Resonance effects in bound state interaction kernels

Ángel Miramontes, Helios Sanchis Alepuz

January 31, 2019

University of Graz

UNI GRAZ

FWF Der Wissenschaftsfonds.

NAWI Graz

Table of contents

Motivation

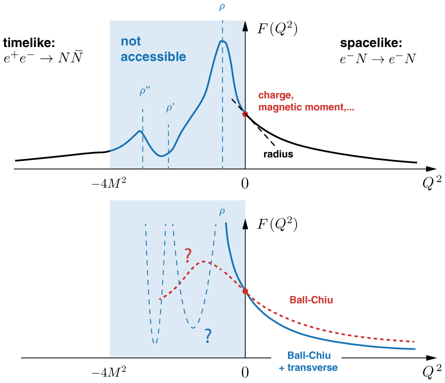
Since quarks and gluons are confined in hadrons, our best chance to learn about their properties is to permform scattering experiments involving hadrons or leptons.

Among the observables that we can extract are:

- hadron spectrum,
- form factors.

The form factors encode their momentum dependent interactions with external currents such as photons and are expressed through electromagnetic and axial form factors.

- The space-like behaviour is experimentally extracted from eN scattering or pion photo- and electroproduction.
- The form factor at $Q^2 = 0$ encode the charge and magnetic moment and the slope defines the charge radius which gives a basic measure of the size of the hadron.



From these considerations it is clear that form factors encode important informations on the substructure of hadrons.

The collection of experimental data on nucleon elastic and transition form factors provides a benchmark test for theoretical approaches.

- Jefferson Lab,
- ELSA,
- MAMI,
- BES III,
- PANDA.

The main task is then to understand and explain the interplay of the various features that enter into such form factors. We are interested in the nonperturbative properties which are enconded in their low momentum.

Mesons in DSE/BSE

Mesons in DSE/BSE

To calculate properties of hadrons we need to:

- Define the hadron as a configuration of quarks and gluons
  - Mesons as a bound states of $q\bar{q}$.
  - Baryons as a bound states of qqq.
- Calculate propagators, vertices, etc.
- Solve a bound-state equation for the system using the calculated elements: This gives bound state masses
- Couple to external current: This gives form factors.

Bethe-Salpeter Equations

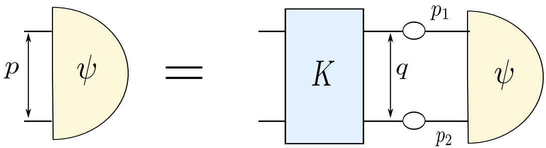
If a system of n-particles form a bound state a pole appears in the Green function for $P^2 = -M^2 + iM\Gamma$,

$$G \to \frac{\Psi \bar{\Psi}}{P^2 + M^2 - iM\Gamma},$$

with Ψ the Bethe-Salpeter amplitude.

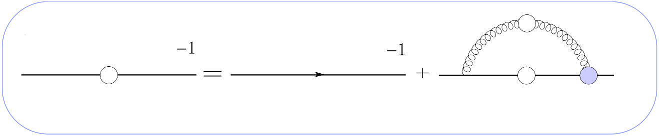
$$\Psi = K G_0 \Psi$$

Diagrammatic BSE



Dyson-Schwinger equations (DSEs)

Quark propagator DSE



- In most phenomenological applications, the quark DSE has been truncated.

We use Rainbow ladder (RL) truncation.

- Preserve chiral symmetry,
- Quark-gluon vertex $\Gamma^\mu \sim \gamma^\mu$
- Collect the dressings in an effective coupling $\alpha(k^2)$.
- One frequently used effective interaction is the Maris-Tandy model[1]

[1] P. Maris and C. Tandy, Phys. Rev. C60 (1999) 055214

Maris-Tandy model

One frequently used effective interaction is the Maris-Tandy model[2]

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 \exp^{-\eta^2 \frac{k^2}{\Lambda^2}} + \alpha_{UV}$$

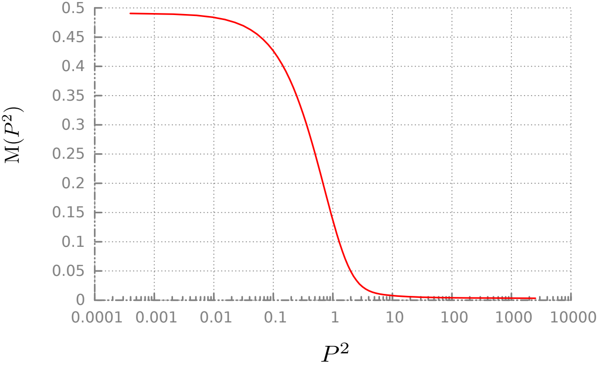
- Reproduces the one-loop QCD behaviour of the quark propagator at large momenta,
- Enough strength for dynamical chiral symmetry breaking to take place.
- Λ and η fitted to reproduce the decay constant from pion BSE.

[2] P. Maris and C. Tandy, Phys. Rev. C60 (1999) 055214

Dressed propagator is given by

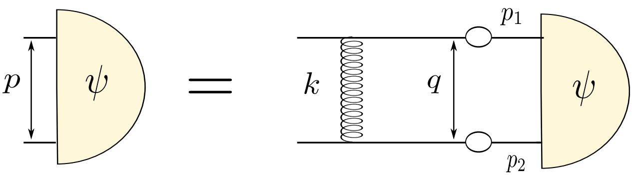
$$S(p) = \frac{1}{A(p^2)} \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)} = -i\not{p}\sigma_v(p^2) + \sigma_s(p^2);$$

Renormalization conditions: $A(\mu^2) = 1, \quad M(\mu^2) = m_q, \quad \mu = 19GeV$



- The quark mass function encodes dynamical chiral symmetry breaking and displays the transition from constituent quark mass to current quark mass.

We need to specify the interaction kernel. We use the BSE with rainbow ladder truncation



- The resulting BSE kernel is a gluon exchange.
- Can be solved numerically.

$$\Psi(p,P) = \int \frac{d^4q}{(2\pi)^4} \gamma^\mu S(p_1) \Psi(q,P) S(p_2) \gamma^\nu D_{\mu\nu}(k)$$

- The BSE is a parametric eigenvalue equation with discrete solutions at $P^2 = -M_n^2$

- Rainbow ladder truncation + Maris-Tandy model works very well for ground states. [3] [4]

	$\Xi(GeV)$	$\Sigma^*(GeV)$	$\Xi^*(GeV)$
BSE	1.235	1.33	1.473
Experiment	1.315	1.385	1.533

- In this truncation, hadrons are stable bound states and they do not decay.
- Nevertheless, most hadrons are resonances and they do decay. In order to get a complete description of hadrons we must incorporate these features.

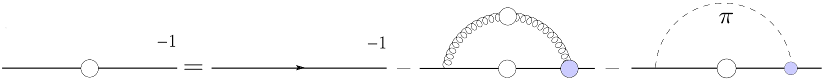
[3] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer. Baryons as relativistic three quark bound states. (arXiv:1606.09602 [hep-ph])

[4] T. Hilger, M. Gmez-Rocha, A. Krassnigg, W. Lucha. Aspects of open-flavour mesons in a comprehensive DSBSE study. arXiv:1702.06262 (2017)

Pion cloud effects

The t-channel pion exchange

We will introduce explicit pionic degrees of freedom in the system, in addition to quarks and gluons. Besides the gluon part of the quark DSE, an emission and absorption of the pion appears[5,6]



- The Bethe-Salpeter kernel can be constrained via the axial Ward-Takahashi identity (axWTI).

$$[\Sigma(p_+)\gamma_5 + \gamma_5\Sigma(p_-)]_{tu} = \int \frac{d^4k}{(2\pi)^4} K_{tu;sr}(p,k;P)\,[\gamma_5 S(k_-) + S(k_+)\gamma_5]_{rs}$$

- The axWTI ensures that chiral symmetry is preserved in the chiral limit.

[5] H. Sanchis-Alepuz, C. S. Fischer, S. Kubrak, Phys. Lett. B733,151 (2014)

[6] C. S. Fischer, R. Williams, Phys. Rev. D78, 074006 (2008)

The Bethe Salpeter vertex of the pion can be represented by

$$\Gamma_\pi^i(p,P) = \tau^i\gamma_5(E_\pi(p,P) - i\not{P}F_\pi(p,P) - i\not{p}p\cdot PG_\pi(p,P) - [\not{P},\not{p}]\,H_\pi(p,P))$$

with four independent dressing function $E_\pi, F_\pi, G_\pi, H_\pi$.

$$\begin{aligned}
K^{pion}_{tu,sr}(q,p;P) &= \frac{1}{4}[\Gamma^j_\pi]_{ru}\left(\frac{p+q-P}{2};p-q\right)[Z_2\tau^j\gamma^5]_{ts}D_\pi(p-q) \\
&+ \frac{1}{4}[\Gamma^j_\pi]_{ru}\left(\frac{p+q-P}{2};q-p\right)[Z_2\tau^j\gamma^5]_{ts}D_\pi(p-q) \\
&+ \frac{1}{4}[\Gamma^j_\pi]_{ts}\left(\frac{p+q-P}{2};p-q\right)[Z_2\tau^j\gamma^5]_{ru}D_\pi(p-q) \\
&+ \frac{1}{4}[\Gamma^j_\pi]_{ts}\left(\frac{p+q-P}{2};q-p\right)[Z_2\tau^j\gamma^5]_{ru}D_\pi(p-q)
\end{aligned}$$

- This is the only kernel that respects the axWTI for the general structure of the Bethe-Salpeter vertex [7]

[7] Beyond the rainbow: Effects from pion back-coupling. Fischer, Christian S. et al. Phys.Rev. D78 (2008) 074006 arXiv:0808.3372 [hep-ph]

- The corresponding Bethe-Salpeter equation that we need to solve is,

$$\Psi(p;P) = \int \frac{d^4k}{(2\pi)^4} \left[K^{RL}(p,q;P) + K^t(p,q;P)\right] \left[S(k_1)\Psi(k;P)S(k_2)\right]$$

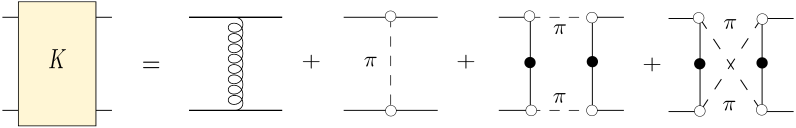
We approximate the pion Bethe-Salpeter amplitude in the quark DSE and the kernel of the BSE by the leading amplitude in the chiral limit,

$$\Gamma_\pi^j(q;P) = \tau^j \gamma_5 \frac{B(p^2)}{f_\pi}$$

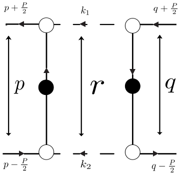
For the corrected kernel we refit the parameters from MT model to reproduce the pion decay constant.

	RL	RL + pion	PDG
m_π(MeV)	140	137	138
f_π(MeV)	93	93	93

s- and u-channel pion exchange



- The kernel corresponding to the new contribution is given by,



$$
\begin{aligned}
K_{da,he}(q,p.r;P) \quad = \quad & \Big[\frac{1}{2}[\Gamma_\pi^j]_{dc}\left(p+\frac{P}{4}-\frac{r}{4};\frac{P+r}{2}\right) S_{cb}\left(p-\frac{r}{2}\right)[\Gamma_\pi^j]_{ba}\left(p-\frac{P}{4}-\frac{r}{4};\frac{P-r}{2}\right) \\
\times \quad & \frac{1}{2}[\Gamma_\pi^j]_{hg}\left(q+\frac{P}{4}-\frac{r}{4};\frac{r-P}{2}\right) S_{gf}\left(q-\frac{r}{2}\right)[\Gamma_\pi^j]_{fe}\left(q+\frac{P}{4}-\frac{r}{4};-\frac{P+r}{2}\right) \\
\times \quad & D_\pi\left(\frac{P+r}{2}\right) D_\pi\left(\frac{P-r}{2}\right)
\end{aligned}
$$

The new BSE to solve is,

$$\begin{aligned}\Psi(p;P) &= \frac{1}{(2\pi)^4}\int r^2 d^2 r \int \sqrt{1-z_r} dz_r \int d\phi_r \int dy_r \Big[K^{RL}(p,q;P) + K^t(p,q;P) \\ &+ K^s(p,q,r;P) + K^u(p,q,r;P)\Big] \left[S(q_1)\Psi(q;P)S(q_2)\right]\end{aligned}$$

- The inclusion of the two kernels in BSE calculations is very challenging, as they have a non-trivial analytic structure.
- Knowing the position of the singularities allows to develop effective algorithms for numerical calculations
- For example, the kernel features now branch cuts corresponding to the virtual pions. Those are determined by the zeroes of the denominators and are parametrized by

$$\begin{aligned}y_1(P,zr) &= -m_\pi^2 - P^2 + 2z_r^2 P^2 - 2\sqrt{-m_\pi^2 z_r^2 P^2 - z_r^2 P^4 + z_r^4 P^4} \\ y_2(P,zr) &= -m_\pi^2 - P^2 + 2z_r^2 P^2 + 2\sqrt{-m_\pi^2 z_r^2 P^2 - z_r^2 P^4 + z_r^4 P^4}\end{aligned}$$

- In order to perform the integration over the relative momentum r to solve the BSE with the new contributions, first we need to deform the contour since the branch cut overlaps the real axis.

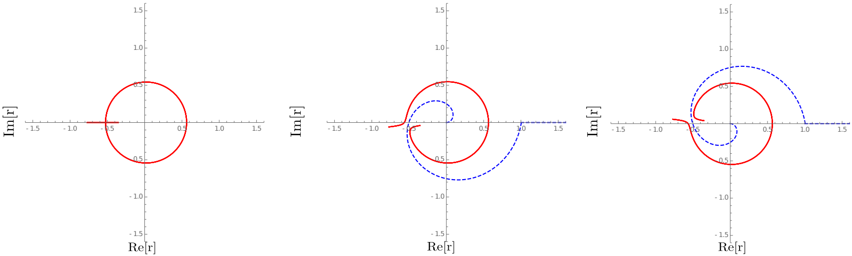
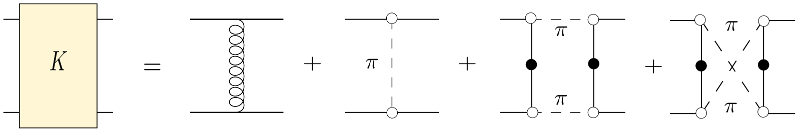


Figure 1: The solid line corresponds to the branch cuts due to the two pion propagators and the dotted line shows a possible integration path.

s- and u-channel pion exchange

- Solving the homogeneous BSE including the different channels we get the following results,

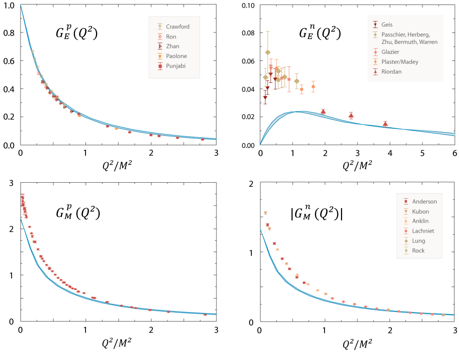


	RL	RL + pion	RL + pion + decay	PDG
m_ρ (MeV)	740	720	645	776
Γ_ρ (MeV)			103	150

Resonance effects in meson FFs

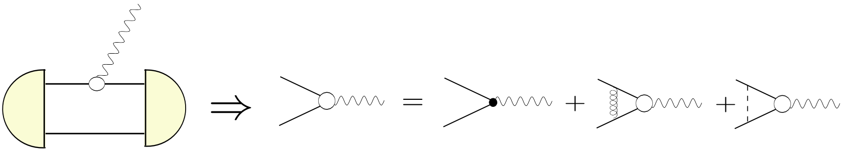
Resonance effects in meson FFs

- We want to explore the effect of pion contributions to meson form factors (FFs).
- The quark photon vertex is needed.
- Rainbow-ladder truncation is useful in describing FFs [8].



[8] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer. Baryons as relativistic three quark bound states. (arXiv:1606.09602 [hep-ph])

- Pion cloud effects are expected to play an important role in the low momentum behavior of form factors.
- The starting point for extracting meson form factors is the same as the extraction to the hadron spectrum.
- We have to solve an inhomogeneous Bethe-Salpeter equation for the vertex; it depends on the kernel where the truncation to rainbow-ladder is made.



The quark photon vertex consists of a longitudinal part and transverse part,

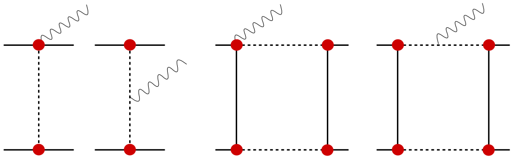
$$\Gamma_q^\mu = \Gamma^\mu_{q,L} + \Gamma^\mu_{q,T}$$

- The longitudinal part is the Ball-Chiu vertex and is fixed by the Ward-Takahashi identity.
- The transverse part receives the dynamical contributions of the meson poles.

- We have additional couplings when we include the pion exchange into DSE/BSE equations.
- In this case the photon also couples with the pion and with the pion vertex.

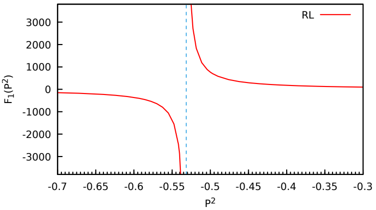
New terms in truncated kernel

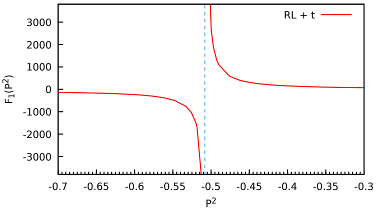
New terms in gauged kernel



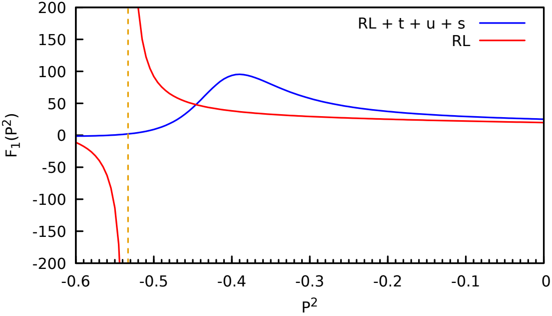
- First we solve the transversal part of the quark-photon vertex using RL truncation and including the t channel.

$$\Gamma^\mu = \Gamma_0^\mu + K G_0 \Gamma^\mu$$





- Then we solve the inhomogeneous BSE including the s and u channels.
- We solve it for $P^2 > 0$ and then we extrapolate using Padé approximant.



Outlook

Summary and Outlook

- In order to include the resonant character of bound states in BSE calculations, virtual decay mechanism must be included.
- The appearance of branch cuts entails that the integration contour must be deformed in order to avoid the crossing of the cuts.
- When we add the s and u channels the poles have an imaginary part and we can extract the width of the resonance.

Next step,

- Solve the inhomogeneous BSE for the quark photon vertex.
- Study meson form factor in space-like and time-like region.