Signs of universal vector-meson coupling constants f_{ρ^0}, f_ω, f_ϕ with photon

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Outline

Universal coupling constants of ρ^0, ω, ϕ

The Review of Particle Physics (2018) **makes known three trinities of neutral vector mesons**

$$\begin{aligned} \rho(770), \omega(782), \phi(1020) \\ \omega'(1420), \rho'(1450)), \phi'(1680) \\ \omega''(1650), \rho''(1700), \phi''(2170), \end{aligned} \tag{1}$$

to be **revealed mainly in** $\sigma_{tot}(e^+e^- \rightarrow hadrons)$ and their lepton decay width $\Gamma(V \rightarrow e^+e^-)$ is **specified by the corresponding universal vector-meson coupling constant** f_V

$$\Gamma(V \rightarrow e^+e^-) = \frac{\alpha^2 m_V}{3}\left(\frac{f_V^2}{4\pi}\right)^{-1} \tag{2}$$

describing **photon-vector-meson transition** in the form $e\frac{m_V^2}{f_V}$.

Universal coupling constants of ρ^0, ω, ϕ

So, **knowing values of the corresponding universal vector-meson coupling constants,**
one could predict $\Gamma(V \to e^+e^-)$ of all abovementioned vector mesons.

However, there is **no theory able to predict** $f_{\rho^0}, f_\omega, f_\phi$ numerically up to now. Therefore we are left only with **"inverse problem"** - i.e. **with numerical estimation of** $f_{\rho^0}, f_\omega, f_\phi$ **from existing data on** $\Gamma(V \to e^+e^-)$.

NOTE:
Still data on $\Gamma(V \to e^+e^-)$, though **very desirable**, are **missing for excited states of neutral vector mesons**
$\omega'(1420), \rho'(1450)), \phi'(1680); \omega''(1650), \rho''(1700), \phi''(2170)$.

Nevertheless, our further **considerations will be concerned of all ground state and excited vector mesons.**

Universal coupling constants of ρ^0, ω, ϕ

Even if one knows $\Gamma(V \to e^+e^-)$ experimentally, it **can provide only the absolute value**, without any sign, of the corresponding f_V, as this is contained in the expression for lepton width $\Gamma(V \to e^+e^-)$ **quadratically**.

However, there are physical quantities in which $f_{\rho^0}, f_\omega, f_\phi$ appear in linear form, so, their **signs play a very important role**.

Despite of this fact, e.g. in text books of

Nguyen van Hieu: *Lectures on the theory of the unitary symmetry of elementary particles (in Russian), ATOMIZDAT, Moscow, (1967).*
F.Renard: *Basics of electron-positron collisions, Editions Frontieres (1981), DREUX, France*
J.P.Perez-y-Jorba and F.Renard, *Phys. Reports 31 (1977) 1-157.*

one finds all coupling constants $f_{\rho^0}, f_\omega, f_\phi$ to be **positive** one.

Universal coupling constants of ρ^0, ω, ϕ

Further we demonstrate:

signs of $f_{\rho^0}, f_\omega, f_\phi$ strongly **depend on the** $\omega - \phi$ **mixing forms**

C.Adamuscin, E.Bartos, S.Dubnicka, A.Z.Dubnickova: **Acta Phys. Polonica B (Proc. Suppl.) 11 (2018) 519**.

$$\begin{array}{lll} 1. & \omega = +\omega_8 \sin\theta + \omega_0 \cos\theta & \phi = -\omega_8 \cos\theta + \omega_0 \sin\theta \\ 2. & \omega = -\omega_8 \sin\theta + \omega_0 \cos\theta & \phi = -\omega_8 \cos\theta - \omega_0 \sin\theta \\ 3. & \omega = +\omega_8 \sin\theta - \omega_0 \cos\theta & \phi = +\omega_8 \cos\theta + \omega_0 \sin\theta \\ 4. & \omega = -\omega_8 \sin\theta - \omega_0 \cos\theta & \phi = +\omega_8 \cos\theta - \omega_0 \sin\theta \\ 5. & \omega = +\omega_8 \sin\theta + \omega_0 \cos\theta & \phi = +\omega_8 \cos\theta - \omega_0 \sin\theta \\ 6. & \omega = -\omega_8 \sin\theta + \omega_0 \cos\theta & \phi = +\omega_8 \cos\theta + \omega_0 \sin\theta \\ 7. & \omega = +\omega_8 \sin\theta - \omega_0 \cos\theta & \phi = -\omega_8 \cos\theta - \omega_0 \sin\theta \\ 8. & \omega = -\omega_8 \sin\theta - \omega_0 \cos\theta & \phi = -\omega_8 \cos\theta + \omega_0 \sin\theta \end{array} \quad (3)$$

where only **1.,4.,5.,8.** are **physically acceptable**.

Signs of $f_{\rho^0}, f_\omega, f_\phi$

The **problem of $f_{\rho^0}, f_\omega, f_\phi$ signs** will be made clear by using:

- **rearranged hadronic electromagnetic (EM) current**

$$J_\mu^{\mathrm{h}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s \qquad (4)$$

into a sum of the ρ^0, ω, ϕ **meson EM currents**
- **application of the $\omega - \phi$ mixing directly to EM currents** of ω and ϕ vector mesons
- and finally by a comparison of the obtained results with the **KLZ hadronic EM current to be identified with a linear combination of the renormalized ρ^0, ω, ϕ fields**

N.M.Kroll, T.D.Lee and B.Zumino, *Phys, Rev. 157 (1967) 1376*

by means of which also $f_{\rho^0}, f_\omega, f_\phi$ **coupling constants are defined**.

Signs of $f_{\rho^0}, f_\omega, f_\phi$

Really, the **hadronic EM current (4) can be formally arranged** to the form

$$J_\mu^{\mathrm{h}} = S_\rho \frac{1}{\sqrt{2}} J_\mu^{\rho^0} + S_\omega \frac{1}{3\sqrt{2}} J_\mu^\omega - S_\phi \frac{1}{3} J_\mu^\phi, \tag{5}$$

where

$$\begin{aligned} J_\mu^{\rho^0} &= \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) \\ J_\mu^\omega &= \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \\ J_\mu^\phi &= \bar{s}\gamma_\mu s \end{aligned} \tag{6}$$

are ρ^0**-meson,** ω**-meson and** ϕ**-meson EM currents** and S_ρ, S_ω, S_ϕ are the ρ**-meson,** ω**-meson and** ϕ**-meson EM current signs**, respectively.

Signs of $f_{\rho^0}, f_\omega, f_\phi$

Now we accede to the $\omega - \phi$ **mixing**.

However, before one writes the well known expressions of ω_8 and ω_0 meson EM currents

$$J_\mu^{\omega_8} = \frac{1}{\sqrt{6}}(u\gamma_\mu\bar{u} + d\gamma_\mu\bar{d} - 2s\gamma_\mu\bar{s}) \tag{7}$$
$$J_\mu^{\omega_0} = \frac{1}{\sqrt{3}}(u\gamma_\mu\bar{u} + d\gamma_\mu\bar{d} + s\gamma_\mu\bar{s}).$$

Then if the $\omega - \phi$ mixing is **applied directly to the** ω, ϕ **meson EM currents in (6)** one finds

Signs of $f_{\rho^0}, f_\omega, f_\phi$

$$
\begin{array}{lll}
1. & J_\mu^\omega = +J_\mu^{\omega_8}\sin\theta + J_\mu^{\omega_0}\cos\theta & J_\mu^\phi = -J_\mu^{\omega_8}\cos\theta + J_\mu^{\omega_0}\sin\theta \\
2. & J_\mu^\omega = -J_\mu^{\omega_8}\sin\theta + J_\mu^{\omega_0}\cos\theta & J_\mu^\phi = -J_\mu^{\omega_8}\cos\theta - J_\mu^{\omega_0}\sin\theta \\
3. & J_\mu^\omega = +J_\mu^{\omega_8}\sin\theta - J_\mu^{\omega_0}\cos\theta & J_\mu^\phi = +J_\mu^{\omega_8}\cos\theta + J_\mu^{\omega_0}\sin\theta \\
4. & J_\mu^\omega = -J_\mu^{\omega_8}\sin\theta - J_\mu^{\omega_0}\cos\theta & J_\mu^\phi = +J_\mu^{\omega_8}\cos\theta - J_\mu^{\omega_0}\sin\theta \\
5. & J_\mu^\omega = +J_\mu^{\omega_8}\sin\theta + J_\mu^{\omega_0}\cos\theta & J_\mu^\phi = +J_\mu^{\omega_8}\cos\theta - J_\mu^{\omega_0}\sin\theta \\
6. & J_\mu^\omega = -J_\mu^{\omega_8}\sin\theta + J_\mu^{\omega_0}\cos\theta & J_\mu^\phi = +J_\mu^{\omega_8}\cos\theta + J_\mu^{\omega_0}\sin\theta \\
7. & J_\mu^\omega = +J_\mu^{\omega_8}\sin\theta - J_\mu^{\omega_0}\cos\theta & J_\mu^\phi = -J_\mu^{\omega_8}\cos\theta - J_\mu^{\omega_0}\sin\theta \\
8. & J_\mu^\omega = -J_\mu^{\omega_8}\sin\theta - J_\mu^{\omega_0}\cos\theta & J_\mu^\phi = -J_\mu^{\omega_8}\cos\theta + J_\mu^{\omega_0}\sin\theta
\end{array}
\tag{8}
$$

Signs of $f_{\rho^0}, f_\omega, f_\phi$

Substituting $J_\mu^{\omega_8}, J_\mu^{\omega_0}$ forms from (7) into (8), by a **detail calculation of J_μ^ω, J_μ^ϕ currents with the ideal mixing angle** $\theta = 35.3^0$, when $\cos\theta = \sqrt{(\frac{2}{3})}$ and $\sin\theta = \sqrt{(\frac{1}{3})}$, one finds that **only 1.,4.,5., and 8. $\omega - \phi$ mixing forms reproduce the J_μ^ω, J_μ^ϕ currents completely** up to the sign.

The 2.,3.,6., and 7. $\omega - \phi$ mixing forms **make always**
- to J_μ^ω **admixture of the J_μ^ϕ current**
- and to J_μ^ϕ **admixture of the J_μ^ω current**

contributions, respectively, **though the ideal mixing angle value has been used**.

Even more, **these admixtures are dominant!**

This is another evidence, that **the 2.,3.,6., and 7. $\omega - \phi$ mixing forms are physically non-acceptable!**

Signs of $f_{\rho^0}, f_\omega, f_\phi$

In order to explain our **previous assertions** in more detail, let us present them here as follows.

Substituting $J_\mu^{\omega_8}, J_\mu^{\omega_0}$ forms from (7) e.g. into 4. of (8), one finds

$$J_\mu^\omega = -\frac{1}{3\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s) - \frac{2}{3\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s)$$
$$= -\frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \quad (9)$$
$$= -J_\mu^\omega$$

$$J_\mu^\phi = \frac{1}{3}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s) - \frac{1}{3}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s)$$
$$= -(\bar{s}\gamma_\mu s) \quad (10)$$
$$= -J_\mu^\phi$$

One obtains **similar results from 1.,5., and 8.** in (8).

Signs of f_{ρ^0}, f_ω, f_ϕ

Now, substituting $J_\mu^{\omega_8}, J_\mu^{\omega_0}$ forms from (7) e.g. into 2. of (8), one finds

$$\begin{aligned} J_\mu^\omega &= -\frac{1}{3\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s) + \frac{2}{3\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s) \\ &= +\frac{1}{3\sqrt{2}}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) + \frac{4}{3\sqrt{2}}(\bar{s}\gamma_\mu s) \quad (11) \\ &= +\frac{1}{3}J_\mu^\omega + \frac{4}{3}\frac{1}{\sqrt{2}}J_\mu^\phi \end{aligned}$$

$$\begin{aligned} J_\mu^\phi &= -\frac{1}{3}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s) - \frac{1}{3}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s) \\ &= +\frac{1}{3}(\bar{s}\gamma_\mu s) - \frac{2}{3}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) \qquad (12) \\ &= +\frac{1}{3}J_\mu^\phi - \frac{2}{3}\sqrt{2}J_\mu^\omega \end{aligned}$$

One obtains **similar results from 3.,6., and 7.** in (8).

Signs of $f_{\rho^0}, f_\omega, f_\phi$

So, as a result the corresponding S_ρ, S_ω, S_ϕ signs in

$$J^{\mathrm{h}}_\mu = S_\rho \frac{1}{\sqrt{2}} J^{\rho^0}_\mu + S_\omega \frac{1}{3\sqrt{2}} J^{\omega}_\mu - S_\phi \frac{1}{3} J^{\phi}_\mu,$$

are as follows

1. $S_\rho = +, \quad S_\omega = +, \quad S_\phi = +$
4. $S_\rho = +, \quad S_\omega = -, \quad S_\phi = -$
5. $S_\rho = +, \quad S_\omega = +, \quad S_\phi = -$
8. $S_\rho = +, \quad S_\omega = -, \quad S_\phi = +$

and one finds **four various forms of the hadronic EM current**

Signs of $f_{\rho^0}, f_\omega, f_\phi$

$$1. \quad J^{\rm h}_\mu = +\frac{1}{\sqrt{2}} J^{\rho^0}_\mu + \frac{1}{3\sqrt{2}} J^{\omega}_\mu - \frac{1}{3} J^{\phi}_\mu, \tag{13}$$

$$4. \quad J^{\rm h}_\mu = +\frac{1}{\sqrt{2}} J^{\rho^0}_\mu - \frac{1}{3\sqrt{2}} J^{\omega}_\mu + \frac{1}{3} J^{\phi}_\mu, \tag{14}$$

$$5. \quad J^{\rm h}_\mu = +\frac{1}{\sqrt{2}} J^{\rho^0}_\mu + \frac{1}{3\sqrt{2}} J^{\omega}_\mu + \frac{1}{3} J^{\phi}_\mu, \tag{15}$$

$$8. \quad J^{\rm h}_\mu = +\frac{1}{\sqrt{2}} J^{\rho^0}_\mu - \frac{1}{3\sqrt{2}} J^{\omega}_\mu - \frac{1}{3} J^{\phi}_\mu, \tag{16}$$

every of which depends on the applied $\omega - \phi$ mixing form **1.,4.,5.,8.** from (3) on the ω, ϕ meson EM currents in (6).

Signs of f_{ρ^0}, f_ω, f_ϕ

On the other hand there is **KLZ hadronic EM current to be a linear combination of the renormalized ρ^0, ω, ϕ fields** as follows

$$(J_\mu^{\mathrm{h}})_{KLZ} = -\frac{m_{\rho^0}^2}{f_\rho}\rho_\mu^0 - \frac{m_\omega^2}{f_\omega}\omega_\mu - \frac{m_\phi^2}{f_\phi}\phi_\mu \tag{17}$$

which is equal to the hadronic EM current J_μ^h expressed by means of the quark currents up to the real constant A, i.e.

$$(J_\mu^{\mathrm{h}})_{KLZ} = A.J_\mu^{\mathrm{h}}. \tag{18}$$

Signs of f_{ρ^0}, f_ω, f_ϕ

If the latter equality is used to **four various forms of the hadronic EM current**, dependent on the applied $\omega - \phi$ mixing form 1.,4.,5.,8., separately, **one finds relations for the universal vector-meson coupling constants**

$$1.\quad -\frac{1}{f_\rho} = +A.\frac{1}{\sqrt{2}};\quad -\frac{1}{f_\omega} = +A.\frac{1}{3\sqrt{2}};\quad -\frac{1}{f_\phi} = -A.\frac{1}{3};$$
$$4.\quad -\frac{1}{f_\rho} = +A.\frac{1}{\sqrt{2}};\quad -\frac{1}{f_\omega} = -A.\frac{1}{3\sqrt{2}};\quad -\frac{1}{f_\phi} = +A.\frac{1}{3};$$
$$5.\quad -\frac{1}{f_\rho} = +A.\frac{1}{\sqrt{2}};\quad -\frac{1}{f_\omega} = +A.\frac{1}{3\sqrt{2}};\quad -\frac{1}{f_\phi} = +A.\frac{1}{3};$$
$$8.\quad -\frac{1}{f_\rho} = +A.\frac{1}{\sqrt{2}};\quad -\frac{1}{f_\omega} = -A.\frac{1}{3\sqrt{2}};\quad -\frac{1}{f_\phi} = -A.\frac{1}{3};$$

or **for their ratios**

$$1.\quad \frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\frac{1}{\sqrt{2}} : -\frac{1}{3\sqrt{2}} : +\frac{1}{3}$$
$$4.\quad \frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\frac{1}{\sqrt{2}} : +\frac{1}{3\sqrt{2}} : -\frac{1}{3}$$
$$5.\quad \frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\frac{1}{\sqrt{2}} : -\frac{1}{3\sqrt{2}} : -\frac{1}{3}$$
$$8.\quad \frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\frac{1}{\sqrt{2}} : +\frac{1}{3\sqrt{2}} : +\frac{1}{3}.$$

Signs of f_{ρ^0}, f_ω, f_ϕ

Now **multiplying the right hand side of the previous relations by** $\sqrt{6}$ one obtains

1. $\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\sqrt{3} : -\frac{1}{\sqrt{3}} : +\sqrt{\frac{2}{3}}$

4. $\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\sqrt{3} : +\frac{1}{\sqrt{3}} : -\sqrt{\frac{2}{3}}$

5. $\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\sqrt{3} : -\frac{1}{\sqrt{3}} : -\sqrt{\frac{2}{3}}$

8. $\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\sqrt{3} : +\frac{1}{\sqrt{3}} : +\sqrt{\frac{2}{3}}$.

Signs of f_{ρ^0}, f_ω, f_ϕ

Finally, if an advantage of the **ideal mixing angle** θ, $\sqrt{\frac{1}{3}} = \sin\theta$ and $\sqrt{\frac{2}{3}} = \cos\theta$, is taken, then the **form of relations demonstrating a dependence of the universal vector-meson coupling constants signs on the $\omega - \phi$ mixing, currently found in the literature,** is specified

1. $\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\sqrt{3} : -\sin\theta : +\cos\theta$
4. $\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\sqrt{3} : +\sin\theta : -\cos\theta$
5. $\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\sqrt{3} : -\sin\theta : -\cos\theta$
8. $\frac{1}{f_\rho} : \frac{1}{f_\omega} : \frac{1}{f_\phi} = -\sqrt{3} : +\sin\theta : +\cos\theta.$

Conclusions

By a **rearrangement of the hadronic electromagnetic (EM) current**

$$J_\mu^{\mathrm{h}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s \qquad (19)$$

into a sum of the ρ^0, ω, ϕ **meson EM currents**,
then by an application of the $\omega - \phi$ mixing directly to EM currents of ω and ϕ vector mesons
and finally by a comparison of the obtained results with the **KLZ hadronic EM current to be identified with a linear combination of the re-normalized ρ^0, ω, ϕ fields**,

we have **elucidated of many years' standing problem of universal vector-meson coupling constants $f_{\rho^0}, f_\omega, f_\phi$ signs**.

Thank you for your attention.