Light scalar/ heavy axialvector tetraquarks in a Dyson-Schwinger, Bethe-Salpeter approach

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Part I: The light scalars
Work on this topic

Diquark antidiquark, large $N_c$, Unitarized ChPT, quark models, ELSM, Lattice, ...

Light mesons in the quark model

- $J = L + S$, $P = (-1)^{L+1}$, $C = (-1)^{L+S}$
- $0^{++}$ scalars are p-waves
- Scalars should have mass around 1.3 GeV

Mass

$\begin{array}{c}
\begin{bmatrix}
P_{0^{--}} \\
0^{--} \\
S=0
\end{bmatrix} < \\
\begin{bmatrix}
V_{1^{--}} \\
1^{--} \\
S=1
\end{bmatrix} < \\
\begin{bmatrix}
A_{1^{--}} \\
1^{--} \\
S=0
\end{bmatrix} \\
S=0
\end{array}$

P: pseudoscalar  V: Vector  A: Axialvector  S: Scalar  T: Tensor
Observed lowest multiplets

- Light scalar octet (red) has unexpectedly low mass
- 0.5-1 GeV
Masses inside scalar octet

Here are the states:
\[ \sigma, \kappa^\pm, \kappa^0, \bar{\kappa}^0, a^\pm, a^0, f_0 \]

\[ S(0^{++}) \]

SU(3) assignment:
\[ 3 \otimes \bar{3} = 1 \oplus 8 \]

...and their experimental masses
Masses inside scalar octet

Here are the states:
\[ \sigma, \kappa^\pm, \kappa^0, \bar{\kappa}^0, a^\pm, a^0, f_0 \]

\[ S(0^{++}) \]

\[ \text{SU(3) assignment:} \quad 3 \otimes \bar{3} = 1 \oplus 8 \]

\[ d\bar{s}, \quad \kappa^0, \quad u\bar{s}, \quad \kappa^+ \]

\[ u\bar{u} - d\bar{d}, \quad a_0, \quad f_0 \]

\[ s\bar{u} \]

\[ \kappa^- \]

\[ \bar{\kappa}^0 \]

\[ s\bar{d} \]

\[ d\bar{u} \]

\[ \kappa \]

\[ a_0 \]

\[ f_0 \]

...and their experimental masses

Iso triplet 1 GeV ?? (no strange quarks!), expect 500 MeV (like sigma)!

Mass ordering inside the multiplet is strange
Are they tetraquarks?

Here are the states:

\[ \sigma, \kappa^0 \bar{\kappa}^0, \kappa^\pm, a_0, a^\pm f_0 \]

\[ S(0^{++}) \]

\[ \begin{array}{c}
 \text{SU(3) assignment:} \\
 (3 \otimes 3) \otimes (\bar{3} \otimes \bar{3}) = 1 + 8 + \ldots
\end{array} \]


\[ \begin{array}{c}
 u\bar{u}d\bar{s} & \kappa^0 \\
 u\bar{s}d\bar{d} & \kappa^+ \\
 s\bar{s}(u\bar{u} - d\bar{d}) & a_0^0 \\
 d\bar{u}s\bar{s} & \sigma u\bar{d}d\bar{u} \\
 s\bar{s}(u\bar{u} + d\bar{d}) & f_0 \\
 \kappa^- & s\bar{u}d\bar{d} \\
 \bar{\kappa}^0 & s\bar{d}u\bar{u}
\end{array} \]
Are they tetraquarks?

- $a_0$ mass is now higher due to internal $s\bar{s}$ pair
- The $a_0$ has $s\bar{s}$ and can have strangeness in decays
- Large width of $\sigma$ and kappa can be justified (fall apart)
- Tetraquark can explain these features
The method
BSE/DSE framework

See talk of M. Huber!

• Calculate quark propagator from DSE

• Model gluon + vertex

• Spectroscopy: solve BSE for meson/baryon/tetraquark… (eigenvalue problem)

• Kernel for $qq/q\bar{q}$ from self energy (cut quark leg)

• $J^{PC} \rightarrow \Gamma$

\[ \Gamma = \tilde{K}\Gamma \]

$\tilde{K}$
Quark propagators

- Two dressing functions ($A, M$)
- Dynamical mass generation
  \[ S^{-1} = A(p^2) (M(p^2) - i p^\mu \gamma^\mu) \]
- Massless chiral pion
Tetraquark BSE(s)


\begin{itemize}
  \item 2 body (simplified meson/diquark propagators, …) -> sigma
  \item 4 body (without 3/4 body forces) -> scalar octet
  \item Currently using 4 body for heavy lights!
\end{itemize}

Heupel et al, PLB 718:545-549, 2012

4 body tetraquark BSE

- Permutations: $\pi_{13} + \pi_{24} - \pi_{1324} + \pi_{14} + \pi_{23} - \pi_{1423}$

- Quark antiquark interactions in these “meson” topologies

- Calculate quark propagators from DSE

- Dressed gluon exchange kernel for RL + MT

- Tetraquark amplitude
tetraquark amplitude

- JPC determine Gamma
- many tensor structures -> restrict to S-waves
- dressing functions (f) depend on 9 Lorentz scalars
- Use S4 variables (better for symmetries)

\[ \Gamma = \sum_i^N f_i(\Omega) \tau_i \otimes \text{color} \otimes \text{flavor} \]

<table>
<thead>
<tr>
<th>J=0</th>
<th>J=1</th>
</tr>
</thead>
<tbody>
<tr>
<td># total</td>
<td>256</td>
</tr>
<tr>
<td># S-wave</td>
<td>32</td>
</tr>
</tbody>
</table>

\[ \Omega = \{p^2, q^2, k^2, p.q, \ldots\} = \{S_0, D, T_0, T_1\} \]

\[ S_0 = \frac{p^2 + q^2 + k^2}{4} \]

\[ T_i = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} \]

\[ D = \frac{1}{4S_0} \left( \frac{\sqrt{3}q^2 - p^2}{p^2 + q^2 - 2k^2} \right) \]

[\( S_4 \)]
Lessons from the sigma
Lessons from the sigma

- Drop different S4 multiplets to see their impact:
  - 4 quark equation (> 1 GeV) without D
  - Much lighter when D is turned on, why?
  - Two body poles are generated in D variables!

\[
S_0 = \frac{p^2 + q^2 + k^2}{4}, \quad T_i = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix} \\
D = \frac{1}{4S_0} \left( \frac{\sqrt{3}q^2 - p^2}{p^2 + q^2 - 2k^2} \right) \quad S_4
\]

(experiment: 500 MeV) $\rightarrow m_\sigma$

$\begin{align*}
&f_i(S_0, \mathcal{D}, \mathcal{Z}_1, \mathcal{Z}_2) \rightarrow 1.4 \text{ GeV} \\
&f_i(S_0, \mathcal{D}, \mathcal{T}_1, \mathcal{T}_2) \rightarrow 1.4 \text{ GeV} \\
&f_i(S_0, \mathcal{D}, \mathcal{T}_1, \mathcal{T}_2) \rightarrow 1.1 \text{ GeV} \\
&f_i(S_0, \mathcal{D}, \mathcal{T}_1, \mathcal{T}_2) \rightarrow \mathbf{0.35} \text{ GeV}
\end{align*}$

:= set to constant value
Lessons from the sigma

1. The 2 body poles are important:
   • Pion - pion bring mass down and create threshold
   • Diquarks have small impact

2. Pole term with residue describes $f$ well, govern the dynamics happening in the doublet $D$

\[ \Gamma = \sum_{i}^{N} f_{i}^{(\Omega)} \tau_{i} \otimes \text{color} \otimes \text{flavor} \]

Structure of dressing functions:

\[ f(\Omega) \approx \frac{f(S_{0})}{(m_{\pi}^{2} + q_{+}^{2})(m_{\pi}^{2} + q_{-}^{2})} + \ldots \]
Results, yes we have a light scalar multiplet!
Results in BSE/DSE

- Box: PDG
  Band: our result

- Correct mass ordering inside scalar octet

- Masses about right

_Eichmann et al PLB 753:282-287, 2016_
Results in BSE/DSE

- $\sigma$ is resonance in 4 body (for physical quark mass)
- Bound state in charm region
- Open question: What is width in our approach?

Williams arXiv:1804.11161, 2018
Miramontes et al, arXiv:1805.03572, 2018
Reminder: Our approach

- Quark gluon interactions only!
- “Internal clustering” (important pion poles) appears dynamically
- We do not have explicit molecules of pions or explicit diquark constituents (→ this would be 2 body equation)
Part II: Charmonium and (axial) vector tetraquarks
Charmonium spectrum

- Prime candidates for non meson states
  - Y states in vector channel
  - X(3872) in axialvector channel
- Charged states cannot be charmonium (Z)

unobserved + predicted
( charmonia Radford, Repko 2007)
observed + predicted
observed
Charmonium spectrum

- Prime candidates for non meson states
  - Y states in vector channel
  - X(3872) in axialvector channel
- Charged states cannot be charmonium (Z)

Padmanath, Lang, Prelovsek, 2015, Giacosa (see talk!)

Esposito et al., I Journal Mod Phys A 30, 2014
What are these states?

- **Molecules** (bound by pion exchange)
- States slightly below constituent thresholds
- **Diquark-anti-diquark**
- Colored diquark constituents
- **Hadro-charmonium**
  - Heavy core + light “cloud”
- 4 quark tetraquark

Maiani et al, PRD 71, 2005
Ebert et al, PRB 634, 2006
Dubynskiy et al, PLB 688, 2008
Tornqvist PRL 67, 1991
Swanson, PLB 588, 2004
Guo et al, arXiv:1705.00141
In our approach

- 4 body equation with quark gluon interaction
- No need to assume structure
- By: “molecule”, “hadro-charmonium”, “diquark” we mean parts of the wave function
A new approach for the axial vectors based on the learnings for the sigma
New method

- Still 4 body with additional simplification:
  - Sigma lessons! (important 2 body poles + residue/pole structure)
  - Which structures develop poles? Physical constituents with respective 2 body poles in f’s
  - Get masses from solving 2 body BSE

\[\Gamma = f_i(\Omega)\tau_i \otimes \text{color} \otimes \text{flavor}\]

- “diquark-anti-diquark” \((Qq)(\bar{Q}\bar{q})\)
- “molecule” \((Q\bar{q})(q\bar{Q})\)
- “hadro-charmonium” \((Q\bar{Q})(q\bar{q})\)

“standard” basis

“physical”
Constructing an amplitude

$\chi_{c1}(3872)$

$I^{G}(J^{PC}) = 0^{+}(1^{++})$

Mass $m = 3871.69 \pm 0.17$ MeV
$m_{\chi_{c1}(3872)} - m_{J/\psi} = 775 \pm 4$ MeV
Full width $\Gamma < 1.2$ MeV, CL = 90%

$\chi_{c1}(3872)$ DECAY MODES

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
<th>$p$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^- J/\psi(1S)$</td>
<td>&gt; 3.2 %</td>
<td>650</td>
</tr>
<tr>
<td>$\omega J/\psi(1S)$</td>
<td>&gt; 2.3 %</td>
<td>†</td>
</tr>
<tr>
<td>$D^0 \bar{D}^0 \pi^0$</td>
<td>&gt;40 %</td>
<td>117</td>
</tr>
<tr>
<td>$\bar{D}^*0 D^0$</td>
<td>&gt;30 %</td>
<td>3</td>
</tr>
<tr>
<td>$\gamma J/\psi$</td>
<td>&gt; 7 x 10^{-3}</td>
<td>697</td>
</tr>
<tr>
<td>$\gamma \psi(2S)$</td>
<td>&gt; 4 %</td>
<td>181</td>
</tr>
<tr>
<td>$\pi^+\pi^- \eta_c(1S)$</td>
<td>not seen</td>
<td>746</td>
</tr>
<tr>
<td>$\pi^+\pi^- \chi_{c1}$</td>
<td>not seen</td>
<td>218</td>
</tr>
<tr>
<td>$\rho \rho$</td>
<td>not seen</td>
<td>1693</td>
</tr>
</tbody>
</table>

Only guideline, not limiting!
New method 1++ tetraquark

\[ \Gamma = (D^0 \bar{D}^{*0} + D^+ \bar{D}^{*-}) + S A_u + S A_d + (J/\Psi \omega) + C.C. \]

- Leading structures + s-waves only \((J = L + S)\) in rest frame
- Assuming residue/pole structure
- Diquarks decouple for this setup (Not for the \(\sigma\))

\[ f_{DD^*}(\Omega) \approx \frac{f_{DD^*}(S_0)}{(m_D^2 + p_-^2)(m_{D^*}^2 + p_+^2)} \]

\[ f_{J/\Psi \omega}(\Omega) \approx \frac{f_{J/\Psi \omega}(S_0)}{(m_{J/\Psi}^2 + q_-^2)(m_\omega^2 + q_+^2)} \]
Questions to be answered

\[ \Gamma = (D^0 \bar{D}^{*0} + D^+ \bar{D}^{*-} + SA_u + SA_d + J/\Psi\omega) + C.C. \]

- What is the mass of the 1++ ground state? (X(3872) ?)
- What is (are) the dominant physical components?
Results for the axial vector tetraquark
Results for the 1++ heavy-light tetraquark
Results for the 1++ heavy-light tetraquark
Results for the 1++ heavy-light tetraquark

\[ M_{cq\bar{q}\bar{c}} \text{ [GeV]} \]

\[ m_q \text{ [MeV]} \]

- \( m_\omega + m_{J/\Psi} \)
- \( m_D + m_{D'} \)
- \( m_A + m_S \)
- \( AS \)
- \( DD^* \)
- \( \omega J/\Psi \)
- \( DD^* + \omega J/\Psi \)
Results for the 1++ heavy-light tetraquark

- 1++ (mol + hadcham) has
  - Strong molecule component
  - Mass around 4.2 GeV
- Decoupled diquarks are bound and a bit heavier (very preliminary)
Summary
Summary

• We find an octet of light scalar tetraquarks in the mass region of the light scalars

• We have an axialvector tetraquark state with a dominant $DD^*$ component in the mass region of the $X(3872)$ (with the new method)

• Current approximation produces a separate, heavier diquark state, general feature?
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