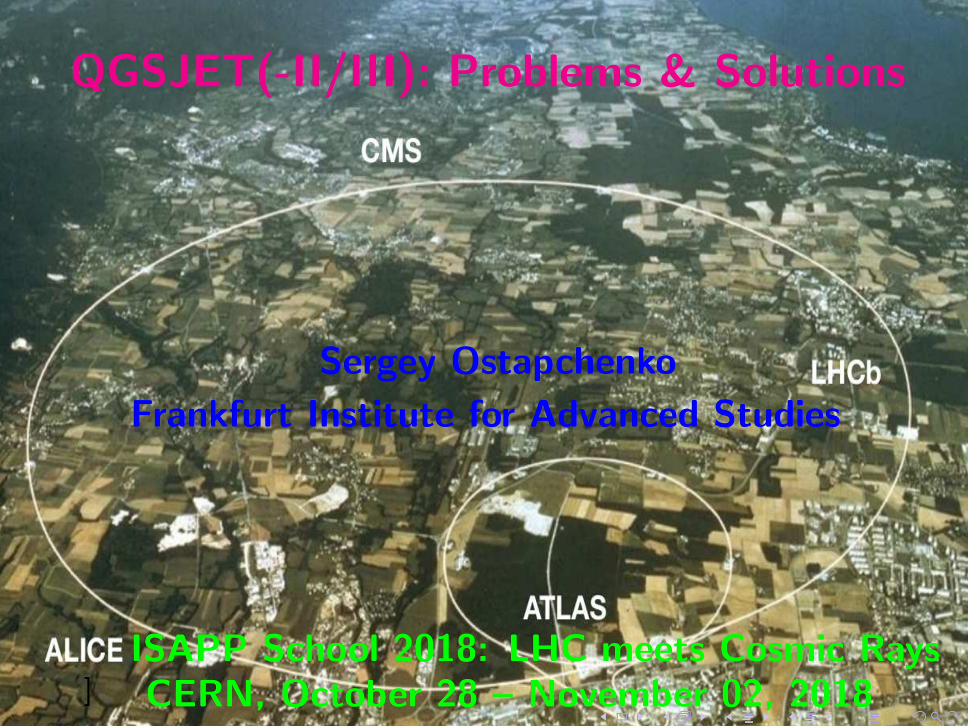
QGSJET(-II/III): Problems & Solutions

Sergey Ostapchenko

Frankfurt Institute for Advanced Studies



ISAPP School 2018: LHC meets Cosmic Rays

CERN, October 28 – November 02, 2018

Getting started...

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- QGSJET: treat "soft" & "hard" within the same scheme *[Kalmykov, SO & Pavlov, 1994, 1997]*
    - $\Rightarrow$ treat both within the Gribov's Reggeon Field Theory (RFT)
    - starting point: Quark-Gluon String model *[Kaidalov & Ter-Martyrosyan, 1982, 1984]*

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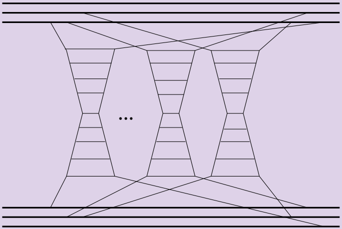
Quark-Gluon String model: RFT-based treatment

High energy interactions $\Rightarrow$ multiple scattering

- elementary interaction = parton cascade $\equiv$ Pomeron exchange



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- Pomeron amplitude:

$$f_{ab}^{\mathbb{P}}(s,b) = \frac{i\gamma_a \gamma_b s^{\alpha_{\mathbb{P}}(0)-1}}{\lambda_{ab}(s)} e^{-\frac{b^2}{4\lambda_{ab}(s)}}$$

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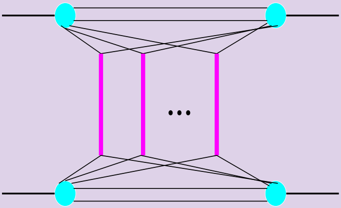

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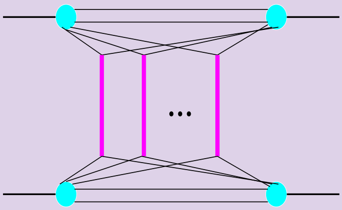

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AGK cutting rules ⇒ partial cross sections for various final states

- "cut" Pomerons ⇒ string formation & hadronization

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- experimental data on $\sigma_{hp}^{\rm tot}$, $\sigma_{hp}^{\rm el}$, $\sigma_{hp}^{\rm difr}$, $B_{hp}^{\rm el}$
  $\Rightarrow$ fixing Pomeron parameters
- experimental data on particle production
  $\Rightarrow$ fixing hadronization parameters
- generalization to hA, AA – parameter free

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NB: original RFT approach - for soft (low $p_{\rm t}$) physics

- high energy behavior governed by the Pomeron intercept $\alpha_{\mathbb{P}}(0)$

QGSJET: (semi-)hard processes within the RFT framework

Robust pQCD prediction: rising contribution of high $p_{\rm t}$ partons

- small $\alpha_s(p_t^2)$ compensated by low-x rise of PDFs $f_I(x,p_t^2)$
  - generated by soft and collinear logs: $\ln\frac{x_n}{x_{n+1}}$, $\ln\frac{p^2_{t_{n+1}}}{p^2_{t_n}}$

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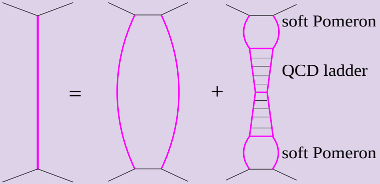
Phenomenological 'semihard Pomeron' approach

[Kalmykov, SO & Pavlov, 1994, 1997; Drescher et al., 1999, 2001]

- Q_0^2 - technical parameter (cutoff between soft & hard physics)
- $|p_t^2| < Q_0^2 \Rightarrow$ soft Pomeron
- $|p_t^2| > Q_0^2 \Rightarrow$ DGLAP ladder



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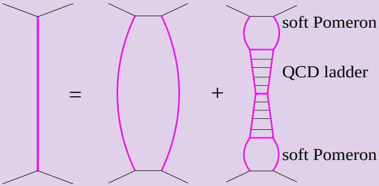
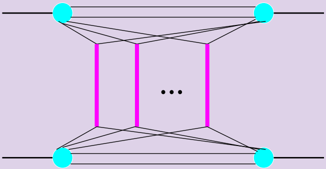


RFT scheme based on a "general Pomeron" (soft + semihard)

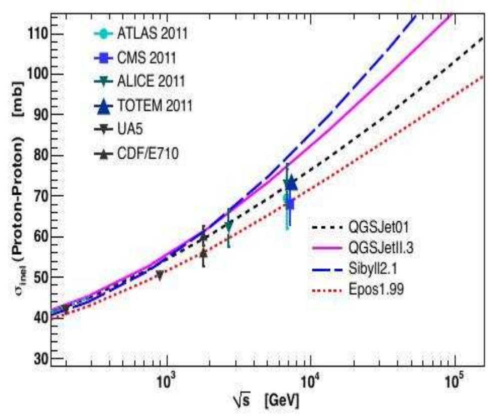
- "semihard Pomeron" combines features of soft & hard ones
  - steep energy-rise: $\propto s^{\Delta_{\rm hard}}$
  - transverse expansion governed by the soft Pomeron slope



- $\Rightarrow$ jet production extends to peripheral collisions

QGSJET: successful predictions for extensive air showers

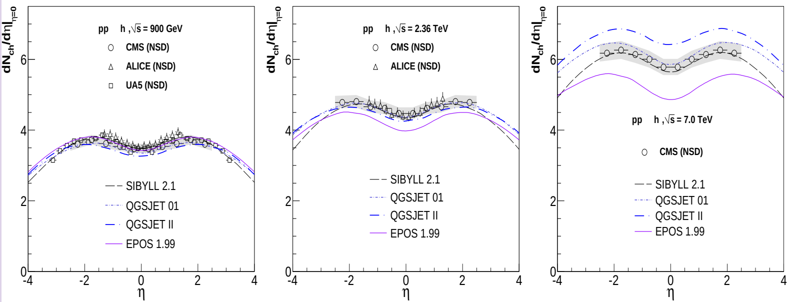
Success aposteriori explained by LHC data, e.g. on $\sigma_{pp}^{\mathrm{tot/el}}$



[from R. Engel, talk at "ICRC-2013"]

QGSJET: successful predictions for extensive air showers

Central production equally well-desribed by QGSJET

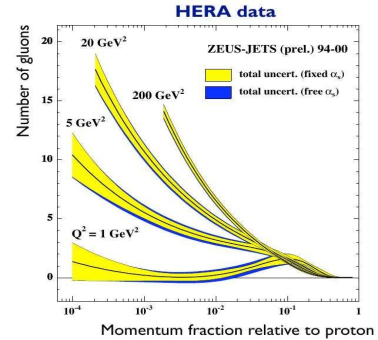


[plots from d'Enterria et al., Astrop. Phys. 35 (2011) 98]

QGSJET: successful predictions for extensive air showers

But: the model was based on flat (pre-HERA) PDFs

- steep low-x increase of gluon density $\Rightarrow$ too fast energy rise of $\sigma_{pp}^{tot}(s)$, $N_{pp}^{ch}(s)$



Total cross section & jet production

- Inclusive cross section for production jets of $p_{\mathrm{t}} > p_{\mathrm{t}}^{\mathrm{cut}}$:

$$\sigma_{pp}^{\mathrm{jet}}(s, p_{\mathrm{t}}^{\mathrm{cut}}) = \sum_{I,J=q,\bar{q},g} \int_{p_{\mathrm{t}}>p_{\mathrm{t}}^{\mathrm{cut}}} dp_t^2 \int dx^+ \, dx^- \, f_{I/p}(x^+, M_{\mathrm{F}}^2) \times \frac{d\sigma_{IJ}^{2\to 2}(x^+x^-s, p_t^2, M_{\mathrm{F}}^2)}{dp_t^2} f_{J/p}(x^-, M_{\mathrm{F}}^2)$$

- Problem: $\sigma_{pp}^{\mathrm{jet}}$ rises quicker than $\sigma_{pp}^{\mathrm{tot}}$ as $s \to \infty$
  - $\sigma_{pp}^{\mathrm{jet}}(s, Q_0) \propto \frac{1}{Q_0^2} s^{\Delta_{\mathrm{eff}}}$, $\Delta_{\mathrm{eff}} \simeq 0.3$
  - $\sigma_{pp}^{\mathrm{tot}}(s) \propto \ln^2 s$

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  - $\sigma_{pp}^{\mathrm{tot}}(s) \propto \ln^2 s$
- $\Rightarrow$ multiple jet production required
  - $=$ multiparton interactions (MPIs)

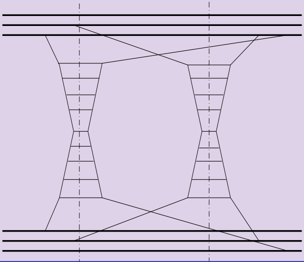
MPIs & generalized parton distributions (GPDs)

- Usual PDFs $f_I(x, Q^2)$ insufficient to describe MPIs
  - multiparton GPDs $F^{(n)}_{I_1 \ldots I_n}(x_1, \ldots x_n, \vec{b}_1, \ldots \vec{b}_n, Q_1^2, \ldots Q_n^2)$ required

E.g., $F^{(2)}$ for double parton scattering (production of 2 dijets)

$$\sigma_{pp}^{4\mathrm{jet(DPS)}}(s, p_{\mathrm{t}}^{\mathrm{cut}}) = \frac{1}{2} \int dx_1^+ dx_2^+ dx_1^- dx_2^- \int_{p_{\mathrm{t}_1}, p_{\mathrm{t}_2} > p_{\mathrm{t}}^{\mathrm{cut}}} dp_{\mathrm{t}_1}^2 \, dp_{\mathrm{t}_2}^2 \sum_{I_1, I_2, J_1, J_2}$$

$$\times \frac{d\sigma_{I_1 J_1}^{2\to 2}}{dp_{\mathrm{t}_1}^2} \frac{d\sigma_{I_2 J_2}^{2\to 2}}{dp_{\mathrm{t}_2}^2} \int d^2\Delta b \, F^{(2)}_{I_1 I_2}(x_1^+, x_2^+, M_{\mathrm{F}_1}^2, M_{\mathrm{F}_2}^2, \Delta b) \, F^{(2)}_{J_1 J_2}(x_1^-, x_2^-, M_{\mathrm{F}_1}^2, M_{\mathrm{F}_2}^2, \Delta b)$$



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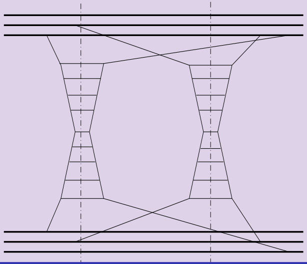
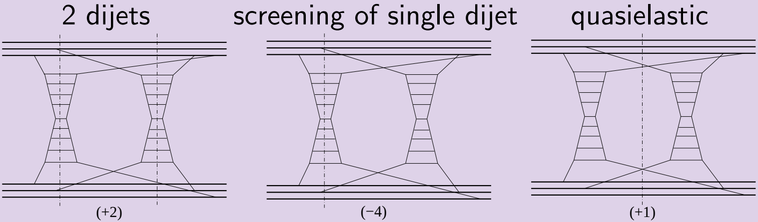
- standard simplification: neglect multiparton correlations
- $\Rightarrow F^{(n)}_{I_1\ldots I_n}(x_1,\ldots x_n,\vec{b}_1,\ldots\vec{b}_n,\ldots) = \prod_{i=1}^n G_{I_i}(x_i,\vec{b}_i,Q_i^2)$
- $\Rightarrow \sigma^{4\mathrm{jet(DPS)}}_{pp}(s,p_{\mathrm{t}}^{\mathrm{cut}}) = \frac{1}{2}\int d^2b\left[G_I\otimes\sigma^{2\to 2}_{IJ}\otimes G_J\right]^2$



Total cross section & multiple scattering

Relation to $\sigma_{pp}^{\mathrm{tot}}$ and $\sigma_{pp}^{\mathrm{inel}}$ comes from the AGK cutting rules



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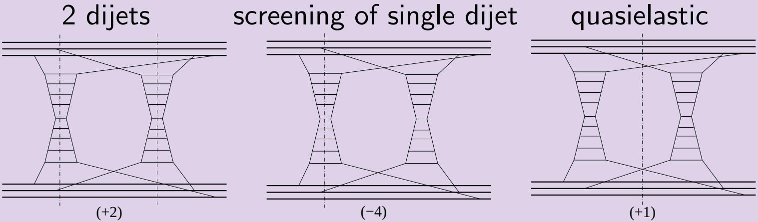

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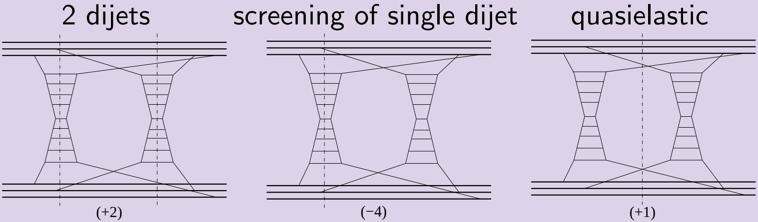

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- $\Rightarrow \Delta^{(2)}\sigma_{pp}^{\rm tot} = -\frac{1}{2}\sigma_{pp}^{\rm 4jet(DPS)}$ (similarly for $n > 2$ dijets)
- this leads to the usual 'minijet' ansatz:
  (for brevity, not discussing soft processes explicitly)
  $\sigma_{pp}^{\rm tot}(s) = 2\int d^2b \left[1 - \exp(-\chi_{pp}^{\rm jet}(s,b,p_{\rm t}^{\rm cut}))\right]$
  $\left(\ \chi_{pp}^{\rm jet}(s,b,p_{\rm t}^{\rm cut}) = \frac{1}{2}\sum_{I,J} G_I \otimes \sigma_{IJ}^{2\to 2} \otimes G_J\ \right)$

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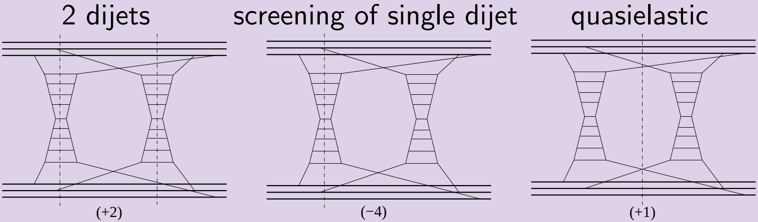

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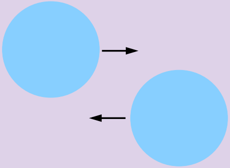
NB: inclusive jet cross section – unmodified by such MPIs

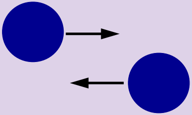
- e.g., summary contribution of the 3 processes: $2*(+2)+1*(-4)+0*(+1)=0$
- $\Rightarrow$ collinear factorization holds: $\frac{d\sigma_{pp}^{\rm jet}}{dp_t^2} = \sum_{I,J} f_I \otimes \frac{d\sigma_{IJ}^{2\to 2}}{dp_t^2} \otimes f_J$

Main message: to reduce σ_{pp}^{tot}, enhance MPIs

Simpliest way to regulate the rise of σ_{pp}^{tot}: denser parton 'packing'



- larger proton size
  $\Rightarrow$ larger σ_{pp}^{tot}
- smaller parton density
  $\Rightarrow$ smaller MPI rate



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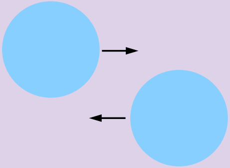

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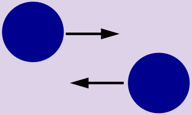

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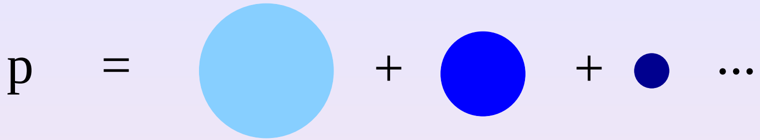
- Unfortunately, not a solution:
  proton size is constrained by data on $B_{pp}^{el}(s) \propto \langle b^2(s) \rangle$
- more generally, $d\sigma_{pp}^{el}/dt$ is related to the transverse profile of the proton (thanks to data of TOTEM & ATLAS ALFA)

Next possibility: color fluctuations in the proton



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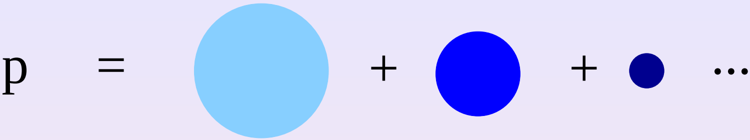

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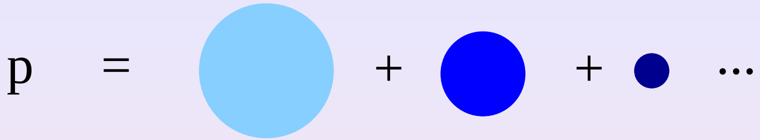

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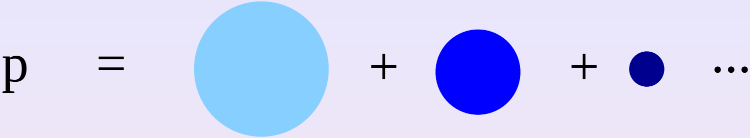

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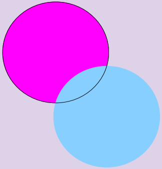
  - small size states contribute sizably to MPIs (e.g., double parton scattering $\propto$ density squared)
- $\Rightarrow$ larger dispersion between the Fock states would reduce σ_{pp}^{tot} for the same σ_{pp}^{jet}
- but: would yield a high cross section for low mass diffraction
  - NB: $\sigma_{pp}^{\text{SD(LM)}}$ – constrained by TOTEM & LHCf data

Nearly last possibility: introduce parton 'clumps'

What is wrong with the uncorrelated parton picture?

- double (multiple) hard scattering results from independent cascades
  - $\Rightarrow$ mostly in central collisions



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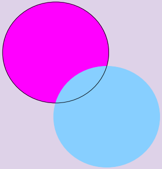
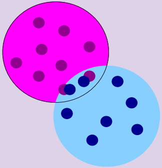


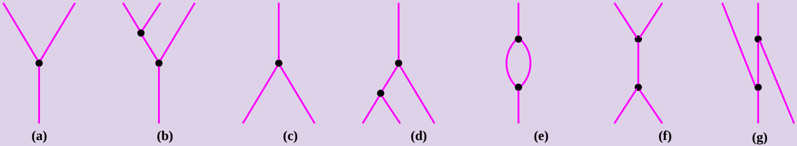
How multiparton correlations help?

- one has to create parton 'clumps' to enhance peripheral multiple scattering (without changing the transverse profile)
  - can be done via 'soft' & 'hard' parton splitting mechanisms



QGSJET-II: interactions between parton cascades $\Rightarrow$ no factorization for ${}^{(n)}$GPDs *[SO, 2006, 2010, 2011]*

Nonlinear processes: Pomeron-Pomeron interactions (scattering of intermediate partons off the proj./target hadrons & off each other)

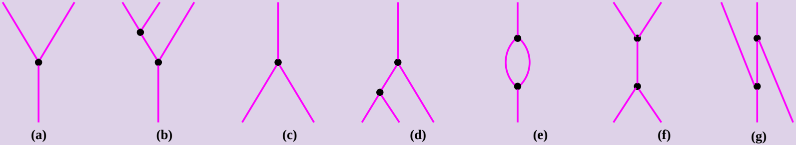


thick lines = Pomerons = 'elementary' parton cascades

- contributions resummed to all orders (sign-altering series)

QGSJET-II: interactions between parton cascades $\Rightarrow$ no factorization for $^{(n)}$GPDs *[SO, 2006, 2010, 2011]*

Nonlinear processes: Pomeron-Pomeron interactions (scattering of intermediate partons off the proj./target hadrons & off each other)



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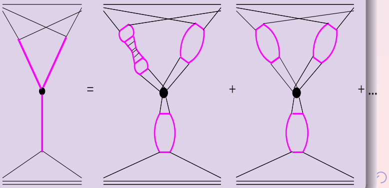
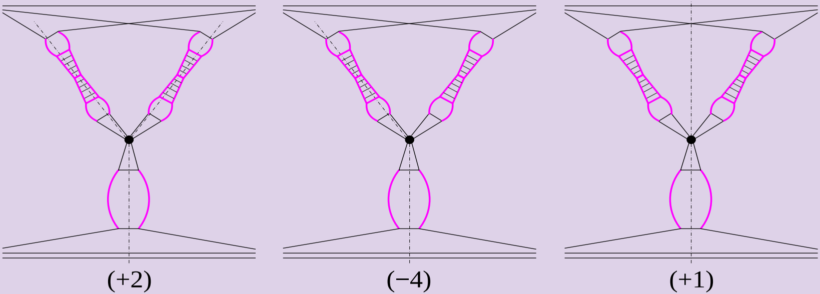
Pomeron-Pomeron interaction: a closer look

- basic assumption: multi-$\mathbb{P}$ vertices – dominated by soft ($|q^2| < Q_0^2$) parton processes
- generates parton 'clumping' *[SO & Bleicher, 2016]*



Parton 'clumping' due to 'soft parton splitting'

E.g., double dijet production from soft Pomeron splitting



- small slope for soft Pomeron ⇒ two hard processes are closeby in b-space
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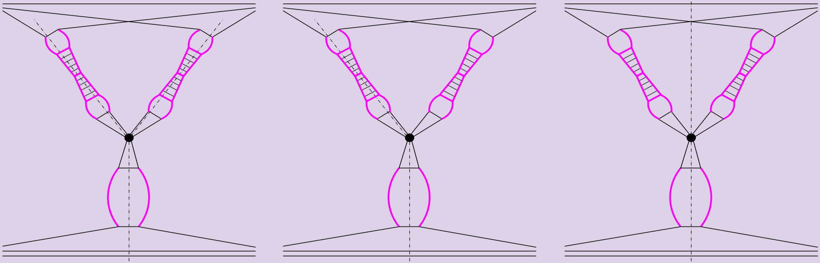

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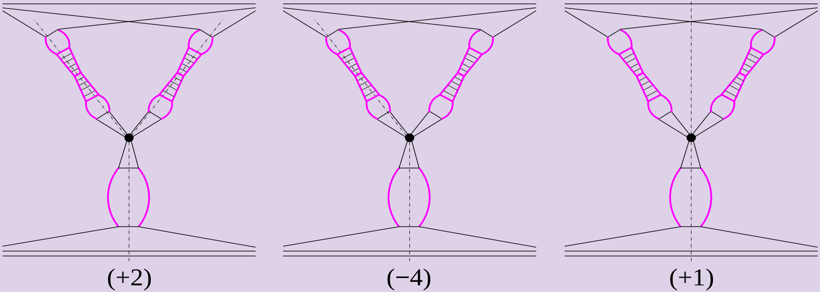

- small slope for soft Pomeron $\Rightarrow$ two hard processes are closeby in b-space
  - $\equiv$ having a parton 'clump' in the target proton
- $\Rightarrow$ enhanced MPI rate in peripheral collisions
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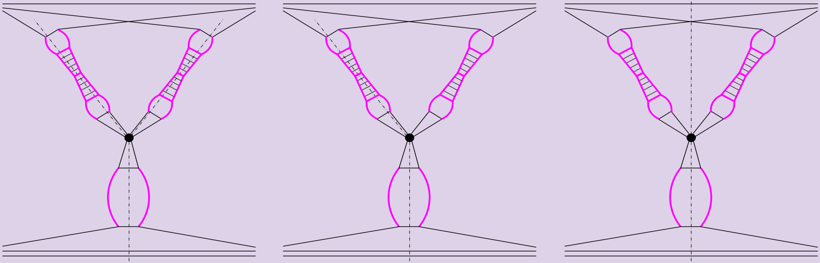

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- adding two other contributions ⇒ negative correction to $\sigma_{pp}^{\rm tot}$
- NB: no impact on inclusive jet cross section $[2*(+2)+1*(-4)+0*(+1)=0]$

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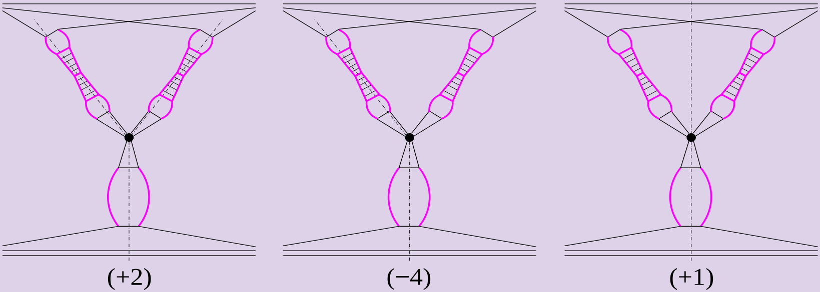

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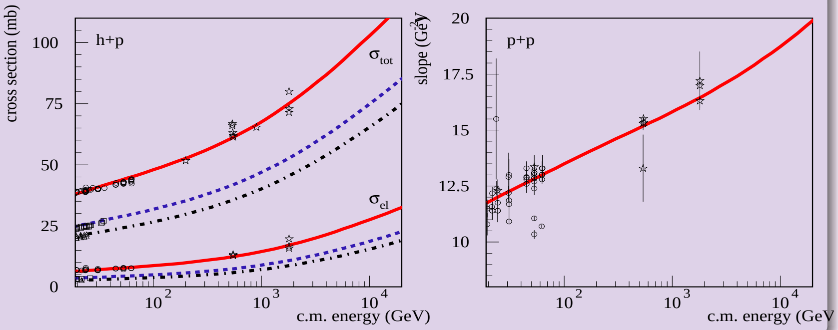
Generic property: thanks to AGK cancellations, collinear factorization holds for inclusive jet cross section

$$\frac{d\sigma_{pp}^{\rm jet}}{dp_t^2} = \sum_{I,J} f_I \otimes \frac{d\sigma_{IJ}^{2\to 2}}{dp_t^2} \otimes f_J$$

QGSJET-II-04: self-consistency seemingly reached *[SO, 2011]*

E.g., $\sqrt{s}$-dependence of $\sigma^{tot/el}_{pp/\pi p/Kp}$ for realistic transverse profiles



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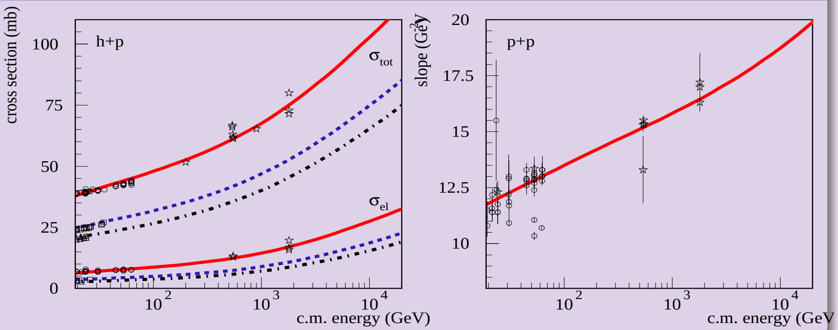
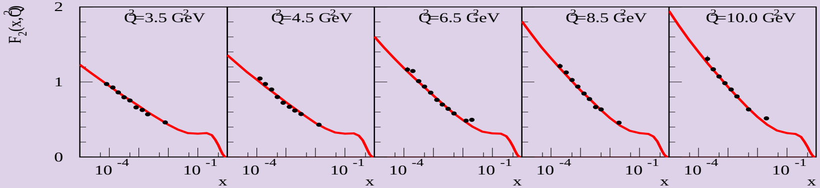


And for realistic PDFs



General major problem with Q_0-cutoff dependence

Applies to any model which respects collinear factorization

- $\sigma_{pp}^{\mathrm{jet}}(s, Q_0) \propto \frac{1}{Q_0^2}\, s^{\Delta_{\mathrm{eff}}}$, $\Delta_{\mathrm{eff}} \simeq 0.3$
  - $\Rightarrow$ $dN_{\mathrm{ch}}/d\eta|_{\eta=0} \propto 1/Q_0^2 \times s^{\Delta_{\mathrm{eff}}}$, $s \to \infty$

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  - with the factorization scale $M_{\rm F}^2 = p_{\rm t}^2/4$, yields $p_{\rm t}^{\rm cut} \simeq 3.4$ GeV

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Wanted: a perturbative mechanism to suppress low $p_{\rm t}$ jet production, without a strong impact on PDFs

Dynamical higher twist corrections as a potential solution?

Power corrections seem to fit in the demand

- can (in principle and to some extent) be treated perturbatively
- come into play at relatively small $p_{\rm t}$ (suppressed as $1/(p_{\rm t}^2)^n$)
- appeared to be significant for nuclear targets
    - hence, may be important for pp as well

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How to proceed?

- basic theoretical approach dates 40 years back *[Shuryak & Vainstein, 1981; Jaffe & Soldate, 1981; Ellis, Furmański & Petronzio, 1982]*
- contributions involve many unknown mutiparton correlators
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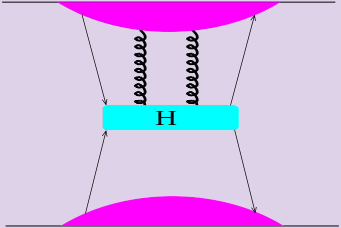
- contributions involve many unknown mutiparton correlators
- generally can't be treated probabilistically
- $\Rightarrow$ brave (wild?) assumptions may be needed

Dynamical higher twist corrections: brave assumptions

Basic assumptions (qq'-scattering as an example)

- restrict oneself with rescattering on soft ($x_g \simeq 0$) gluons
- neglect color octet contributions



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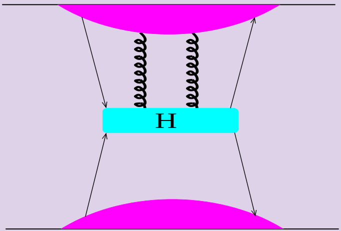

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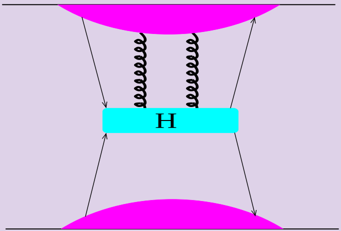

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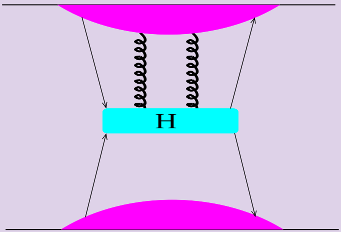

Some justifications

- dominant contributions in the small x limit usually from gluons
- soft gluon contributions proved important for p_t-broadening and suppression of SFs & jet spectra on nuclear targets *[Guo, 1998; Guo & Qiu, 2000; Qiu & Vitev, 2004, 2006]*
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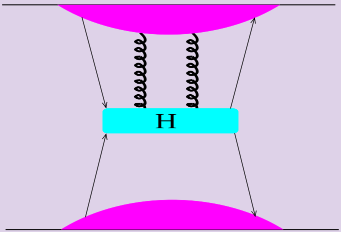

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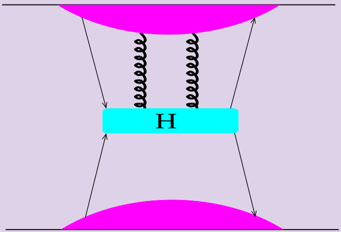

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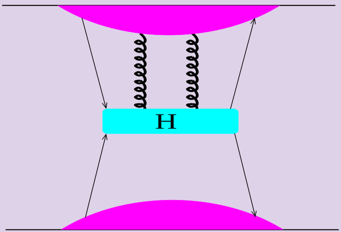

More technical (model implementation)

- describe low x GPDs by Pomeron asymptotics
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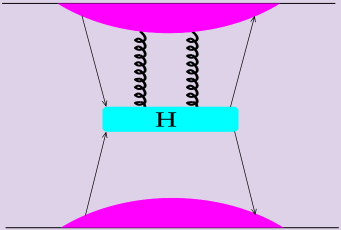

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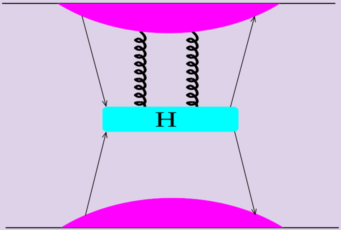

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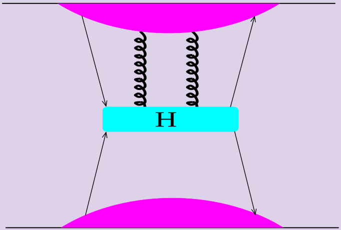

More technical (model implementation)

- describe low x GPDs by Pomeron asymptotics
- account for multiparton correlations due to the "soft splitting" mechanism
- account for absorptive corrections to GPDs due to enhanced Pomeron diagrams
- i.e., incorporate the mechanism in the Pomeron framework
    - NB: AGK rules not applicable for HT contributions

Dynamical higher twist corrections: hevristic reasoning

Consider as an example corrections to qq' scattering in LC gauge

Twist 4 contribution to the cross section:

$$\Delta\sigma_{\mathrm{HT}}(s) = \frac{1}{2s}\int \frac{d^4k_q}{(2\pi)^4}\frac{d^4k_{q'}}{(2\pi)^4}\frac{d^4k_{g_1}}{(2\pi)^4}\frac{d^4k_{g_2}}{(2\pi)^4}\, H^{\alpha\beta}_{ijkl}(k_q,k_{q'},k_{g_1},k_{g_2})$$
$$\times\left[\int d^4z_q d^4z_{g_1} d^4z_{g_2}\, e^{ik_qz_q+ik_{g_1}z_{g_1}-ik_{g_2}z_{g_2}}\langle p|\bar\psi_j(0)A_\alpha(z_{g_2})A_\beta(z_{g_1})\psi_i(z_q)|p\rangle\right]$$
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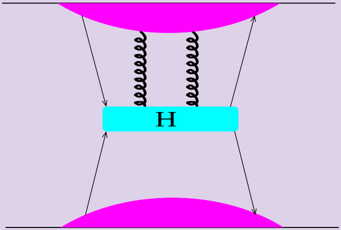

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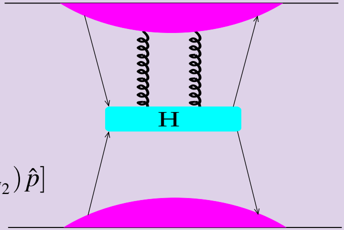
$$\times\left[\int d^4z_{q'}\, e^{ik_{q'}z_{q'}}\ \langle p|\bar\psi_l(0)\,\psi_k(z_{q'})|p\rangle\right]$$

- doing collinear factorization, one obtains *[Ellis et al., 1982; Qiu, 1990]*

$$\Delta\sigma_{\rm HT}(s) = \int dx_{q'}\,dx_q\,dx_{g_1}\,dx_{g_2}$$
$$\times\quad q(x_{q'})\,T_{qg}(x_q,x_{g_1},x_{g_2})$$
$$\times\quad \frac{1}{2s}\,d^{\perp}_{\alpha\beta}\,\mathbf{Tr}[\hat p'\,H^{\alpha\beta}(x_q,x_{q'},x_{g_1},x_{g_2})\,\hat p]$$



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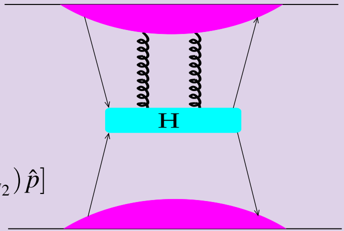

- here the quark-gluon correlation function:

$$T_{qg}(x_q, x_{g_1}, x_{g_2}) = \frac{1}{x_{g_1} x_{g_2}} \int \frac{dy_q^-}{4\pi} \frac{dy_{g_1}^-}{2\pi} \frac{dy_{g_2}^-}{2\pi}$$
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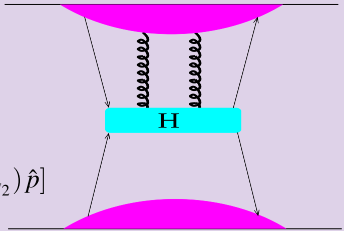

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- now: assume the integrals to be dominated by $x_{g_1}, x_{g_2} \simeq 0$
  - e.g., converting $1/x_{g_i}$ into poles & doing residies *[Guo & Qiu, 2001]*

Dynamical higher twist corrections: hevristic reasoning

Most radical assumptions

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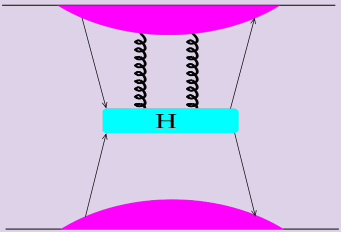

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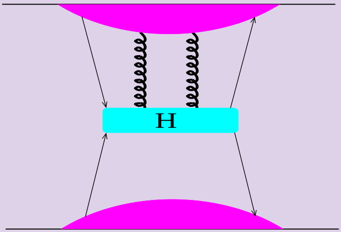

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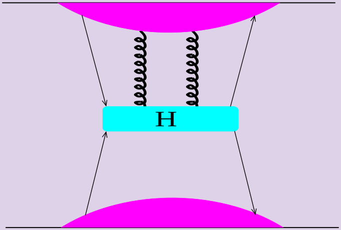

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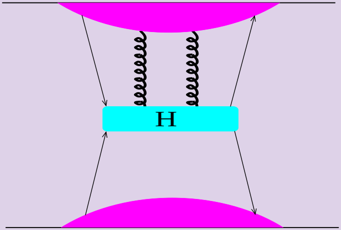

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- what about using even a smaller cutoff?
  - generally possible but would require higher order corrections (multiple exchanges of soft gluons)
  - $\Rightarrow$ additional assumptions

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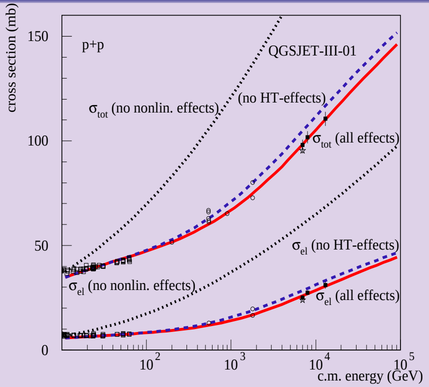

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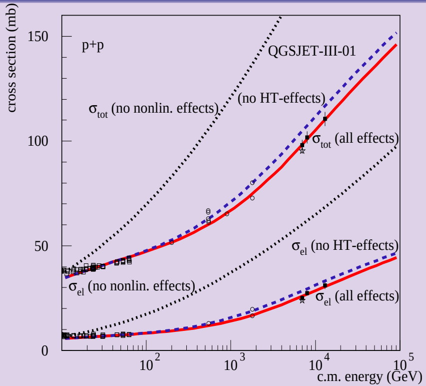

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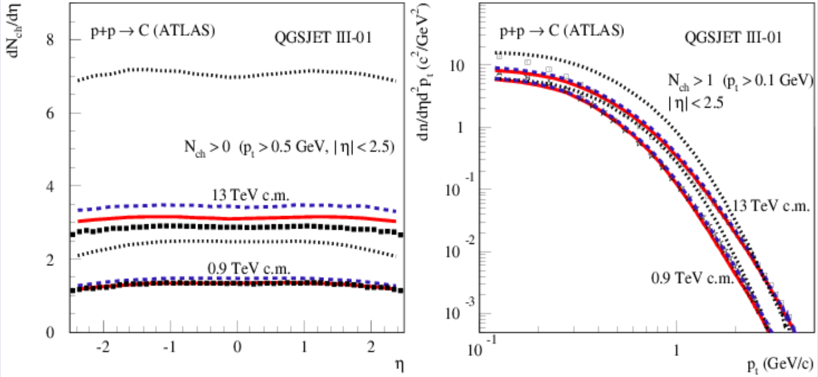
$\sqrt{s}$-dependence of $dN_{\rm ch}/d\eta$ & $dN_{\rm ch}/dp_{\rm t}$



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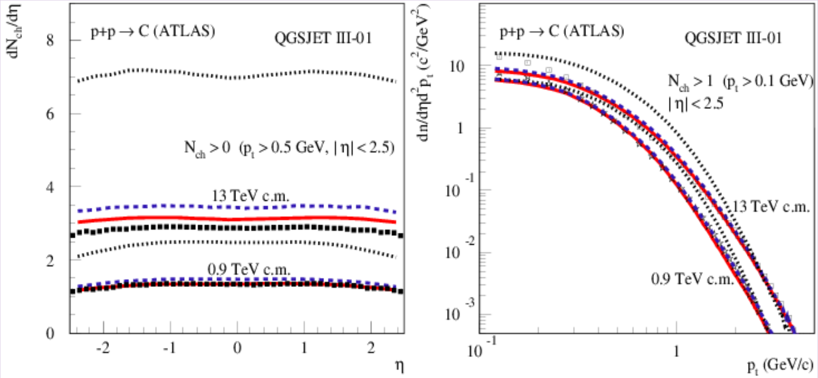

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