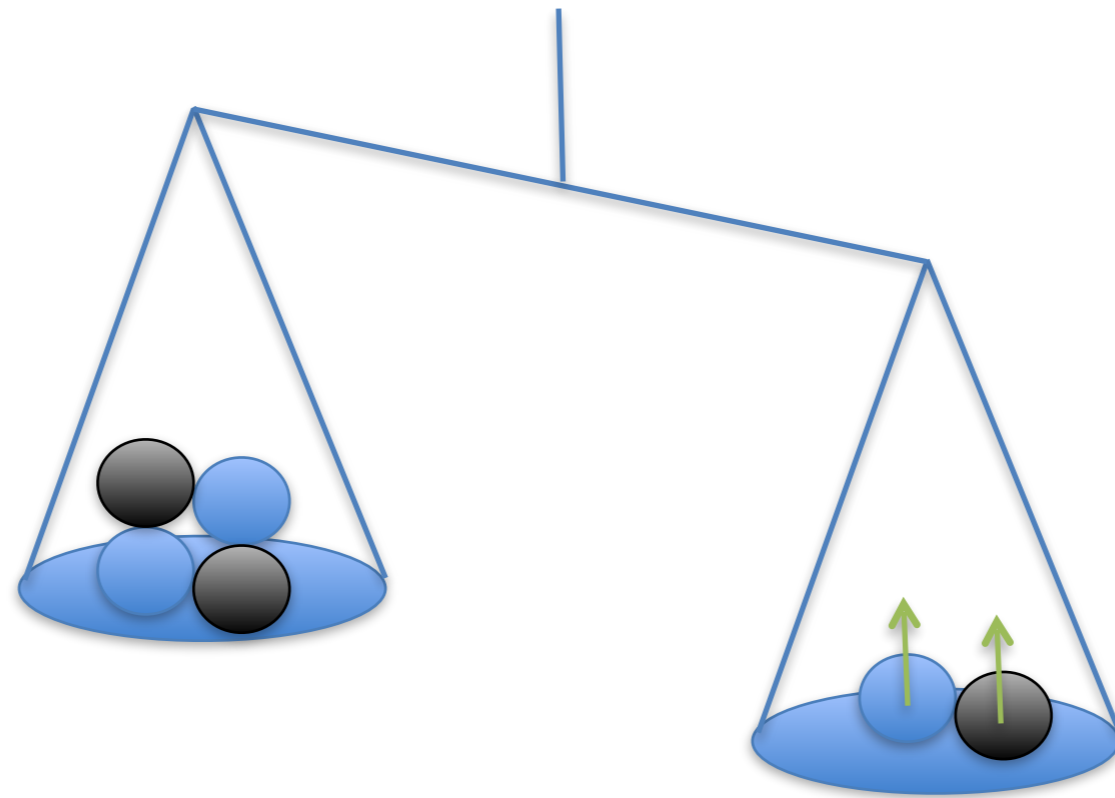


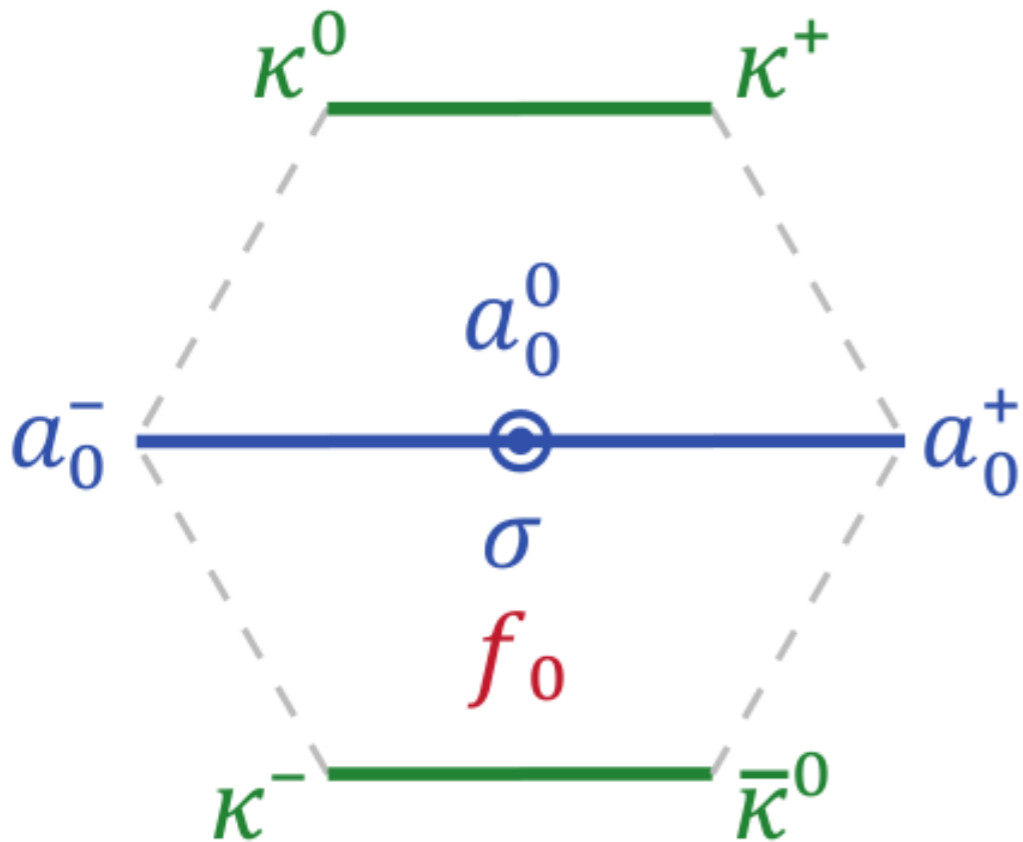
Light scalar/ heavy axialvector tetraquarks in a Dyson-Schwinger, Bethe-Salpeter approach



P. C. Wallbott, G. Eichmann, C. S. Fischer

Part I: The light scalars

Work on this topic



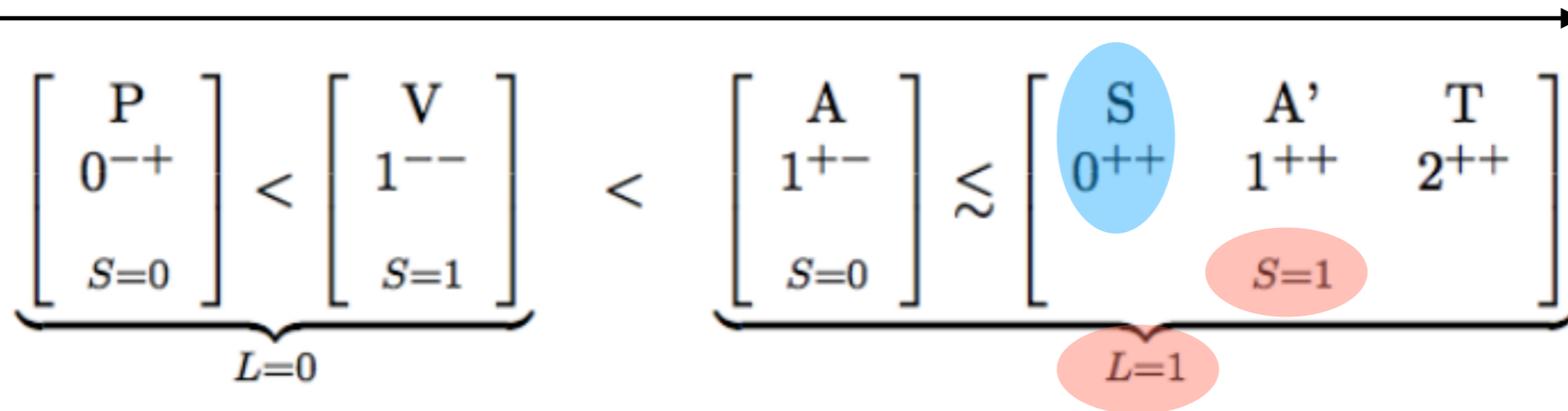
Diquark antidiquark, large N_c , Unitarized ChPT, quark models, ELSM, Lattice, ...

Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004, Pelaez 2004, Weinberg 2013, Cohen et al, 2014, Londergan et al 2013, Bicudo et al, 2010, Giacosa 2006, Parganlija et al, 2010, van Beveren et al, 2007, Briceno et al 2017

Light mesons in the quark model

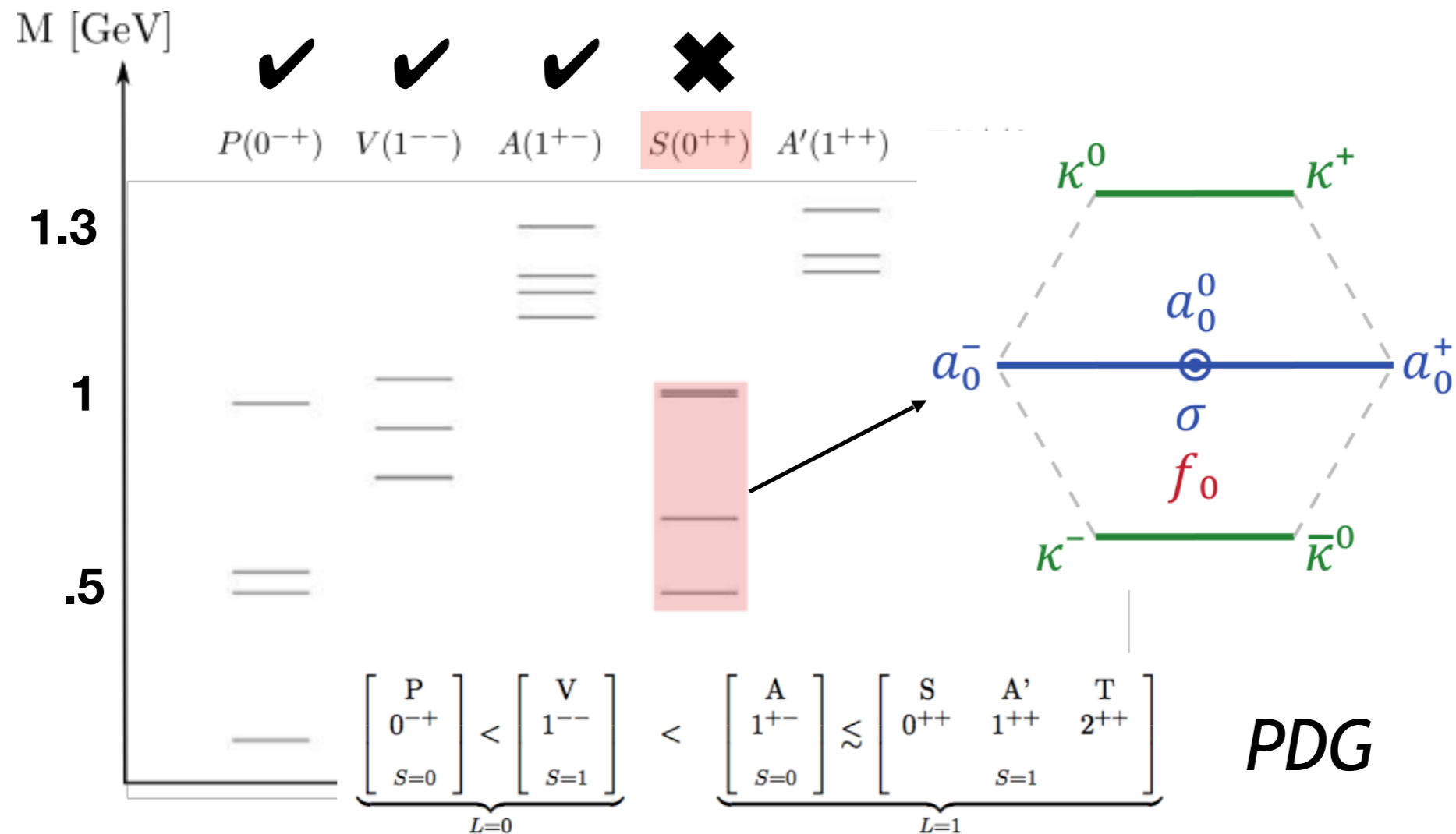
- $\mathbf{J} = \mathbf{L} + \mathbf{S}$, $P = (-1)^{L+1}$, $C = (-1)^{L+S}$
- 0^{++} scalars are p-waves
 - Scalars should have mass around 1.3 GeV

Mass



P: pseudoscalar V: Vector A: Axialvector S: Scalar T: Tensor

Observed lowest multiplets

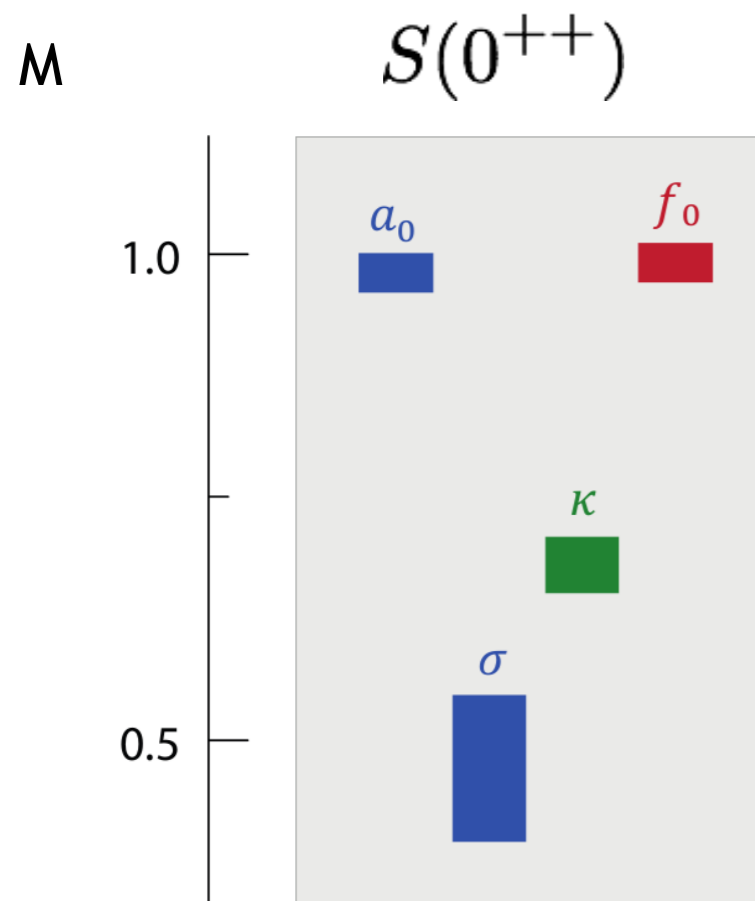


- Light scalar octet (red) has unexpectedly low mass
- 0.5-1 GeV

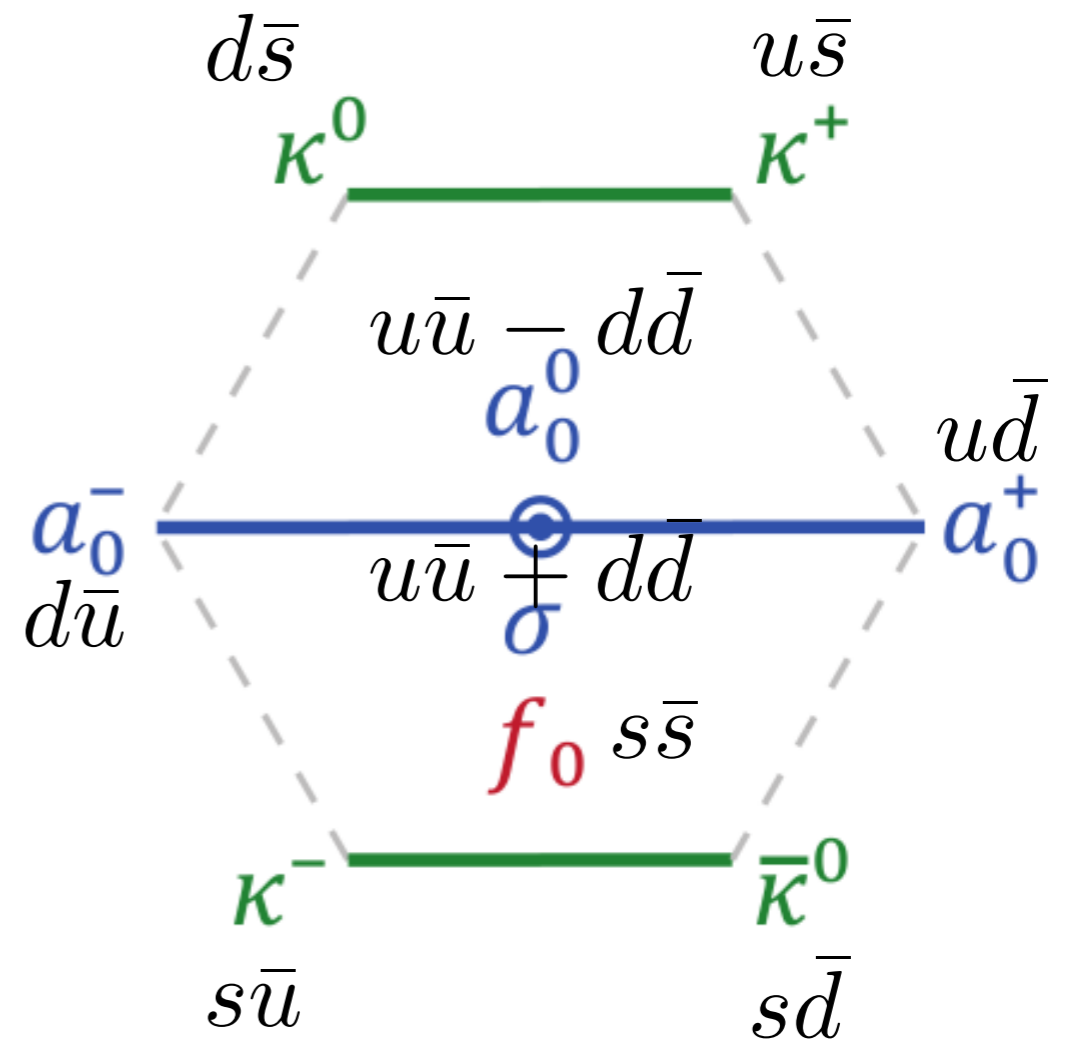
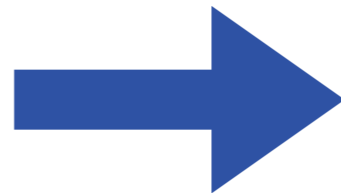
Masses inside scalar octet

here are the states:

$$\sigma, \kappa^\pm, \kappa^0, \bar{\kappa}^0, a^\pm, a^0, f_0$$



SU(3) assignment:
 $3 \otimes \bar{3} = 1 \oplus 8$

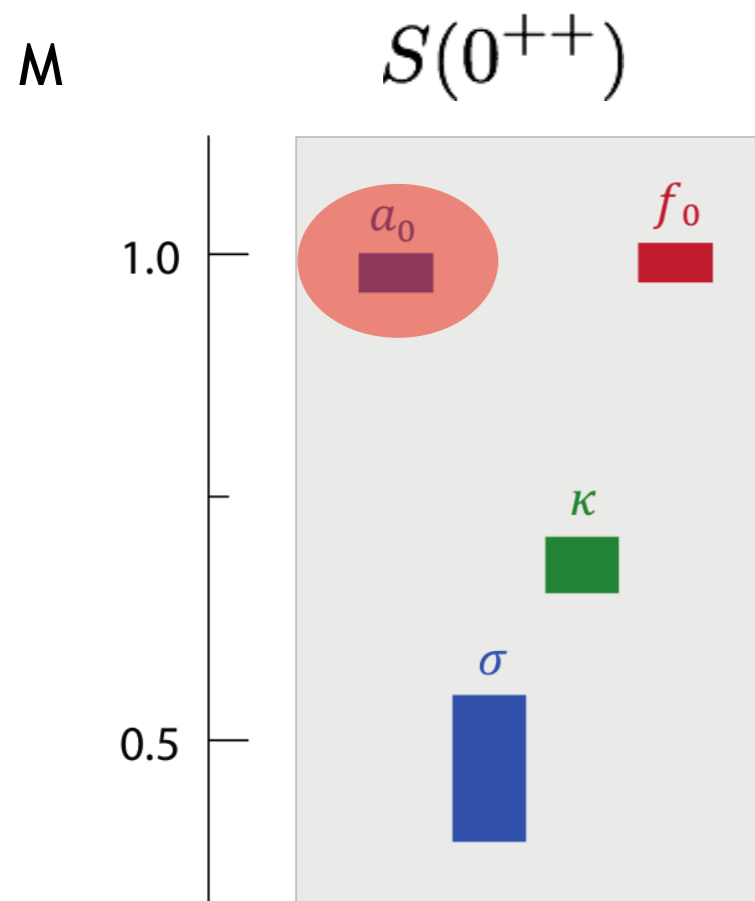


...and their experimental masses

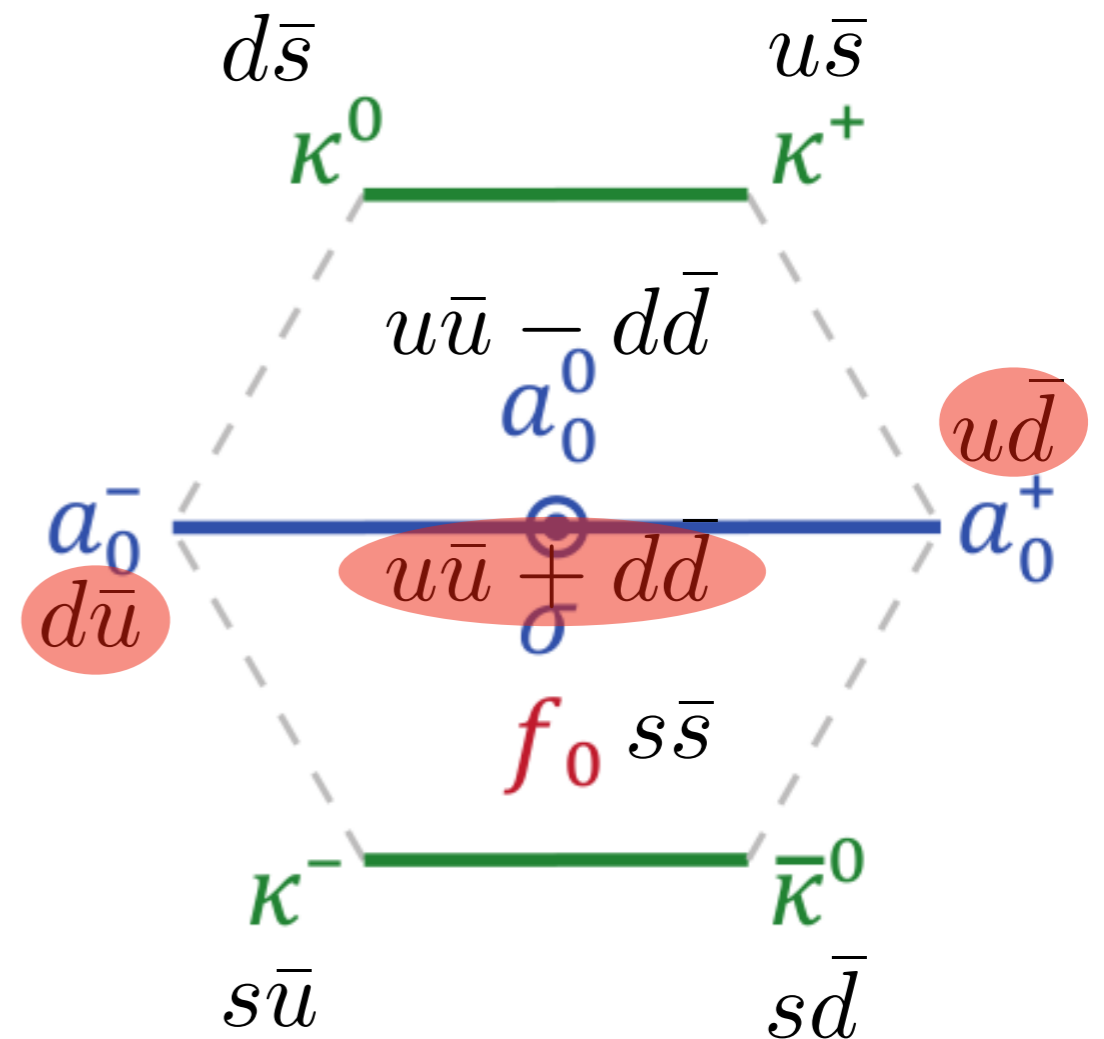
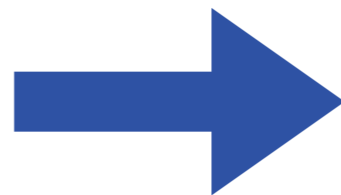
Masses inside scalar octet

Here are the states:

$$\sigma, \kappa^\pm, \kappa^0, \bar{\kappa}^0, a^\pm, a^0, f_0$$



SU(3) assignment:
 $3 \otimes \bar{3} = 1 \oplus 8$



...and their experimental masses

Iso triplet 1 GeV ?? (no strange quarks!), expect 500 MeV (like sigma)!

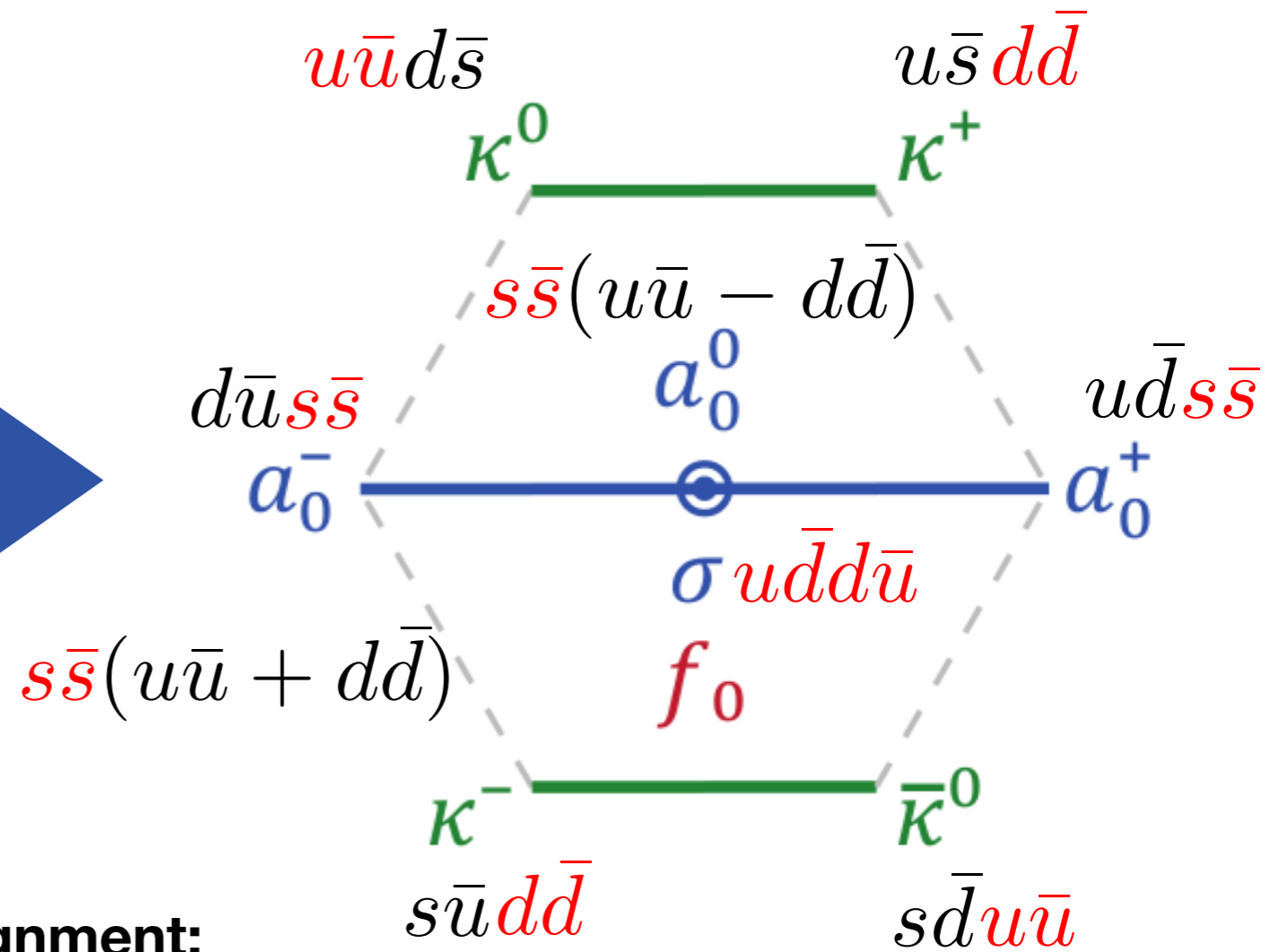
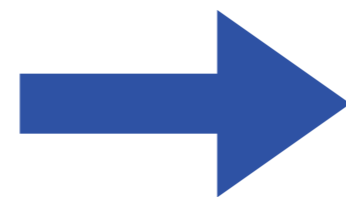
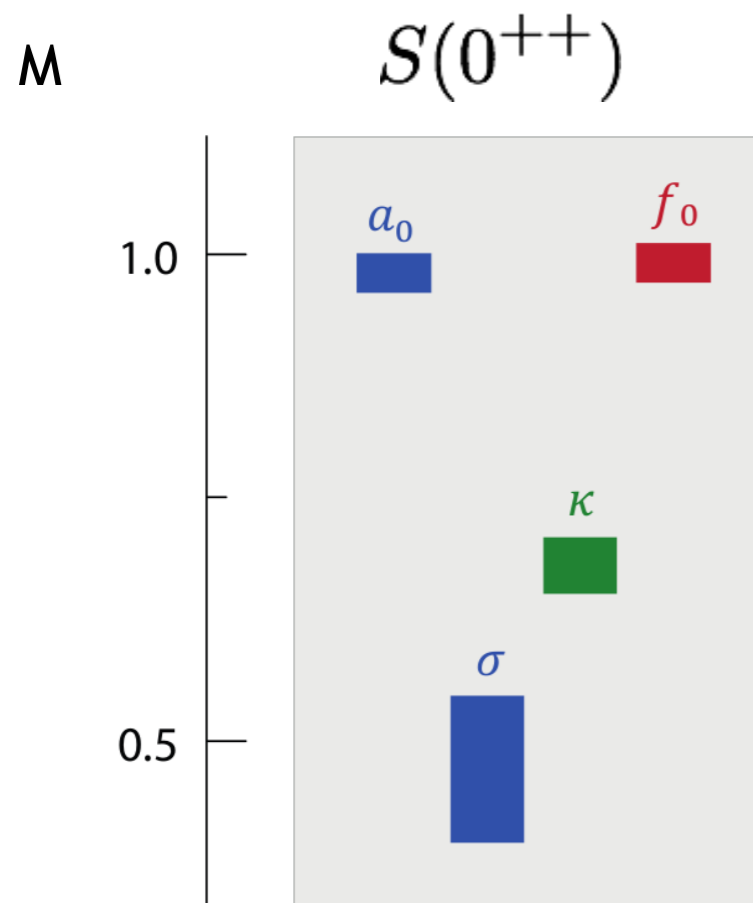
Mass ordering inside the multiplet is strange

Are they tetraquarks?

Here are the states:

$$\sigma, \kappa^0 \bar{\kappa}^0, \kappa^\pm, a_0, a^\pm, f_0$$

Jaffe, RJ: PRD, 15(1):267, 1977

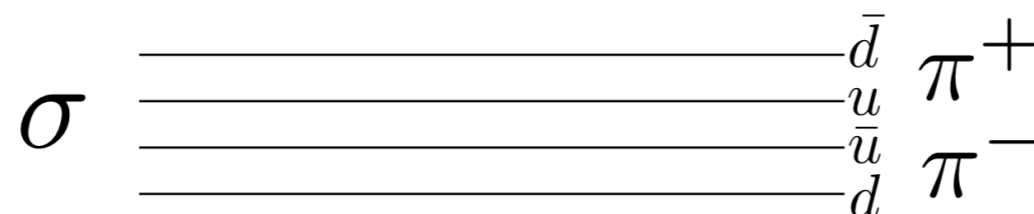
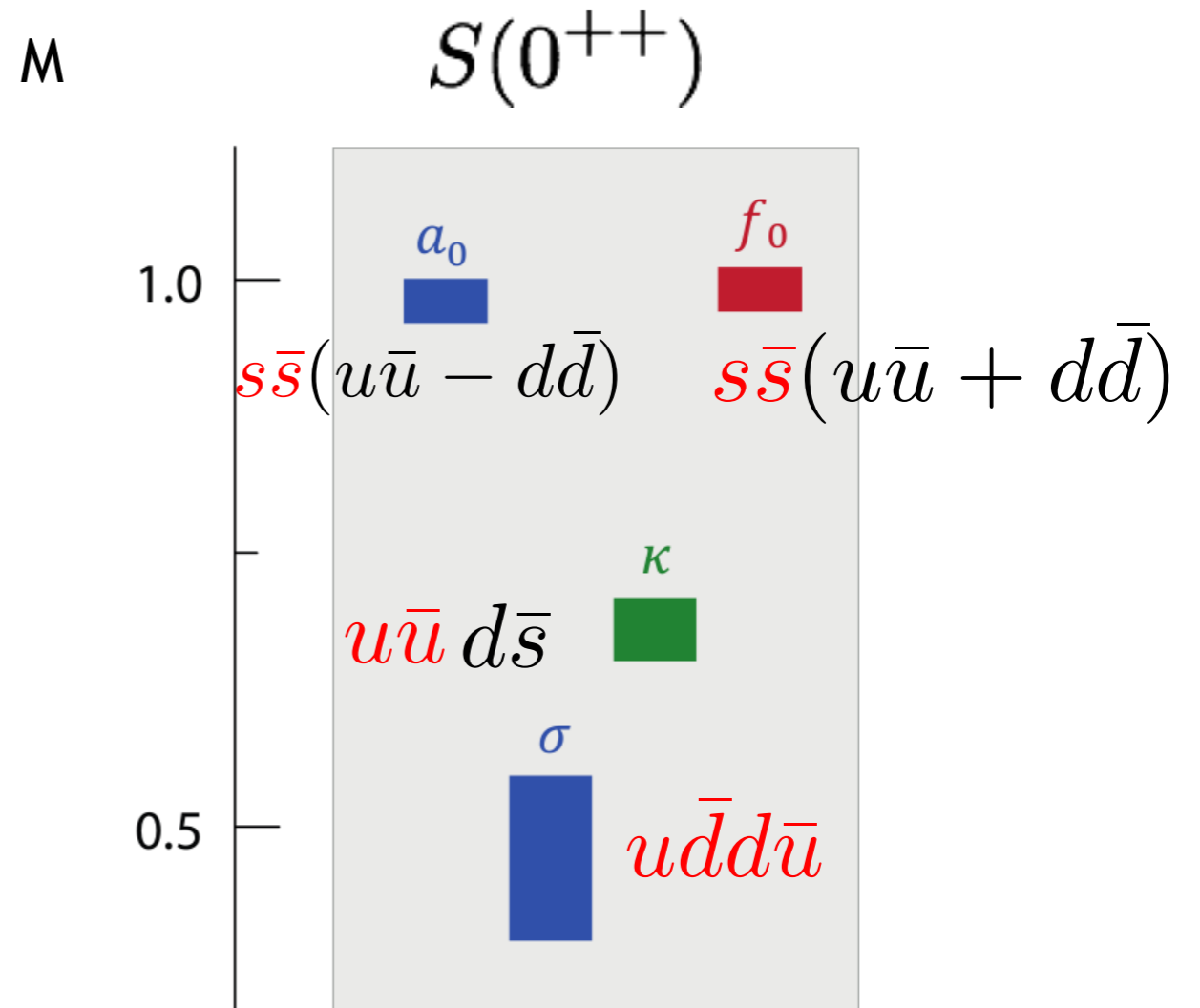


SU(3) assignment:

$$(3 \otimes 3) \otimes (\bar{3} \otimes \bar{3}) = 1 + 8 + \dots$$

Are they tetraquarks?

- a_0 mass is now higher due to internal $s\bar{s}$ pair
- The a_0 has $s\bar{s}$ and can have strangeness in decays
- Large width of σ and kappa can be justified (fall apart)
- Tetraquark can explain these features



The method

BSE/DSE framework

See talk of M. Huber!

- Calculate **quark propagator from DSE**
 - **Model gluon + vertex**
- Spectroscopy: **solve BSE for meson/baryon/tetraquark...** (eigenvalue problem)
 - Kernel for $qq/q\bar{q}$ from self energy (cut quark leg)
 - $J^{PC} \rightarrow \Gamma$

$$\text{---} \circ \text{---}^{-1} = \text{---}^{-1} + \text{---} \circ \text{---} \circ \text{---}$$

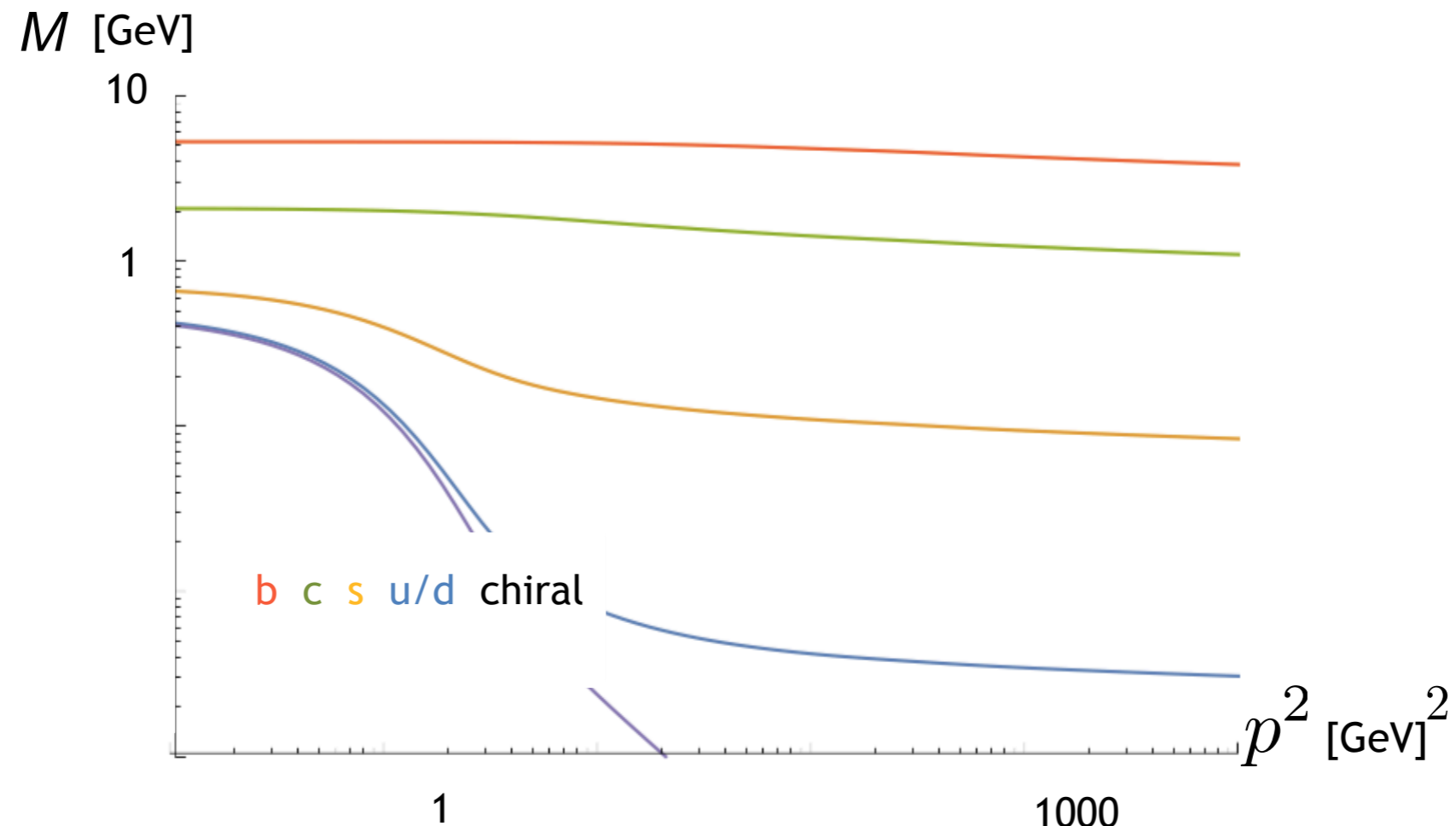
$$\text{---} \circ \text{---} \circ \text{---} = \text{---} \circ \text{---} \circ \text{---}$$

$$\Gamma = \tilde{K} \Gamma$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

$$\Gamma = color \otimes dirac \otimes flavor$$

Quark propagators

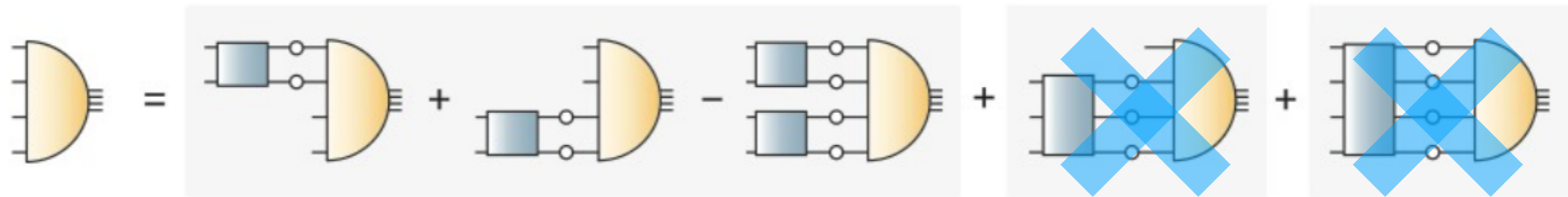


- Two dressing functions (A,M)
- **Dynamical mass generation**
- Massless chiral pion

$$S^{-1} = A(p^2)(M(p^2) - ip^\mu \gamma^\mu)$$

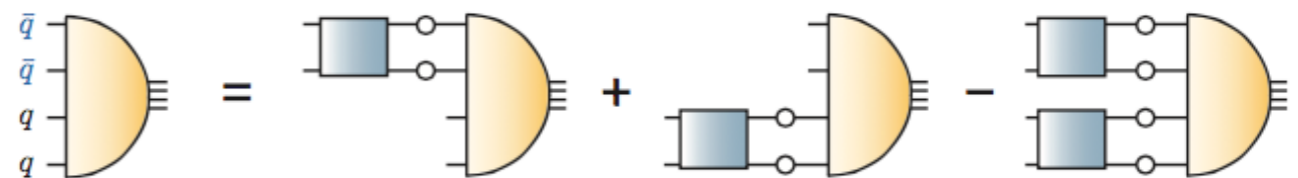
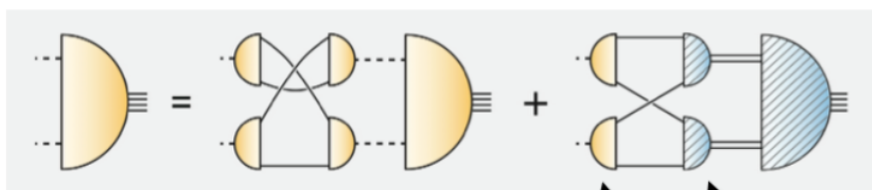
Tetraquark BSE(s)

A. Khvedelidze, A. Kvinikhidze, *Theor. Math. Phys.* 90, 62 (1992)



2 body approx

4 body



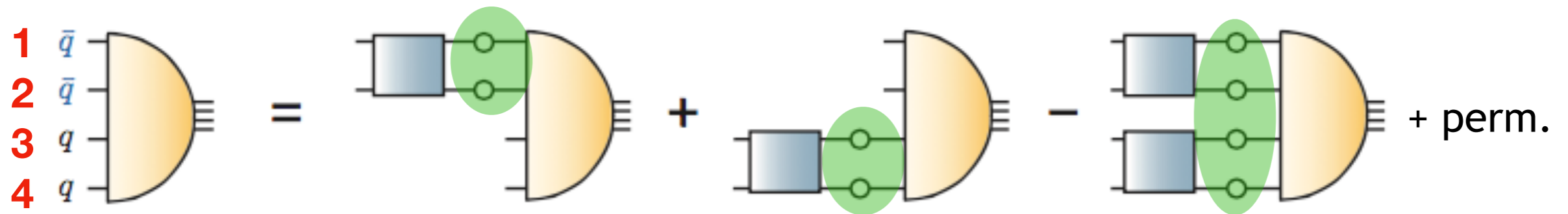
Meson Diquark

Eichmann et al, *PLB* 753:282-287, 2016
PCW, master thesis, 2016

Heupel et al, *PLB* 718:545-549, 2012

- 2 body (simplified meson/diquark propagators, ...) -> sigma
- 4 body (without 3/4 body forces) -> scalar octet
- Currently using 4 body for heavy lights!

4 body tetraquark BSE



- Permutations: $(13)+(24)-(13)(24) + (14) +(23) - (14)(23)$
 - Quark antiquark interactions in these “meson” topologies
- Calculate **quark propagators** from DSE
- Dressed **gluon exchange kernel** for RL + MT
- Tetraquark **amplitude**

tetraquark amplitude

- JPC determine Gamma
 - many tensor structures -> restrict to S-waves
 - dressing functions (f) depend on 9 Lorentz scalars
 - Use S4 variables (better for symmetries)

$$\Gamma = \sum_i^N f_i(\Omega) \tau_i \otimes color \otimes flavor$$

	J=0	J=1
# total	256	768
# S-wave	32	48

$$\Omega = \{p^2, q^2, k^2, p \cdot q, \dots\} = \{S_0, D, T_0, T_1\}$$

$$S_0 = \frac{p^2 + q^2 + k^2}{4} \quad T_i = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$$D = \frac{1}{4S_0} \begin{pmatrix} \sqrt{3}q^2 - p^2 \\ p^2 + q^2 - 2k^2 \end{pmatrix} \quad \boxed{S_4}$$

Lessons from the sigma

Lessons from the sigma

- Drop different S4 multiplets to see their impact:
 - 4 quark equation (> 1 GeV) without D
 - Much lighter when D is turned on, why?
 - Two body poles are generated in D variables!

$$S_0 = \frac{p^2 + q^2 + k^2}{4} \quad T_i = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$$D = \frac{1}{4S_0} \begin{pmatrix} \sqrt{3}q^2 - p^2 \\ p^2 + q^2 - 2k^2 \end{pmatrix} \quad \boxed{S_4}$$

(experiment: 500 MeV) $\longrightarrow m_\sigma$

$$f_i(S_0, \cancel{D}, \cancel{T}_1, \cancel{T}_2) \rightarrow 1.4 \text{ GeV}$$

$$f_i(S_0, \cancel{D}, \cancel{T}_1, T_2) \rightarrow 1.4 \text{ GeV}$$

$$f_i(S_0, \cancel{D}, T_1, \cancel{T}_2) \rightarrow 1.1 \text{ GeV}$$

$$f_i(S_0, D, T_1, \cancel{T}_2) \rightarrow \mathbf{0.35} \text{ GeV}$$

$\cancel{}$:= set to constant value

Lessons from the sigma

- We take away the main lessons:

1. The 2 body poles are important:

- Pion - pion bring mass down and create threshold
- Diquarks have small impact

2. Pole term with residue describes f well, govern the dynamics happening in the doublet D

$$\Gamma = \sum_i^N f_i(\Omega) \tau_i \otimes color \otimes flavor$$

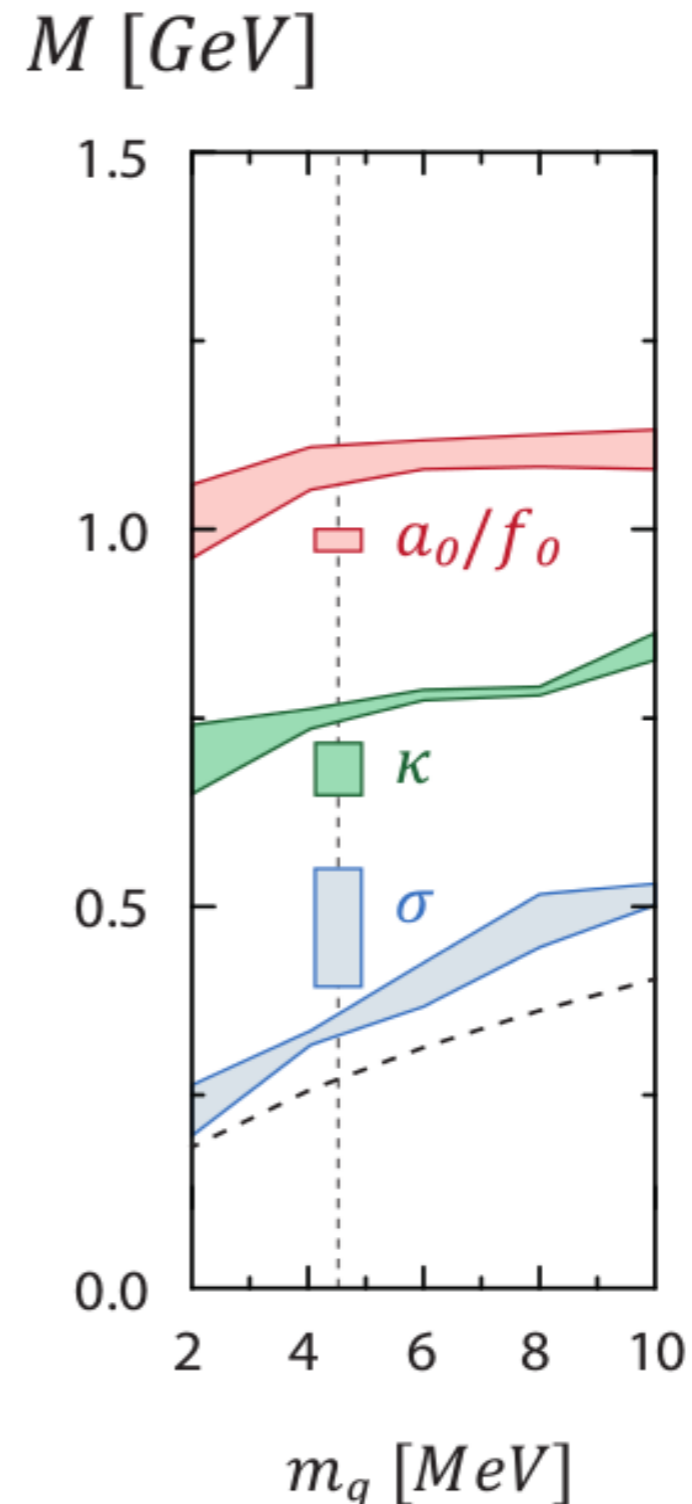
Structure of dressing functions:

$$f(\Omega) \approx \frac{f(S_0)}{(m_\pi^2 + q_+^2)(m_\pi^2 + q_-^2)} + \dots$$

**Results, yes we have
a light scalar multiplet!**

Results in BSE/DSE

- Box: PDG
Band: our result
- Correct mass ordering inside scalar octet
- Masses about right

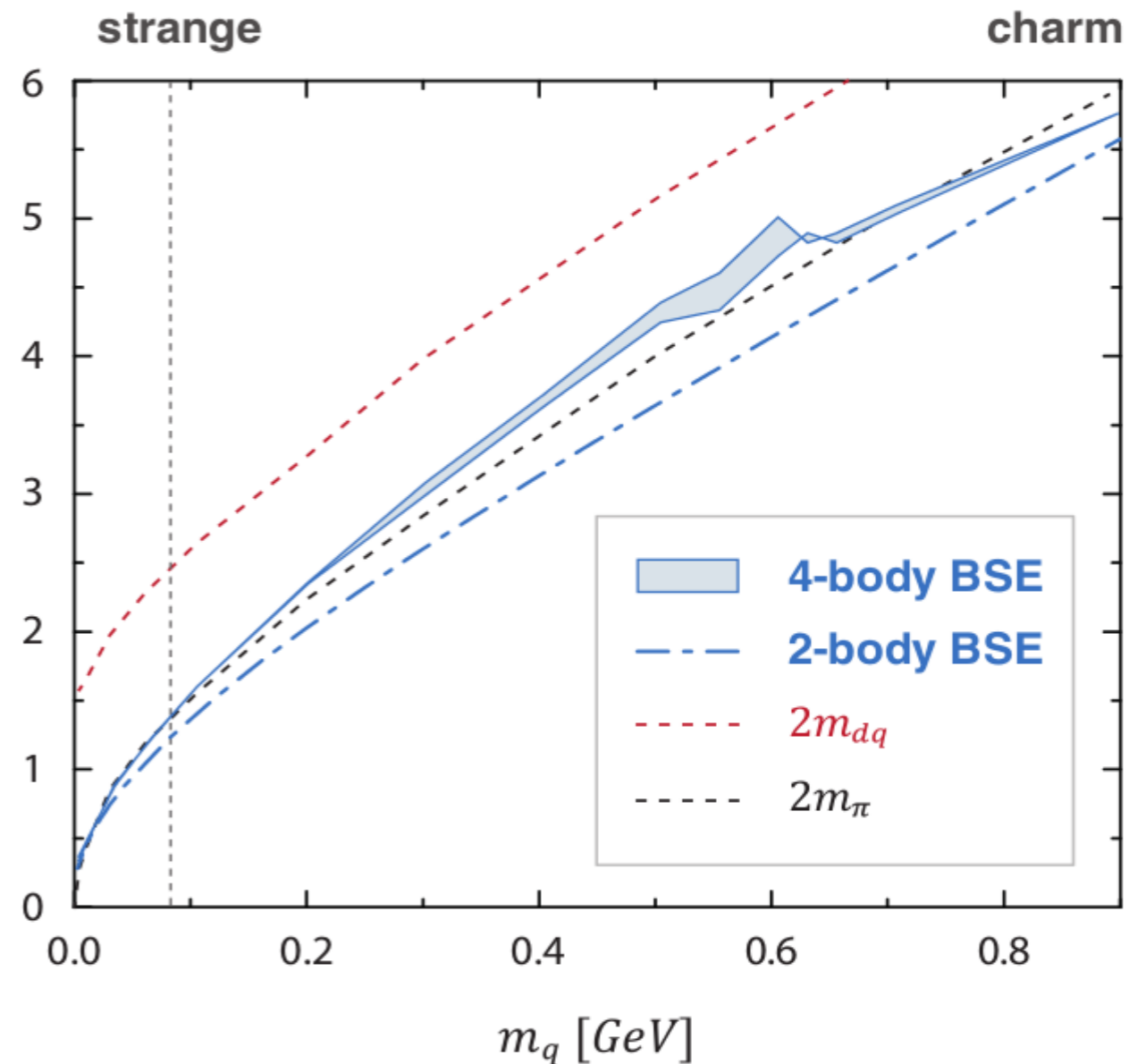


Eichmann et al PLB 753:282-287, 2016

Results in BSE/DSE

- σ is **resonance** in 4 body (for physical quark mass)
- Bound state in charm region
- Open question: What is width in our approach?

Williams arXiv:1804.11161, 2018
Miramontes et al, arXiv:1805.03572, 2018



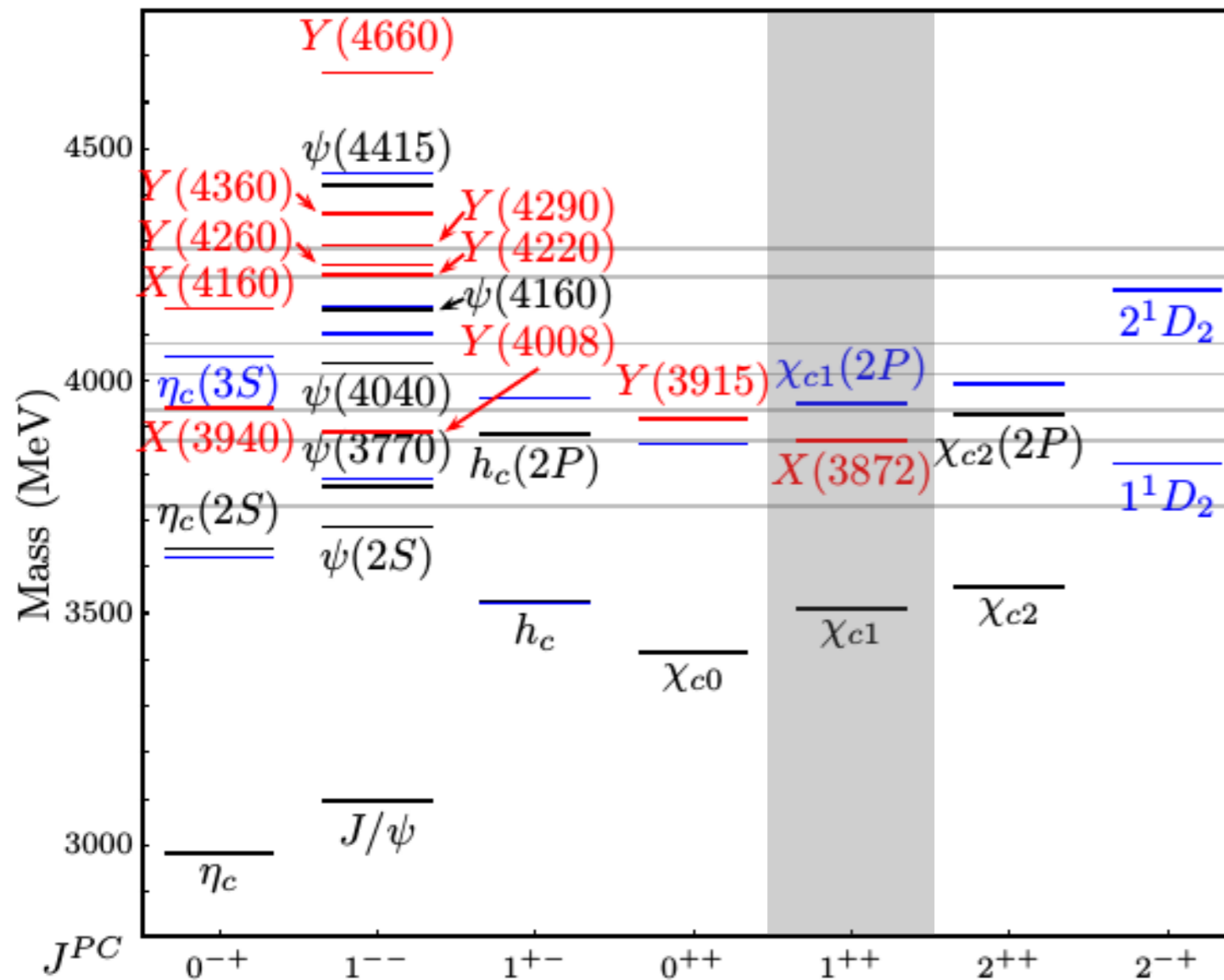
Eichmann et al, PLB 753:282-287, 2016

Reminder: Our approach

- Quark gluon interactions only!
- “Internal clustering” (important pion poles) appears dynamically
- We do not have explicit molecules of pions or explicit diquark constituents (-> this would be 2 body equation)

Part II: Charmonium and (axial) vector tetraquarks

Charmonium spectrum

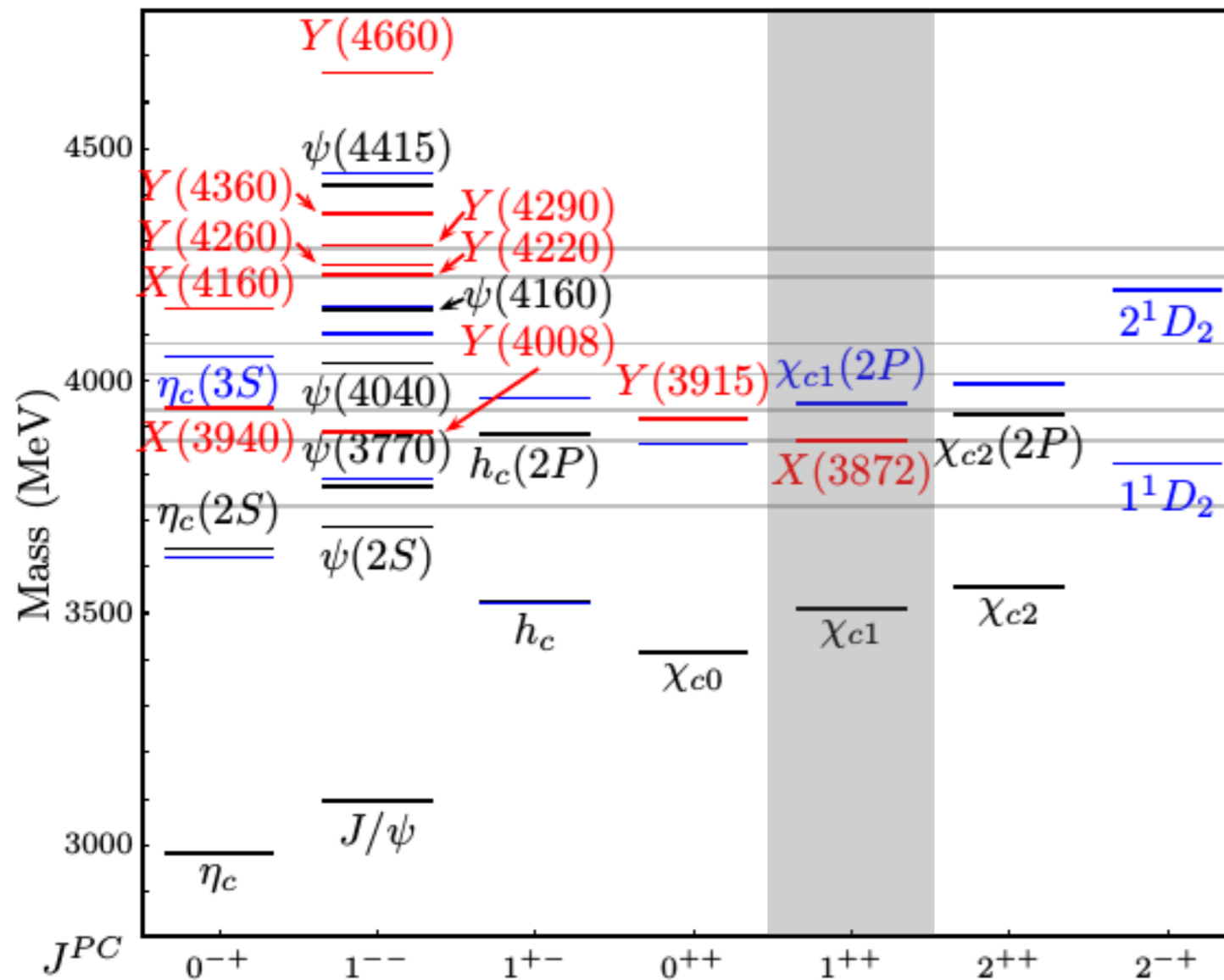


Esposito et al., *Journal Mod Phys A* 30, 2014

- Prime candidates for non meson states
 - Y states in vector channel
 - X(3872) in axialvector channel
- Charged states cannot be charmonium (Z)

unobserved + predicted
(charmonia Radford, Repko 2007)
observed + predicted
observed

Charmonium spectrum



Esposito et al., I Journal Mod Phys A 30, 2014

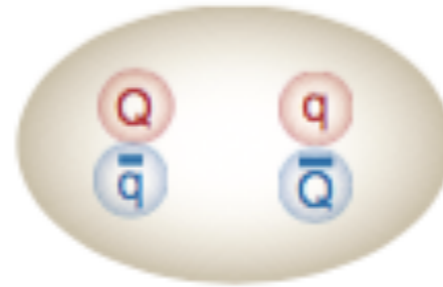
- Prime candidates for non meson states
 - Y states in vector channel
 - X(3872) in axialvector channel
- Charged states cannot be charmonium (Z)

*Godfrey 0910.3409, Szczepaniak 1110.0647,
Ali et al, 2018, Esposito 2014
Padmanath, Lang, Prelovsek, 2015,
Giacosa (see talk!)*

....

What are these states?

- **Molecules** (bound by pion exchange)



Tornqvist PRL 67, 1991
Swanson, PLB 588, 2004
Guo et al, arXiv:1705.00141

- States slightly below constituent thresholds

- **Diquark-anti-diquark**



Maiani et al, PRD 71, 2005
Ebert et al, PRB 634, 2006

- Colored diquark constituents

- **Hadro-charmonium**

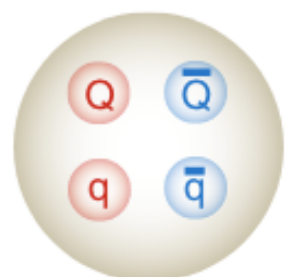


Dubynskiy et al, PLB 688, 2008

- Heavy core + light “cloud”

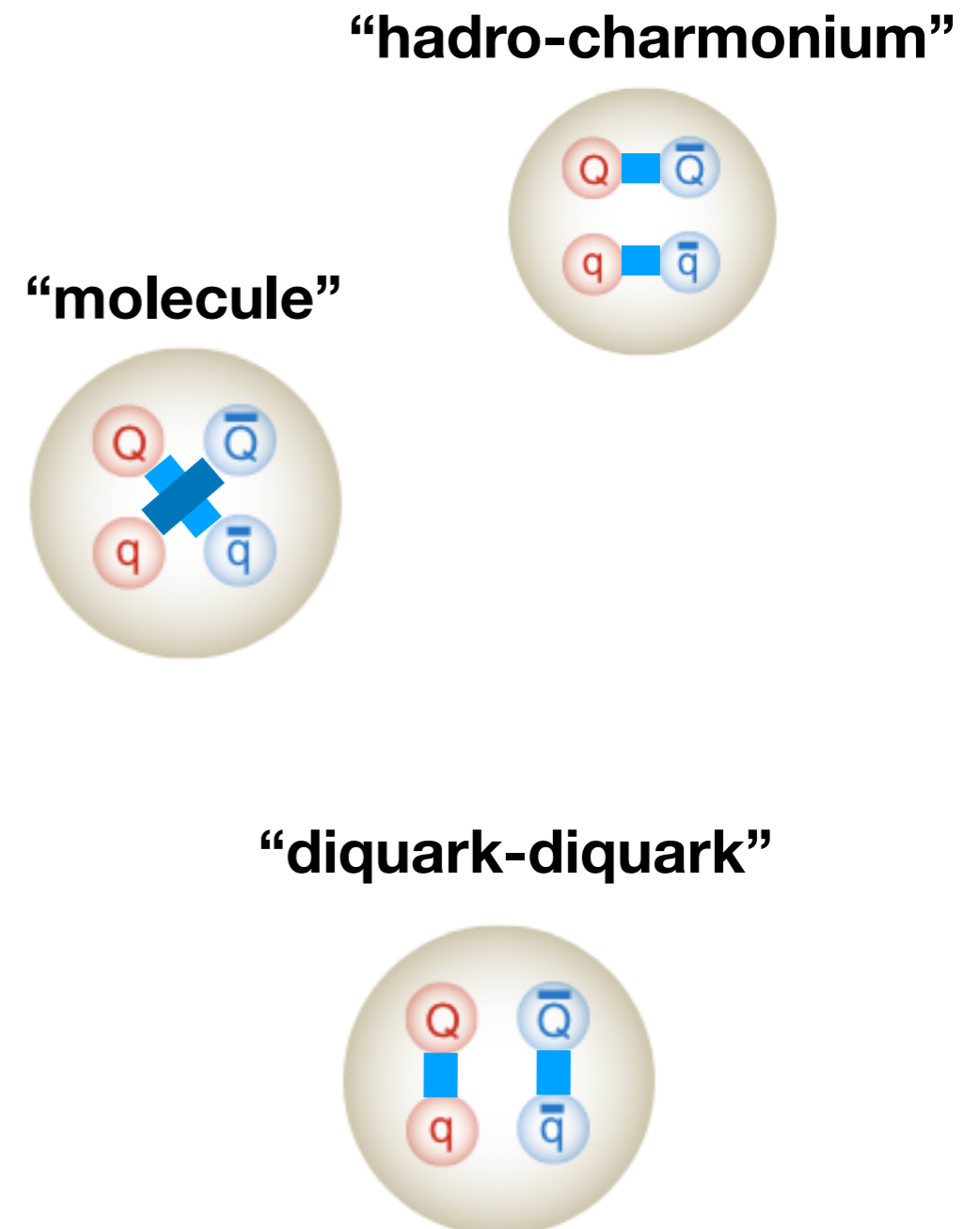
- 4 quark tetraquark

Stancu, arXiv:0607077v2, 2007



In our approach

- 4 body equation with quark gluon interaction
- No need to assume structure
- By:
“molecule”,
“hadro-charmonium”,
“diquark”
we mean parts of the wave function



A new approach for the axial vectors based on the learnings for the sigma

New method

- Still 4 body with additional **simplification**:
 - Sigma lessons! (important 2 body poles + residue/pole structure)
 - Which structures develop poles? **Physical constituents with respective 2 body poles in f's**
 - Get masses from solving 2 body BSE

$$\Gamma = f_i(\Omega) \tau_i \otimes color \otimes flavor$$

“standard” basis

$$= \sum_i f_i^{(da)} (d \otimes a)_i + \sum_i f_i^{(\tilde{m}\tilde{m})} (\tilde{m} \otimes \tilde{m})_i + \sum_i f_i^{(mm)} (m \otimes m)_i$$

“diquark-anti-diquark” $(Qq)(\bar{q}\bar{Q})$ “molecule” $(Q\bar{q})(q\bar{Q})$ “hadro-charmonium” $(Q\bar{Q})(q\bar{q})$

“physical”

Constructing an amplitude

$\chi_{c1}(3872)$

$$J^{PC} = 0^+(1^{++})$$

Mass $m = 3871.69 \pm 0.17$ MeV

$$m_{\chi_{c1}(3872)} - m_{J/\psi} = 775 \pm 4$$
 MeV

Full width $\Gamma < 1.2$ MeV, CL = 90%

$\chi_{c1}(3872)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi^+ \pi^- J/\psi(1S)$	> 3.2 %	650
$\omega J/\psi(1S)$	> 2.3 %	†
$D^0 \bar{D}^0 \pi^0$	>40 %	117
$\bar{D}^{*0} D^0$	>30 %	3
$\gamma J/\psi$	> 7 $\times 10^{-3}$	697
$\gamma \psi(2S)$	> 4 %	181
$\pi^+ \pi^- \eta_c(1S)$	not seen	746
$\pi^+ \pi^- \chi_{c1}$	not seen	218
$p \bar{p}$	not seen	1693

Only guideline, not limiting!

New method 1++ tetraquark

$$\Gamma = (D^0 \bar{D}^{*0} + D^+ \bar{D}^{*-}) + SA_u + SA_d + J/\Psi\omega + C.C.$$

- Leading structures + s-waves only ($\mathbf{J} = \mathbf{L} + \mathbf{S}$ in rest frame)
- Assuming residue/pole structure
- Diquarks decouple for this setup (Not for the σ)

$$f_{DD^*}(\Omega) \approx \frac{f_{DD^*}(S_0)}{(m_D^2 + p_-^2)(m_{D^*}^2 + p_+^2)}$$

$$f_{J/\Psi\omega}(\Omega) \approx \frac{f_{J/\Psi\omega}(S_0)}{(m_{J/\Psi}^2 + q_-^2)(m_\omega^2 + q_+^2)}$$



molecule
diquark diquark
hadro-charmonium

Questions to be answered

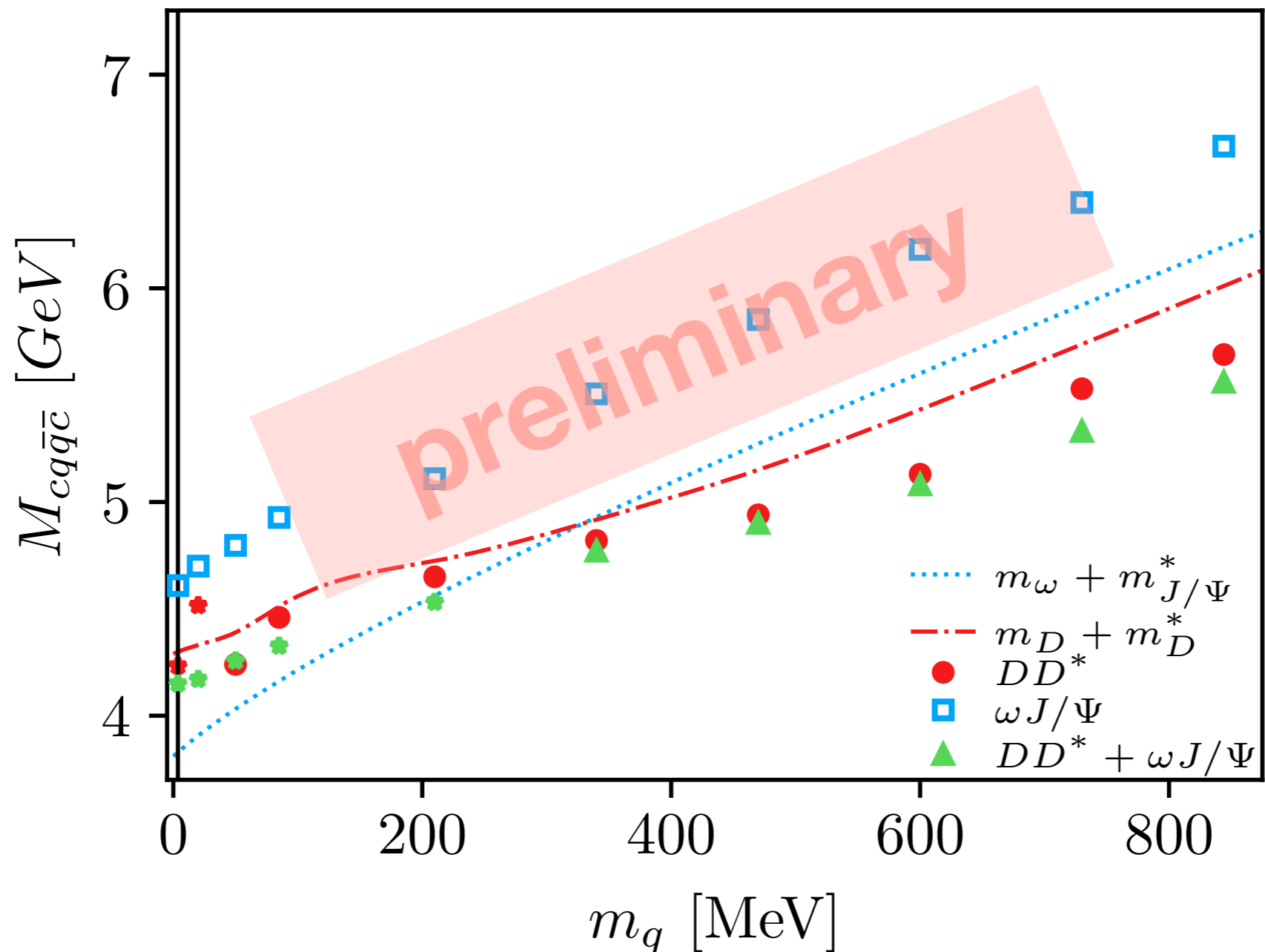
$$\Gamma = (D^0 \bar{D}^{*0} + D^+ \bar{D}^{*-}) + SA_u + SA_d + (J/\Psi\omega) + C.C.$$

- What is the mass of the 1++ ground state? (X(3872) ?)
- What is (are) the dominant physical components?

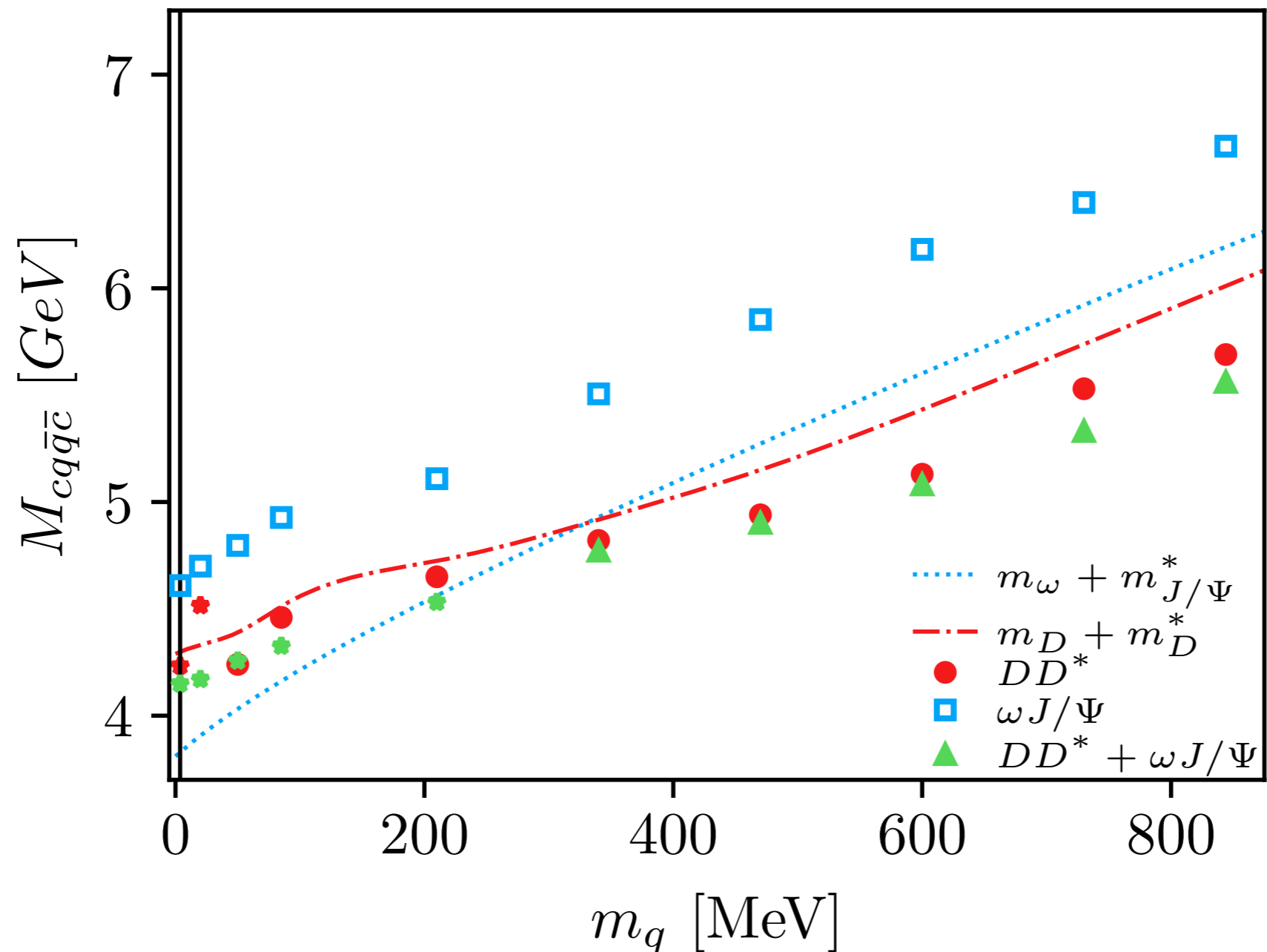
molecule
diquark diquark
hadro charmonium

Results for the axial vector tetraquark

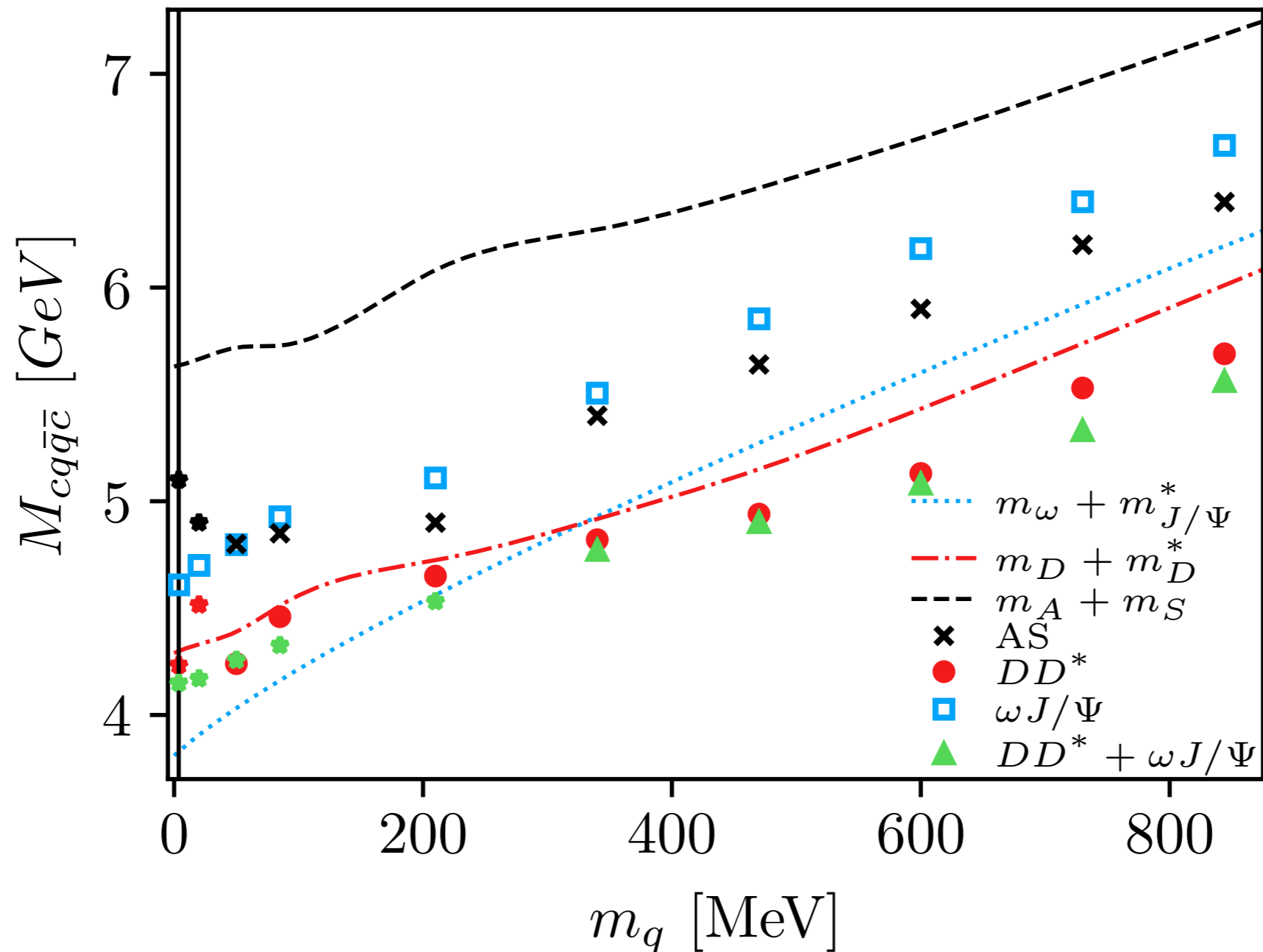
Results for the 1^{++} heavy-light tetraquark



Results for the 1^{++} heavy-light tetraquark

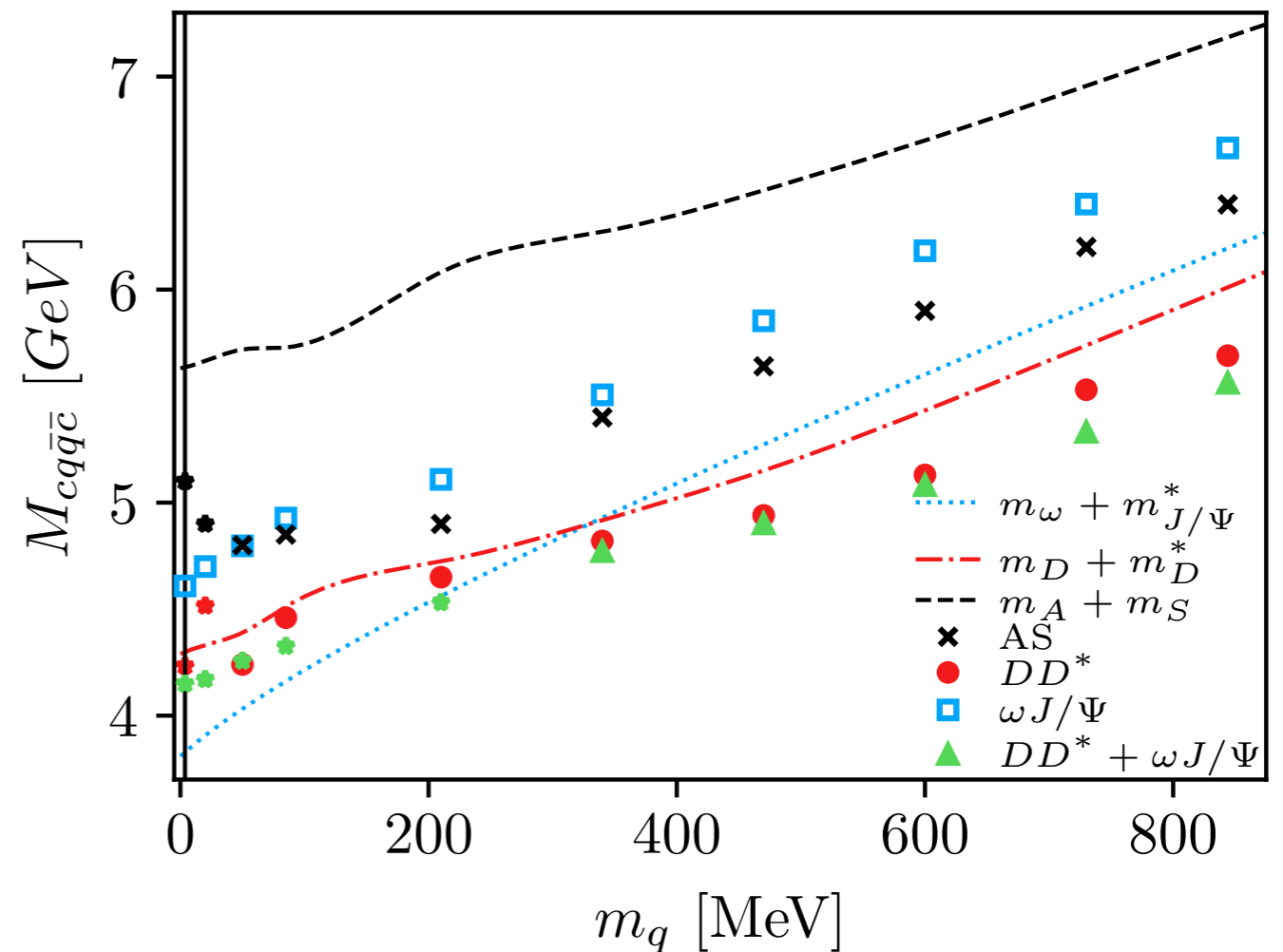


Results for the 1^{++} heavy-light tetraquark



Results for the 1^{++} heavy-light tetraquark

- 1^{++} (mol + hadcham) has
 - Strong molecule component
 - Mass around 4.2 GeV
- Decoupled diquarks are bound and a bit heavier (very preliminary)



Summary

Summary

- We find an octet of light scalar tetraquarks in the mass region of the light scalars
- We have an axialvector tetraquark state with a dominant DD^* component in the mass region of the $X(3872)$ (with the new method)
- Current approximation produces a separate, heavier diquark state, general feature?

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