Excited QCD 2019,
Schladming, Jan.30 – Feb. 3, 2019

Deconfinement temperature in AdS/QCD from the spectrum of scalar glueballs

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Based on: S.S. Afonin and A.D. Katanaeva,
Phys. Rev. D98, 114027 (2018) [arXiv:1809.07730]

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A brief introduction

AdS/CFT correspondence (= gauge/gravity duality = holographic duality)
is a conjectured equivalence between a quantum gravity (in terms of string theory or M-theory) compactified on anti-de Sitter space (**AdS**) and a Conformal Field Theory (**CFT**) on AdS boundary

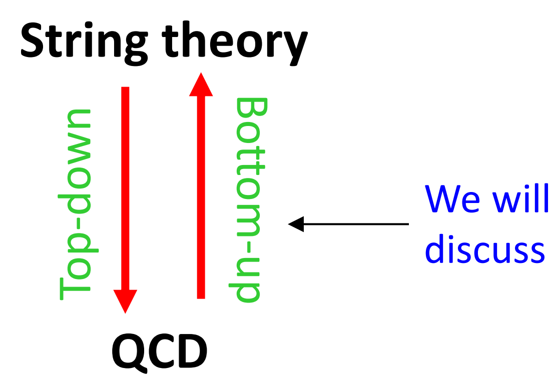
The most promoted example (Maldacena, 1997 - *the most cited work in theoretical physics*!):

Type IIB string theory on $AdS_5 \times S^5$ in the low-energy (i.e. supergravity) approximation	$\Longleftrightarrow$	$\mathcal{N}=4$ SYM theory with *SU(N)* gauge group on AdS_5 boundary (= 4D Minkowski) in the limit $g^2 N \gg 1$

S^5: $X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 + X_6^2 = R^2$

AdS_5: $X_1^2 + X_2^2 - X_3^2 - X_4^2 - X_5^2 - X_6^2 = R^2$

AdS/QCD correspondence – a program for implementation of such a duality for QCD following some recipies from the AdS/CFT correspondence



5D Anti-de Sitter space

$$\tau^2 + y_0^2 - y_1^2 - y_2^2 - y_3^2 - y_4^2 = R^2$$

$$u = \tau + y_4 \qquad v = \tau - y_4$$

$$uv + y_\mu^2 = R^2$$

$$ds^2 = du dv + dy_\mu dy^\mu$$

Exclude v and introduce

$$z = \frac{R^2}{u}, \qquad x_\mu = \frac{z}{R} y_\mu$$

$$\boxed{ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)}$$

invariant under dilatations

$$x_\mu \to \rho x_\mu, \quad z \to \rho z$$

4D Minkovski space at $z \to 0$

$$p_x = -i\partial_x = \frac{R}{z} p_y$$

<u>Physical meaning of *z*: Inverse energy scale</u>

holographic coordinate

[Witten; Gubser, Polyakov, Klebanov (1998)]

Essence of the holographic method

$$\langle e^{\int d^d x J(x)\mathcal{O}(x)}\rangle_{\mathrm{CFT}} = \int \mathcal{D}\phi \; e^{-S[\phi,\, g]}\Big|_{\phi(x,\partial AdS)=J(x)}$$

generating functional

action of dual gravitational theory
evaluated on classical solutions

AdS boundary

$$\Pi_n \equiv \langle\, \mathcal{O}_{I_1}(x_1)\ldots\mathcal{O}_{I_n}(x_n)\,\rangle = \frac{1}{\sqrt{g}}\frac{\delta}{\delta\phi^{I_1}(x_1)}\cdots\frac{1}{\sqrt{g}}\frac{\delta}{\delta\phi^{I_n}(x_n)} S[\phi\,,\,g]$$

The output of the holographic models: **Correlation functions**

Poles of the 2-point correlator → mass spectrum

Residues of the 2-point correlator → decay constants

Residues of the 3-point correlator → transition amplitudes

Alternative way for finding the mass spectrum is to solve e.o.m. $\phi(x_\mu, z) = e^{ixp}\phi(z)$

Bottom-up AdS/QCD models

Typical ansatz:

$$S = \int d^4x dz \sqrt{g}\, F(z) \mathcal{L} \qquad F(0) = 1$$

$$ds^2 = \frac{R^2}{z^2}(dx_\mu dx^\mu - dz^2), \qquad z > 0$$

- operators $\mathcal{O}(x)$ in $4D$ theory $\Leftrightarrow$ fields $\Phi(x,z)$ in $5D$ dual theory
- canonical dimension Δ of the p-form operator $\mathcal{O}(x)$ $\Leftrightarrow$ $5D$ mass of $\Phi(x,z)$:
  $m_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$,

  Example 1: vector mesons $\Leftrightarrow$ $\bar{q}\gamma^\mu t^a q$ with $p = 1$ and $\Delta = 3$ $\Leftrightarrow$ $A_\mu^a(x,z)$ with $m_5^2 R^2 = 0$

  Example 2: 0^{++} glueballs $\Leftrightarrow$ $G_{\mu\nu}G^{\mu\nu}$ with $p = 0$ and $\Delta = 4$ $\Leftrightarrow$ $\varphi(x,z)$ with $m_5^2 R^2 = 0$
- the desired matter content defines $\mathcal{L}_{matter}$ in $5D$

$\longrightarrow$ radial Regge trajectories $\Leftrightarrow$ finding normalizable solutions of $5D$ equations of motion with $q^2 = M^2(n)$, $n = 0, 1, 2, \ldots$, that match certain boundary conditions,

$$\Phi(x,z) = \sum_{n=0}^{\infty} \phi_n(z) \phi^{(n)}(x)$$

Hard wall model

(Erlich et al., PRL (2005); Da Rold and Pomarol, NPB (2005))

The AdS/CFT dictionary dictates: local symmetries in 5D → global symmetries in 4D

The chiral symmetry: $SU_L(2) \times SU_R(2)$

The typical model describing the chiral symmetry breaking and meson spectrum:

$$\boxed{S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}} \qquad 0 < z \leq z_m$$

$$D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu},\ A_{L,R} = A^a_{L,R}t^a,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

The pions are introduced via $X = X_0 \exp(i2\pi^a t^a)$ $\quad t^a = \sigma^a/2$

$$V = (A_L + A_R)/2 \qquad A = (A_L - A_R)/2 \qquad m_5^2 R^2 = (\Delta - J)(\Delta + J - 4)$$

At $z = z_m$ one imposes certain gauge invariant boundary conditions on the fields.

Soft wall model (Karch et al., PRD (2006))

$$\boxed{I = \int d^5x \sqrt{g}\, e^{-\Phi}\, \mathcal{L}} \qquad \Phi = \Phi(z)$$

Usually

$$\boxed{\Phi = az^2}$$

The IR boundary condition is that the action is finite at $z = \infty$

Holographic description of thermal and finite density effects

Basic ansatz

$$A_t = A_t(z),$$
$$A_i = 0 \qquad (i = 1, \cdots, 3, z),$$
$$ds^2 = \frac{R^2}{z^2}\left(f(z)dt^2 - d\vec{x}^2 - \frac{dz^2}{f(z)}\right)$$

- corresponds to $\bar{q}\gamma^0 q$

One uses the Reissner-Nordstrom AdS black hole solution

$$f(z) = 1 - (1 + Q^2)\left(\frac{z}{z_h}\right)^4 + Q^2\left(\frac{z}{z_h}\right)^6,$$
$$A_t(z) = \mu - \kappa \frac{Q}{z_h^3} z^2,$$

where $0 \leqslant Q \leqslant \sqrt{2}$ is the charge of the gauge field.

The hadron temperature is identified with the Hawking one: $\boxed{T_H = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z\to z_h} = \frac{1}{\pi z_h}\left(1 - \frac{Q^2}{2}\right)}$

The chemical potential is defined by the condition $A_t(z_h) = 0$ $\boxed{\mu = \kappa \frac{Q}{z_h}}$

Deconfinement temperature from the Hawking-Page phase transition

The main idea: Transition to deconfined phase is dual to formation of black hole in dual gravitational theory

Critical temperature in AdS/QCD

Consider evaluating $S_{gravity}$ on different AdS backgrounds:

(1) thermal AdS metric: $ds^2 = \frac{R^2}{z^2}\left(dt^2 - d\vec{x}^2 - dz^2\right)$, $t \in [0, \beta]$;

(2) metric of AdS with a black hole: $ds^2 = \frac{R^2}{z^2}\left(h(z)dt^2 - d\vec{x}^2 - \frac{dz^2}{h(z)}\right)$,

where $h(z) = 1 - (z/z_h)^4$ and R denotes the AdS radius.

Transition between two backgrounds
$\Longleftrightarrow$ (De)confinement transition

The Hawking temperature is related to the black hole horizon z_h via the relation $T_c = 1/(\pi z_h)$.
The free action densities V identified with the regularized gravitational action are:

$$V_{\text{Th}}(\epsilon) = \frac{4R^3}{k_g}\int_0^{\beta} dt \int_{\epsilon}^{z_{max}} dz f^2(z) z^{-5}, \quad V_{\text{BH}}(\epsilon) = \frac{4R^3}{k_g}\int_0^{\pi z_h} dt \int_{\epsilon}^{\min(z_{max}, z_h)} dz f^2(z) z^{-5}.$$

The two geometries are compared at a radius $z = \epsilon$ where the periodicity in the time direction is locally the same $\Rightarrow \beta = \pi z_h \sqrt{h(\epsilon)}$.
The order parameter of the phase transition is defined by ΔV:

$$\Delta V = \lim_{\epsilon \to 0}\left(V_{\text{BH}}(\epsilon) - V_{\text{Th}}(\epsilon)\right)$$

The Hawking-Page phase transition occurs at a point where $\Delta V = 0$.
However, $\Delta V = 0$ yields z_h as a function of model dependent parameters – z_{max} and/or those possibly introduced in $f(z)$. We must appeal to $\mathcal{L}_{matter}$ to give physical meaning to these parameters and to connect T_c to a particular type of a holographic model.

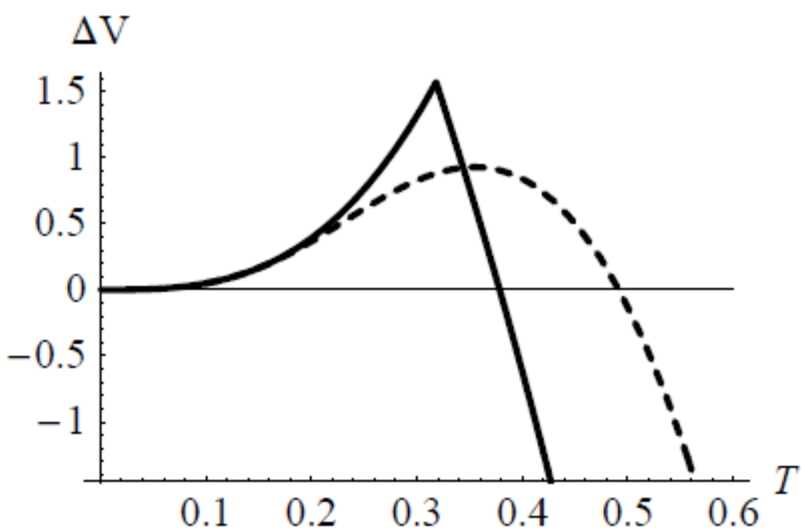


FIG. 1: The solid line is the free energy difference in the hard wall model, the dashed line the difference in the soft wall model.

HW: $T_c = \dfrac{2^{1/4}}{\pi z_0} \approx 0.157 m_\rho = 122\ \mathrm{MeV}$

SW: $T_c \approx 0.49\sqrt{a} \approx 0.246 m_\rho = 191\ \mathrm{MeV}$

Entropy density $\begin{cases} \mathcal{O}(1) & \text{- confined phase} \\ \mathcal{O}(N_c^2) & \text{-deconfined phase} \end{cases}$

Bottom-up AdS/QCD models

What matter content should we consider?

- vector case: reproducing the masses of non-strange vector mesons
  $J=1 \quad \mathcal{L}_V = -\frac{1}{4g_5^2} g^{MP} g^{NQ} \left(\partial_M A_N - \partial_N A_M\right) \left(\partial_P A_Q - \partial_Q A_P\right),$
- scalar case: achieving accordance with assumed glueball masses
  $J=0 \quad \mathcal{L}_{sc} = \frac{1}{2k_s} \left(g^{MN} \partial_M \varphi \partial_N \varphi - m_5^2 \varphi^2 \right),$

ρ meson
vs
0^{++} glueball

Which holographic model to choose?

- Hard Wall (HW) model [Erlich *et al.* (2005)]: z_{max} is finite and $f^2(z) = 1$

$$m_{HW}^{J=1} = M_{J=1}(0) = \frac{2.405}{z_{max}}, \quad m_{HW}^{J=0} = M_{J=0}(0) = \frac{3.832}{z_{max}}$$
$$T_{HW}^{J=1} = 0.1574 m_{HW}^{J=1}, \quad T_{HW}^{J=0} = 0.0988 m_{HW}^{J=0}.$$

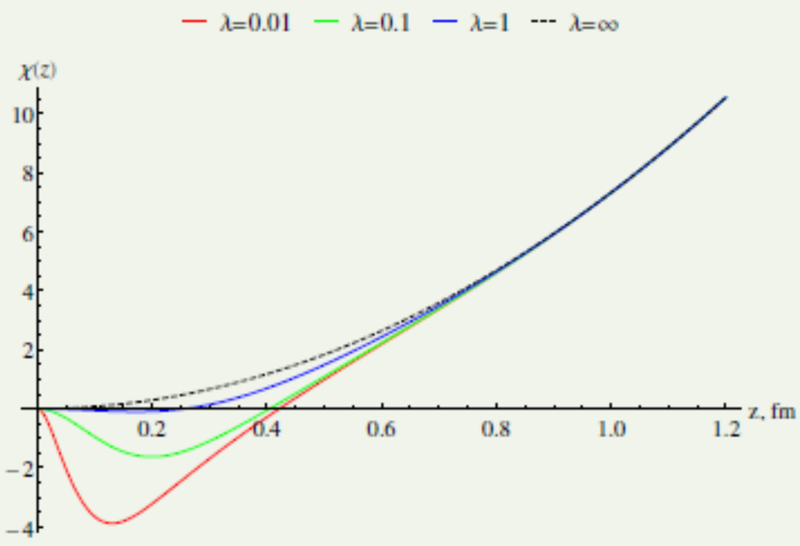
- Soft Wall (SW) model [Karch *et al.* (2006)]: $z_{max} = \infty$ and $f^2(z) = e^{-\kappa^2 z^2}$

$$M_{J=1}^2(n) = 4\kappa^2 (n+1), \quad M_{J=0}^2(n) = 4\kappa^2 (n+2),$$
$$T_{SW}^{\text{any } J} \simeq 0.4917 \cdot \kappa.$$

- Generalized Soft Wall (GSW) model [Afonin (2013)]:
  $z_{max} = \infty$ and $f^2(z) = e^{-\kappa^2 z^2} U^2(b, J-1; \kappa^2 z^2)$, where U is the Tricomi hypergeometric function introducing a free intercept parameter b in the spectrum while keeping SW asymptotes in UV and IR, $b=0$ reduces GSW to SW

$$M_{J=1}^2(n) = 4\kappa^2 (n+1+b), \quad M_{J=0}^2(n) = 4\kappa^2 (n+2+b)$$
$$T_{GSW}^{J=1}/\kappa \simeq 0.670 \cdot b + 0.496, \quad T_{GSW}^{J=0}/\kappa \simeq 0.123 \cdot b + 0.314.$$

Isospectral models



An idea from SUSY quantum mechanics $\Rightarrow$ a family of strictly isospectral potentials associated with a given potential for Schrödinger-type equations:

$$-\psi_n''(z) + \widehat{\mathcal{V}}_\lambda(z)\psi_n(z) = M^2(n)\psi_n(z)$$

The family members are distinguished through a parameter λ, with $\lambda = \infty$ corresponding to the original potential.

Usage: generating a family of the dilaton profiles realted to the original SW $\chi(z) = \kappa^2 z^2$, thus constructing new models while keeping the spectrum fixed [Vega and Cabrera (2016)].

$$\hat{\mathcal{V}}_J(z) = \mathcal{V}_J(z) - 2\frac{d^2}{dz^2}\ln[I_J(z) + \lambda].$$

$$I_J(z) \equiv \int_0^z \psi_0^2(z')dz'$$

How does this affect the critical temperature?

Our analysis: Isospectrality, in general, does not entail isothermality!

TABLE II. The isospectral parameter λ affecting the deconfinement temperature estimations from vector meson fits in (G)SW models.

	T_{SW} (MeV)		T_{GSW} (MeV)
	Universal trajectory	Lightest ρ meson	ρ: $n = 1, 2, 3, 4, 5$ [3]
λ	$m^2 = 4 \cdot (534\ \mathrm{MeV})^2 \cdot (n+1)$	$m^2 = 4 \cdot (388\ \mathrm{MeV})^2 \cdot (n+1)$	$m^2 = 4 \cdot (446\ \mathrm{MeV})^2 \cdot (n+1.7)$
100	261.1 ± 1.2	189.66 ± 0.06	422.5 ± 34.2
20	256.1 ± 1.1	185.97 ± 0.06	419.0 ± 34.5
1	194.9 ± 0.9	141.54 ± 0.05	339.2 ± 39.2
0.1	159.1 ± 0.7	115.53 ± 0.04	189.2 ± 11.4
0.01	154.3 ± 0.7	112.03 ± 0.04	172.4 ± 8.0

varies significantly!

The gravitational part of holographic models is supposed to come from pure gluodynamics. This suggests that the scalar glueball (as the lightest one) should be more relevant for estimations of deconfinement temperature.

Lattice and experimental data

(I) the deconfinement temperature T_c

- → lattice with physical quarks [Borsanyi *et al.* (2010)]: 150 – 170 MeV;
- → lattice with non-dynamical quarks and $N_c \to \infty$ [Lucini *et al.* (2012)]: ~ 250 MeV;
- → lattice for $SU(3)$ theory [Boyd *et al.* (1996); Iwasaki *et al.* (1997)]: 260 – 270 MeV;
- → experimental results favour the range of 150 – 160 MeV.

(II) 0^{++} glueball masses

- → quenched lattice: 1.5 or 1.7 GeV;
- → unquenched lattice: 1.8 GeV;
- → lattice approximation for $N_c \to \infty$;
- → radial excitations from lattice - not more than 2 states usually;
- → state among $f_0(1370)$, $f_0(1500)$, $f_0(1710)$.

(III) identification of ρ radial excitations; the idea of the universal slope for the light mesons

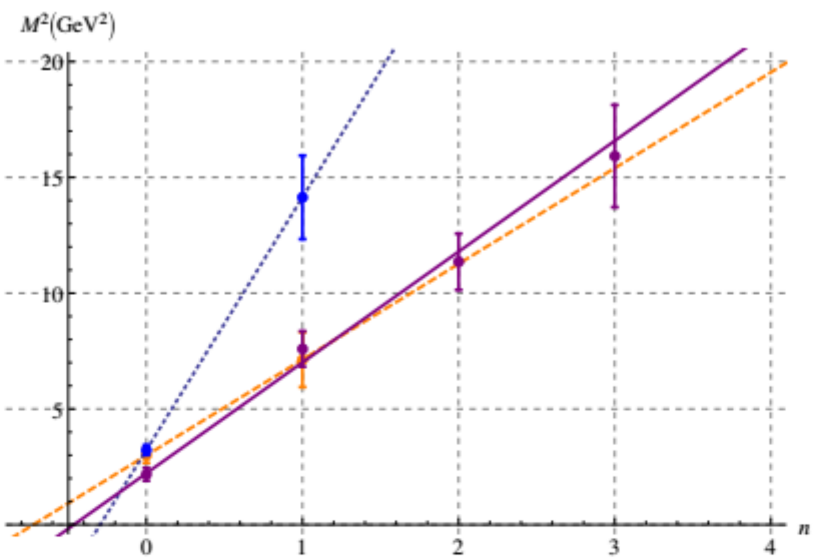


FIG. 2. The radial Regge trajectories of glueball states: dashed is Morningstar and Peardon [35], solid is Meyer [36], and dotted is unquenched [38].

Fit	m_{gl} (MeV)	T_{HW} (MeV)	T_{SW} (MeV)		
			$\lambda = \infty$	$\lambda = 1$	$\lambda = 0.1$
M & P [31]	1730(100)	171(10)	301(17)	253(15)	173(10)
Meyer [32]	1475(75)	146(7)	256(13)	215(11)	147(8)
Chen *et al.* [33]	1710(95)	169(9)	297(17)	250(14)	171(10)
Large N_c [35]	1455(70)	144(7)	253(12)	212(10)	145(7)
Unquenched [34]	1795(60)	177(6)	312(10)	262(9)	179(6)
$f_0(1500)$ meson [38]	1464(47)	145(5)	255(8)	214(7)	146(5)
	1519(41)	150(4)	264(7)	222(6)	152(4)
$f_0(1710)$ meson [40]	1674(14)	165(1)	291(2)	244(2)	167(1)

TABLE III. One glueball state predictions in HW and SW models.

strongly varies again

Fit	$\sqrt{\sigma}$ or r_0^{-1} (MeV)	$m^2 = 4\kappa^2(n+2+b)$		T_{GSW} (MeV)		
		κ (MeV)	b	$\lambda = \infty$	$\lambda = 1$	$\lambda = 0.1$
M & P [31]	410	1017(151)	−1.28(0.23)	153.6(39.2)	151.4(36.1)	149.9(34.1)
Meyer [32]	440	1094(49)	−1.54(0.07)	132.5(9.7)	132.3(9.5)	132.1(9.4)
Unquenched [34]	420	1652(138)	−1.71(0.05)	174.6(16.4)	174.6(16.4)	174.5(16.4)
Large N_c [35]	440	1120(88)	−1.58(0.08)	131.3(13.5)	131.1(13.4)	131.0(13.3)
Large N_c [32]	440	735(121)	−1.00(0.35)	142.8(55.3)	134.8(43.7)	129.7(36.6)

TABLE IV. Predictions of T_c from different fits for glueball towers in GSW model with isospectrality.

Almost no variation!

CONCLUSION

We have demonstrated that, although for the holographic estimations of deconfinement temperature there exists a huge number of possibilities, some sensible theoretical and phenomenological restrictions on holographic models lead to sensible and rather stable predictions for the range of temperatures in the deconfinement crossover region at small baryon densities.

Our analysis may be viewed from the opposite side: We scrutinized how the requirement to reproduce this range efficiently restricts the possible bottom-up holographic models and may have a serious predictive power for the hadron spectroscopy.

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