rev. Lett. 106 032301 (2011) pi+, K+, and (anti)protons in Pb+Pb: Phys. Rev. C 88 044910 (2013) Lambda in Pb+Pb: Phys. Rev. Lett. 111 222301 (2013) Xi- and Omega in p+Pb: Phys. Lett. B 758 389-401 (2016) pi+, K+, (anti)protons, and Lambda in p+Pb: Phys. Lett. B 728 25-38 (2014)

<dNvh/deta> in p+Pb: Fur. Phys. J. C 76 245 (2016) Xi- and Omega in p+Pb: Phys. Lett. B 758 389-401 (2016) <dNvh/deta> in p+p 7 TeV: Eur. Phys. J. C 68 345-354 (2010) pi+, K+, and (anti)protons in p+p 7 TeV: Eur. Phys. J. C 75 226 (2015)

Xi- and Omega in p+p 7 TeV: Phys. Lett. B 712 309 (2012) and data points from Rafael Derradi de Souza, SQM2016

Proton to pion ratio (sofar GC)



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Omega to pion ratio (GC)



4 EPOS Summary

- □ Based on "semihard Pomerons" (parton ladders)
- □ Multiple scattering via Gribov-Regge approach, employing cutting rules
- □ Realizing configurations via Markov chains
- □ Consistent "outside to inside" parton production
- □ "Natural" parton-string mapping
- Core-corona separation,
 hydrodynamic expansion of core (=> flow)
- □ Statistical hadronization (new: microcanonical)