QGSJET(-II/III): Problems & Solutions

Sergey Ostapchenko
Frankfurt Institute for Advanced Studies

ISAPP School 2018: LHC meets Cosmic Rays
CERN, October 28 – November 02, 2018
High energy hadron scattering ⇒ copious production of (mini-)jets [e.g., Gaisser & Halzen, 1985]

⇒ standard approach in MC generators: concentrate on the “hard” scattering, add something for the “soft”
• High energy hadron scattering ⇒ copious production of (mini-)jets \[e.g., \text{Gaisser} \& \text{Halzen, 1985}\]

• ⇒ standard approach in MC generators: concentrate on the “hard” scattering, add something for the “soft”

• \textbf{QGSJET:} treat “soft” \& “hard” within the same scheme \[\text{Kalmykov, SO \& Pavlov, 1994, 1997}\]
  
  • ⇒ treat both within the Gribov’s Reggeon Field Theory (RFT)

  • starting point: Quark-Gluon String model \[\text{Kaidalov \& Ter-Martyrosyan, 1982, 1984}\]
High energy hadron scattering ⇒ copious production of (mini-)jets \cite[e.g.,][]{GaisserHalzen1985}

⇒ standard approach in MC generators: concentrate on the “hard” scattering, add something for the “soft”

QGSJET: treat “soft” & “hard” within the same scheme \cite[Kalmykov, SO \& Pavlov, 1994, 1997]

⇒ treat both within the Gribov’s Reggeon Field Theory (RFT)

starting point: Quark-Gluon String model \cite[Kaidalov \& Ter-Martyrosyan, 1982, 1984]
Quark-Gluon String model: RFT-based treatment

- High energy interactions ⇒ multiple scattering
  - elementary interaction = parton cascade ≡ Pomeron exchange
Quark-Gluon String model: RFT-based treatment

High energy interactions $\Rightarrow$ multiple scattering

- elementary interaction $=$ parton cascade $\equiv$ Pomeron exchange

Pomeron amplitude:

$$ f_{ab}^{\mathcal{P}}(s,b) = \frac{i\gamma_a\gamma_b s^{\alpha_{\mathcal{P}}(0)-1}}{\lambda_{ab}(s)} e^{-\frac{b^2}{4\lambda_{ab}(s)}} $$

$$ \lambda_{ab}(s) = R_a^2 + R_b^2 + \alpha_{\mathcal{P}}'(0) \ln s $$

- Pomeron intercept $\alpha_{\mathcal{P}}(0) > 1 \Rightarrow$ energy-rise of multiple scattering
- Pomeron slope $\alpha_{\mathcal{P}}'(0) > 0 \Rightarrow$ energy-rise of the interaction radius (transverse diffusion)
Quark-Gluon String model: RFT-based treatment

High energy interactions $\Rightarrow$ multiple scattering

- elementary interaction $=$ parton cascade $\equiv$ Pomeron exchange

- Pomeron amplitude:

$$f_{ab}^P(s, b) = \frac{i\gamma_a\gamma_b s^{\alpha_P(0)-1}}{\lambda_{ab}(s)} e^{-\frac{b^2}{4\lambda_{ab}(s)}}$$

$$\lambda_{ab}(s) = R_a^2 + R_b^2 + \alpha'_P(0) \ln s$$

- Pomeron intercept $\alpha_P(0) > 1 \Rightarrow$ energy-rise of multiple scattering

- Pomeron slope $\alpha'_P(0) > 0 \Rightarrow$ energy-rise of the interaction radius (transverse diffusion)

AGK cutting rules $\Rightarrow$ partial cross sections for various final states

- “cut” Pomerons $\Rightarrow$ string formation & hadronization
Quark-Gluon String model: RFT-based treatment

- experimental data on $\sigma_{hp}^{\text{tot}}, \sigma_{hp}^{\text{el}}, \sigma_{hp}^{\text{difr}}, B_{hp}^{\text{el}}$
  $\Rightarrow$ fixing Pomeron parameters

- experimental data on particle production
  $\Rightarrow$ fixing hadronization parameters

- generalization to $hA, AA$ – parameter free
Quark-Gluon String model: RFT-based treatment

- Experimental data on $\sigma_{hp}^{\text{tot}}$, $\sigma_{hp}^{\text{el}}$, $\sigma_{hp}^{\text{difr}}$, $B_{hp}^{\text{el}}$
  $\Rightarrow$ fixing Pomeron parameters

- Experimental data on particle production
  $\Rightarrow$ fixing hadronization parameters

- Generalization to $hA$, $AA$ – parameter free

NB: Original RFT approach - for soft (low $p_t$) physics
- High energy behavior governed by the Pomeron intercept $\alpha_{\text{P}}(0)$
Robust pQCD prediction: rising contribution of high $p_t$ partons

- small $\alpha_s(p_t^2)$ compensated by low-$x$ rise of PDFs $f_i(x, p_t^2)$
- generated by soft and collinear logs: $\ln \frac{x_n}{x_{n+1}}$, $\ln \frac{p_{t,n+1}^2}{p_{t,n}^2}$
Robust pQCD prediction: rising contribution of high $p_t$ partons

- small $\alpha_s(p_t^2)$ compensated by low-$x$ rise of PDFs $f_I(x, p_t^2)$
- generated by soft and collinear logs: $\ln\frac{x_n}{x_{n+1}}$, $\ln\frac{p_{t_{n+1}}^2}{p_{t_n}^2}$

Phenomenological 'semihard Pomeron' approach

- $Q_0^2$ - technical parameter (cutoff between soft & hard physics)

  - $|p_t^2| < Q_0^2 \Rightarrow$ soft Pomeron
  - $|p_t^2| > Q_0^2 \Rightarrow$ DGLAP ladder

$Q_2^0$ - technical parameter (cutoff between soft & hard physics)
QGSJET: (semi-)hard processes within the RFT framework

Phenomenological 'semihard Pomeron' approach


- $Q_0^2$ - technical parameter (cutoff between soft & hard physics)

- $|p_t^2| < Q_0^2 \Rightarrow$ soft Pomeron
- $|p_t^2| > Q_0^2 \Rightarrow$ DGLAP ladder

RFT scheme based on a "general Pomeron" (soft + semihard)

- "semihard Pomeron" combines features of soft & hard ones
  - steep energy-rise: $\propto s^{\Delta_{\text{hard}}}$
  - transverse expansion governed by the soft Pomeron slope
- $\Rightarrow$ jet production extends to peripheral collisions
QGSJET: successful predictions for extensive air showers

Success aposteriori explained by LHC data, e.g. on $\sigma_{pp}^{\text{tot/el}}$

[from R. Engel, talk at “ICRC-2013”]
QGSJET: successful predictions for extensive air showers

Central production equally well-described by QGSJET

[plots from d’Enterria et al., Astrop. Phys. 35 (2011) 98]
QGSJET: successful predictions for extensive air showers

But: the model was based on flat (pre-HERA) PDFs

- steep low-$x$ increase of gluon density $\Rightarrow$ too fast energy rise of $\sigma_{pp}^{\text{tot}}(s), N_{pp}^{\text{ch}}(s)$
Total cross section & jet production

- Inclusive cross section for production jets of $p_t > p_t^{\text{cut}}$:

$$
\sigma_{pp}^{\text{jet}}(s, p_t^{\text{cut}}) = \sum_{I,J=q,\bar{q},g} \int_{p_t > p_t^{\text{cut}}} dp_t^2 \int dx^+ dx^- f_{I/p}(x^+, M_F^2)
\times \frac{d\sigma_{IJ}^{2\rightarrow2}(x^+ x^- s, p_t^2, M_F^2)}{dp_t^2} f_{J/p}(x^-, M_F^2)
$$

- Problem: $\sigma_{pp}^{\text{jet}}$ rises quicker than $\sigma_{pp}^{\text{tot}}$ as $s \to \infty$

$$
\sigma_{pp}^{\text{jet}}(s, Q_0) \propto \frac{1}{Q_0^2} s^{\Delta_{\text{eff}}}, \quad \Delta_{\text{eff}} \approx 0.3
$$

$$
\sigma_{pp}^{\text{tot}}(s) \propto \ln^2 s
$$
Total cross section & jet production

- Inclusive cross section for production jets of $p_t > p_t^{\text{cut}}$:

$$\sigma_{pp}(s, p_t^{\text{cut}}) = \sum_{I,J=q,\bar{q},g} \int_{p_t>p_t^{\text{cut}}} dp_t^2 \int dx^+ dx^- f_{I/p}(x^+, M_F^2)$$

$$\times \frac{d\sigma_{IJ}^{2\rightarrow2}(x^+ x^- s, p_t^2, M_F^2)}{dp_t^2} f_{J/p}(x^-, M_F^2)$$

- Problem: $\sigma_{\text{jet}}^{pp}$ rises quicker than $\sigma_{\text{tot}}^{pp}$ as $s \rightarrow \infty$

  - $\sigma_{\text{jet}}^{pp}(s, Q_0) \propto \frac{1}{Q_0^2} s^{\Delta_{\text{eff}}}$, $\Delta_{\text{eff}} \approx 0.3$

  - $\sigma_{\text{tot}}^{pp}(s) \propto \ln^2 s$

  $\Rightarrow$ multiple jet production required

  - = multiparton interactions (MPIs)
MPIs & generalized parton distributions (GPDs)

- Usual PDFs $f_I(x, Q^2)$ insufficient to describe MPIs
- Multiparton GPDs $F_{I_1...I_n}^{(n)}(x_1,...x_n, \vec{b}_1,...\vec{b}_n, Q_1^2,...Q_n^2)$ required

E.g., $F^{(2)}$ for double parton scattering (production of 2 dijets)

$$\sigma_{4 \text{jet}(\text{DPS})}^{pp}(s, p_{t \text{cut}}^\text{cut}) = \frac{1}{2} \int dx_1^+ dx_2^+ dx_1^- dx_2^- \int_{p_{t_1}, p_{t_2} > p_{t \text{cut}}} dp_{t_1}^2 dp_{t_2}^2 \sum_{I_1, I_2, J_1, J_2} d\sigma_{I_1 J_1}^{2 \to 2} \frac{dp_{t_1}^2}{dp_{t_1}^2} \frac{dp_{t_2}^2}{dp_{t_2}^2} \int d^2\Delta b F_{I_1 I_2}^{(2)}(x_1^+, x_2^+, M_{F_1}^2, M_{F_2}^2, \Delta b) F_{J_1 J_2}^{(2)}(x_1^-, x_2^-, M_{F_1}^2, M_{F_2}^2, \Delta b)$$
MPIs & generalized parton distributions (GPDs)

- Usual PDFs $f_I(x, Q^2)$ insufficient to describe MPIs
- multiparton GPDs $F_{I_1...I_n}^{(n)}(x_1, ... x_n, \vec{b}_1, ... \vec{b}_n, Q_1^2, ... Q_n^2)$ required

E.g., $F^{(2)}$ for double parton scattering (production of 2 dijets)

\[
\sigma_{pp}^{4\text{jet(DPS)}}(s, p_t^{\text{cut}}) = \frac{1}{2} \int dx_1^+ dx_2^+ dx_1^- dx_2^- \int_{p_{t1},p_{t2} > p_t^{\text{cut}}} dp_{t1}^2 dp_{t2}^2 \sum_{I_1,I_2,J_1,J_2} d\sigma^{2\rightarrow 2}_{I_1J_1} d\sigma^{2\rightarrow 2}_{I_2J_2} \int d^2\Delta b F^{(2)}_{I_1I_2}(x_1^+, x_2^+, M_{F_1}^2, M_{F_2}^2, \Delta b) F^{(2)}_{J_1J_2}(x_1^-, x_2^-, M_{F_1}^2, M_{F_2}^2, \Delta b)
\]

- standard simplification:
  - neglect multiparton correlations

\[
\Rightarrow F_{I_1...I_n}^{(n)}(x_1, ... x_n, \vec{b}_1, ... \vec{b}_n, ...) = \prod_{i=1}^{n} G_{I_i}(x_i, \vec{b}_i, Q_i^2)
\]

\[
\Rightarrow \sigma_{pp}^{4\text{jet(DPS)}}(s, p_t^{\text{cut}}) = \frac{1}{2} \int d^2b \left[ G_I \otimes \sigma_{IJ}^{2\rightarrow 2} \otimes G_J \right]^2
\]
Total cross section & multiple scattering

Relation to $\sigma_{pp}^{tot}$ and $\sigma_{pp}^{inel}$ comes from the AGK cutting rules

<table>
<thead>
<tr>
<th>2 dijets</th>
<th>screening of single dijet</th>
<th>quasielastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(+1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- partial contributions of the 3 processes are related as $(+2):(-4):(+1)$
Relation to $\sigma_{pp}^{\text{tot}}$ and $\sigma_{pp}^{\text{inel}}$ comes from the AGK cutting rules

- partial contributions of the 3 processes are related as $(+2):(-4):(+1)$
- $\Rightarrow \Delta^{(2)} \sigma_{pp}^{\text{tot}} = -\frac{1}{2} \sigma_{pp}^{4\text{jet}(\text{DPS})}$ (similarly for $n > 2$ dijets)
Total cross section & multiple scattering

Relation to $\sigma_{pp}^{tot}$ and $\sigma_{pp}^{inel}$ comes from the AGK cutting rules

2 dijets screening of single dijet quasielastic

partial contributions of the 3 processes are related as
(+2):(-4):(+1)

$\Rightarrow \Delta^{(2)} \sigma_{pp}^{tot} = -\frac{1}{2} \sigma_{pp}^{4jet(DPS)}$ (similarly for $n > 2$ dijets)

this leads to the usual 'minijet' ansatz:
(for brevity, not discussing soft processes explicitly)

$\sigma_{pp}^{tot}(s) = 2 \int d^2 b \left[ 1 - \exp(-\chi_{pp}^{jet}(s, b, p_t^{cut})) \right]$

( $\chi_{pp}^{jet}(s, b, p_t^{cut}) = \frac{1}{2} \sum_{I,J} G_I \otimes \sigma_{IJ}^{2 \to 2} \otimes G_J$ )
Total cross section & multiple scattering

Relation to $\sigma_{pp}^{tot}$ and $\sigma_{pp}^{inel}$ comes from the AGK cutting rules

- partial contributions of the 3 processes are related as $(+2):(-4):(+1)$

  $\Rightarrow \Delta^{(2)} \sigma_{pp}^{tot} = -\frac{1}{2} \sigma_{pp}^{4jet(DPS)}$ (similarly for $n > 2$ dijets)

NB: inclusive jet cross section – unmodified by such MPIs

- e.g., summary contribution of the 3 processes:
  $2 \times (+2) + 1 \times (-4) + 0 \times (+1) = 0$

  $\Rightarrow$ collinear factorization holds: $\frac{d\sigma_{pp}^{jet}}{dp_t^2} = \sum_{I,J} f_I \otimes \frac{d\sigma_{II}^{2\rightarrow 2}}{dp_t^2} \otimes f_J$
Main message: to reduce $\sigma_{pp}^{\text{tot}}$, enhance MPIs

Simpliest way to regulate the rise of $\sigma_{pp}^{\text{tot}}$: denser parton 'packing'

- larger proton size $\Rightarrow$ larger $\sigma_{pp}^{\text{tot}}$
- smaller parton density $\Rightarrow$ smaller MPI rate

- smaller proton size $\Rightarrow$ smaller $\sigma_{pp}^{\text{tot}}$
- larger parton density $\Rightarrow$ larger MPI rate
Main message: to reduce $\sigma_{pp}^{\text{tot}}$, enhance MPIs

Simpliest way to regulate the rise of $\sigma_{pp}^{\text{tot}}$: denser parton 'packing'

- Larger proton size $\Rightarrow$ larger $\sigma_{pp}^{\text{tot}}$
- Smaller parton density $\Rightarrow$ smaller MPI rate
- Smaller proton size $\Rightarrow$ smaller $\sigma_{pp}^{\text{tot}}$
- Larger parton density $\Rightarrow$ larger MPI rate

Unfortunately, not a solution:
proton size is constrained by data on $B_{pp}^{\text{el}}(s) \propto \langle b^2(s) \rangle$

More generally, $d\sigma_{pp}^{\text{el}}/dt$ is related to the transverse profile of the proton (thanks to data of TOTEM & ATLAS ALFA)
Next possibility: color fluctuations in the proton

\[ p = |p\rangle = \sum_i \sqrt{C_i} |i\rangle \]

Generally, proton is a superposition of different parton Fock states (of different size & parton density): \[ |p\rangle = \sum_i \sqrt{C_i} |i\rangle \]
Next possibility: color fluctuations in the proton

\[ p = \sum \sqrt{C_i} |i\rangle \]

Generally, proton is a superposition of different parton Fock states (of different size & parton density): \(|p\rangle = \sum_i \sqrt{C_i} |i\rangle\)

- Larger size states dominate \(\sigma_{pp}^{\text{tot}}\)
- Small size states contribute sizably to MPIs (e.g., double parton scattering \(\propto\) density squared)
Next possibility: color fluctuations in the proton

\[ p = \sum_i \sqrt{C_i} |i\rangle \]

- Generally, proton is a superposition of different parton Fock states (of different size & parton density): \[ |p\rangle = \sum_i \sqrt{C_i} |i\rangle \]
  - larger size states dominate \( \sigma_{pp}^{\text{tot}} \)
  - small size states contribute sizably to MPIs
    (e.g., double parton scattering \( \propto \) density squared)
Next possibility: color fluctuations in the proton

\[ p = \begin{align*} 
  &\quad + \quad + \quad \ldots 
\end{align*}\]

Generally, proton is a superposition of different parton Fock states (of different size & parton density):

\[ |p\rangle = \sum_i \sqrt{C_i} |i\rangle \]

- larger size states dominate \( \sigma_{pp}^{\text{tot}} \)
- small size states contribute sizably to MPIs
  (e.g., double parton scattering \( \propto \) density squared)

\[ \Rightarrow \text{larger dispersion between the Fock states would reduce } \sigma_{pp}^{\text{tot}} \]

for the same \( \sigma_{pp}^{\text{jet}} \)
Next possibility: color fluctuations in the proton

\[ p = \text{larger size states dominate } \sigma_{\text{tot}}^{pp} \]
\[ \text{small size states contribute sizably to MPIs (e.g., double parton scattering } \propto \text{ density squared)} \]
\[ \Rightarrow \text{larger dispersion between the Fock states would reduce } \sigma_{\text{tot}}^{pp} \]
\[ \text{for the same } \sigma_{\text{jet}}^{pp} \]

\[ \text{but: would yield a high cross section for low mass diffraction} \]

\[ \text{NB: } \sigma_{\text{pp}}^{\text{SD(LM)}} \text{ - constrained by TOTEM & LHCf data} \]
Nearly last possibility: introduce parton 'clumps'

What is wrong with the uncorrelated parton picture?

- double (multiple) hard scattering results from independent cascades
  - ⇒ mostly in central collisions
Nearly last possibility: introduce parton 'clumps'

What is wrong with the uncorrelated parton picture?
- double (multiple) hard scattering results from independent cascades
  - ⇒ mostly in central collisions

How multiparton correlations help?
- one has to create parton 'clumps' to enhance peripheral multiple scattering (without changing the transverse profile)
  - can be done via 'soft' & 'hard' parton splitting mechanisms
QGSJET-II: interactions between parton cascades
⇒ no factorization for \( (n)^{\text{GPDs}} \) [SO, 2006, 2010, 2011]

Nonlinear processes: Pomeron-Pomeron interactions (scattering of intermediate partons off the proj./target hadrons & off each other)

(a) (b) (c) (d) (e) (f) (g)

thick lines = Pomerons = 'elementary' parton cascades

contributions resummed to all orders (sign-altering series)
QGSJET-II: interactions between parton cascades

⇒ no factorization for \((n)\)GPDs [SO, 2006, 2010, 2011]

Nonlinear processes: Pomeron-Pomeron interactions (scattering of intermediate partons off the proj./target hadrons & off each other)

thick lines = Pomerons = ’elementary’ parton cascades

• contributions resummed to all orders (sign-altering series)

Pomeron-Pomeron interaction: a closer look

• basic assumption: multi-\(\mathbb{P}\)
  vertices – dominated by soft \((|q^2| < Q_0^2)\) parton processes

• generates parton ’clumping’
  [SO & Bleicher, 2016]
Parton 'clumping' due to 'soft parton splitting'

E.g., double dijet production from soft Pomeron splitting

- small slope for soft Pomeron $\Rightarrow$ two hard processes are closeby in $b$-space
  - $\equiv$ having a parton 'clump' in the target proton
Parton 'clumping' due to 'soft parton splitting'

E.g., double dijet production from soft Pomeron splitting

\[ +1 \] \[ -4 \] \[ +2 \]

- small slope for soft Pomeron \( \Rightarrow \) two hard processes are closeby in \( b \)-space
  - \( \equiv \) having a parton 'clump' in the target proton

\( \Rightarrow \) enhanced MPI rate in peripheral collisions
Parton 'clumping' due to 'soft parton splitting' 

E.g., double dijet production from soft Pomeron splitting

- small slope for soft Pomeron $\Rightarrow$ two hard processes are closeby in $b$-space
  - $\equiv$ having a parton 'clump' in the target proton
- $\Rightarrow$ enhanced MPI rate in peripheral collisions
- adding two other contributions $\Rightarrow$ negative correction to $\sigma_{pp}^{tot}$
Parton 'clumping' due to 'soft parton splitting'

E.g., double dijet production from soft Pomeron splitting

- small slope for soft Pomeron $\Rightarrow$ two hard processes are closeby in $b$-space
  - $\equiv$ having a parton 'clump' in the target proton
- $\Rightarrow$ enhanced MPI rate in peripheral collisions
- adding two other contributions $\Rightarrow$ negative correction to $\sigma_{pp}^{\text{tot}}$
- NB: no impact on inclusive jet cross section

$[2 \times (+2) + 1 \times (-4) + 0 \times (+1) = 0]$
Parton 'clumping' due to 'soft parton splitting'

E.g., double dijet production from soft Pomeron splitting

- small slope for soft Pomeron $\Rightarrow$ two hard processes are closeby in $b$-space
  - $\equiv$ having a parton 'clump' in the target proton

Generic property: thanks to AGK cancellations, collinear factorization holds for inclusive jet cross section

$$\frac{d\sigma_{pp}^{\text{jet}}}{dp_t^2} = \sum_{I,J} f_I \otimes \frac{d\sigma_{IJ}^{2\rightarrow2}}{dp_t^2} \otimes f_J$$
E.g., $\sqrt{s}$-dependence of $\frac{\sigma_{\text{tot}}}{\sigma_{\text{el}}}$ for realistic transverse profiles
QGSJET-II-04: self-consistency seemingly reached [SO, 2011]

E.g., $\sqrt{s}$-dependence of $\sigma_{\text{tot/el}}^{\text{pp/\pi p/Kp}}$ for realistic transverse profiles

And for realistic PDFs
General major problem with $Q_0$-cutoff dependence

Applies to any model which respects collinear factorization

$$\sigma_{pp}^\text{jet}(s, Q_0) \propto \frac{1}{Q_0^2} s^{\Delta_{\text{eff}}}, \quad \Delta_{\text{eff}} \approx 0.3$$

$$\Rightarrow \quad dN_{\text{ch}}/d\eta|_{\eta=0} \propto 1/Q_0^2 \times s^{\Delta_{\text{eff}}}, \quad s \to \infty$$
General major problem with $Q_0$-cutoff dependence

Applies to any model which respects collinear factorization

- $\sigma_{pp}^{\text{jet}}(s, Q_0) \propto \frac{1}{Q_0^2} s^{\Delta_{\text{eff}}}, \Delta_{\text{eff}} \simeq 0.3$
- $\Rightarrow dN_{\text{ch}}/d\eta|_{\eta=0} \propto 1/Q_0^2 \times s^{\Delta_{\text{eff}}}, s \to \infty$

- the normalization depends on the chosen $p_t$-cutoff
  - in QGSJET-II-04, a rather large value ($3 \text{ GeV}^2$) is used
  - with the factorization scale $M_F^2 = p_t^2/4$, yields $p_t^{\text{cut}} \simeq 3.4 \text{ GeV}$
General major problem with $Q_0$-cutoff dependence

Applies to any model which respects collinear factorization

- $\sigma_{pp}(s, Q_0) \propto \frac{1}{Q_0^2} s^{\Delta_{\text{eff}}}$, $\Delta_{\text{eff}} \simeq 0.3$
  - $\Rightarrow dN_{\text{ch}}/d\eta|_{\eta=0} \propto 1/Q_0^2 \times s^{\Delta_{\text{eff}}}$, $s \to \infty$

- the normalization depends on the chosen $p_t$-cutoff
  - in QGSJET-II-04, a rather large value (3 GeV$^2$) is used
    - with the factorization scale $M_F^2 = p_t^2/4$, yields $p_t^{\text{cut}} \simeq 3.4$ GeV

- ideally, $p_t$-cutoff should be just a technical parameter, without a strong impact on the results
General major problem with $Q_0$-cutoff dependence

Applies to any model which respects collinear factorization

$\sigma_{pp}^{\text{jet}}(s, Q_0) \propto \frac{1}{Q_0^2} s^{\Delta_{\text{eff}}}, \Delta_{\text{eff}} \simeq 0.3$

$\Rightarrow \frac{dN_{\text{ch}}}{d\eta}|_{\eta=0} \propto \frac{1}{Q_0^2} \times s^{\Delta_{\text{eff}}}, s \to \infty$

the normalization depends on the chosen $p_t$-cutoff

in QGSJET-II-04, a rather large value (3 GeV$^2$) is used

with the factorization scale $M_F^2 = p_t^2/4$, yields $p_t^{\text{cut}} \simeq 3.4$ GeV

ideally, $p_t$-cutoff should be just a technical parameter, without a strong impact on the results

$\Rightarrow$ some important perturbative mechanism seems missing
General major problem with $Q_0$-cutoff dependence

 Applies to any model which respects collinear factorization

\[ \sigma_{pp}(s, Q_0) \propto \frac{1}{Q_0^2} s^{\Delta_{\text{eff}}}, \quad \Delta_{\text{eff}} \simeq 0.3 \]

\[ \Rightarrow \frac{dN_{\text{ch}}}{d\eta}|_{\eta=0} \propto 1/Q_0^2 \times s^{\Delta_{\text{eff}}}, \quad s \to \infty \]

the normalization depends on the chosen $p_t$-cutoff

- in QGSJET-II-04, a rather large value (3 GeV$^2$) is used
- with the factorization scale $M_F^2 = p_t^2/4$, yields $p_t^{\text{cut}} \simeq 3.4$ GeV

ideally, $p_t$-cutoff should be just a technical parameter, without a strong impact on the results

\[ \Rightarrow \text{some important perturbative mechanism seems missing} \]

Wanted: a perturbative mechanism to suppress low $p_t$ jet production, without a strong impact on PDFs
Dynamical higher twist corrections as a potential solution?

Power corrections seem to fit in the demand

- can (in principle and to some extent) be treated perturbatively
- come into play at relatively small $p_t$ (suppressed as $1/(p_t^2)^n$)
- appeared to be significant for nuclear targets
  - hence, may be important for $pp$ as well
Dynamical higher twist corrections as a potential solution?

Power corrections seem to fit in the demand

- can (in principle and to some extent) be treated perturbatively
- come into play at relatively small $p_t$ (suppressed as $1/(p_t^2)^n$)
- appeared to be significant for nuclear targets
  - hence, may be important for $pp$ as well
Power corrections seem to fit in the demand

- can (in principle and to some extent) be treated perturbatively
- come into play at relatively small $p_t$ (suppressed as $1/(p_t^2)^n$)
- appeared to be significant for nuclear targets
  - hence, may be important for $pp$ as well
Power corrections seem to fit in the demand

- can (in principle and to some extent) be treated perturbatively
- come into play at relatively small $p_t$ (suppressed as $1/(p_t^2)^n$)
- appeared to be significant for nuclear targets
  - hence, may be important for $pp$ as well

How to proceed?

- basic theoretical approach dates 40 years back [Shuryak & Vainstein, 1981; Jaffe & Soldate, 1981; Ellis, Furmański & Petronzio, 1982]
- contributions involve many unknown multiparton correlators
- generally can’t be treated probabilistically
Dynamical higher twist corrections as a potential solution?

Power corrections seem to fit in the demand
- can (in principle and to some extent) be treated perturbatively
- come into play at relatively small $p_t$ (suppressed as $1/(p_t^2)^n$)
- appeared to be significant for nuclear targets
  - hence, may be important for $pp$ as well

How to proceed?
- basic theoretical approach dates 40 years back [*Shuryak & Vainstein, 1981; Jaffe & Soldate, 1981; Ellis, Furmański & Petronzio, 1982*]
- contributions involve many unknown multiparton correlators
- generally can’t be treated probabilistically
Dynamical higher twist corrections as a potential solution?

Power corrections seem to fit in the demand
- can (in principle and to some extent) be treated perturbatively
- come into play at relatively small $p_t$ (suppressed as $1/(p_t^2)^n$)
- appeared to be significant for nuclear targets
  - hence, may be important for $pp$ as well

How to proceed?
- basic theoretical approach dates 40 years back [Shuryak & Vainstein, 1981; Jaffe & Soldate, 1981; Ellis, Furmański & Petronzio, 1982]
- contributions involve many unknown multiparton correlators
- generally can’t be treated probabilistically
Power corrections seem to fit in the demand

- can (in principle and to some extent) be treated perturbatively
- come into play at relatively small $p_t$ (suppressed as $1/(p_t^2)^n$)
- appeared to be significant for nuclear targets
  - hence, may be important for $pp$ as well

How to proceed?

- basic theoretical approach dates 40 years back [Shuryak & Vainstein, 1981; Jaffe & Soldate, 1981; Ellis, Furmański & Petronzio, 1982]
- contributions involve many unknown multiparton correlators
- generally can’t be treated probabilistically
  - ⇒ brave (wild?) assumptions may be needed
Basic assumptions ($qq'$-scattering as an example)

- restrict oneself with rescattering on soft ($x_g \simeq 0$) gluons
- neglect color octet contributions
Dynamical higher twist corrections: brave assumptions

Basic assumptions ($qq'$-scattering as an example)

- restrict oneself with rescattering on soft ($x_g \simeq 0$) gluons
- neglect color octet contributions
Basic assumptions ($qq'$-scattering as an example)

- restrict oneself with rescattering on soft ($x_g \sim 0$) gluons
- neglect color octet contributions
- interprete the respective correlators as GPDs
Dynamical higher twist corrections: brave assumptions

Basic assumptions ($qq'$-scattering as an example)

- restrict oneself with rescattering on soft ($x_g \approx 0$) gluons
- neglect color octet contributions
- interprete the respective correlators as GPDs

Some justifications

- dominant contributions in the small $x$ limit usually from gluons
- soft gluon contributions proved important for $p_t$-broadening and suppression of SFs & jet spectra on nuclear targets
  
- Pomeron exchange (color singlet) dominates at large $s$
Basic assumptions (qq'-scattering as an example)

- restrict oneself with rescattering on soft \(x_g \sim 0\) gluons
- neglect color octet contributions
- interpret the respective correlators as GPDs

Some justifications

- dominant contributions in the small \(x\) limit usually from gluons
- soft gluon contributions proved important for \(p_t\)-broadening and suppression of SFs & jet spectra on nuclear targets
  
- Pomeron exchange (color singlet) dominates at large \(s\)
Dynamical higher twist corrections: brave assumptions

Basic assumptions ($qq'$-scattering as an example)

- restrict oneself with rescattering on soft ($x_g \approx 0$) gluons
- neglect color octet contributions
- interprete the respective correlators as GPDs

Some justifications

- dominant contributions in the small $x$ limit usually from gluons
- soft gluon contributions proved important for $p_t$-broadening and suppression of SFs & jet spectra on nuclear targets
  
- Pomeron exchange (color singlet) dominates at large $s$
Dynamical higher twist corrections: brave assumptions

Basic assumptions ($qq'$-scattering as an example)

- restrict oneself with rescattering on soft ($x_g \simeq 0$) gluons
- neglect color octet contributions
- interprete the respective correlators as GPDs

More technical (model implementation)

- describe low $x$ GPDs by Pomeron asymptotics
- account for multiparton correlations due to the “soft splitting” mechanism
- account for absorptive corrections to GPDs due to enhanced Pomeron diagrams
Dynamical higher twist corrections: brave assumptions

Basic assumptions (qq'-scattering as an example)

- restrict oneself with rescattering on soft \(x_g \simeq 0\) gluons
- neglect color octet contributions
- interpret the respective correlators as GPDs

More technical (model implementation)

- describe low \(x\) GPDs by Pomeron asymptotics
- account for multiparton correlations due to the “soft splitting” mechanism
- account for absorptive corrections to GPDs due to enhanced Pomeron diagrams
Dynamical higher twist corrections: brave assumptions

Basic assumptions ($qq'$-scattering as an example)

- restrict oneself with rescattering on soft ($x_g \approx 0$) gluons
- neglect color octet contributions
- interpret the respective correlators as GPDs

More technical (model implementation)

- describe low $x$ GPDs by Pomeron asymptotics
- account for multiparton correlations due to the "soft splitting" mechanism
- account for absorptive corrections to GPDs due to enhanced Pomeron diagrams
Dynamical higher twist corrections: brave assumptions

Basic assumptions \((qq')\)-scattering as an example

- restrict oneself with rescattering on soft \((x_g \approx 0)\) gluons
- neglect color octet contributions
- interprete the respective correlators as GPDs

More technical (model implementation)

- describe low \(x\) GPDs by Pomeron asymptotics
- account for multiparton correlations due to the "soft splitting" mechanism
- account for absorptive corrections to GPDs due to enhanced Pomeron diagrams
  - i.e., incorporae the mechanism in the Pomeron framework
- NB: AGK rules not applicable for HT contributions
Consider as an example corrections to $qq'$ scattering in LC gauge

Twist 4 contribution to the cross section:

$$\Delta \sigma_{HT}(s) = \frac{1}{2s} \int \frac{d^4k_q}{(2\pi)^4} \frac{d^4k_{q'}}{(2\pi)^4} \frac{d^4k_{g_1}}{(2\pi)^4} \frac{d^4k_{g_2}}{(2\pi)^4} \ H^{\alpha\beta}_{ijkl}(k_q, k_{q'}, k_{g_1}, k_{g_2})$$

$$\times \left[ \int d^4z_q d^4z_{g_1} d^4z_{g_2} e^{ik_q z_q + ik_{g_1} z_{g_1} - ik_{g_2} z_{g_2}} \langle p | \bar{\psi}_j(0) A_{\alpha}(z_{g_2}) A_{\beta}(z_{g_1}) \psi_i(z_q) | p \rangle \right]$$

$$\times \left[ \int d^4z_{q'} e^{ik_{q'} z_{q'}'} \langle p | \bar{\psi}_l(0) \psi_k(z_{q'}) | p \rangle \right]$$

Diagram: dynamical higher twist corrections
Consider as an example corrections to $qq'$ scattering in LC gauge

Twist 4 contribution to the cross section:

$$\Delta \sigma_{HT}(s) = \frac{1}{2s} \int \frac{d^4k_q}{(2\pi)^4} \frac{d^4k_{q'}}{(2\pi)^4} \frac{d^4k_{g1}}{(2\pi)^4} \frac{d^4k_{g2}}{(2\pi)^4} H^{\alpha\beta}_{ijkl}(k_q, k_{q'}, k_{g1}, k_{g2})$$

$$\times \left[ \int d^4z_q d^4z_{g1} d^4z_{g2} e^{ik_qz_q + ik_{g1}z_{g1} - ik_{g2}z_{g2}} \langle p | \bar{\psi}_j(0) A^\alpha(z_{g2}) A^\beta(z_{g1}) \psi_i(z_q) | p \rangle \right]$$

$$\times \left[ \int d^4z_{q'} e^{ik_{q'}z_{q'}} \langle p | \bar{\psi}_l(0) \psi_k(z_{q'}) | p \rangle \right]$$

Doing collinear factorization, one obtains [Ellis et al., 1982; Qiu, 1990]

$$\Delta \sigma_{HT}(s) = \int dx_{q'} dx_q dx_{g1} dx_{g2}$$

$$\times q(x_{q'}) T_{qg}(x_q, x_{g1}, x_{g2})$$

$$\times \frac{1}{2s} d^\perp_{\alpha\beta} \text{Tr}[\hat{p}' H^{\alpha\beta}(x_q, x_{q'}, x_{g1}, x_{g2}) \hat{p}]$$
Consider as an example corrections to $qq'$ scattering in LC gauge

$$\Delta \sigma_{\text{HT}}(s) = \int dx' dx d x' d x_1 d x_2$$

$$\times q(x_{q'}) T_{qg}(x_q, x_{g1}, x_{g2})$$

$$\times \frac{1}{2s} d_{\alpha\beta}^\perp \text{Tr}[\hat{p}' H^{\alpha\beta}(x_q, x_{q'}, x_{g1}, x_{g2}) \hat{p}]$$

here the quark-gluon correlation function:

$$T_{qg}(x_q, x_{g1}, x_{g2}) = \frac{1}{x_{g1} x_{g2}} \int \frac{dy_q^-}{4\pi} \frac{dy_{g1}^-}{2\pi} \frac{dy_{g2}^-}{2\pi}$$

$$\times e^{ip^+ x_q y_q^- + ip^+ x_{g1} y_{g1}^- - ip^+ x_{g2} y_{g2}^-} \langle p| \bar{\psi}(0) \gamma^+ F_+^{\alpha}(y_{g2}^-) F_+^\alpha(y_{g1}^-) \psi(y_q^-) |p \rangle$$
Consider as an example corrections to $qq'$ scattering in LC gauge

\[ \Delta \sigma_{HT}(s) = \int dx_q' \ dx_q \ dx_{g_1} \ dx_{g_2} \]
\[ \times \ q(x_q') \ T_{qg}(x_q, x_{g_1}, x_{g_2}) \]
\[ \times \ \frac{1}{2s} \ d^\perp_{\alpha\beta} \ \text{Tr}[\hat{p}' H^{\alpha\beta}(x_q, x_{q'}, x_{g_1}, x_{g_2}) \hat{p}] \]

here the quark-gluon correlation function:

\[ T_{qg}(x_q, x_{g_1}, x_{g_2}) = \frac{1}{x_{g_1} x_{g_2}} \int \frac{dy_q^- \ dy_{g_1}^- \ dy_{g_2}^-}{4\pi \ 2\pi \ 2\pi} \]
\[ \times e^{ip^+ x_q y_q^- + ip^+ x_{g_1} y_{g_1}^- - ip^+ x_{g_2} y_{g_2}^-} \langle p | \bar{\psi}(0) \gamma^+ F_+^\alpha(y_{g_2}^-) F_+^\alpha(y_{g_1}^-) \psi(y_q^-) | p \rangle \]

now: assume the integrals to be dominated by $x_{g_1}, x_{g_2} \simeq 0$

- e.g., converting $1/x_{g_i}$ into poles & doing residues

[Guo & Qiu, 2001]
observe that
\[
\lim_{x_{g1,2}\to 0} x_{g1} x_{g2} T_{qg}(x_q, x_{g1}, x_{g2}) \propto \\
\lim_{x_g\to 0} x_g F_{qg}^{(2)} (x_q, x_g, \Delta b = 0)
\]
Most radical assumptions

- observe that
  \[ \lim_{x_{g_1,2} \to 0} x_{g_1} x_{g_2} T_{qg}(x_q, x_{g_1}, x_{g_2}) \propto \lim_{x_g \to 0} x_g F_{qg}^{(2)}(x_q, x_g, \Delta b = 0) \]
- note that \( x_g \simeq \langle k_{\perp}^2 \rangle/(x_q' s) \)
Dynamical higher twist corrections: heuristics reasoning

Most radical assumptions

- observe that
  \[
  \lim_{x_{g_1,2} \to 0} x_{g_1} x_{g_2} T_{qg}(x_q, x_{g_1}, x_{g_2}) \propto \lim_{x_g \to 0} x_g F_{qg}^{(2)}(x_q, x_g, \Delta b = 0)
  \]

- note that \( x_g \simeq \langle k_{\perp}^2 \rangle / (x_q' s) \)

- finally the model:
  \[
  \Delta \sigma_{HT}(s) = K_{HT} \int dx_{q'} dx_q \, \sigma_{qg}^{HT}(x_q x_{q'} s, M_F^2) \\
  \times \, q(x_{q'}, M_F^2) F_{qg}^{(2)}(x_q, x_g = Q_0^2 / (x_q' s), M_F^2, Q_0^2, \Delta b = 0)
  \]
  involves 1 adjustable parameter \( K_{HT} \) which governs the magnitude of the contribution
Dynamical higher twist corrections: heuristic reasoning

Most radical assumptions

- observe that

\[
\lim_{x_{g1,2} \to 0} x_{g1} x_{g2} T_{qg}(x_q, x_{g1}, x_{g2}) \propto \lim_{x_g \to 0} x_g F_{qg}^{(2)}(x_q, x_g, \Delta b = 0)
\]

- note that \( x_g \simeq \langle k_{\perp}^2 \rangle / (x_q') \)

- finally the model:

\[
\Delta \sigma_{HT}(s) = K_{HT} \int dx_{q'} dx_q \sigma_{qg}^{HT}(x_q, x_{q'}, s, M_F^2) \\
\times q(x_{q'}, M_F^2) F_{qg}^{(2)}(x_q, x_g = Q_0^2 / (x_q' s), M_F^2, Q_0^2, \Delta b = 0)
\]

- involves 1 adjustable parameter \( K_{HT} \) which governs the magnitude of the contribution
Currently, implementation of the HT-effects is the main difference to QGSJET-II-04

- now twice smaller cutoff for hard processes: $Q_0^2 = 1.5 \text{ GeV}^2$
  ($\Rightarrow p_t^{\text{cut}} \simeq 2.4 \text{ GeV}$)
- additionally, I enhanced the rate of high mass diffraction by $\sim 30\%$ and reduced low mass diffraction
Currently, implementation of the HT-effects is the main difference to QGSJET-II-04

- now twice smaller cutoff for hard processes: $Q_0^2 = 1.5$ GeV$^2$  
  ($\Rightarrow p_t^{\text{cut}} \simeq 2.4$ GeV)
- additionally, I enhanced the rate of high mass diffraction by $\sim 30\%$ and reduced low mass diffraction
Currently, implementation of the HT-effects is the main difference to QGSJET-II-04

- now twice smaller cutoff for hard processes: $Q_0^2 = 1.5$ GeV$^2$ ($\Rightarrow p_t^{\text{cut}} \simeq 2.4$ GeV)
- additionally, I enhanced the rate of high mass diffraction by $\sim 30\%$ and reduced low mass diffraction

what about using even a smaller cutoff?

- generally possible but would require higher order corrections (multiple exchanges of soft gluons)
- $\Rightarrow$ additional assumptions
\[ \sqrt{s}\text{-dependence of } \sigma_{pp}^{\text{tot/el}} : \text{different effects} \]

- the main effect on \( \sigma_{pp}^{\text{tot/el}} \) is due to enhanced diagrams (already in QGSJET-II)
- higher twist effects: additional correction

\[ \text{c.m. energy (GeV)} \]
\[ \text{cross section (mb)} \]

- \( p+p \)
- \( \sigma_{\text{tot}} \) (no HT-effects)
- \( \sigma_{\text{tot}} \) (all effects)
- \( \sigma_{\text{el}} \) (no HT-effects)
- \( \sigma_{\text{el}} \) (no nonlin. effects)
- \( \sigma_{\text{el}} \) (all effects)
the main effect on $\sigma_{pp}^{\text{tot/el}}$ is due to enhanced diagrams (already in QGSJET-II)

higher twist effects: additional correction
\( \sqrt{s}\)-dependence of \( dN_{\text{ch}}/d\eta \) & \( dN_{\text{ch}}/dp_t \)

- **soft production**: mostly affected by enhanced diagrams (shadowing & saturation of soft \( p_t < p_t^{\text{cut}} \) parton cascades)
  - reduction of jet production \( (p_t > p_t^{\text{cut}}) \) by HT effects: \( \simeq 25\% \)
  - the effect fades away at high \( p_t \) \( (\propto 1/|q|^2) \)
soft production: mostly affected by enhanced diagrams (shadowing & saturation of soft \( p_t < p_t^{\text{cut}} \) parton cascades)
- reduction of jet production \( (p_t > p_t^{\text{cut}}) \) by HT effects: \( \approx 25\% \)
  - the effect fades away at high \( p_t \) \( \propto 1/|q|^2 \)
1. Treatment of nonlinear effects due to enhanced Pomeron graphs remains the key element of the model
   - with 2 adjustable parameters, provides a very rich formalism
   - plays central role for timing the energy-rise of cross sections & soft hadron production

2. Treatment of higher twist effects – a useful complement
   - with 1 additional adjustable parameter, provides a dynamical scheme which mimics energy-dependent $p_t$-cutoff
   - offers additional flexibility for the model tuning
   - however, not a perturbative approach: involves numerous phenomenological assumptions
   - ⇒ independent cross checks & calibration desirable
Outlook

1. **Treatment of nonlinear effects due to enhanced Pomeron graphs remains the key element of the model**
   - with 2 adjustable parameters, provides a very rich formalism
   - plays central role for timing the energy-rise of cross sections & soft hadron production

2. **Treatment of higher twist effects – a useful complement**
   - with 1 additional adjustable parameter, provides a dynamical scheme which mimics energy-dependent $p_t$-cutoff
   - offers additional flexibility for the model tuning
   - however, not a perturbative approach: involves numerous phenomenological assumptions
   - ⇒ independent cross checks & calibration desirable