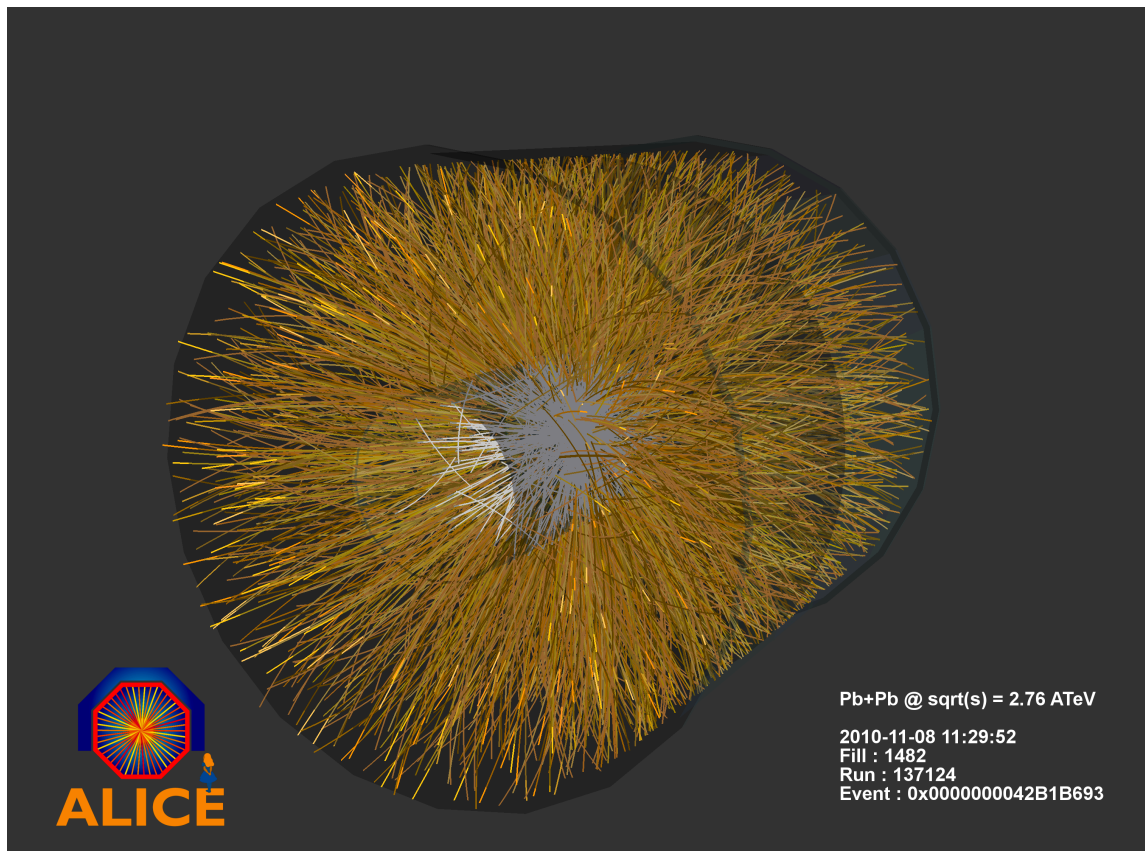


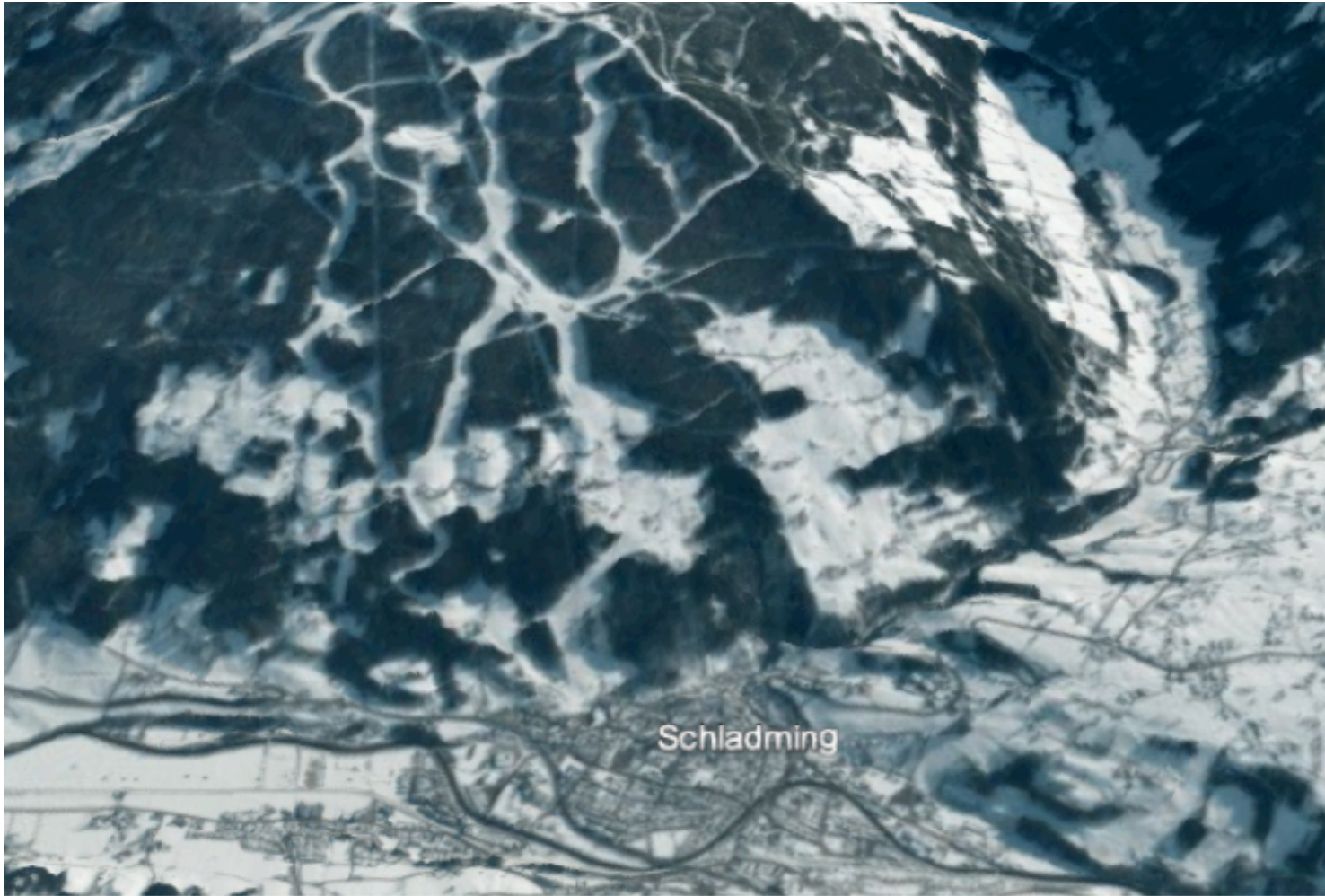
What do the yields of light nuclei tell us about heavy ion collisions?



Yiming Cai, TDC, Boris Gelman and Yukari Yamauchi

An Overview

An Overview



An Overview

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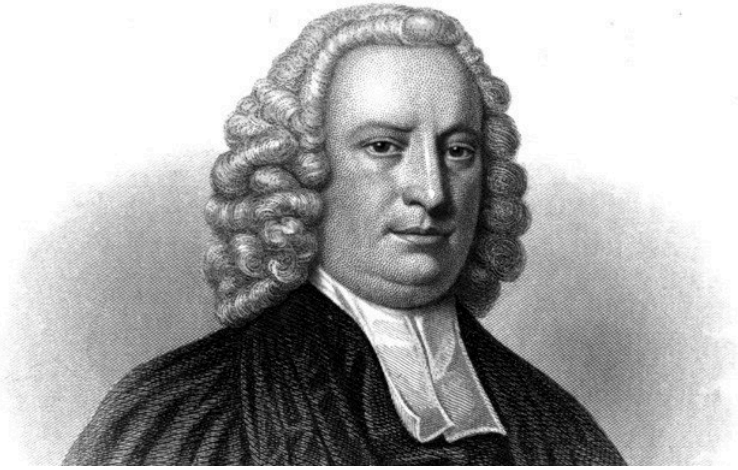
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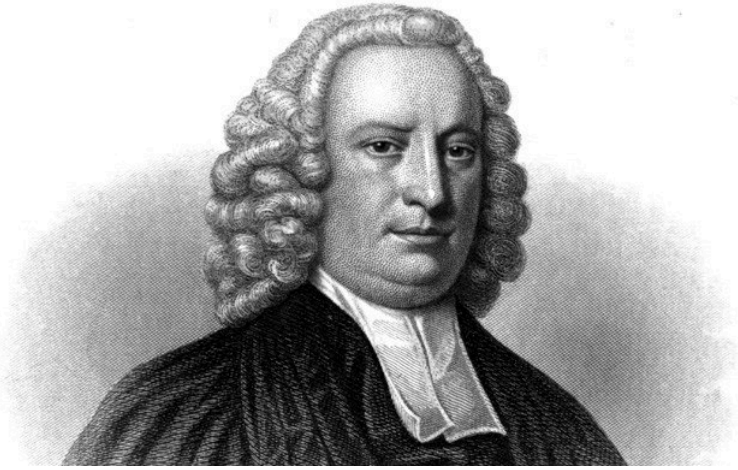
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- This talk focuses on the yield of light nuclei, which indicate the assumptions of the SHM cannot be justified.

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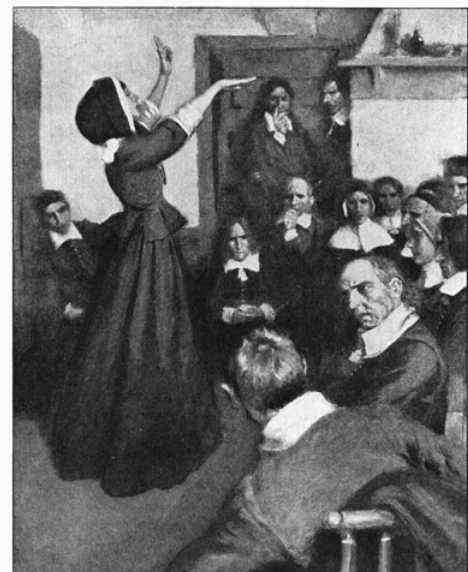


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"Sir, a woman's preaching is like a dog's walking on his hinder legs. It's not done well; but you are surprised to find it done at all." July 31, 1763 (as recorded by Thomas Boswell)



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Key question is what—if anything—one can learn from the phenomenological success, about the underlying dynamics of heavy ion physics.

Basic Assumptions of SHM

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2. Then the system expands and cools and becomes an equilibrated hadronic gas (including light nuclei) with the bulk of the system contained in a large volume at a nearly uniform temperature.
 - a. In this regime, the system is sufficiently dilute enough so that hadrons (and light nuclei) are sufficiently well-separated as to be discernible.
 - b. The system is sufficiently dilute so that the relevant properties of the hadronic gas (densities of each species of hadrons, their momentum distributions as well as thermodynamic properties such energy density and pressure) are well-approximated by a gas of noninteracting hadrons with a mass given by the zero temperature value.
 - c. The system is sufficiently dense so that interactions maintain both chemical and kinetic equilibrium for all species of hadron

Basic Assumptions of SHM

3. As the system cools further it falls out of chemical equilibrium with the hadronic species freezing out chemically
 - a. All species of hadron freeze out at the same temperature to good approximation
 - b. The yields seen in the detectors are given by the primordial yields given by the model for (strong-interaction) stable species at the freeze out temperature plus yields due to the decay products due to unstable hadrons with totals given by the model at the freeze out temperature folded with branching ratios given by their free space values.
 - c. The chemical freeze out temperature depends on the energy of the heavy ion reactions with increasing freeze out temperatures.

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 - c. The chemical freeze out temperature depends on the energy of the heavy ion reactions with increasing freeze out temperatures.
4. Following chemical freeze out, the system will remain in kinetic equilibrium with cooling temperatures until the hadronic species subsequently kinetically freeze out and free stream to the detector.
 - a. Strong interaction unstable hadrons decay prior to reaching the detector into stable hadrons with branching ratios given by their free space values.

The model has three parameters:

T_{cf} Temperature at chemical freeze out

μ Baryon chemical potential at chemical freeze out

V Volume of hadronic gas at chemical freeze out

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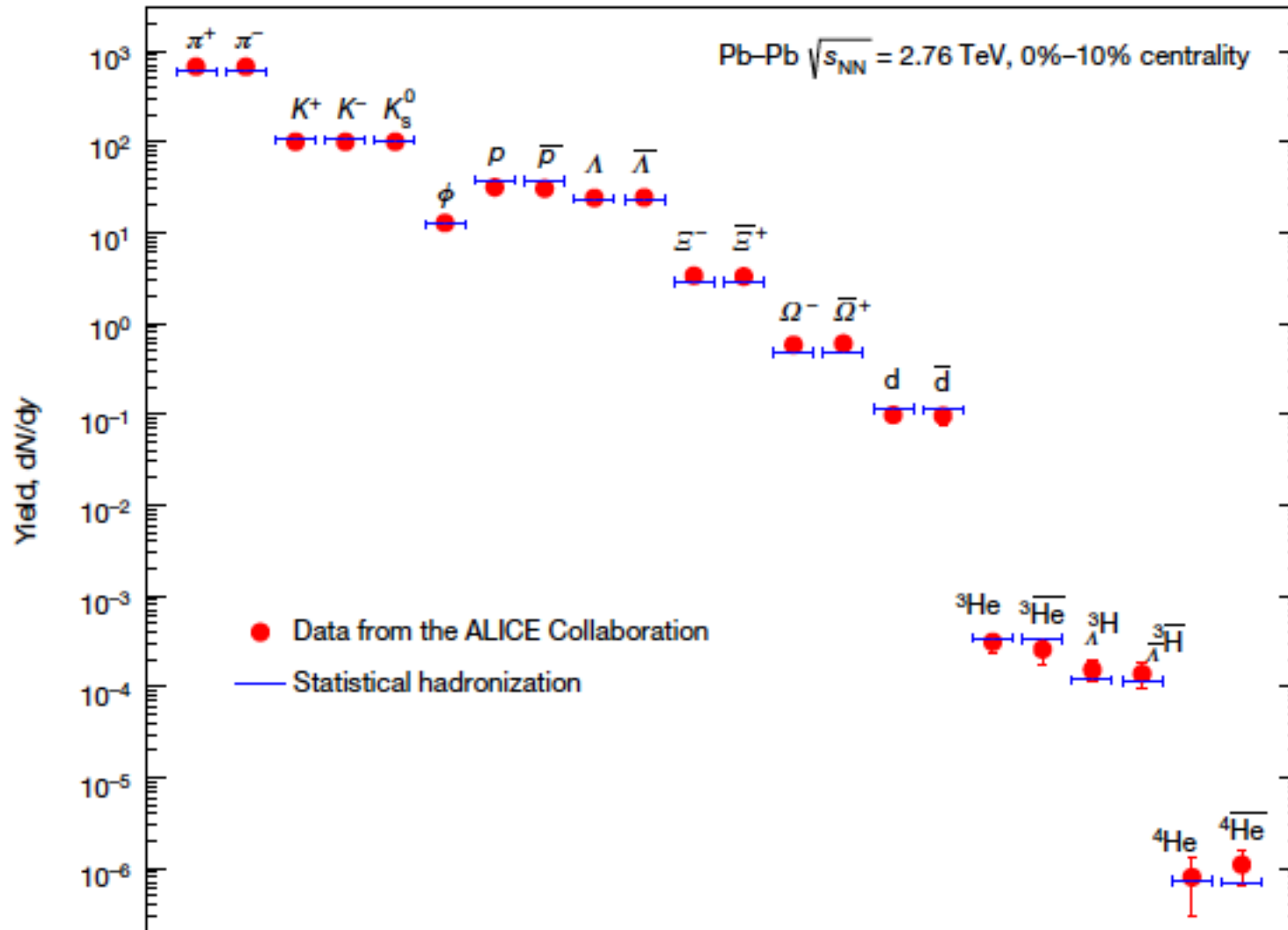
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Predicts yields of hadrons (and light nuclei) for midrapidity in central collisions

Note that relative yields at high beam energy effectively only depend on T_{cf} : V does not affect relative yields (only absolute) and that $\mu \rightarrow 0$ as the beam energy gets high. (Only thing distinguishing baryons from antibaryons is the baryons in initial state which is a tiny fraction of baryons seen at midrapidity).

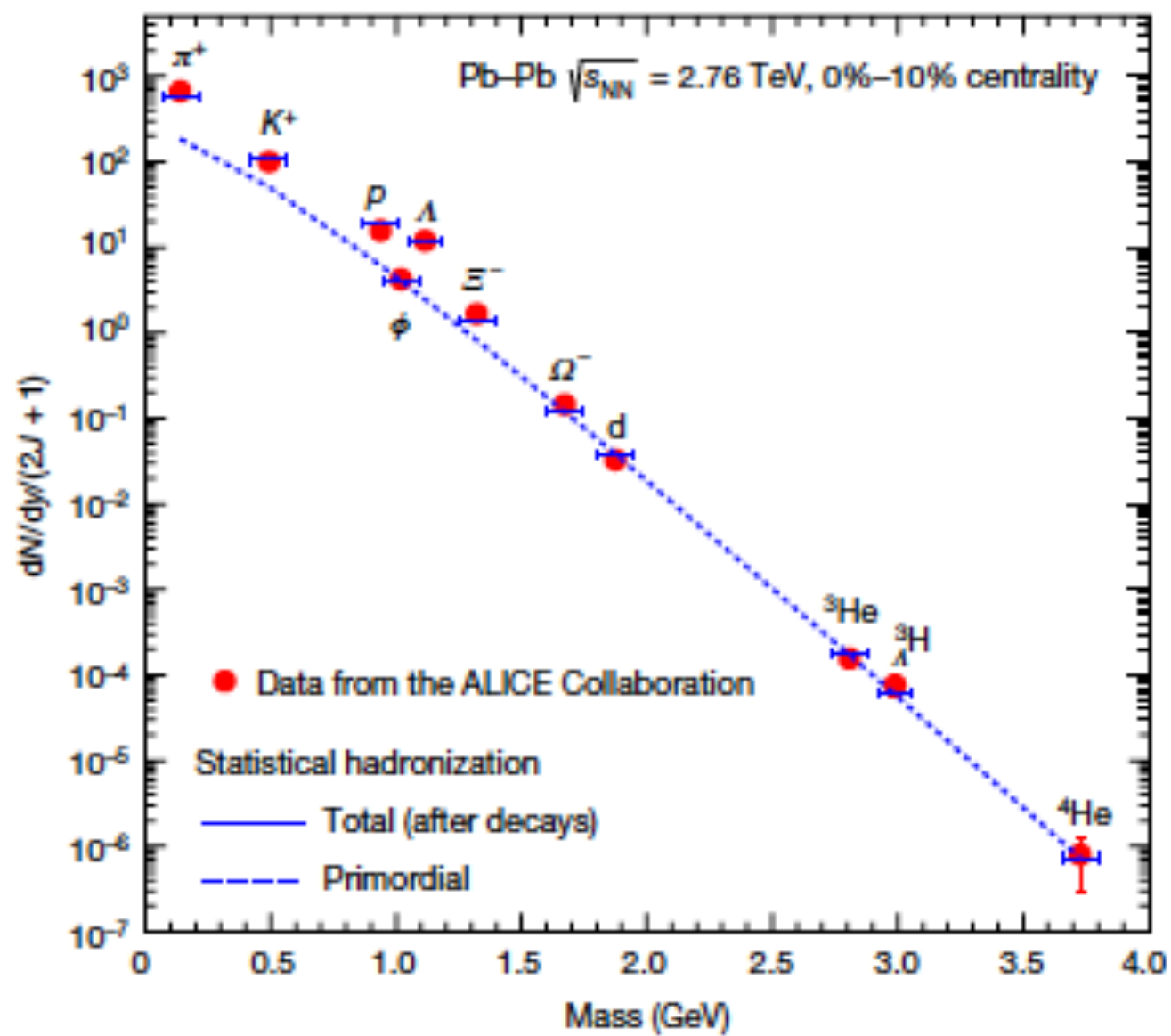


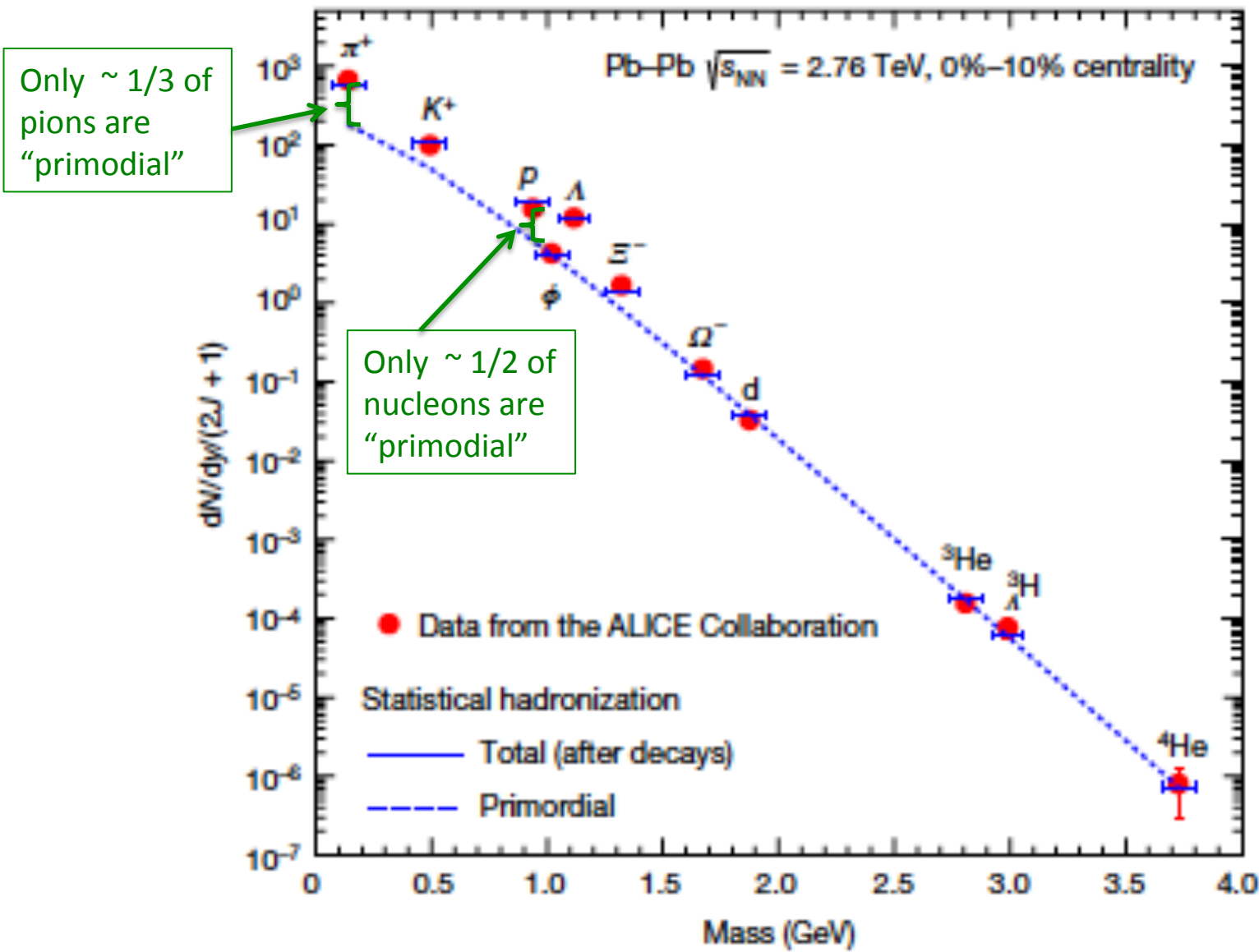
$$T_{cf} = 156.5 \pm 1.5 \text{ MeV}$$

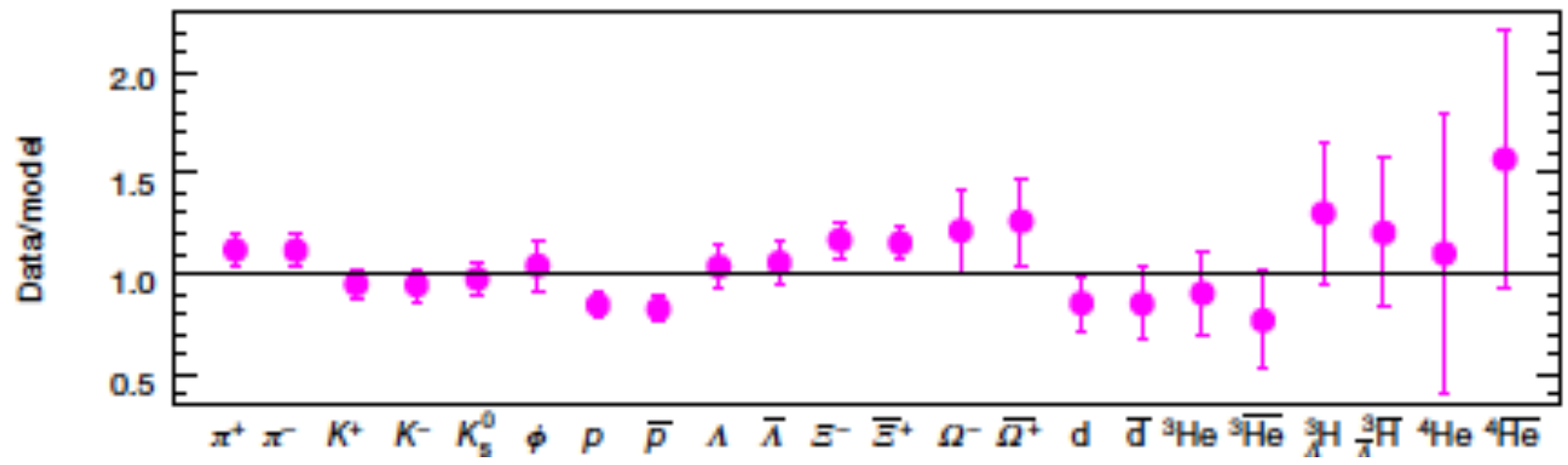
$$\mu_b = .7 \pm 3.8 \text{ MeV (Consistent with zero)}$$

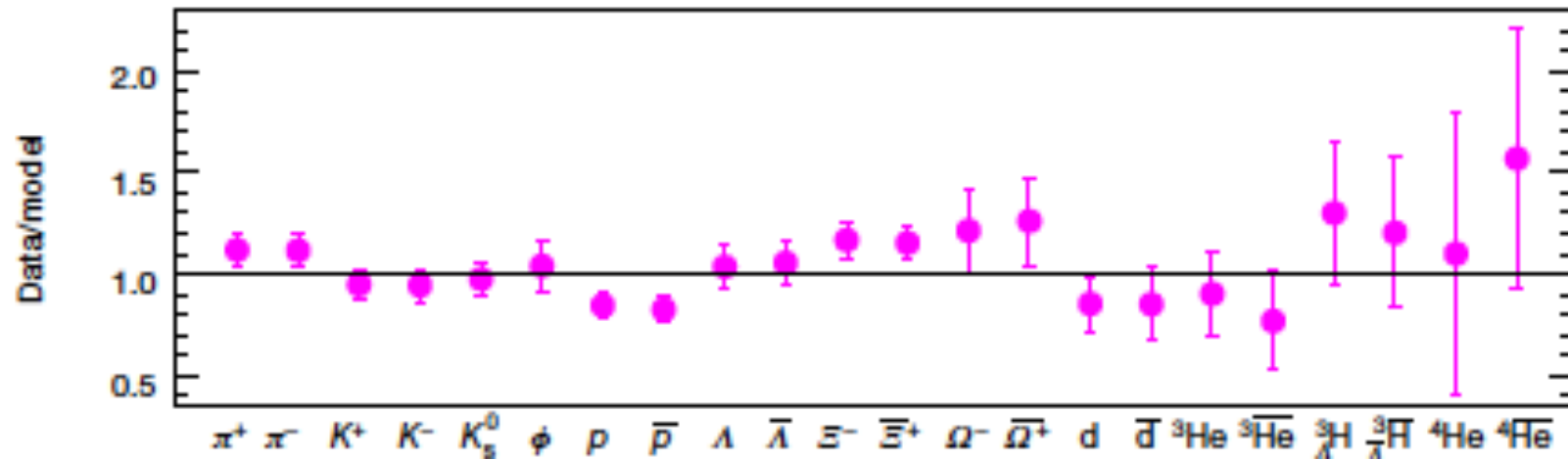
$$V = 5280 \pm 410 \text{ fm}^3 = (17.4 \pm .4 \text{ fm})^3$$

From A. Andronic, P. Braun-Muniziger, Krzysztof Redlich & J. Stachel, Nature 561, 312 (2018)



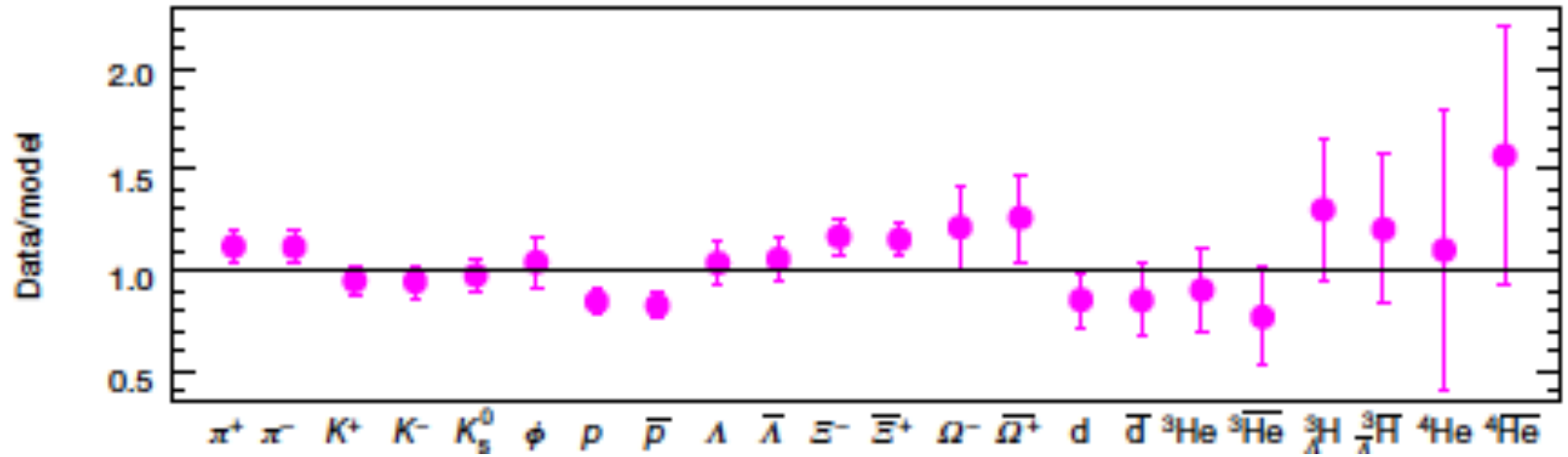






Not perfect, as seen above.

But relative yields of 11 quantities (ignoring difference of isospin and particles vs antiparticles) covering 9 orders of magnitude are fit to better than .12 orders of magnitude with one parameter, T_{cf} .

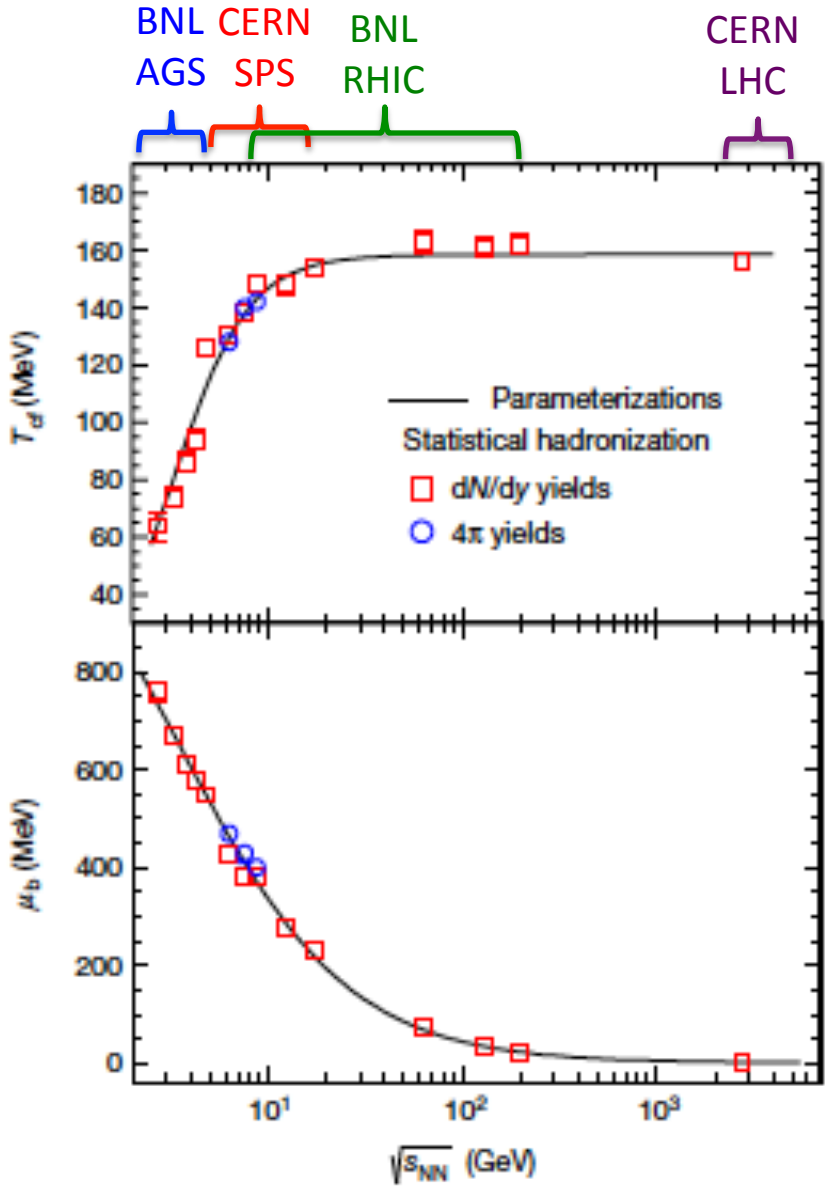


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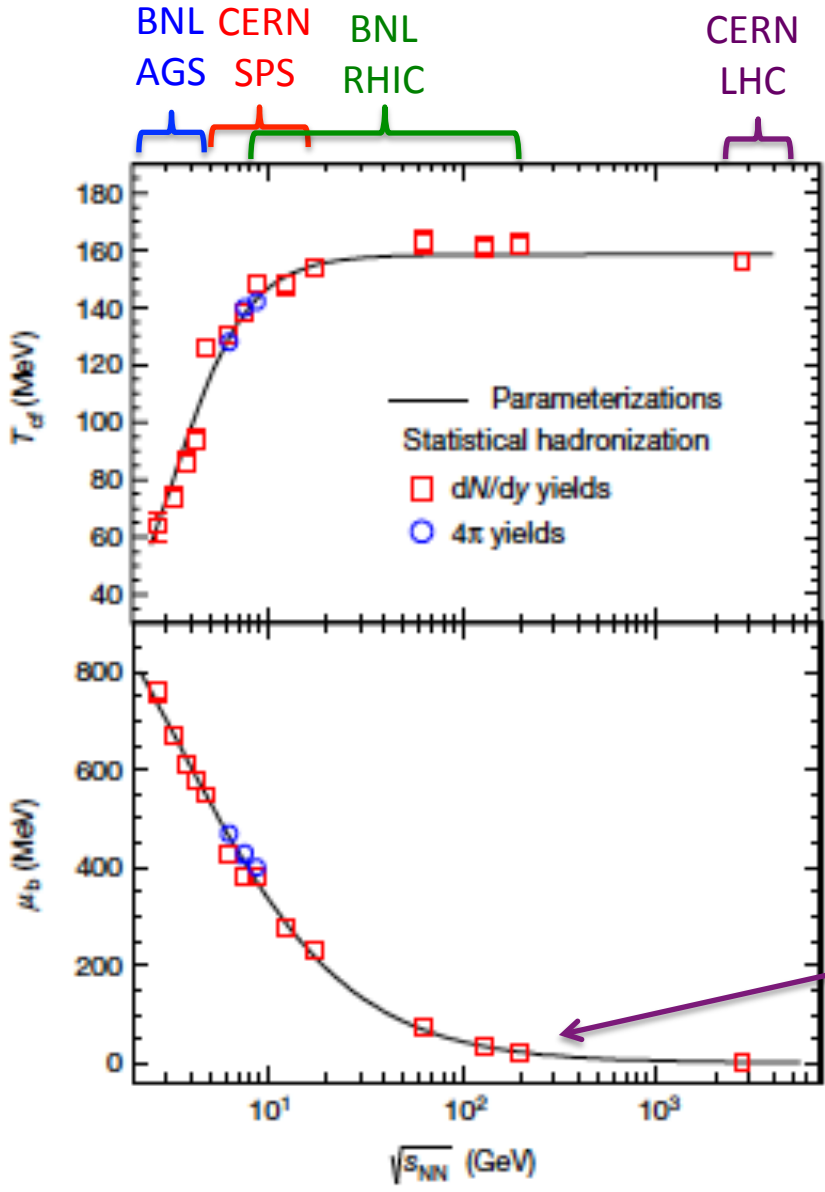
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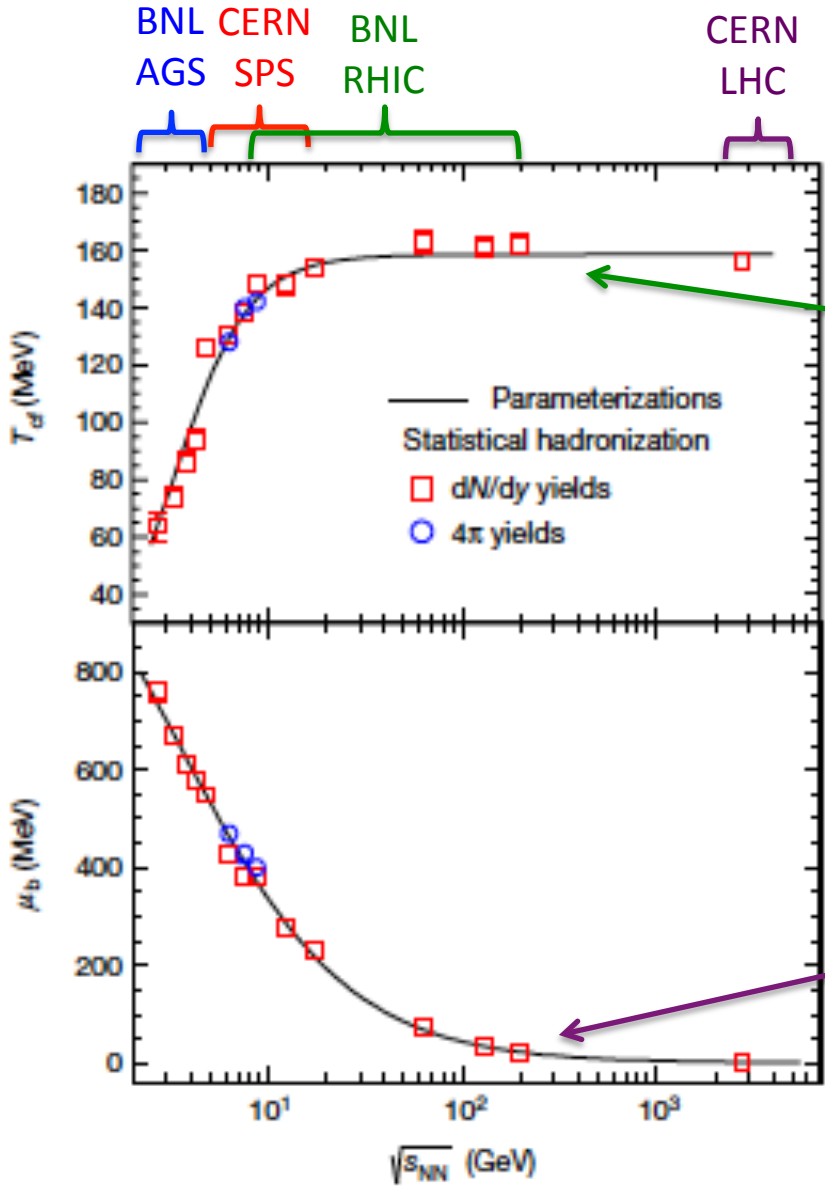


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Model allows one to track T_{cf} as a function of energy



The chemical freeze out temperature appears to saturate with increasing beam energy

As advertised, baryon chemical potential goes to zero as energy increases

- Saturating behavior allows one to “Decode the Phase Structure of QCD ”*

* Title of [Nature](#) 561, 312 (2018) by A. Andronic, P. Braun-Muniziger, Krzysztof Redlich & J. Stachel

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 - Clearly in real QCD notion of “phase structure” is fuzzy as there is a cross-over region rather than a phase transition at $\mu_b = 0$.
 - However there is a remarkable fact: the saturating value of T_{cf} is consistent with the “cross-over temperature”, or pseudocritical temperature T_c , as determined from lattice studies in which T_c is identified as the maximum of the chiral susceptibility. (Note the chiral susceptibility would diverge at a true 2nd order chiral phase transition)

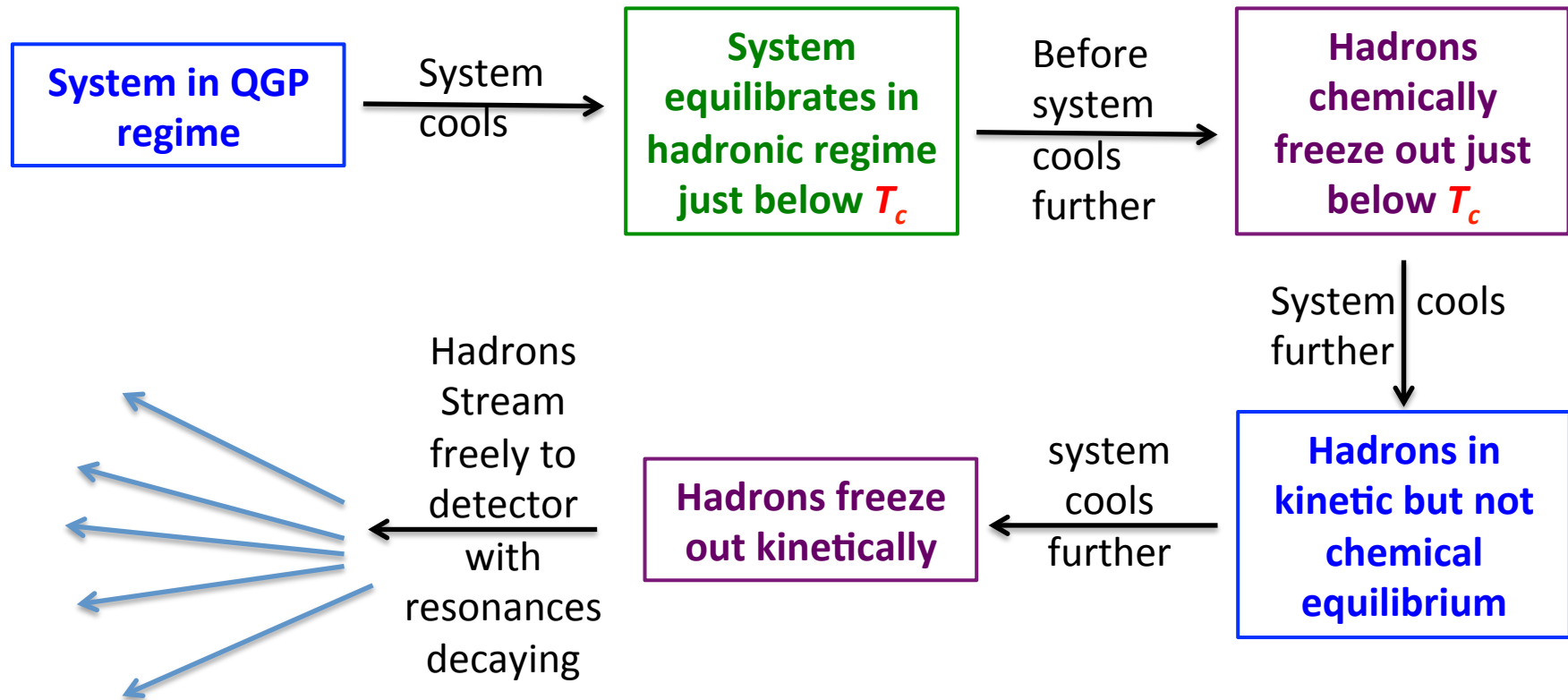
$$T_c = 154 \pm 9 \text{ Mev} \star \text{ compared with } T_{cf} = 156.5 \pm 1.5 \text{ Mev}$$

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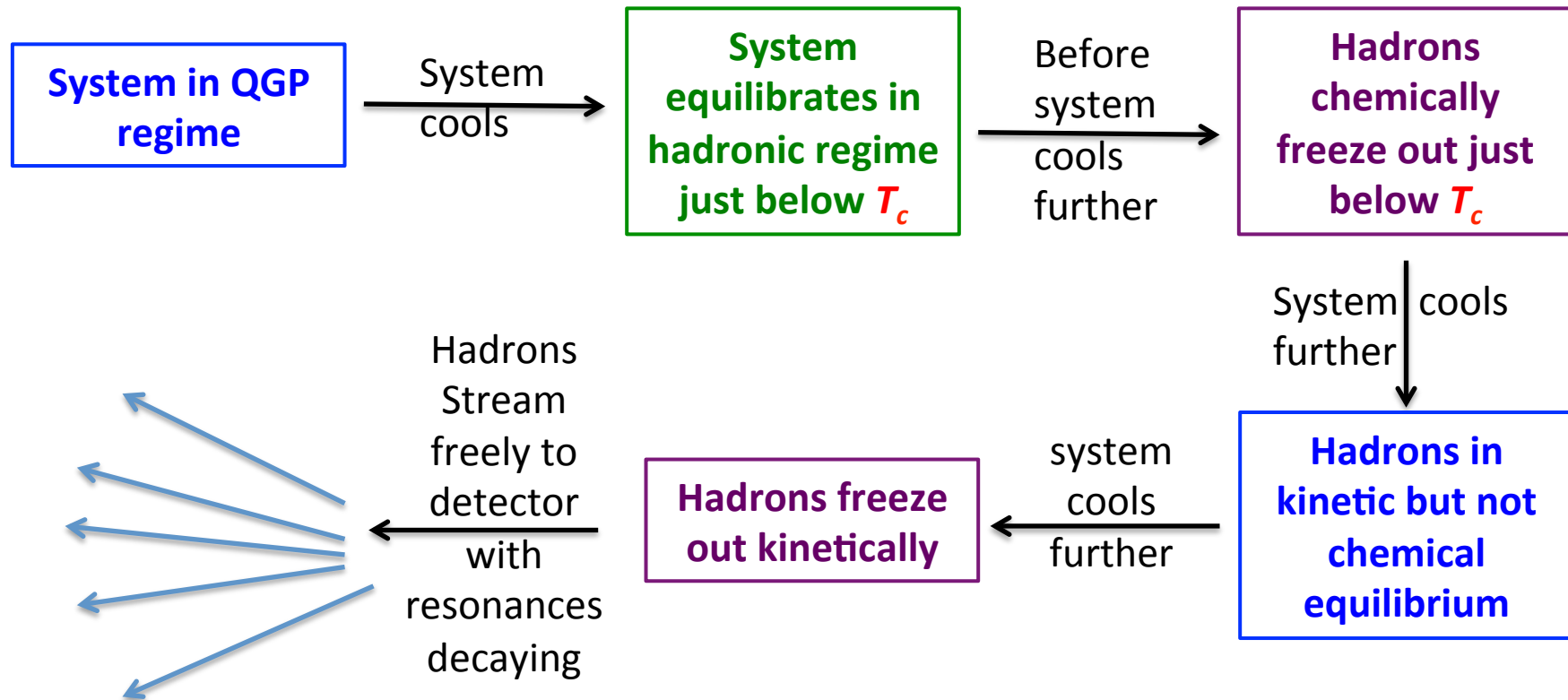
★ Hot QCD Collaboration, Phys Rev. D 90, 094503 (2014).

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If the assumptions of the SHM are trustworthy this implies a truly remarkable scenario



Remarkable thing is that $T_c = T_{cf}$. Logically nothing relates the two in the SHM beyond requirement $T_c \geq T_{cf}$: T_c is a thermodynamic property of equilibrated matter, while T_{cf} depends on the dynamics of expansion and how things fall out of equilibrium.

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- The assumptions of the model have been questioned for some time*.
 - Typical concern involves the time scales of the dynamics.
 - Concern that chemical equilibrium depends on processes with very different time scales so that one does not expect universal chemical freezeout temp.
 - Concern that all hadrons in the system do not have time to chemically equilibrated in hadronic phase.
 - A “born in equilibrium” dynamical scenario has been considered as a way around these problems.

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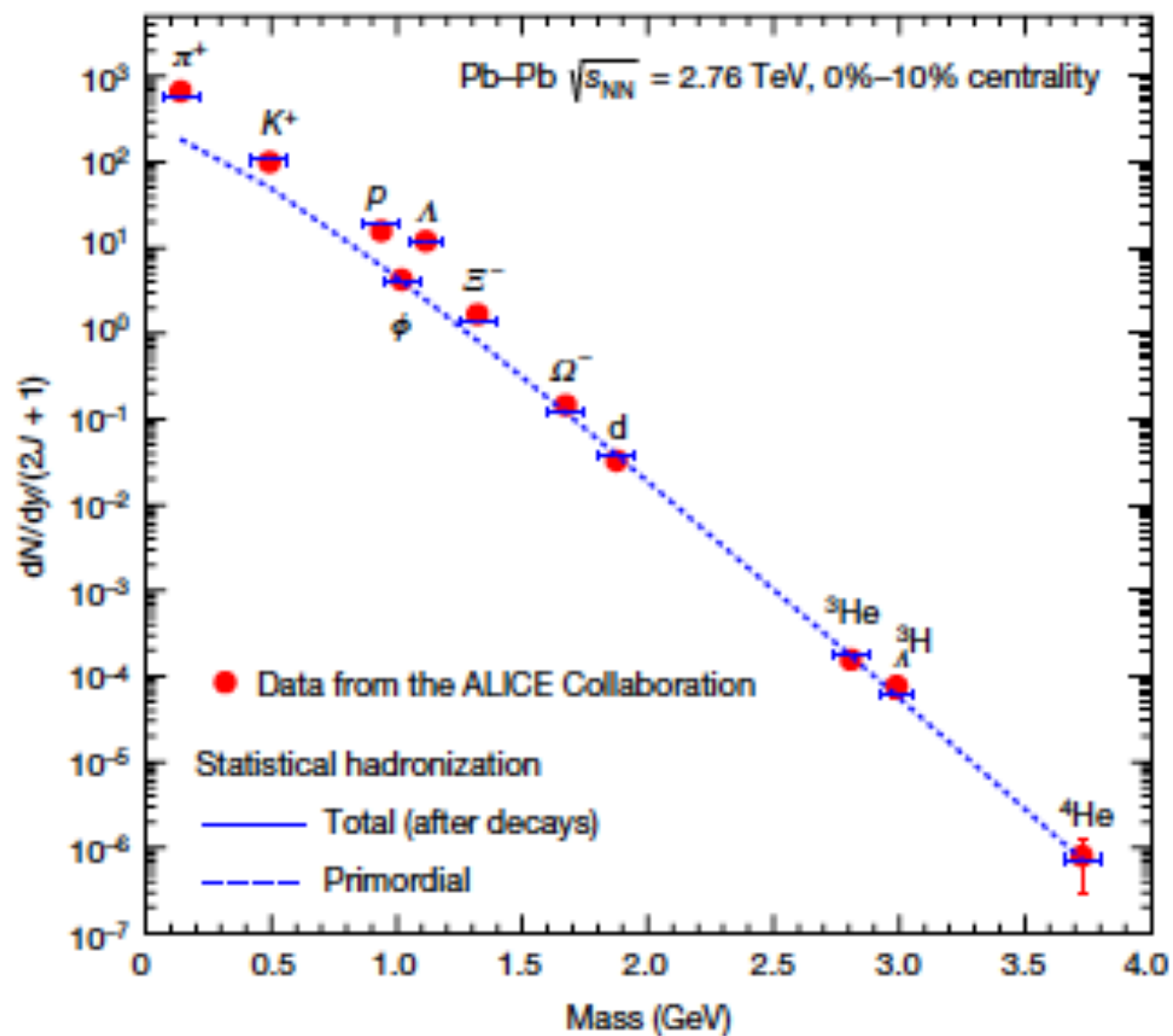
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- To test this, focus on the yield of light nuclei.
 - The binding energies of light nuclei are very much smaller than both typical hadronic scales and the temperature. This could cause tensions with the assumptions.
 - Much of the phenomenological success of the model come from the light nuclei:
 - Of the 9 orders of magnitudes in yields, 5 of them come from light nuclei.
 - The yields of light nuclei come entirely from the primordial densities (rather than feed down from decaying resonances as seen in pions, kaons, nucleons, Lambdas etc.) So the yields directly probe the putative equilibrated matter.



Recall model assumptions about equilibrated hadronic regime

2. Then the system expands and cools and becomes an equilibrated hadronic gas (including light nuclei) with the bulk of the system contained in a large volume at a nearly uniform temperature.

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Let us probe these in more detail

Assumptions 2.a, 2.b & 2.c are the basic assumptions underlying the validity of kinetic theory.

Assumption 2a: *In this regime, the system is sufficiently dilute enough so that hadrons (and light nuclei) are sufficiently well-separated as to be discernible.*

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The physical picture:

- Almost all of the energy is in the mass and kinetic energy for discernible hadrons.
- Hadrons are freely propagating almost all of their time with their energies fixed via free space standard dispersion relation.
- The hadrons occasionally exchange energy via elastic collisions enabling the establishment and maintenance of kinetic equilibrium.
- Chemical equilibrium established and maintained via rare inelastic interactions; “interactions” includes spontaneous decay of an unstable hadrons as well as inelastic collisions .

symbol	quantity	mass dimension
T	temperature	mass ¹
n_i	density of hadrons of species i	mass ³
ϵ_i	energy density of hadrons of species i	mass ⁴
$v_i \equiv \sqrt{1 - \left(\frac{m_i n_i}{\epsilon_i}\right)^2}$	characteristic velocity of hadrons of species i	mass ⁰
C_i	rate per unit volume for hadrons of species i to be created in a collision	mass ⁴
A_i	rate per unit volume for hadrons of species i to be destroyed in a collision	mass ⁴
$\tau_i^C \equiv \frac{n_i}{C_i}$	characteristic creation time for hadrons of species i	mass ⁻¹
$\tau_i^A \equiv \frac{n_i}{A_i}$	characteristic destruction time for hadrons of species i	mass ⁻¹
$\tau_i^{\text{int inel}}$	characteristic duration of an inelastic interaction that creates or annihilates hadron of species i	mass ⁻¹
τ_i^{elastic}	characteristic time between elastic collisions involving species i	mass ⁻¹
$\tau_i^{\text{int el}}$	characteristic duration of interaction in which hadron of species i elastically scatters	mass ⁻¹
$l_i^{\text{mfp}} \equiv v_i \tau_i^{\text{elastic}}$	mean free path for elastic collisions involving species i	mass ⁻¹

Some useful quantities characterizing interactions in a hadronic gas

Some conditions for validity of the picture

$$A_i = C_i \quad \text{Equilibrium condition. This implies}$$

$$\tau_i^A \stackrel{\downarrow}{=} \tau_i^C \equiv \tau_i$$

τ_i is the lifetime of the hadron in the equilibrated medium. It is also the characteristic chemical equilibration time if decay rate per volume of a species is linear in density and its decay products do not substantially disturb equilibrium of other species.

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$$n \text{Vol}^{\text{hadron}} \ll 1$$

Natural conditions for hadrons to be discernable; "hadrons" in this context include light nuclei, n is total density of hadrons

Model predictions for equilibrium

$$n_i(T) = g_i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\exp((m_i^2 + p^2)^{1/2}/T) \pm 1}$$

↑
degeneracy
factor

$$\varepsilon_i(T) = g_i \int \frac{d^3 p}{(2\pi)^3} \frac{(m_i^2 + p^2)^{1/2}}{\exp((m_i^2 + p^2)^{1/2}/T) \pm 1}$$

↑
+fermions
-bosons

Model predictions for equilibrium

For $T=156.5$ MeV

$$n_i(T) = g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp((m_i^2+p^2)^{1/2}/T) \pm 1}$$

↑ degeneracy factor ↑ +fermions
↓ ↓ -bosons

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meson	n (fm^{-3})	$\langle \gamma \rangle =$ $\varepsilon/(n m)$	v
pions	.143	3.62	.96
kaons	.052	1.61	.78
$f_0(500)$.013	1.60	.78
η	.010	1.55	.76
$K_0(700)$.010	1.41	.70
ρ	.032	1.37	.68
ω	.010	1.36	.68
K^*	.024	1.31	.65

	n (fm^{-3})
All mesons with mass < 1250 MeV	.302

baryons	n (fm^{-3})*	$\langle \gamma \rangle =$ $\varepsilon/(n m)$	v
nucleon	.0124	1.29	.63
Λ	.0025	1.24	.59
Σ	.0051	1.23	.58
Δ	.0107	1.21	.57
All baryons with mass < 1250 MeV	.0254		
*includes antibaryons			

Note that the τ_i is the lifetime of the hadron in the medium. In general one can use **Boltzmann equations** to determine τ_i . To implement one needs knowledge of rates for all interaction processes.

For **unstable hadrons** the lifetime is shorter than in free space. Resonances can decay spontaneously as in free space and can also be destroyed in a collision with another hadron in the gas. This provides an **upper bound** for τ_i for without full knowledge of interaction rates.

$$\tau_i < \frac{\langle \gamma_i \rangle}{\Gamma_i} \quad C_i = A_i > \frac{n_i \Gamma_i}{\langle \gamma_i \rangle}$$

Time dilation factor Width of resonance

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Time dilation factor

Width of resonance

Stable hadrons have finite lifetimes in the hadron resonance gas. *Eg.* When two pions resonate into a ρ meson, they cease to be pions.

Remarkably, even without full knowledge of interaction rates one can also deduce a lower bound for their lifetimes given the equilibrium assumption of the SHM. This will prove useful in studying weakly bound nuclei

$$C_{i\text{stable}} = C_i^{\text{resonance decays}} + C_i^{\text{collisions}} > C_i^{\text{resonance decays}}$$

$$= \sum_{j=\text{resonances}} \langle N_{i,j} \rangle A_j > \sum_{j=\text{resonances}} \langle N_{i,j} \rangle \frac{n_j \Gamma_j}{\langle \gamma_j \rangle} > \langle N_{i,k} \rangle \frac{n_k \Gamma_k}{\langle \gamma_k \rangle}$$

Average number of particles of type i produced in decay of resonance j

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Therefore

$$\tau_{i\text{stable}} < \frac{n_i}{C_i} < \frac{n_i}{\sum_{j=\text{resonances}} \langle N_{i,j} \rangle \frac{n_j \Gamma_j}{\langle \gamma_j \rangle}} < \frac{n_i \langle \gamma_k \rangle}{n_k \langle N_{i,k} \rangle \Gamma_k}$$

$$C_{i\text{stable}} = C_i^{\text{resonance decays}} + C_i^{\text{collisions}} > C_i^{\text{resonance decays}}$$

$$= \sum_{j=\text{resonances}} \langle N_{i,j} \rangle A_j > \sum_{j=\text{resonances}} \langle N_{i,j} \rangle \frac{n_j \Gamma_j}{\langle \gamma_j \rangle} > \langle N_{i,k} \rangle \frac{n_k \Gamma_k}{\langle \gamma_k \rangle}$$

Average number of particles of type i produced in decay of resonance j

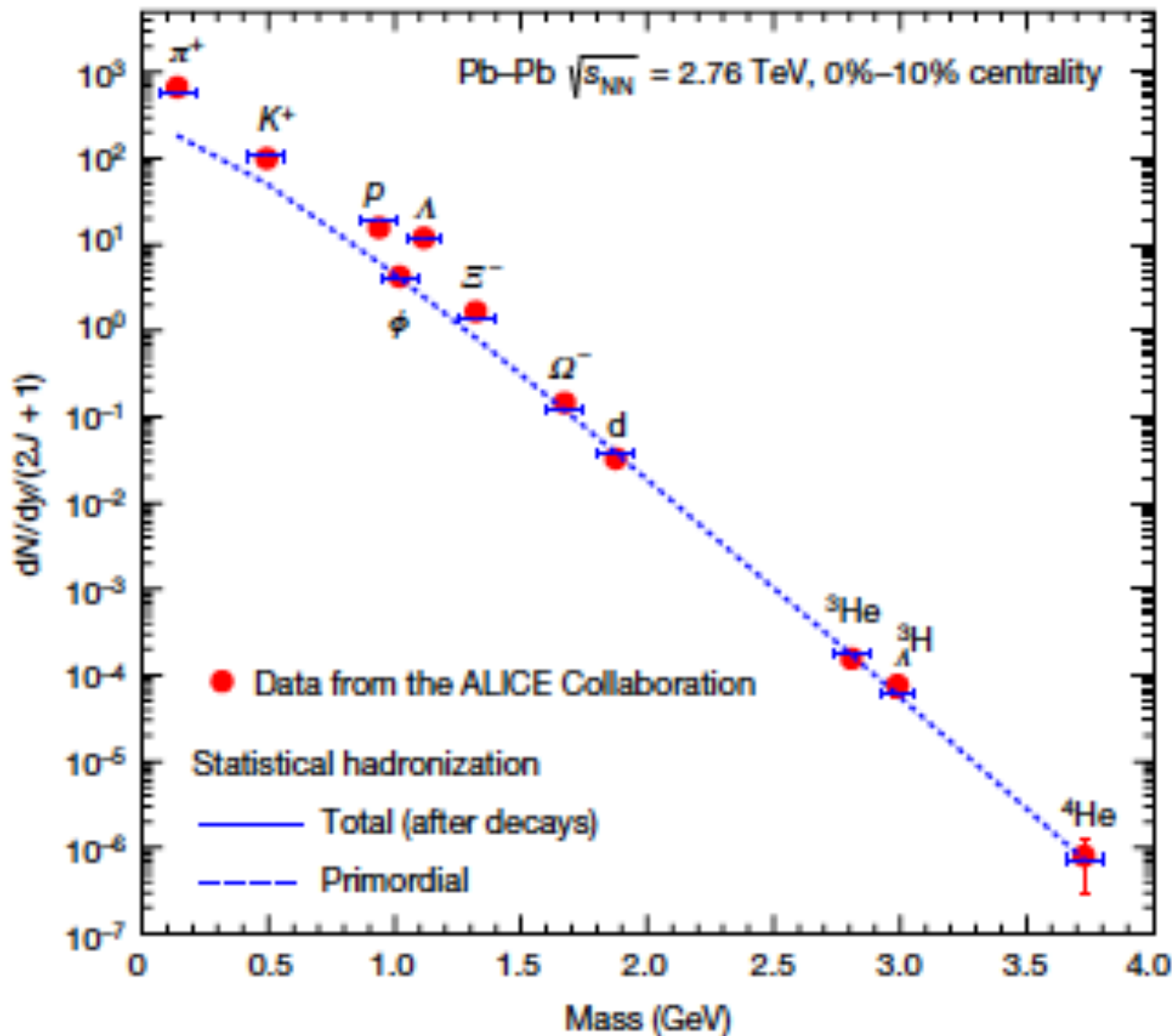
k is any of the resonances

Therefore

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For the nucleon using the Δ for k with densities and $\langle \gamma_k \rangle$ from the tables above, $\langle N_{\text{nucleon},\Delta} \rangle \approx .994$, $\Gamma_{\Delta} \approx 117 \text{ MeV}$

$$\tau_{\text{nucleon}} < 2.38 \text{ fm}$$



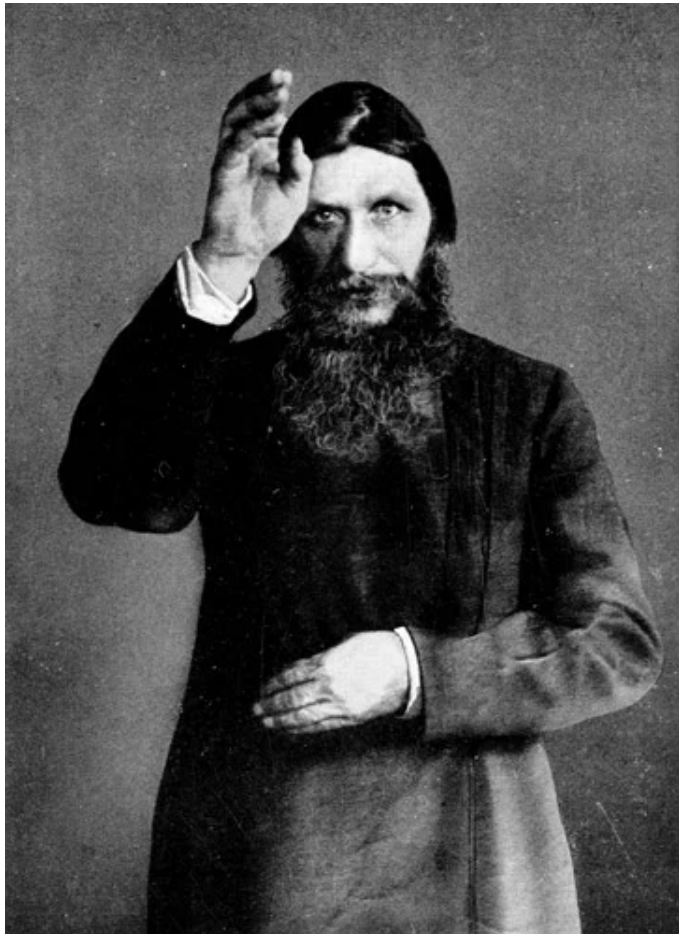
Note that the SHM describes the light nuclei rather well.

A fit to just the light nuclei rather than the whole set yields $T_{cf} = 159 \pm 5$ MeV
 Consistent with full fit of $T_{cf} = 156.5 \pm 5$ MeV

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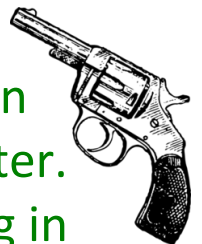
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Russian nobles lead by Prince Yusupov concluded that Rasputin was a threat to the empire and decided to kill him. The plot unfolded Dec. 29-30 1916.

The murder involved

- A poisoned cake (cyanide).
- Poisoned Madeira wine(cyanide).
- A pistol shot to chest believe by the conspirators to be fatal.
- Two subsequent pistol shots when Rasputin attempt to flee hours later.
- Ultimate cause of death drowning in the Neva river where his body was thrown.



Some light nuclei properties; velocity uses T=156.5 MeV

Nucleus	velocity	Effective Volume (fm ³)*	$t^{\text{int inel}}$ (fm) ⁺
D	.48	88.3	> 89
³He	.40	69.7	>35
³_ΛH	.38	9940	>1500
⁴He	.34	42.7	>10

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*Effective volume for D, ³He and ⁴He defined for simplicity as volume of a uniform sphere whose RMS radius is the electric charge radius .

*For the hypertriton the extremely small binding energy relative to a D + Λ (~130 KeV) implies the wave function is dominated by these two bodies outside the range of interaction.

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$$\langle r^2 \rangle_{\Lambda H} \approx \frac{1}{2} \sqrt{\frac{m_D + m_\Lambda}{b m_D m_\Lambda}}$$

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[†] The time the interaction to create a bound state must be long enough to clearly resolve whether one has the bound state of interest rather than unbound constituents . $\tau^{\text{int inel}} \gg \sim 1/B$ This is essential the energy time uncertainty relation

$$\tau_i^{\text{intinelas}} \ll \tau_i$$

Combined with $\frac{1}{B} \ll \tau_i^{\text{intinelas}}$ implies

$$1 \gg \frac{1}{B \tau_{\text{bound state}}}$$

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Thus for SHM to make sense

$$\frac{1}{\tau_{\text{bound state}}} < \sum_{j=\text{constituents}} \frac{1}{\tau_j}$$

It is $<$ rather than $=$ as the bound state could dissociate the bound state leaving constituents intact.

For deuteron condition is : $1 \gg \frac{2}{B_D \tau_{\text{nucleon}}}$

$$B_D = 2.22 \text{ MeV} \quad \frac{2}{\tau_{\text{nucleon}}} > \frac{2}{2.38 \text{ fm}} = 166 \text{ Mev}$$

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For hypertriton the situation is even worse $\frac{2}{B_{\Lambda^3 H} \tau_{\text{nucleon}}} > 1200$

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Asking a hadron gas to produce a D that lasts for less than 2.38 fm is like asking a violin virtuoso to play a middle A (440 Hz= $2.3 \times 10^{-3} \text{ s}^{-1}$) for less than $4.85 \times 10^{-3} \text{ s}^{-1}$.



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Even if there was skill to play a note that short, it would not be a middle A, but a muddle: to resolve A from the G[#] below, the note's duration must satisfy $t_{\text{duration}} \gg 4. \times 10^{-2} \text{ s}^{-1}$

- This means that using the numbers fit by the SHM, the assumptions underlying the SHM lead to a contradiction
 - the fact the Δ is assumed to be in equilibrium implies that the lifetime of a nucleon in the medium is much shorter than the time needed for the existence of light nuclei. Light nuclei cannot form and equilibrate in the medium at the putative freeze out temperature as assumed by the SHM.

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By itself, this kills off picture of light nuclei propagating in a dilute gas at the time of chemical freeze out



Recall the condition required for hadron to be produced inside the putative hadronic volume.

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$n Vol^{nucleus} \ll 1$ Is required for SHM assumption of a dilute gas of hadrons and nuclei

$n Vol^{eff}$	pions	kaons	mesons with $m < 1250$ MeV	nucleons	baryons with $m < 1250$ MeV	Hadrons with $m < 1250$ MeV	$\ll 1$
D	12.6	4.6	26.7	1.1	2.2	28.9	No
^3He	10.0	3.6	21.0	.9	1.7	22.7	No
$^3_{\Lambda}\text{H}$	1421	516	3001	123	252	3253	No
^4He	6.1	2.2	12.9	.52	1.1	14.0	No

Condition is badly violated for all light nuclei

By itself, this kills off picture of light nuclei propagating in a dilute gas of hadrons time of chemical freeze out



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- T_{cf} effectively parameterizes the yield of light nuclei as a function of mass but **cannot** be interpreted as a chemical freeze out temp.

- Given that light nuclei yields have same T_{cf} as hadronic yields, this raises the question as to whether there is any reason to believe that hadronic yields are due to the chemical freeze out from a equilibrated hadronic gas.



Implications

- This talk is entitled “what do the yields of light nuclei tell us about heavy ion collisions?”
 - The answer is I really do not know. The remarkable thing is not that a model based on such obviously inconsistent assumptions describes the data well—the remarkable thing is that it describes the data at all

