Inhomogeneous phases in the 1+1 dimensional Gross-Neveu model at finite number of fermion flavors

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02.02.2019, excited QCD 2019

Outline

Intro

- Phase diagram of QCD poses an interesting problem with a lot of open questions
- no first principle approach can explore it
- several toy models for QCD have inhomogeneous phases in the large N_f limit
- This research focuses on these phases in the Gross-Neveu (GN) Model in 1+1 dimensions at finite number of flavors

GN model (1)

- The GN Model serves as a toy model for QCD
  - The fermion interactions are approximated by 4-point interaction
  - A discrete chiral symmetry is realized in the action
  - This symmetry can be spontaneously broken
  - It is asymptotically free
- Euclidean action of GN model

$$S_{\mathrm{E}} = -\int \mathrm{d}^2 x \left(\bar{\psi}_f (\partial_0 \gamma_{E,0} + \partial_1 \gamma_{E,1}) \psi_f - \frac{\lambda}{2N_{\mathrm{f}}} (\bar{\psi}_f \psi_f)^2 \right) \quad (1)$$

GN model (2)

- After Hubbard-Stratonovich(HS)-transformation

$$Z = \mathcal{N} \int \mathrm{D}\psi_f \, \mathrm{D}\bar{\psi}_f \mathrm{D}\sigma \exp\left[-\int \mathrm{d}^2 x \left(\bar{\psi}_f \mathrm{D}\psi_f + \frac{N_\mathrm{f}}{2\lambda}\sigma^2\right)\right] \tag{2}$$

  with $D = \partial\!\!\!/ + \sigma$
- $\langle\bar{\psi}(x)\psi(x)\rangle = \frac{-N_\mathrm{f}}{\lambda}\langle\sigma(x)\rangle$

Spontaneous symmetry breaking

- The action is invariant under chiral symmetry $\sigma \to -\sigma$
- To illustrate spontaneous symmetry breaking consider $\sigma(t,x) = \text{const}$
- The effective action has a single minimum in the chirally symmetric phase at $\sigma = 0$
- In the broken phase the minimum splits into two minima

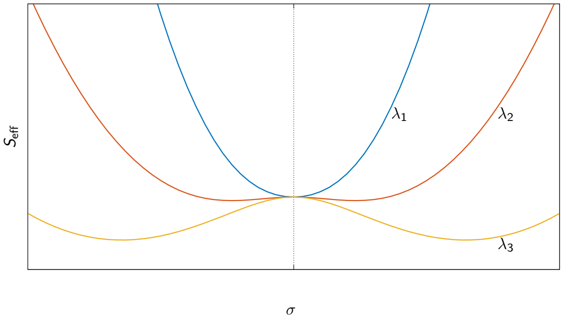
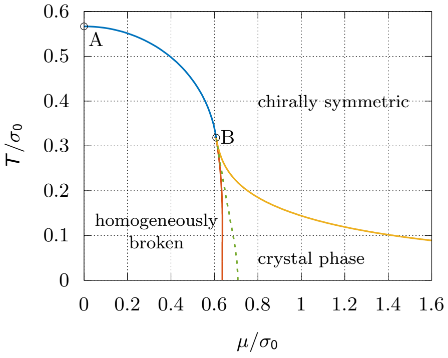
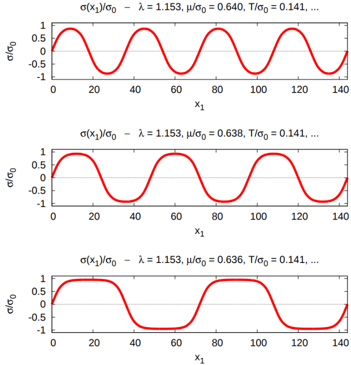


Figure: S_{eff} for homogeneous σ for different couplings λ. $\lambda_1 < \lambda_{\text{crit}} \lessapprox \lambda_2 \ll \lambda_3$

Phase diagram in the large N limit



(a) Revised phase diagram of the Gross-Neveu model. A: Critical temperature, B: tricritical point, blue line: 2nd order homogeneous phase boundary, orange line: 1st order phase boundary, yellow line: 2nd order boundary, dashed green line: the former phase first order boundary from Wolff. [M. Thies, K. Urlichs, *Phys. Rev.* **D67**, 125015 (2003)]



(b) The shape of the chiral condensate for $T/\sigma_0 = 0.141$ and different values of μ/σ_0. [M. Wagner, *PoS* **LATTICE2007**, 339 (2007)]

Why lattice simulations?

- Analytic solution of the GN model phase diagram in the large N_f-limit is known
- Exhibits 3 phases: homogeneously broken, restored (or chirally symmetric) and inhomogeneous phase
- Interesting, but unknown, whether inhomogeneous phase survives at finite number of flavors in 1+1 dimensions

Why lattice simulations?

- Analytic solution of the GN model phase diagram in the large N_f-limit is known
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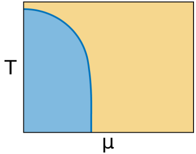
- Interesting, but unknown, whether inhomogeneous phase survives at finite number of flavors in 1+1 dimensions
- Not only the minimum of the action contributes
  $\rightarrow$ lattice simulations of the path integral
- Results shown are computed with the naive discretization of the derivative, cross check via identical computation with SLAC derivative
- Scale setting is done via σ_0 - the value of σ at very low temperatures and $\mu = 0$. All quantities are expressed in units of σ_0

Outline

Observables to detect homogeneous phases



- $\sigma_n = \frac{1}{V} \int_V \sigma(t,x)\, \mathrm{d}t\, \mathrm{d}x$
- $\langle|\sigma|\rangle = \frac{1}{N_{\text{conf}}} \sum_{n=0}^{N_{\text{conf}}} |\sigma_n|$
  with N_{conf}: Number of configurations.
  → serves as an **order parameter** to distinguish broken phase ($\sigma = \text{const} \neq 0$) and restored phase ($\sigma \approx 0$)
- $\chi = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$
  → peaks at the **phase transition**

Phase diagram of $\langle|\sigma|\rangle$ and χ

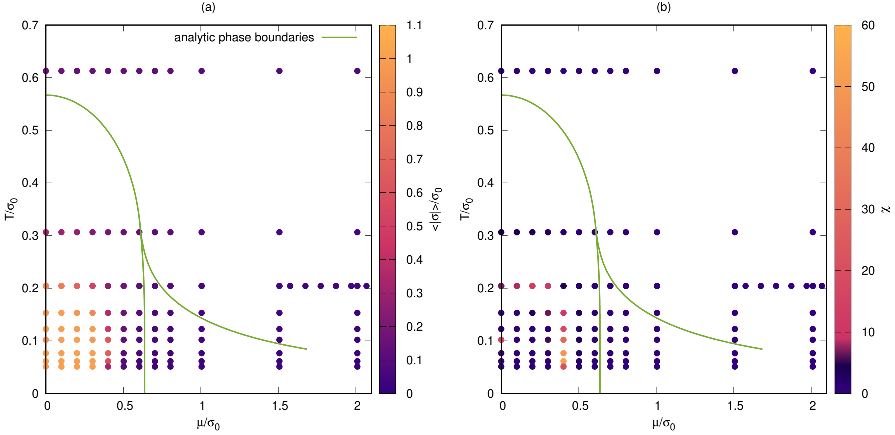


Figure: Phase diagram in (a) for $\langle|\sigma|\rangle/\sigma_0$ and in (b) χ with $\sigma_0 = 0.4080$, $L = 64$ and $N_{\mathrm{f}} = 2 \cdot 4$. Large-N limit boundaries are depicted in green.

Dependence on number of flavors

- Qualitatively similar for different number of flavors, but quantitative differences
- Results for increasing N_f approach large-N_f result

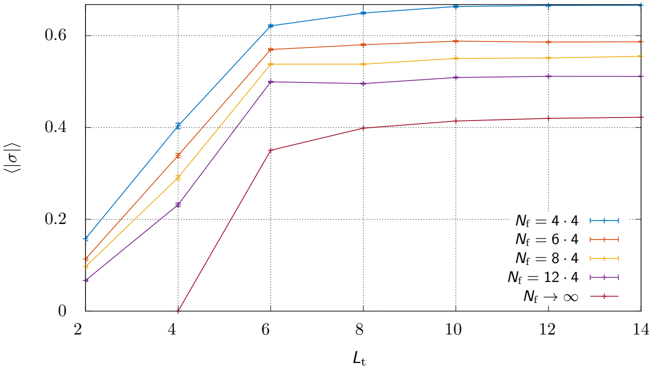
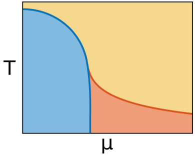


Figure: Scan of $\langle|\sigma|\rangle$ over L_t for different N_f and $\mu = 0$ with $L = 14$. λ is tuned so that T_c corresponds to $L_\mathrm{t} = 4$

Outline

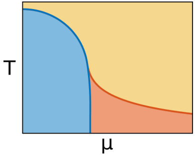
Observables to detect spatial inhomogeneities

- $\langle|\sigma|\rangle$ not able to detect inhomogeneous phase
  $\rightarrow$ need for new observable

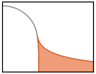


Observables to detect spatial inhomogeneities

- $\langle|\sigma|\rangle$ not able to detect inhomogeneous phase
  $\rightarrow$ need for new observable
- $C_n(x) = \frac{1}{V}\int_V \sigma(t,y)\cdot\sigma(t,y+x)\,\mathrm{d}t\,\mathrm{d}y$
- $C(x) = \frac{1}{N_{\text{conf}}}\sum_{n=0}^{N_{\text{conf}}} C_n(x)$
- $\tilde{C}_n(k) = \mathcal{F}[C_n](k)$
- $\tilde{C}(k) = \frac{1}{N_{\text{conf}}}\sum_{n=0}^{N_{\text{conf}}} \tilde{C}_n(k)$
- $k_{\text{max}} = \arg\max(\mathcal{F}[C](k))$



Observable C - inhomogeneous phase



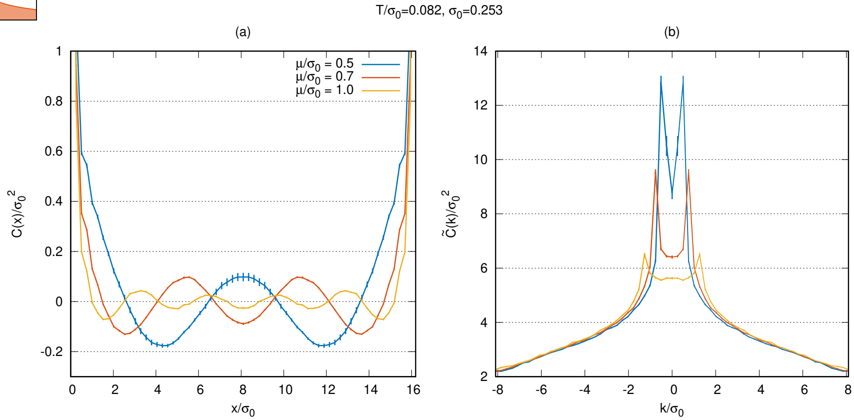


Figure: $C(x)$ in (a) and $\tilde{C}(k)$ in (b) for the **inhomogeneous phase** at $T/\sigma_0 = 0.082$ and $\mu/\sigma_0 = 0$ with $\sigma_0 = 0.253$, $L = 64$ and $N_\mathrm{f} = 2 \cdot 4$.

Observable C - restored phase

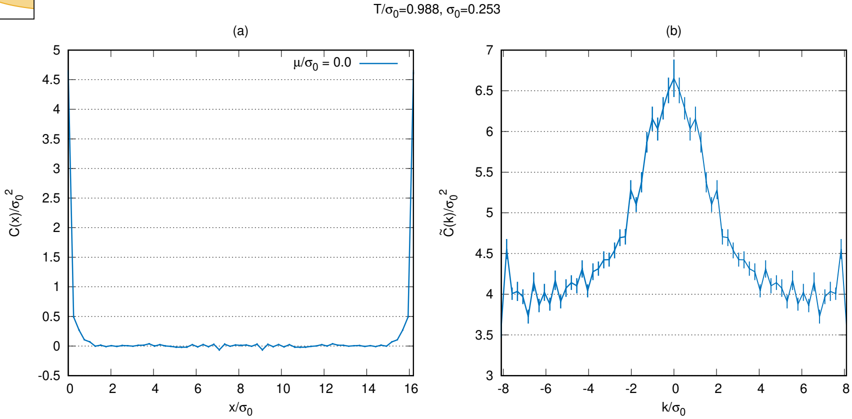
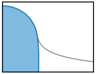


Figure: $C(x)$ in (a) and $\tilde{C}(k)$ in (b) for the **restored phase** at $T/\sigma_0 = 0.988$ and $\mu/\sigma_0 = 0$ with $\sigma_0 = 0.253$, $L = 64$ and $N_f = 2 \cdot 4$.



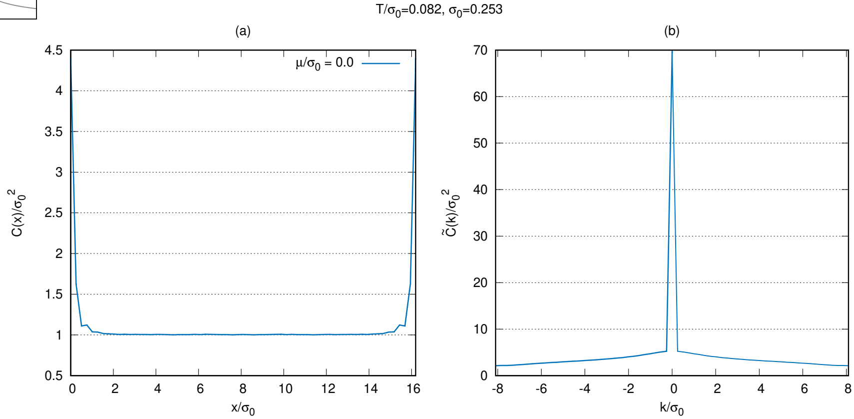


Figure: $C(x)$ in (a) and $\tilde{C}(k)$ in (b) for the **homogeneously broken phase** at $T/\sigma_0 = 0.082$ and $\mu/\sigma_0 = 0$ with $\sigma_0 = 0.253$, $L = 64$ and $N_\mathrm{f} = 2 \cdot 4$.

Phase diagram of k_{max}

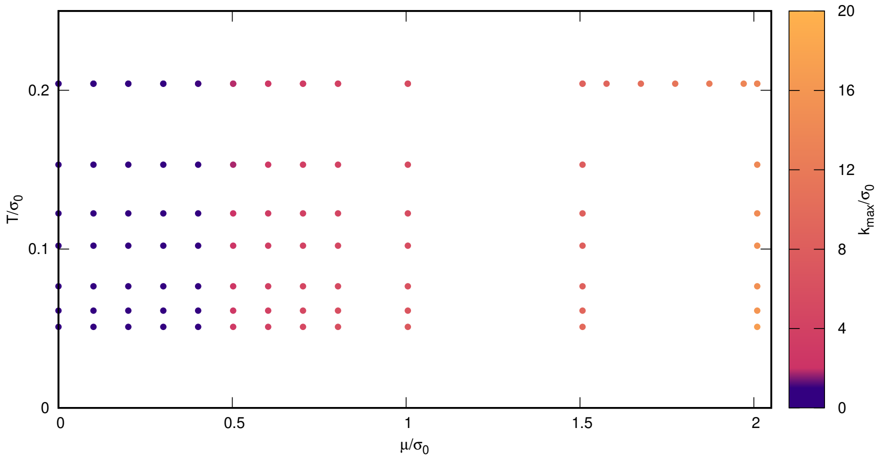


Figure: Phase diagram for k_{max}/σ_0 with $\sigma_0 = 0.4080$, $L = 64$ and $N_f = 2 \cdot 4$.

Field observable to detect spatial inhomogeneities

- Absolute of σ in frequency space is invariant under spatial shift $x \to x + x_{\text{shift}}$
-

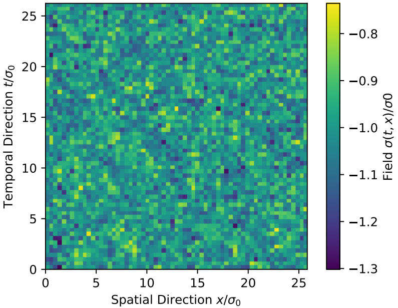
$$\tilde{\sigma}(\omega, k) := \left\langle \left| \sum_{(t,x)\in\Lambda} \sigma(t,x) \mathrm{e}^{-\mathrm{i}\frac{2\pi}{N_\mathrm{t}} t\omega} \mathrm{e}^{-\mathrm{i}\frac{2\pi}{N_\mathrm{s}} xk} \right| \right\rangle \tag{3}$$

$$\to \left\langle \left| \mathrm{e}^{-\mathrm{i}\frac{2\pi}{N_\mathrm{s}} x_{\text{shift}} k} \sum_{(t,x)\in\Lambda} \sigma(t, x + x_{\text{shift}}) \mathrm{e}^{-\mathrm{i}\frac{2\pi}{N_\mathrm{t}} t\omega} \mathrm{e}^{-\mathrm{i}\frac{2\pi}{N_\mathrm{s}} xk} \right| \right\rangle \tag{4}$$

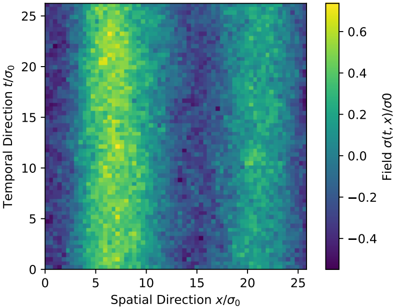
$$= \tilde{\sigma}(\omega, k). \tag{5}$$

- extract x_{shift} for every configuration
- $\sigma(t,x) := \langle \sigma(t, x - x_{\text{shift}}) \rangle$

Shifted field results

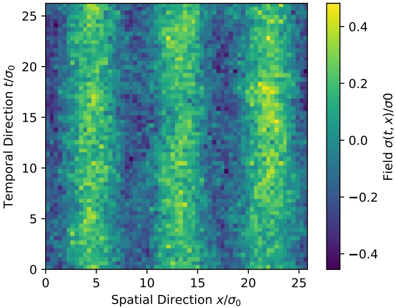


(a) Shifted field observable for $\mu/\sigma_0 = 0$ and $T/\sigma_0 \approx 0.038$ [J.Lenz]

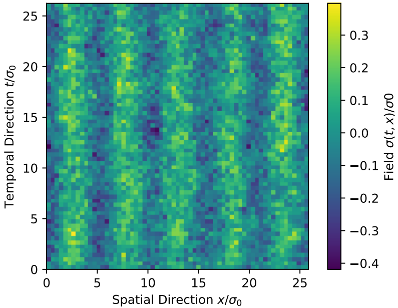


(b) Shifted field observable for $\mu/\sigma_0 = 0.4$ and $T/\sigma_0 \approx 0.038$ [J.Lenz]

Shifted field results



(c) Shifted field observable for $\mu/\sigma_0 = 0.5$ and $T/\sigma_0 \approx 0.038$ [J.Lenz]



(d) Shifted field observable for $\mu/\sigma_0 = 0.7$ and $T/\sigma_0 \approx 0.038$ [J.Lenz]

Summary and next steps

- Inhomogeneous phase found at finite numbers of fermion flavors for the first time

Next steps:

- Map out the phase boundaries
- Study order of phase transition
- Explore flavor dependence further
- Examine other models (NJL, ...) which are closer to real situation of QCD