Correlations in ultra-relativistic nuclear collisions with strings

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Radiation via strings

Emission from Strings $\Rightarrow$ Correlations

Fluctuations of strings $\Rightarrow$ Fluctuations in $\eta$!

Lund model [Andersson], dual parton model [Capella et al.]
Monte-Carlo codes: HIJING, PYTHIA, AMPT...

Goal: Describe fluctuations semianalytically in a generic approach $\Rightarrow$ Constraints for models?
Description of rapidity spectra (1/2)

Motivation
Wounded Quark Model
String Models
Correlations
Summary

exp. data: [PHOBOS-collaboration].

Wounded Quark Model: \[ \frac{dN_{\text{ch}}}{d\eta} = \langle N_A \rangle f(\eta) + \langle N_B \rangle f(-\eta) \]

[Białas, Czyż, Furmański, Fiałkowski, Słomiński, Zieliński].
Universality of \( f(\eta) \)

\[ f_{\text{AuAu}}(\eta) \]

**symmetric part:**

\[ f_{\text{AuAu}}(\eta) \]

**antisymmetric part:**

\[ f_{\text{dAu}}(\eta) \]

**entire profile:**

\[ f_{\text{dAu}}(\eta) \]

\[ \langle N_A \rangle \langle N_B \rangle \] radiate independently:

\[ \frac{dN_{\text{ch}}}{d\eta} = \langle N_A \rangle f(\eta) + \langle N_B \rangle f(-\eta) \]

\[ \langle N_A \rangle, \langle N_B \rangle \ldots \text{ from Glauber Model} \]

Monte-Carlo generator GLISSANDO.

[Broniowski, Rybczyński, Bożek, Stefanek].

cf. [Barej, Bzdak, Gutowski]

[Broniowski, Rybczyński, Bożek, Stefanek].

Motivation

Wounded Quark Model

String Models

Correlations

Summary
Wounded Quark Model: \[
\frac{dN_{\text{ch}}}{d\eta} = \langle N_A \rangle f(\eta) + \langle N_B \rangle f(-\eta)
\]

[Białas, Czyż, Furmański, Fiałkowski, Słomiński, Zieliński].
Description of $f(\eta)$ (1/2)

**Original Model:** 1 end-point fluctuates! [Broniowski, Bożek]

end point 1: rapidity $y_1$ fluctuates;
distribution $g(y_1)$

end point 2: rapidity fixed to beam rapidity $y_b$

**String:**
uniform profile of radiation:
$s(\eta, y_1) = \theta(y_1 < \eta < y_b)$

\[
f(\eta) = \int_{-y_b}^{y_b} dy_1 g(y_1) \theta(y_1 < \eta < y_b)
= \int_{-y_b}^{\eta} g(y_1) dy_1
\]

$\Rightarrow f(\eta) = G(\eta) - G(-y_b)$

with $G(y) = \int_{-y}^{y} g(x) dx$. 
Alternative model: 2 end-points fluctuate:

For end points $y_1$ and $y_2$,
Analogously:

Distributions: $g_1(y_1)$ and $g_2(y_2)$,
CDF’s $G_1(y_1)$ and $G_2(y_2)$.

$\Rightarrow f(\eta) = \omega \{ G_1(\eta)[1 - G_2(\eta)] + G_2(\eta)[1 - G_1(\eta)] \}$

$\Rightarrow$ Ambiguous solutions $G_1$ and $G_2$!
Ambiguity of string end points

CDF of string-end points:

string-end-point distribution:

Possible cases:

- \( g_1 = g_2 \)
- intermediate
  here: \( g_1 \) from PDF parametrization; \( g_2 \) matched to \( f(\eta) \)!
- disjoint support of \( g_1 \) and \( g_2 \)

Disjoint case: Limiting case of all possibilities!
Ambiguity of string end points

CDF of string-end points:

string-end-point distribution:

Possible cases:

- $g_1 = g_2$
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  - here: $g_1$ from PDF parametrization; $g_2$ matched to $f(\eta)$!
- disjoint support of $g_1$ and $g_2$

Disjoint case: Limiting case of all possibilities!
2-particle-emission

\[ f(\eta) = \omega \{ G_1(\eta)[1 - G_2(\eta)] + G_2(\eta)[1 - G_1(\eta)] \} \]

\[ \downarrow \]

\[ \text{generalize} \]

\[ \downarrow \]

\[ f_2(\eta_1, \eta_2) = \omega^2 \{ G_1(\min(\eta_1, \eta_2))[1 - G_2(\max(\eta_1, \eta_2))] + G_2(\min(\eta_1, \eta_2))[1 - G_1(\max(\eta_1, \eta_2))] \} \]
Correlations

\[ C_{AB}(\eta_1, \eta_2) = 1 + \frac{\text{cov}_{AB}(\eta_1, \eta_2)}{f_{AB}(\eta_1)f_{AB}(\eta_2)} \]

\[ g_1 = g_2 : \]

**disjoint case:**

\[
\text{string-end-point fluctuations} \equiv \text{cov}_{AB}^*(\eta_1, \eta_2)
\]

\[
\text{cov}_{AB}(\eta_1, \eta_2) = \langle N_A \rangle \text{cov}(\eta_1, \eta_2) + \langle N_B \rangle \text{cov}(-\eta_1, -\eta_2) + \text{var}(N_A)f(\eta_1)f(\eta_2) + \text{var}(N_B)f(-\eta_1)f(-\eta_2) + \text{cov}(N_A, N_B)[f(\eta_1)f(-\eta_2) + f(-\eta_1)f(\eta_2)]
\]

**event by event fluctuations**

\[
f_{AB}(\eta) = \langle N_A \rangle f(\eta) + \langle N_B \rangle f(-\eta)
\]
Legendre coefficient $a_{11}$

[Bzdak, Teany]

$$a_{nm} = \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} \frac{1}{N_C} C(\eta_1, \eta_2) T_n \left( \frac{\eta_1}{Y} \right) T_m \left( \frac{\eta_2}{Y} \right)$$

with $Y = 1$ and $N_C = \int_{-Y}^{Y} \frac{d\eta_1}{Y} \int_{-Y}^{Y} \frac{d\eta_2}{Y} C(\eta_1, \eta_2)$

$$N_+ = N_A + N_B$$
Contributions from string fluctuations

\[ a^*_{nm} = \int_{-Y}^Y \frac{d\eta_1}{Y} \int_{-Y}^Y \frac{d\eta_2}{Y} \frac{1}{N_C} C^*(\eta_1, \eta_2) T_n \left( \frac{\eta_1}{Y} \right) T_m \left( \frac{\eta_2}{Y} \right) \]

\[ C^*_{AB}(\eta_1, \eta_2) = 1 + \frac{\text{cov}^*_{AB}(\eta_1, \eta_2)}{f_{AB}(\eta_1)f_{AB}(\eta_2)} \]

\[ \text{cov}_{AB}(\eta_1, \eta_2) = \text{cov}^*_{AB}(\eta_1, \eta_2) + \text{var}(N_A)f(\eta_1)f(\eta_2) + \text{var}(N_B)f(-\eta_1)f(-\eta_2) + \text{cov}(N_A, N_B)[f(\eta_1)f(-\eta_2) + f(-\eta_1)f(\eta_2)] \]

event by event fluctuations
Results for LHC-energies:


\[ g_1 = g_2 \]

disjoint case

ATLAS

ATLAS w/o SRC

\begin{align*}
0.0 & \quad 0.2 & \quad 0.4 & \quad 0.6 & \quad 0.8 \\
5. \times 10^{-5} & \quad 1. \times 10^{-4} & \quad 5. \times 10^{-5} & \quad 0.001 & \quad 0.005 & \quad 0.010
\end{align*}

centrality 

\( a_{11} \)

2-end fluctuations necessary!

Pb-Pb collisions:

- ATLAS data at \( \sqrt{s_{NN}} = 2.76 \) TeV
- model predictions for \( \sqrt{s_{NN}} = 5.02 \) TeV

SRC...short range correlations
• Semianalytic description of radiation in heavy-ion collisions.
• String fluctuations are dominant contribution to correlations.
• 2-end point fluctuations contribute to correlations → necessary for description of Pb-Pb data from the LHC!

Some details: [arXiv:1809.08666 [nucl-th]]

Thank you for your attention!
BACK-UP SLIDES
Correlations from string fluctuations

\[ C_{AB}^\ast(\eta_1, \eta_2) = \frac{\langle N_A \rangle \text{cov}(\eta_1, \eta_2) + \langle N_B \rangle \text{cov}(-\eta_1, -\eta_2)}{f_{AB}(\eta_1)f_{AB}(\eta_2)} \]

both \( g_1 = g_2 \):

with \( f_{AB}(\eta) = \langle N_A \rangle f(\eta) + \langle N_B \rangle f(-\eta) \)
Constraints $G_1$ & $G_2(1/3)$

\[ f(\eta) = \omega \left\{ G_1(\eta)[1 - G_2(\eta)] + G_2(\eta)[1 - G_1(\eta)] \right\} \]

$G_{1,2}(\eta) \in [0, 1]$ and continuous

\[ \Rightarrow \exists \eta_1^{(0)}, \eta_2^{(0)} : \]

\[ \eta_1^{(0)} : G_1(\eta_1^{(0)}) = \frac{1}{2}, \]

\[ \eta_2^{(0)} : G_2(\eta_2^{(0)}) = \frac{1}{2}. \]

\[ \Rightarrow \]

\[ \omega = 2f(\eta_1^{(0)}) = 2f(\eta_2^{(0)}). \]
Constraints $G_1$ & $G_2(2/3)$

With $H_i(\eta) = G_i(\eta) - \frac{1}{2}$:

$$f(\eta) = \omega \{ G_1(\eta)[1 - G_2(\eta)] + G_2(\eta)[1 - G_1(\eta)] \}$$

$$\Leftrightarrow$$

$$H_1(\eta)H_2(\eta) = \frac{1}{4} - \frac{1}{2\omega}f(\eta)$$

$$\Rightarrow$$

$$G_{1,2}(\eta) \in [0, 1]$$

$$\Leftrightarrow$$

$$H_{1,2}(\eta) \in [-1/2, 1/2]$$

$$\Rightarrow$$

$$f(\eta) \in [0, \omega]$$

$$f(\eta_{max}) \in [\omega/2, \omega]$$
Constraints $G_1$ & $G_2(3/3)$

\[ f(\eta_{\text{max}}) = \omega/2: \]
\[
H_1(\eta) = \sqrt{\frac{1}{4} - \frac{1}{2\omega} f(\eta) \text{sgn}(\eta - \eta_0) s(\eta)}
\]
\[
H_2(\eta) = \sqrt{\frac{1}{4} - \frac{1}{2\omega} f(\eta) \text{sgn}(\eta - \eta_0) / s(\eta)}
\]
We used $s(\eta) = 1$.

\[ f(\eta_{\text{max}}) = \omega: \]
\[
H_1(\eta) = -\frac{1}{2} \theta(\eta_0 - \eta) + \left[ \frac{1}{2} - \frac{1}{\omega} f(\eta) \right] \theta(\eta - \eta_0)
\]
\[
H_2(\eta) = -\left[ \frac{1}{2} - \frac{1}{\omega} f(\eta) \right] \theta(\eta_0 - \eta) + \frac{1}{2} \theta(\eta - \eta_0)
\]

intermediate case ($\omega/2 < f(\eta_{\text{max}}) < \omega$):
\[
H_2(\eta) = \frac{\frac{1}{4} - \frac{1}{2\omega} f(\eta)}{H_1(\eta)}
\]
for a given $H_1(\eta)$. 

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Scaling with $\langle N_+ \rangle$
Covariances

\[ \text{cov}(\eta_1, \eta_2) = f_2(\eta_1, \eta_2) - f(\eta_1)f(\eta_2) \]

**disjoint case:**

\[ g_1 = g_2 : \]

**intermediate:**