NEW APPROACH IN KNOWLEDGE OF $a_\mu^{(LO)had}$ VALUE TO THE MUON g-2 ANOMALY

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INTRODUCTION

The **anomalous magnetic moment of the negative muon** μ^-

$$a_\mu = \frac{g-2}{2},$$

to be measured and simultaneously evaluated theoretically, provides an **extremely clean test of the Standard Model (SM)** of elementary particle physics, validity of which is now confirmed experimentally with a precision of 1%.
Therefore - it is constantly important to **achieve in its theoretical evaluation** the inequality

$$(a_\mu^{exp} - a_\mu^{th}) < \Delta(a_\mu^{exp} - a_\mu^{th}).$$

INTRODUCTION

All **charged leptons**, including the μ^- (and also their antiparticles), are **described by the Dirac equation**.
The **magnetic moments** of these particles are **related to the spin** by means of very well known expression

$$\vec{\mu} = g\left(\frac{e}{2m_l}\right)\vec{s} \tag{1}$$

where the value of **gyromagnetic ratio** g is predicted theoretically

I.J.R.Aitchison and A.J.G.Hey: **Gauge theories in particle physics, Bristol and Philadelphia, 2003**

to be $g = 2$.

INTRODUCTION

However, interactions existing in nature **modify g to be exceeding the value "2"** because of the emission and absorption of

- virtual photons (EM effects)
- intermediate vector and Higgs bosons (weak interaction effects)
- vacuum polarization into virtual hadronic states (strong interaction effects).

MUON g-2 ANOMALY

In order to describe this modification of g for μ^- theoretically, the **magnetic anomaly** has been introduced by the relation

$$a_\mu = \frac{g-2}{2} = a_\mu^{(1)}\left(\frac{\alpha}{\pi}\right) + \left(a_\mu^{(2)QED} + a_\mu^{(LO)had}\right)\left(\frac{\alpha}{\pi}\right)^2 + a_\mu^{(2)weak} + O\left(\frac{\alpha}{\pi}\right)^3 \quad (2)$$

where α is the **fine structure constant of QED** to be $\alpha(0) = \frac{1}{137{,}036}$.

MUON g-2 ANOMALY

Dominant sources of the **total uncertainties in theoretical predictions** of a_μ is **the "Leading Order" hadronic contribution** $a_\mu^{(LO)had}$ represented by the vacuum-polarization diagram in Fig.1,

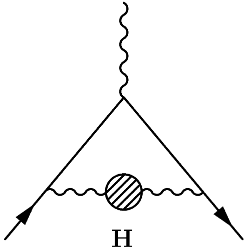

Fig.1: The leading-order hadronic vacuum-polarization contribution to a_μ.

MUON g-2 ANOMALY

the **contribution of which is** given

M.Gourdin, E. de Rafael: **Nucl. Phys. B10 (1969) 667**

by the sum of two dispersion integrals

$$a_\mu^{(LO)had} = \frac{\alpha^2(0)}{3\pi^2}\left(\int_{m^2_{\pi^0}}^{s_{(cut)}} \frac{ds}{s} R^{data}(s)K(s) + \int_{s_{(cut)}}^{\infty} \frac{ds}{s} R^{pQCD}(s)K(s)\right), \tag{3}$$

where $s_{(cut)}$ is usually **taken around the value 2 GeV^2**, in which highly fluctuating (due to hadronic resonances and threshold effects) $R^{data}K(s)$ **function of the first integral is changed to smoother one in the second integral**.

The $R^{data}(s) = \sigma_{tot}(e^+e^- \to had)/\sigma_{tot}(e^+e^- \to \mu^+\mu^-)$ with $\sigma_{tot}(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha(s)^2}{3s}$ and $K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)\frac{s}{m_\mu^2}}$.

MUON g-2 ANOMALY

The **first integral** is evaluated by the integration **directly through experimental data points** on $\sigma_{tot}(e^+e^- \to hadrons)$ and the **second integral** by the integration **through existing experimental points on** $R(s)$ up to some energy s_{max} and next by using the perturbative QCD (pQCD) result

$$R^{pQCD}(s) = 3\sum_f Q_f^2\sqrt{(1-4m_f^2/s)}(1+2m_f^2/s)(1+\frac{\alpha_s(s)}{\pi}+c_1(\frac{\alpha_s(s)}{\pi})^2+c_2(\frac{\alpha_s(s)}{\pi})^3+...)$$

where Q_f and m_f are the charge and mass of the quarks, respectively, $\alpha_s(s)$ the **running strong coupling constant of QCD**, $c_1 = 1.9857 - 01153N_f$
$c_2 = -6.6368 - 1.2002N_f - 0.0052N_f^2 - 1.2395(\sum Q_f)^2/(3\sum Q_f^2)$
with the number of active flavors N_f,
for **the high energy tail contribution evaluation**.

RUNNING FINE STRUCTURE CONSTANT OF QED

The EM constant $\alpha(s)$ appearing in $\sigma_{tot}(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha(s)^2}{3s}$ is the **running fine structure constant of QED**, which can be expressed as

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)} \tag{4}$$

whereby **one distinguishes contributions** in $\Delta\alpha(s)$

- from leptons (e, μ, τ)
- from 5 light quarks $u, d, c, s, b \quad (mass < 5GeV)$
- from "top" - quark t $(mass \approx 175GeV)$

Then

$$\Delta\alpha(s) = \Delta\alpha_l(s) + \Delta^{(5)}_{had}(s) + \Delta\alpha_{top}(s). \tag{5}$$

RUNNING FINE STRUCTURE CONSTANT OF QED

The **leptonic contributions** are calculable in perturbation theory, where at leading order the free leptons yield

$$\Delta\alpha_l(s) = \frac{\alpha(0)}{3\pi} \sum_{l=e,\mu,\tau} [ln \frac{s}{m_l^2} - \frac{5}{3}]. \quad (6)$$

Since the t**-quark is heavy**, one can not use the light fermion approximation for it and **it behaves like**

$$\Delta\alpha_{top}(s) \approx -\frac{\alpha(0)}{3\pi} \frac{4}{15} \frac{s}{m_t^2}. \quad (7)$$

RUNNING FINE STRUCTURE CONSTANT OF QED

A serious problem is the 5 light quarks, u, d, s, c, b contribution $\Delta\alpha_{had}^{(5)}(s)$, **due to the light masses of these quarks it can not be calculated** in the framework of the pQCD.

Fortunately - **one can evaluate it from** $e^+e^- \to hadrons$ **data** through the corresponding dispersion integral **in time-like region**,

$$\Delta\alpha_{had}^{(5)}(s) = -\frac{\alpha(0)s}{3\pi} Re \int_{4m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s'-s-i\varepsilon)} \qquad (8)$$

like in the muon g-2 anomaly, however, **without the QED kernel function** $K(s)$.

NEW APPROACH IN KNOWLEDGE OF $a_\mu^{(LO)had}$ VALUE

Recently Pavia-Padova-Parma-Frascati group of theoretical physicists

C.M.Carloni Calame, M.Passera, L.Trentadue, G.Venanconi,
Phys. Letter B746 (2015) 325.
suggested a **novel approach to determine the LO of hadronic contribution to muon g-2 anomaly** $a_\mu^{(LO)had}$, represented by Fig.1, in a measurement

G.Abbiendi at al, **Eur. Phys. J C (2017) 77:139**
of the **QED running fine structure constant** $\alpha(t)$ **in space-like region**, extracted from elastic scattering $\mu e \to \mu e$ data, obtained by the CERN North Area muon beam scattered on atomic electrons of Be and C.

NEW APPROACH IN KNOWLEDGE OF $a_\mu^{(LO)had}$ VALUE

In order to demonstrate it practically, let us first rewrite the relation for $a_\mu^{(LO)had}$ (3) in the abbreviated form

$$a_\mu^{(LO)had} = \frac{\alpha^2(0)}{3\pi^2} \int_{m_{\pi^0}^2}^{\infty} \frac{ds}{s} R(s) K(s). \tag{9}$$

Then, if we **exchange the x and s integrations** and rearrange the function under the integral in $K(s)$ by a suitable way, one gets

$$a_\mu^{(LO)had} = \frac{\alpha^2(0)}{3\pi^2} \int_0^1 dx(1-x) \int_{m_{\pi^0}^2}^{\infty} \frac{ds'}{s'} R(s') \frac{x^2 m_\mu^2}{x^2 m_\mu^2 + s'(1-x)}, \tag{10}$$

NEW APPROACH IN KNOWLEDGE OF $a_\mu^{(LO)had}$ VALUE

from where one obtains

$$a_\mu^{(LO)had} = \frac{\alpha(0)}{\pi}\int_0^1 dx(1-x)\frac{\alpha(0)}{3\pi}\int_{m^2_{\pi^0}}^\infty \frac{ds'}{s'}R(s')\frac{\frac{x^2 m_\mu^2}{x-1}}{\frac{x^2 m_\mu^2}{x-1}-s'}, \tag{11}$$

or introducing new variable, the **momentum transfer squared** $t(x)=\frac{x^2 m_\mu^2}{x-1}$, to be **negative in the integration interval** $0<x<1$, one finally finds

$$a_\mu^{(LO)had} = \frac{\alpha(0)}{\pi}\int_0^1 dx(1-x)[-\frac{\alpha(0)t(x)}{3\pi}\int_{m^2_{\pi^0}}^\infty ds'\frac{R(s')}{s'(s'-t(x))}]. \tag{12}$$

Here the term in the angular brackets

$$\Delta\alpha^{(5)}_{had}(t(x)) = [-\frac{\alpha(0)t(x)}{3\pi}\int_{m^2_{\pi^0}}^\infty ds'\frac{R(s')}{s'(s'-t(x))}] \tag{13}$$

NEW APPROACH IN KNOWLEDGE OF $a_\mu^{(LO)had}$ VALUE

is actually **hadronic contribution** $\Delta\alpha_{had}^{(5)}(t(x))$ **(8) to the running fine structure constant of QED**, however, now as a function of $t(x)$ **in the space-like region**.
So, knowing the **behavior of** $\Delta\alpha_{had}^{(5)}(t(x))$ from the elastic scattering of μ^- on atomic electrons of Be and C **in the space-like region**, one can, by means of the following integral,

$$a_\mu^{(LO)had} = \frac{\alpha(0)}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}^{(5)}(t(x)) \qquad (14)$$

calculate the "Leading Order" hadronic contribution $a_\mu^{(LO)had}$ to the **anomalous magnetic moment of the muon** a_μ.

NEW APPROACH IN KNOWLEDGE OF $a_\mu^{(LO)had}$ VALUE

Further, our aim is:
- first, to find **behavior of the hadronic contribution to the QED running fine structure constant** $\Delta\alpha_{had}^{(5)}(t(x))$ **in the space-like region** by means of the integral (13), however by a **classical approach through existing experimental data on** $\sigma_{tot}(e^+e^- \to hadrons)$**,** $R^{data}(s)$ **and** R^{pQCD} as in this approach we have abundant experience from the past

L.Martinovic, S,Dubnicka, **Phys. Rev. D42 (1990) 884**,
S.Dubnicka, A.Z.Dubnickova, P.Strizenec, **Acta Phys. Slovaca 45 (1995) 467**
A.Z.Dubnickova, S.Dubnicka, A.Liptaj, **Acta. Phys. Polonica B(Proc. Suppl.) 9 (2016) 407**

NEW APPROACH IN KNOWLEDGE OF $a_\mu^{(LO)had}$ VALUE

and to predict this behavior at the interval $0 < x < 1$, where
$x = \frac{t}{2m_\mu^2}\left[1 - \sqrt{(}1 - \frac{4m_\mu^2}{t})\right]$,
prior to experimental results at CERN

- second, then to evaluate $a_\mu^{(LO)had}$, represented in Fig.1, by using the integral relation (14) and to **compare obtained result with values of other authors to be found by the classical approach.**

In evaluation of the contributions of $\sigma_{tot}(e^+e^- \to M\bar{M})$ and $\sigma_{tot}(e^+e^- \to \gamma M)$ cross-sections, which **always can be expressed through EM FFs squared**, to $\Delta\alpha^{(5)}_{had}(t(x))$, the *Unitary&Analytic* model of pseudoscalar meson EM structure is applied.

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

The **corresponding EM FFs are split into isoscalar and isovector parts** as follows

$$F_{\pi^\pm}(s) = F_\pi^{I=1}[W(s)]$$

$$F_{K^\pm}(s) = F_K^{I=0}[V(s)] + F_K^{I=1}[W(s)]$$

$$F_{K^0}(s) = F_K^{I=0}[V(s)] - F_K^{I=1}[W(s)]$$

$$F_{\pi^0\gamma}(s) = F_{\pi^0\gamma}^{I=0}[V(s)] + F_{\pi^0\gamma}^{I=1}[W(s)]$$

$$F_{\eta\gamma}(s) = F_{\eta\gamma}^{I=0}[V(s)] + F_{\eta\gamma}^{I=1}[W(s)]$$

$$F_{\eta'\gamma}(s) = F_{\eta'\gamma}^{I=0}[V(s)] + F_{\eta'\gamma}^{I=1}[W(s)].$$

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

The *Unitary&Analytic* model takes into account **all known properties of FFs**:

- normalization of FFs
- asymtotic behaviour as predicted by the quark model
- analytic properties of FFs
- unitarity conditions of FFs
- reality conditions of FFs
- experimental fact of a creation of vector mesons in $e^+e^- \to had$ process
- $F^{I=1}(s)$ are **saturated** by ρ, ρ', ρ'', etc and $F^{I=0}(s)$ by $\omega, \phi, \omega', \phi', etc.$

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

Its explicit form **for the charged pion EM FF** can be found in

E.Bartos, S.Dubnicka, A.Z.Dubnickova and A.Liptaj, **Nucl. Physics B (Proc. Suppl.) 198 (2010) 194**
explicit forms **for charged and neutral kaons EM FFs** in

S.Dubnicka, A.Z.Dubnickova and A.Liptaj, **Nucl. Physics B (Proc. Suppl.) 219-220 (2011) 271**
and explicit forms **for neutral pseudoscalar meson transition FFs** in

A.Z.Dubnickova, S.Dubnicka, A.Liptaj, G.Pancheri and R.Pekarik, **Nucl. Physics B (Proc. Suppl.) 131 (2004) 176**.

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

Then every $F^{I=1}[W(s)]$ and $F^{I=0}[V(s)]$ **represents one analytic function** in the whole complex s-plane, **besides two cuts on the positive real axis**, to be defined on the four-sheeted Riemann surface and **depends on only physically interpretable parameters**.

Their **predictions** for the complete **nonet of pseudoscalar mesons** are presented in the following Figs.

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

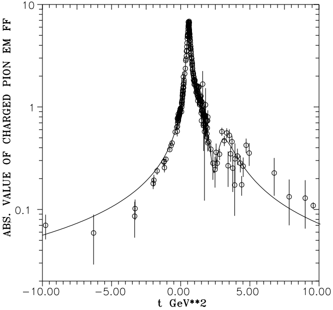

Fig.2 Prediction of pion EM FF behavior by *U&A* model.

$U\&A$ EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

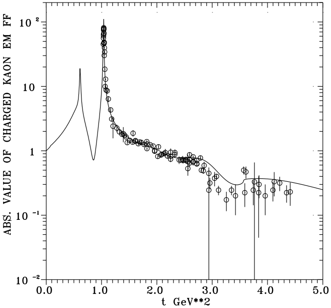

Fig.3 Prediction of charge kaon EM FF behavior by $U\&A$ model.

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

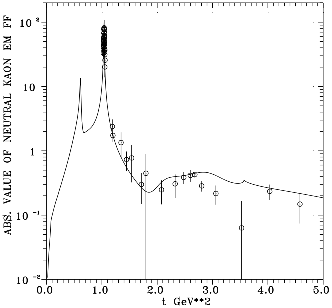

Fig.4 Prediction of neutral kaon EM FF behavior by *U&A* model.

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

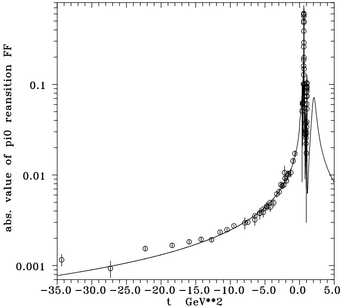

Fig.5 Prediction of $\pi^0\gamma$ transition EM FF behavior by *U&A* model.

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

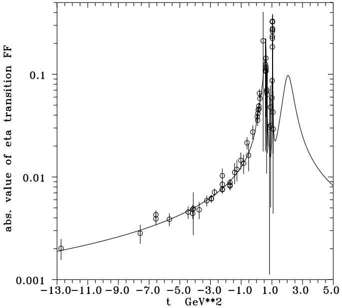

Fig.6 Prediction of $\eta\gamma$ transition EM FF behavior by *U&A* model.

U&A EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

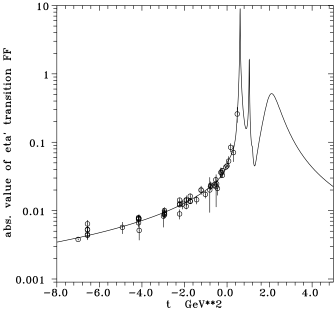

Fig.7 Prediction of $\eta'\gamma$ transition EM FF behavior by *U&A* model.

EVALUATION OF CONTRIBUTIONS TO $\alpha(t(x))$

Similarly one can calculate the contribution of R^{data} from $2.0449 GeV^2 < s < s_{max}$ and also contribution of the R^{pQCD} behind s_{max}.

Every such contribution to $\Delta\alpha_{had}^{(5)}(t(x))$ **is given by a similar curve** as is presented for the contribution of the process $e^+e^- \to \pi^+\pi^-$ in Fig.8.

EVALUATION OF CONTRIBUTIONS TO $\alpha(t(x))$

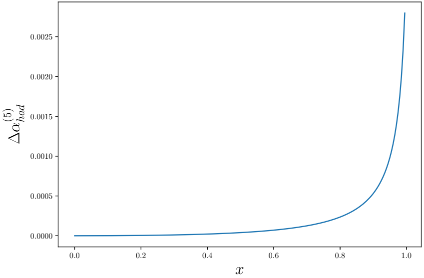

Fig.8 Predicted curve representing contribution of $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ to $\Delta\alpha^{(5)}_{had}(t(x))$.

EVALUATION OF CONTRIBUTIONS TO $\alpha(t(x))$

The **sum of all obtained curves corresponding to the separate contributions will give the curve** which is expected to be obtained from measurement of the $\alpha(t(x))$ QED running fine structure constant by μ^- elastic scattering on atomic electrons of Be and C at CERN-COMPASS.

The **complete evaluation of contributions to** $\Delta\alpha_{had}^{(5)}(t(x))$ **and then also the** $a_\mu^{(LO)had}$ is in progress.

Conclusions

We have presented, a **novel approach to determine the LO of hadronic contribution to muon g-2 anomaly** $a_\mu^{(LO)had}$, and how to find **behavior of the hadronic contribution to the QED running fine structure constant** $\Delta\alpha_{had}^{(5)}(t(x))$ **in the space-like region**, prior to experimental results at CERN-COMPASS, which is crucial in evaluation of the $a_\mu^{(LO)had}$ by means of the integral

$$a_\mu^{(LO)had} = \frac{\alpha(0)}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}^{(5)}(t(x)). \tag{15}$$

Thank you for your attention.