Inhomogeneous phases in the 1+1 dimensional Gross-Neveu model at finite number of fermion flavors

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Outline

1. Gross-Neveu Model
2. Homogeneous phases
3. Inhomogeneous phase

Inhomogeneous phases in the GN model at finite $N_f$
Phase diagram of QCD poses an interesting problem with a lot of open questions

no first principle approach can explore it

several toy models for QCD have inhomogeneous phases in the large $N_f$ limit

This research focuses on these phases in the Gross-Neveu (GN) Model in $1+1$ dimensions at finite number of flavors
The GN Model serves as a toy model for QCD
- The fermion interactions are approximated by 4-point interaction
- A discrete chiral symmetry is realized in the action
- This symmetry can be spontaneously broken
- It is asymptotically free

Euclidean action of GN model

$$S_E = -\int d^2x \left( \bar{\psi}_f (\partial_0 \gamma_{E,0} + \partial_1 \gamma_{E,1}) \psi_f - \frac{\lambda}{2N_f} (\bar{\psi}_f \psi_f)^2 \right)$$ (1)
After Hubbard-Stratonovich (HS)-transformation

\[ Z = \mathcal{N} \int D\psi_f D\bar{\psi}_f D\sigma \exp \left[ - \int d^2x \left( \bar{\psi}_f D\psi_f + \frac{N_f}{2\lambda} \sigma^2 \right) \right] \quad (2) \]

with \( D = \partial + \sigma \)

\[ \langle \bar{\psi}(x)\psi(x) \rangle = -\frac{N_f}{\lambda} \langle \sigma(x) \rangle \]
Spontaneous symmetry breaking

- The action is invariant under chiral symmetry $\sigma \rightarrow -\sigma$
- To illustrate spontaneous symmetry breaking consider $\sigma(t, x) = \text{const}$
- The effective action has a single minimum in the chirally symmetric phase at $\sigma = 0$
- In the broken phase the minimum splits into two minima

**Figure:** $S_{\text{eff}}$ for homogeneous $\sigma$ for different couplings $\lambda$. $\lambda_1 < \lambda_{\text{crit}} \lesssim \lambda_2 \ll \lambda_3$

The shape of the chiral condensate for $T/\sigma_0 = 0.141$ and different values of $\mu/\sigma_0$. [ M. Wagner, PoS LATTICE2007, 339 (2007) ]
Why lattice simulations?

- Analytic solution of the GN model phase diagram in the large $N_f$-limit is known
- Exhibits 3 phases: homogeneously broken, restored (or chirally symmetric) and inhomogeneous phase
- Interesting, but unknown, whether inhomogeneous phase survives at finite number of flavors in 1+1 dimensions
Why lattice simulations?

- Analytic solution of the GN model phase diagram in the large $N_f$-limit is known.
- Exhibits 3 phases: homogeneously broken, restored (or chirally symmetric) and inhomogeneous phase.
- Interesting, but unknown, whether inhomogeneous phase survives at finite number of flavors in $1+1$ dimensions.
- Not only the minimum of the action contributes → lattice simulations of the path integral.
- Results shown are computed with the naive discretization of the derivative, cross check via identical computation with SLAC derivative.
- Scale setting is done via $\sigma_0$ - the value of $\sigma$ at very low temperatures and $\mu = 0$. All quantities are expressed in units of $\sigma_0$. 
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1 Gross-Neveu Model

2 Homogeneous phases

3 Inhomogeneous phase
Observables to detect homogeneous phases

- $\sigma_n = \frac{1}{V} \int_V \sigma(t, x) \, dt \, dx$
- $\langle |\sigma| \rangle = \frac{1}{N_{\text{conf}}} \sum_{n=0}^{N_{\text{conf}}} |\sigma_n|$
  with $N_{\text{conf}}$: Number of configurations.
  - → serves as an order parameter to distinguish broken phase ($\sigma = \text{const} \neq 0$) and restored phase ($\sigma \approx 0$)
- $\chi = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$
  - → peaks at the phase transition
Phase diagram of $\langle|\sigma|\rangle$ and $\chi$

Figure: Phase diagram in (a) for $\langle|\sigma|\rangle/\sigma_0$ and in (b) $\chi$ with $\sigma_0 = 0.4080$, $L = 64$ and $N_f = 2 \cdot 4$. Large-N limit boundaries are depicted in green.
Dependence on number of flavors

- Qualitatively similar for different number of flavors, but quantitative differences
- Results for increasing $N_f$ approach large-$N_f$ result

Figure: Scan of $\langle |\sigma| \rangle$ over $L_t$ for different $N_f$ and $\mu = 0$ with $L = 14$. $\lambda$ is tuned so that $T_c$ corresponds to $L_t = 4$
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Observables to detect spatial inhomogeneities

\[ \langle |\sigma| \rangle \] not able to detect inhomogeneous phase
\[ \rightarrow \] need for new observable
Observables to detect spatial inhomogeneities

- $\langle |\sigma| \rangle$ not able to detect inhomogeneous phase
  $\rightarrow$ need for new observable

- $C_n(x) = \frac{1}{V} \int_V \sigma(t, y) \cdot \sigma(t, y + x) \, dt \, dy$

- $C(x) = \frac{1}{N_{\text{conf}}} \sum_{n=0}^{N_{\text{conf}}} C_n(x)$

- $\tilde{C}_n(k) = \mathcal{F}[C_n](k)$

- $\tilde{C}(k) = \frac{1}{N_{\text{conf}}} \sum_{n=0}^{N_{\text{conf}}} \tilde{C}_n(k)$

- $k_{\text{max}} = \arg\max(\mathcal{F}[C](k))$
Figure: $C(x)$ in (a) and $\tilde{C}(k)$ in (b) for the inhomogeneous phase at $T/\sigma_0 = 0.082$ and $\mu/\sigma_0 = 0$ with $\sigma_0 = 0.253$, $L = 64$ and $N_f = 2 \cdot 4$. 
Figure: $C(x)$ in (a) and $\tilde{C}(k)$ in (b) for the restored phase at $T/\sigma_0 = 0.988$ and $\mu/\sigma_0 = 0$ with $\sigma_0 = 0.253$, $L = 64$ and $N_f = 2 \cdot 4$. 

$C(x)/\sigma_0^2$ vs $x/\sigma_0$ and $\tilde{C}(k)/\sigma_0^2$ vs $k/\sigma_0$.

Observable $C$ - restored phase
Figure: $C(x)$ in (a) and $\tilde{C}(k)$ in (b) for the homogeneously broken phase at $T/\sigma_0 = 0.082$ and $\mu/\sigma_0 = 0$ with $\sigma_0 = 0.253$, $L = 64$ and $N_f = 2 \cdot 4$. 
Phase diagram of $k_{\text{max}}$

Figure: Phase diagram for $k_{\text{max}}/\sigma_0$ with $\sigma_0 = 0.4080$, $L = 64$ and $N_f = 2 \cdot 4$. 
Field observable to detect spatial inhomogeneities

- Absolute of $\sigma$ in frequency space is invariant under spatial shift $x \rightarrow x + x_{\text{shift}}$

$$
\tilde{\sigma}(\omega, k) := \left\langle \left| \sum_{(t,x) \in \Lambda} \sigma(t, x) e^{-\frac{i 2 \pi}{N_t} t \omega} e^{-\frac{i 2 \pi}{N_s} x k} \right| \right\rangle \tag{3}
$$

$$
\rightarrow \left\langle \left| \sum_{(t,x) \in \Lambda} e^{-\frac{i 2 \pi}{N_s} x_{\text{shift}} k} \sigma(t, x + x_{\text{shift}}) e^{-\frac{i 2 \pi}{N_t} t \omega} e^{-\frac{i 2 \pi}{N_s} x k} \right| \right\rangle \tag{4}
$$

$$
= \tilde{\sigma}(\omega, k). \tag{5}
$$

- extract $x_{\text{shift}}$ for every configuration

- $\sigma(t, x) := \langle \sigma(t, x - x_{\text{shift}}) \rangle$
Shifted field results

(a) Shifted field observable for $\mu/\sigma_0 = 0$ and $T/\sigma_0 \approx 0.038$ [J.Lenz]

(b) Shifted field observable for $\mu/\sigma_0 = 0.4$ and $T/\sigma_0 \approx 0.038$ [J.Lenz]
(c) Shifted field observable for $\mu/\sigma_0 = 0.5$ and $T/\sigma_0 \approx 0.038$ [J.Lenz]

(d) Shifted field observable for $\mu/\sigma_0 = 0.7$ and $T/\sigma_0 \approx 0.038$ [J.Lenz]
Summary and next steps

- Inhomogeneous phase found at finite numbers of fermion flavors for the first time

Next steps:
- Map out the phase boundaries
- Study order of phase transition
- Explore flavor dependence further
- Examine other models (NJL, ...) which are closer to real situation of QCD