The FLUKA code
Some extra material/examples

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The FLUKA hadronic Models
The FLUKA hadronic Models

Elastic, exchange
Phase shifts
data, eikonal

hadron
hadron
The FLUKA hadronic Models

Elastic, exchange
Phase shifts
data, eikonal

\( p < 3-5 GeV/c \)
Resonance prod
and decay
The FLUKA hadronic Models

Elastic, exchange Phase shifts data, eikonal
P<3-5GeV/c Resonance prod and decay

low E \pi, K Special

hadron

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The FLUKA hadronic Models

- Elastic, exchange
- Phase shifts data, eikonal
- Low E $\pi, K$
- Special

- $P < 3-5\text{GeV/c}$
- Resonance prod and decay

- High Energy
- DPM hadronization

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The FLUKA hadronic Models

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Low E
π, K
Special

Hadron-nucleus: PEANUT

High Energy
DPM
hadronization

Sophisticated
G-Intranuclear Cascade

Gradual onset of
Glauber-Gribov multiple
interactions

Preequilibrium
Coalescence

Evaporation/Fission/Fermi break-up
γ deexcitation

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Hadron-nucleon interaction models: NN

\[ N_1 + N_2 \rightarrow N_1' + N_2' + \pi \]

threshold at 290 MeV, important above 700 MeV

Examples:

\[ N_1 + N_2 \rightarrow N_1' + \Delta(1232) \]
\[ \rightarrow N_1' + N_2' + \pi \]

\[ N_1 + N_2 \rightarrow N_1' + N^*(1440) \]
\[ \rightarrow N_1' + N_2' + \pi \]

or

\[ N_1 + N_2 \rightarrow N_1' + N_2' + \pi_1 + \pi_2 \]

\[ N_1 + N_2 \rightarrow \Delta_1(1232) + \Delta_2(1232) \]
\[ \rightarrow N_1' + \pi_1 + N_2' + \pi_2 \]

Elastic, charge exchange and strangeness exchange reactions:

- Available phase-shift analysis and/or fits of experimental differential data
- At high energies, standard eikonal approximations are used

Particle production interactions:

two kinds of models

Those based on "resonance" production and decays, cover the energy range up to 3-5 GeV/c

Those based on quark/parton string models, which provide reliable results up to several tens of TeV
Nonelastic hN interactions at intermediate energies: $\pi N$

$\pi + N \rightarrow \pi' + \pi'' + N' \quad \text{opens at 170 MeV.}$

- Dominance of the $\Delta$ resonance and of the $N^*$ resonances → isobar model
- All reactions proceed through an intermediate state containing at least one resonance.

Resonance energies, widths, cross sections, branching ratios from data and conservation laws, whenever possible. Inferred from inclusive cross sections when needed.

**Examples:**

- $N_1 + N_2 \rightarrow N'_1 + \Delta(1232) \quad \rightarrow N'_1 + N'_2 + \pi$
- $\pi + N \rightarrow \Delta(1600) \rightarrow \pi' + \Delta(1232) \rightarrow \pi' + \pi'' + N'$
- $N'_1 + N'_2 \rightarrow \Delta_1(1232) + \Delta_2(1232) \quad \rightarrow N'_1 + \pi_1 + N'_2 + \pi_2$
Hadron-Nucleon resonance production: summary

Summarizing, all reactions are thought to proceed through channels like:

\[ h + N \rightarrow X \rightarrow x_1 + \ldots + x_n \rightarrow \ldots \]
\[ h + N \rightarrow X + Y \rightarrow x_1 + \ldots + x_n + y_1 + \ldots + y_m \rightarrow \ldots \]

where \( X \) and \( Y \) can be real resonances or “stable” particles (\( \pi, n, p, K \)) directly

**Resonances can be treated as real particles:** they can be transported and then transformed into secondaries according to their lifetime and decay branching ratios

Reactions of 1\(^{st}\) kind: s-channel reactions direct resonance production \( \rightarrow \) **bumps** in the isospin cross section around a centre-of-mass energy \( \sqrt{s} = M_X \)

2\(^{nd}\) kind: the extra degree of freedom associated to the \( X, Y \) relative motion \( \rightarrow \) **NO resonant behaviour**, rather a relatively fast increase from \( \sqrt{s} \approx M_X + M_Y \) followed by a smooth behaviour

**\( N \ N \) reactions are all of type 2, while \( \pi \ N \) reactions can be of both types**
Inelastic hN at high energies: (DPM, QGSM, …)

- Problem: “soft” interactions → QCD perturbation theory cannot be applied.
- Interacting strings (quarks held together by the gluon-gluon interaction into the form of a string)
- Interactions treated in the Reggeon-Pomeron framework
- each of the two hadrons splits into 2 colored partons → combination into 2 colourless chains → 2 back-to-back jets
- each jet is then hadronized into physical hadrons
Inelastic hN interactions at high energies (DPM, QGSM)

Problem: “soft” interactions → no perturbation theory

**Solution! Interacting strings (quarks held together by the gluon-gluon interaction into the form of a string)**

- Interactions treated in the Reggeon-Pomeron framework
- At sufficiently high energies the leading term corresponds to a Pomeron ($P$) exchange (a closed string exchange)
- Each colliding hadron splits into two colored partons → a combination into two color neutral chains → two back-to-back jets
- Physical particle (Reggeon, $R$) exchanges produce single chains at low energies, treated explicitly as such in Fluka
- Higher order contributions with multi-Pomeron exchanges important at $E_{lab} \gg 1$ TeV
Nonelastic $hN$ at high energies (DPM)

Parton and color concepts, Topological expansion of QCD, Duality

Reggeon exchange

Optical theorem

Pomeron exchange

\[
\frac{d\sigma_{el}(t, s)}{dt} = \frac{\pi}{k^2} |f(t, s)|^2
\]

\[
\sigma_T(s) = \frac{4\pi}{k} \text{Im} f(0, s)
\]
Hadron-hadron collisions: chain examples

Leading two-chain diagram in DPM for p-p scattering. The color (red, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities.

Leading two-chain diagram in DPM for pbar-p scattering. The color (red, antired, blue, antiblue, green, and antigreen) and quark combination shown in the figure is just one of the allowed possibilities.
Hadron-hadron collisions: chain examples II

Single chain (s-channel) diagram for $\pi^+-p$ scattering. The color (red, antired, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities.

Leading two-chain diagram in DPM for $\pi^+-p$ scattering. The color (red, antired, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities.
The “hadronization” of color strings

An example:

Low mass chain: just 2-3 meson/(anti)baryon resonances

- $u\bar{d}$, $\pi^+, \rho^+, \ldots$
- $d\bar{u}$, $\pi^-, \rho^-, \ldots$
- $uud$, $p, \Delta^-, \ldots$
- $udd$, $n, \Delta^0, \ldots$
- $us$, $K^+, K^{*+}, \ldots$
- $sd$, $K^0, \bar{K}^{*0}, \ldots$
- $u\bar{d}$, $\pi^+, \rho^+, \ldots$
- $\ldots$
- $d\bar{u}$, $\pi^-, \rho^-, \ldots$
Dual Parton Model# and hadronization

From DPM:
- Number of chains
- Chain composition
- Chain energies and momenta
- Diffractive events

Almost No Freedom

Chain hadronization
- Assumes chain universality*
- Fragmentation functions from hard processes and $e^+e^-$ scattering
- Transverse momentum from uncertainty considerations
- Mass effects at low energies

*Chain formation and "decay" (hadronization) processes are assumed to be decoupled

Does it sound familiar?

The same functions and (a few) parameters for all reactions and energies

#For a review: Physics Report 236
Inclusive cross section for the production of $\pi^0$ (blue), $\pi^+$ (red), and $\pi^-$ (green) in p-p collisions as a function of the proton kinetic energy. Lines: simulations, symbols exp. Data. (figure from AstrPhys81, 21 (2016))
DualPartonModel at its lower limit

- Strong interest in particle production from beams in the few to tens of GeV range
- Important for *neutrino beams and interactions*
- **Challenging:** too high energy for resonance formation, too low for quark gluon string models
- FLUKA high energy hadron-hadron interaction model - DPM: *chain production and chain hadronization*
- **Strong mass effects for low energy chains** → “standard” hadronization outside its validity region
DualPartonModel at its lower limit

- Strong interest in particle production from beams in the few to tens of GeV range
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(Possible) solution! gradual transition of low energies chains to “phase space explosion” constrained in $p_T$, including baryons, mesons, resonances (examples later)
Effect of low $\sqrt{s_{\text{chain}}}$ "phase-space" like explosion

"standard" FLUKA hadronization

With low-mass chain "phase-space" like explosion

With low-mass chain explosion: much better agreement for forward emission!!

$\frac{d^2\sigma}{dp_d\Omega}$ (µb/sr/GeV/c)

Fluka: histos
Data: symbols

Pion+ and Pion- emission from proton-proton interactions at 12.2 GeV/c.

Longitudinal momentum distributions at different transverse momenta

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Transverse momentum:

$\pi^- + p \rightarrow \pi^+\pi^- + X (16 \text{ GeV/c})$

$\pi^- \times 2$

$\pi^+$

Don't be cheated by the different $p_T^2$ horizontal scales!

The distributions left and right are ~ the same, with a characteristic $p_T$ scale of ~ 300 MeV/c $\approx \eta/r_{\text{hadron}}$

*ZPC39, 311 (1988)

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Other (G)INC related concepts/examples
FLUKA (PEANUT) modeling of nuclear interactions

Target nucleus description (density, Fermi motion, etc)

Glauber-Gribov cascade with formation zone

(Generalized) IntraNuclear cascade

Preequilibrium stage with current exciton configuration and excitation energy (all non-nucleons emitted/decayed + all nucleons below 30-100 MeV)

Evaporation/Fragmentation/Fission model

γ deexcitation

α (s)

$10^{-23}$

$10^{-22}$

$10^{-20}$

$10^{-16}$
Nuclei: charge density distributions, radii, etc

Central density: \( \rho_0 \approx 0.17 \text{ fm}^{-3} \)

RMS radius: \( R \approx 1.2 \cdot A^{1/3} \text{ fm} \)

(1 fm = 10^{-13} \text{ cm}, \ 1 \text{ fm}^2 = 10^{-26} \text{ cm}^2 = 10 \text{ mb})

\[
\rho_{NN} = \left( \frac{3}{4\pi\rho} \right)^{1/3} \approx 1.1 \text{ fm}
\]

\[
r_{NN} \geq \left( \frac{3}{4\pi\rho_0} \right)^{1/3} \approx 1.1 \text{ fm}
\]
"Classical" IntraNuclearCascade (INC) model:

50 MeV nucleon: $\lambda = \hbar / p = 0.64$ fm, MFP $\sim 1.2$ fm at $\rho \sim 0.08$

200 MeV nucleon: $\lambda = \hbar / p = 0.31$ fm, MFP $\sim 4$ fm at $\rho \sim 0.08$

Both Mean Free Path’s without accounting for Pauli blocking which would increase them by a significant factor ($\sim 2$ at 50 MeV)

Hence at intermediate energies, nucleon-nuclear reactions can be described as the passage of the incoming nucleon through the nucleus, undergoing individual nucleon-nucleon collisions (IntraNuclear Cascade).

Main assumptions:

- Target nucleons occupy states of a cold Fermi gas;
- Incoming nucleon follows a classical (straight) trajectory;
- Given a nucleon-nucleon interaction cross section and N, Z and density profile of the target nucleus, one can evaluate the mean free path (MFP) of the incoming nucleon;
- The nucleon trajectory can be simulated as subsequent nucleon-nucleon collisions between straight-line trajectory segments, governed by the calculated MFP;
- Collision products must be above the Fermi level ("Pauli blocking") and can either escape or get "captured" if their energy is insufficient versus the binding or Coulomb barrier;
- "Captured" nucleon energies above $E_F$ and the holes in the Fermi gas both contribute to a residual excitation to be spent through the statistical model.
Sketch* of IntraNuclearCascade (INC):

Nuclei: charge density distributions, radii, etc in Peanut

Woods-Saxon approximation for the nuclear (charge) density profile

- Valid for medium-heavy nuclei
- Shell model density distributions should be used for light nuclei
- Parameters for total density (p+n)

\[
\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{a}\right)}
\]

central density: \( \rho_0 \approx 0.17 \text{ fm}^{-3} \)

1/2 radius: \( R \approx 1.1 \cdot A^{1/3} \text{ fm} \)

"skin thickness": \( t \approx 4.4 a \quad a \approx 0.55 \text{ fm} \)
Validity of the semiclassical trajectory assumption*

- Justified when $\lambda << r_{NN}/R_{Nuc}$. It is also interesting to note the following:

  Schroedinger equation for the incoming nucleon in the nucleus:

  $$
  \left[-\nabla^2 + U(r)\right] \psi(r) = k^2 \psi(r)
  $$

- For $U(r)$ varying slowly in the scale of $1/k$ (as is the case of OPM for nucleons of intermediate energies), an eikonal approximation suggests:

  $$
  \psi(r) \approx A(r)e^{iS(r)}
  $$

- The connection between the quantum and semi-classical description can be formally made by interpreting $S(r)$ as the classical action function, yielding the momentum as a function of $r$:

  $$
  k(r) = \nabla S(r)
  $$

- Thus, one can assign a momentum at each point without conceptual issues.

(Generalized) IntraNuclear Cascade in PEANUT

- Primary and secondary particles moving in the nuclear medium
- Target nucleons motion and nuclear well according to the Fermi gas model
- Interaction probability
  \[ \sigma_{\text{free}} + \text{Fermi motion} \times \rho(r) + \text{exceptions (ex. } \pi) \]
- Glauber cascade at higher energies
- Classical trajectories (+) nuclear mean potential (resonant for \( \pi \))
- Curvature from nuclear potential → refraction and reflection throughout the nucleus
- Interactions are incoherent and uncorrelated
- Interactions in projectile-target nucleon CMS → Lorentz boosts
- Multibody absorption for \( \pi, \mu^-, K^- \)
- Quantum effects (Pauli, formation zone, coherence length, correlations...)
- Preequilibrium step
- Energetic light ion production by coalescence
- Exact conservation of energy, momenta and all additive quantum numbers, including nuclear recoil
Nucleon Fermi motion:

Fermi gas model: nucleons as non-interacting fermions constrained in a cubic box of linear dimension $a$.

The nucleon wave functions are those of a free-particle with the condition $\Psi(x, y, z) = 0$ at the nuclear boundary, thus with constraints on the three components of the momentum:

$$K_x a = n_x \pi \quad K_y a = n_y \pi \quad K_z a = n_z \pi$$

where the $n$'s are positive integers. Each set of integers defines a solution of the Schrödinger equation with energy given by:

$$E(n_x, n_y, n_z) = \frac{\hbar^2}{2m_N} \left| K \right|^2$$

$$\left| K \right|^2 = \left( K_x^2 + K_y^2 + K_z^2 \right)$$

Nucleons must obey the Pauli principle, → for each set of $n$'s, → for a given momentum state, there can be at most four nucleons, two protons and two neutrons, each pair with opposite spin.
Each solution occupies a box in momentum space given by $\left(\frac{\pi}{a}\right)^3$; thus the number of states with momentum comprised between $K$ and $K + dK$ is given by the volume of the spherical shell of radius $K$ and thickness $dK$ divided by the volume occupied by each single solution, and divided again by eight since we consider only positive values of $n_x, n_y, n_z$:

$$dN = \frac{K^2}{2\pi^2} \Omega \, dK$$

$$dN = \frac{\Omega m_N}{2\pi^2 \hbar^3} \sqrt{2m_N E} \, dE$$

Thus, the total number of states with momentum smaller than $|K_F|$ is given by:

$$n(K_F) = \frac{4\pi}{3} \frac{K_F^3}{8(\pi/a)^3}$$

If we take $N = Z = A/2$, and place 4 nucleons on each level, we get:

$$A = \frac{2\Omega}{3\pi^2} K_F^3$$

⇒ the momentum of the highest occupied state depends only on the nuclear density $\rho = A/\Omega$:

$$\rho = \frac{2}{3\pi^2} K_F^3 \quad \Rightarrow \quad K_F = \sqrt[3]{\frac{3\pi^2}{2} \rho}$$
Nucleon Fermi motion III:

The observed central/saturation density of nuclei, \( \rho \approx 0.17 \text{ fm}^{-3} \times (1.7 \times 10^{38} \text{ nucleons/cm}^3) \), implies:

\[
K_F = 1.36 \text{ fm}^{-1} \quad E_F = 38 \text{ MeV}
\]

*These are called the Fermi momentum and Fermi Energy*

The probability distribution for the momentum/energy of a nucleon are therefore given by:

\[
P(K) dK = \frac{K^2}{3 K_F^3} dK \quad P(E_k) dE_k = \frac{2 \sqrt{E_k}}{3 E_F^3} dE_k
\]

In nuclei with \( N \neq Z \), two different values of the Fermi energy can be defined:

\[
\rho^p(r) = \frac{Z}{A} \rho = \frac{1}{3 \pi^2} \left( K_F^p \right)^3 \quad \rho^n(r) = \frac{N}{A} \rho = \frac{1}{3 \pi^2} \left( K_F^n \right)^3
\]

The so defined Fermi energies are obviously kinetic energies, that is energies counted from the bottom of a potential well that in this model must be input from outside. This gives an average potential depth of about \( 38 + 8 = 46 \text{ MeV} \). The Fermi energy can be made radius-dependent in a straightforward way, through the so called *local density approximation*:

\[
\rho(r) = \frac{2}{3 \pi^2} K_F^3(r)
\]
Nuclear potential for n/p: schematic drawing

$^{208}\text{Pb}$:

- Blue: neutron
- Red: proton
Nuclear potential for n/p: schematic drawing

208Pb:

- Blue: neutron
- Red: proton
Nuclear potential for n/p: schematic drawing

\[208\text{Pb}:\]

- Blue: neutron
- Red: proton
Nuclear potential for n/p: schematic drawing

208Pb:

- Blue: neutron
- Red: proton

Incident neutron

\( S_n \) \( S_p \)

\( E_{Fn} \) \( E_{Fp} \)

Target proton

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Nuclear potential for n/p: schematic drawing

208Pb:

- Blue: neutron
- Red: proton
Nuclear potential for n/p: schematic drawing

\[ \text{\textbf{208Pb:}} \]

- Blue: neutron
- Red: proton
Nucleon correlation function:

Correlation function: it can be computed within the Fermi-gas model

Due to the anti-symmetrization of the fermion's wave function, given a nucleon in a position $\vec{r}$ in a nucleus with density $\rho_0$, the probability of finding another like nucleon in a position $\vec{r}'$ is decreased for small values of the distance $d = |\vec{r} - \vec{r}'|$ by a factor

$$g(x) = 1 - \frac{1}{2} \left[ \frac{3}{x^2} \left( \frac{\sin x}{x} - \cos x \right) \right]^2$$

where $x = K_F d$, and the factor $1/2$ in front of the parenthesis accounts for the two possible spin orientations.
hA at high energies: Glauber-Gribov cascade with formation zone

- **Glauber cascade**
  - Quantum mechanical method to compute Elastic, Quasi-elastic and Absorption hA cross sections from Free hadron-nucleon scattering + nuclear ground state
  - *Multiple Collision* expansion of the scattering amplitude

- **Glauber-Gribov**
  - Field theory formulation of Glauber model
  - Multiple collisions ↔ Feynman diagrams
  - High energies: exchange of one or more Pomerons with one or more target nucleons (a closed string exchange)

- **Formation zone (=materialization time)**
Glauber formalism/cascade (R. Glauber, 2005 Physics Nobel prize)

Quantum mechanical method to compute all relevant hadron-nucleus cross sections from hadron-nucleon scattering:

and nuclear ground state wave function $\Psi_i$

Total

$$\sigma_{hA_T}(s) = 2 \int d^2 b \int d^3 u |\Psi_i(u)|^2 \left[ 1 - \prod_{j=1}^A \text{Re} S_{hN}(b-r_{j\perp},s) \right]$$

Elastic

$$\sigma_{hA_{el}}(s) = \int d^2 b \int d^3 u |\Psi_i(u)|^2 \left[ 1 - \prod_{j=1}^A S_{hN}(b-r_{j\perp},s) \right]^2$$

Scattering

$$\sigma_{hA_{sf}}(s) = \sum_f \sigma_{hA_{sf}}(s) = \int d^2 b \int d^3 u |\Psi_i(u)|^2 \left[ 1 - \prod_{j=1}^A S_{hN}(b-r_{j\perp},s) \right]^2$$

Absorption (particle prod.)

$$\sigma_{hA_{abs}}(s) = \sigma_{hA_T}(s) - \sigma_{hA_{sf}}(s)$$

$$= \int d^2 b \int d^3 u |\Psi_i(u)|^2 \left[ 1 - \prod_{j=1}^A \left[ 1 - S_{hN}(b-r_{j\perp},s)^2 \right] \right]$$

Absorption probability over a given $b$ and nucleon configuration

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Gribov interpretation of Glauber multiple collisions

The absorption cross section can be shown to be just the integral in the impact parameter plane of the probability of getting at least one non-elastic hadron-nucleon collision

and the overall average number of collision integrated over the impact parameter space is given by

- Glauber-Gribov model = Field theory formulation of Glauber model
- Multiple collision terms \( \Rightarrow \) Feynman graphs
- At high energies: exchange of one or more pomeron with one or more target nucleons

- In the Dual Parton Model language: (neglecting higher order diagrams):

  Interaction with \( n \) target nucleons \( \Rightarrow 2n \) chains
  - Two chains from projectile valence quarks + valence quarks of one target nucleon \( \Rightarrow \) valence-valence chains
  - \( 2(n-1) \) chains from sea quarks of the projectile + valence quarks of target nucleons \( \Rightarrow 2(n-1) \) sea-valence chains

\[
\langle \nu \rangle = \frac{Z \sigma_{hp r} + N \sigma_{hn r}}{\sigma_{hA \text{ abs}}}
\]
Glauber-Gribov: chain examples

Leading two-chain diagrams in DPM for $p$-A Glauber scattering with 4 collisions. The color (red blue green) and quark combinations shown in the figure are just one of the allowed possibilities.

Leading two-chain diagrams in DPM for $\pi^+$-A Glauber scattering with 3 collisions.

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From one to many: Glauber cascade

At energies below a few GeV hA interactions can be described by a single primary collision hN (elastic or non-elastic), followed by reinteraction of the secondary particles (INC).
From one to many: Glauber cascade

At energies below a few GeV $hA$ interactions can be described by a single primary collision $hN$ (elastic or non-elastic), followed by reinteraction of the secondary particles (INC).

At higher energies, the Glauber calculus predicts explicit multiple primary collisions.
From one to many: Glauber cascade

At energies below a few GeV hA interactions can be described by a single primary collision hN (elastic or non-elastic), followed by reinteraction of the secondary particles (INC).

At higher energies, the Glauber calculus predicts explicit multiple primary collisions due to the relativistic length contraction and the uncertainty principle, at high energy most of the newly produced particles escape the nucleus without further reinteraction.
# The Transition

<table>
<thead>
<tr>
<th>High energy: the Glauber regime ($E \approx 10$ GeV)</th>
<th>Low energy: the “single collision” regime ($E \approx 5$ GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The first interaction involves many target nucleons coherently</td>
<td>- The first interaction involves one target nucleon (exc. pions)</td>
</tr>
<tr>
<td>- Quasi-Elastic* cross section separated from non-elastic $\sigma$ (experimentally is added to elastic)</td>
<td>- Quasi-Elastic is considered as a contribution to non-elastic</td>
</tr>
<tr>
<td>- QE is suppressed since $h$-$N$ inelastic is “integrated” over the projectile path in the nucleus</td>
<td>- QE fraction comes from single nucleon cross section ratio</td>
</tr>
<tr>
<td>- Mass effects, energy losses, are small</td>
<td>- Mass effects and energy losses are essential</td>
</tr>
</tbody>
</table>

* $Q.E.$ = elastic interaction at the hadron-nucleon level
Formation zone* (→ classical INC will never work)

Naively: "materialization" time (J. Ranft, L. Stodolski).
Qualitative estimate:

* J. Ranft applied the concept, originally proposed by Stodolski, to hA and AA nuclear interactions
Formation zone*  (→ classical INC will never work)

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Qualitative estimate:

In the frame where $p_{||} = 0$

$$\bar{t} = \Delta t \approx \frac{\hbar}{E_T} = \frac{\hbar}{\sqrt{p_T^2 + M^2}}$$

* J.Ranft applied the concept, originally proposed by Stodolski, to hA and AA nuclear interactions
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Qualitative estimate:

\[ \bar{t} = \Delta t \approx \frac{\hbar}{E_T} = \frac{\hbar}{\sqrt{p_T^2 + M^2}} \]

\[ \tau = \frac{M}{E_T} \bar{t} = \frac{\hbar M}{p_T^2 + M^2} \]

In the frame where \( p_{||} = 0 \)

Particle proper time

* J. Ranft applied the concept, originally proposed by Stodolski, to hA and AA nuclear interactions
Formation zone* (*bold*) (→ classical INC will never work)

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\[
\bar{t} = \Delta t \approx \frac{\hbar}{E_T} = \frac{\hbar}{\sqrt{p_T^2 + M^2}}
\]

\[
\tau = \frac{M}{E_T} \bar{t} = \frac{\hbar M}{p_T^2 + M^2}
\]

\[
\Delta x_{for} \equiv \beta c \cdot t_{lab} \approx \frac{p_{lab}}{E_T} \bar{t} \approx \frac{p_{lab}}{M} \tau = k_{for} \frac{\hbar p_{lab}}{p_T^2 + M^2}
\]

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\]

In the frame where \( p_{||} = 0 \)

Particle proper time

Going to the nucleus system

\[
\Delta x_{for} \equiv \beta \cdot c \cdot t_{lab} \approx \frac{p_{lab}}{E_T} \bar{t} \approx \frac{p_{lab}}{M} \tau = k_{for} \frac{\hbar p_{lab}}{p_T^2 + M^2}
\]

Condition for possible reinteraction inside a nucleus:

\[
\Delta x_{for} \leq R_A \approx r_0 A^{\frac{1}{3}}
\]

* J.Ranft applied the concept, originally proposed by Stodolski, to hA and AA nuclear interactions
Formation zone: early evidence

Many early experiments with high energy beams were using emulsions as target and recording media at the same time.

**Charged particle tracks were classified into:**

- **Shower or fast tracks**: weakly ionizing particles, around the ionization minimum ($E > 2-3$ times their mass), *mostly pions* → a measure of the particles produced in the primary collisions
- **Grey tracks**: mildly ionizing particles, typically *intermediate energy protons*, → a measure of re-interaction products
- **Black tracks**: heavily ionizing particles, typically *evaporation products* ($p, \alpha, ...$), → a measure of the *excitation energy* left in the residual nucleus

From the plots it is evident that while the shower particles continue to grow as expected, the re-interactions, responsible for cascade and evaporation, are fully saturated above 10 GeV, indicating that only the slowest fragment(s) of each primary interaction have a chance to re-interact and feed the intranuclear cascade and the excitation energy.
Effect of formation zone on residuals

Experimental and computed residual nuclei mass distribution

Ag(p,x)X at 300 GeV (top)  Au(p,x)X at 800 GeV (bottom)

Data from:
(The heavy fragment evaporation model is key for FLUKA predictions for A< 30)

Ag with and without formation zone:
\(<\pi> = 21.1, \ <E_{\pi}> = 7.3 \text{ GeV}\)
\(<\pi> = 49.7, \ <E_{\pi}> = 3.4 \text{ GeV}\)

Au with and without formation zone:
\(<\pi> = 30.1, \ <E_{\pi}> = 12.5 \text{ GeV}\)
\(<\pi> = 96.0, \ <E_{\pi}> = 4.6 \text{ GeV}\)
Setting the formation zone: no Glauber, no formation zone

(Pseudo)rapidity $\eta$ ($\eta$) distribution of charged particles produced in 250 GeV $\pi^+$ collisions on Aluminum (left) and Gold (right).

Points: exp. data (Agababyan et al., ZPC50, 361 (1991))

$$ y = \cosh^{-1}\left(\frac{E}{m_T}\right) = \tanh^{-1}\left(\frac{p_\perp}{E}\right) = \frac{1}{2} \ln \left(\frac{E + p_\perp}{E - p_\perp}\right) $$

$$ \eta = -\ln \left(\tan\frac{\vartheta}{2}\right) = \frac{1}{2} \ln \left(\frac{p + p_\parallel}{p - p_\parallel}\right) $$
Setting the formation zone: **no** Glauber, **yes** formation zone

(Pseudo)rapidity $\eta$ ($\pi^+$) distribution of charged particles produced in 250 GeV $\pi^+$ collisions on Aluminum (left) and Gold (right).

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$$\eta = -\ln\left(\tan\frac{\vartheta}{2}\right) = \frac{1}{2} \ln\left(\frac{p + p_\parallel}{p - p_\parallel}\right)$$
pBe, pAl @ 14.6 GeV/c

π⁺⁻, K⁺⁻, p, d, rapidity distributions for pBe @ 14.6 GeV/c (left)

π⁺ production double differential cross section for pAl @ 14.6 GeV/c as a function of the transverse mass, for different rapidity intervals

Symbols: exp. data
Histos: FLUKA
Coalescence: typical approach

- d, t, $^3$He, and alpha’s (and maybe heavier IMF’s) generated during the (G)INC and pre-equilibrium stage
- All possible combinations of (unbound) nucleons and/or light fragments checked at each stage of system evolution
- FOM evaluation based on phase space “closeness” at the nucleus periphery used to decide whether a light fragment is formed rather than not

◊ It works reasonably well at medium/high energies
◊ Recently extended up to IMF (mass $\sim$10) (INCL) (Boudard et al., PRC87 014606)
◊ Problems at lower energies: is it really the right mechanism or should it be complemented/replaced by something else?

... (p,d) reactions at energies below few tens of MeV on light nuclei: badly reproduced by coalescence. A **direct deuteron formation** mechanism at the first pn elementary step in the cascade or pre-equilibrium stage can fix the issue
Coalescence: PEANUT examples

Double differential $d$ production (left) by 383 MeV neutrons on Copper, and double differential $t$ production (right) from 542 MeV neutrons on Copper (data Nucl. Phys. A510, 774 (1990))
Resonance reinteractions:

Resonance cross sections are a problem because of the lack of any direct exp. data and the complication of the variable mass.

Resonance-nucleon → nucleon-nucleon cross-sections can be obtained from the extended balance principle, i.e.:

\[
\frac{d\sigma^{N\Delta\to NN}}{d\Omega} = \frac{1}{N_f} \frac{p_f^2}{p_i^2} \frac{d\sigma^{NN\to N\Delta}}{d\Omega} \left[ \int_{(m_N-m_x)^2}^{\sqrt{s}-m_N} dM^2 \frac{1}{\pi} \frac{M^2 \Gamma(M)}{(M^2-M^2)^2+M^2 \Gamma^2(M)} \right]^{-1}
\]

As a further example, the elastic nucleon-resonance cross section can be approximated using the NN invariant cross section matrix element as:

\[
\sigma^{N\Delta\to N\Delta} \approx \frac{M_{NN\to NN}^2}{16\pi p_s} \left[ \int_{(m_N-m_x)^2}^{\sqrt{s}-m_N} dM^2 \frac{1}{\pi} \frac{M\Gamma(M)}{(M^2-M^2)^2+M^2 \Gamma^2(M)} \right]
\]

However recent papers suggest that the conditions for the detailed balance to be valid are not met by resonances, and indeed a semi-quantitative correction factor which enhances the cross section by a factor ~3 is suggested.
Coherence length (~ as the Landau-Pomeranchuk-Migdal effect!)

Coherence length \(\approx\) formation time for elastic, charge exchange, or quasi-elastic interactions.
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Coherence length $\approx$ formation time for elastic, charge exchange, or quasi-elastic interactions.

Given a two body interaction with four-momentum transfer

$$q = p_{1i} - p_{1f}$$
Coherence length (~ as the Landau-Pomeranchuk-Migdal effect!)

Coherence length ≈ formation time for elastic, charge exchange, or quasi-elastic interactions.

Given a two body interaction with four-momentum transfer

\[ q = p_{1i} - p_{1f} \]

the energy transfer seen in a frame where the particle 2 is at rest is given by

\[ \Delta E_2 = \nu_2 = \frac{q^2}{2m_2} = \frac{q \cdot p_{2i}}{m_2} \]
Coherence length (~ as the Landau-Pomeranchuk-Migdal effect!)

Coherence length \( \approx \) formation time for elastic, charge exchange, or quasi-elastic interactions.

Given a two-body interaction with four-momentum transfer

\[ q = p_{1i} - p_{1f} \]

the energy transfer seen in a frame where the particle 2 is at rest is given by

\[ \Delta E_2 = v_2 = \frac{q^2}{2m_2} = \frac{q \cdot p_{2i}}{m_2} \]

From the uncertainty principle this \( \Delta E \) corresponds to an indetermination in proper time given by

\[ \Delta \tau \cdot \Delta E_2 \approx \hbar \]
**Coherence length (~ as the Landau-Pomeranchuk-Migdal effect!)**

Coherence length $\approx$ formation time for elastic, charge exchange, or quasi-elastic interactions.

Given a two-body interaction with four-momentum transfer

$$q = p_{1i} - p_{1f}$$

the energy transfer seen in a frame where the particle 2 is at rest is given by

$$\Delta E_2 = \nu_2 = \frac{q^2}{2m_2} = \frac{q \cdot p_{2i}}{m_2}$$

From the uncertainty principle, this $\Delta E$ corresponds to an indetermination in proper time given by

$$\Delta \tau \cdot \Delta E_2 \approx \hbar$$

that boosted to the nucleus frame gives a coherence length

$$\Delta x_{coh} \approx \frac{p_{2lab}}{m_2} \cdot \Delta \tau = k_{coh} \frac{p_{2lab} \hbar}{m_2 \nu_2}$$
Coherence length \( (~ as \ the \ Landau-Pomeranchuk-Migdal \ effect!) \)

Coherence length \( \approx \) formation time for elastic, charge exchange, or quasi-elastic interactions.

Given a two-body interaction with four-momentum transfer

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From the uncertainty principle this \( \Delta E \) corresponds to an indetermination in proper time given by

\[
\Delta \tau \cdot \Delta E_2 \approx \hbar
\]

that boosted to the nucleus frame gives a coherence length

\[
\Delta x_{coh} \approx \frac{p_{2lab}}{m_2} \cdot \Delta \tau = k_{coh} \frac{p_{2lab}}{m_2 \nu_2} \hbar
\]

Interactions with small momentum transfers are somewhat suppressed because it takes a long time/space for them to take place and other scatterings over that space can destroy the coherence.
Nuclear potential for pions

For pions, a complex resonant nuclear potential can be defined out of the $\pi$-nucleon scattering amplitude to be used in conjunction with the Klein-Gordon equation

$$\left[(\omega-V_c)^2 - 2\omega U_{\text{opt}} - K^2 \right] \Psi = m_\pi^2 \Psi$$

In coordinate space (the upper/lower signs refer to $\pi^+$/ $\pi^-$):

$$2\omega U_{\text{opt}}(\omega,r) = -\beta(\omega,r) + \frac{\omega}{2M} \nabla^2 \alpha(\omega,r) - \nabla \cdot \frac{\alpha}{1 + g\alpha(\omega,r)} \nabla$$

$$\beta = 4\pi \left[ \left(1 + \frac{\omega}{M}\right) b_0(\omega) + b_1(\omega) \frac{N-Z}{A} \right] \rho(r) + \left(1 + \frac{\omega}{2M}\right) B_0(\omega) \rho^2(r)$$

$$\alpha = 4\pi \left[ \left(1 + \frac{\omega}{M}\right) c_0(\omega) + c_1(\omega) \frac{N-Z}{A} \right] \rho(r) + \left(1 + \frac{\omega}{2M}\right) C_0(\omega) \rho^2(r)$$

Using standard methods to get rid of the non-locality, in momentum space

$$2\omega U_{\text{opt}}(\omega,r) = -\beta - \frac{K^2}{1 + g\alpha} + \frac{\omega}{2M} \nabla^2 \alpha$$

$$K^2 = k_0^2 + V_c^2 - 2\omega V_c^2 - 2\omega U_{\text{opt}}(\omega,r) = \frac{k_0^2 + V_c^2 - 2\omega V_c^2 + \beta - \frac{\omega}{2M} \nabla^2 \alpha}{1 - \overline{\alpha}}$$

$$\overline{\alpha} = \frac{\alpha}{1 + g\alpha}$$
Nuclear potential for pions: examples

The real part of the pion optical potential for $\pi^+$ on $^{16}$O (left) and $\pi^-$ on $^{208}$Pb (right) as a function of radius for various pion energies (MeV)
Pions: nuclear medium effects

Free $\pi N$ interactions $\Rightarrow$ Non resonant channel
$\Rightarrow$ $P$-wave resonant $\Delta$ production

$\pi + N \rightarrow \Delta$ in nuclear medium $\Rightarrow$ decay
$\Rightarrow$ elastic scattering, charge exchange

$\Rightarrow$ reinteraction $\Rightarrow$ Multibody pion absorption

Assuming for the free resonant $\sigma$ a Breit-Wigner form with width $\Gamma_F$

$$\sigma_{\text{Free}} = \frac{8\pi}{p_{\text{cms}}^2} \frac{M_\Delta^2 \Gamma_F^2 (p_{\text{cms}})}{(s - M_\Delta^2)^2 + M_\Delta^2 \Gamma_F^2 (p_{\text{cms}})}$$

An "in medium" resonant $\sigma$ ($\sigma_{\text{res}}^A$) can be obtained adding to $\Gamma_F$ the imaginary part of the (extra) width arising from nuclear medium

$$\frac{1}{2} \Gamma_T = \frac{1}{2} \Gamma_F - \text{Im} \Sigma = \Sigma_{\text{qe}} + \Sigma_t + \Sigma_3$$

Two-body absorption

The in-nucleus $\sigma_t^A$ takes also into account a two-body $s$-wave absorption $\sigma_s^A$ derived from the optical model

$$\sigma_t^A = \sigma_{\text{res}}^A + \sigma_t^{\text{Free}} - \sigma_{\text{res}}^{\text{Free}} + \sigma_s^A$$

$$\sigma_s^A (\omega) = \frac{4\pi}{p} \left(1 + \frac{\omega}{2m}\right) \text{Im} B_0 (\omega)$$
On the left, the total absorption cross section (red curve) for $\pi^+$ on an isoscalar target at central nuclear density. The various contributions, both 2- and 3-body resonant ($p$-wave $\leftrightarrow \Delta$) and 2-body s-wave are also shown.
Pion absorption:

Pion absorption cross section on Gold and Bismuth in the $\Delta$ resonance region (Black dots: exp data, colored ones: model)

Emitted proton spectra at different angles, 160 MeV $\pi^+$ on Ni
Proton spectra extend up to 300 MeV!!

Symbols: exp. Data
Histos: model
Pion-Nucleus scattering:

Angular distribution for $^{58}\text{Ni}(\pi^+,\pi^0X)$ (charge exchange, left) and $^{58}\text{Ni}(\pi^+,\pi^+X)$ (inelastic scattering, right) at 160 MeV. **Histos model, symbols** exp. data
Pion production close to DPM thr.

Pion production from proton interactions on Be and Cu at 12.3 GeV. Emitted pion spectra at different angles in the range $\cos\theta = 1 - 0.5$. Dots: data (BNL910 expt.), histograms: Fluka.

$\pi^-$ from pBe

$\pi^+$ from pCu
Pion production examples:

\[ p + \text{Pb} \rightarrow \pi^+ + X \ (1.6 \text{ GeV}) \]

Double differential pion production cross sections \( \frac{d^2\sigma}{dE d\Omega} \) for 1.6 GeV p on Pb (left), and for 4 GeV/c p on Al (right)

Symbols: exp. data*  
Histograms: model

*CEA-N-2670  
PLB159, 1 (1985)
Preequilibrium emission

For $E > \pi$ production threshold $\rightarrow$ only (G)INC models

At lower energies a variety of preequilibrium models $\Rightarrow$ share the excitation energy among many nucleons/holes

Two leading approaches

The quantum-mechanical multistep model:
Very good theoretical background
Complex, difficulties for multiple emissions

The semiclassical exciton model
Statistical assumptions
Simple and fast
Suitable for MC

Statistical assumption:
any partition of the excitation energy $E^*$ among $N$, $N = N_h + N_p$, excitons has the same probability to occur

Step: nucleon-nucleon collision with $N_{n+1} = N_n + 2$ ("never come back approximation")
Chain end = equilibrium = $N_n$ sufficiently high or excitation energy below threshold

$N_1$ depends on the reaction type and cascade history
**Preequilibrium in Peanut:** (based on M. Blann GDH cast in a Monte Carlo form)

Preequilibrium emission probability for particle type $x$ at energy $\varepsilon$ in $n$-th step:
$(n_{px}=\text{number of particle-like excitons of type } x)$

$$P_{x,n}(\varepsilon) d\varepsilon = n_{px} \frac{\rho_n(U,\varepsilon) gd\varepsilon}{\rho_n(E)} \frac{r_c(\varepsilon)}{r_c(\varepsilon) + r_+(\varepsilon)}$$

where the density (MeV$^{-1}$) of exciton states is given by:

$$\rho_n(E) = \frac{g(gE)^{n-1}}{n!(n-1)!}$$

($(g=$single state density$)$

the emission rate in the continuum:

$$r_c = \sigma_{\text{inv}} \frac{\varepsilon}{g_x} \frac{(2s+1)8\pi m}{\hbar^3}$$

and the reinteraction rate:

$$r_+(\varepsilon) = f_{\text{Pauli}}(\varepsilon, E_F)[\rho_p \sigma_{xp} + \rho_n \sigma_{xn}] \left[ \frac{2(\varepsilon + V)}{m} \right]^{1/2}$$

(or from optical potential)

**GDH:** $\rho$, $r$, $E_F$ "local" averages on the trajectory

constrained exciton state densities are used for small exciton numbers.
Modified GDH in PEANUT

- $\sigma_{\text{inv}}$ from systematics
- Correlation/formation zone/hardcore effects on reinteractions

$$\frac{r_c(\varepsilon)}{r_c(\varepsilon) + r_+(\varepsilon)} \Rightarrow P_c^{h\tau} + P_c^{co} + P_c^{\text{std}}$$

$P_c^{h\tau}$ = escape probability in zone $h\tau \leq \max(\tau, \text{hardcore})$

$P_c^{co}$ = escape/total prob. in zone (correlation - $h\tau$) (reinteraction only on non-correlated nucleon specie)

$P_c^{\text{std}}$ = standard escape/total in remaining zone

- Constrained exciton state densities configurations 1p-1h, 2p-1h, 1p-2h, 2p-2h, 3p-1h and 3p-2h
- Energy dependent form for $g$
- Angular distributions: fast particle approximation
Modified GDH in PEANUT II

- Position dependent parameters = point-like values:
  - First step: \( n_h \) holes generated in the INC step at positions \( x_i \)

\[
\rho_{n_h}^{loc} = \sum_{i=1}^{n_h} \rho(\bar{x}_i) \quad n_h
\]

- When looking at reinteractions: consider neighborhood:

\[
\rho_{n_h}^{nei} = \frac{n_h \rho_{n_h}^{loc} + \rho^{ave}}{n_h + 1} \quad n_h
\]

\[
E_{F_{n_h}}^{loc} = \sum_{i=1}^{n_h} E_F(\bar{x}_i) \quad n_h
\]

- Subsequent steps: go towards average quantities

\[
\rho_{n_h+1}^{loc} = \rho_{n_h}^{nei} \quad \rho_{n_h+1}^{nei} = \rho_{n_h}^{nei}
\]
Compound nucleus:

Early theories of nuclear reactions based on compound-nucleus formation and decay:

- Nucleon-(target) nucleon interactions not described individually, but on the basis of a mean-field potential (parametrized optical-potential model, OPM)
- Captured incoming nucleon occupies a (single-particle) state of the nucleon+nucleus system
- Decay of the compound nucleus available through all open channels, competing on equal footing according to statistical considerations
- Compound-nucleus formation and decay processes are decoupled

*Successful at low energies, however inadequate for projectile energies > 10-20 MeV*
Compound nucleus: evaporation

- After many collisions and possibly particle emissions, the residual nucleus is left in a highly excited “equilibrated” state.
- De-excitation can be described by statistical models which resemble the evaporation of “droplets”, actually low energy particles (p, n, d, t, 3He, alphas...) from a “boiling”soup characterized by a “nuclear temperature”.
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- De-excitation can be described by statistical models which resemble the evaporation of “droplets”, actually low energy particles (p, n, d, t, 3He, alphas...) from a “boiling” soup characterized by a “nuclear temperature”.

- Formation and decay are supposed to be decoupled → residuals with the same A, Z, U (exc. Energy), (and Jπ) will decay the same regardless of the process which generated them.

- The process is terminated when all available energy is spent → the leftover nucleus, possibly radioactive, is now “cold”, with typical recoil energies ~ MeV.

- For heavy nuclei the excitation energy can be large enough to allow breaking into two major chunks (fission).

- Since only neutrons have no barrier to overcome, neutron emission is strongly favoured.
Equilibrium particle emission in Fluka

- Evaporation: Weisskopf-Ewing approach
  - ~600 possible emitted particles/states ($A<25$) with an extended (heavy) evaporation/fragmentation formalism
  - Full level density formula with level density parameter $A,Z$ and excitation dependent
  - Inverse cross section with proper sub-barrier
  - Analytic solution for the emission widths (neglecting the level density dependence on $U$, taken into account by rejection
  - Emission energies from the width expression with no. approx.

- Fission: past, improved version of the Atchison algorithm, now
  - $\Gamma_{\text{fis}}$ based of first principles, full competition with evaporation
  - Improved mass and charge widths
  - Myers and Swiatecki fission barriers, with exc. en. dependent level density enhancement at saddle point

- Fermi Break-up for $A<18$ nuclei
  - ~50000 combinations included with up to 6 ejectiles

- $\gamma$ de-excitation: statistical + rotational + tabulated levels
Equilibrium particle emission (evaporation, fission and nuclear break-up)

From statistical considerations and the detailed balance principle, the probabilities for emitting a particle of mass $m_j$, spin $S_j\hbar$ and energy $E$, or of fissioning are given by*:

(i, f for initial/final state, Fiss for fission saddle point)

\[ P_j = \frac{(2S_j + 1)m_j c}{\pi^2 \hbar^3} \int_{V_j}^{U_i - Q_j - \Delta_f} \rho_f(U_f) \rho_i(U_i) \sigma_{\text{inv}}(E) EdE \]

\[ P_{\text{Fiss}} = \frac{1}{2 \pi \hbar} \int_0^{U_i - B_{\text{Fiss}}} \rho_{\text{Fiss}}(U_i - B_{\text{Fiss}} - E) \rho_i(U_i) dE \]

• $\rho$'s: nuclear level densities
• $U$'s: excitation energies
• $V$'s: possible Coulomb barrier for emitting a particle type $j$
• $B_{\text{Fiss}}$: fission barrier
• $Q$’s: reaction $Q$ for emitting a particle type $j$
• $\sigma_{\text{inv}}$: cross section for the inverse process
• $\Delta$’s: pairing energies

*Weisskopf-Ewing approach

November, 1st, 2018 Alfredo Ferrari, ISAPP
Equilibrium particle emission (evaporation, fission and nuclear break-up)

From statistical considerations and the detailed balance principle, the probabilities for emitting a particle of mass $m_j$, spin $S_j$, $\hbar$ and energy $E$, or of fissioning are given by*:

(i, f for initial/final state, Fiss for fission saddle point)

- Probability per unit time of emitting a particle $j$ with energy $E$

\[
P_j = \frac{(2S_j + 1)m_j c}{\pi^2 \hbar^3} \int_{V_j}^{U_i - Q_j - \Delta_f} \frac{\rho_f(U_f)}{\rho_i(U_i)} \sigma_{inv}(E) EdE
\]

- Probability per unit time of fissioning

\[
P_{Fiss} = \frac{1}{2 \pi^3 \hbar} \int_{0}^{U_i - B_{Fiss}} \frac{\rho_{Fiss}(U_i - B_{Fiss} - E)}{\rho_i(U_i)} dE
\]

- $\rho$'s: nuclear level densities
- $U$'s: excitation energies
- $V_j$'s: possible Coulomb barrier for emitting a particle type $j$
- $B_{Fiss}$: fission barrier
- $Q_j$'s: reaction Q for emitting a particle type $j$
- $\sigma_{inv}$: cross section for the inverse process
- $\Delta$'s: pairing energies

Neutron emission is strongly favoured because of the lack of any barrier

Heavy nuclei generally reach higher excitations because of more intense cascading
Statistical (evaporation) models are known to work poorly for light nuclei. An alternative, better performing, description of light nuclei de-excitation can be obtained with the Fermi break-up mechanism. The probability of splitting a nucleus \( A, Z \), with excitation \( U \) into \( n \) fragments of given masses, \( m_i \), spins, \( s_i \), … is given by:

\[
P_n(E_{\text{kin}}) = S_n G_n \left( \frac{V}{(2\pi \hbar)^3} \right)^{n-1} \left( \prod_{i=1}^{n} \frac{m_i}{M_{A,Z} + U} \right)^{3/2} \frac{(2\pi)^{3/2(n-1)}}{\Gamma \left[ \frac{3}{2}(n-1) \right]} \left( E_{\text{kin}} - E_{\text{Coul}} \right)^{3n/2-5/2}
\]

\[
S_n = \prod_{i=1}^{n} (2s_i + 1) \quad G_n = \prod_{k=1}^{l} \frac{1}{n_k!} \quad \sum_{k=1}^{l} n_k = n
\]

… however it implicitly assumes that the emission takes place in \( L=0 \).

Significant improvement can be obtained when the compound nucleus spin and parity, \( J^\pi \), are known:
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$$P_n(E_{kin}) = S_n G_n \left( \frac{V}{(2\pi\hbar)^3} \right)^{n-1} \left( \prod_{i=1}^{n} m_i \right)^{3/2} \left( \frac{2\pi}{M_{A,Z} + U} \right)^{3/2} \frac{(2\pi)^{3/2}(n-1)}{\Gamma\left[\frac{3}{2}(n-1)\right]} (E_{kin} - E_{Coul})^{3n/2-5/2}$$

$$S_n = \prod_{i=1}^{n} (2S_i + 1) \quad G_n = \prod_{k=1}^{l} \frac{1}{n_k !} \quad \sum_{k=1}^{l} n_k = n$$

...however it implicitly assumes that the emission takes place in $L=0$.

Significant improvement can be obtained when the compound nucleus spin and parity, $J^\pi$, are known:

- The minimum orbital momentum, $L_{min}$, required to match $J^\pi$ is computed
Fermi Break-up in FLUKA:

Statistical (evaporation) models are known to work poorly for light nuclei. An alternative, better performing, description of light nuclei de-excitation can be obtained with the Fermi break-up mechanism. The probability of splitting a nucleus $A$, $Z$, with excitation $U$ into $n$ fragments of given masses, $m_i$, spins, $s_i$, ... is given by:

$$P_n(E_{\text{kin}}) = S_n G_n \left( \frac{V}{(2\pi\hbar)^3} \right)^{n-1} \left( \frac{n!}{\prod_{i=1}^{n} m_i} \right)^{3/2} \left( \frac{(2\pi)^{3/2}(n-1)}{\Gamma(3/2(n-1))} \right) \left( E_{\text{kin}} - E_{\text{Coul}} \right)^{3n/2-5/2}$$

$$S_n = \prod_{i=1}^{n} (2s_i + 1) \quad G_n = \prod_{i=1}^{n} \frac{1}{n_i!} \quad \sum_{k=1}^{l} n_k = n$$

... however it implicitly assumes that the emission takes place in $L=0$.

Significant improvement can be obtained when the compound nucleus spin and parity, $J^\pi$, are known:

- The minimum orbital momentum, $L_{\text{min}}$, required to match $J^\pi$ is computed
- $S_n$ is restricted to the subset of spin combinations compatible with $L_{\text{min}}$
Fermi Break-up in FLUKA:

Statistical (evaporation) models are known to work poorly for light nuclei. An alternative, better performing, description of light nuclei de-excitation can be obtained with the Fermi break-up mechanism. The probability of splitting a nucleus $A$, $Z$, with excitation $U$ into $n$ fragments of given masses, $m_i$, spins, $s_i$, ... is given by:

$$P_n(E_{kin}) = S_n G_n \left( \frac{V}{(2\pi\hbar)^3} \right)^{-1} \left( \prod_{i=1}^{n} m_i \right)^{3/2} \left( \frac{2\pi}{M_{A,Z} + U} \right)^{3/2} \left( \frac{2\pi}{3/2(n-1)} \right)^{3n/2-5/2} \left( E_{kin} - E_{Coul} \right)^3$$

where:

- $S_n = \prod_{i=1}^{n} (2s_i + 1)$
- $G_n = \prod_{k=1}^{l} \frac{1}{n_k !}$
- $\sum_{k=1}^{l} n_k = n$

... however it implicitly assumes that the emission takes place in $L=0$.

Significant improvement can be obtained when the compound nucleus spin and parity, $J^\pi$, are known:

- The minimum orbital momentum, $L_{min}$, required to match $J^\pi$ is computed
- $S_n$ is restricted to the subset of spin combinations compatible with $L_{min}$
- If $L_{min} > 0$, then $E_{Coul} \rightarrow E_{Coul} + B_{centrifugal}$
Spin-parity in Fermi-Break-up: example

In FLUKA, for $A<16$, evaporation is substituted by Fermi break-up.
In cases where spin and parity of the residual nucleus are known, conservation laws, constraints on available configurations and centrifugal barrier (if $L=0$ is forbidden), are enforced in the fragment production.

Straightforward example: photonuclear reaction in the GDR region.

Effect: residual nuclei production.

Application: background from $\mu$ induced showers in underground experiments.

$^{12}\text{C} + \gamma$ in GDR
$\rightarrow J^\pi = 1^-$
$\rightarrow 3\alpha$ and $\alpha + ^{9}\text{Be}$ impossible in $L=0$
$\rightarrow$ Factor 3 on $^{11}\text{C}$ production.
From INC to (G)INC: $^{90}$Zr(p,xp) @ 80.5 MeV
From INC to (G)INC: $^{90}$Zr(p,xp) @ 80.5 MeV
From INC to (G)INC: $^{90}$Zr(p,xp) @ 80.5 MeV

Plain INC (a la Bertini)

$d^2\sigma/dE_d\Omega$

Data: connected symbols
Model: “isolated” symbols
From INC to (G)INC: $^{90}$Zr(p,xp) @ 80.5 MeV

Plain INC (a la Bertini)

Plain INC plus preequilibrium

Data: connected symbols
Model: “isolated” symbols

d$^2\sigma$/dEdΩ
From INC to (G)INC: $^{90}\text{Zr}(p,\text{xp}) @ 80.5\text{ MeV}$

Plain INC (a la Bertini)

Plain INC plus preequilibrium

GINC calculation with no quantum effect, apart Pauli blocking

d$^2\sigma$/d$E$d$\Omega$

Data: connected symbols
Model: “isolated” symbols
From INC to (G)INC: $^{90}$Zr(p,xp) @ 80.5 MeV

Plain INC (a la Bertini)

Plain INC plus preequilibrium

GINC calculation with no quantum effect, apart Pauli blocking

Full GINC calculation

d$^2\sigma$/dEd$\Omega$

Data: connected symbols
Model: “isolated” symbols
Pion production from proton interactions on Cu ($\pi^+$, 585 MeV) and Be ($\pi^-$, 730 MeV). Emitted pion spectra at different angles.

Thin target examples: neutrons

Double differential cross section $d^2\sigma/dE d\Omega$ for $C(p,xn) @ 113$ MeV, thin target
Data: NSE112, 78 (1992)

Double differential cross section $d^2\sigma/dE d\Omega$ for $C(p,xn) @ 113$ MeV, thin target
Data: NSE112, 78 (1992)

Double differential cross section $d^2\sigma/dE d\Omega$ for $Pb(p,xn) @ 3$ GeV → thin target
Data: NST32, 827 (1995)
Thin target examples: neutrons

Cascade, preequilibrium neutrons
Thin target examples: neutrons

Cascade, preequilibrium neutrons

Evaporation neutrons
Thin target examples: neutrons

Remember!! Up to a few hundred's MeV non-elastic p/n-nucleus interactions are made up by individual elastic nucleon-nucleon interactions!
Example of fission/evaporation


Data
Model
Model after cascade
Model after preeq

Quasi-elastic
Spallation
Deep spallation
Fission
Evaporation
Fragmentation
Example of fission/evaporation

1 A GeV $^{208}\text{Pb} + p$ reactions


For 1 GeV $p$ (n very similar) on Pb (out of 50000 trials):

- $<n> = 14$ (~4 “fast”) (p,xn), “x” up to (p,33n...)
- $<p> = 2.3$ (~2 “fast”) (p,xp), “x” up to (p,8p...)
- $<\pi> = 0.33$ (p,x$\pi$), “x” up to (p,3$\pi$...)
- $<d> = 0.5$ (~0.4 “fast”) $P_{\text{fiss}} \sim 9\%$
- $<t> = 0.25$ (~0.2 “fast”)  
- $<\alpha> = 0.7$ (~0.15 “fast”)
As soon as the energy is going above few tens of MeV, models must deal, besides neutrons and γ’s, with protons (and to lesser extent d and α), above 200 MeV with π’s …
Heavy ion interaction models in FLUKA

DPMJET-III
DPMJET (R. Engel, A. Fedynitch, J. Ranft, S. Roesler\textsuperscript{1}): Nucleus-Nucleus interaction model. Used in many Cosmic Ray shower codes. Based on the Dual Parton Model and formation zone Glauber cascade, like the high-energy FLUKA h-A event generator.

Modified and extended version\textsuperscript{3} of rQMD-2.4
rQMD-2.4 (H. Sorge et al\textsuperscript{2}) Cascade-Relativistic QMD model
Successfully applied to relativistic A-A particle production

BME (BoltzmannMasterEquation)
FLUKA implementation of BME from E. Gadioli et al (Milan)

FLUKA Evaporation-fission-fragmentation module handles fragment deexcitation

Tested and benchmarked in h-A reactions

(Projectile-like evaporation is responsible for the most energetic fragments)

3ASR 34, 1302 (2004)
FLUKA with heavy ion generators:

(Projectile) fragmentation is critical for heavy ion interactions!!!

Fragment charge cross section for 1.05 GeV/n Fe ions on Al (left) and Cu (right).

Ion fragmentation: examples

Experimental (red symbols) and computed (Fluka, blue symbols) charge changing cross sections for $^{56}\text{Fe} @ 3$ GeV/n and $^{35}\text{Cl} @ 1$ GeV/n on Carbon and Aluminum, as a function of the fragment atomic number $Z$.
Fragmentation: 400 MeV/n Ar ions

Exp. and MC (FLUKA) charge fragment cross sections for 1 GeV/n $^{48}$Ti ions

November, 1st, 2018

Alfredo Ferrari, ISAPP
ElectroMagneticDissociation cross section:

- The EMD cross section can be expressed as (N_i being the equivalent photon number for the i_th multipolarity already integrated over all impact parameters):

\[
\sigma_{EMD} \approx \sum_{i} \int_{E_{min}}^{E_{max}} \sigma_{i, \gamma A} (E_\gamma) n_i(E_\gamma) \frac{dE_\gamma}{E_\gamma} \quad \sigma_{\gamma A} (E_\gamma) = \sum_{i} \sigma_{i, \gamma A} (E_\gamma)
\]

- In practice it can be shown (e.g., Bertulani PhysRep163) that the dominant components are E1 and E2, with E2 being (for ions) important mostly at low energies, while M1 is always negligible:

\[
\sigma_{EMD} \approx \int_{E_{min}}^{E_{max}} \left[ \sigma_{E_1, \gamma A} (E_\gamma) n_{E_1} (E_\gamma) + \sigma_{E_2, \gamma A} (E_\gamma) n_{E_2} (E_\gamma) \right] \frac{dE_\gamma}{E_\gamma}
\]

- ...the equivalent (virtual) photon number is customarily expressed as a function of the adiabacity parameter \( \xi \), defined as (\( \omega = E_\gamma, b = \) impact parameter):

\[
\xi = \frac{\omega b}{\hbar \beta \gamma} \equiv \frac{\omega}{\omega_{max}} \quad \omega_{max} = \frac{\hbar \beta \gamma}{b_{min}} \quad b^{ion}_{min} = R_{AB} + \delta_{Ruth} \quad b^{lepton}_{min} = \lambda_c = \frac{\hbar}{\beta \gamma m_{lepton}}
\]

- ...finally:

\[
\xi^{ion} = \frac{\omega (R_{AB} + \delta_{Ruth})}{\hbar \beta \gamma} \quad \xi^{ion}_{min} = \frac{\omega_{thresh} (R_{AB} + \delta_{Ruth})}{\hbar \beta \gamma} \quad \xi^{lepton} = \frac{\omega}{\beta^2 \gamma^2 m_{lepton}}
\]

\[
\xi^{lepton}_{min} = \frac{\omega_{thresh}}{\beta^2 \gamma^2 m_{lepton}}
\]
ElectroMagneticDissociation cross section:

- The EMD cross section can be expressed as \( N_i \) being the equivalent photon number for the \( i \)th multipolarity already integrated over all impact parameters:

\[
\sigma_{\text{EMD}} \approx \sum_i E_{\text{max}}^E \int \sigma_{i\gamma A} \left( E_\gamma \right) n_i \left( E_\gamma \right) \frac{dE_\gamma}{E_\gamma} \sigma_{\gamma A} \left( E_\gamma \right) = \sum_i \sigma_{i\gamma A} \left( E_\gamma \right)
\]

- In practice it can be shown (e.g. Bertulani PhysRep163) that the dominant components are E1 and E2, with E2 being (for ions) important mostly at low energies, while M1 is always negligible:

- …the equivalent (virtual) photon number is customarily expressed as a function of the adiabacity parameter \( \xi \), defined as \( \xi = \frac{\omega b}{\hbar \beta \gamma} \equiv \frac{\omega}{\omega_{\text{max}}} \frac{\hbar \beta \gamma}{b_{\text{min}}} \), with \( b_{\text{min}} = R_{\text{AB}} + \delta_{\text{Ruth}} \), \( b_{\text{lepton}} = \lambda_e = \frac{\hbar}{\beta \gamma m_{\text{lepton}}} \):

- … finally:

\[
\xi_{\text{ion}} = \frac{\omega \left( R_{\text{AB}} + \delta_{\text{Ruth}} \right)}{\hbar \beta \gamma} \quad \xi_{\text{lepton}} = \frac{\omega}{\beta^2 \gamma^2 m_{\text{lepton}}} \quad \xi_{\text{min}} = \frac{\omega_{\text{thresh}}}{\beta^2 \gamma^2 m_{\text{lepton}}}
\]

This is the part which needed to be modified (mostly for E2) to ~match the 2nd Born and nuclear finite size calculations.
The equivalent photon numbers for E1 and E2 can be expressed as a function of $\xi$ as (PhysRep163) $(N_{E1} = n_{E1}/E_\gamma, N_{E2} = n_{E2}/E_\gamma)$:

$$N_{E1}(E_\gamma) = \frac{2Z^2 \alpha_{fsc}}{E_\gamma \pi \beta^2} \left[ \xi K_0(\xi)K_1(\xi) - \frac{1}{2} \xi^2 \beta^2 (K_1^2(\xi) - K_0^2(\xi)) \right]$$

$$N_{E2}(E_\gamma) = \frac{2Z^2 \alpha_{fsc}}{E_\gamma \pi \beta^4} \left[ 2(1-\beta)^2 K_1(\xi)^2 + \xi(2-\beta^2)^2 K_0(\xi)K_1(\xi) - \frac{1}{2} \xi^2 \beta^4 (K_1^2(\xi) - K_0^2(\xi)) \right]$$

Please note the $1/\beta^4$ wrt $1/\beta^2$ low energy dependence of $N_{E2}$ wrt $N_{E1}$ which makes E2 important at low energies even though in general $\sigma_{E2} \ll \sigma_{E1}$.
**Electromagnetic dissociation**

... nuclear and, mostly, Electromagnetic Dissociation collisions on LHC machine elements or at IP’s produce a variety of (excited), possibly radioactive, fragments in flight.

![Graph](image)

**Symbols:** exp. data
**Lines:** Fluka

- Very peripheral collisions
- Break-up of one of the colliding nuclei in the electromagnetic field of the other nucleus

**Total EMD, 1 n, 2 n, and nuclear cross sections as a function of the effective $\gamma$ factor**

**Alice:** $\sqrt{s_{nn}}=2.8$ TeV

**November, 1st, 2018 Alfredo Ferrari, ISAPP**
158 GeV/n Pb ion fragmentation: EMD and nuclear

Fragment charge cross section for 158 AGeV Pb ions on various targets.


and from C.Scheidenberger et al. PRC70, 014902 (2004), (red squares),

yellow histos are FLUKA (with DPMJET-III) predictions: purple histos are the EMD
181\text{Ta}(e^-, n)180\text{Ta}: E1+E2 (FLUKA)/EPA vs exp.

Examples of electronuclear reaction:

181\text{Ta}(e^-, n)180\text{Ta}:

- Blue symbols: data from JPG13,515
- Blue line: FLUKA E1+E2+HO
- Red symbols: FLUKA with EPA (Equivalent Photon Approximation)
Examples of electronuclear reaction:

$^{181}\text{Ta}(e^-,n)^{180}\text{Ta}$:
- Blue symbols: data from JPG13,515
- Blue line: FLUKA E1+E2+HO
- Red symbols: FLUKA with EPA (Equivalent Photon Approximation)

$C(e^-,n)^{11}\text{C}$:
- Green symbols: data from ZPA281,35
- Green curve: FLUKA E1+E2+HO
Further test of LeptoNuclear interactions:

Electro(Positro)Fission on $^{238}$U

- Blue symbols exp. data for $^{238}$U($e^{-},f$) from various sources (ZPA292,285; PRC14,1499; NPA378,237), blue line FLUKA
- Red symbols exp. data (NPA378,237) for $^{238}$U($e^{+},f$), red line FLUKA
(Anti)Neutrinos in FLUKA:

- $\nu N$ QuasiElastic (from $\sim 0.1$ GeV upward):
  - Following Llewellyn Smith formulation
  - $M_A = 1.03$, $M_V = 0.84$
  - Lepton masses accounted for
- $\nu N$ Resonance production
  - From Rein-Sehgal formulation
  - Keep only $\Delta$ production
  - Non-resonant background term assumed to come from DIS
- $\nu N$ Deep Inelastic Scattering
  - NunDIS model (developed ad hoc for FLUKA)
- $\nu N$ interactions embedded in PEANUT for $\nu A$ (Initial State and Final State effects)
- Only for Argon: Fermi/GT absorption of few-MeV (solar) neutrinos on $^{40}\text{Ar}$
- Products of the neutrino interactions can be directly transported in the detector (or other) materials
- Used for all ICARUS simulations/publications

Comparison with data on total cross section

Isoscalar $\nu_\mu$ - Nucleon total CC cross section

Fluka (lines) with two pdf options

vs

Experimental data

FLUKA can currently manage (anti)$\nu$-$A$ interactions from $\sim0.1$ GeV up to 1000 TeV

November, 1st, 2018

Alfredo Ferrari, ISAPP
Charm production in neutrino interactions

- Ratio of the charm to total cross sections
- Results of NUNDIS simulation with $M_c = 1.35$ GeV (curves) and experimental data: E531 (open circles) and CHORUS-2011 (filled squares).
Extrapolation from $Q^2 = 1.0 \text{ GeV}^2$ to $Q^2 = 0$

Solid lines: M. Bertini et al. 1996 (Default in NUNDIS)

$$F_2(x, Q^2) = A\left[1 + \epsilon \ln\left(Q^2(1/x - 1) + M^2\right)\right] \ln\left(1 + Q^2/(Q^2 + a^2)\right).$$

Dashed lines: Donnachie-Landshoff 1994

$$F_2(x, Q^2) \sim A x^{-0.0808} \left(\frac{Q^2}{Q^2 + a}\right)^{1.0808} + B x^{0.4525} \left(\frac{Q^2}{Q^2 + b}\right)^{0.5475}.$$

Data/cuves scaled for clarity, factors from 1 to 128

November, 1st, 2018
Alfredo Ferrari, ISAPP
Single pion production in $\nu N$ CC interactions:

Chains from $\nu$ DIS: One quark-diquark chain if interaction on valence quark
One quark-diquark plus one $q$-$q\bar{q}$ chain if int. on sea quark

Low-mass chain treatment of hadronization
improvements in the RES-DIS transition

Strong DIS contribution/sensitivity in 1pion
Distribution of total deposited energy in the ICARUS T600 detector

- CNGS numuCC events (~20 GeV $E_\nu$ peak)
- Same reconstruction in MC (FLUKA) and Data
- Neutrino fluxes from FLUKA CNGS simulations
- Absolute agreement on neutrino rate within 6%

$\langle E_{\text{Data}} \rangle = 10.1 \pm 0.5 \text{ GeV}$

$\text{rms} = 9.6 \text{ GeV}$

$\langle E_{\text{MC}} \rangle = 10.1 \pm 0.2 \text{ GeV}$

$\text{rms} = 11.6 \text{ GeV}$

Other GCR related examples
Negative muons at floating altitudes: CAPRICE94

Open symbols: CAPRICE data
Full symbols: FLUKA

primary spectrum normalization ~AMS-BESS

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Alfredo Ferrari, ISAPP
The contributions of the various Z groups include the primary ion contributions and those of all products generated in their interactions.
Solar Energetic Particle (SEP) events

- Integrated fluence: up to $10^{11}$ (nucleon/cm$^2$), $E > 1$ MeV / n
- Large variations in spectra
- Variable composition: mostly protons (~90%) and α’s (~9%), but ions up to Iron are not negligible
- Variable duration, from hours to days
- Rise time from minutes to hours
- Dose equivalent up to ~Sv, highly dependent on organ, shielding, and SEP intensity/spectrum
- Unpredictable
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Nightmare scenarios for (manned) missions beyond Earth low orbits

(from Mewaldt, ICRC2005)
20-Sep-2005: open space doses after 1 g/cm² Al, Skin

Organ Dose: 1.363±0.004 Gy
(Uncollided contr.: 1.250±0.004 Gy)

Organ Dose Equivalent: 6.16±0.03 Sv
(Uncollided contr.: 5.61±0.02 Sv)
20-Sep-2005: open space doses after 1 g/cm² Al, Skin

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Whole body Dose Eq.: 1.83±0.05 Sv
20-Sep-2005: open space doses after 1 g/cm² Al, Skin

- Organ Dose: \(1.363 \pm 0.004\) Gy
  - (Uncollided contr.: \(1.250 \pm 0.004\) Gy)

- Organ Dose Equivalent: \(6.16 \pm 0.03\) Sv
  - (Uncollided contr.: \(5.61 \pm 0.02\) Sv)

- Whole body Dose Eq.: \(1.83 \pm 0.05\) Sv

SEP 28-Oct-2003:

- Whole body Dose Eq.: \(4.9 \pm 0.1\) Sv !!
Neutrons on the ER-2 plane at 21 km altitude

Measurements:
Goldhagen et al., NIM A476, 42 (2002)

Note one order of magnitude difference depending on latitude

FLUKA calculations:
Aircraft dosimetry applications:


Ambient dose equivalent from neutrons at solar maximum on commercial flights from Seattle to Hamburg and from Frankfurt to Johannesburg.

Solid lines: FLUKA simulation

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Dosimetry applications: doses to aircrew and passengers

Commercial flight doses: (Pelliccioni et al. RPD93, 101 (2001))

Simulated (FLUKA, red) and measured (blue, NIMA422, 621, 1999) ambient dose equivalent for various altitudes (scaled by one decade) and geomagnetic cut-off's.
The neutron albedo from GCR’s at 400 km altitude*

Computed with a full 3D FLUKA simulation of the whole Earth, the atmosphere, and of the geomagnetic field

Important for satellites