T-dependence of the axion mass when the $U_A(1)$ and chiral symmetry breaking are tied

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Introduction

- QCD has the Strong CP problem: no experimental evidence of any CP-symmetry violation in strong interactions, in spite of its θ-term:

$$\mathcal{L}_{\rm QCD} = \mathcal{L}^{\rm QCD}_{\rm CPsymmetric} + \theta \, \frac{g^2}{32\pi^2} \, F^b_{\mu\nu} \widetilde{F}^{b\mu\nu} \qquad (\widetilde{F}^{b\mu\nu} \equiv \tfrac{1}{2} \, \epsilon^{\mu\nu\rho\sigma} F^b_{\rho\sigma})$$

θ-term cannot be discarded (unlike in QED), although it is a total divergence, due to nontrivial topological structures in QCD – *e.g.*, **instantons** (probably yielding anomalously large $M_{\eta'}$) **important for solving the $U_A(1)$ problem**.

- In spite of this, the experimental bound is only $|\theta| < 10^{-10}$
- Thus, a mystery: **why is θ so small ?**
- Nowadays preferred solution: a new particle beyond SM: **axion** a
- **Axions are very interesting also for cosmology as candidates for dark matter.**

Axions as quasi-Goldstone bosons

• Peccei & Quinn introduced a new axial global symmetry $U(1)_{PQ}$ which is broken spontaneously at some scale f_a
(f_a = free parameter of axion theories, determines absolute value of the axion mass m_a, but cancels from combinations such as $m_\mathrm{a}(T)/m_\mathrm{a}(0)$.)

• the pseudoscalar axion field $\mathrm{a}(x)$ is the (would-be massless) Goldstone boson of this spontaneous breaking. Then,

$$\mathcal{L}_{\mathrm{axion}}^{\mathrm{QCD+}} = \mathcal{L}_{\mathrm{CPsymmetric}}^{\mathrm{QCD}} + \left(\theta + \frac{\mathrm{a}}{f_\mathrm{a}}\right) \frac{g^2}{32\pi^2} F_{\mu\nu}^b \widetilde{F}^{b\mu\nu} + \tfrac{1}{2}\partial_\mu \mathrm{a}\, \partial^\mu \mathrm{a} + \mathcal{L}_{\mathrm{int}}^{\mathrm{a}\,\psi}$$

• But, the $U(1)_{PQ}$ symmetry is also broken explicitly by the gluon axial anomaly through axion's coupling with gluons $\Rightarrow m_\mathrm{a} \neq 0$. Gluons generate an effective axion potential, which leads to the axion expectation value $\langle \mathrm{a} \rangle$ such that $(\theta + \langle \mathrm{a} \rangle / f_\mathrm{a}) = 0$, minimizing the potential $\Rightarrow$ **strong CP problem solved**, irrespective of the initial θ.

("Misalignment production" is relaxation from any value in the early Universe towards the effective potential minimum at $\theta = -\langle \mathrm{a} \rangle / f_\mathrm{a}$. The resulting axion oscillation energy is a "cold dark matter" candidate.)

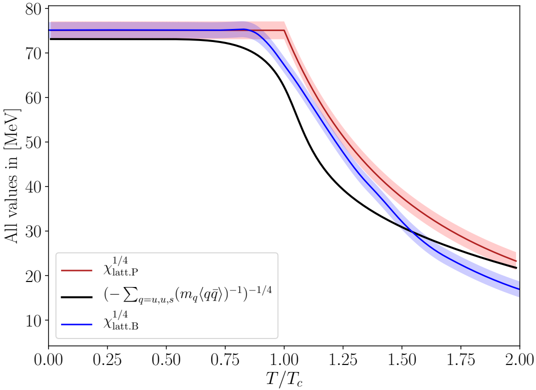
Axion mass

- For all temperatures: $m_a^2(T)\, f_a^2 = \chi(T) =$ QCD topological susceptibility
- At $T = 0$, $$m_a^2\, f_a^2 = \frac{m_u\, m_d}{(\,m_u + m_d\,)^2}\, f_\pi^2\, M_\pi^2 \to \frac{\text{isospin}}{\text{limit}} \to (78.9\,\text{MeV})^4$$

- This agrees well with results, including $\chi(T)$, from "our" DS-BSE chirally well-behaved model (separable: simplified, but phenomenologically successful)

- $\chi(T)$ from lattice: Petreczky & al. PLB (2016) and Borsany & al. Nature (2016)

- $\chi(T)$ from our usual DS-BSE model: successful at $T = 0$, no additional fitting for $T > 0$: condensates $\langle\bar{q}q\rangle(T)$ of massive $q = u, d, s$ essential to yield good T-dependence of $\chi(T)$ for good T-dependence of η and η' masses.



Our various $\chi(T)$ – from our various calculations of anomalous η'-η masses

QCD chiral behavior (reproduced by, e.g., DS approach) of the non-anomalous parts of masses of light $q\bar{q}'$ pseudoscalars: $M^2_{q\bar{q}'} = \mathrm{const}\,(m_q + m_{q'})$.

$\Rightarrow$ non-anomalous parts of the masses cancel in Witten-Veneziano rel. (WVR):

$$M_{\eta'}{}^2 + M_\eta{}^2 - 2\,M_K{}^2 = \chi_{\mathrm{YM}}\,\frac{2N_f}{f_\pi^2} = \mathrm{anomalous\ \ mass}^2 \equiv M_{U_A(1)}{}^2 \approx \Delta M_{\eta_0}{}^2\,,$$

$$\chi = \int d^4x\,\langle 0|Q(x)Q(0)|0\rangle\,, \qquad Q(x) = \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x)$$

QCD topological susceptibility χ = a direct measure of $U_A(1)$ breaking $\Rightarrow$ (partial) $U_A(1)$ restoration is indicated by vanishing or asymptotic reduction of χ and quantities related to it, like $M_{U_A(1)} \approx \Delta M_{\eta_0} \approx \Delta M_{\eta'}$.

- $Q(x)$ = topological charge density operator
- In WVR, χ is pure-gauge, YM one, $\chi_{\mathrm{YM}} \leftrightarrow \chi_{quench}$, obtained long ago by lattice - harder for χ of light-flavorQCD, but can use DiVecchia-Veneziano

relation:
$$\chi = -\frac{\langle\bar{q}q\rangle_0}{\sum\limits_{q=u,d,s}\frac{1}{m_q}} + \mathcal{C}_m(\mathrm{unknown\ corrections,\ higher}\ \mathcal{O}\ \mathrm{in\ small}\ m_q)$$

.

The 1st example of tied breaking of $U_A(1)$ and chiral symmetries:

Leutwyler-Smilga relation (LS), also connecting YM and full QCD quantities (like WVR), "making" $\chi_{\rm YM}$ out of much smaller χ:

$$\chi_{\rm YM} = \frac{\chi}{1 + \frac{\chi}{\langle\bar{q}q\rangle_0} \sum\limits_{q=u,d,s} \frac{1}{m_q}} \equiv \widetilde{\chi} \to \widetilde{\chi}(T)$$

where for the light quarks $$\chi = -\frac{1}{\sum\limits_{q=u,d,s} \frac{1}{m_q \langle\bar{q}q\rangle_0}} + \mathcal{C}_m$$

.

- $\mathcal{C}_m$ = small corrections of higher orders in small m_q, ... but $\mathcal{C}_m$ should not be neglected, since $\mathcal{C}_m = 0$ would imply that $\chi_{\rm YM} = \infty$. However, note that for axions, the leading term of χ would suffice.
- LS relation fixes the value of the correction at $T = 0$:

$$\frac{1}{\mathcal{C}_m} = \sum_{q=u,d,s} \frac{1}{m_q \langle\bar{q}q\rangle_0} - \chi_{\rm YM}(0) \left(\sum_{q=u,d,s} \frac{1}{m_q \langle\bar{q}q\rangle_0} \right)^2 .$$

Shore's generalization of WV = 2$^{\rm nd}$ example of this tying

$$(f^0_{\eta'})^2 M^2_{\eta'} + (f^0_{\eta})^2 M^2_{\eta} = \frac{1}{3}\left(f^2_\pi M^2_\pi + 2 f^2_K M^2_K\right) + 6A \quad (1)$$

$$f^0_{\eta'} f^8_{\eta'} M^2_{\eta'} + f^0_{\eta} f^8_{\eta} M^2_{\eta} = \frac{2\sqrt{2}}{3}\left(f^2_\pi M^2_\pi - f^2_K M^2_K\right) \quad (2)$$

$$(f^8_{\eta'})^2 M^2_{\eta'} + (f^8_{\eta})^2 M^2_{\eta} = -\frac{1}{3}\left(f^2_\pi M^2_\pi - 4 f^2_K M^2_K\right) \quad (3)$$

(Large N_c limit, $f^0_{\eta'}, f^8_{\eta}, f_K \to f_\pi$ & 'off-diagonal' $f^0_{\eta}, f^8_{\eta'} \to 0$ recovers standard WV.)
The role of $\chi_{\rm YM}$ taken over by the full QCD topological charge parameter A ,

$$A = \frac{\chi}{1 + \chi\left(\frac{1}{\langle \bar{u}u \rangle\, m_u} + \frac{1}{\langle \bar{d}d \rangle\, m_d} + \frac{1}{\langle \bar{s}s \rangle\, m_s}\right)} \quad (4)$$

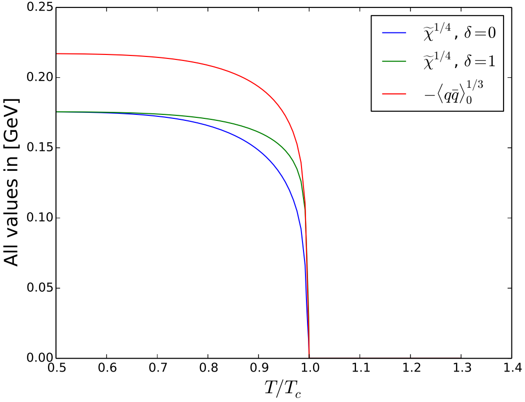
A behaves with T as a full QCD quantity, **but**, at $T = 0$, $A = \chi_{\rm YM} + \mathcal{O}(\frac{1}{N_c})$
Gives the now established crossover behavior around $T \sim T_{\rm Ch}$ when also

$$\chi = \frac{-1}{\frac{1}{m_u \langle \bar{u}u \rangle} + \frac{1}{m_d \langle \bar{d}d \rangle} + \frac{1}{m_s \langle \bar{s}s \rangle}} + \mathcal{C}_m \quad (5)$$

Large N_c limit & approximating 3 condensates by $\langle \bar{q}q \rangle_0$, returns the LS relation.

Chiral condensate $\langle q\bar{q}\rangle_0(T)$ and resulting $\widetilde{\chi}(T)$

Would something go wrong in axion theory because of $\chi(T) \to 0$ as $T \to T_c$?

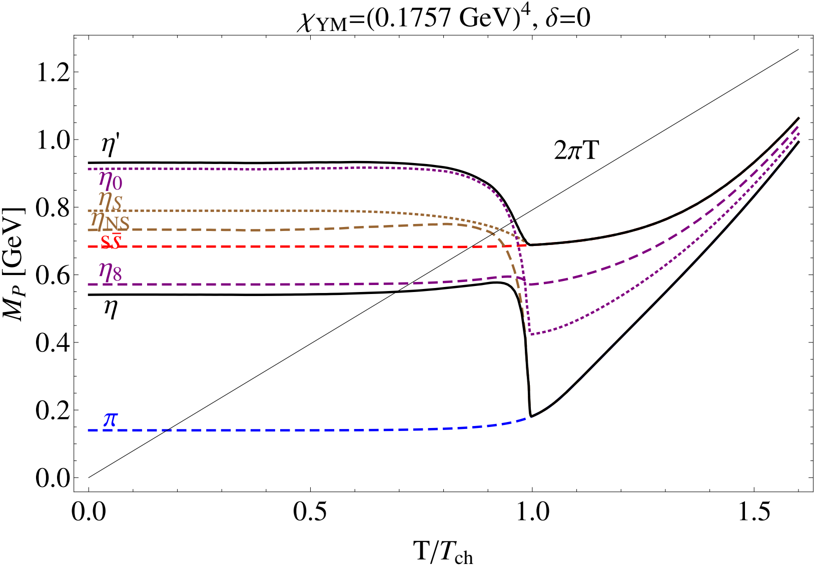


Anyhow, this sharp chiral transition enforces at $T = T_c \equiv T_{\rm Ch}$ the abrupt transition to the NS-S asymptotic regime of vanishing $U_A(1)$ anomaly influence: $M_{\eta'}(T) \to M_{s\bar{s}}(T)$, and $M_\eta(T) \to M_{\rm NS}(T) \to M_\pi(T)$, and $\phi(T) \to 0$.

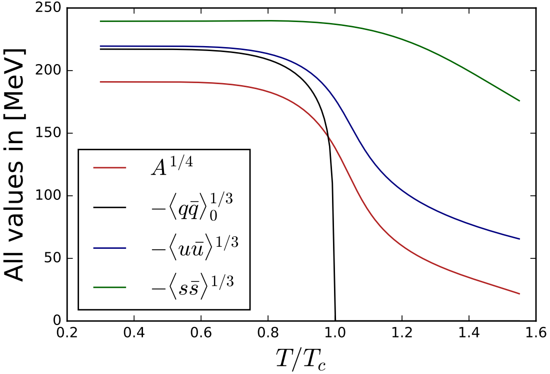
Prediction good for η', but for η not supported by any experiment

[Benić, Horvatić, Kekez and Klabučar, Phys. Rev. D **84** (2011) 016006.]:

Anomalous contribution obtained via LS with chiral-limit condensate $\langle \bar{q}q \rangle_0$:



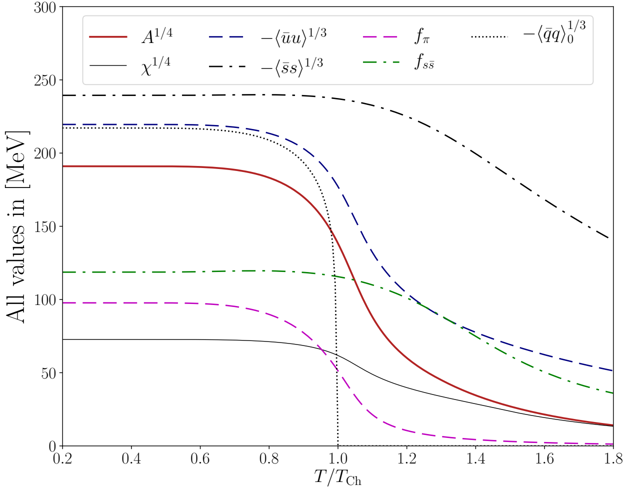
A solution: $U_A(1)$ breaking from realistic condensates



Instead of the fast-falling chiral-limit condensate $\langle \bar{q}q \rangle_0$, try $\langle \bar{q}q \rangle$ condensates with realistic explicit chiral symmetry breaking: replace $m_q \langle \bar{q}q \rangle_0 \to m_q \langle \bar{q}q \rangle$, $(q = u, d, s)$ in χ, like in the original A.

- Introduces crossover behavior, also in $\chi(T)$, and thus in $m_a(T)$

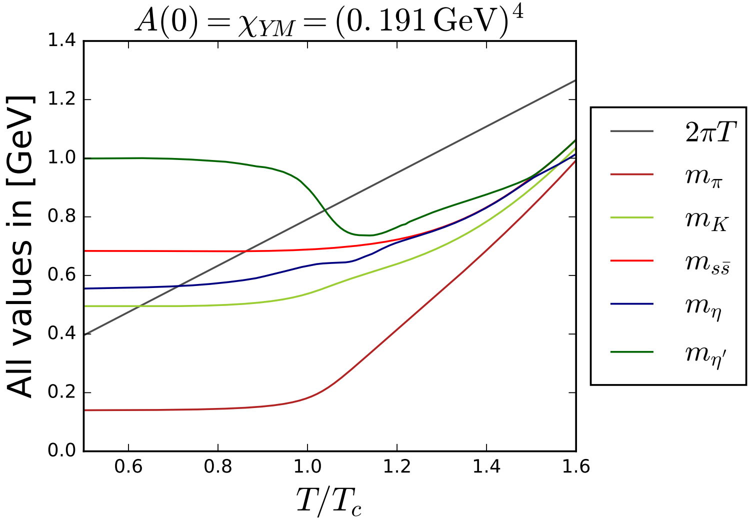
Compare T-dependence of $\langle\bar{q}q\rangle$ & decay const's f_P with χ & A



FKS scheme on Shore $\Rightarrow$ how f_P influence elements of the η-η' mass matrix:

$$X = \frac{f_\pi}{f_{s\bar{s}}}\,, \qquad M_{\rm NS}^2 = M_\pi^2 + \frac{4A}{f_\pi^2}\,, \qquad M_{\rm NSS}^2 = \frac{2\sqrt{2}A}{f_\pi f_{s\bar{s}}}\,, \qquad M_{\rm S}^2 = M_{s\bar{s}}^2 + \frac{2A}{f_{s\bar{s}}^2}$$

⇒ T dependence of light pseudoscalars including η and η'



Simplest Ansatz $\mathcal{C}(T) = \mathcal{C}(0)$ fails above $T > 1.6\,T_c$, as $\chi(T)$ and $A(T) < 0$.

⇒ Try different $\mathcal{C}(T)$ ...

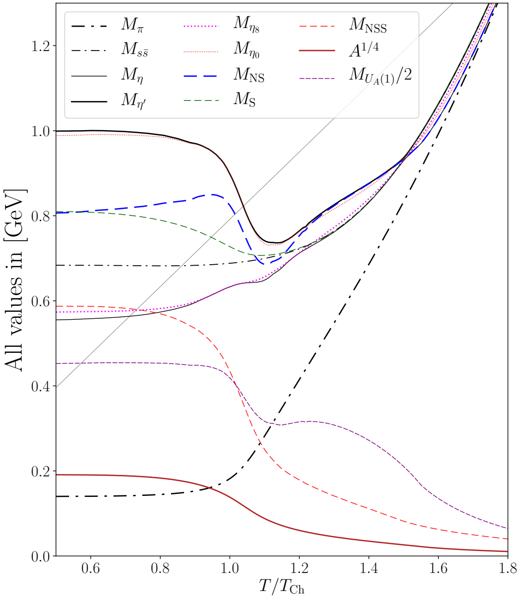
T-dependence of $M_P(T)$ up to $T = 1.8\,T_{\rm Ch}$ [Horvatić& al. Phys.Rev. D99 (2019) 014007]

- $\mathcal{C}(T) \neq const$, adjusted to enable reaching arbitrary high T's, results otherwise very similar to previous case with $\mathcal{C}(T) = \mathcal{C}(0)$.
- Other limitations of rank-2 separable model make it hard to find solutions beyond $\sim 1.8\,T_c$.

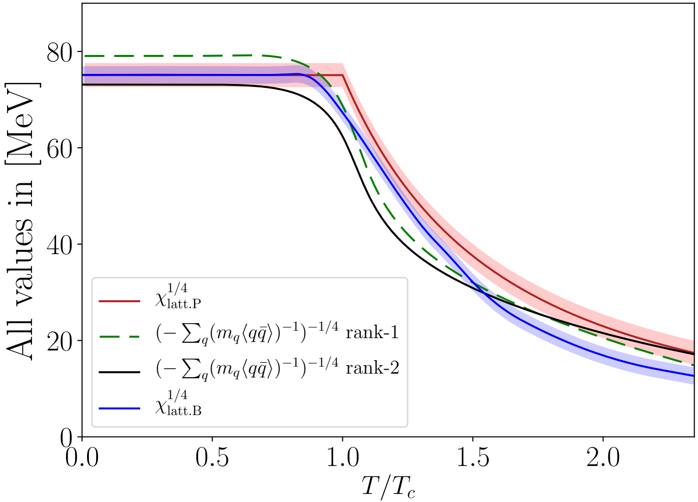
But it is enough to exhibit cleanly the asymptotic regime beyond anticrossing at $1.5\,T_{\rm Ch}$.

Along with A, influence on anomalous masses is given by $M_{\rm NS\,S}$ and $(\frac{1}{2})M_{U_A(1)}$.

Utopistic in practice? - but in principle, accurate experimental knowledge of $M_{\eta'}(T)$ would tell about $A(T)$ and thus about $\chi(T) \propto m_a(T)^2$.



Issue of model dependence: compare even more simplified model:



Model dependence seems not too big, at least at not too high temperatures.
In the future, we would like to employ condensates, *etc.*, from the lattice.

Summary

- The axion mass (up to the free scale parameter $f_{\rm a}$) in terms of the topological susceptibility can be calculated in an effective model of nonperturbative QCD, presently in the SD-BSE approach.
- Our approach to η'-η complex, tying the $U_A(1)$ SB to the DChSB very closely, supports that also for axions $\chi(T)$ should be in terms of the "massive" condensates exhibiting crossover around $T \sim T_{\rm Ch}$, and there exhibits qualitative agreement with lattice results.
- Our $\chi(T)$ for axions can be less modeled than for η'-η, since it can stop at the leading term; *i.e.*, the presently unknown correction term $\mathcal{C}_m$ is not mandatory and can be omitted, thus avoiding assumptions about its T-dependence.
- Our calculations have so far reached $T's$ of only a couple of $T_{\rm Ch}$, reaching the power-law regime $1/T^b$ with smaller exponent ($b \approx 5$) than the lattice ($b \approx 6$ Petreczky et al. and $b \approx 8$ Borsany at al.) But some models yield larger b – more work is needed on the issue of model dependence.