Mesonic bound states in a nearly-conformal gauge theory

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Overview

Outline for Section 1

Introduction

Dyson-Schwinger and Bethe-Salpeter equations : powerful tools for QCD. (DSEs and BSEs)

We can also use them to study BSM theories.

Which theory? Technicolor (TC).

Outline

Outline for Section 2

Motivation for TC

[Lecture Notes by Axel Maas, University of Graz, summer semester 18]

The SM has a **hierarchy problem** linked to the SM Higgs.

Possible solution to the problem :

If the Higgs is **not** the SM one but is composite, with constituents without a hierarchy problem :

problem solved !

Need strong interactions to create a composite Higgs.

$\rightarrow$ Upscaled-QCD suggested for technicolor.

Principle of Simple TC

We keep the structure and particles of the SM, except for its Higgs.

We add an upscaled QCD with :

- a more general gauge group G (SU(N_T) or Sp(2n)),
- N_f additional fermions : the techniquarks,
- Some additional gauge bosons : the technigluons, (N_T^2 - 1) of them in the case of a SU(N_T) gauge group.

The total gauge group is :

$$G \times SU(3) \times SU(2) \times U(1).$$

Principle of Simple TC

A few properties of Simple TC

- The techniquarks have a chiral symmetry.
- The TC Higgs condensate generates EW symmetry breaking. Success.
- $\Lambda_{\mathrm{TC}} \sim 1\ \mathrm{TeV}$ (upscaled QCD).
- Possible to give mass to the W and Z (spontaneous breaking of chiral symmetry). Success.

Principle of Simple TC

Problems in Simple TC

So there are many successes in simple TC.

But there are also some problems with this theory.

In particular, we notice some when we investigate its possible realizations.

(Susskind-Weinberg TC, Farhi-Susskind TC)

ex: no SM fermion masses generation. Problem !

$\rightarrow$ Extended Technicolor (ETC) proposed.

Walking TC

Motivation for walking TC

In QCD

α running : goes from strong to weak very quickly and all non-trivial dynamics happen in a narrow window around Λ_{QCD}.

In the EW sector

Dynamics spread out over a large energy range.
(masses of SM fermions and EW symmetry breaking)

Technicolor

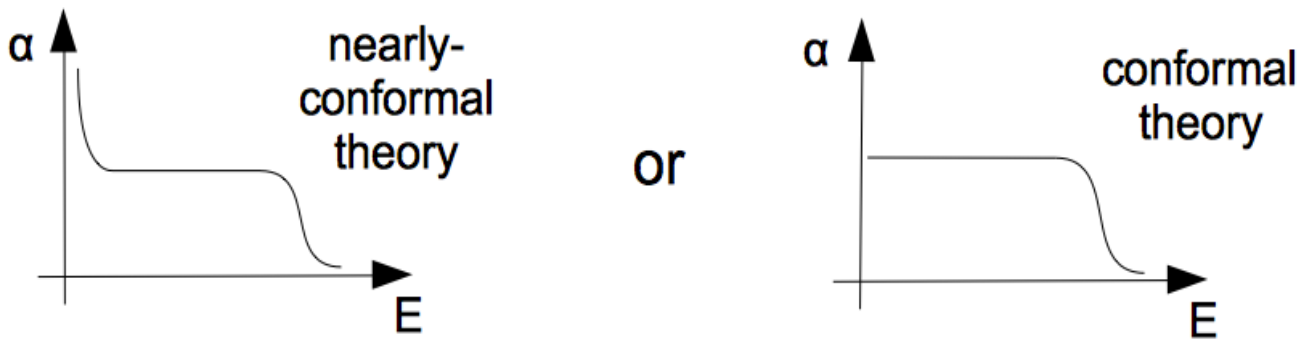
Has strong dynamics but should recreate elementary Higgs physics.
$\rightarrow$ Possible if the coupling has a **walking** behavior instead of a running one.

Moreover, a walking coupling suppresses the FCNC and lepton number violation (arise in ETC). [D.D. Dietrich, F. Sannino, 2006]
$\rightarrow$ Other advantage of walking TC

Walking TC

Realization of a walking TC

How to make a walking coupling happen?



It is the $\beta(g)$ function that describes $g(\mu)$:

$$\beta(g) = \mu \ \frac{\partial g}{\partial \mu}$$

A coupling flat at g_0 entails $\beta(g_0) \simeq 0$.

Walking TC

Fixed points and the conformal window

Definition : A fixed point of $\beta(g)$ is a g_0 such that $\beta(g_0) = 0$.

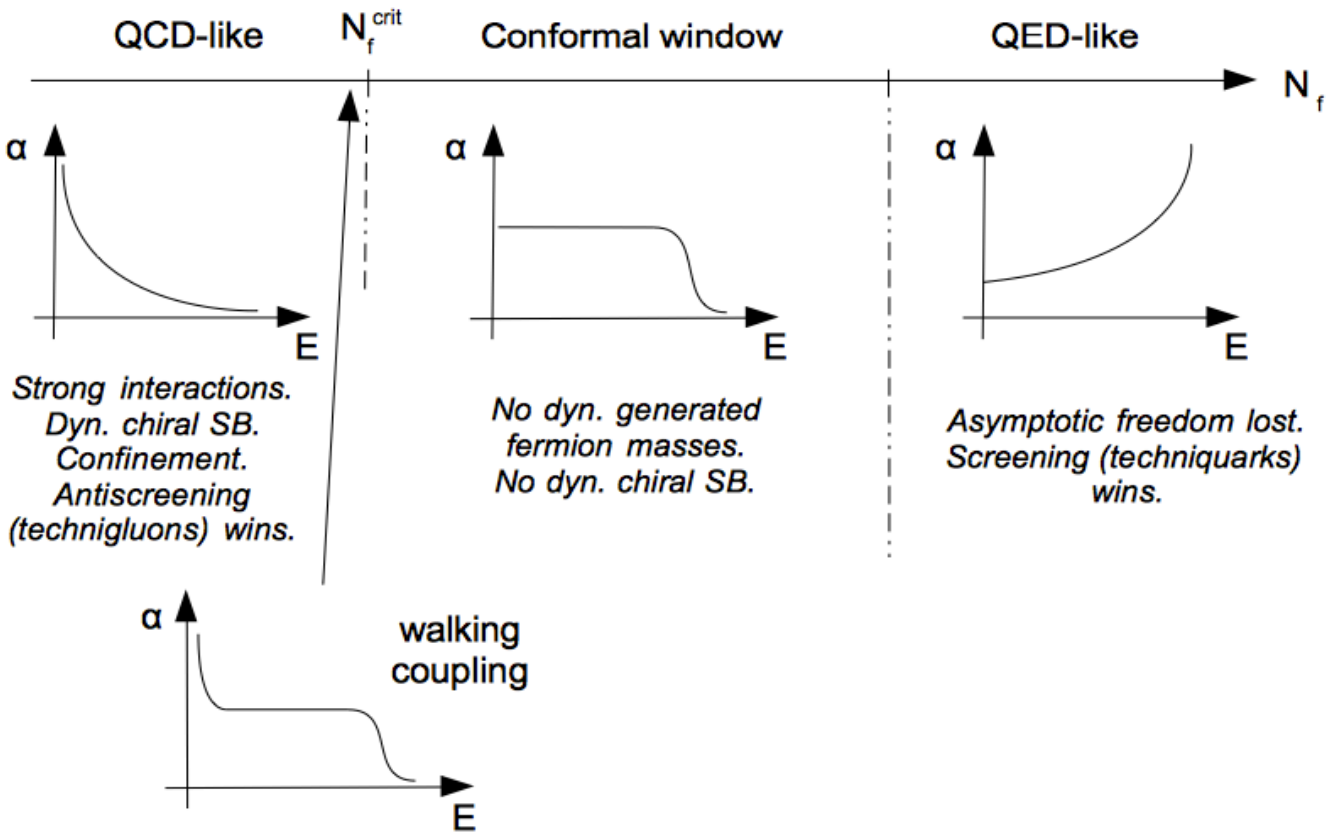
If we have an IR fixed point, $g(\not{\mu})$.

$\rightarrow$ no dynamically generated masses for the techniquarks and no dynamical chiral symmetry breaking.

Definition : The **conformal window** is the range of N_f for which we have an IR fixed point in $\beta(g)$ <u>and</u> asymptotic freedom.

Walking TC

We know : for $m_f = 0$,



For a TC theory : both nearly-conformal and conformal theories work.

Phase transition $\rightarrow$ $\mathrm{N}_f^{\mathrm{crit}} = ?$

Goals working with a TC theory

- Study the conformal window and nearly-conformal theories (ie. study theories close-by to the phase transition)

  How do the Greens functions behave ?

- Study the phase transition : value for $\mathrm{N}_f^{\mathrm{crit}} = ?$

- Build a theory where the scalar has $\sim 1/10^{th}$ of the mass of the closest vector.

Goals working with a TC theory

- Build a theory where the scalar has $\sim 1/10^{th}$ of the mass of the closest vector.

  $\longrightarrow$ Why ? To match with observations.

  $\longrightarrow$ How ? By changing :

  - The gauge group : SU(N_T), Sp (2n)
  - N_f (enough to be tuned to a conformal or nearly-conformal theory $\rightarrow$ liberty for N_f)
  - The representation of the fermions

Outline for Section 3

Introduction

DSEs and BSEs :

- non-perturbative tools
- large energy range studied (IR $\rightarrow$ UV)
- robust
- give results sometimes too demanding for lattice simulations, especially for a walking theory (multiple scales)

<u>But</u> their drawbacks are :

- they are an infinite set of coupled equations $\rightarrow$ truncations will be necessary.
- comparison with lattice results is necessary to verify the suitability of the truncations.

The quarks, gluons and ghosts in the following sections refer to the fermions and gauge bosons of this added gauge sector. ("shorthand name")

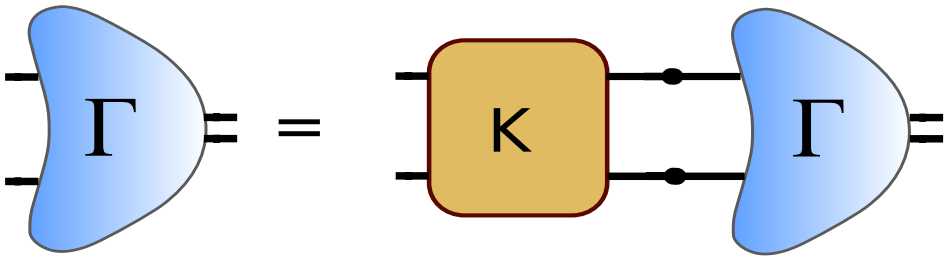
The Dyson-Schwinger equation (DSE)

Definition: a Dyson-Schwinger equation (DSE) is an equation for a dressed n-point function.

$\oplus$ handy method : when we change G, the changes in the DSEs are factors (Casimirs) and sometimes the diagrams.

The Bethe-Salpeter equation (BSE)

Definition: The BSE is a bound state equation for mesons. It gives the mass and the amplitude of the meson. It is the same equation for all meson particles.



$$\Gamma_{ij} = \int \frac{d^4q}{(2\pi)^4} K_{ij,kl}(p,q,P)\{S(q_+)\Gamma(q,P)S(-q_-)\}_{kl},$$

where Γ is the bound state amplitude,
$K_{ij,kl}$ is the kernel of the BSE.

$\oplus$ handy method : when we change the gauge group, the changes in the BSE is a factor (Casimir) and sometimes the diagrams.

The Bethe-Salpeter equation (BSE)

The kernel K of the BSE represents all possible interactions between the meson constituents.

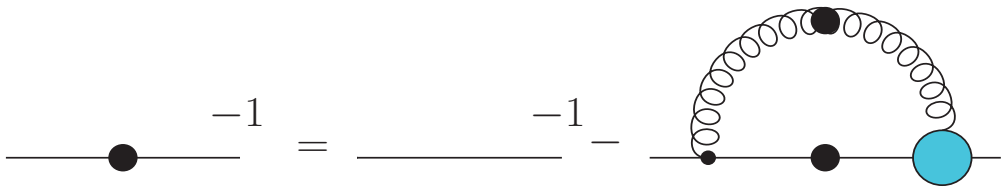
$\longrightarrow$ Necessary to truncate it to solve the BSE.

Next, the BSE can be solved as an eigenvalue equation, and the mass M and amplitude of the meson are obtained.

A coupled system

The quark propagator DSE

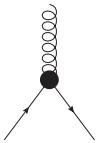
The quark propagator DSE :



Necessary to solve it explicitly, because :

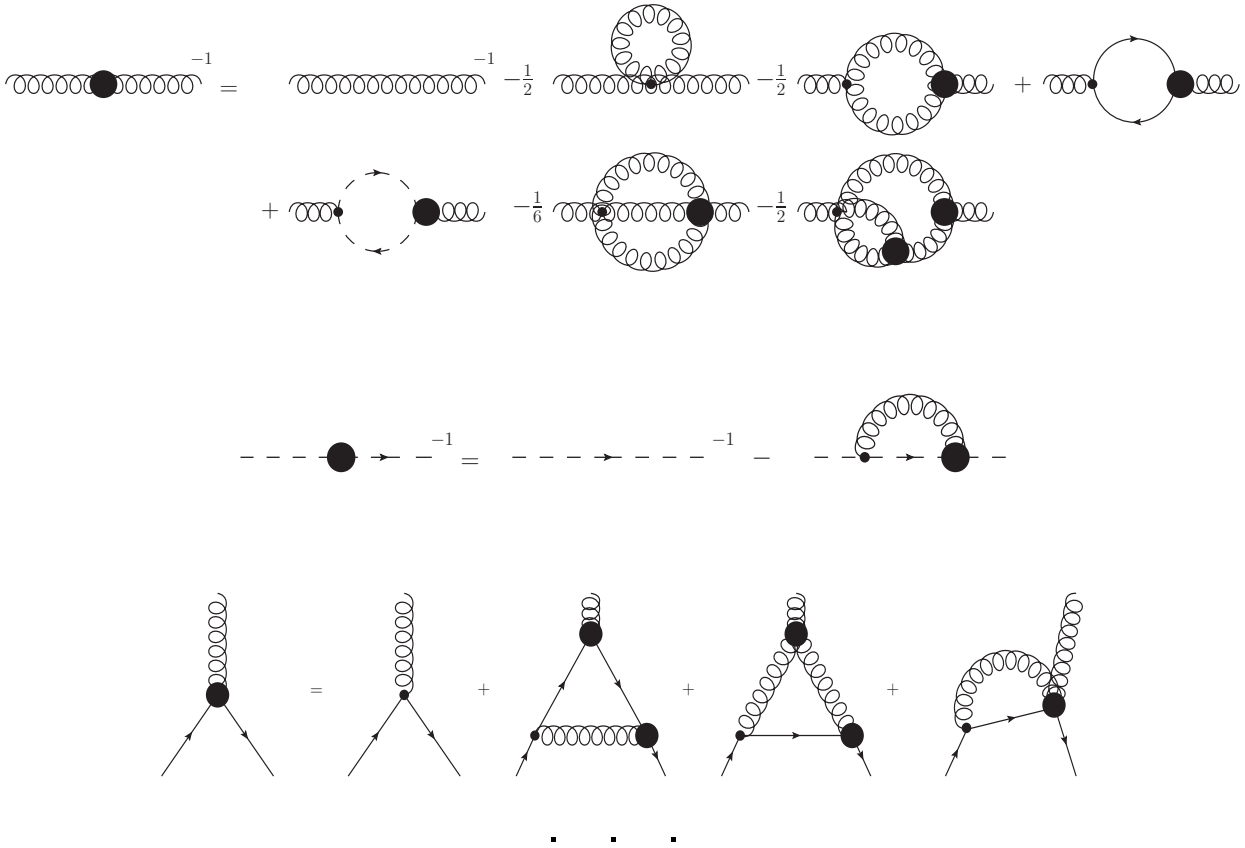
- the quarks are the main elements of our phase diagram,
- the quark propagator appears explicitly in the BSE,
- solving it gives us $M(p^2)$ and $\langle\bar{\psi}\psi\rangle$ (dynamical quark mass and chiral condensate) : order parameters of the phase transition.

  $\longrightarrow$ they give us $\mathrm{N}_f^{\mathrm{crit}}$! Good.

<u>But</u> to solve it, we need  and  ...

A coupled system

[M.Q. Huber, 2018] The system of DSEs : (internal propagators dressed)



$\longrightarrow$ Infinite coupled system.

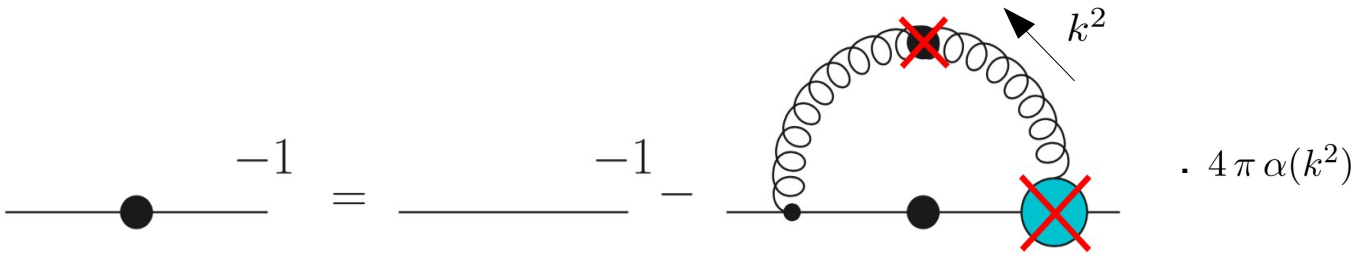
$\longrightarrow$ Need for a truncation of the DSE system.

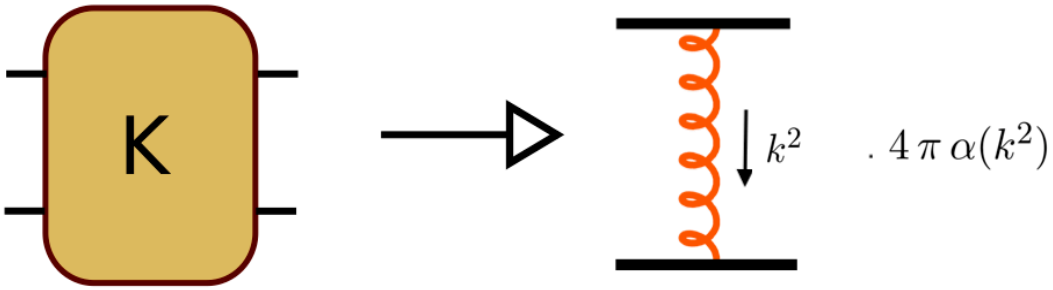
Rainbow-ladder truncation (RL)

The Rainbow-ladder truncation (RL) is the most basic truncation for the DSE system and the BSE kernel that respects the axWTI.

($\rightarrow$ resulting truncated system without explicit chiral symmetry breaking)

It acts on the DSE and the kernel of the BSE :





Rainbow-ladder truncation (RL)

Drawbacks of RL :

- very successful in hadronic studies (QCD) ... <u>but</u> its parameters were tuned for that !
  How much physics is in RL really?
- not possible to separate the influence of the different physical processes to RL.
- not possible to determine the influence of each Green's function.
- not possible to adjust RL when we change N_f or the gauge group, from QCD.

$\longrightarrow$ RL is not the optimal truncation for our study.

Outline for Section 4

Goal of the paper

Current state of the topic: [M. Hopfer, C.S. Fischer and R. Alkofer, 2014].

Calculation of

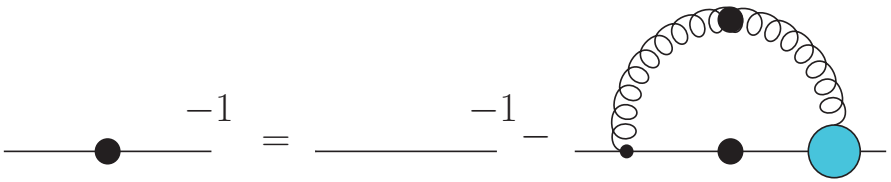
- N_f^{crit}
- the running coupling
- the propagators

for a QCD-like gauge theory (N_T=3), with N_f variable.

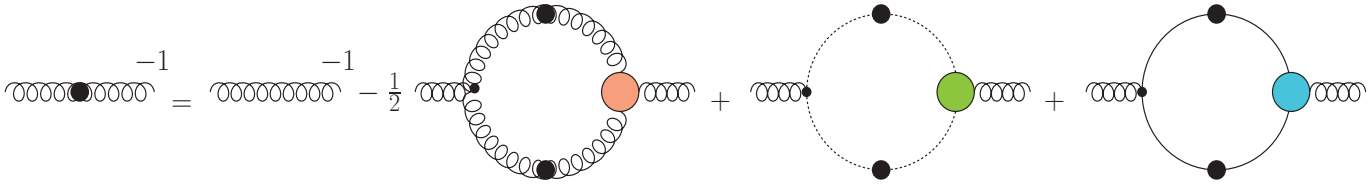
The system

The quark, gluon and ghost DSEs are solved simultaneously (no ansatz for  because of increased effects of the quark loop on its DSE when N_f increases).

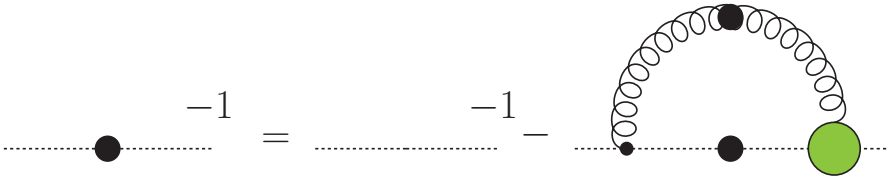
quark DSE :



gluon DSE :

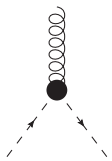
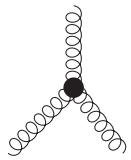
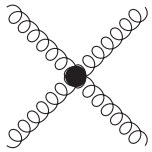
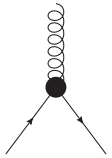


ghost DSE :



The truncation scheme

The approximations in the system :

-  : bare vertex approximation
-  : ansatz
-  : neglected in the system (already in the above DSEs)
-  : ansatz

The phase transition

Once the coupled system of DSEs is solved (with truncations) ...

... we can see the value of N_f^{crit}, and study the **different phases** obtained when N_f varies in this study ..

The phase transition

Results [M. Hopfer, C.S. Fischer and R. Alkofer, 2014]

They found : $N_f^{\text{crit}} \simeq 4.5$. They plotted : for $N_f = 0, 4, (4.4), 5$

$\alpha(p^2)$: running coupling ; $G(p^2)$: ghost dressing function

$\frac{Z(p^2)}{p^2}$: gluon propagator ; $A^{-1}(p^2)$: inverse vector self-energy

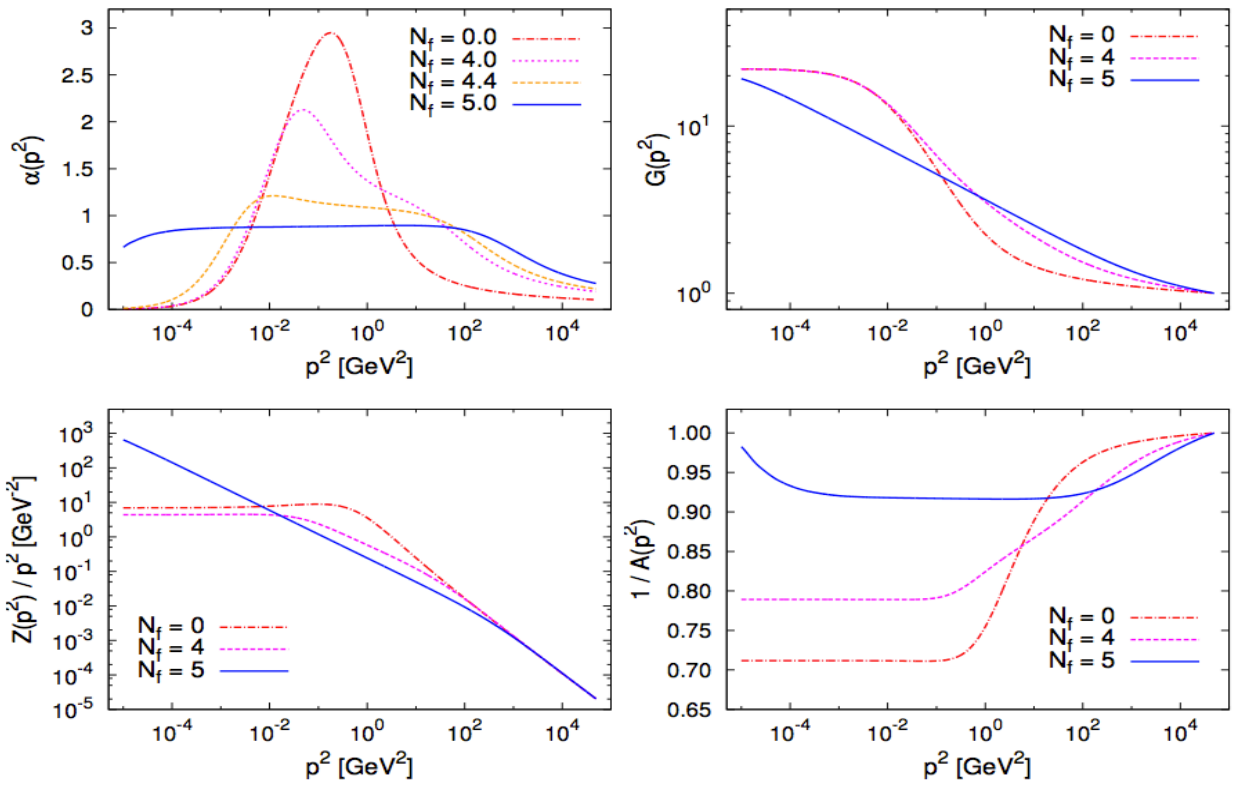


Figure: $\alpha(p^2)$, $G(p^2)$, $\frac{Z(p^2)}{p^2}$ and $A^{-1}(p^2)$ (with $\mu^2 = 5.10^4$ GeV^2).

The phase transition

Results [M. Hopfer, C.S. Fischer and R. Alkofer, 2014]

- Main result : The behaviors of all the functions change drastically when entering the conformal window

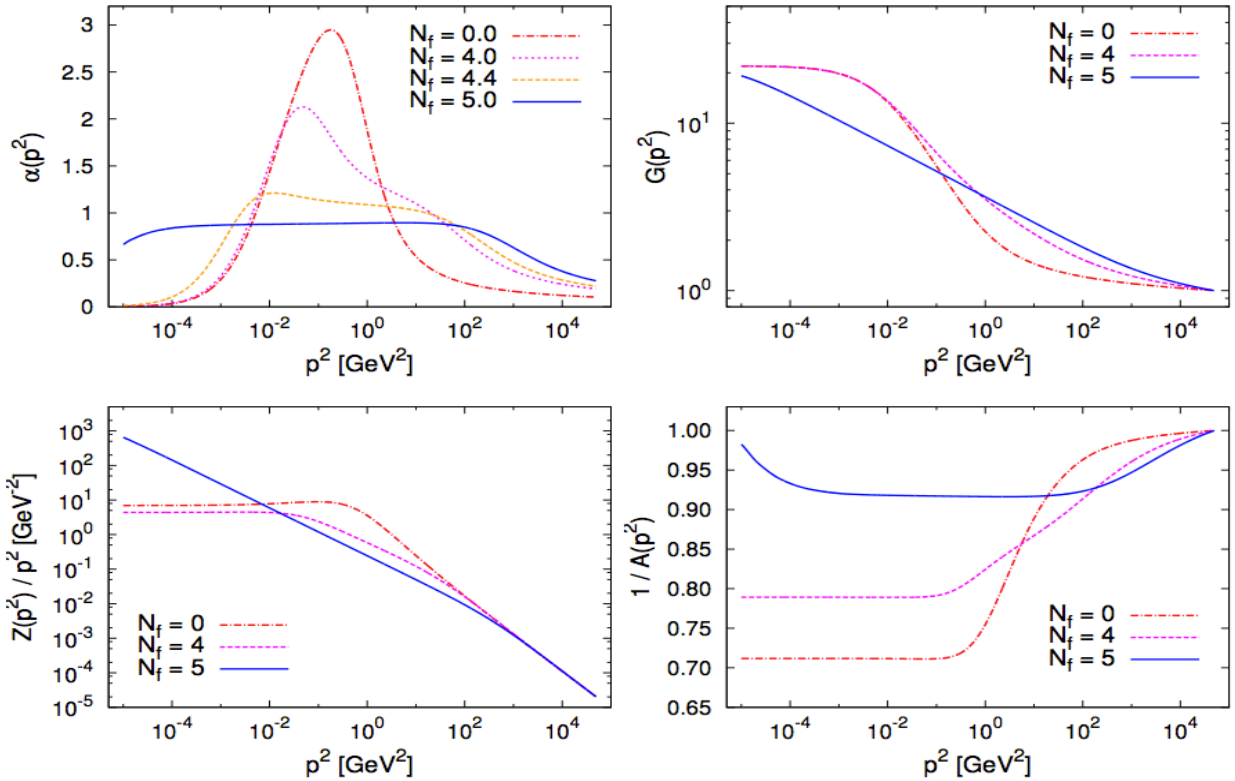


Figure: $\alpha(p^2)$, $G(p^2)$, $\frac{Z(p^2)}{p^2}$ and $A^{-1}(p^2)$ (with $\mu^2 = 5.10^4$ GeV^2).

Conclusion of this study

Successes: Evidence that the results are correct qualitatively:

- Plateau observed for α in the conformal window.
- When changing models for the GF's, the qualitative results are maintained.

Elements to work on:

Value of N_f^{crit} obtained in this paper : $N_f^{\mathrm{crit}} \simeq 4.5$.

$\rightarrow$ Way smaller than its value from other studies.

$\rightarrow$ Reason: the truncations used are too simple to give good quantitative results.

(big influence expected from the overly simple ansatz for the q-gl vertex...)

Our plan: We want first to reproduce these results, then we want to do better quantitatively.

M. Adrien, R. Alkofer, H. Sanchis-Alepuz, M. Vujinovic, F. Zierler, in preparation.

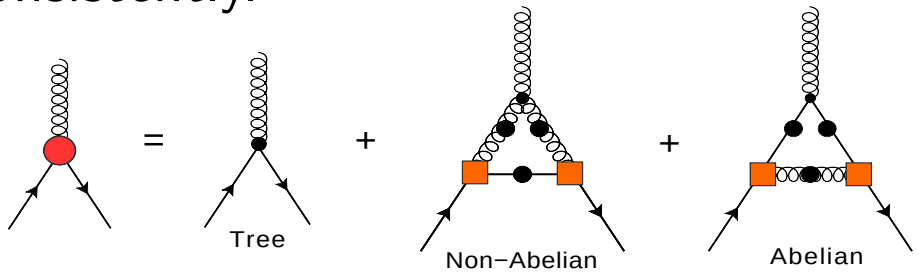
Outline for Section 5

Two ways to treat a Green's function

There are two possible types of handling for an unknown Green's function :

- Using an ansatz (or model) :
  - function of the momenta,
  - basically independant of N_f, of the gauge group and of the other Green's functions behaviors.
- Solve its DSE explicitly, usually as we solve the quark propagator DSE (and possibly others), at least semi-self-consistently.

e.g.



[M. Vujinovic and R. Alkofer, 2018]

Treating the Green's functions

However, it is very difficult numerically to solve explicitly many DSEs at the same time, even semi-self consistently.

Solution: We will proceed by steps :

- First, use ansaetze for the main Greens functions, to reproduce the results of [M. Hopfer, C.S. Fischer and R. Alkofer, 2014].
- Then, **progressively** include the explicit solving (at least semi-self consistent) of more and more DSEs.

Outline for Section 6

Conclusion of the presentation

We have

- introduced the theory,
- introduced the tools,
- seen what has been done on the topic already.

Our goals are to

- improve the results quantitatively ; $N_f^{\mathrm{crit}} = ?$
- Study conformal and nearly-conformal theories : Greens functions behaviors
- Build a theory with scalar much lighter than closest vector (factor $\sim 1/10$)

Thank you for your attention!

Back-Up slides

Extended TC

- Extended Technicolor is the **most successful proposal** to solve some of the problems of simple TC while keeping its successes.

- One reason why simple TC theories are not enough :

  no SM fermion masses generation. Problem !

- To generate the SM fermion masses :

  the TC condensate has to be coupled differently to all the different SM fermions.

Extended TC

SM fermion masses generation is achieved by adding to the simple TC general theory :

- an extended TC gauge group
- with massive gauge bosons
- with masses $\sim \Lambda_{\mathrm{ETC}} > \Lambda_{\mathrm{TC}}$
- coupled to the SM **and** the TC fermions.

It is the breaking of the ETC gauge group which provides a mechanism for the **generation** of the different fermion masses.

Extended TC

is not without problems..

There remain problems in extended TC :

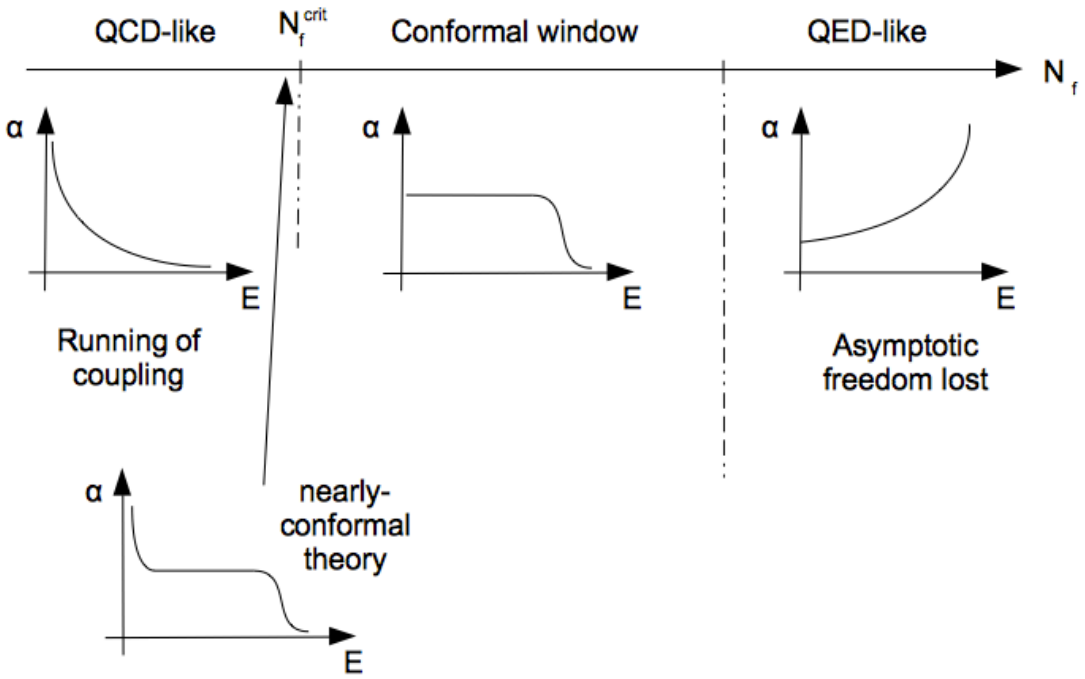
- how to generate the different $m_{\text{SM fermions}}$ without introducing new parameters or fields
- which value should we choose for Λ_{ETC} to recreate all the correct $m_{\text{SM fermions}}$
- Extended technicolor generates Flavor Changing Neutral Currents (FCNC), whose size depends on Λ_{ETC} as well.
  $\rightarrow$ further bound on this constant

However, extended TC does seem a **promising** theory for BSM.

Walking TC

Diagram when $m_f \neq 0$

For $m_f = 0$:



For $m_f \neq 0$:

Similar pattern of theories when N_f varies

<u>but</u> no conformal theories

and walking is only possible at energies $E \gg m_f$.

Walking TC

Superficial argumentation

Argumentation with the β-function and the running coupling however superficial :

- The $\beta(g)$ function is renormalization scheme dependant.
  $\rightarrow$ We may be investigating an artificial IR fixed point.
- Ambiguity in the definition of α.

$\rightarrow$ More subtle than it seems to show that a theory is walking or conformal.

<u>But</u> we do think to have captured the gist of the idea, even with imperfect arguments.

Goals working with a TC theory

To create the gap in the spectrum, we can change:

- The representation of the fermions:

  When some fermions are placed in higher-dimensional representations, they **screene more**. [D.D. Dietrich and F. Sannino, 2006]

$\longrightarrow$ N_f^{crit} decreases (fewer fermions necessary to create enough screening ($><$ antiscreening technigluons) to not have chiral symmetry breaking).

$\longrightarrow$ Change in the phase diagram.

$\longrightarrow$ Might imply a change in the spectrum...

Goals working with a TC theory

To create the gap in the spectrum, we can change:

- The gauge group :

  In particular, we plan to investigate some simplectic groups $Sp\,(2n)$:

Why ?

They are pseudo-real $\Rightarrow$ they show a different kind of behavior than unitary groups $SU(N_{\mathrm{T}})$.

$\longrightarrow$ Worth investigating the effect on the spectrum...

Article studied: The phase transition

Order parameters

Which quantities are suitable to study the phase transition ?

(order parameters)

From the (solved) quark propagator, we deduce :

- $M(p^2 \to 0)$: quark mass function at vanishing p
- $\langle \bar{\psi}\psi \rangle$: chiral condensate
  obtained by integration of the quark propagator
  in the chiral limit.

Article studied: The phase transition

Results

Left: $M(p^2 \to 0)$ and $\langle \bar{\psi}\psi \rangle$ for different N_f $(m_f = 0)$.

Phase transition observed for $N_f^{\mathrm{crit}} \simeq 4.5$.

This value seems unnaturally small (compared with other results).

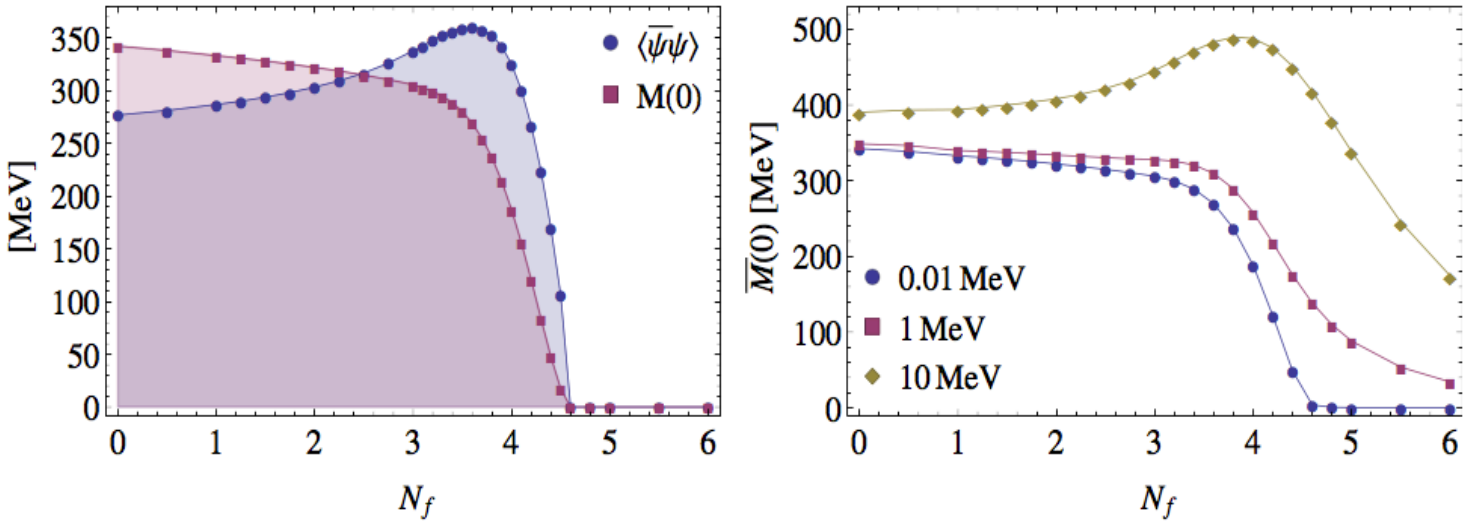


Figure: Left: $M(p^2 \to 0)$ and $\langle \bar{\psi}\psi \rangle$ for different N_f and for $m_f = 0$. Right: $\bar{M}(p^2 \to 0) = M(p^2 \to 0) - m_f@2\mathrm{GeV}$ for different N_f and m_f.

Article studied: The phase transition

Results

Right: Impact of finite bare fermion masses :

$$\bar{M}(p^2 \to 0) = M(p^2 \to 0) - m_f@2\mathrm{GeV} \ , \text{ for different } \mathrm{N}_f.$$

$\to$ No more phase transition but a crossover.

Value for $\mathrm{N}_f^{\mathrm{crit}}$ unchanged.

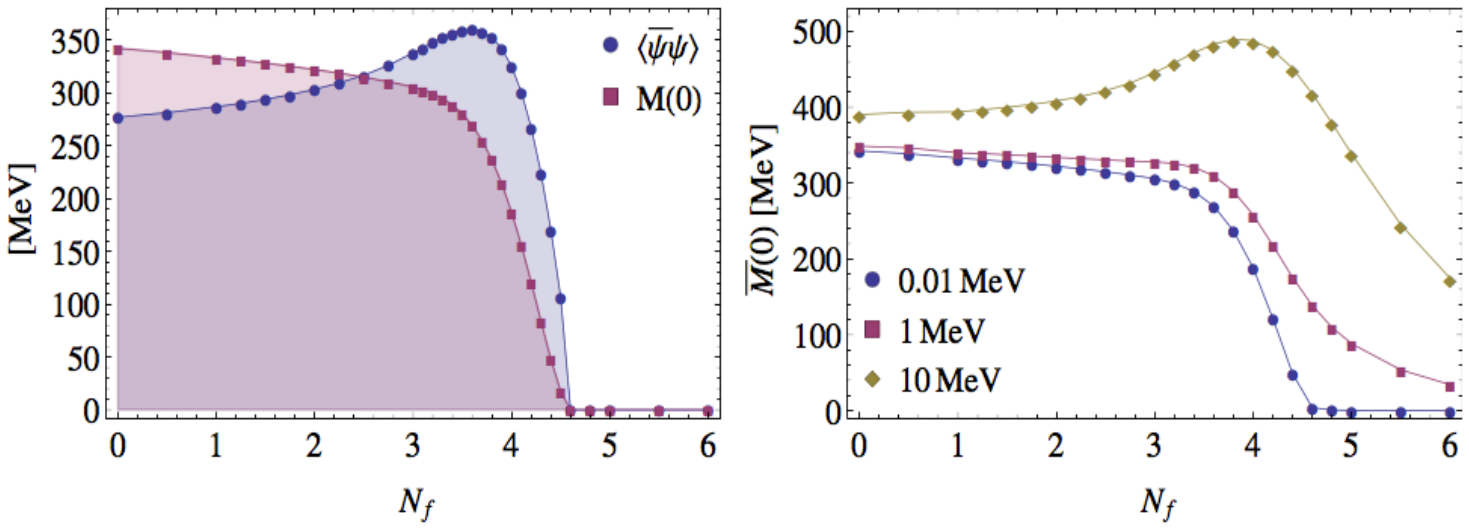


Figure: Left: $M(p^2 \to 0)$ and $\langle\bar{\psi}\psi\rangle$ for different N_f and for $m_f = 0$. Right: $\bar{M}(p^2 \to 0) = M(p^2 \to 0) - m_f@2\mathrm{GeV}$ for different N_f and m_f.

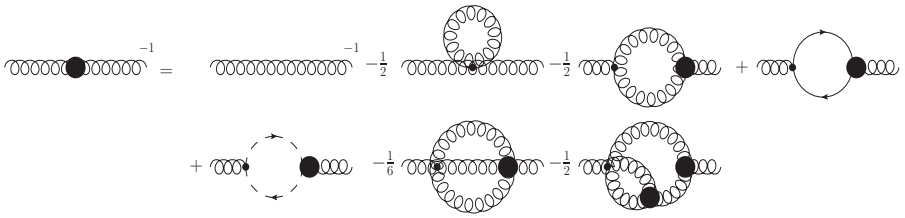
The quark propagator ; the gluon and ghost propagators

Our plan

-  : its DSE is solved explicitly.
-  and  :

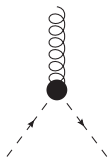
  Need to solve the gluon DSE explicitly in our study <u>because</u>, when N_f increases, the effects of the quark loop on the DSE increase. [M. Hopfer, C.S. Fischer and R. Alkofer, 2014]

  $\longrightarrow$ A simple ansatz basically independant of N_f does not convey that. Therefore, explicit resolution of the gluon DSE needed in our study.



The ghost-gluon vertex

Our plan

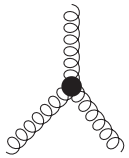
-  : in general, tree-level value used.

(in studies of SU(2) and SU(3) with N_f changing)

<u>Plan</u> : check if that is a valid approximation when N_f and the gauge group change (SU(N_T) : first gauge groups considered).

The three-gluon vertex

Our plan

-  : often in studies, an ansatz is used.

  Different possibilities can be found in [M. Hopfer, C.S. Fischer and R. Alkofer, 2014].

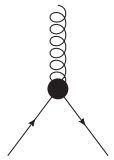
  All gave similar results.

<u>Plan</u> :

- First, use an ansatz and redo the calculation of [M. Hopfer, C.S. Fischer and R. Alkofer, 2014].
- Then, try to solve its DSE explicitly (at leat semi-self-consistently), as part of the system of the DSEs we decide to solve.

The quark-gluon vertex

Our plan

-  :

  - Using its symmetries, we find 12 tensors in a covariant gauge.

    We choose the Landau gauge because only 8 remain.

    (because 4 are such that $D_{\mu\nu}\,\Gamma_i^{\mu} = 0$)

    Other advantage of the Landau gauge : we know better in it other correlation functions.

  - RL truncation : just keeps 1 of the 8 tensors (the tree-level one) , times a function of the momenta.

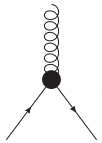
The quark-gluon vertex

Our plan

- What we plan to do with it :
  - First use an ansatz (redo calculation from [M. Hopfer, C.S. Fischer and R. Alkofer, 2014]).
  - This ansatz gives good qualitative results but not good quantitative ones.

    For that, we need to solve its DSE explicitly too, at least semi-self-consistently, and include a more complete tensor structure..

Partial reason why not good quantitative results with an ansatz :

Half of the  -tensors have an even number of γ's.

$\longrightarrow$ Not chiral-symmetry invariants.

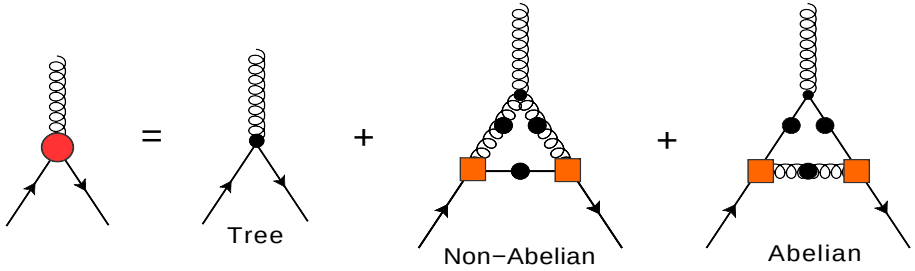
$\longrightarrow$ In the conformal window their T_i's drop to 0.

$\longrightarrow$ This is not accounted for in general by the ansatz.

The quark-gluon vertex

Our plan

- Example for semi-self consistent resolution of its DSE : [M. Vujinovic and R. Alkofer, 2018]



$\longrightarrow$ Better than an ansatz because it takes into account :

- how the other Green's functions change with N_f and the gauge group ($\rightarrow$ that influences the vertex !)
- partially, the full tensor structure of the vertex.

Conformality and bound states?

For a conformal theory, we expect that the BSE will potentially **not** have any results (no bound states).

Why ?

$\longrightarrow$ A bound state has an energy, and that is **not** possible in a conformal theory (no scale).

Conformality and bound states?

However, maybe the coupling still has asymptotic freedom in the conformal window.

$\rightarrow$ There is some scale.

$\rightarrow$ Maybe not **all** the bound states are suppressed by the walking.

In any way, we do expect the walking to suppress (some) many bound states in a (nearly)-conformal theory.