Departamento de Física Teórica and IPARCOS
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Strange resonances from analyticity and dispersion relations

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In collaboration with
A. Rodas, J. Ruiz de Elvira

Phys.Rev. D93 (2016) no.7, 074025
Eur.Phys.J. C77 (2017) no.2, 91
Eur.Phys.J. C78 (2018) no.11, 897
and work in preparation

Excite QCD, Schladming, Austria. January 30th-February 3rd, 2019

Motivation to study πK scattering with Dispersion Relations

- π,K appear as final products of almost all hadronic strange processes:
  Examples: B,D, decays, CP violation studies, etc…
- π,K are Goldstone Bosons of QCD → Test Chiral Symmetry Breaking
- Main or relevant source for PDG parameters of:
  K_0^* (700), $K_0^*(1430)$, ${}^{*}_{1}K_1*(892)$,$K_1*(1410)$,$K_2^{*0}K_2*(1410)$, K_3* (1780)
- Extracted frequently with strong model dependences (Breit Wigners,….)

Analytic Methods reduce model independence
Dispersion Relations model **in**dependent

Particularly controversial:
κ/ K_0^* (700) light scalar meson. “needs confirmation” @PDG.
Light scalar mesons **longstanding candidates for non-ordinary mesons**.
Needed to identify the lightest scalar nonet

Was $K_0^*K_0*(800)$ until last 2018 PDG revision!
Triggered by our 2017 work

Overview of the K_0*(800) or “kappa” meson until 2018 @PDG

- Omittted from the 2017PDG summary table since, “needs confirmation”

But, all descriptions of data respecting unitarity and chiral symmetry find a pole at M=650-770 MeV and Γ~550 MeV or larger.

Best determination comes from a SOLUTION of a Roy-Steiner dispersive formalism, consistent with UChPT

Decotes Genon et al 2006

2017PDG dominated by such a SOLUTION

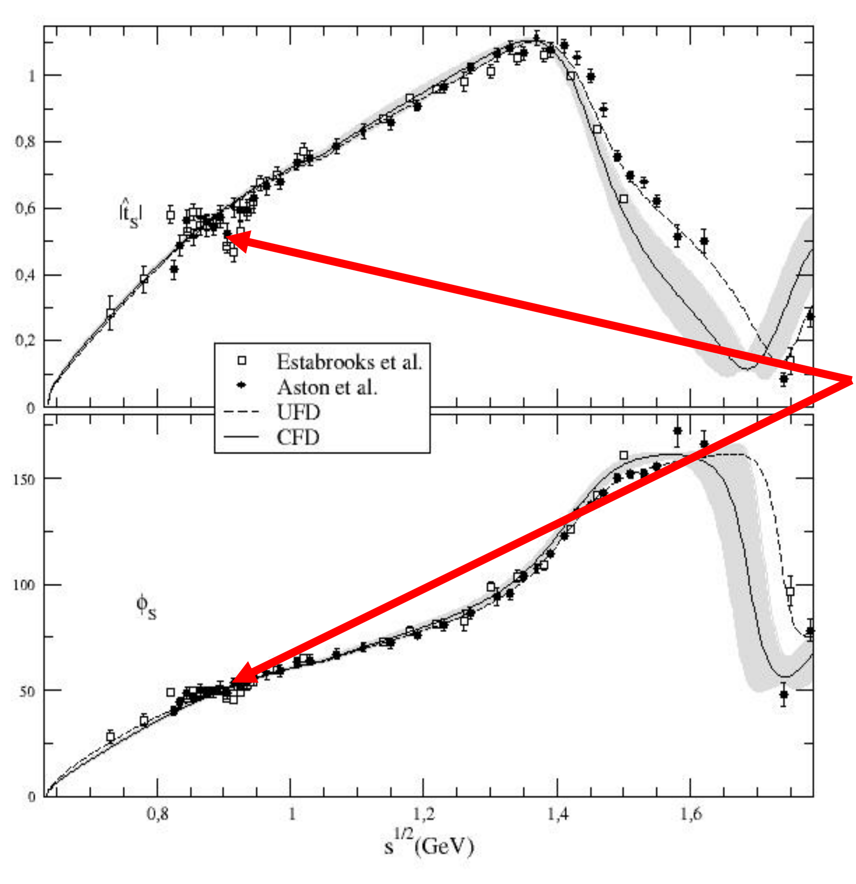
$$M-i\,\Gamma/2=(682\pm29)-i(273\pm i12)\ \text{MeV}\quad @\text{PDG2017}$$

PDG willing to reconsider situation.. **if** additional independent dispersive DATA analysis.

We were encouraged

by PDG members to do it.

Data on πK scattering: S-channel



Most reliable sets:

Estabrooks et al. 78 (SLAC)
Aston et al.88 (SLAC-LASS)

I=1/2 and 3/2 combination

No clear "peak" or phase movement of $\kappa\kappa/K_0^*K_0{}^*(800)$ resonance

Definitely NO BREIT-WIGNER shape

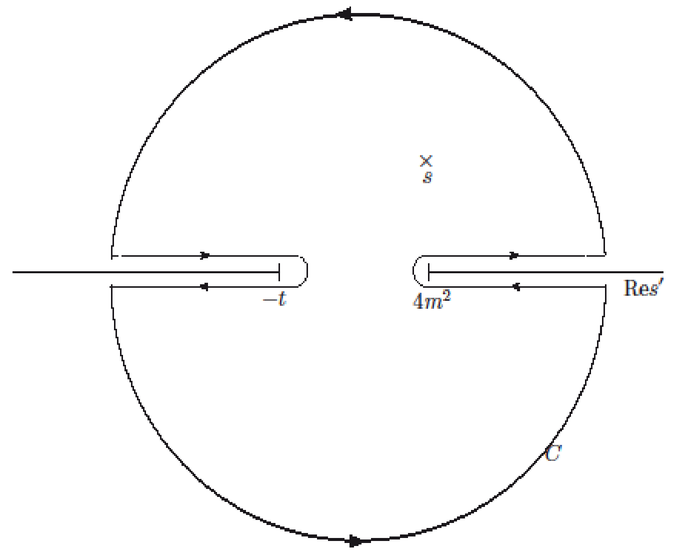
Mathematically correct to use POLES

Strong support for K_0*(700) from decays of heavier mesons, but rigorous model-independent extractions absent. Often inadequate Breit-Wigner formalism

POLE extraction rigorous when using Dispersion Relations
or complex-analyticity properties

Why use dispersion relations?

CAUSALITY:
Amplitudes T(s,t) are ANALYTIC in
complex s plane but for cuts for thresholds.
Crossing implies left cut from u-channel threshold



Cauchy Theorem determines T(s,t) at ANY s,
from an INTEGRAL on the contour

If T->0 fast enough at high s, curved part vanishes

$$T(s,t,u) = \underbrace{\frac{1}{\pi}\int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\, T(s',t,u')}{s'-s}}_{\text{Right cut}} + \underbrace{\frac{1}{\pi}\int_{-\infty}^{-t} ds' \frac{\mathrm{Im}\, T(s',t,u')}{s'-s}}_{\text{Left cut}}$$

Otherwise, determined up to
a polynomial (subtractions)
Left cut usually a problem

Good for:

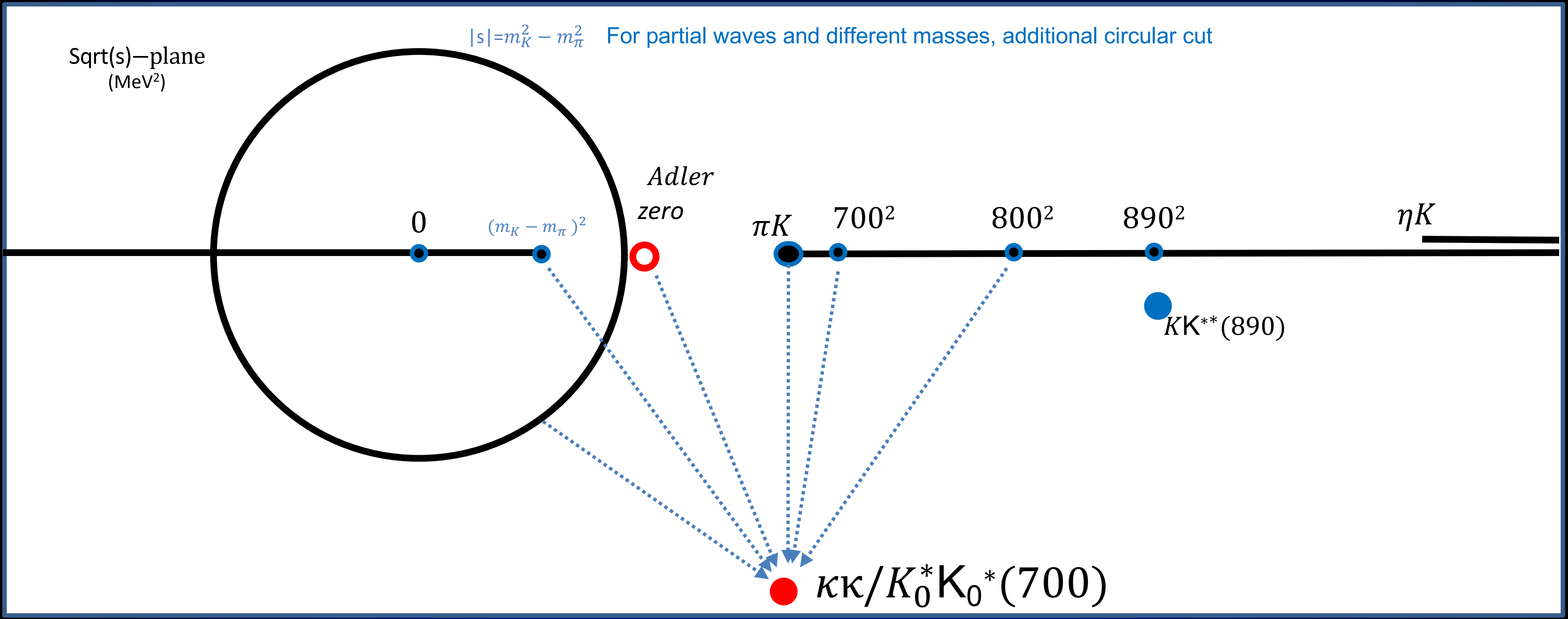
1) Calculating T(s,t) where there is not data

2) Constraining data analysis

3) ONLY MODEL INDEPENDENT extrapolation to complex s-plane
without extra assumptions

Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the *s*-variable, not in Sqrt(s)



Important for the $\kappa\kappa\kappa/K_0^*K_0^*(700)$

- Threshold behavior (chiral symmetry)
- Subthreshold behavior (chiral symmetry →Adler zeros)
- Other cuts (Left & circular)
- Avoid spurious singularities

Less important for other resonances…

Our Dispersive/Analytic Approach for πK and strange resonances

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

- As πK checks: Small inconsistencies.
- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)
- Analytic methods to extract poles: reduced model dependence on strange resonances

JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits up to 1.5 GeV

JRP, A.Rodas, Eur.Phys.J. C78 (2018)

- From fixed-t DR:
  ππ→KK influence small.
  K_0^*κ/ ${}^{*}_{0}$K$_0$*(700) out of reach
- From Hyperbolic DR:
  ππ→KK influence important.

JRP, A.Rodas, in progress.
PRELIMINARY results
shown here

- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV
- **Rigorous κ/ ${}^{*}_{0}$K$_0$*(700) pole**

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Forward dispersion relations for K π scattering.

Since interested in the resonance region, we use minimal number of subtractions

Defining the s↔u symmetric
and anti-symmetric amplitudes
at t=0

$$T^{+}(s)=\frac{T^{1/2}(s)+2T^{3/2}(s)}{3}=\frac{T^{I_t-0}(s)}{\sqrt{6}},$$

$$T^{-}(s)=\frac{T^{1/2}(s)-T^{3/2}(s)}{3}=\frac{T^{I_t-1}(s)}{2}.$$

We need one subtraction for the symmetric amplitude

$$\mathrm{Re}T^{+}(s)=T^{+}(s_{\mathrm{th}})+\frac{(s-s_{\mathrm{th}})}{\pi}P\int_{s_{\mathrm{th}}}^{\infty}ds'\left[\frac{\mathrm{Im}T^{+}(s')}{(s'-s)(s'-s_{\mathrm{th}})}-\frac{\mathrm{Im}T^{+}(s')}{(s'+s-2\Sigma_{\pi K})(s'+s_{\mathrm{th}}-2\Sigma_{\pi K})}\right],$$

And none for the antisymmetric

$$\mathrm{Re}T^{-}(s)=\frac{(2s-2\Sigma_{\pi K})}{\pi}P\int_{s_{\mathrm{th}}}^{\infty}ds'\frac{\mathrm{Im}T^{-}(s')}{(s'-s)(s'+s-2\Sigma_{\pi K})}.$$

where $\Sigma_{\pi K}=m_\pi^2+m_K^2$

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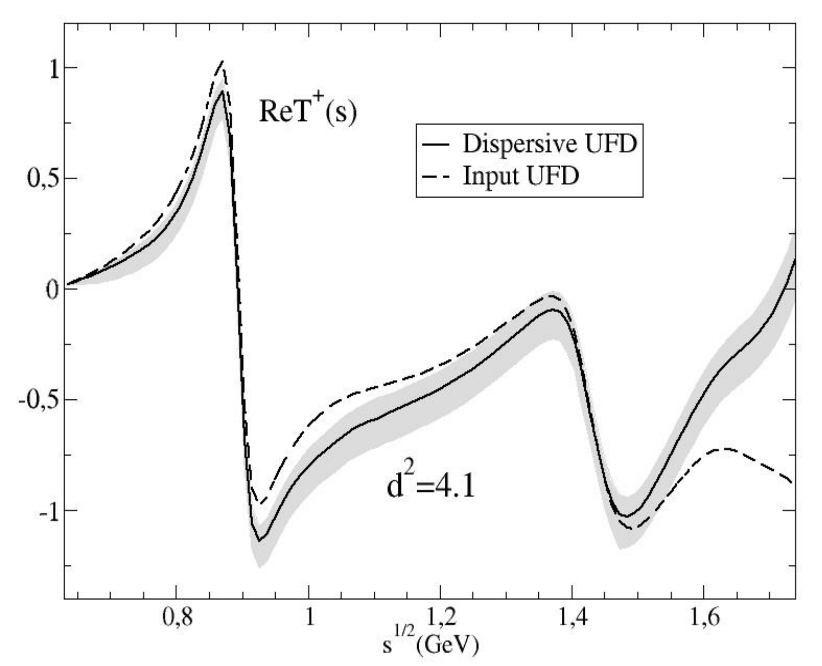

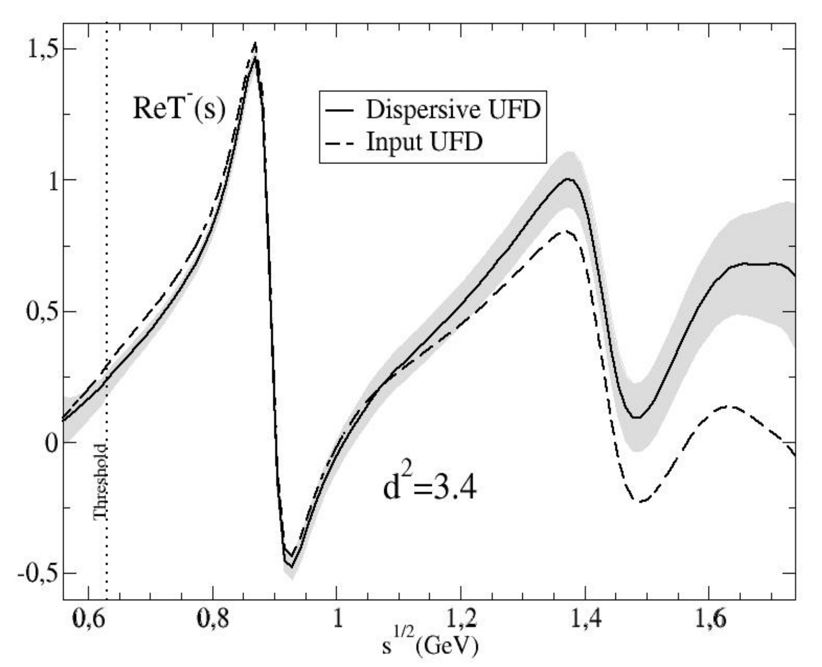

Forward Dispersion Relation analysis of πK scattering DATA up to 1.6 GeV

(**not a solution** of dispersión relations,
but a constrained fit)
A.Rodas & JRP, PRD93,074025 (2016)

First observation:
Forward Dispersion relations
Not well satisfied by data
Particularly at high energies

So we use
Forward Dispersion Relations
as CONSTRAINTS on fits

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- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)

How well Forward Dispersion Relations are satisfied by unconstrained fits

Every 22 MeV calculate the difference between both sides of the DR /uncertainty

Define an averaged χ^2 over these points, that we call d^2

d^2 close to 1 means that the relation is well satisfied

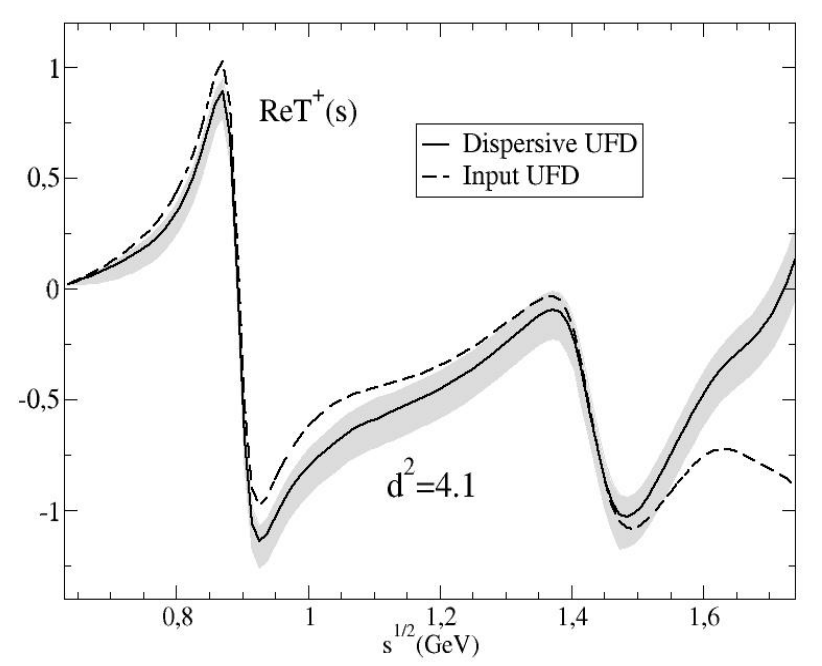
$d^2 >> 1$ means the data set is inconsistent with the relation.

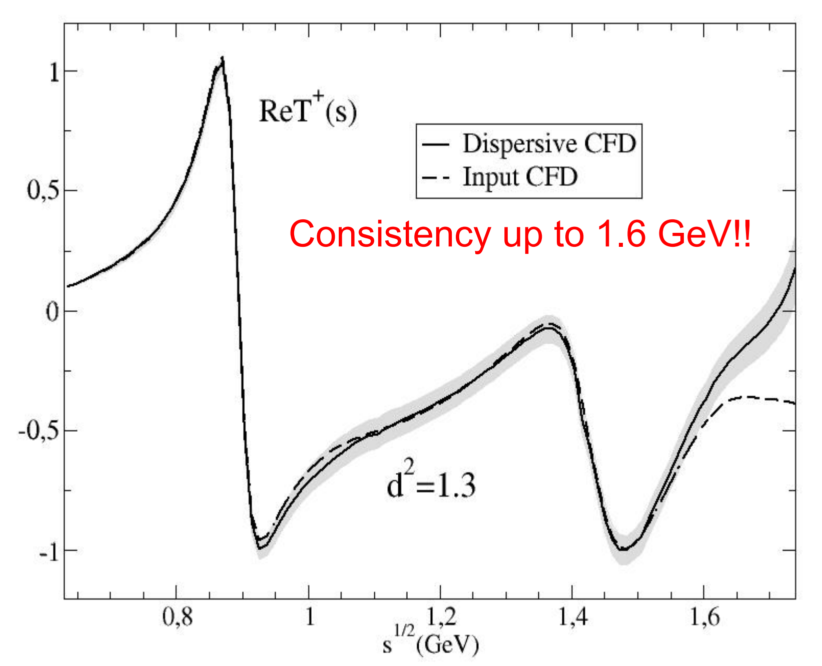
This can be used to check DR

To obtain CONSTRAINED FITS TO DATA (CFD) we minimize:

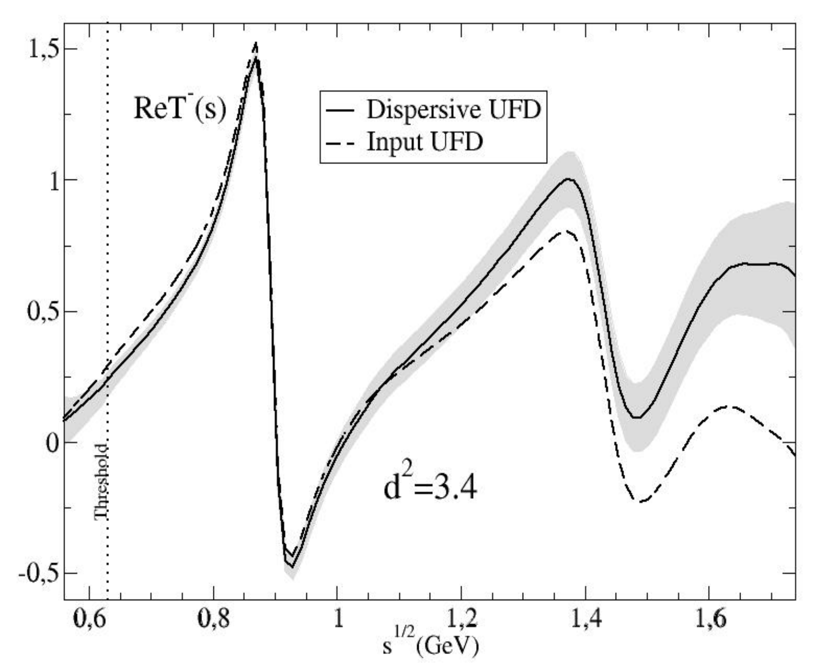
$$W^2\underbrace{(d^2_{T^+} + d^2_{T^-})}_{\text{2 FDR's}} + \underbrace{\sum_{I=\frac{1}{2},\frac{3}{2}} \left(\frac{\Delta_I}{\delta\Delta_I}\right)^2}_{\text{Sum Rules threshold}} + \underbrace{\sum_k \left(\frac{p_k^{UFD} - p_k}{\delta p_k^{UFD}}\right)^2}_{\text{Parameters of the unconstrained data fits}},$$

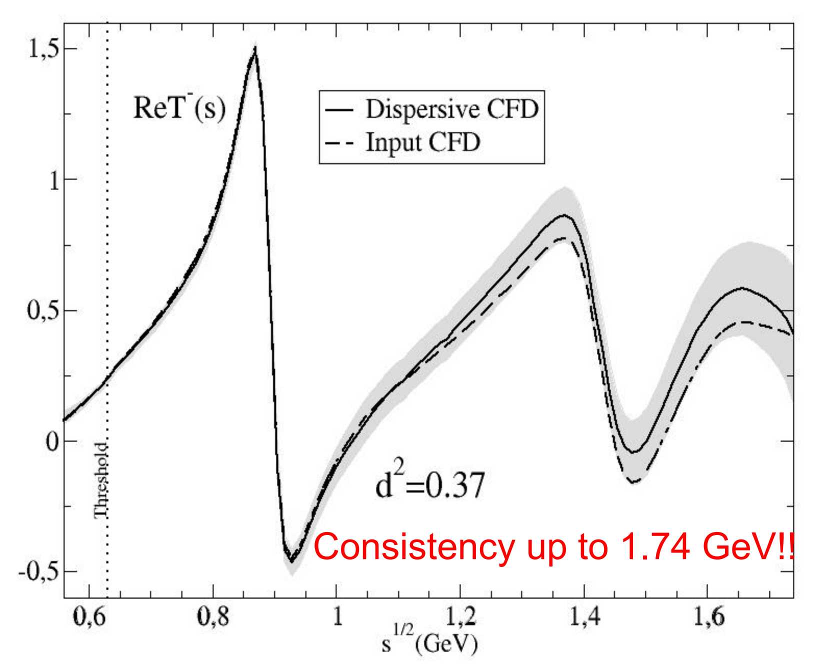
W roughly counts the number
of effective degrees of freedom
(sometimes we add weight on certain energy regions)





Consistency up to 1.6 GeV!!





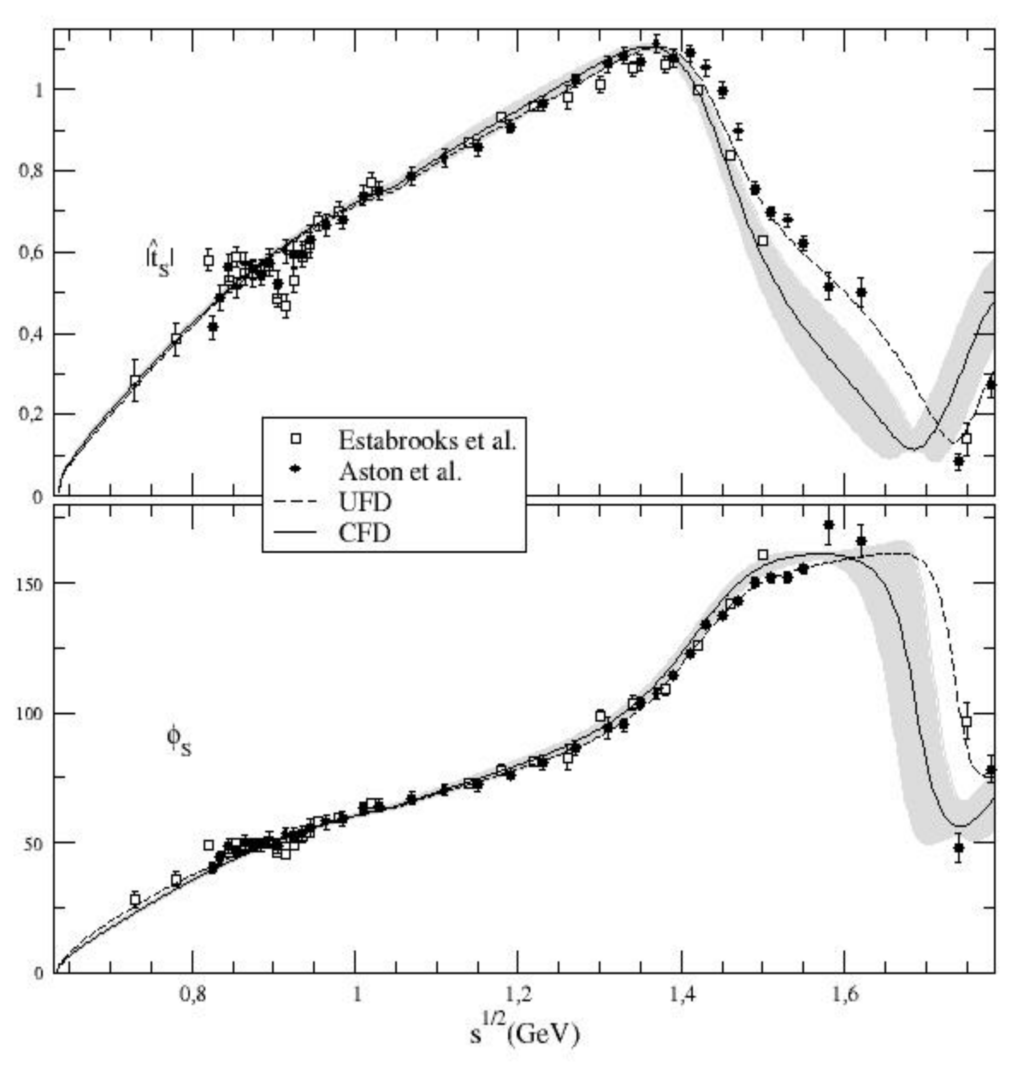
Consistency up to 1.74 GeV!!

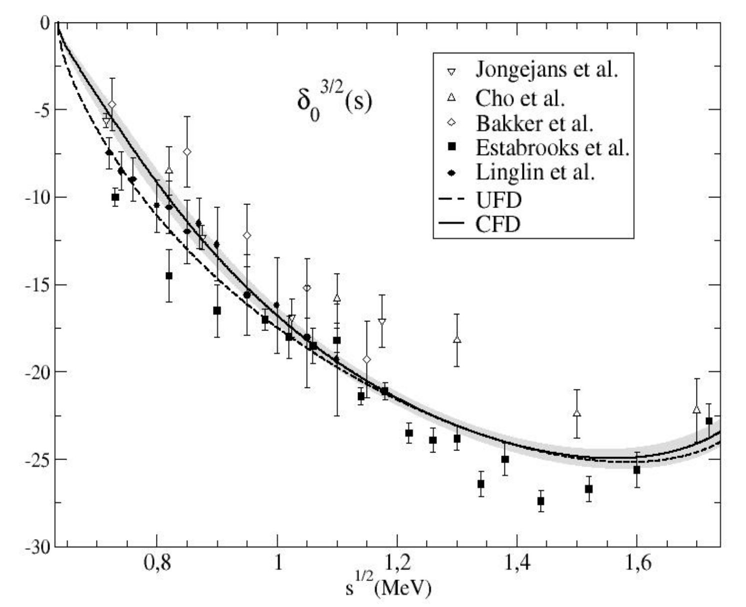
From Unconstrained (UFD) to Constrained Fits to data (CFD)

S-waves. The most interesting for the K_0^* resonances

Largest changes from UFD to CFD

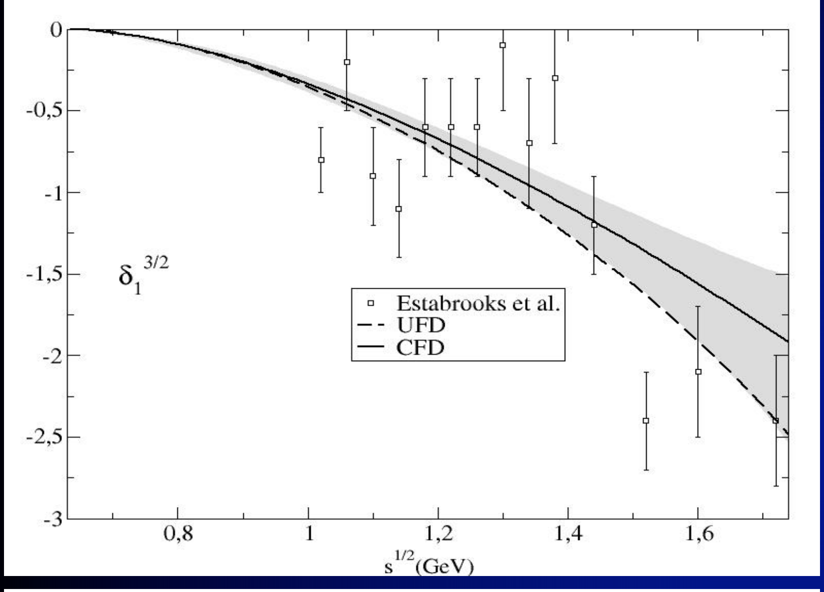
at higher energies

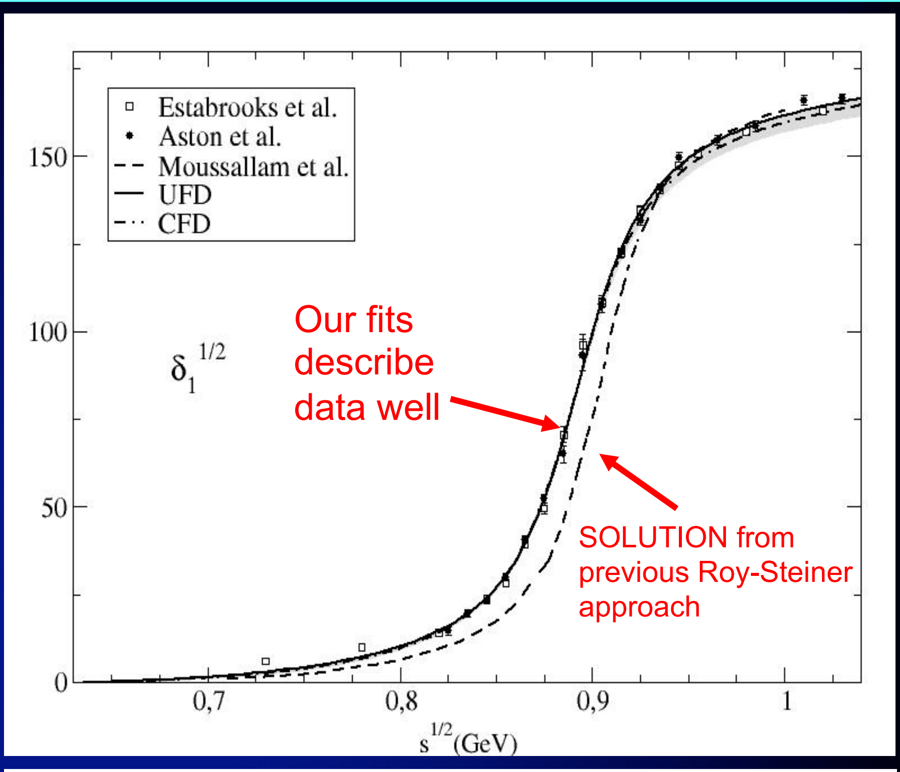




From Unconstrained (UFD) to Constrained Fits to data (CFD)

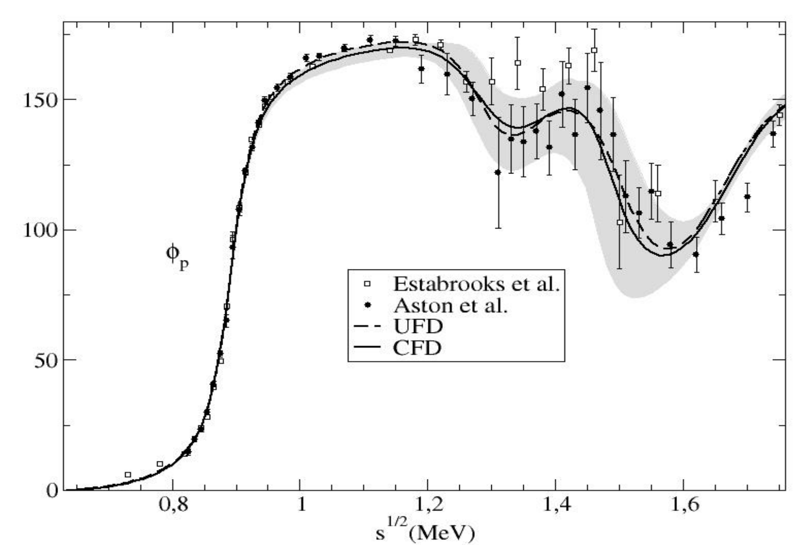
P-waves: Small changes

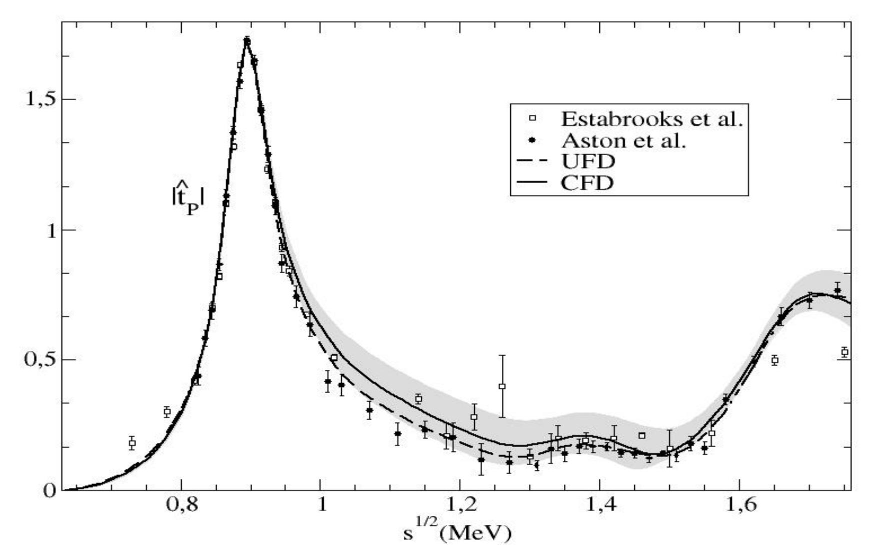




Our fits describe data well

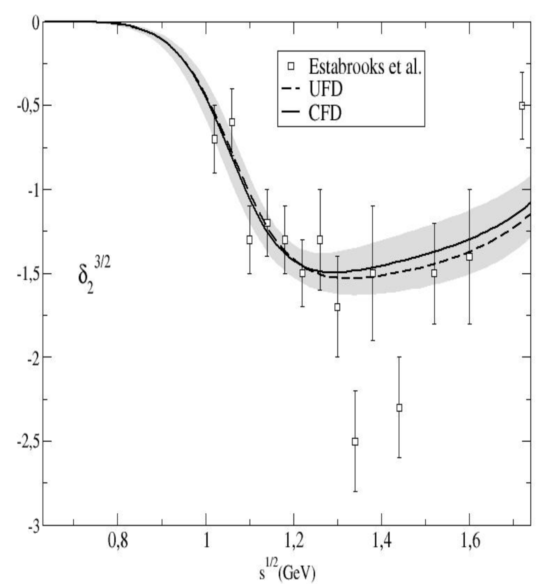
SOLUTION from previous Roy-Steiner approach

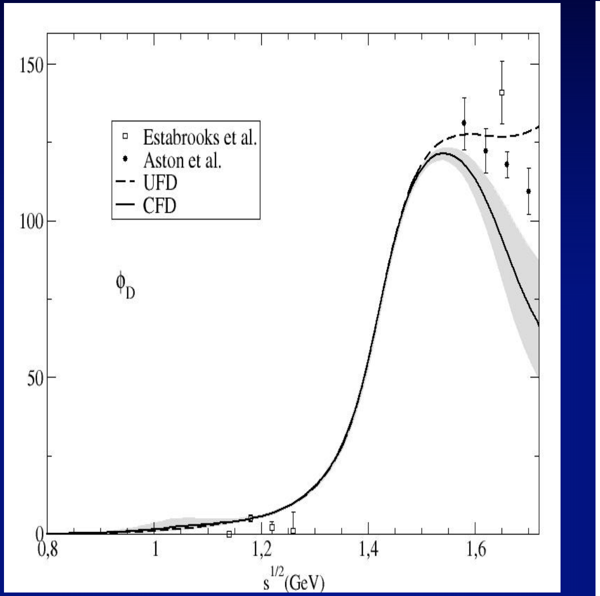


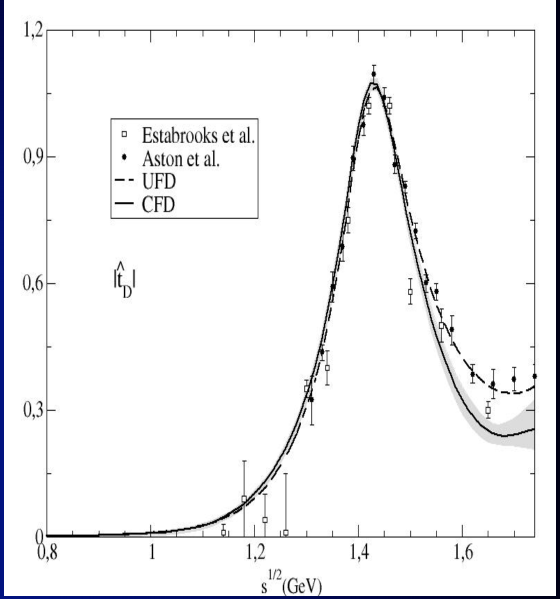


From Unconstrained (UFD) to Constrained Fits to data (CFD)

D-waves: Largest changes of all, but at very high energies

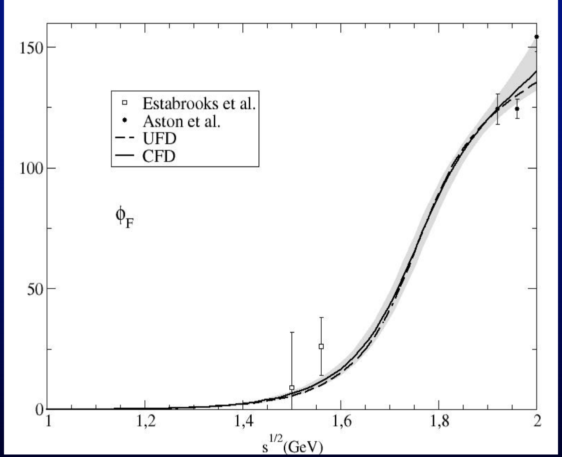


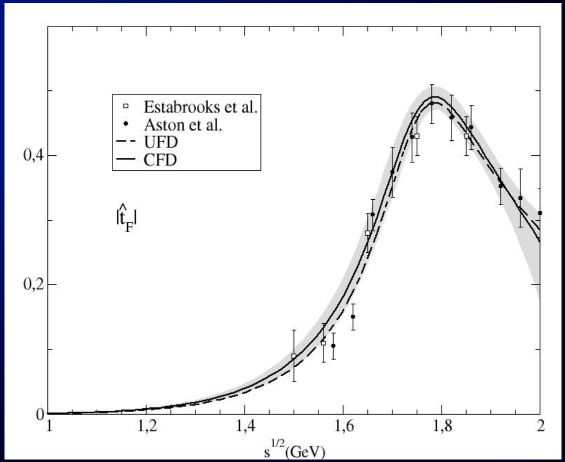




F-waves:

Imperceptible changes





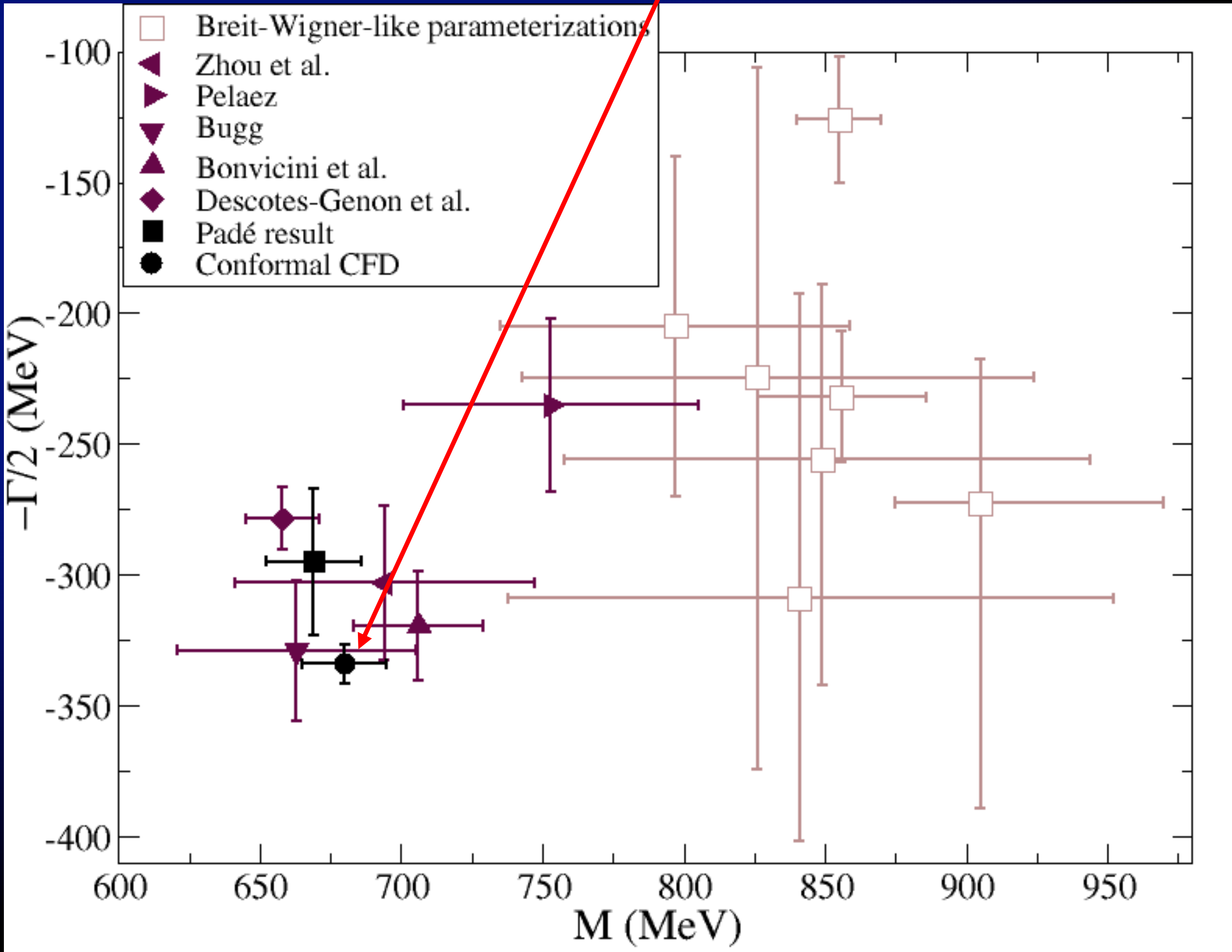
Regge parameterizations allowed to vary: Only πK-ρ residue changes by 1.4 deviations

Kappa pole from CFD

1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

Fantastic analyticity properties,
but not model independent

(680±15)-i(334±7.5) MeV



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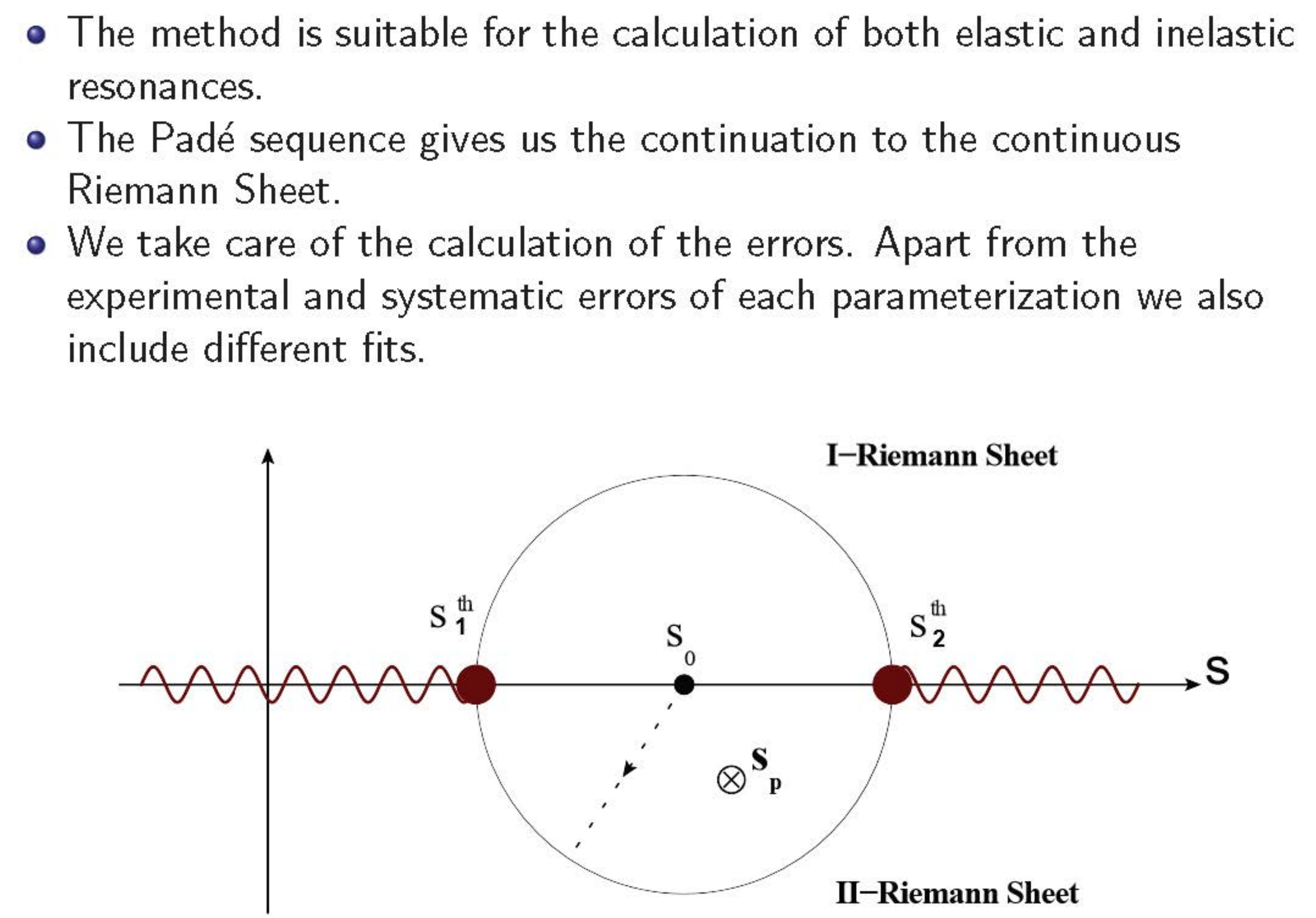
JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Strange resonance poles from CFD: Using Padé sequences

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

Almost model independent: Does not assume any particular functional form
(but local determination)

Based on previous works by P.Masjuan, J.J. Sanz Cillero, I. Caprini, J.Ruiz de ELvira



- The method is suitable for the calculation of both elastic and inelastic resonances.
- The Padé sequence gives us the continuation to the continuous Riemann Sheet.
- We take care of the calculation of the errors. Apart from the experimental and systematic errors of each parameterization we also include different fits.

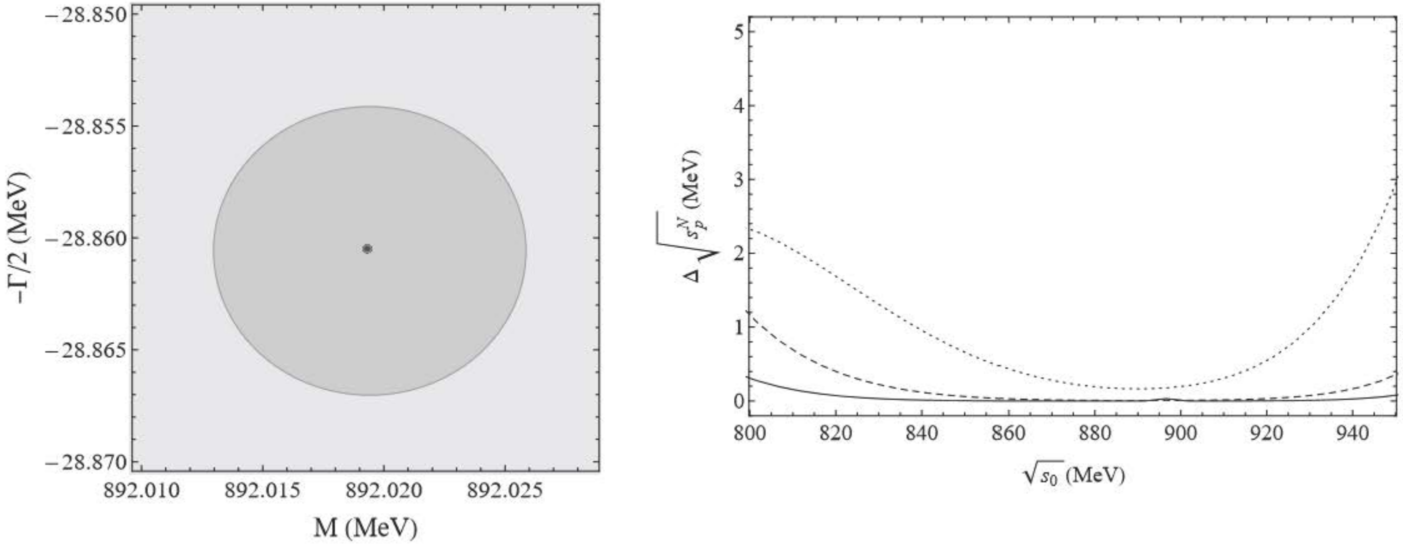
Strange resonance poles from CFD: Using Padé sequences

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

Almost model independent: Does not assume any particular functional form (but local determination) **CAN BE USED FOR INELASTIC RESONANCES TOO**

Based on previous works by P.Masjuan, J.J. Sanz Cillero, I. Caprini, J.Ruiz de ELvira

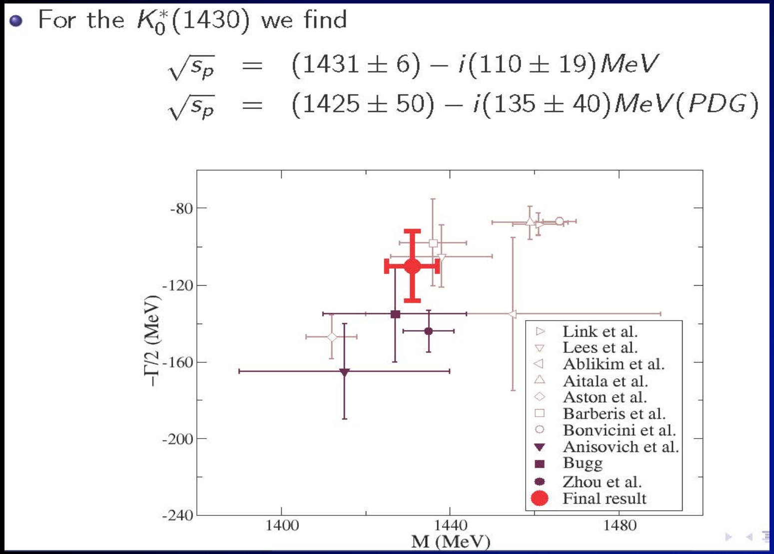
- For every fit we search the s_0 thats gives the minimum difference for the truncation of the sequence.
- We stop at a N ($N+1$ derivatives) where the systematic uncertainty is smaller than the statistical one (usually $N=4$ is enough).
- Run a montecarlo for every fit to calculate the parameters an errors of every resonance.



Strange resonance poles from CFD: Using Padé sequences

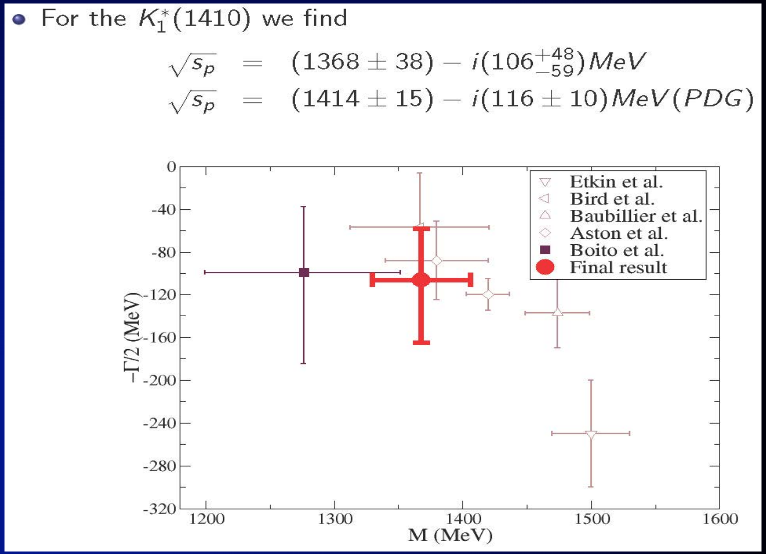
JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017)

The method can be used for inelastic resonances too. Provides resonance parameters WITHOUT ASSUMING SPECIFIC FUNCTIONAL FORM



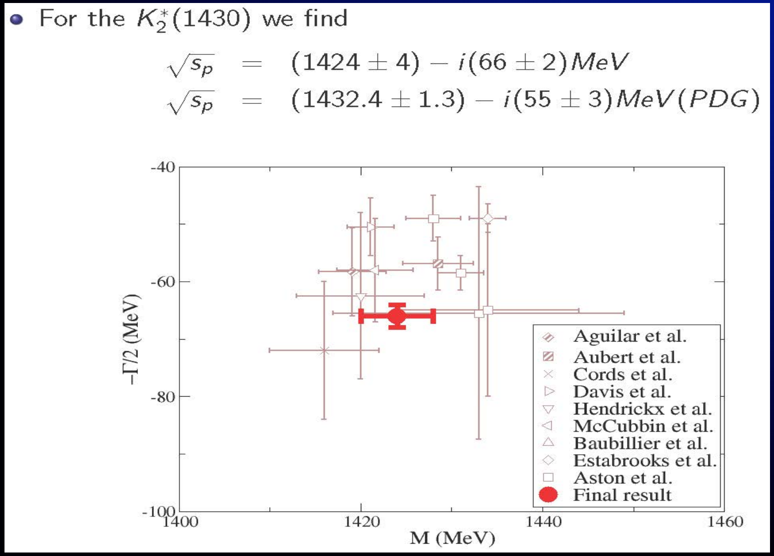
- For the $K_0^*(1430)$ we find

$$\sqrt{s_p} = (1431 \pm 6) - i(110 \pm 19) MeV$$
$$\sqrt{s_p} = (1425 \pm 50) - i(135 \pm 40) MeV(PDG)$$



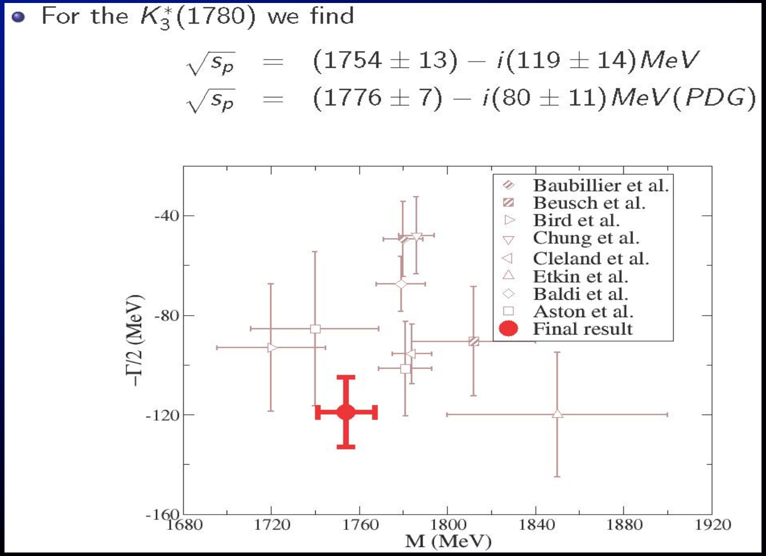
- For the $K_1^*(1410)$ we find

$$\sqrt{s_p} = (1368 \pm 38) - i(106^{+48}_{-59}) MeV$$
$$\sqrt{s_p} = (1414 \pm 15) - i(116 \pm 10) MeV(PDG)$$



- For the $K_2^*(1430)$ we find

$$\sqrt{s_p} = (1424 \pm 4) - i(66 \pm 2) MeV$$
$$\sqrt{s_p} = (1432.4 \pm 1.3) - i(55 \pm 3) MeV(PDG)$$



- For the $K_3^*(1780)$ we find

$$\sqrt{s_p} = (1754 \pm 13) - i(119 \pm 14) MeV$$
$$\sqrt{s_p} = (1776 \pm 7) - i(80 \pm 11) MeV(PDG)$$

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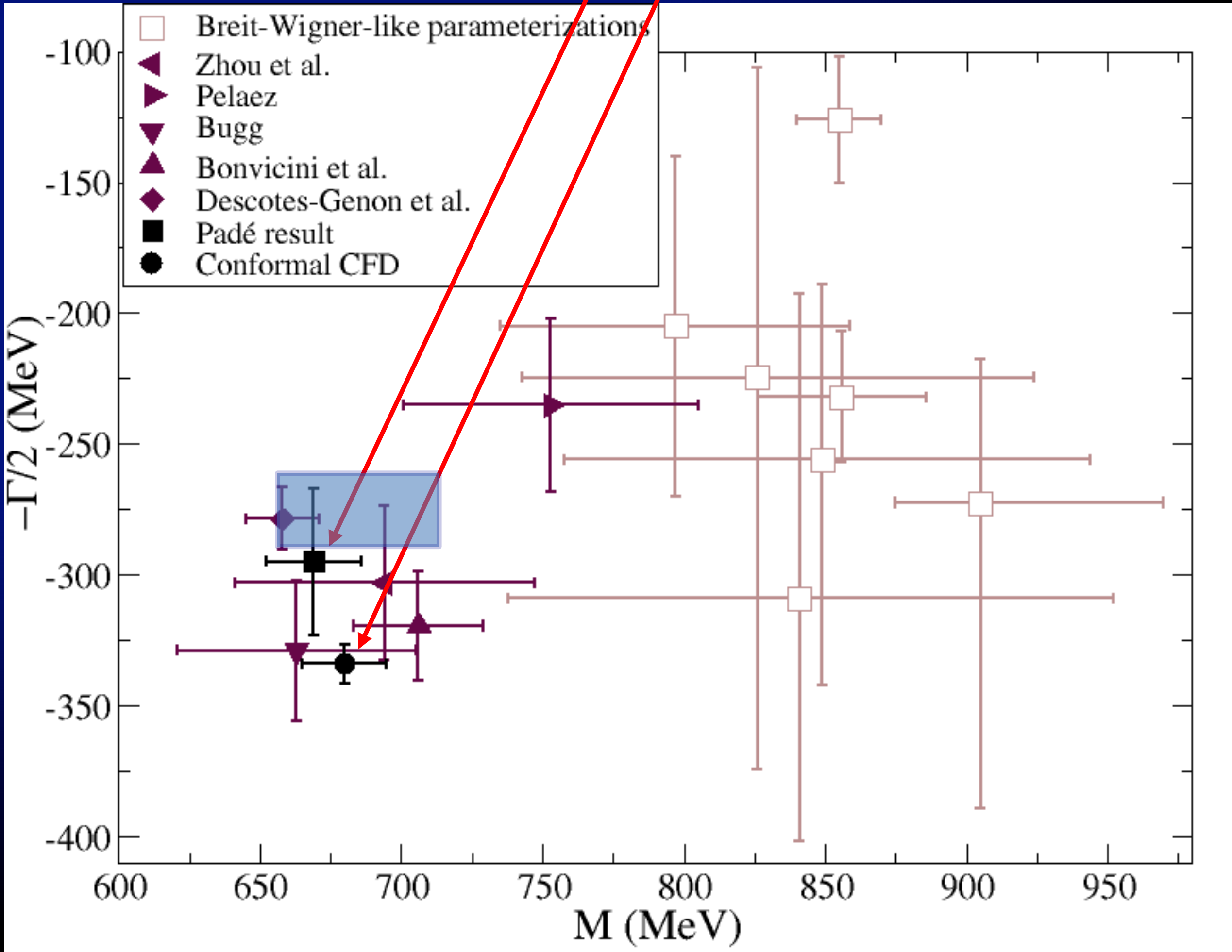
Fantastic analyticity properties,
but not model independent

(680±15)-i(334±7.5) MeV

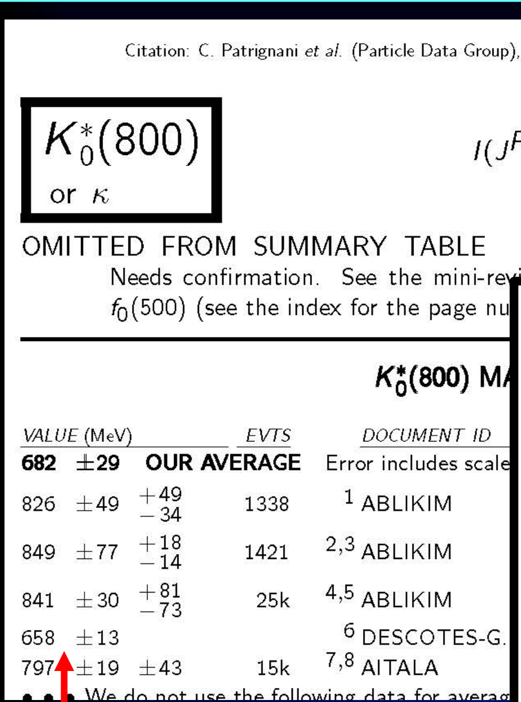
2) Using Padé Sequences…
JRP, A. Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

(670±18)-i(295± 28) MeV

Compare to PDG2017:
(682±29)-i(273±12) MeV



The resonance is NO LONGER the κ nor the $K_0^*K_0*(800)$



Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

$K_0^*(800)$
or κ

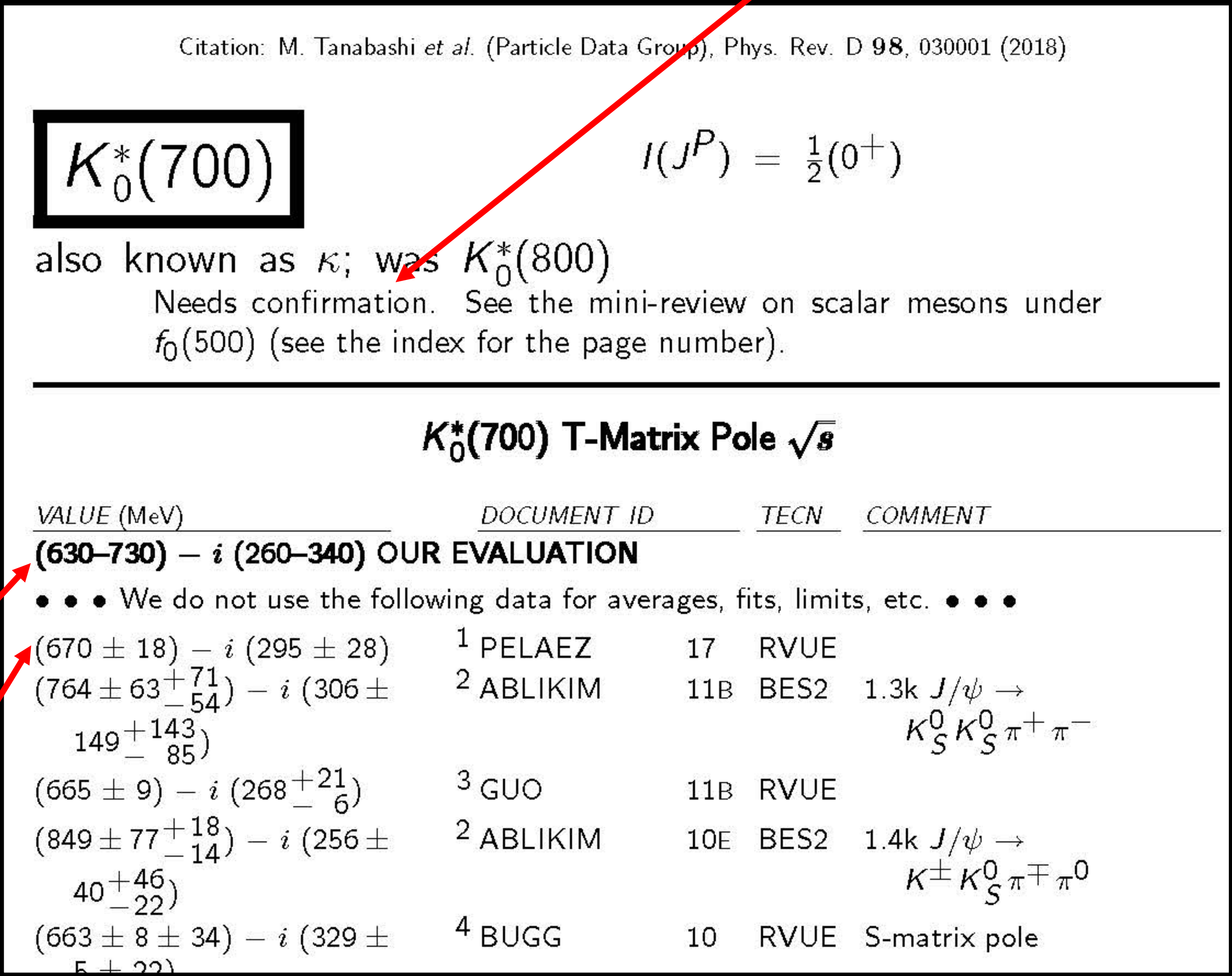
$I(J^P) = \frac{1}{2}(0^+)$

OMITTED FROM SUMMARY TABLE
Needs confirmation. See the mini-review on scalar mesons under
$f_0(500)$ (see the index for the page nu

$K_0^*(800)$ MA

VALUE (MeV)			*EVTS*	*DOCUMENT ID*
682	**±29**	**OUR AVERAGE**		Error includes scale
826	±49	$^{+49}_{-34}$	1338	[1] ABLIKIM
849	±77	$^{+18}_{-14}$	1421	[2,3] ABLIKIM
841	±30	$^{+81}_{-73}$	25k	[4,5] ABLIKIM
658	±13			[6] DESCOTES-G.
797	±19	±43	15k	[7,8] AITALA

• • • We do not use the following data for averag



Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

$K_0^*(700)$

$I(J^P) = \frac{1}{2}(0^+)$

also known as κ; was $K_0^*(800)$
Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$K_0^*(700)$ T-Matrix Pole $\sqrt{s}$

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(630–730) − *i* (260–340) OUR EVALUATION			
• • • We do not use the following data for averages, fits, limits, etc. • • •			
$(670 \pm 18) - i\,(295 \pm 28)$	[1] PELAEZ 17	RVUE	
$(764 \pm 63^{+71}_{-54}) - i\,(306 \pm 149^{+143}_{-85})$	[2] ABLIKIM 11B	BES2	1.3k $J/\psi \to K^0_S K^0_S \pi^+ \pi^-$
$(665 \pm 9) - i\,(268^{+21}_{-6})$	[3] GUO 11B	RVUE	
$(849 \pm 77^{+18}_{-14}) - i\,(256 \pm 40^{+46}_{-22})$	[2] ABLIKIM 10E	BES2	1.4k $J/\psi \to K^\pm K^0_S \pi^\mp \pi^0$
$(663 \pm 8 \pm 34) - i\,(329 \pm 5 \pm 22)$	[4] BUGG 10	RVUE	S-matrix pole

But Still “Needs Confirmation” !

Best analysis so far: Roy-Steiner dispersion relations

Plenty of room for improvement on parameters

Our Pade sequences

Kappa pole analytic determinations from constrained fits

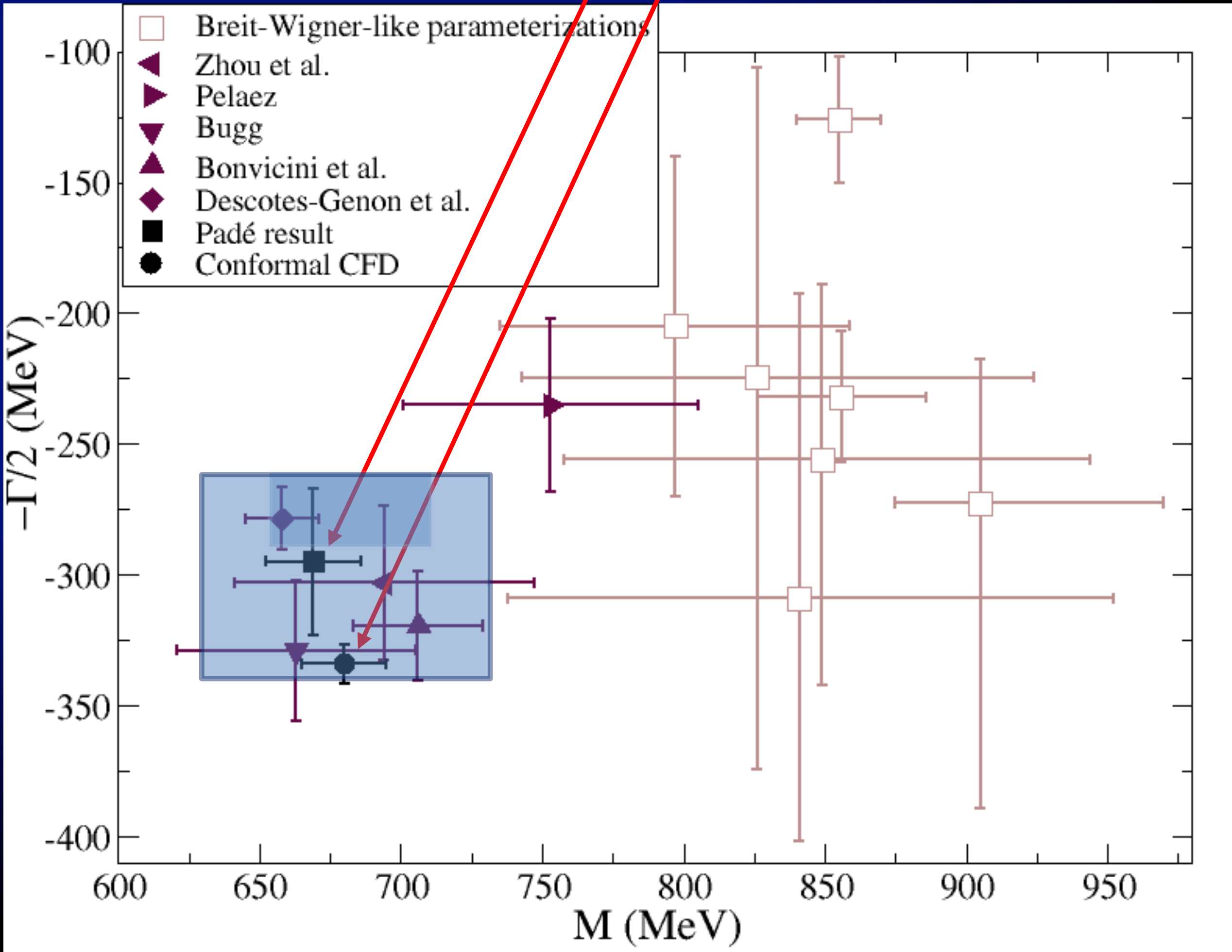
1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

Fantastic analyticity properties,
but not model independent

(680±15)-i(334±7.5) MeV

2) Using Padé Sequences…
JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

(670±18)-i(295± 28) MeV



Compare to PDG2017:
(682±29)-i(273±12) MeV

New PDG2018:
(630-730)-i(260-340) MeV
And name changed
K_0^*(700)
Still "Needs Confirmation"

Our Dispersive/Analytic Approach for πK and strange resonances

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

- As πK checks: Small inconsistencies.
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  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)
- Analytic methods to extract poles: reduced model dependence on strange resonances
  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

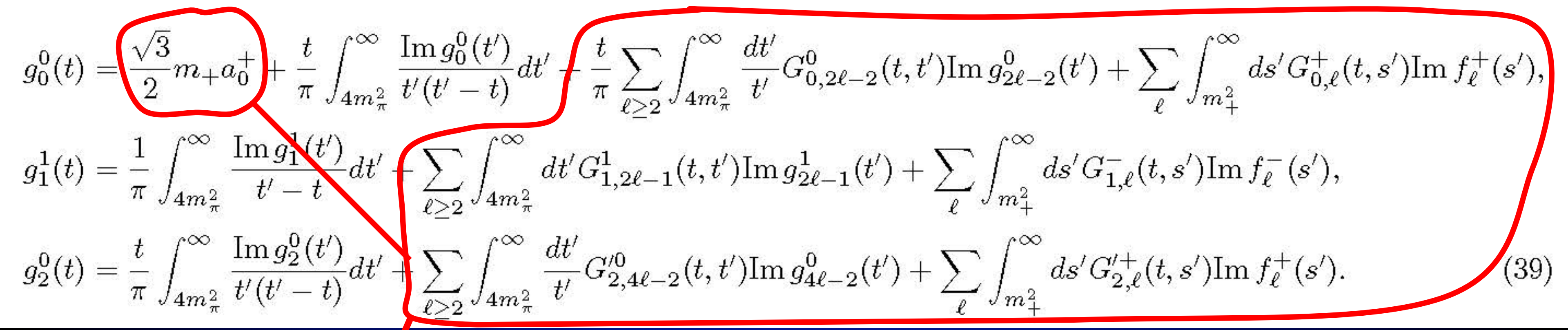
- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits up to 1.5 GeV
  JRP, A.Rodas, Eur.Phys.J. C78 (2018)

ππ→KK HDR

g^I_J =ππ → KK partial waves. We study (I,J)=(0,0),(1,1),(0,2)

f^I_J = Kπ → Kπ partial waves. Taken from previous dispersive study

JRP, A. Rodas PRD 2016



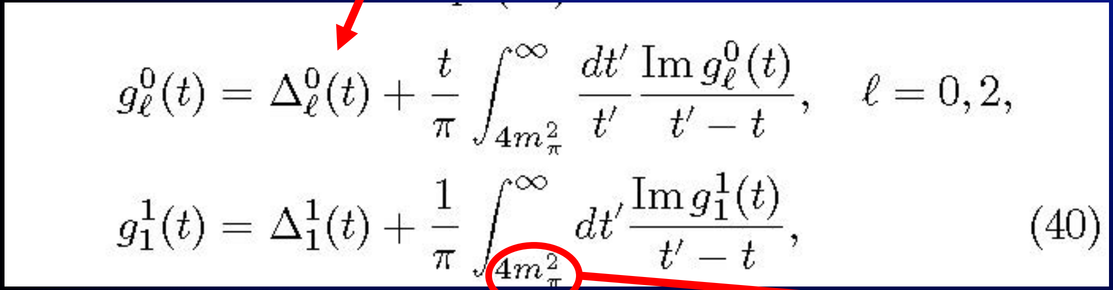
$$g_0^0(t) = \frac{\sqrt{3}}{2} m_+ a_0^+ + \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_0^0(t')}{t'(t'-t)}dt' + \frac{t}{\pi}\sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'} G^0_{0,2\ell-2}(t,t')\mathrm{Im}\, g^0_{2\ell-2}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G^+_{0,\ell}(t,s')\mathrm{Im}\, f^+_\ell(s'),$$

$$g_1^1(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_1^1(t')}{t'-t}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} dt' G^1_{1,2\ell-1}(t,t')\mathrm{Im}\, g^1_{2\ell-1}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G^-_{1,\ell}(t,s')\mathrm{Im}\, f^-_\ell(s'),$$

$$g_2^0(t) = \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, g_2^0(t')}{t'(t'-t)}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'} G'^0_{2,4\ell-2}(t,t')\mathrm{Im}\, g^0_{4\ell-2}(t') + \sum_{\ell}\int_{m_+^2}^{\infty} ds' G'^+_{2,\ell}(t,s')\mathrm{Im}\, f^+_\ell(s'). \qquad (39)$$

$G^I_{J,J'}$(t,t') =integral kernels, depend on a parameter

Lowest # of subtractions. Odd pw decouple from even pw.



$$g_\ell^0(t) = \Delta_\ell^0(t) + \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{dt'}{t'}\frac{\mathrm{Im}\, g_\ell^0(t)}{t'-t}, \qquad \ell = 0, 2,$$

$$g_1^1(t) = \Delta_1^1(t) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} dt' \frac{\mathrm{Im}\, g_1^1(t)}{t'-t}, \qquad (40)$$

Δ(t) depend on higher waves or on Kπ→Kπ.

Integrals from 2π threshold !

Solve in descending J order

We have used models for higher waves, but give very small contributions

ππ→KK HDR

For unphysical region below KK threshold, we used Omnés function

$$\Omega^I_\ell(t) = \exp\left(\frac{t}{\pi}\int_{4m_\pi^2}^{t_m}\frac{\phi^I_\ell(t')dt'}{t'(t'-t)}\right),$$

$$\Omega^I_\ell(t) \equiv \Omega^I_{l,R}(t)e^{i\phi^I_\ell(t)\theta(t-4m_\pi^2)\theta(t_m-t)},$$

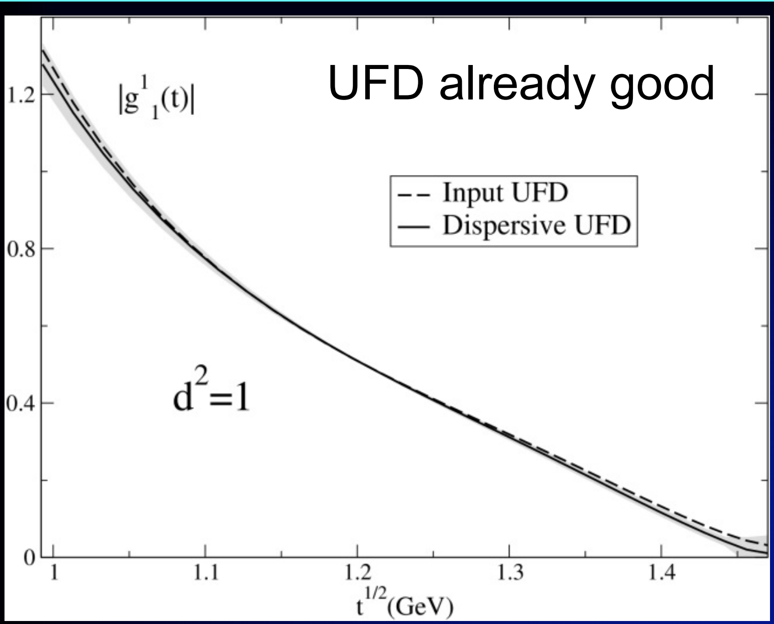
This is the form of our HDR: Roy-Steiner+Omnés formalism

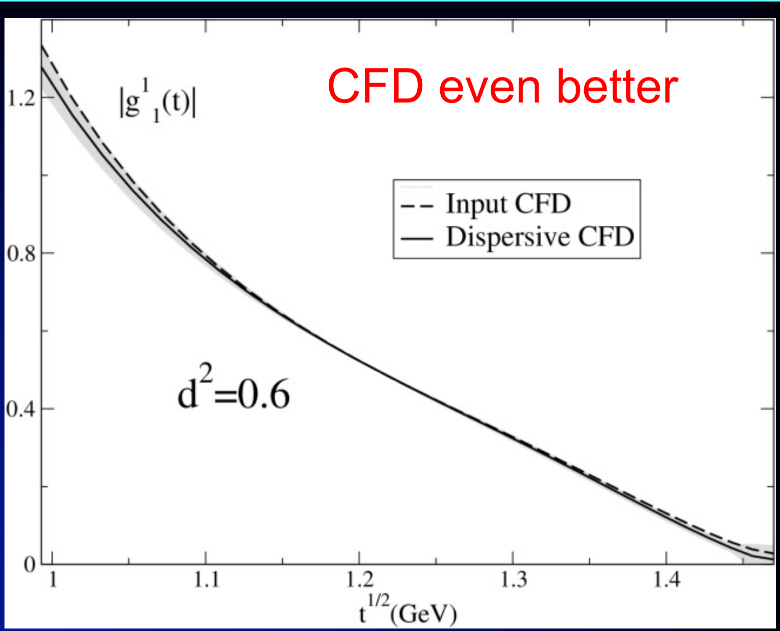
$$\begin{aligned}
g^0_0(t) &= \Delta^0_0(t) + \frac{t\Omega^0_0(t)}{t_m - t}\left[\alpha + \frac{t}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{(t_m - t')\Delta^0_0(t')\sin\phi^0_0(t')}{\Omega^0_{0,R}(t')t'^2(t'-t)} + \frac{t}{\pi}\int_{t_m}^{\infty} dt' \frac{(t_m - t')|g^0_0(t')|\sin\phi^0_0(t')}{\Omega^0_{0,R}(t')t'^2(t'-t)}\right] \\
g^1_1(t) &= \Delta^1_1(t) + \Omega^1_1(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta^1_1(t')\sin\phi^1_1(t')}{\Omega^1_{1,R}(t')(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g^1_1(t')|\sin\phi^1_1(t')}{\Omega^1_{1,R}(t')(t'-t)}\right], \\
g^0_2(t) &= \Delta^0_2(t) + t\Omega^0_2(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta^0_2(t')\sin\phi^0_2(t')}{\Omega^0_{2,R}(t')t'(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g^0_2(t')|\sin\phi^0_2(t')}{\Omega^0_{2,R}(t')t'(t'-t)}\right].
\end{aligned}$$

We can now check how well these HDR are satisfied

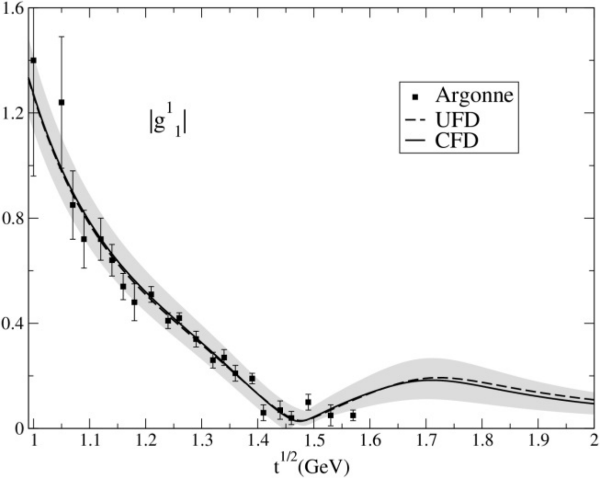
ππ→KK Hiperbolic Dispersion Relations I=1,J=1, UFD vs.CFD

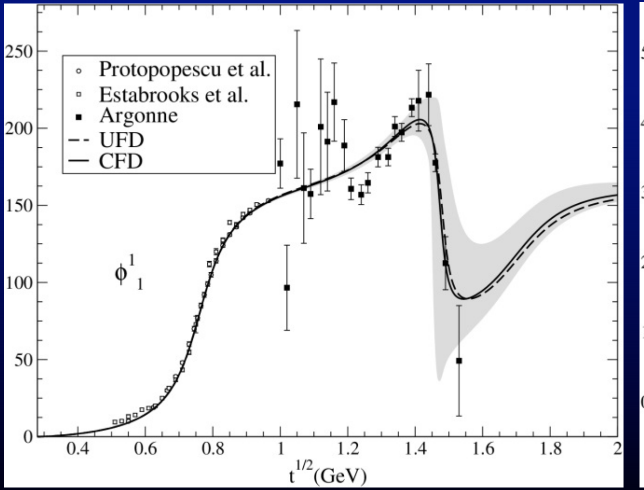
JRP, A.Rodas, Eur.Phys.J. C78 (2018)

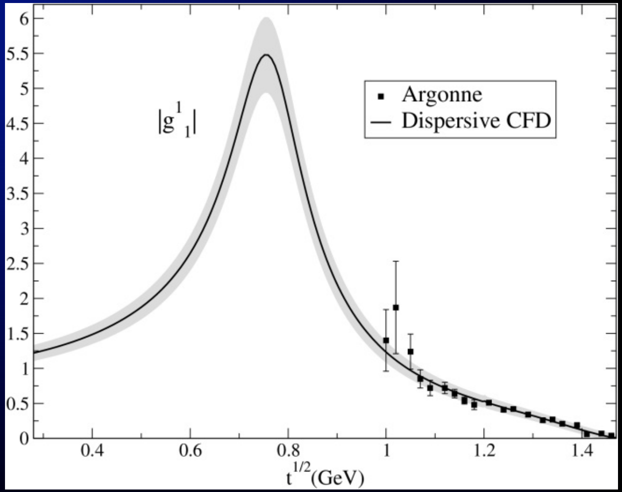

UFD already good


CFD even better

Requires almost imperceptible change from UFD to CFD

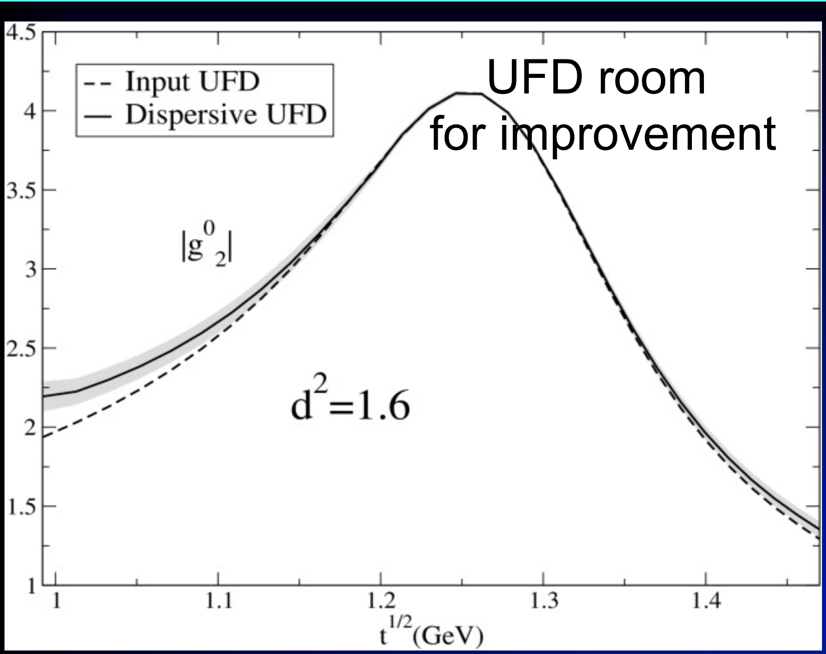




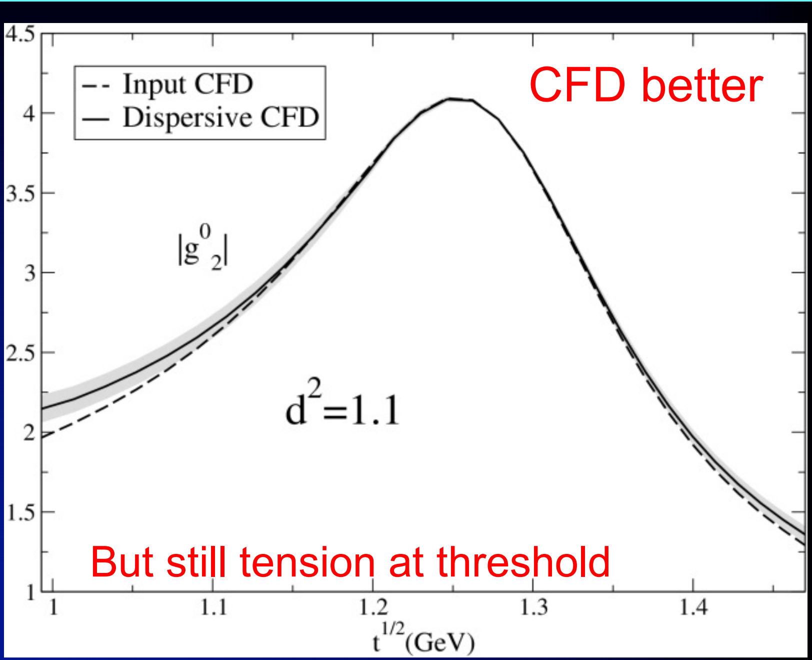


ππ→KK Hiperbolic Dispersion Relations I=2,J=2, UFD vs. CFD

JRP, A.Rodas, Eur.Phys.J. C78 (2018)



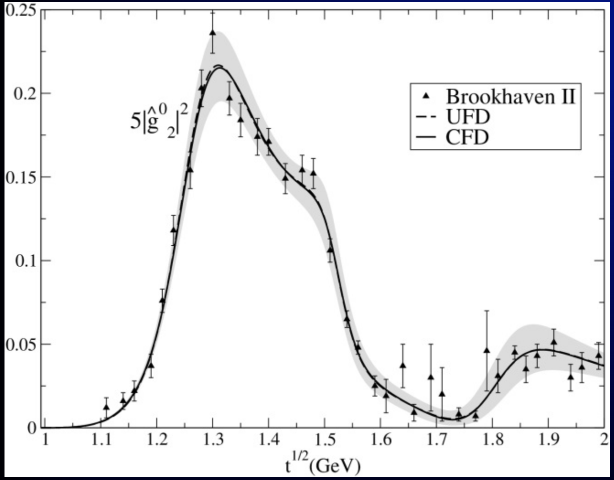
UFD room for improvement

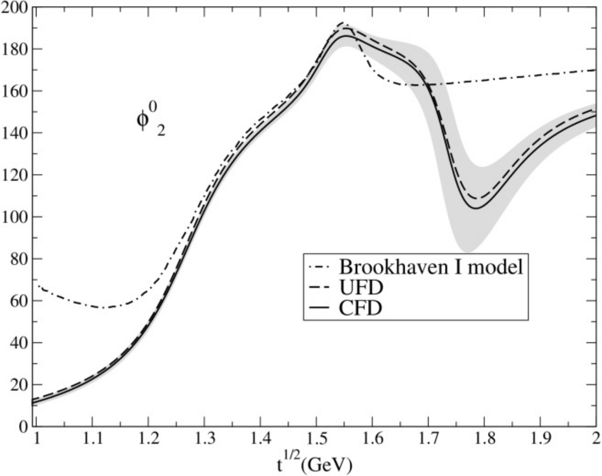


CFD better

But still tension at threshold

Very small change from UFD to CFD. Only significant at threshold and high energies





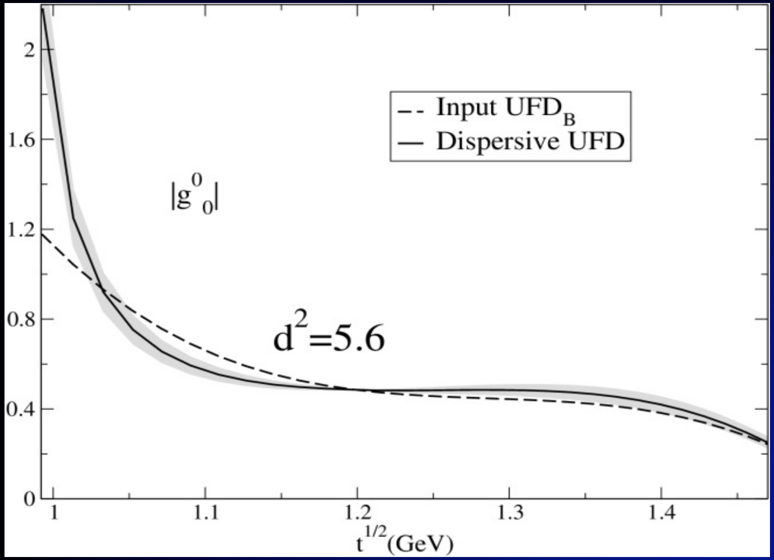
Other parameterizations (BW…), worse.

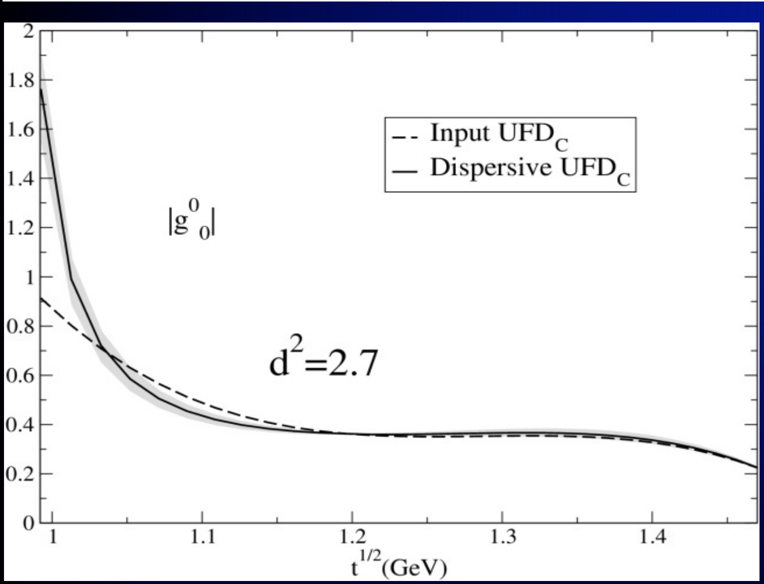
ππ→KK Hiperbolic Dispersion Relations I=0,J=0, UFD vs. CFD

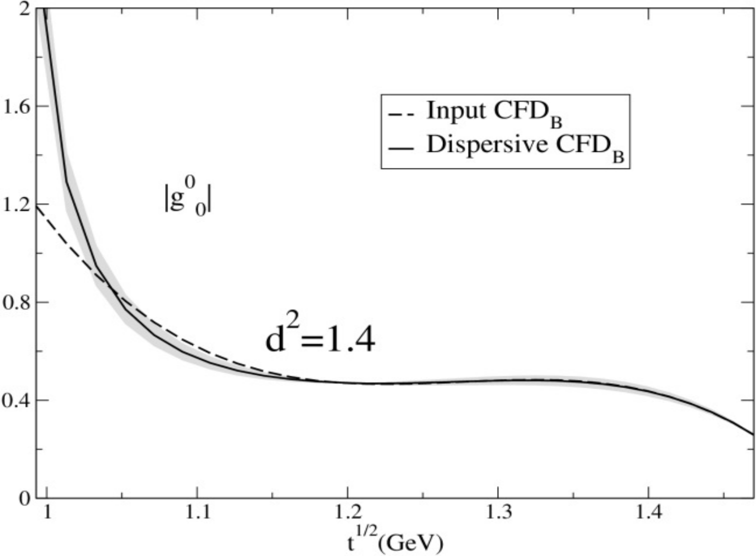
JRP, A.Rodas, Eur.Phys.J. C78 (2018)

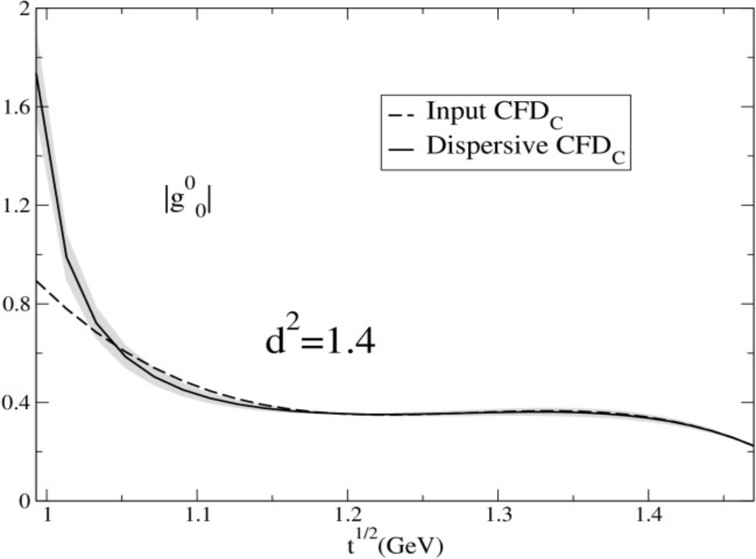
Two possible sets of data

We use I=0,J=2 CFD as input.







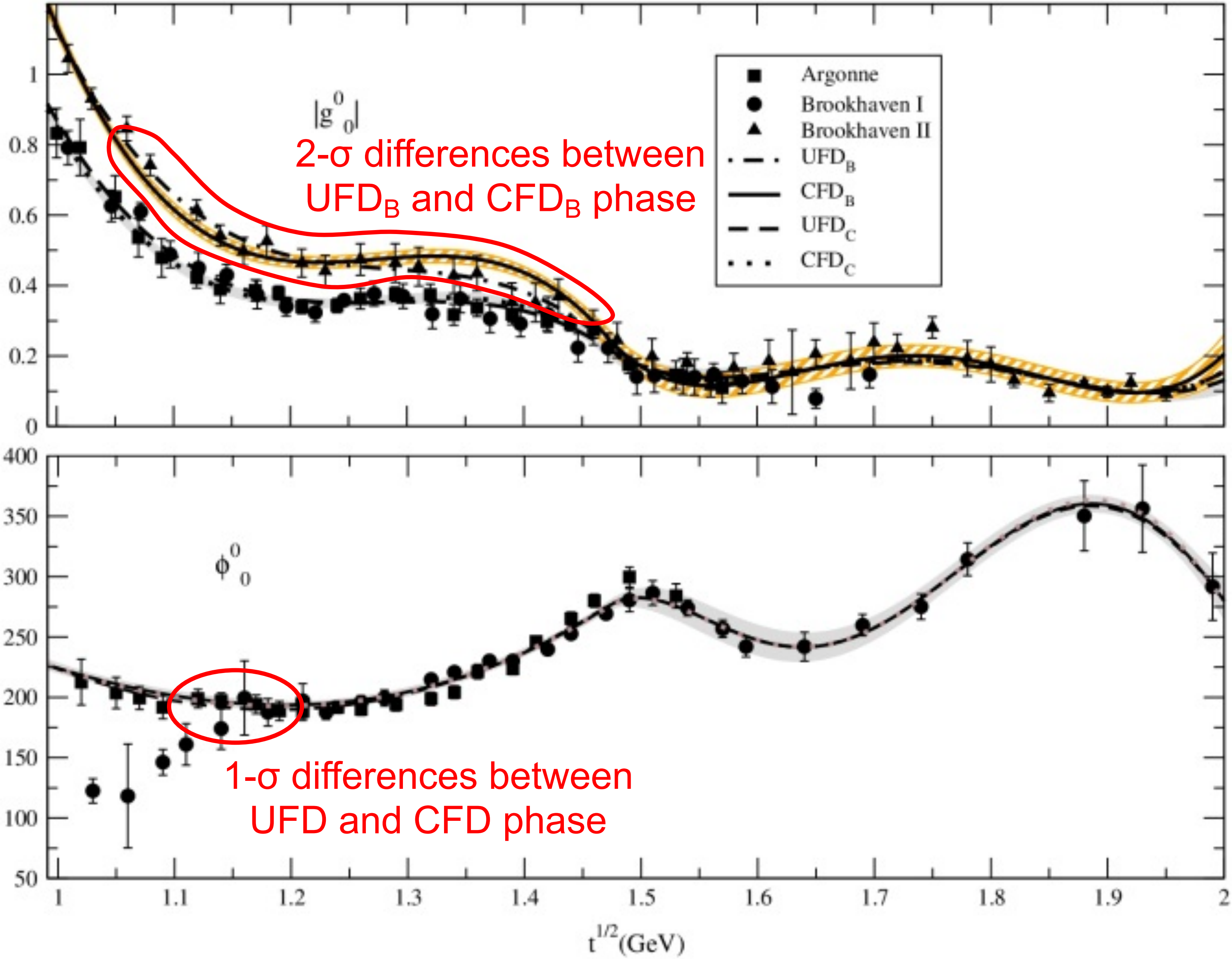


Remarkable improvement from UFD to CFD, except at threshold.
Both data sets equally acceptable now.

$\pi\pi \to KK$ Hiperbolic Dispersion Relations   I=0,J=0, CFD

JRP, A.Rodas, Eur.Phys.J. C78 (2018)

Some 2-σ level differences between UFD_B and CFD_B between 1.05 and 1.45 GeV
CFD_C consistent within 1-σ band of UFD_C


2-σ differences between UFD_B and CFD_B phase

1-σ differences between UFD and CFD phase

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Need ππ→KK to rewrite left cut. Range optimized.

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- From fixed-t DR:
  ππ→KK influence small.
  K_0^*κ/ $^*_0$K$_0$*(700) out of reach
- From Hyperbolic DR:
  ππ→KK influence important.
  JRP, A.Rodas, in progress. PRELIMINARY results shown here

- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV

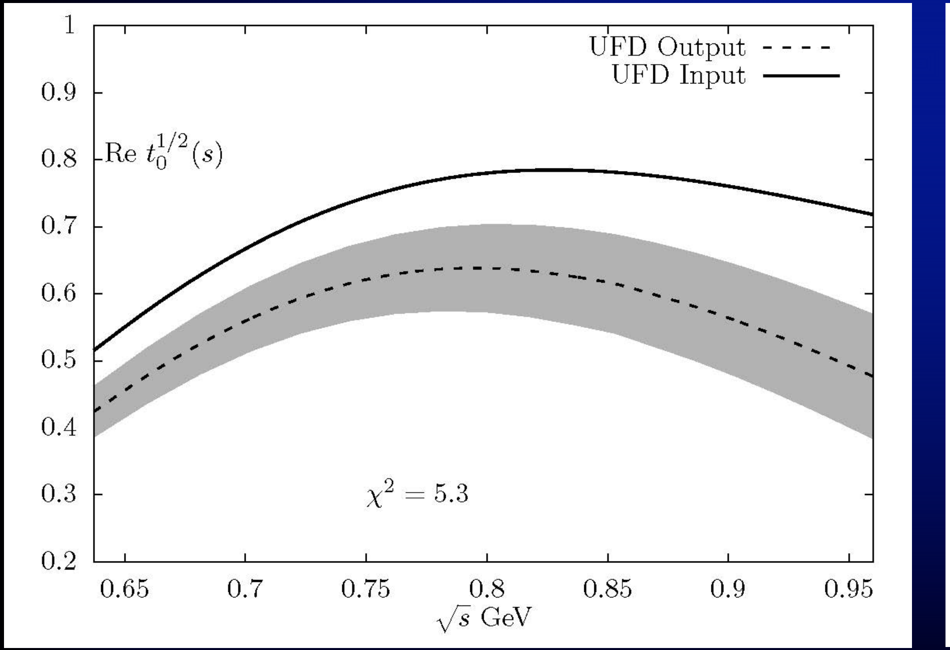
πK Hiperbolic Dispersion Relations I=1/2, J=0

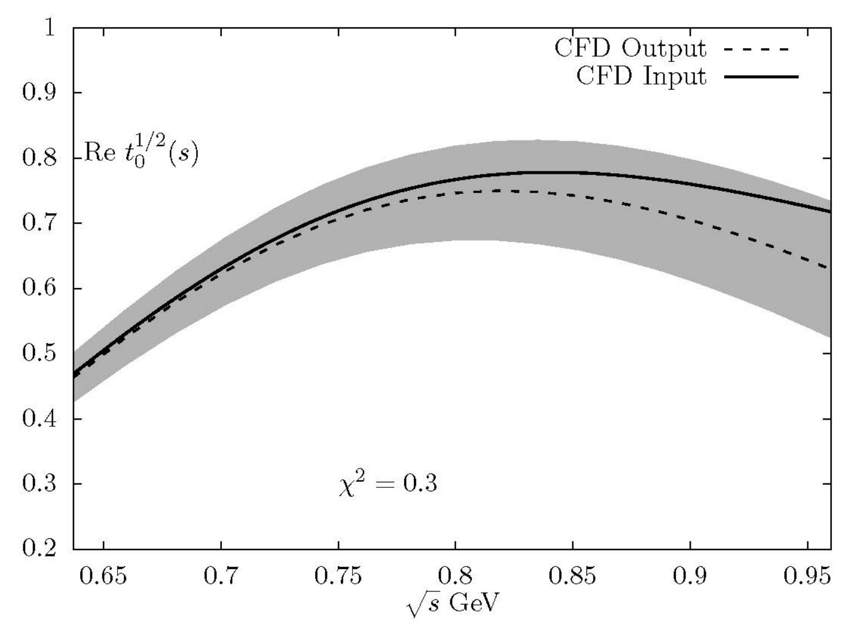
Preliminary!!

LARGE inconsistencies of unconstrained fits with the minimal number of subtractions (shown here)

Fairly consistent with one more subtraction for F^-

Consistent within uncertainties if we use the DR as constraints



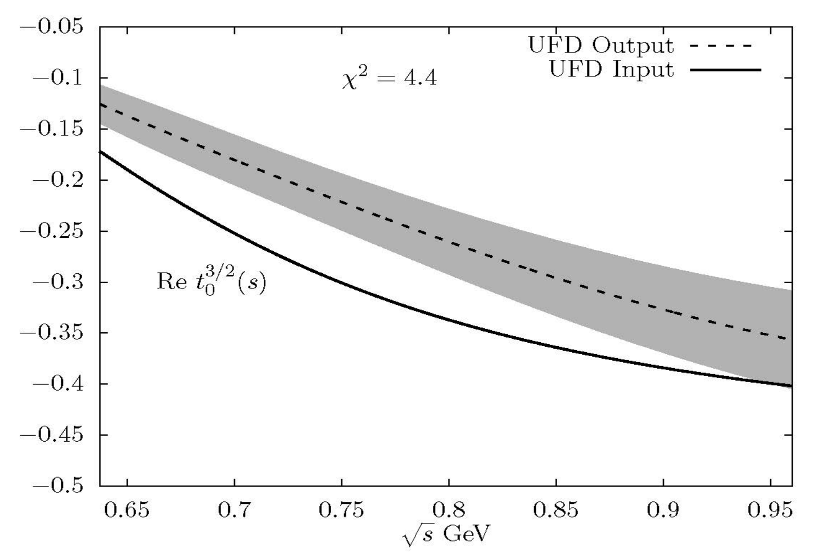


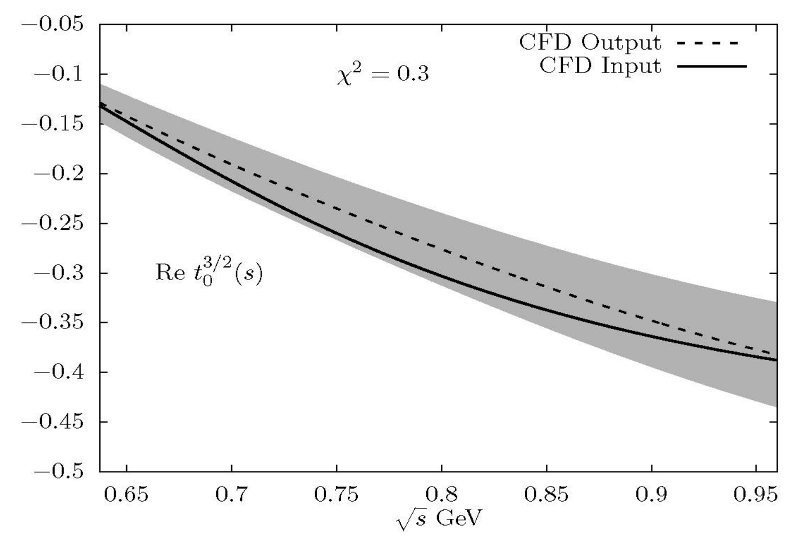
πK Hiperbolic Dispersion Relations I=3/2, J=0 and I=1/2, J=0

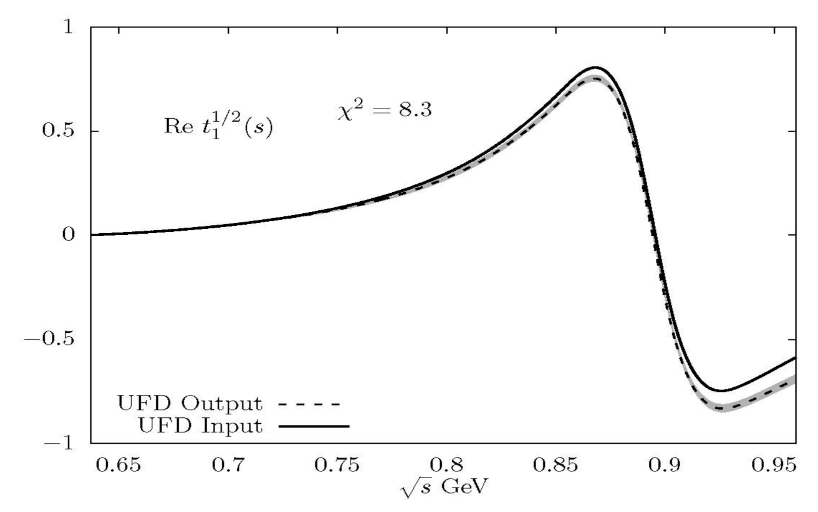
Preliminary!!

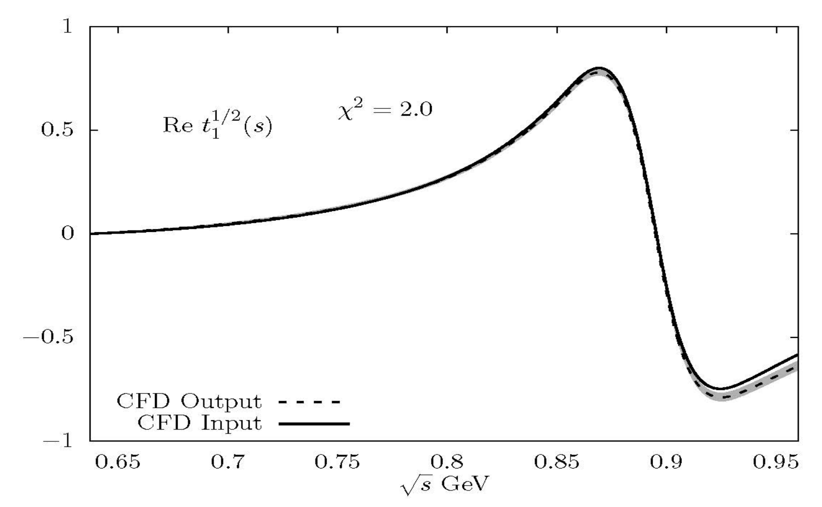
SIZABLE inconsistencies of unconstrained fits with the minimal number of subtractions (shown here). Fairly consistent with one more subtraction for F^-

Made consistent within uncertainties when we use the DR as constraints





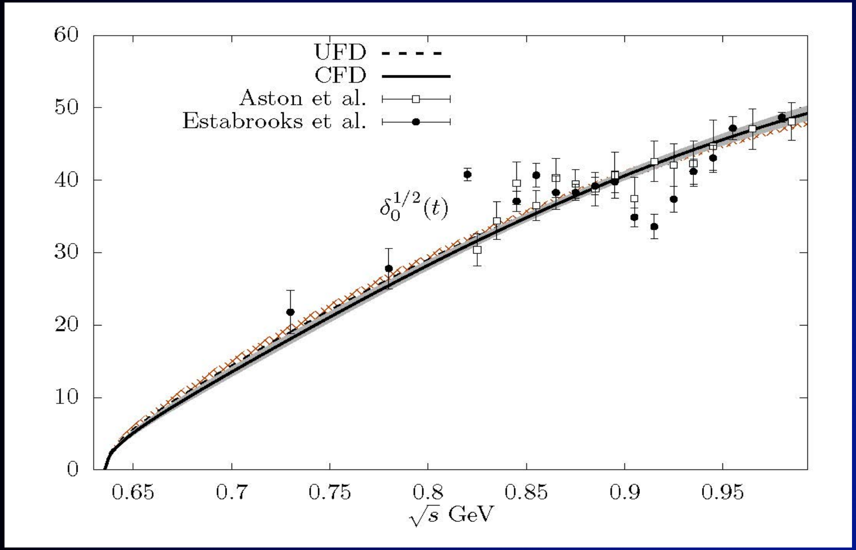


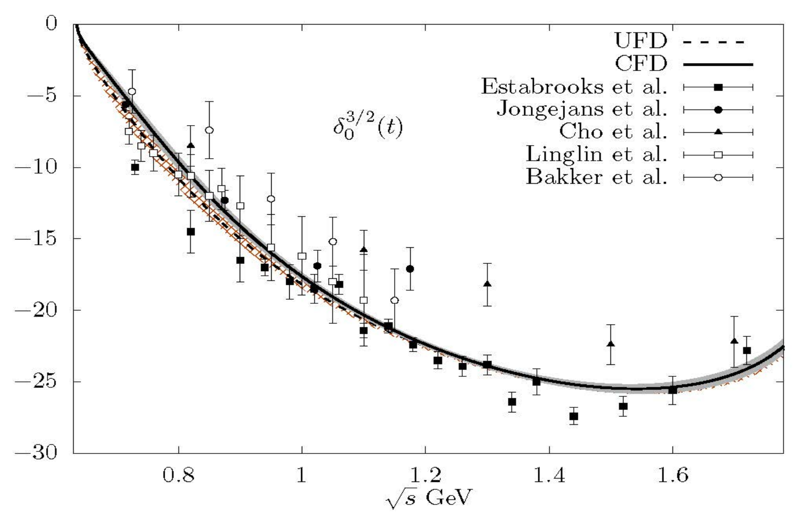


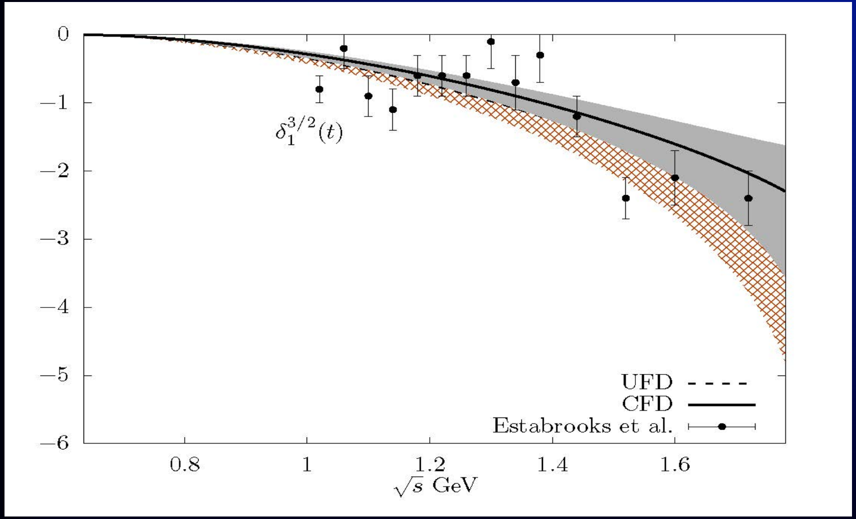
πK CFD vs. UFD

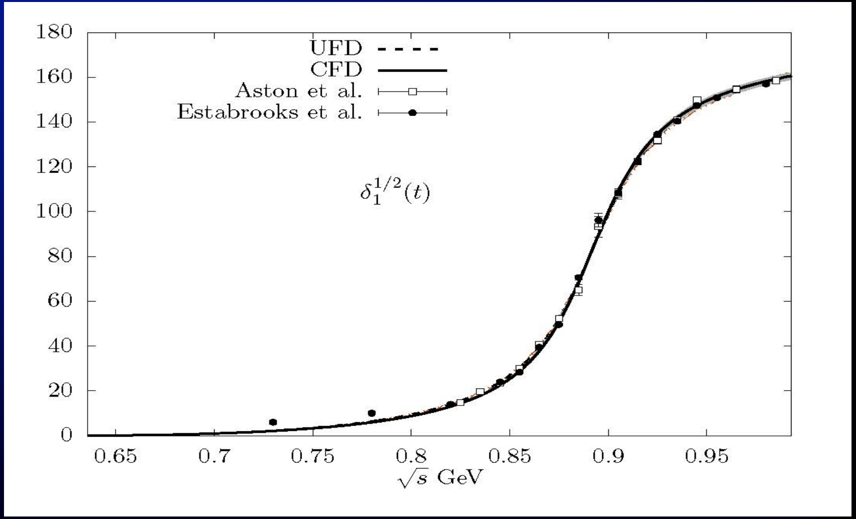
Preliminary!!

Constrained parameterizations suffer minor changes but still describe πK data fairly well. Here we compare the unconstrained fits (UFD) versus the constrained ones (CFD)









The "unphysical" rho peak in ππ→KK grows by 10% from UFD to CFD

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  JRP, A.Rodas, Eur.Phys.J. C78 (2018)

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- From Hyperbolic DR:
  ππ→KK influence important.

- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV
- **Rigorous κ/ $\overset{*}{}_0$K₀*(700) pole** JRP, A.Rodas, in progress. PRELIMINARY results shown here

Dispersive pole analysis from constrained fit to data JRP, A. Rodas, in preparation

Preliminary!!

Recall Roy-Steiner **SOLUTION** from Paris group (658±13)-i(278.5±12) MeV

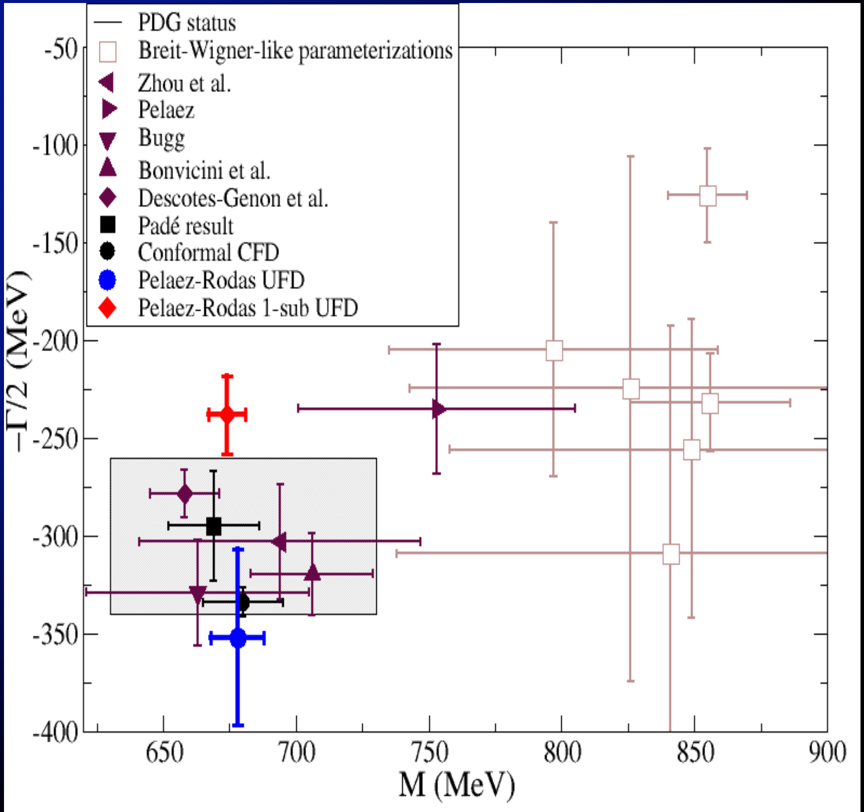
Decotes-Genon-Moussallam 2006

Before imposing Roy Eqs.
incompatible results with
different# of subtractions

Now we have:

- Constrained **FIT TO DATA** (not solution but fit)
- Improved P-wave (consistent with data)
- Realistic ππ→KK uncertainties (none before)
- Improved Pomeron

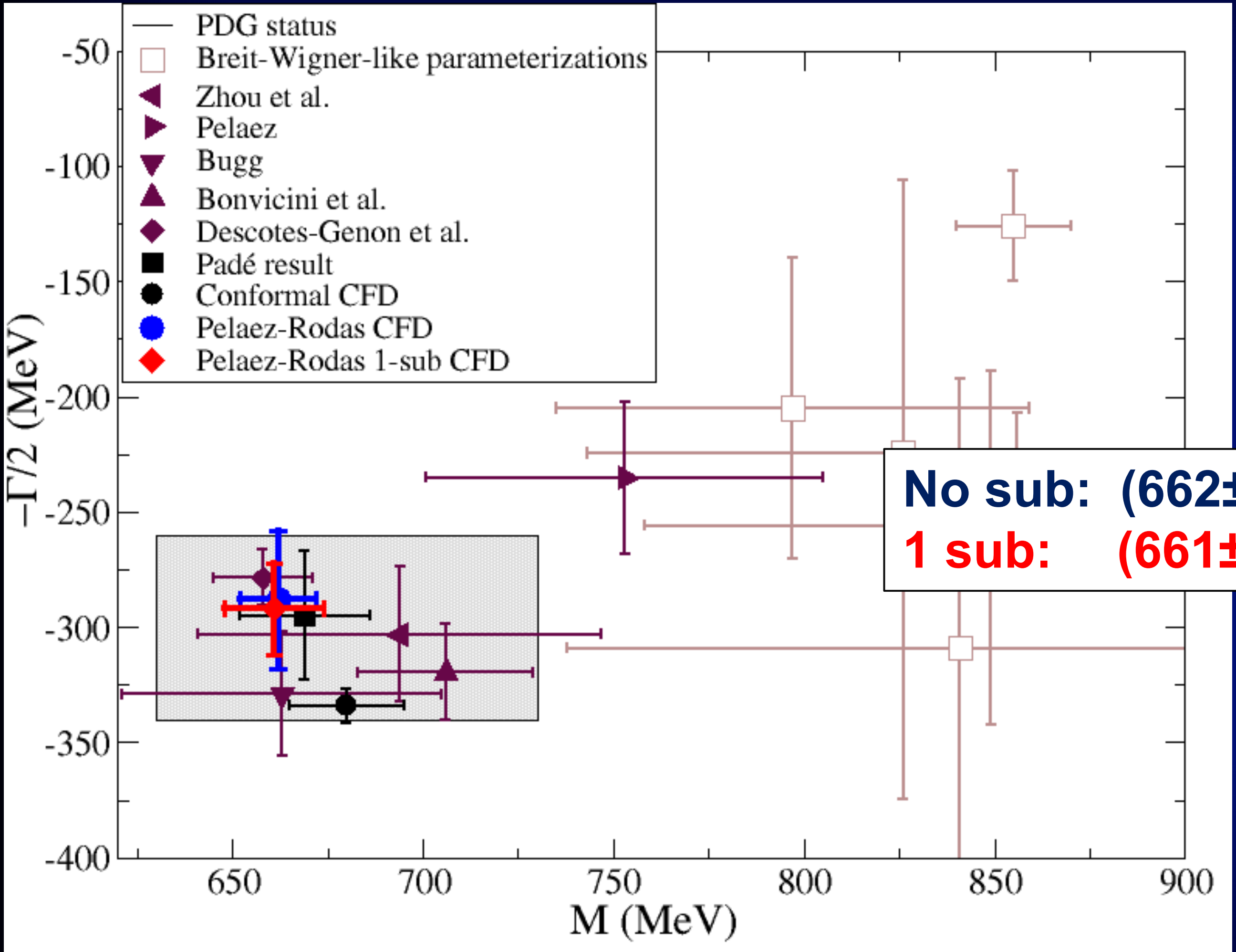
- Constrained ππ→KK input with DR
- FDR up to 1.6 GeV
- Fixed-t Roy-Steiner Eqs.
- Hyperbolic Roy Steiner Eqs.
  both in real axis (not before)
  and complex plane
- Both one and no-subtraction for F- HDR
  (only the subtracted one before)



Dispersive pole analysis from constrained fit to data JRP, A. Rodas, in preparation

Preliminary!!

When using the constrained fit to data both poles come out nicely compatible



No sub: (662± 9)-i(288±31) MeV

1 sub: (661±13)-i(293±20) MeV

Compatible with Paris group

Decotes-Genon-Moussallam 2006

(658±13)-i(278.5±12) MeV

Summary

- The πK and π π →KK data do not satisfy well basic dispersive constraints

- Using dispersion relations as constraints we provide simple and consistent data parameterizations.

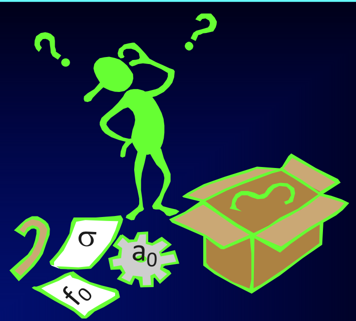
- Simple analytic methods of complex analysis can then reduce the model dependence in resonance parameter determinations.

- We are implementing partial-wave dispersion relations whose applicability range reaches the kappa pole. Our preliminary results confirm previous studies. We believe this resonance should be considered “well-established”, completing the nonet of lightest scalars.

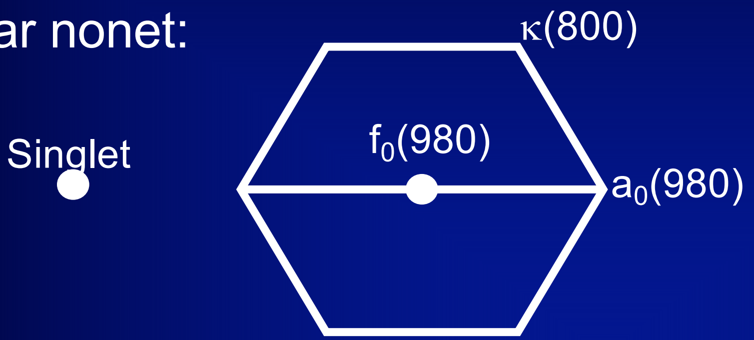
SPARE SLIDES

The light scalar controversy. The theory side... classification

- Scalar SU(3) multiplets identification controversial
  - Too many resonances for many years….
    But there is an emerging picture



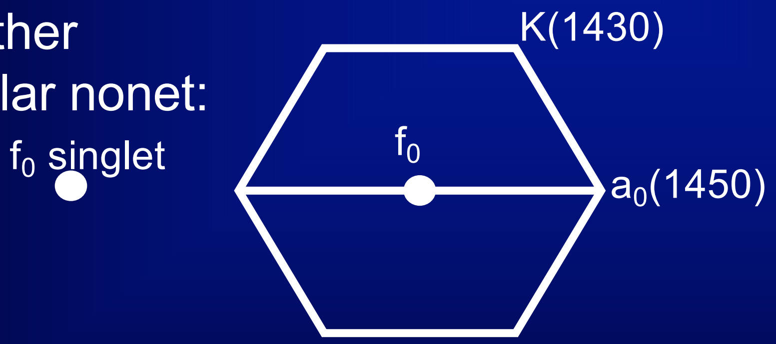
A Light scalar nonet:



Non-strange heavier!!
Inverted hierarchy problem
For quark-antiquark

f_0(500) and f_0(980) are
really octet/singlet mixtures

+ Another
heavier scalar nonet:



+ glueball

f_0 ●

Enough f_0 states have been observed: f_0(1370), f_0(1500), f_0(1700).
The whole picture is complicated by mixture between them (lots of works here)

Only the κ(800) or K0*(800) “Needs Confirmation” @ PDG

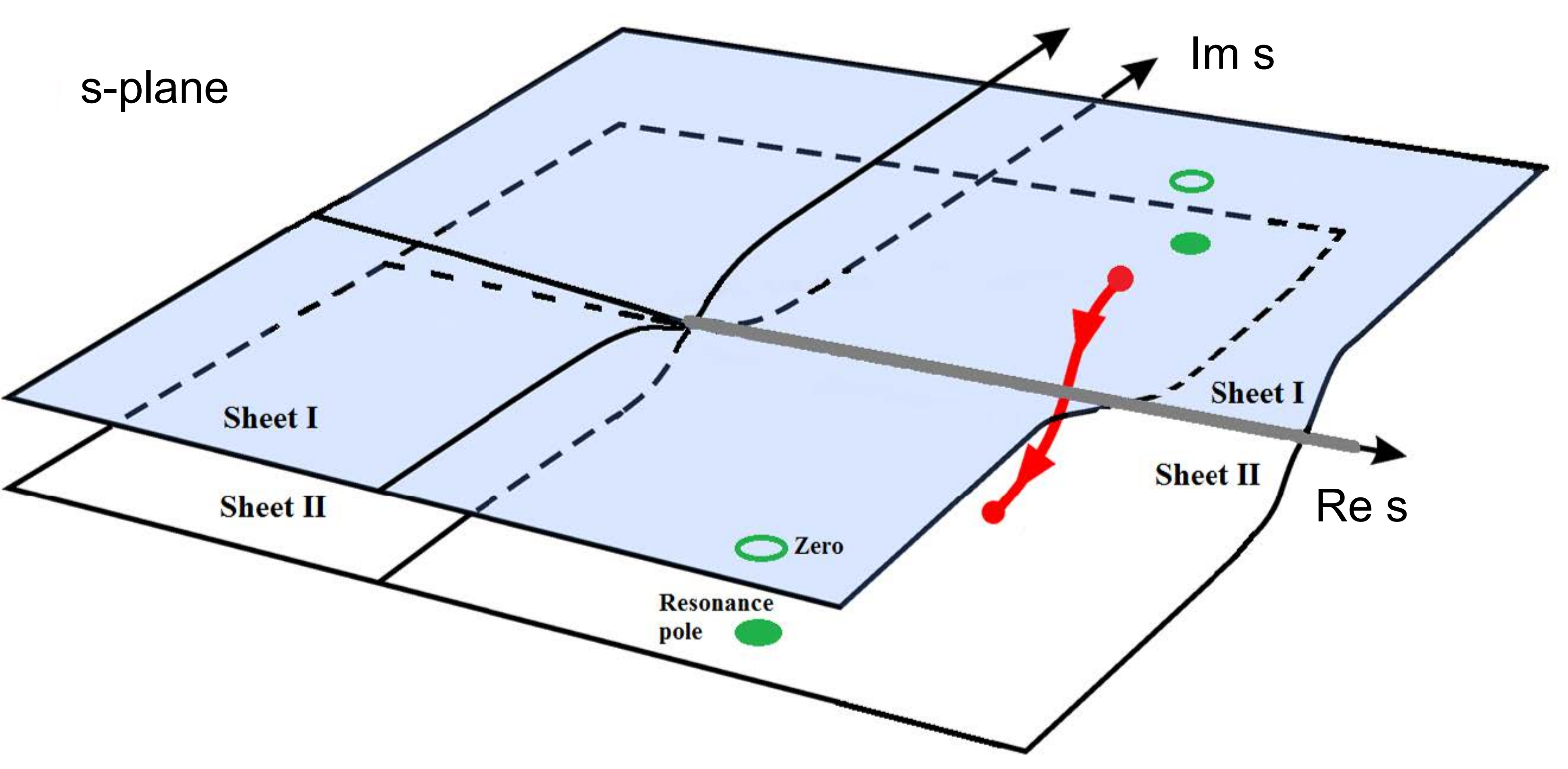
Resonances as poles

The Breit-Wigner shape is just an approximation for narrow and isolated resonances

The universal features of resonances are their
pole positions and residues *

$\sqrt{s_{pole}} \approx M\text{-}i\,\Gamma/2$

*in the Riemann sheet obtained from an analytic continuation through the physical cut



Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the s-variable, not in $\sqrt{s}$

Partial—wave vs. fixed-variable Dispersion Relations

So, we need to get rid of ONE VARIABLE to write CAUCHY THEOREM in terms of the other one

TWO MAIN APPROACHES

1) Integrate one variable and keep the other
(partial wave dispersion relations)

Partial Wave Dispersion Relations: General

- Analytic structure complicated if unequal masses (Circular cuts)

- For **elastic** region second Riemann sheet is easy to obtain.

Due to elastic unitarity:

$$S^{II}(s) = \frac{1}{S^{I}(s)}$$

Recalling $\mathrm{s}(s) = 1 + 2i\sigma\, t(s), \quad \sigma(s) = \frac{k}{2\sqrt{s}}$

The second sheet is then:

$$\boxed{t^{II}(s) = \frac{t^{I}(s)}{1 + 2i\sigma\, t^{I}(s)}}$$

Looking for resonance poles
is nothing but looking for a zero in that denominator
on the first Riemann sheet accesible with the pw DR

The problem is the left (and circular) cut

Partial Wave Dispersion Relations: Unitarized ChPT

- **Unitarized ChPT**

90's Truong, Dobado, Herrero, JRP, Oset, Oller, Ruiz Arriola, Nieves, Meissner,…

Uses Chiral Perturbation Theory amplitudes inside dispersion relation.

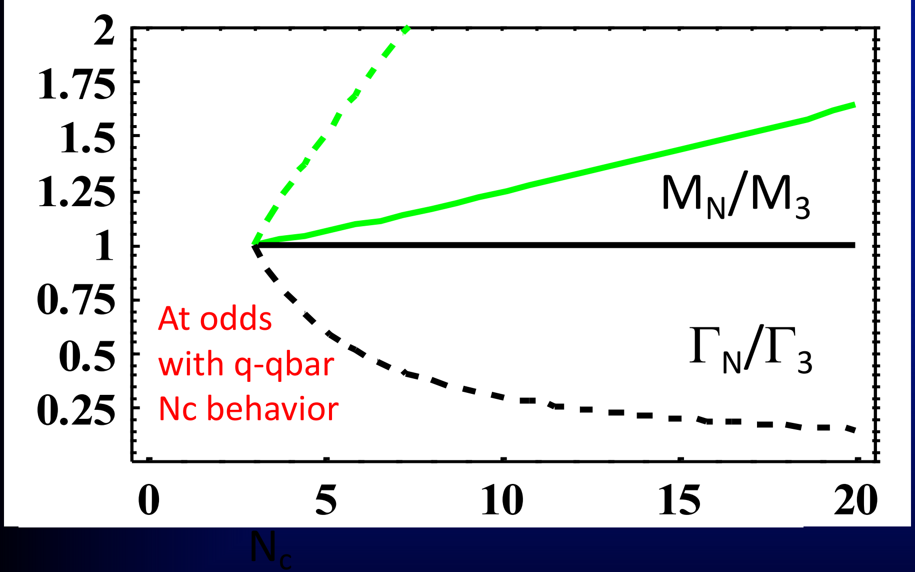
Relatively simple, although different levels of rigour. Generates all scalars

LEFT CUT APPROXIMATED, not so good for precision: (753 ± 52)-i(235 ±33)MeV

But good for connecting with QCD. Strong hints of non-ordinary nature:

Nc behavior

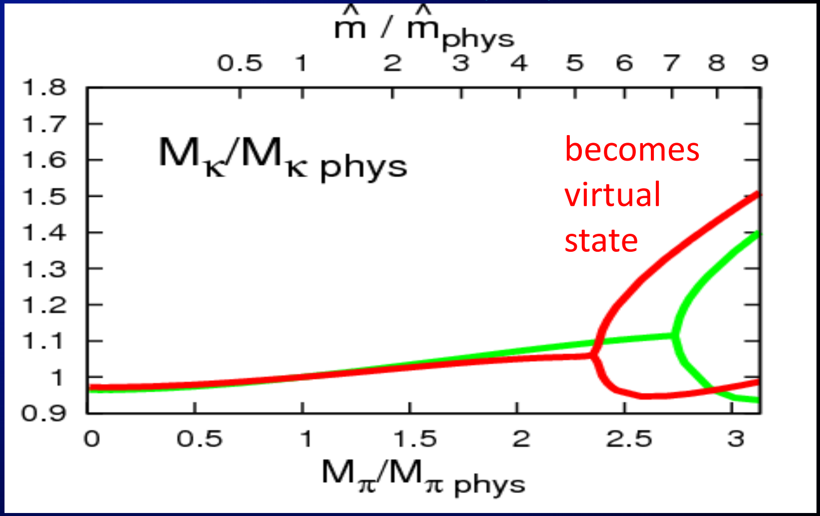
JRP, PRL. 92:102001,2004



Correct behavior obtained for vectors

m_q dependence

Nebreda, JRP, PRD81 (2010) 054035



Virtual state recently found on lattice

Dudek,Edwards, Thomas, Wilson, PRL. 113 (2014) 18, 182001

Both suggest important "molecular" component

Partial Wave Dispersion Relations: Partial Wave DR -3

- **Roy-like equations.** 70's Roy, Basdevant, Pennington, Petersen…

  00's Ananthanarayan, Caprini, Colangelo, Gasser, Leutwyler, Moussallam, Decotes Genon, Lesniak, Kaminski, JRP, Ruiz de Elvira, Yndurain…

  LEFT CUT WITH PRECISION.

  PRICE: Infinite set of coupled integral equations. VALIDITY LIMITED at ~1.1 GeV

  Use data on all waves + high energy . Optional: ChPT predictions for subtraction constants

  The most precise and model independent pole determinations

$f_0(500)$ and $K_0^*(800)$ existence, mass and width

firmly established with precision

(658±13)-i(278.5±12) MeV

Descotes-Genon, B. Moussallam

Listed @PDG, but not enough for PDG

We have been asked for an independent

dispersive analysisto trigger the PDG revision

Two strategies

- SOLVE equations: (Ananthanarayan, Colangelo, Gasser, Leutwyler, Caprini, Moussallam, Stern…)

  S and P wave solution for Roy-like equations unique at low energy if high-energy, higher waves and scattering lengths known. (in isospin limit)

  NO scattering DATA used at low energies ($\sqrt{s} \leq 1\ GeV$)

  Good if interested in low energy scattering and do not trust data.

  Uses ChPT/other input for threshold parameter

  Already followed by Paris group (B. Moussallam et al.) . Most reliable determination so far.

- Impose Dispersion Relations on fits to data. (García-Martín, Kaminski,JRP, Ruiz de Elvira, Ynduráin)

  Use any functional form and fit to DATA imposing DR within uncertainties.

  Also needs input on other waves and high energy.

  (But you can use physical inspiration for clever choices of parameterizations)

  THIS IS OUR APPROACH

Partial—wave vs. fixed-variable Dispersion Relations

So, we need to get rid of ONE VARIABLE to write CAUCHY THEOREM in terms of the other one

TWO MAIN APPROACHES

1) Integrate one variable and keep the other
(partial wave dispersion relations)

2) Fix one variable in terms of the other
(fixed-t, hyperbolic relations…)

Fixed-t Dispersion Relations (DR)

Simple analytic structure in s-plane, simple derivation and use

Left cut: With crossing can be rewritten in terms of physical region

Most popular: t_0=0, **FORWARD DISPERSION RELATIONS** (FDRs).
(Kaminski, Pelaez , Yndurain, Garcia Martin, Ruiz de Elvira, Rodas)

One equation per amplitude.
High Energy part known since Forward Amplitude~ Total cross section

Calculated up **1.7 GeV for πK** (and 1400 MeV for ππ)
JRP, A .Rodas, Phys.Rev. D93 (2016) no.7, 074025

Not directly usable for unphysical sheets but very useful to constraint physical amplitudes up to relatively high energies

Summary of this part

- We have used FORWARD DISPERSION RELATIONS to constraint πK scattering amplitudes **up to 1.6 GeV**:
  - Simple parameterizations. Easy to use
  - Still describe data
  - Consistent with unitarity, ANALYTICITY and crossing

In progress:

We are about to finish the ππ→KK Roy-Steiner analysis up to 1.5 GeV
Working on the Roy-Steiner analysis for πK→πK. See final slides

Strange scalar resonances from dispersive analysis and analytiicty

J. R. Peláez, A. Rodas, J. Ruiz de Elvira

Eur.Phys.J. C77 (2017) no.2, 91

JRP, A. Rodas in preparation

Kappa pole from CFD

We have amplitudes that describe data and satisfy dispersion relations up to 1.6 GeV

There is also a κ POLE in the elastic piece of our CFD parameterizations

Unconstrained Fits (UFD):Elastic region

- We use the unitary functional form for the partial waves

$$t_l^I(s) = \frac{1}{\sigma(s)} \frac{1}{cot\delta_l^I(s) - i} \tag{5}$$

- Where

$$cot\delta_l^I(s) = \frac{\sqrt{s}}{2q^{2l+1}} \sum B_n \omega(s)^n \tag{6}$$

- with $\omega(s) = \frac{\sqrt{y(s)} - \alpha\sqrt{y(s0) - y(s)}}{\sqrt{y(s)} + \alpha\sqrt{y(s0) - y(s)}}$ as our new variable (conformal mapping).
- Here $y(s) = (\frac{s - su}{s + su})^2$ defines the circular cut on the next figure.
- ω used to maximize the analyticity domain.

Unconstrained Fits (UFD):Elastic region

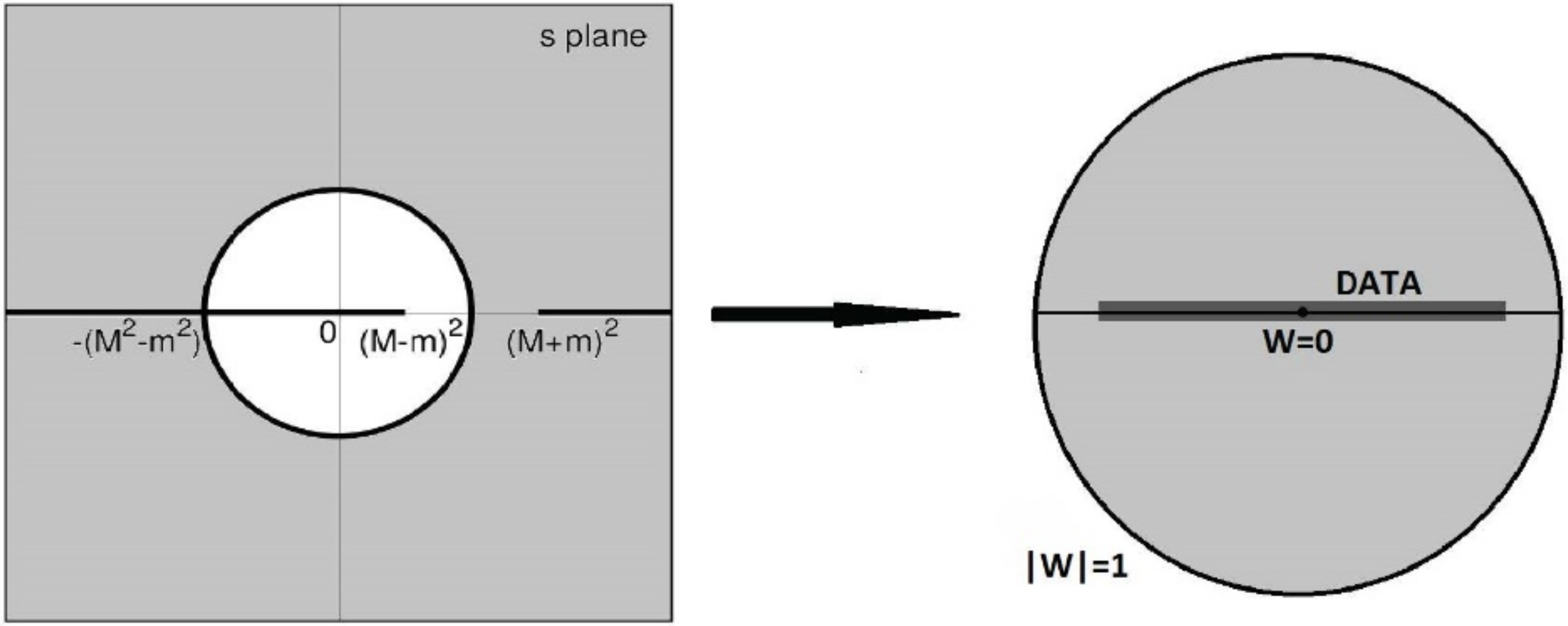


Figure: Structure of the PW.

- α is used to center the point of energy s_c for the expansion.

ππ→KK HDR

$g^I_J(t)$ = ππ → KK partial waves. We study (I,J)=(0,0),(1,1),(0,2)

$f^I_J(s)$ = Kπ → Kπ partial waves. Taken from previous dispersive study JRP, A. Rodas PRD 2016

$$\begin{aligned}
g_0^0(t) &= \boxed{\frac{\sqrt{3}}{2} m_+ a_0^+} + \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{\operatorname{Im} g_0^0(t')}{t'(t'-t)}dt' + \frac{t}{\pi}\sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty \frac{dt'}{t'} G^0_{0,2\ell-2}(t,t')\operatorname{Im} g^0_{2\ell-2}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G^+_{0,\ell}(t,s')\operatorname{Im} f^+_\ell(s'),\\
g_1^1(t) &= \frac{1}{\pi}\int_{4m_\pi^2}^\infty \frac{\operatorname{Im} g_1^1(t')}{t'-t}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty dt' G^1_{1,2\ell-1}(t,t')\operatorname{Im} g^1_{2\ell-1}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G^-_{1,\ell}(t,s')\operatorname{Im} f^-_\ell(s'),\\
g_2^0(t) &= \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{\operatorname{Im} g_2^0(t')}{t'(t'-t)}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty \frac{dt'}{t'} G'^0_{2,4\ell-2}(t,t')\operatorname{Im} g^0_{4\ell-2}(t') + \sum_\ell \int_{m_+^2}^\infty ds' G'^+_{2,\ell}(t,s')\operatorname{Im} f^+_\ell(s').
\end{aligned} \tag{39}$$

$G^I_{J,J'}$(t,t') =integral kernels, depend on a parameter
Lowest # of subtractions. Odd pw decouple from even pw.

$$\begin{aligned}
g_\ell^0(t) &= \Delta_\ell^0(t) + \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{dt'}{t'}\frac{\operatorname{Im} g_\ell^0(t)}{t'-t}, \qquad \ell = 0, 2,\\
g_1^1(t) &= \Delta_1^1(t) + \frac{1}{\pi}\int_{\boxed{4m_\pi^2}}^\infty dt' \frac{\operatorname{Im} g_1^1(t)}{t'-t},
\end{aligned} \tag{40}$$

Δ(t) depend on higher waves or on Kπ→Kπ.

Integrals from 2π threshold !

Solve in descending J order

We have used models for higher waves, but give very small contributions

ππ→KK HDR

For unphysical region below KK threshold, we used Omnés function

(see G.Colangelo's talk)

$$\Omega_\ell^I(t) = \exp\left(\frac{t}{\pi}\int_{4m_\pi^2}^{t_m}\frac{\phi_\ell^I(t')dt'}{t'(t'-t)}\right),$$

$$\Omega_\ell^I(t) \equiv \Omega_{l,R}^I(t)\, e^{i\phi_\ell^I(t)\theta(t-4m_\pi^2)\theta(t_m-t)},$$

This is the form of our HDR: Roy-Steiner+Omnés formalism

$$\begin{aligned}
g_0^0(t) &= \Delta_0^0(t) + \frac{t\Omega_0^0(t)}{t_m-t}\left[\alpha + \frac{t}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{(t_m-t')\Delta_0^0(t')\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)} + \frac{t}{\pi}\int_{t_m}^{\infty} dt' \frac{(t_m-t')|g_0^0(t')|\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)}\right]\\
g_1^1(t) &= \Delta_1^1(t) + \Omega_1^1(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_1^1(t')\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_1^1(t')|\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)}\right],\\
g_2^0(t) &= \Delta_2^0(t) + t\Omega_2^0(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_2^0(t')\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_2^0(t')|\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)}\right].
\end{aligned}$$

We can now check how well these HDR are satisfied