Multiple Particles Interactions in Herwig

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Outline

1. Motivation

2. Basic building blocks of Monte Carlo Event Generators

3. Multiple Particles Interactions in Herwig

4. Summary and outlook
Motivation: What is the universe made of?

Standard Model
(Forces Mediated by Gauge Bosons)

Standard Model Lagrangian

\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + i \bar{D} \gamma_\mu \psi - \frac{g}{c} W^\mu \bar{\psi} \gamma_\mu \gamma_5 \psi + \frac{g}{c} Z^\mu \bar{\psi} \gamma_\mu \gamma_5 \psi + \frac{\lambda}{c} \phi^2 - V(\phi) \]
Motivation: What is the universe made of?

Standard Model Interactions
There is a huge gap between a one-line formula of a fundamental theory, like the Lagrangian of the SM, and the experimental reality that it implies.

Standard Model Lagrangian

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{D}_\mu y + h.c. \\
+ \lambda_i (\bar{\psi}_i \gamma_5 \psi_j \phi + h.c.) \\
+ |D_\mu \phi|^2 - V(\phi)
\]

Experimental reality
What Virtual Colliders are and why they are useful?

Lagrangian
Gauge invariance
QCD
Partons
NLO
Resummation

DATA MAKES YOU SMARTER

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.

Richard P. Feynman

Detector simulation
Pions, Kaons, ...
Reconstruction
B-tagging efficiency
Boosted decision tree
Neural network

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What Virtual Colliders are and why they are useful?

Theory

- Lagrangian
- Gauge invariance
- QCD
- Partons
- NLO
- Resummation
- ...

Experiment

- Detector simulation
- Pions, Kaons, ...
- Reconstruction
- B-tagging efficiency
- Boosted decision tree
- Neural network
- ...

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What Virtual Colliders are and why they are useful?

- General Purpose Monte Carlo (GPMC) event generators are designed to bridge that gap.

![Image of CMS Experiment at the LHC, CERN](image)

- One can think of a GPMC as a “Virtual Collider” ➞ Direct comparison with the data.
- Almost all HEP measurements and discoveries in the modern era have relied on GPMC generators, most notably the discovery of the Higgs boson.
Hadron colliders and the importance of strong interactions

Relative strength of the forces at $10^{-15}$m (= proton radius):

Strong : Electromagnetic : Weak

1 : 1 / 100 : 1/10000

QCD: Quantum field theory of strong interactions
(C.N. Yang, R. Mills; H. Fritzsch, M. Gell-Mann, H. Leutwyler)

- interaction carried by gluons acting on quarks and gluons
- QCD-charge: colour: of three types (`colours`: red, blue, green)

QCD coupling strength $\alpha_s$ depends on energy $\alpha_s(Q)$

- low energy ( = long distance or time)
  $\rightarrow \alpha_s$ is large (confinement): non-perturbative regime of QCD
- high energy ( = short distance or time)
  $\rightarrow \alpha_s$ is small (asymptotic freedom): perturbative regime of QCD

[see Ralph Engel lecture]
Complex structure of Quantum Chromodynamics – three faces of QCD

**Perturbative:** \( \alpha_s \ll 1 \)

\[
\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \alpha_s^3 \sigma_3 \ldots
\]

\[
\sigma_0 > \alpha_s \sigma_1 > \alpha_s^2 \sigma_2 > \alpha_s^3 \sigma_3 \ldots
\]

LO NLO NNLO N3LO

**State of the art:**


Example of one of hundreds of diagram

**Non-Perturbative:** \( \alpha_s \gg 1 \)

- Perturbative techniques break down
- Non-pertubative models inspired by physical motivations
- Lattice QCD?

**Perturbative resummation:**

- enhanced terms
  \[
  \sigma_i \supset L^i
  \]
  \[
  \sigma_0 \sim \alpha_s L \sim \alpha_s^2 L^2 \sim \alpha_s^3 L^3 \ldots
  \]
- Resum them
  \[
  \sum_i \alpha_i^s L^i
  \]
What do MC event generators do?

- An “event“ is a list of particles (pions, protons, ...) with their momenta.
- The MCs generate events.
- The probability to generate an event is proportional to the (approximate!) cross section for such an event.
- Calculate Everything ~ solve QCD (1M $ prize) → requires compromise!
- Improve lowest-order perturbation theory, by including the ”most significant“ corrections → complete events (can evaluate any observable you want)

The Workhorses: What are the Differences?

All offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

**PYTHIA**: Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.


**SHERPA**: Begun in 2000. Originated in ”matching” of matrix elements to showers: CKKW.
Basic building blocks of Monte Carlo Event Generators

Parton Distribution Function
Basic building blocks of Monte Carlo Event Generators

Hard process (exact fixed-order perturbation theory)
Basic building blocks of Monte Carlo Event Generators

Parton Shower (Approximate all-order perturbation theory)
Basic building blocks of Monte Carlo Event Generators

Parton Shower (Approximate all-order perturbation theory)
Basic building blocks of Monte Carlo Event Generators

Hadronization (non-perturbative semi-empirical models)
Basic building blocks of Monte Carlo Event Generators

Multiple Interactions and beam remnants
How do we know MPI exists? Data makes you smarter!

**FIG. 3.** Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low $p_T$ only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

**FIG. 12.** Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs multiple-interaction model with variable impact parameter: solid line, double-Gaussian matter distribution; dashed line, with fix impact parameter [i.e., $O_0(b)$].


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How do we know MPI exists? Data makes you smarter!

Direct observation of multiple interactions


Order 4 jets $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > p_{\perp 4}$ and define $\varphi$
as angle between $p_{\perp 1} + p_{\perp 2}$ and $p_{\perp 3} + p_{\perp 4}$ for AFS/CDF

Double Parton Scattering

\[
|p_{\perp 1} + p_{\perp 2}| \approx 0 \\
|p_{\perp 3} + p_{\perp 4}| \approx 0 \\
d\sigma/d\varphi \text{ flat}
\]

Double Bremsstrahlung

\[
|p_{\perp 1} + p_{\perp 2}| \gg 0 \\
|p_{\perp 3} + p_{\perp 4}| \gg 0 \\
d\sigma/d\varphi \text{ peaked at } \varphi \approx 0/\pi \text{ for AFS/CDF}
\]
How do we know MPI exists? Data makes you smarter!

Direct observation of multiple interactions

CDF: Double parton scattering in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$


![Graph showing CDF 16 GeV $\gamma/\pi^0 + 3$ Jets 1-Vertex Events]

- Data
- DP component, from background subtraction method (52.6%)
- Monte Carlo admixture: 52.6%DP + 47.4%PYTHIA

$\Delta S, \varphi$-angle between pairs (radians)
How do we know MPI exists? Minimum Bias Measurements

- “Zero bias” - Every event in a perfect $4\pi$ detector.
- A “minimum bias” event is what one would see with a totally inclusive trigger. All events, with a minimum bias from restricted trigger conditions.
- In practice this definition depends on the experiment’s trigger!
  Two examples:
  1. ATLAS, Minimum Bias Trigger Scintillator ($2.1 < |\eta| < 3.8$), single arm MBTS trigger fired, primary vertex reconstructed, phase space: $p_T > 500(100)$ MeV, $|\eta| < 2.5$, $n_{ch} \geq 1$ (2, 6, 20)
  2. CDF (2009), Minimum bias trigg. BBC ($3.2 < |\eta| < 5.9$), coincidence in time of signals in both forward and backward modules, primary vertex reconstructed, phase space: $p_T > 400$ MeV, $|\eta| < 1.0$

- Typical observables:

  \[
  \frac{1}{N_{ev}} \cdot \frac{dN_{ch}}{d\eta}, \quad \frac{1}{N_{ev}} \cdot \frac{1}{2\pi p_T} \cdot \frac{d^2N_{ch}}{d\eta dp_T}, \quad \frac{1}{N_{ev}} \cdot \frac{dN_{ev}}{dn_{ch}} \quad \text{and} \quad \langle p_T \rangle \text{ vs. } n_{ch},
  \]

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How do we know MPI exists? Underlying event measurements

- Everything except the hard/interesting process

**On event-by-event basis:**

1) Identify the leading object in the event

2) Build **TRANSVERSE REGIONS** w.r.t. it

3) Compute $\Sigma p_T$ of charged particles (or multiplicity) in the different regions

**SETTINGS:**

- $p_T > 0.5$ GeV/c (tracks and leading-track)

- leading-track not included in distributions

- The transverse regions are most sensitive to the underlying event, since they are perpendicular to the axis of hardest scattering
How do we know MPI exists? Underlying event measurements

Only $p_T^{\text{jet}} > 20\text{GeV}$. 
MPI Motivation - is it really important?

Motivation:

- The minimum bias/underlying event is an unavoidable background to most collider observables and having good understand of it leads to more precise collider measurements!

- First LHC results are Minimum Bias and Underlying Event! Alice: [0911.5430], CMS [1002.0621], ATLAS [1003.3124] so it must be important ;)

- These will be particularly relevant for the LHC as, when it is operated at design luminosity, rare signal events will be embedded in a background of more than 20 near-simultaneous minimum-bias collisions.

- Any realistic experiment simulation event generator needs to be able to model these effects.

- “Don’t worry, we will measure and subtract it” But... fluctuations and correlations on an event-by-event basis are crucial.
MPI Motivation - is it really important?

- “Don’t worry, we will measure and subtract it” But... fluctuations and correlations on an event-by-event basis are crucial.

- Steep distribution ⇒ small sideways shift = large vertical
- Rare fluctuations can have a huge influence
MPI model basics

Inclusive hard jet cross section in pQCD:

\[
\sigma^{\text{inc}}(s, p_{t}^{\text{min}}) = \sum_{i,j} \int_{p_{t}^{\text{min}}^2} dp_t^2 \int dx_1 dx_2 \ f_i(x_1, Q^2) f_j(x_2, Q^2) \ \frac{d\hat{\sigma}_{ij}}{dp_t^2}
\]

\(\sigma^{\text{inc}} > \sigma_{\text{tot}} \) eventually

Interpretation:

- \(\sigma^{\text{inc}}\) counts all partonic scatters in a single \(pp\) collision
- more than a single interaction

\[
\sigma^{\text{inc}} = \langle n_{\text{dijets}} \rangle \sigma_{\text{inel}}
\]
MPI Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number $m$ of additional scatters,

$$P_m(\vec{b}, s) = \frac{\bar{n}(\vec{b}, s)^m}{m!} e^{-\bar{n}(\vec{b}, s)} .$$

Then we get $\sigma_{\text{inel}}$:

$$\sigma_{\text{inel}} = \int d^2\vec{b} \sum_{n=1}^{\infty} P_m(\vec{b}, s) = \int d^2\vec{b} \left( 1 - e^{-\bar{n}(\vec{b}, s)} \right) .$$

Cf. $\sigma_{\text{inel}}$ from scattering theory in eikonal approx. with scattering amplitude $a(\vec{b}, s) = \frac{1}{2i} (e^{-\chi(\vec{b}, s)} - 1)$

$$\sigma_{\text{inel}} = \int d^2\vec{b} \left( 1 - e^{-2\chi(\vec{b}, s)} \right) \Rightarrow \chi(\vec{b}, s) = \frac{1}{2} \bar{n}(\vec{b}, s) .$$

$\chi(\vec{b}, s)$ is called eikonal function.
MPI Eikonal model basics – Overlap function

Assumptions:

- the distribution of partons in hadrons factorizes with respect to the $b$ and $x$ dependence $\Rightarrow$ average number of parton collisions:

\[
\bar{n}(\vec{b}, s) = L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int dp_t^2 \frac{d\sigma_{ij}}{dp_t^2} \\
= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2 \vec{b}' \int dp_t^2 \frac{d\sigma_{ij}}{dp_t^2} \\
\times D_{i/A}(x_1, p_t^2, |\vec{b}'|) D_{j/B}(x_2, p_t^2, |\vec{b} - \vec{b}'|) \\
= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2 \vec{b}' \int dp_t^2 \frac{d\sigma_{ij}}{dp_t^2} \\
\times f_{i/A}(x_1, p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2, p_t^2) G_B(|\vec{b} - \vec{b}'|) \\
= A(\vec{b}) \sigma_{\text{inc}}^{\text{inc}}(s; p_t^{\text{min}}).
\]

- at fixed impact parameter $b$, individual scatterings are independent (leads to the Poisson distribution)
MPI Eikonal model basics – Overlap function

From assumptions:

- at fixed impact parameter $b$, individual scatterings are independent,
- the distribution of partons in hadrons factorizes with respect to the $b$ and $x$ dependence.

we get the average number of partonic collisions at a given $b$ value is

$$\bar{n}(b, s) = A(b)\sigma^{inc}(s; p_t^{min}) = 2\chi(b, s)$$

where $A(b)$ is the partonic overlap function of the colliding hadrons

$$A(b) = \int d^2\vec{b}' G_A(|\vec{b}'|) G_B(|\vec{b} - \vec{b}'|)$$

$G(\vec{b})$ from electromagnetic FF:

$$G_P(\vec{b}) = G_\bar{P}(\vec{b}) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{e^{i\vec{k}\cdot\vec{b}}}{(1 + \frac{\vec{k}^2}{\mu^2})^2}$$

But $\mu^2$ not fixed to the electromagnetic 0.71 GeV$^2$.
Free for colour charges.

$\Rightarrow$ Two main parameters: $\mu^2, p_t^{min}$. 
MPI Eikonal model basics – Semihard MPI and UE data

Good description of Run I Underlying event data ($\chi^2 = 1.3$).

Only $p_T^{\text{jet}} > 20 \text{GeV}$. 

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MPI Eikonal model basics – extension to soft MPI

So far only hard MPI. Now extend to soft interactions with

\[
\chi_{\text{tot}}(\vec{b}, s) = \frac{1}{2} \left( A(\vec{b}; \mu)\sigma_{\text{inc hard}}(s; p_t^{\text{min}}) + A(\vec{b}; \mu_{\text{soft}})\sigma_{\text{soft inc}} \right)
\]

Fix the two parameters \(\mu_{\text{soft}}\) and \(\sigma_{\text{soft inc}}\) from two constraints

\[
\sigma_{\text{tot}}(s) \overset{!}{=} 2 \int d^2 \vec{b} \left( 1 - e^{-\chi_{\text{tot}}(\vec{b}, s)} \right),
\]

\[
b_{\text{el}}(s) \overset{!}{=} \int d^2 \vec{b} \frac{b^2}{\sigma_{\text{tot}}} \left( 1 - e^{-\chi_{\text{tot}}(\vec{b}, s)} \right).
\]

(measured/well predicted)
MPI Eikonal model basics – extension to soft MPI

- data, uncorrected
  - $p_t^{\text{min}} = 3.5$, $\mu^2 = 1.50$, $\chi^2_{\text{tot}}/N = 3.1$
  - $p_t^{\text{min}} = 3.5$, $\mu^2 = 1.25$, $\chi^2_{\text{tot}}/N = 2.9$
  - $p_t^{\text{min}} = 4.0$, $\mu^2 = 1.50$, $\chi^2_{\text{tot}}/N = 2.8$

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What we have so far:

- Unitarized jet cross sections
- Fulfil constraints from $\sigma_{\text{tot}}$ and $\sigma_{\text{el}}$.
- Simple model with similar overlap functions.
- No additional (explicit) energy dependence.
- Left with freedom in parameter space.

→ Look at LHC results (900 GeV).

- ATLAS charged particles in Min Bias.
ATLAS charged particles in Min Bias ($N_{ch} \geq 1$, $p_T > 500 MeV$, $|\eta| < 2.5$)

- On to the LHC

- Average transverse momentum as function of $N_{ch}$

- Charged particle multiplicity as function of $\eta$

- oops, not so nice...

- despite very good agreement with Rick Field’s CDF UE analysis

- choice of PDF set (CTEQ611 vs MSTW LO** (our default))

- Failure of a physically motivated model usually points to more, interesting physics ... colour structure?
Colour reconnection (CR) in Herwig

Extending the hadronization model in Herwig(++):

- QCD parton showers provide *pre-confinement*
  - colour-anticolour pairs form highly excited hadronic states, the *clusters*
Colour reconnection (CR) in Herwig

Extending the hadronization model in Herwig(++):

- QCD parton showers provide pre-confinement
  $\Rightarrow$ colour-anticolour pairs form highly excited hadronic states, the *clusters*

- CR in the cluster hadronization model: allow *reformation* of clusters, e.g. $(il) + (jk)$

- Physical motivation: exchange of soft gluons during non-perturbative hadronization phase
Colour reconnection (CR) in Herwig – Minimum Bias data

Charged particle multiplicity as function of $\eta$ (0.9 TeV, $N_{ch} \geq 6$)

Charged particle density (0.9 TeV, $N_{ch} \geq 6$)
**MPI – key components**

**Matter distribution** ($\mu^2$)

Based on electromagnetic form factor (radius of the proton free parameter)

**Extension to soft MPI**

($p_t < p_t^{\text{min}}$)

Gaussian extension below $p_t^{\text{min}}$

Energy dependent $p_t^{\text{min}}$

**Colour structure** ($p_{\text{reco}}$, $p_{\text{CD}}$)

Possibility of change of color structure (color reconnection)

The least understood part of modeling

---

**Main parameters:**

- $\mu^2$ - inverse hadron radius squared (parametrization of overlap function)
- $p_t^{\text{min}}$ - transition scale between soft and hard components $\Rightarrow p_t^{\text{min}} = p_t^{\text{min}} \left( \frac{\sqrt{s}}{E_0} \right)^b$
- $p_{\text{reco}}$ - colour reconnection

---

[Gieseke, Röhr, AS, EPJC C72 (2012)]
MPI – recent progress

Baryonic Colour Reconnection

Idea:
- Allow gluon to strange-pairs
- Allow recombination of mesonic to baryonic clusters with probability derived in proximity in momentum space.


[ALICE, EPJC75 (2015) 226]
MPI – recent progress

Soft Physics

- Inclusion of diffractive topologies
- New soft peripheral MPI model
- The rapidity bump disappears

A lot of progress in Soft Physics → important to use up-to-date models and tunes!

More is coming:

- Colour Reconnection from Soft Gluon Evolution
- Space-time Colour Reconnection


[Bellm, Blok, Duncan, Gieseke, Myska, AS]
Summary and outlook

• Almost all HEP measurements and discoveries in the modern era have relied on GPMC generators, most notably the discovery of the Higgs boson.

• Complex structure of Quantum Chromodynamics:
  - Perturbative techniques (hard process)
  - Resummation techniques (Parton Shower – well established)
  - Non-perturbative models (crucial to obtain fully exclusive simulation of the collisions)

• Tremendous amount of new developments in GPMCs because we need more precise results.

• Constant improvements of MPI models in Herwig

• Good first round of LHC data well described...

• ... but still a lot of space for improvements (collective effects in pp – CR? Hydro? Rope models? Mixture of all the effect?)

• In both the Cosmic Rays and LHC experiments we study the same physics! Space for cross-took and progress!!
Monte Carlo training studentships

3-6 month fully funded studentships for current PhD students at one of the MCnet nodes. An excellent opportunity to really understand and improve the Monte Carlos you use!

Application rounds every 3 months.

MCnet projects
- Pythia+Vincia
- Herwig
- Sherpa
- MadGraph
- "Plugin" – Ariadne+HEJ
- CEDAR – Rivet+Professor
- +Contur+hepforge+…

for details go to: www.montecarlonet.org

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Thank you for your attention!
Since these n-clusters can lie at very different rapidities (the extreme case being the two opposite beam remnants), the strings or clusters spanned between them can have very large invariant masses (though normally low $pT$), and give rise to large amounts of (soft) particle production.
\[ f_a(m_{\text{cut}}) \equiv \frac{N_a(m_{\text{cut}})}{\sum_{b=h,i,n} N_b(m_{\text{cut}})} = \frac{N_a(m_{\text{cut}})}{N_{\text{cl}}}, \quad (1) \]

Since these n-clusters can lie at very different rapidities (the extreme case being the two opposite beam remnants), the strings or clusters spanned between them can have very large invariant masses (though normally low \( pT \)), and give rise to large amounts of (soft) particle production.
Monte Carlo methods why and how?

- We want to compute expectation values of observables
  \[ \langle O \rangle = \sum_n \int d\phi_n P(\phi_n) O(\phi_n), \]
  where \( \phi_n \) - Point in n-particle phase-space, \( P(\phi_n) \) Probability to produce \( \phi_n \), Value of observable at \( O(\phi_n) \).

- large \( n \) \( O(100 \div 1000) \) \( \Rightarrow \) Monte Carlo is the only choice.
Monte Carlo methods why and how?

\[ \langle O \rangle = \sum_n \int d\phi_n P(\phi_n) O(\phi_n) \]

Problems:

- Integrate a multi dimensional function

<table>
<thead>
<tr>
<th>Efficiencies of integration methods (MC with numerical quadrature):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty as a function of number of points $n$</td>
</tr>
<tr>
<td>-------------------------------------------------------------</td>
</tr>
<tr>
<td>Monte Carlo</td>
</tr>
<tr>
<td>Trapezoidal rule</td>
</tr>
<tr>
<td>Simpson’s rule</td>
</tr>
</tbody>
</table>

- Pick a point at random according to a probability distribution.
Monte Carlo methods why and how?

Wikipedia

Monte Carlo methods are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results.

History:

- G. Comte de Buffon (1777) – perhaps the earliest documented use of random sampling to find the solution to the integral (by throwing a needle onto horizontal plane ruled with straight lines).
- Marquis Pierre-Simon de Laplace (1886) – use of Buffon’s method to evaluate $\pi$.

Calculate $\pi$ by dropping a needle onto the floor.

$\approx 34/11 \sim 3.1$ based on 17 throws

- Lord Kelvin (1901) – use random sampling (drawing numbered pieces of paper from a bowl) to aid in evaluating some integrals in the kinetic theory of gases.
Monte Carlo methods why and how?

History – cont.

➤ Enrico Fermi (1930s) – numerical sampling experiments on neutron diffusion and transport in nuclear reactors (deviced FERMIAC – a mechanical sampling device).

➤ J. von Neumann, S. Ulam, N. Metropolis, R. Feynman (1940s) – first large-scale random-numbers based calculations of neutron scattering and absorption during the “Manhattan” project (work on a nuclear bomb). Name Monte Carlo refers to the Monte Carlo Casino in Monaco where Ulam’s uncle would borrow money from relatives to gamble.

➤ ...

➤ In Particle Physics we have to solve multidimensional integrals (many particles) MC methods very efficient! So we play a roulette to understand the law of the nature :)