The road to solving the Gribov problem of the centre vortex model in quantum chromo dynamics.

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The centre vortex model - some of the successes:

- behaviour of Wilson and Polyakov loops → phase transition
  → DelDebbio, 1998; Nishimura, 2017

- Casimir scaling of heavy-quark potentials (due to thick vortices)
  → M.F, 1997

- spontaneous breaking of scale invariance
  → Langfeld, 1997

- color structure of vortices → instantons, lumps of topological charge
  → Schweigler; Nejad, 2016; Höllwieser, 2011

- quark condensate → chiral symmetry breaking
  (requires smooth configurations)
  → Höllwieser; M.F, 2017

⇒ Thick vortices need to be identified in smooth configurations.

$Z_N$ vortex condensation theory:

⇒ ’t Hooft, Vinciarelli, Yoneya, 1978; Cornwall, Nielsen, Olesen, Mack, Petkova, 1979
Detection of thick centre vortices

Finding the best fit to a configuration of thick vortices by P-vortices:

P-vortices correlating to thick vortices?

✔ → Vortex finding property
✗ → bad things happening
Direct maximal centre gauge and centre projection

Gauge fixing:

\[
R = \sum_{x} \sum_{\mu} | \text{Tr}[U_\mu(x)] |^2 \rightarrow \text{maximize}
\]

Numerical methods can only find local maxima.

→ Gribov copies: Vortex finding property?

Centre projection:

\[
U_\mu(x) \rightarrow Z_\mu(x) = \text{sign} \text{ Tr}[U_\mu(x)]
\]

Non-trivial Links build up the Dirac volume, whose surface is the vortex (transparent). They are detected by non-trivial plaquettes.

P-vortices and the string tension

\[ \langle W(R, T) \rangle = [(-1)\varrho + (+1)(1 - \varrho)]^{A}_{R \times T} = e^{-\ln(1-2\varrho)A} = e^{-\sigma A} \]

\[ \rightarrow \sigma = -\ln(1 - 2\varrho) \]
Gribov copy problem

Search absolut maximum of gauge fixing functional with simulated annealing $\Rightarrow$ P-vortex string tension is too small

$\Rightarrow$ Bornaykov et al. (2000)

Laplacian center gauge is free of Gribov copies but does not scale appropriately in the continuum limit

$\Rightarrow$ Langfeld (2004)

SU(3): centre vortices reproduce only 66% of the string tension

$\Rightarrow$ Langfeld (2004)
Problems with smoothing or cooling ("the bad things")

Figure: $18^4$-lattice, 458 Wilson-configurations at $\beta = 2.2$. Cooling with strength 0.05.

→ Caused by a loss of the vortex finding property?
Non-abelian Stokes law & behaviour of Wilson loops

Centre regions

Area law

Perimeter law

Perimeter law $\rightarrow$ Coulombic behaviour for small Wilson loops

Area law $\rightarrow$ Linear rising potential for big Wilson loops

Centre regions useful to check the vortex finding property?

$= (-1)^{22} \times$
Projection and gauging independent quantities

The assumption: Centre regions or arbitrary Wilson loops evaluating to centre elements ("centre loops") should neither be modified by gauge fixing nor by projection $\iff$ Vortex finding property.

The idea: use this to identify failing configurations.

Two possibilities:

- Adapt the single gauge fixing procedure: $R \rightarrow \dot{R} = R - $ penalty, with a penalty for each centre loop that gets modified by gauge fixing and projection.
- Reject failed configurations from the ensemble used in Monte Carlo calculations.

Requirement: identifying centre regions.
Identifying centre regions

...
Growing regions.

After a Wilson loop is opened, the opening is moved around to find the best direction for enlargement.
Comparing growing algorithms

→ strong influence of cooling on the identified -1 loops

→ less influence of cooling on the identified -1 loops
Use growing towards negativity for vortex detection?

\[ \beta = 2.2 : \sim 5000 \text{ non-trivial loops} \]

- Expected P-plaquettes: \( \rightarrow 16800 \pm 400 \)
- Identified after 10 cooling steps: 4794
- Non-trivial loops: \( \sim 5000 \)

\[ \beta = 2.4 : \sim 3000 \text{ non-trivial loops} \]

- Expected P-plaquettes: \( \rightarrow 7300 \pm 300 \)
- Identified after 10 cooling steps: 1416
- Non-trivial loops: \( \sim 3000 \)
Centre regions might be an approach to improve vortex detection.

Thank you for listening!
Sizes of centre loops (no cooling)

Histogram of loop sizes and loop traces taken from 5 configurations with a $14^3 \times 8$ lattice. Left: Growing towards centre. Right: growing towards trivial centre.
Sizes of centre loops (10 cooling steps)

Histogram of loop sizes and loop traces taken from 5 configurations with a $14^3 \times 8$ lattice. Left: Growing towards centre. Right: growing towards trivial centre.
Justification of cooling

The effect of cooling on the Creutz-Ratios. 470 Wilson-Configurations with $20^4$ lattices, $18^4$ for 20 cooling steps
An alternative approach: flux collimation

Collimating flux by a transformation that combines gauge fixing and projection.
Flux collimation, the transformation

Shuffle flux from a **weak plaquette** to a **stronger neighbouring plaquette** by modifying their **shared link**.
Homogeneity of a $2 \times 2$-Wilson-Loop

\[ W_j = \sum_{\mu=0}^{3} (u_j)_\mu \sigma_\mu \in SU(2), \]
\[ u_j \in S^3, \quad |u_j| = 1, \quad \sigma_0 = 1_2, \quad \text{Pauli matrices } \sigma_k, \]

**Definition: S3-homogeneity:**
\[
h_{S3} := \frac{1}{4} | \sum_{j=1}^{4} u_j | \in [0, 1].
\]

\[ W_j = \cos(\alpha_j) \sigma_0 + i \sum_{k=1}^{3} \sin(\alpha_j) (n_j)_k \sigma_k, \]
\[ n_j \in S^2, \quad |n_j| = 1, \]

**Definition: S2-homogeneity:**
\[
h_{S2} := \frac{1}{4} | \sum_{j=1}^{4} n_j | \in [0, 1].
\]
Influence of cooling on the homogeneity-distribution

Histogram of Homogeneity. 400 Simulations per cooling step, $18^4$ lattice, $\beta = 2.2$. 

cooling:
- 0
- 5
- 10
Vortex sensitivity in cooled configurations?

Difference in homogeneity of vortex-containing $2 \times 2$-loops and the whole lattice. $12^4$ lattice, 25 simulations per cooling step, $\beta = 2.2$