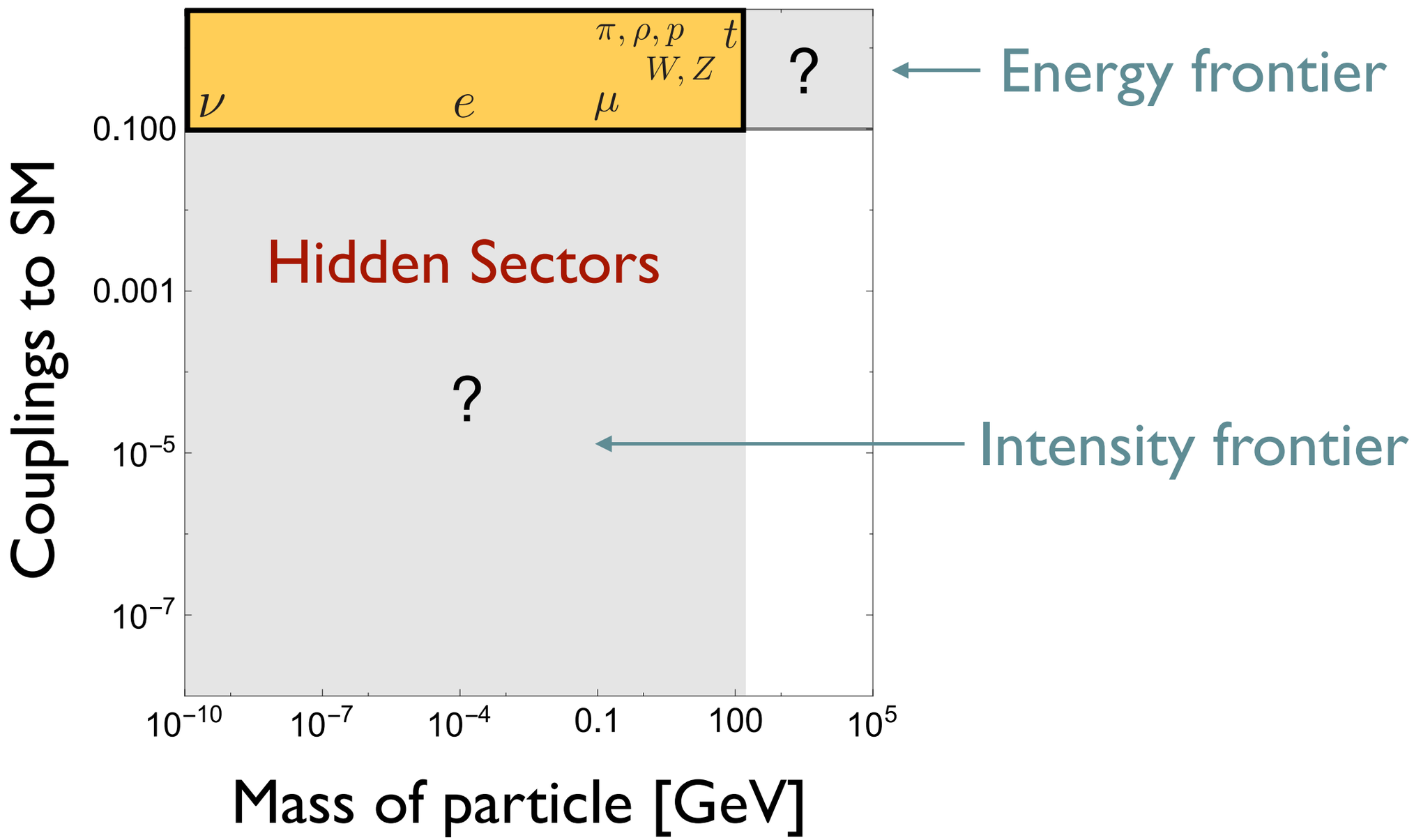
Light (and dark) Mediators

Ian M. Shoemaker
University of South Dakota

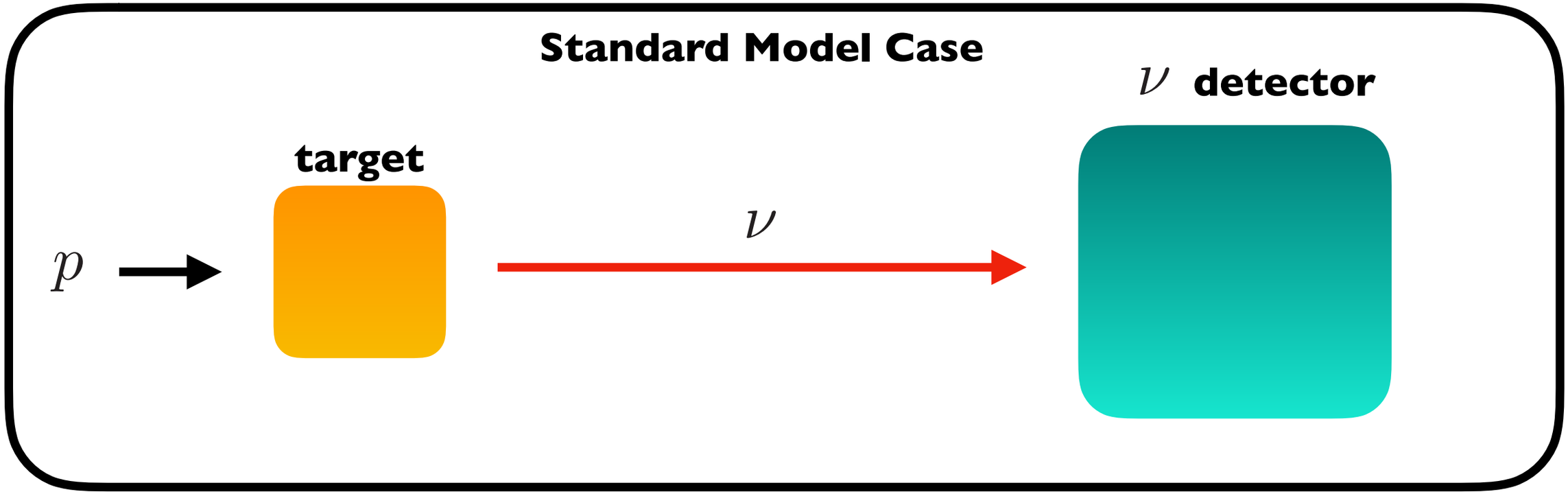
Near Detector Physics at Neutrino Experiments
CERN
June 19th, 2018

Where is new physics?



Need a multi-pronged effort to find new physics.

Qualitatively Distinct Classes



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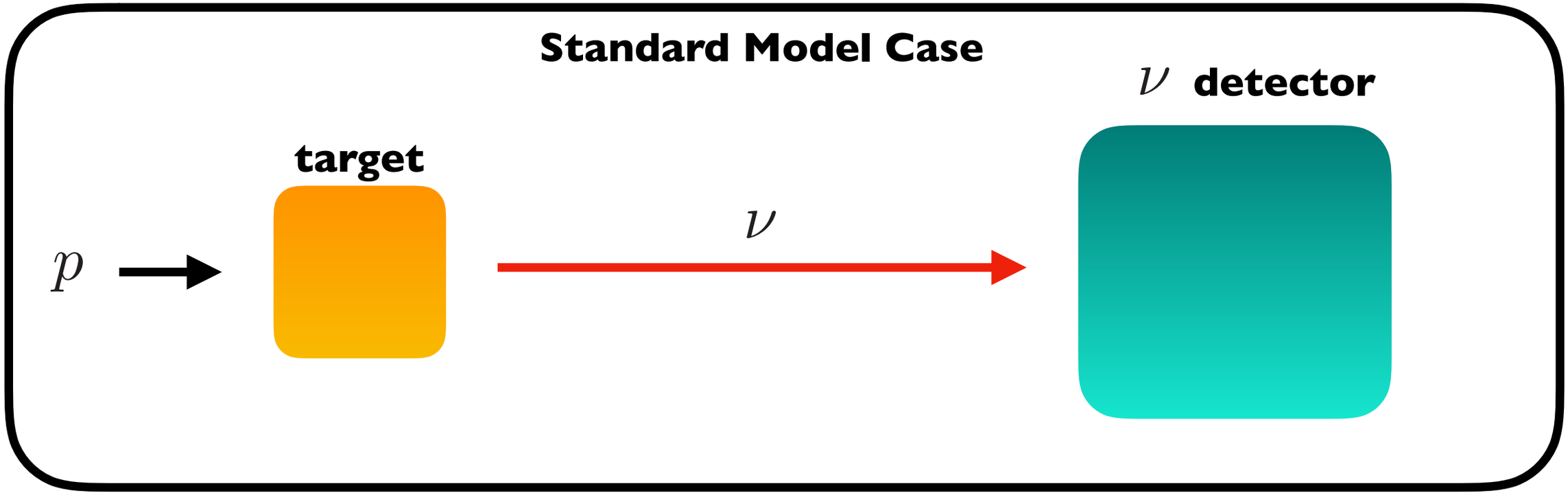

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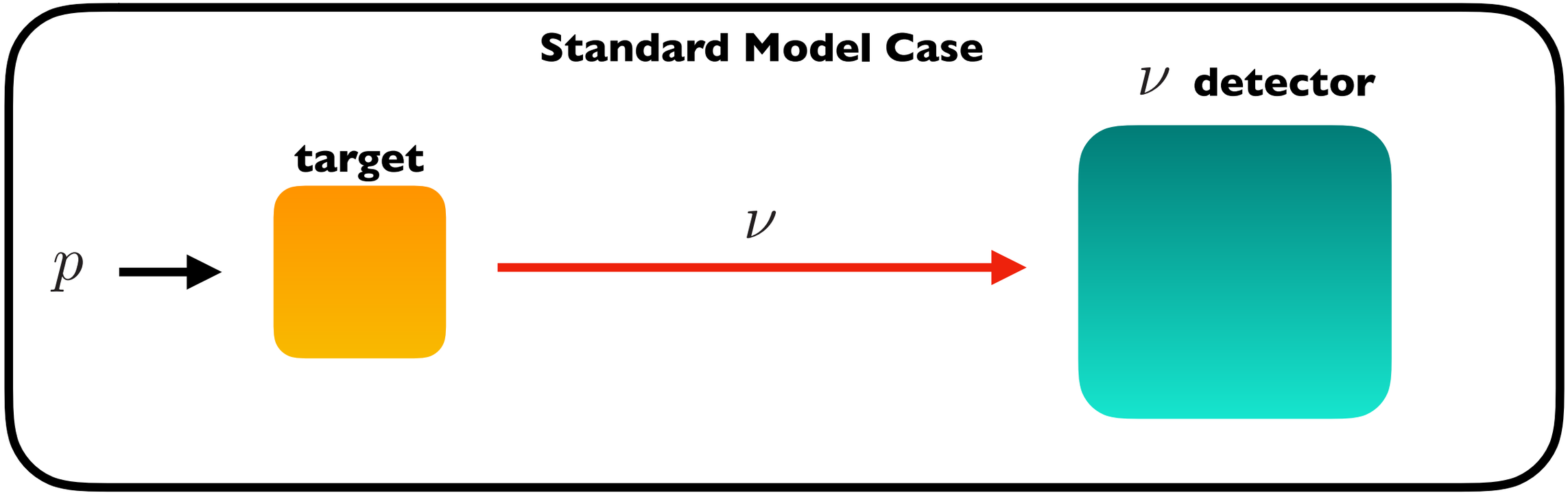

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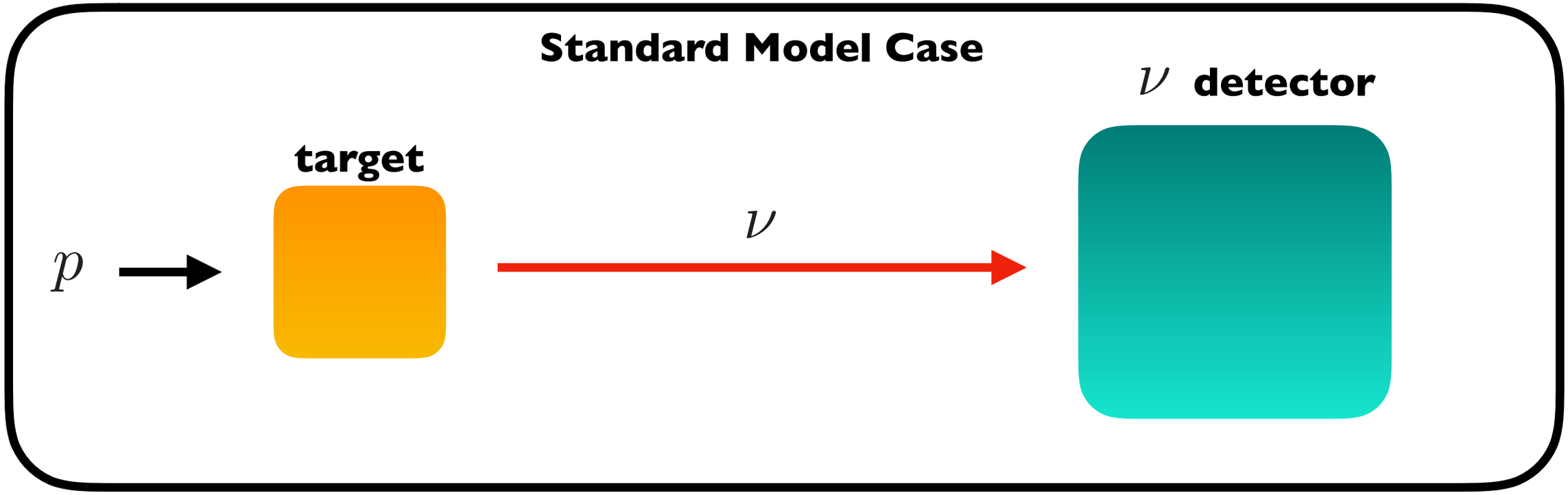

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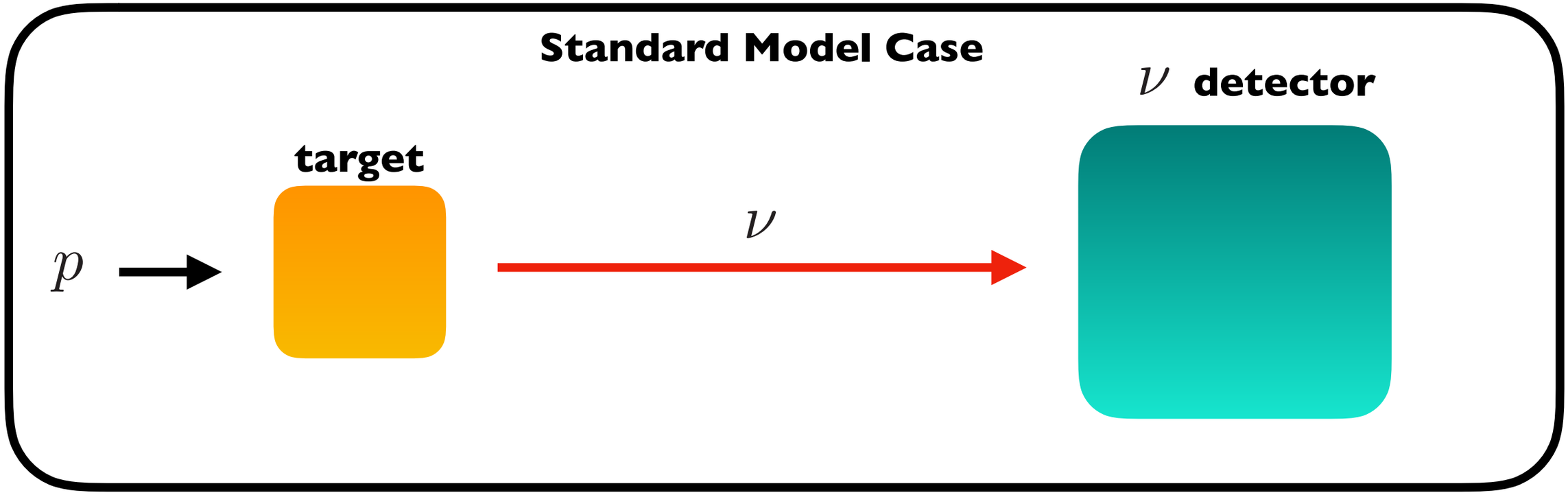

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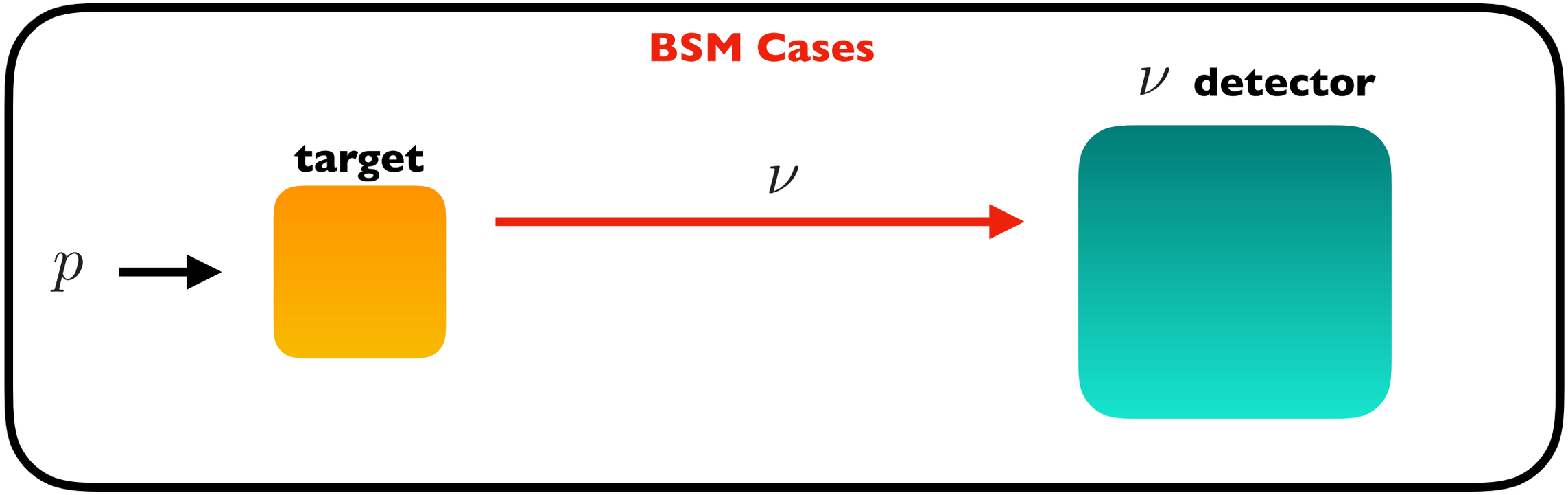

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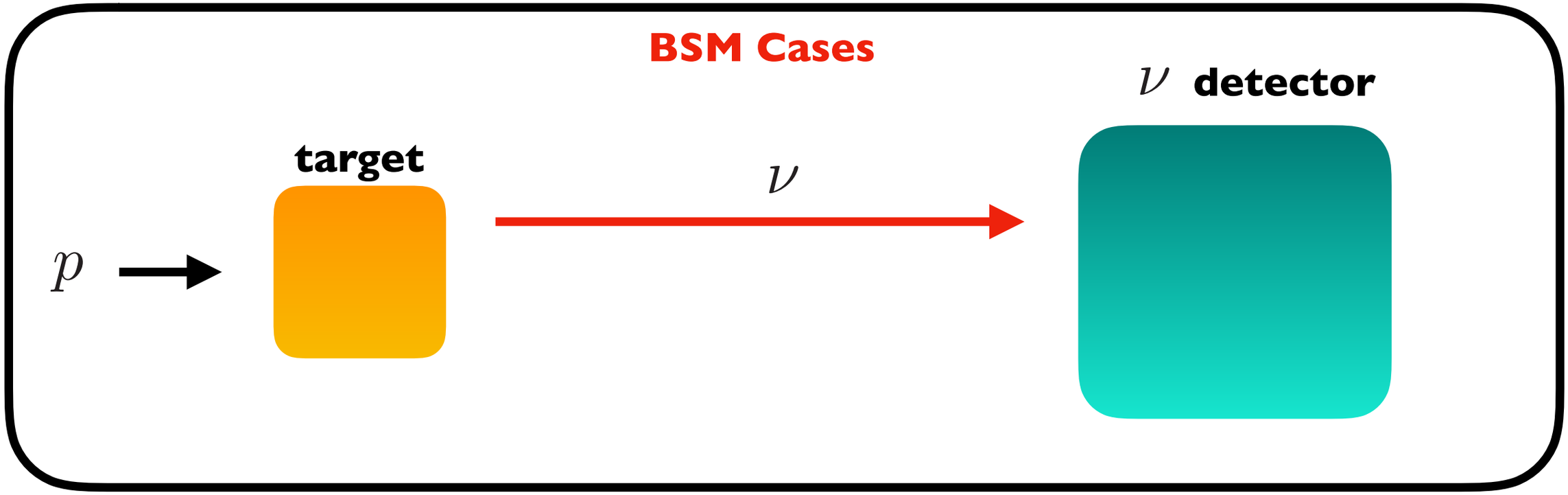

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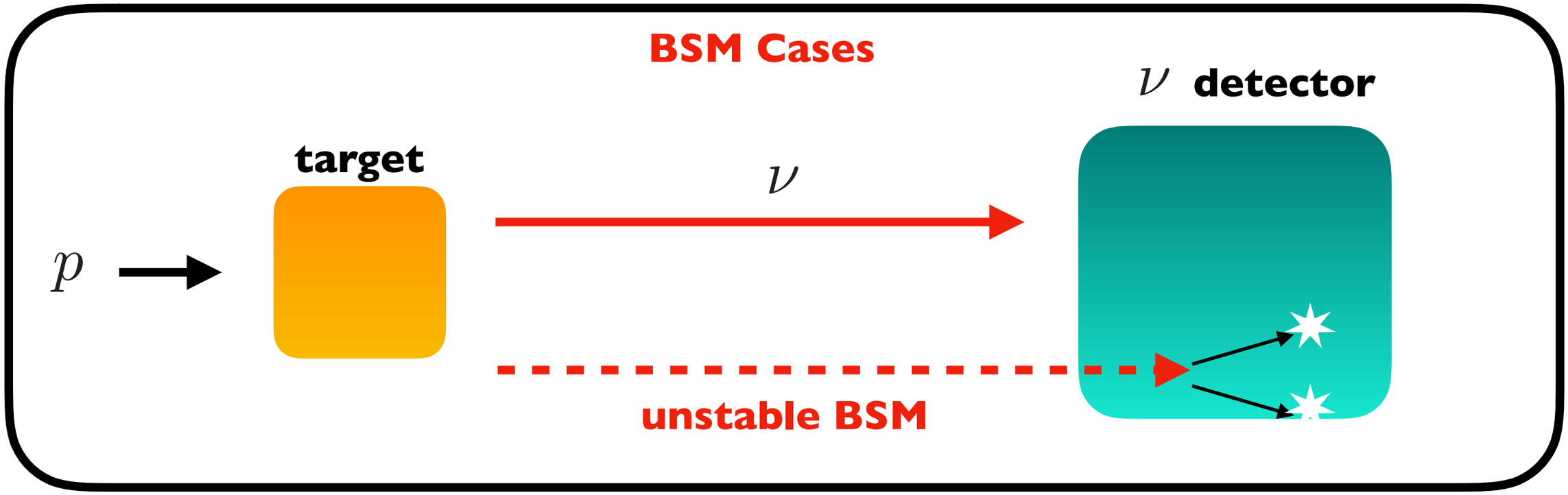

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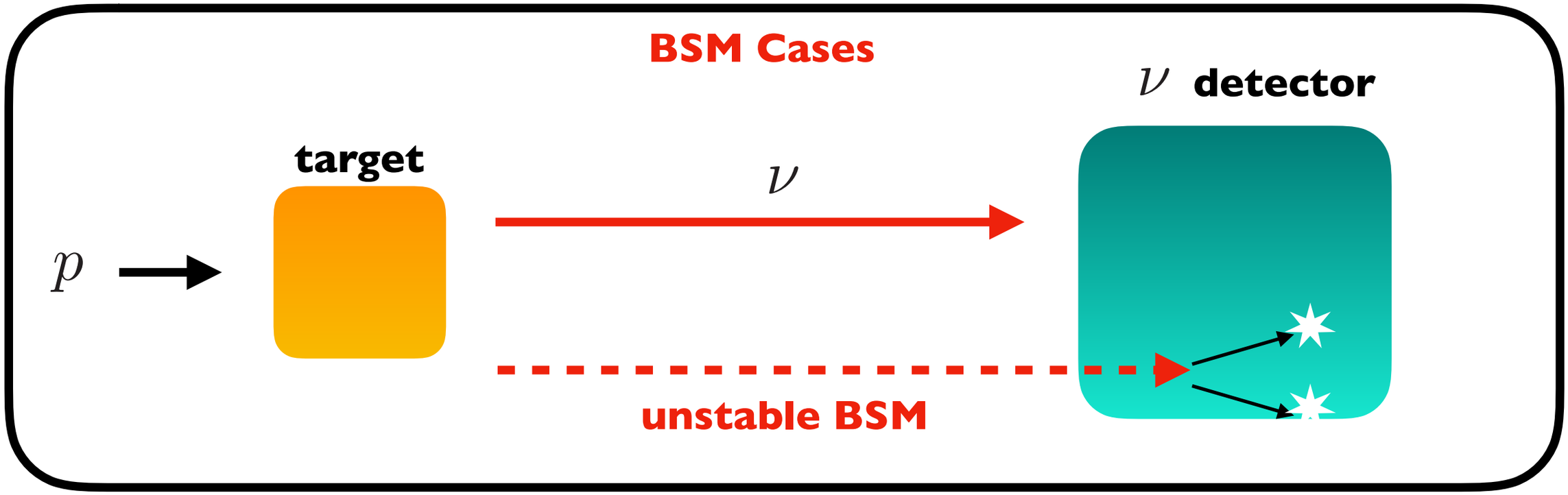

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dark photons,
heavy RH neutrinos, ...

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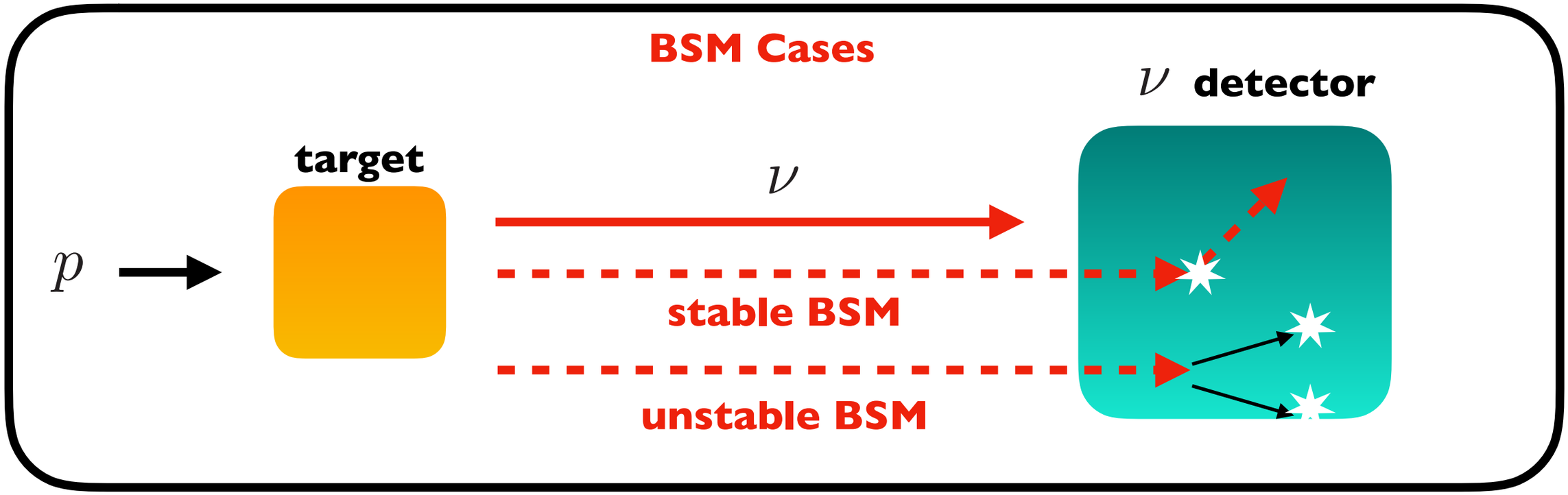

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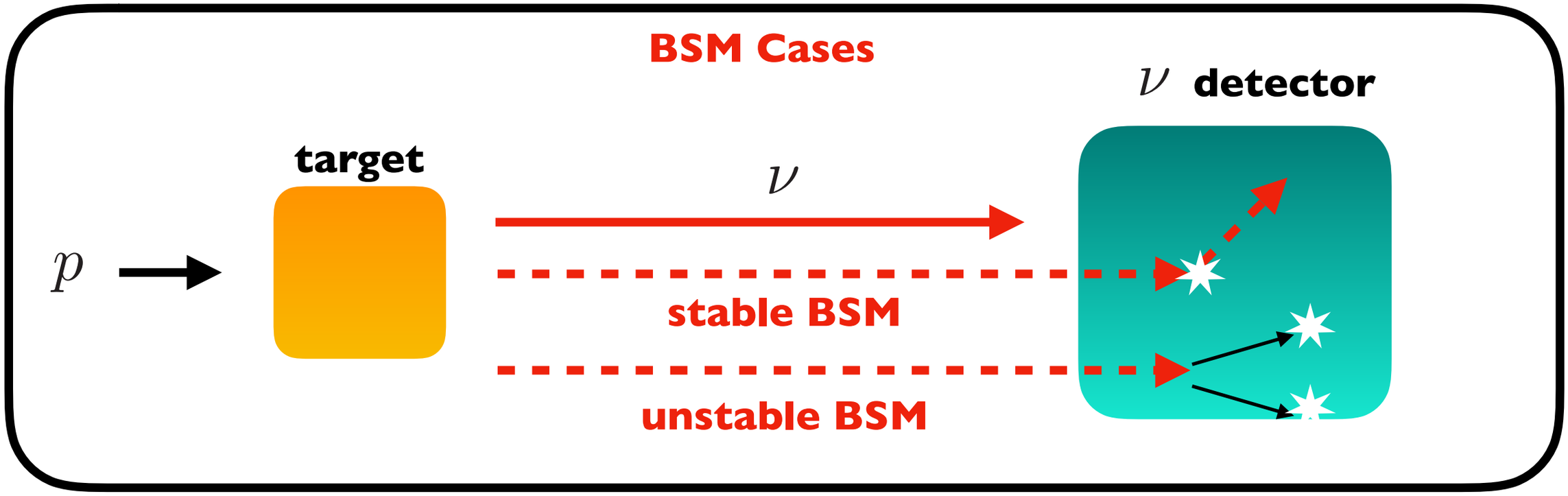

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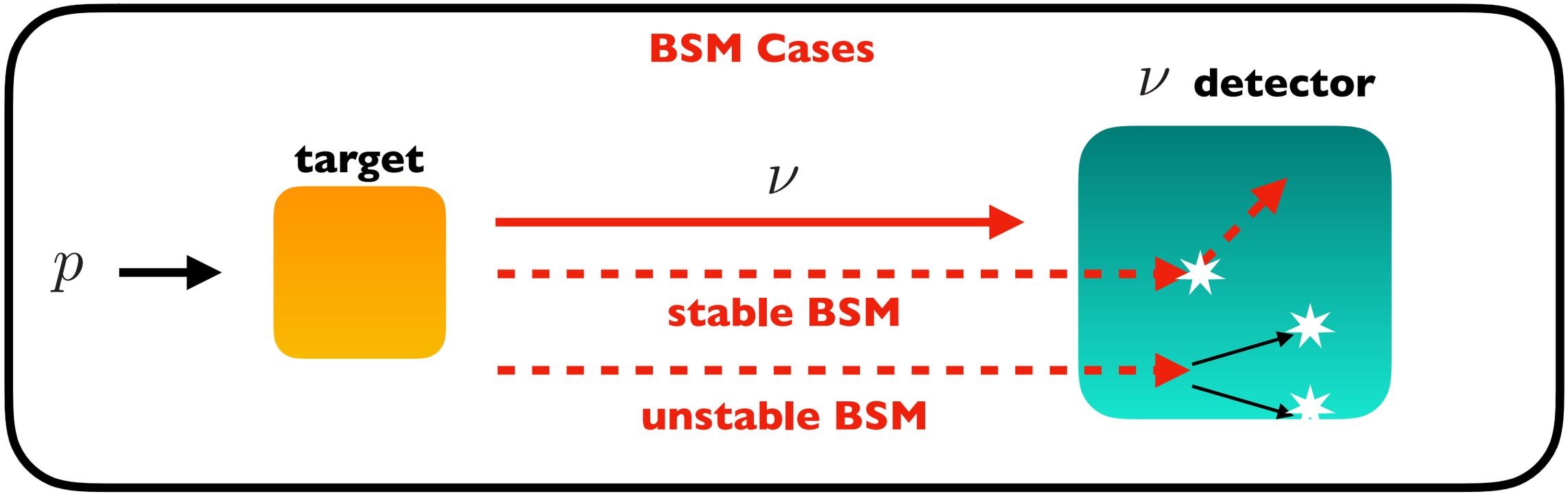

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Marco Drewes & Peter Ballett on Wed.

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Yu-Dai Tsai on Thurs.

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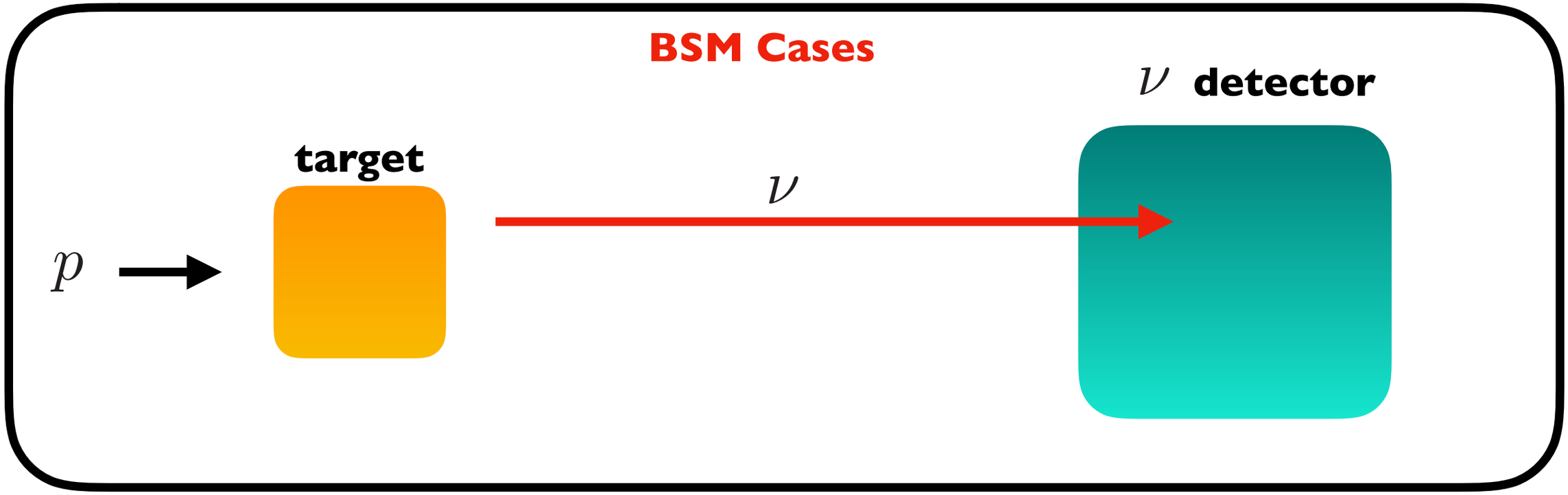

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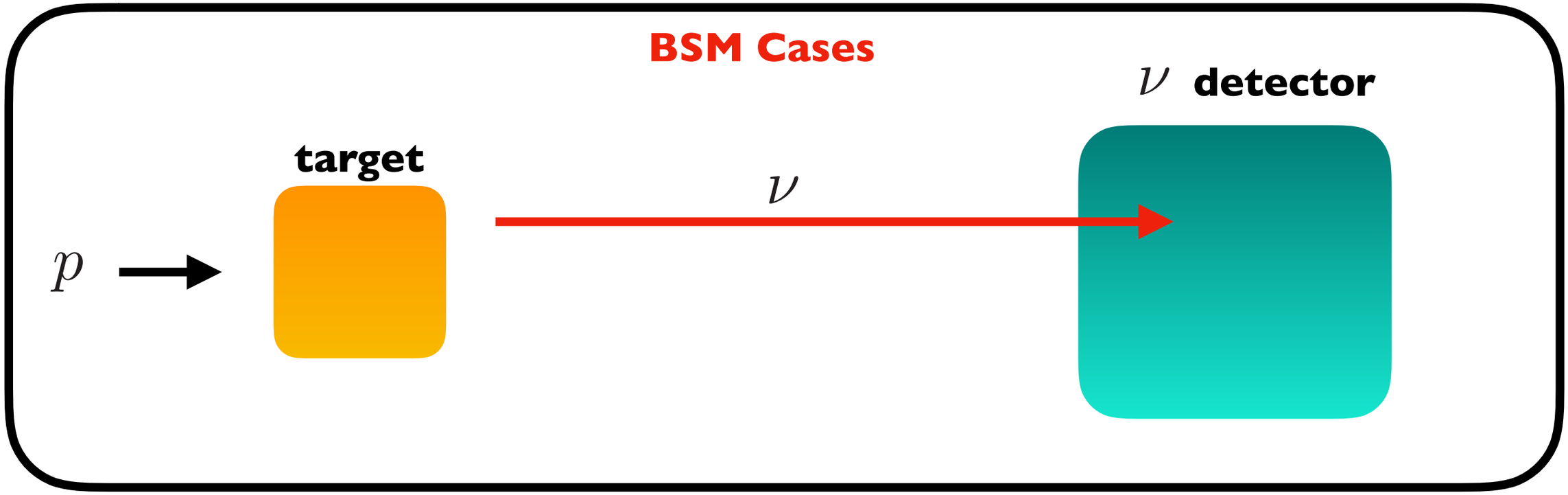

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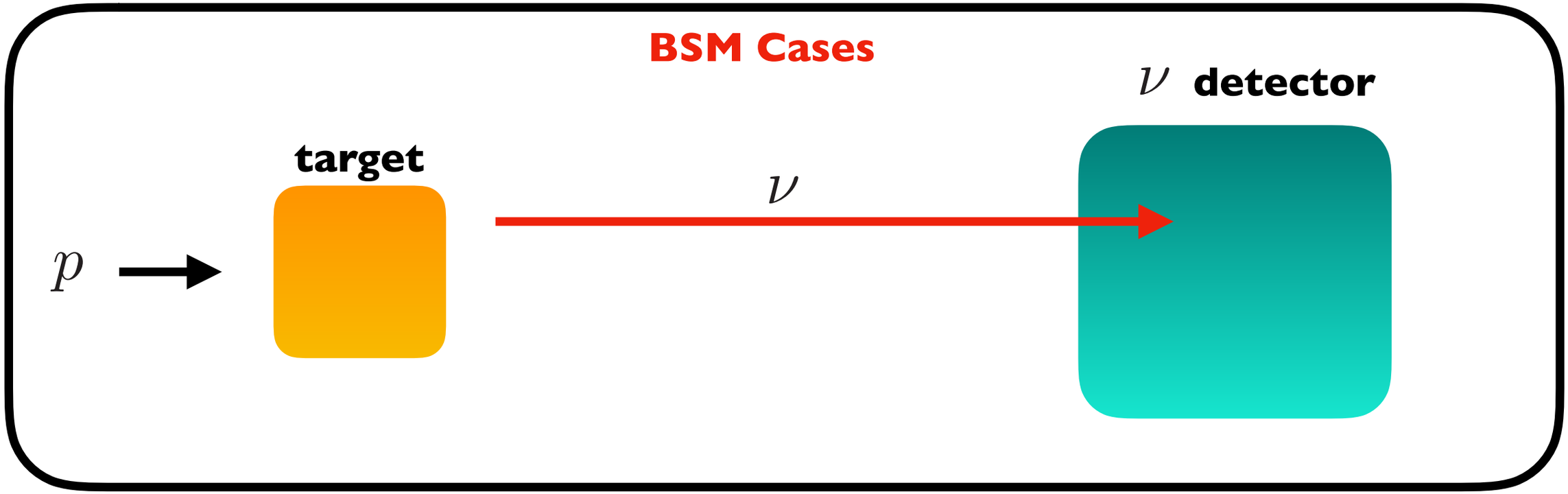

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Gabriel Magill on Thursday

NSI, dipole portal, Z' (tridents) etc..

Jacobo Lopez Pavon on Fri.

Matheus Hostert on Wed.

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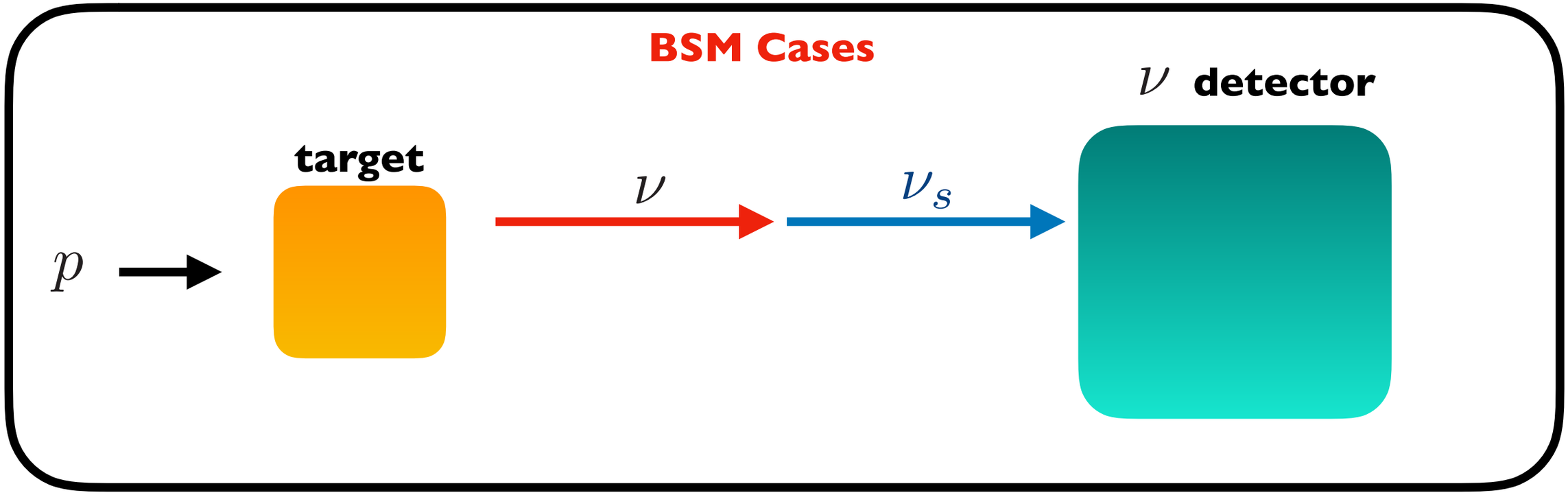

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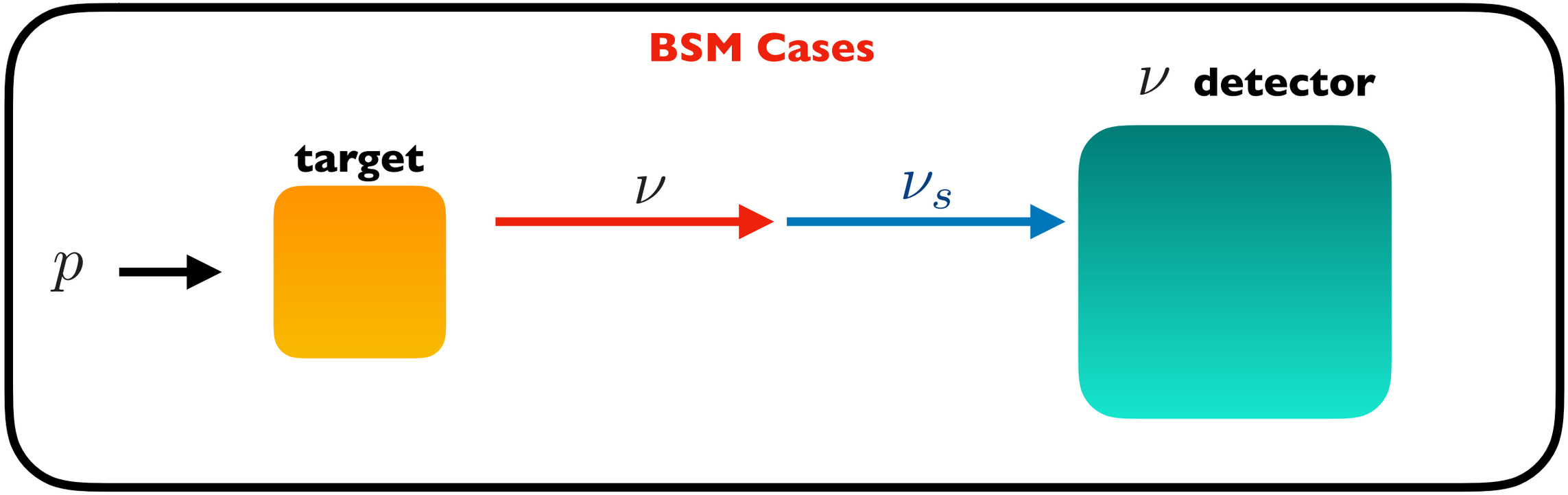

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Sterile Neutrinos

Jacobo Lopez Pavon on Fri.

Plan

- Have we found all of the forces and matter in Nature yet?
    - ***No!*** New Physics might be hiding beneath our noses at relatively low energies=> ***light mediators***.
- Light Dark Matter: thermal relic, asymmetric DM, …
    - Lots of room for **light DM** to hide, **but needs to be accompanied by a light(er) mediator if thermal**.
    - Can produce a DM beam in neutrino experiments.
- Even w/o DM connection, there exist light mediator models that predict new signals at near detectors.

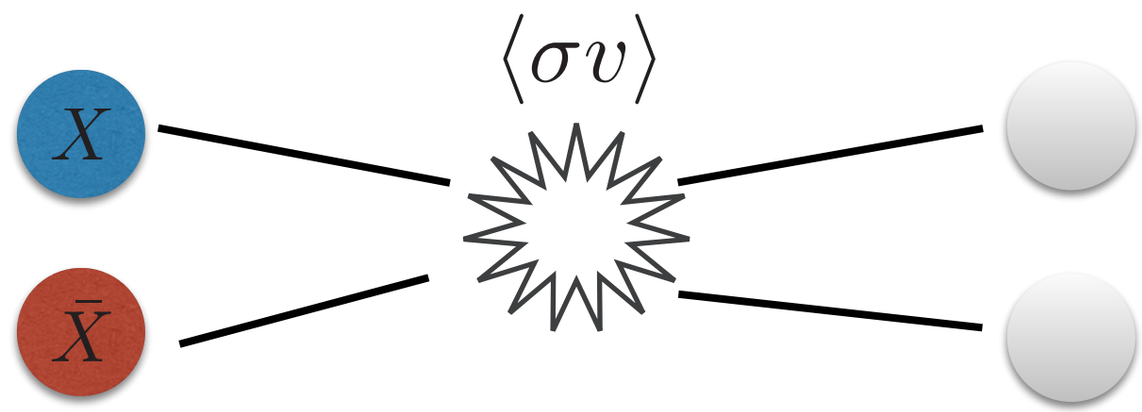
Why light DM?
And why light mediators?

(Wouldn’t Occam be disappointed?)

DM as a Thermal Relic

- The early Universe was a hot/dense place.

DM annihilation



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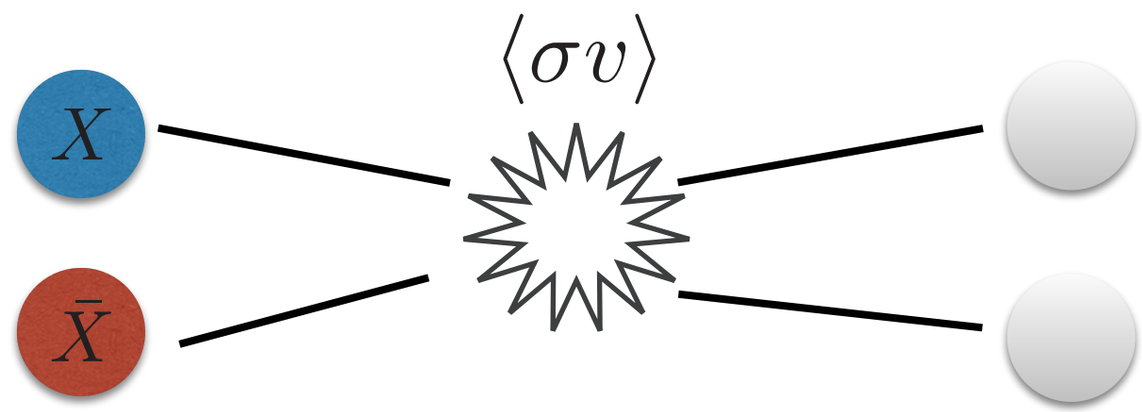

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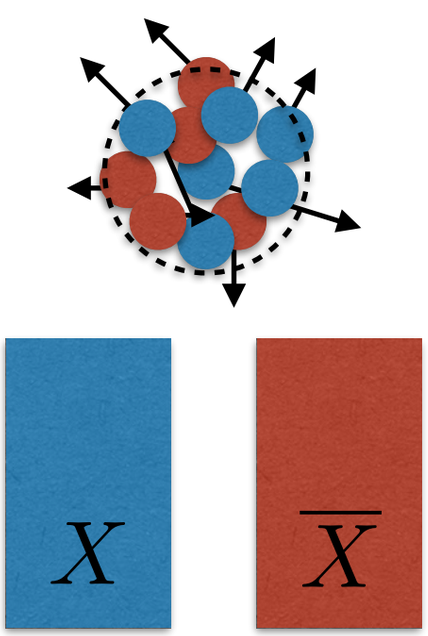

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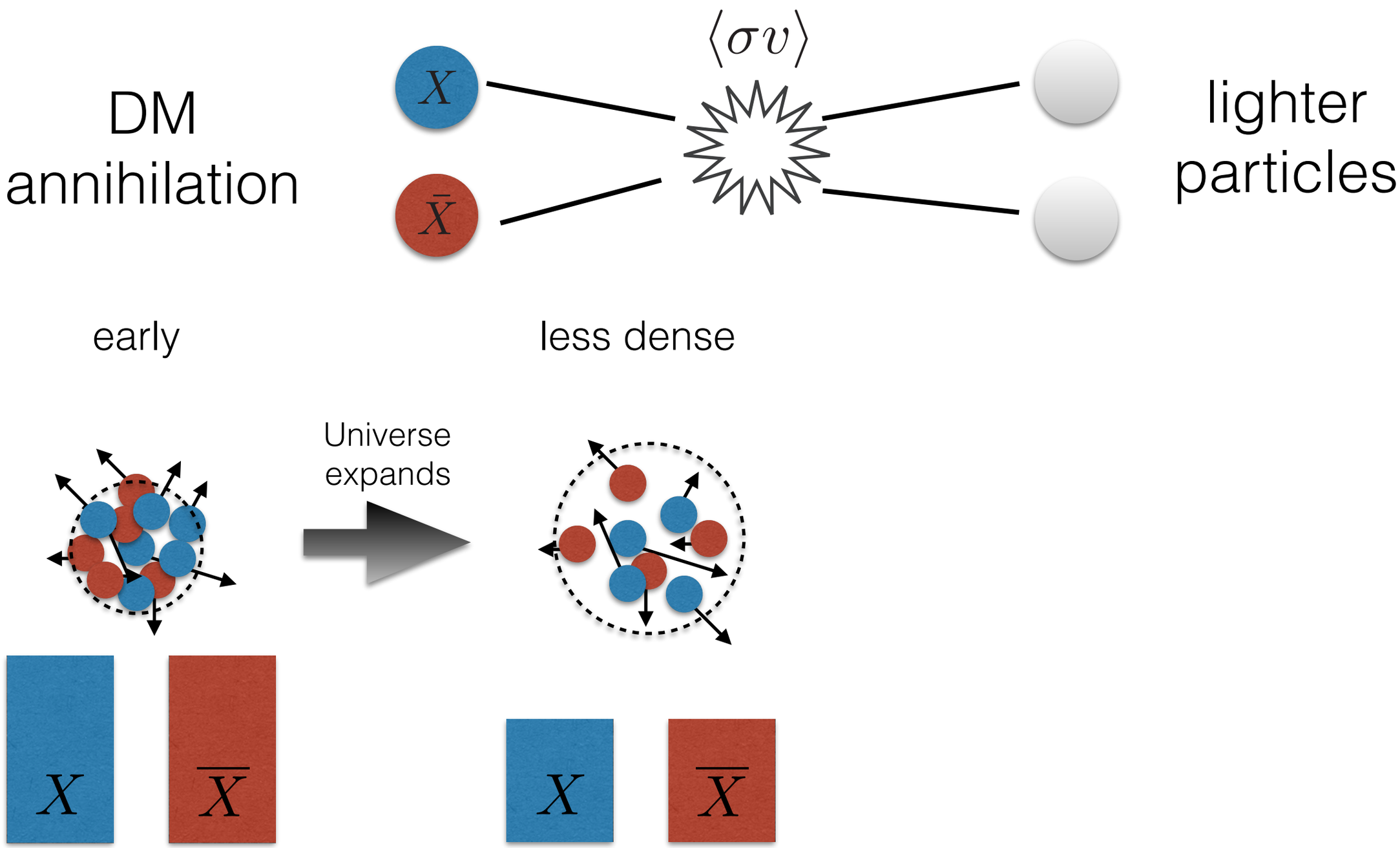

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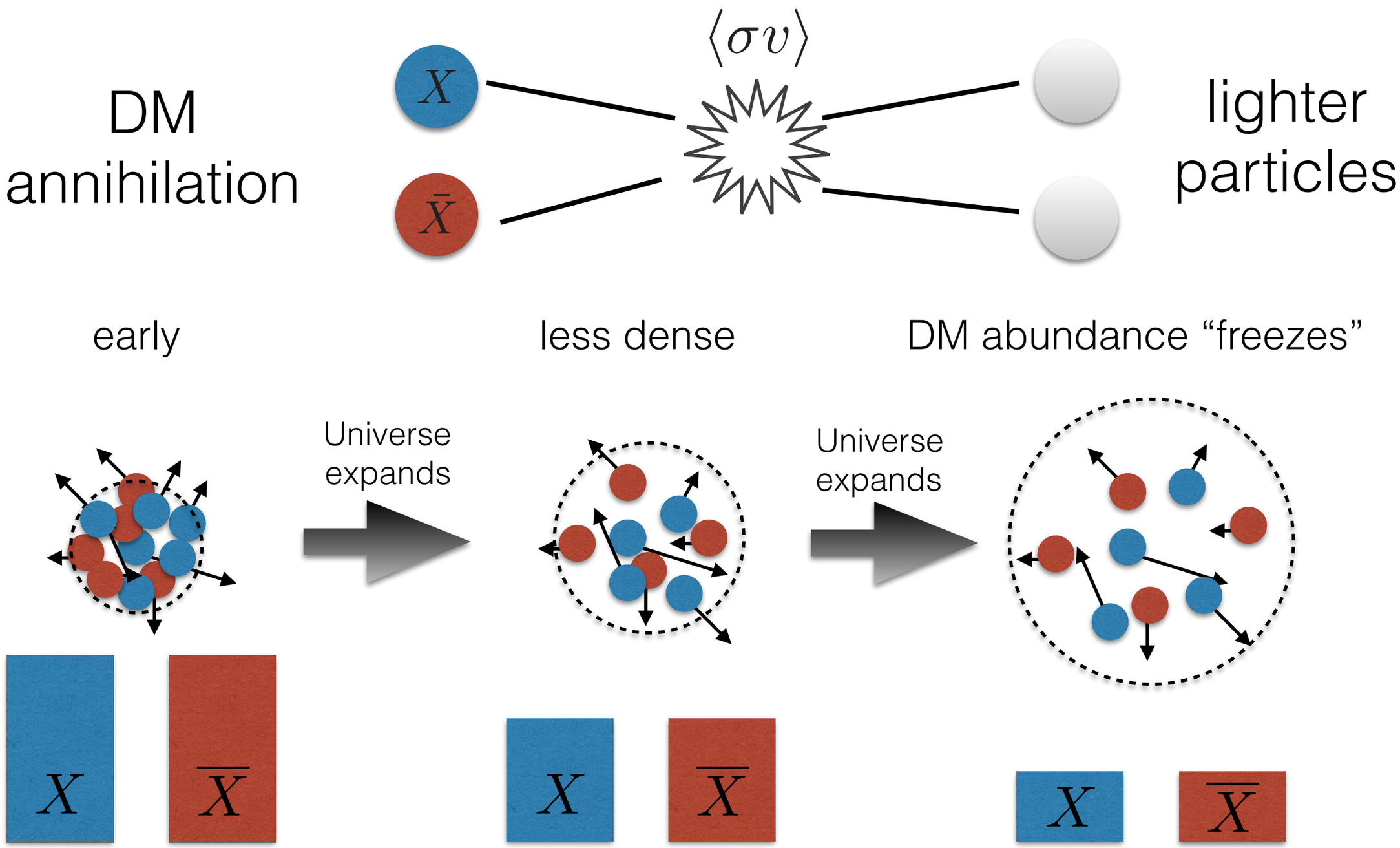

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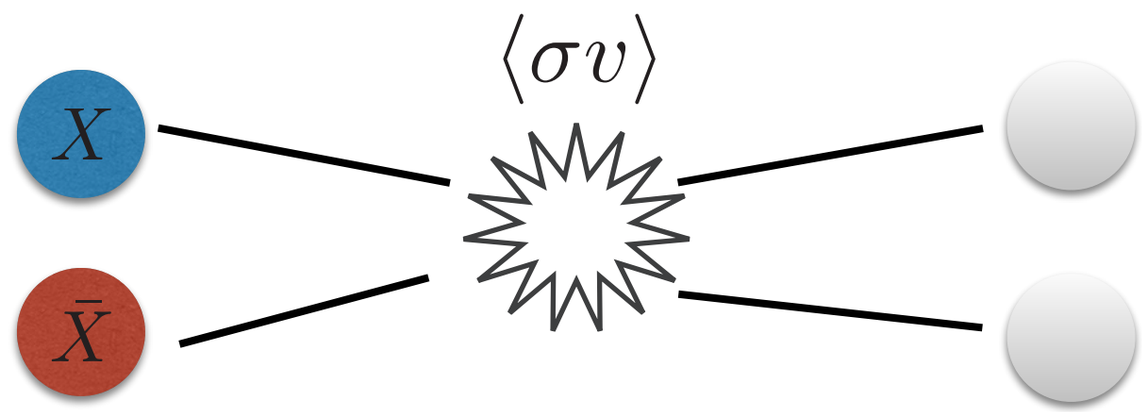

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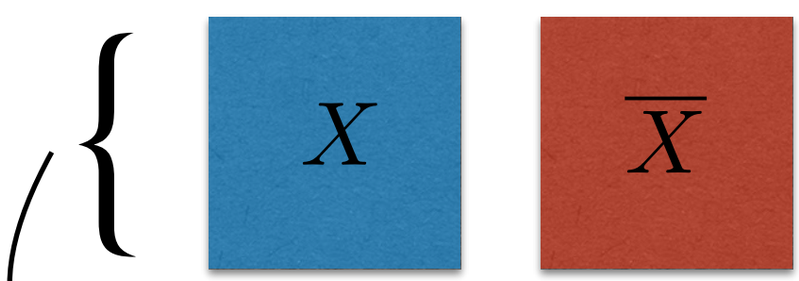
Final "freeze-out" abundance



$$\rho_{DM} \propto \frac{1}{\langle \sigma v \rangle}$$

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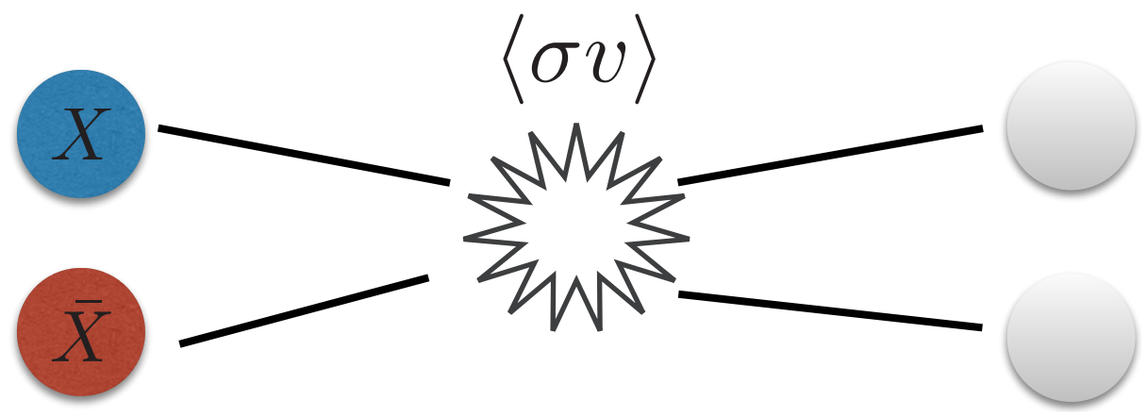

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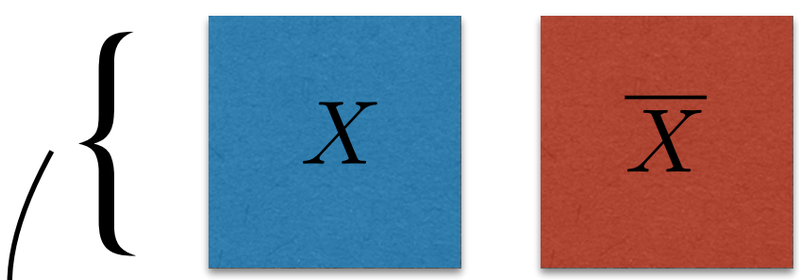

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A thermal relic has the observed DM abundance if:

$$\langle \sigma v \rangle = 3 \times 10^{-26}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$$

"WIMP miracle"

WIMP = Weakly-Interacting Massive Particle

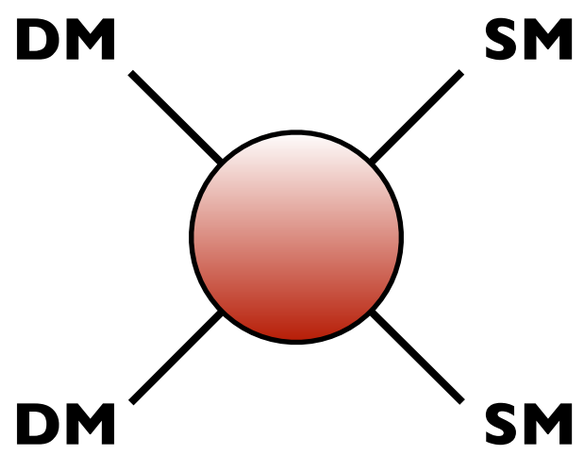
Search Complementarity

"Break it" - Indirect Detection

Search for products of DM annihilation in regions of high DM density.

"Wait for it" Direct Detection

DM-SM scattering in detector



"Make it" - Colliders

Produce DM and find anomalous missing energy.

Elegant, compelling,
but not **unique.**

What about baryons?

- The amounts of dark and visible matter are **comparable**:

$$\left.\begin{aligned} \Omega_{DM} h^2 &= 0.1109 \pm 0.0056 \\ \Omega_B h^2 &= 0.002258^{+0.00057}_{-0.00056} \end{aligned}\right\} \quad \frac{\Omega_{DM}}{\Omega_B} \simeq 5$$

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  - An indication of an underlying origin.

Asymmetric DM

(Reviews: Petraki, Volkas [1305.4939]; Zurek [1308.0338])

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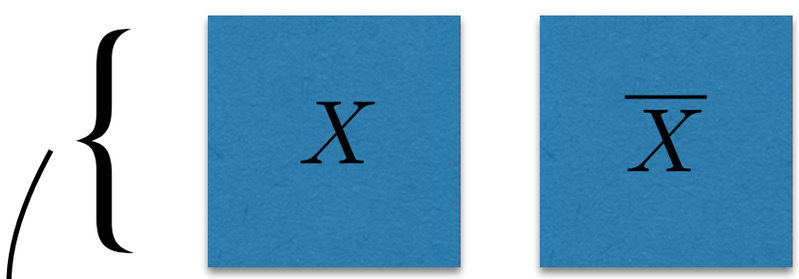

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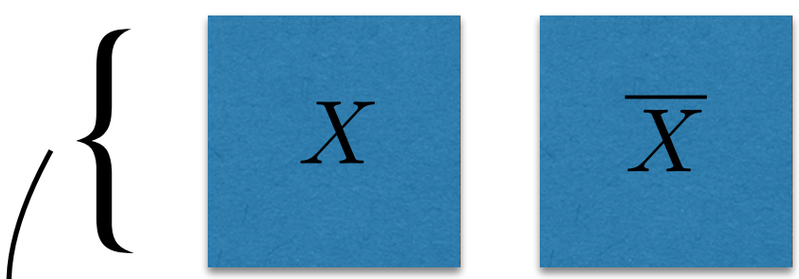

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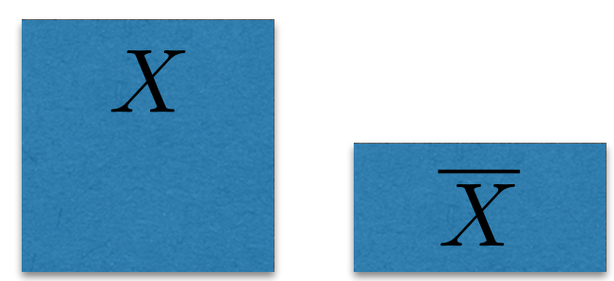

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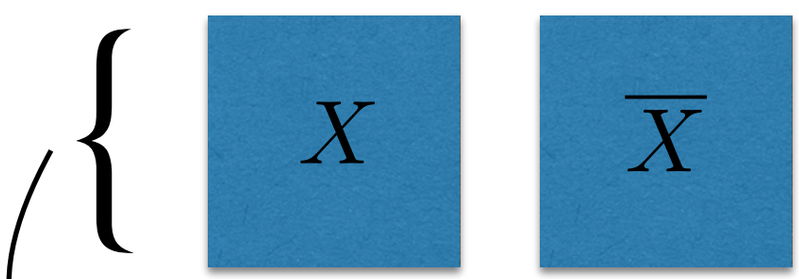

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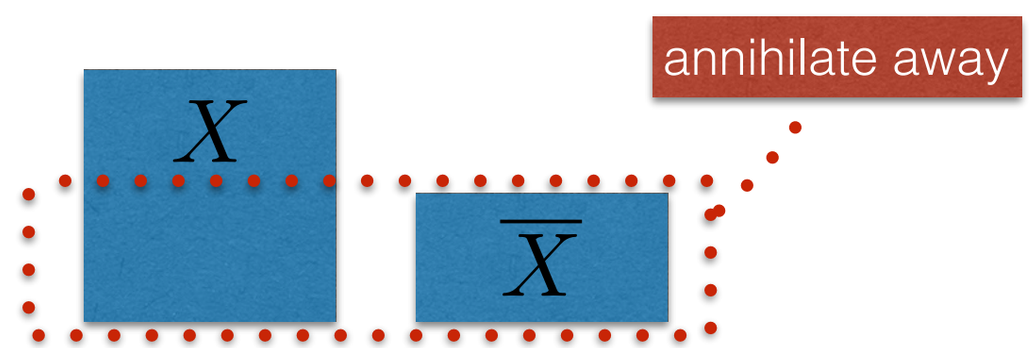

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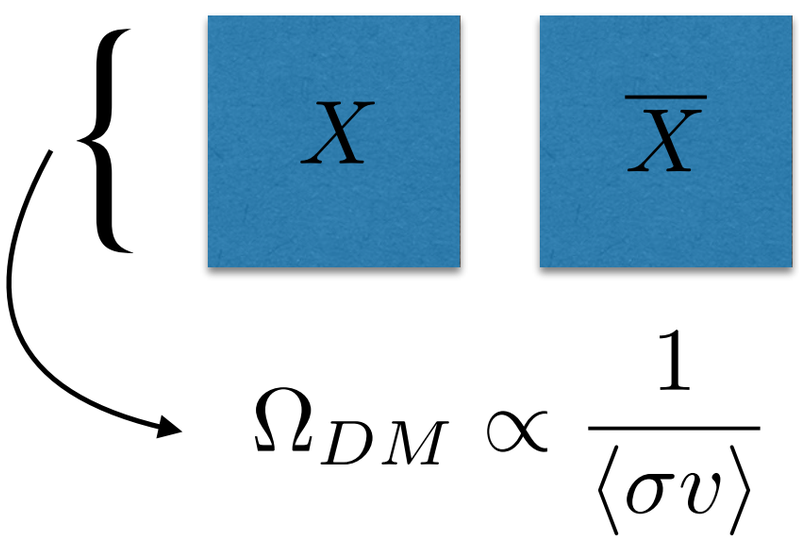

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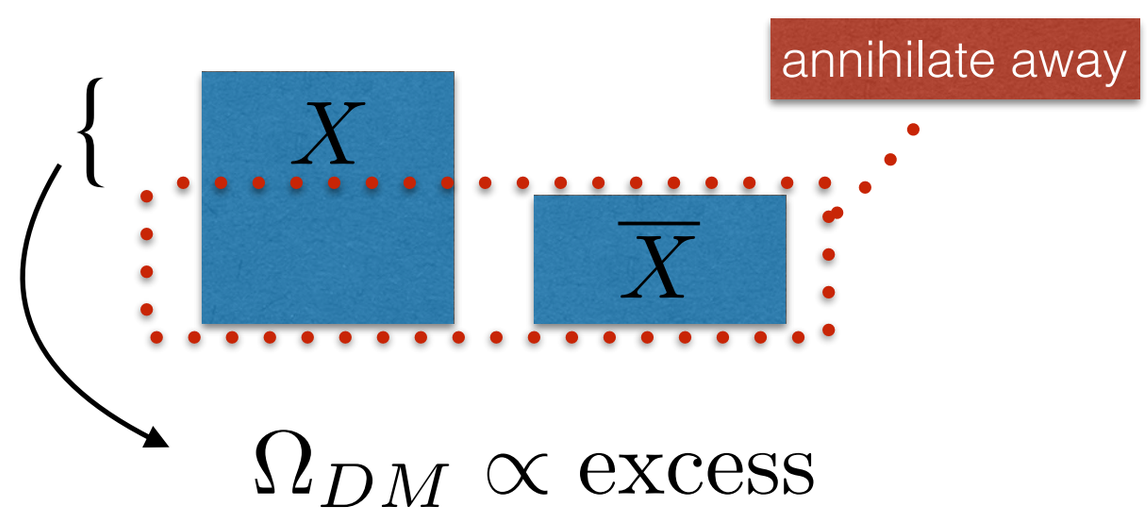
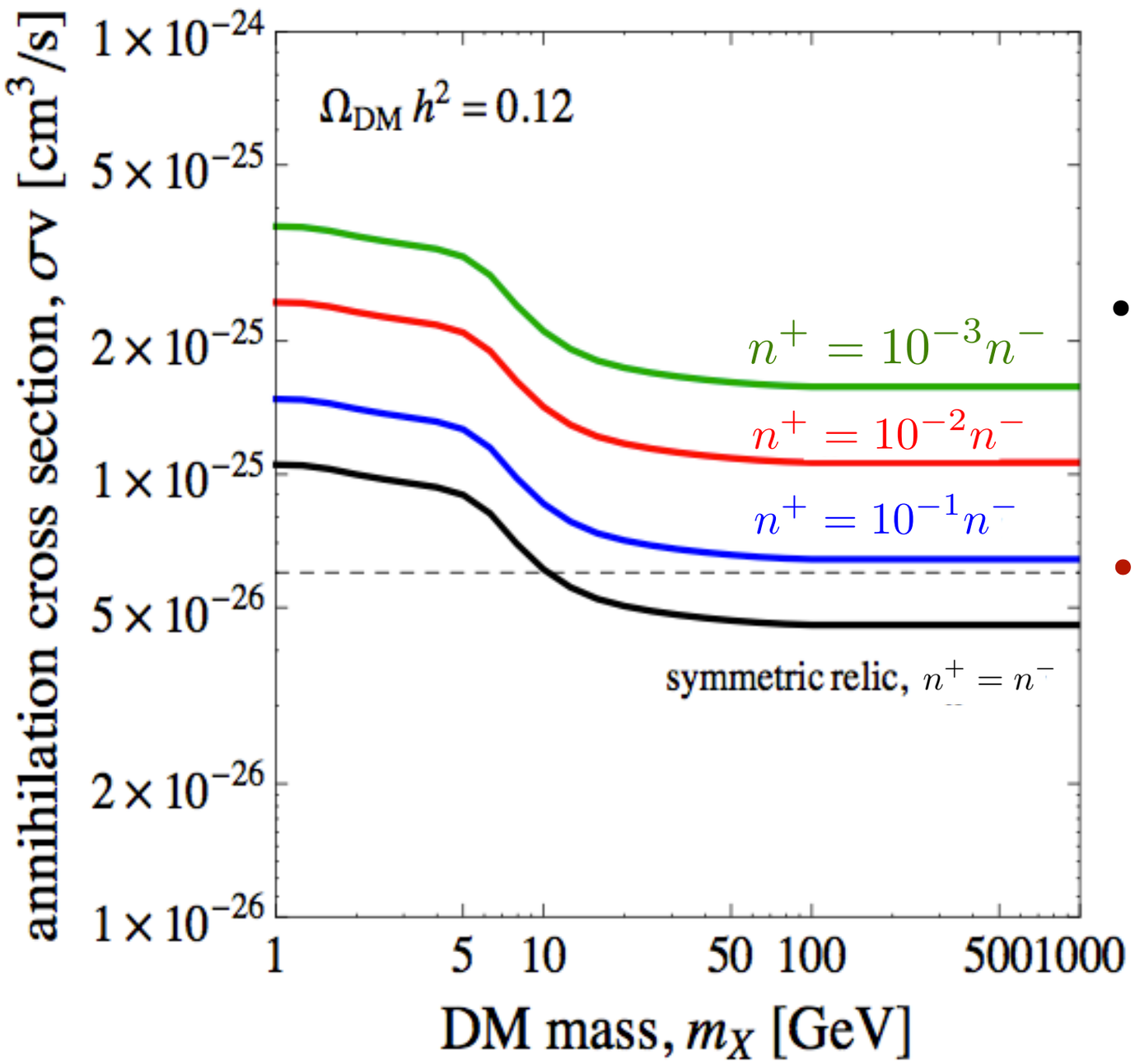


ADM "Miracle" Cross Sections

Michael Graesser, **IMS**, and Luca Vecchi, JHEP 1110 (2011) 110.

Lin, Yu, Zurek, Phys.Rev. D85 (2012) 063503 .

Nicole Bell, Shunsaku Horiuchi, **IMS**, *Phys.Rev.* D91 (2015) 2, 023505.

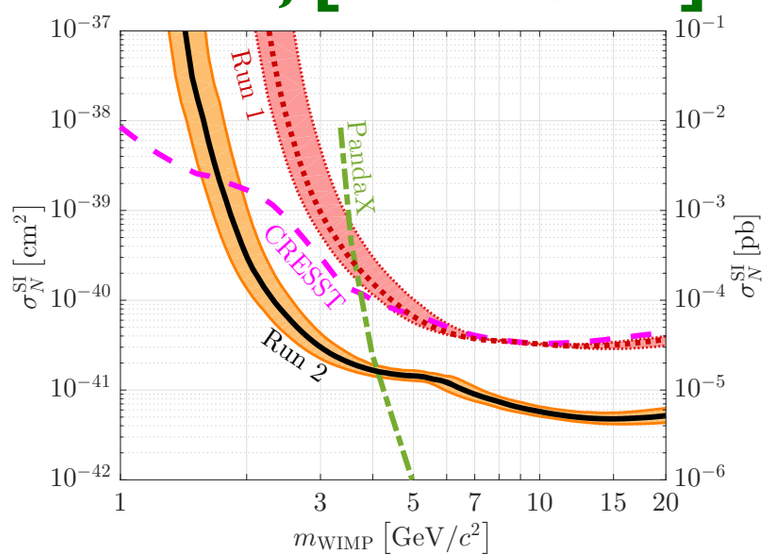


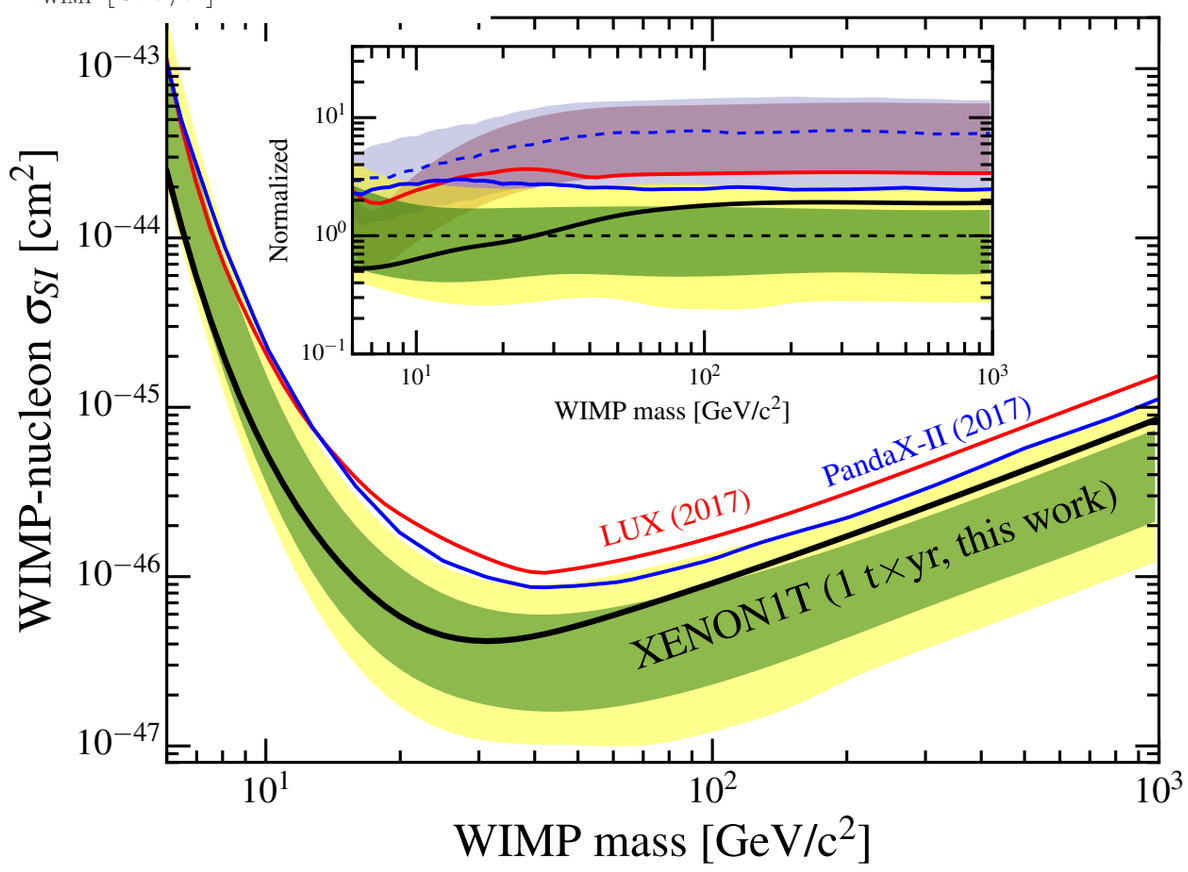
- **Size of cross section, will yield different anti-particle abundances.**
- **Cross sections needed are larger than the symmetric case.**

Where can DM still be hiding?

Direct Detection vs. DM

CDMSlite, [1707.01632]





XENON1T, [1805.12562]

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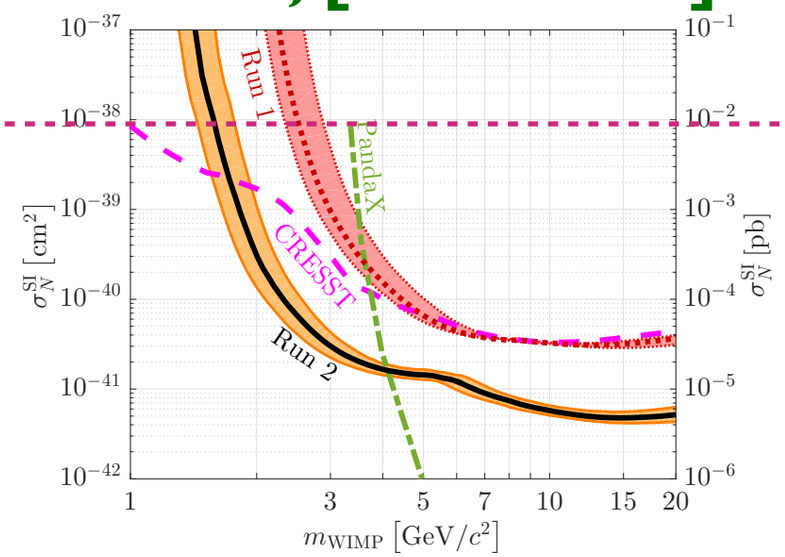
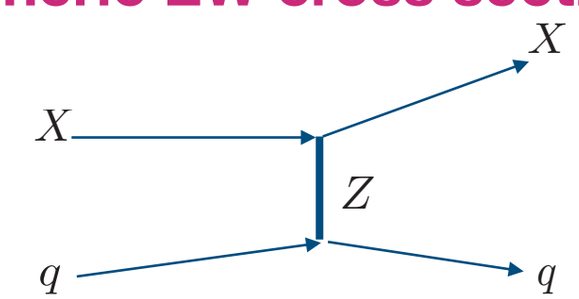
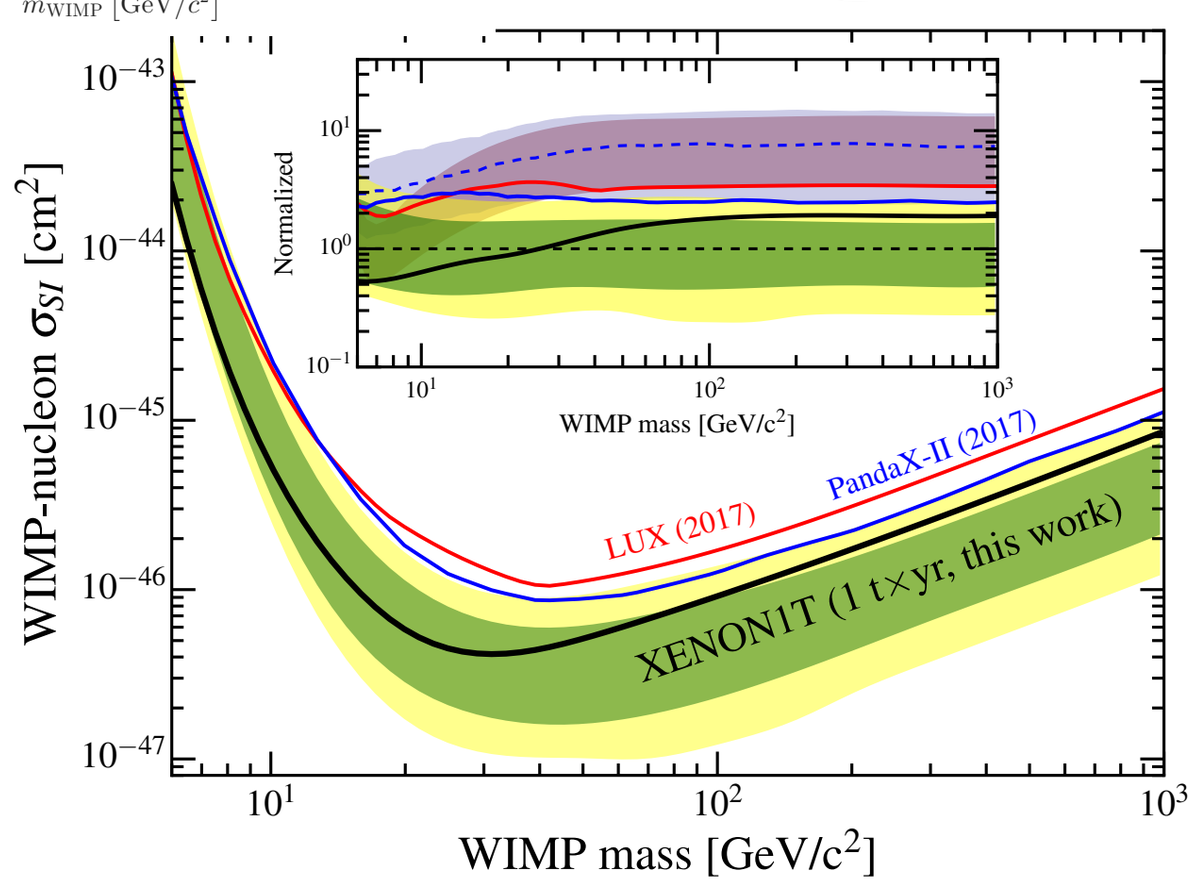


Generic EW cross section:

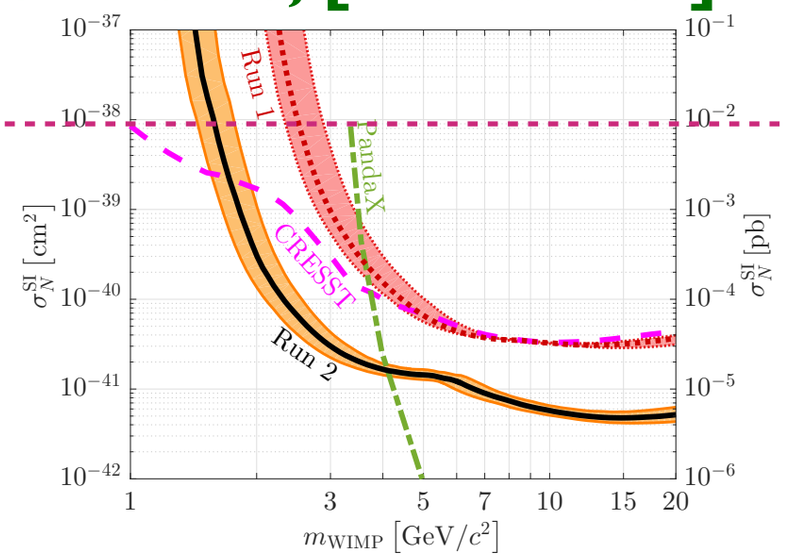


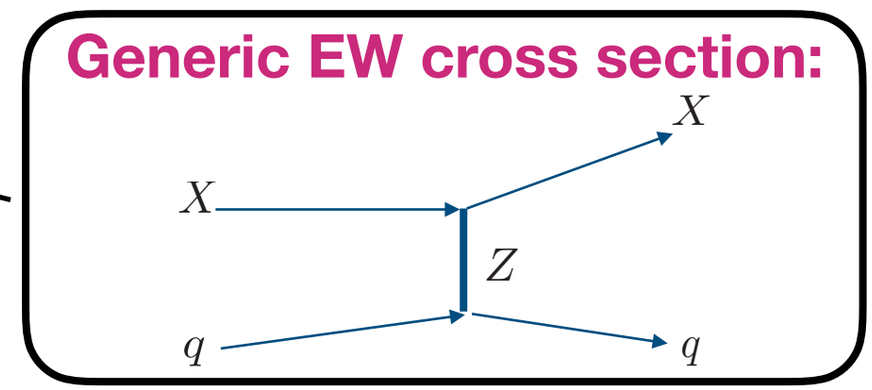


XENON1T, [1805.12562]

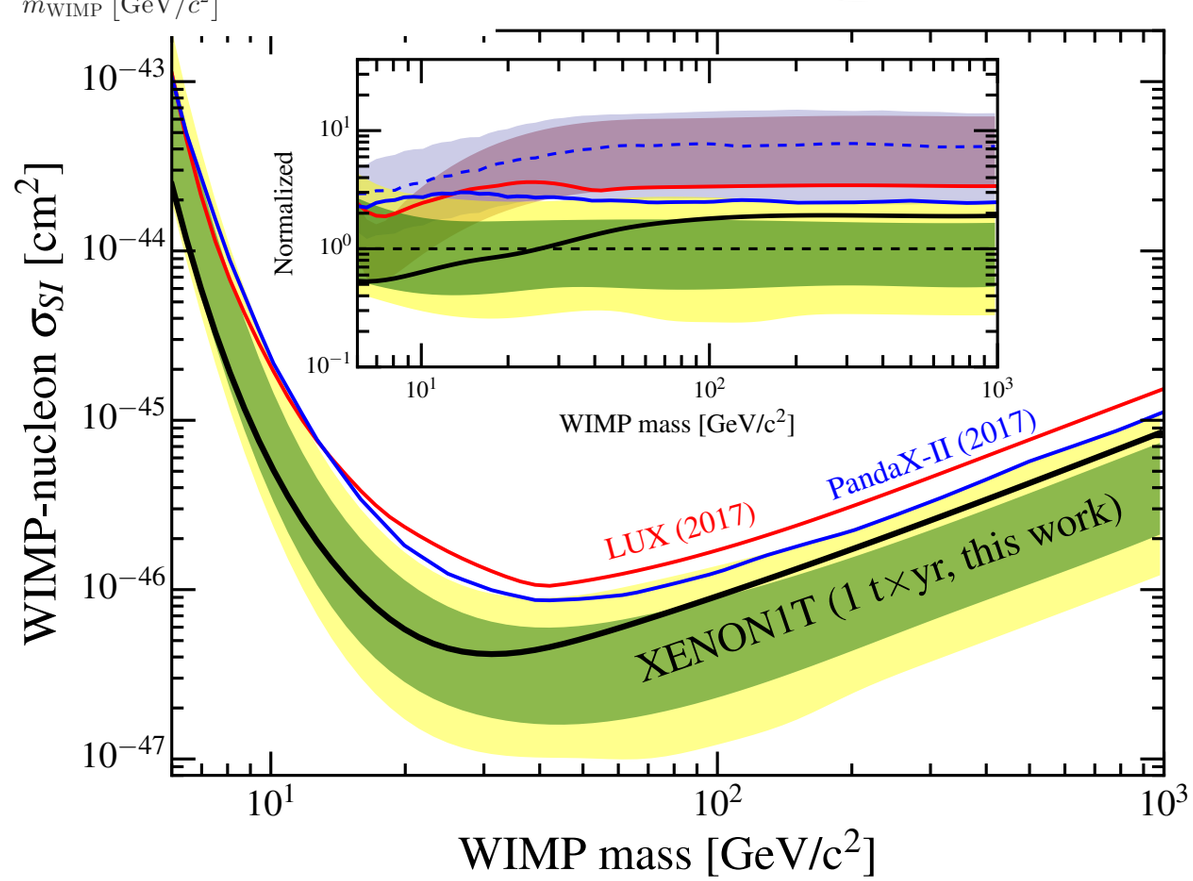
Direct Detection vs. DM

CDMSlite, [1707.01632]





- **Sub-GeV DM is challenging given Milky Way DM speeds < 500 km/s.**



XENON1T, [1805.12562]

Light DM via Light Mediators

[Lee, Weinberg (1977)]

- Suppose we like **sub-GeV** DM but also like Occam, and want to just use the **SM weak force** to yield the relic abundance of DM.

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- To regimes for annihilation:

$$\langle \sigma v \rangle \sim \frac{g^4 m_X^2}{m_W^4}$$

[light DM, (<<W mass)]

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[heavy DM]

Fails for sub-GeV DM.

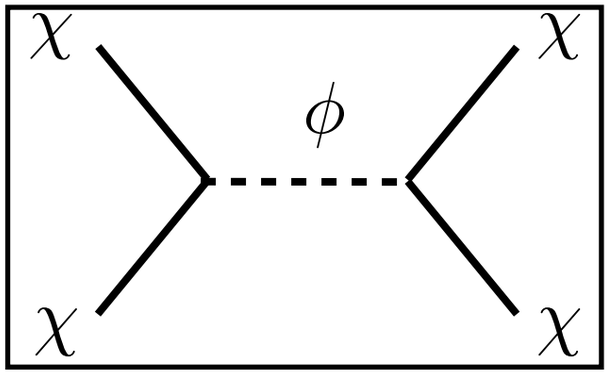
Simple escape route for sufficient annihilation: light DM is non-Occam! Comes with a <u>light mediator</u> to facilitate annihilation.

[Boehm, Fayet (2003)]

Consequences of a light mediator for annihilation

1) **Potentially sizable self-interactions.**
2) **Distinct experimental search strategies.**

Small-scale structure problems?



- To get rid of cold cores, bring them in contact with hotter components of the halo via **self-scattering**. (Spergel & Steinhardt, PRL 2000, +100s more papers).

- DM-DM scattering. Required cross section is

$$\sigma \sim 10^{-24}\ \mathrm{cm}^2 \left(\frac{m_X}{1\ \mathrm{GeV}}\right)$$

- Needs a $\lesssim 10\ \mathrm{MeV}$ force carrier for large DM self-interactions.

See also Loeb, Weiner (2010); Tulin, Yu, Zurek (2013).

How does the requisite annihilation occur?

Secluded DM

Dark (Hidden) ((Secluded)) Sector Models

[Batell, Pospelov, Ritz (2009)]

Secluded DM

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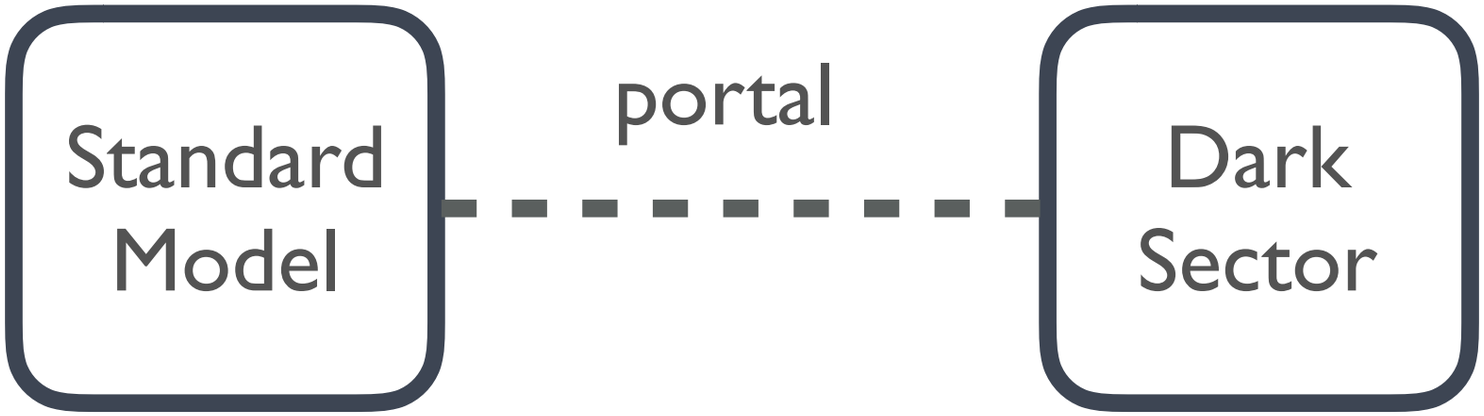

$$\mathscr{L}_{\text{portal}} = \begin{cases} \epsilon F_{\mu\nu} F_h'^{\mu\nu} & \text{(photon portal)} \\ h|H^2||H_h^2| & \text{(Higgs portal)} \\ y(LH)N & \text{(neutrino portal)}, \end{cases}$$

Only 3 renormalizable portals!

Photon Portal DM

[Holdom 1986; Batell, Pospelov, Ritz, 0906.5614]

$$\mathcal{L}_{V,\chi} = |D_\mu \chi|^2 - m_\chi^2 |\chi|^2 - \frac{1}{4} V_{\mu\nu}^2 + \frac{1}{2} m_V^2 V_\mu^2 + \epsilon V_{\mu\nu} F^{\mu\nu} + \ldots$$

$$D_\mu = \partial_\mu - i g_D V_\mu \ , \quad g_D = \sqrt{4\pi\alpha_D}$$

4 parameters: $m_\chi,\ m_V,\ \epsilon,\ \alpha_D$

- **For scalar DM, annihilation to SM particles is velocity-dependent (p-wave).**

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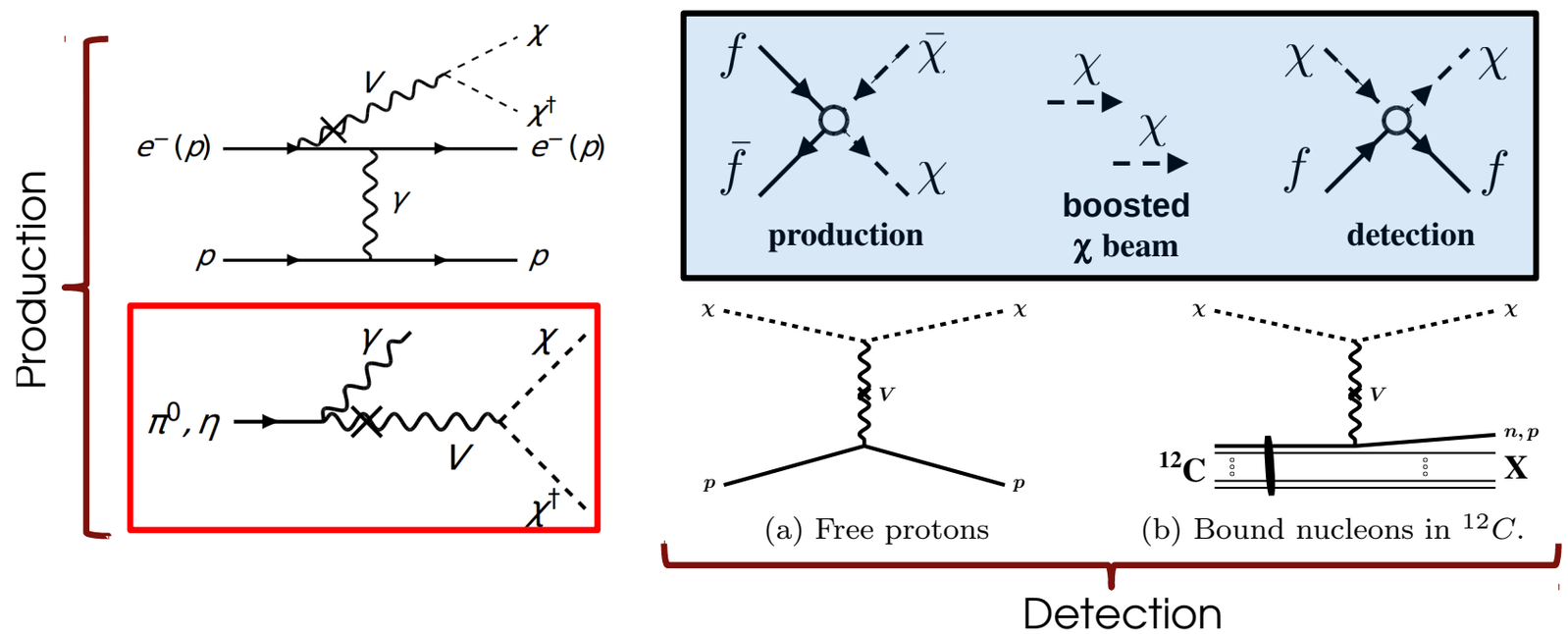
→ **Safe from strong CMB bounds on DM annihilation to EM states.**

- **Simple modification with Fermion DM works if Asymmetric (i.e. antiparticles << particles).**

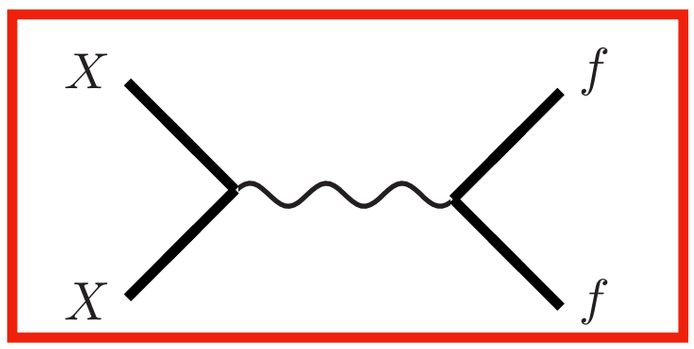
A Light DM Beam

[Batell, Pospelov, Ritz, 0906.5614, MiniBooNE 1702.02688]



Total event rate~ (branching)x(DM-N cross section) : $\sim \epsilon^4 \alpha_D$

Main assumption, light mediator can decay to DM: $m_V > 2m_\chi$

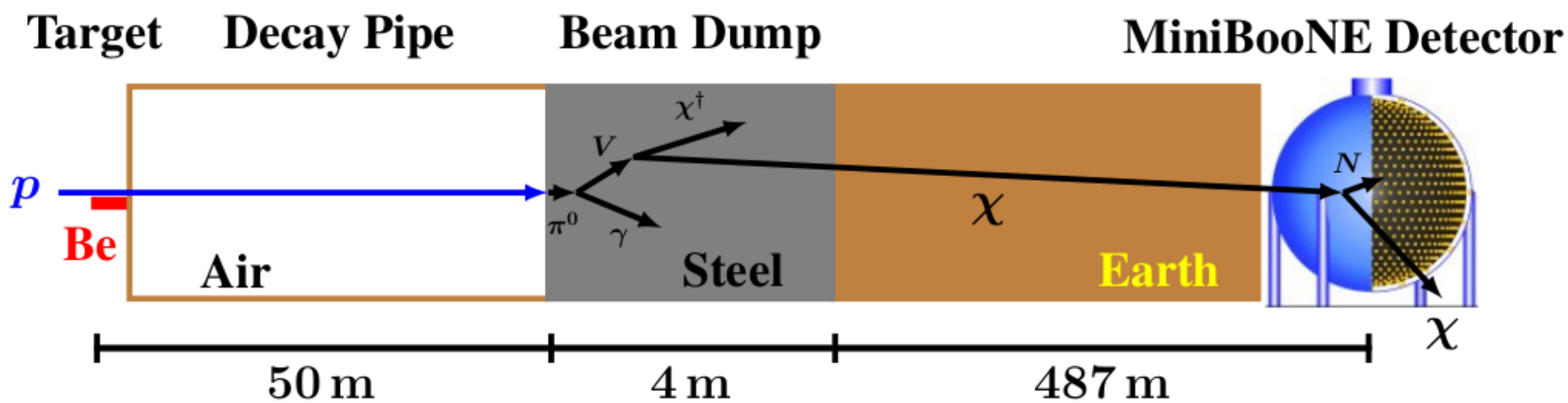


DM annihilation rate: $\sim \epsilon^2 \alpha_D$

A Light DM Beam

[1702.02688]

MiniBooNE in "off-target" mode

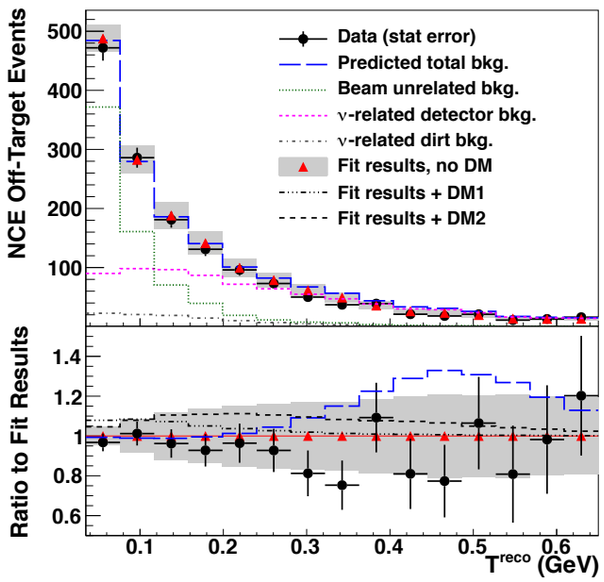


- **Rather than reanalyze old data, this was first dedicated search of this type!**

- **Instead of impacting the Beryillium target, the 8 GeV protons are steered off-target to steel target.**
  -> Greatly suppresses nu's from in-flight meson decay

MiniBooNE DM results

[1702.02688]



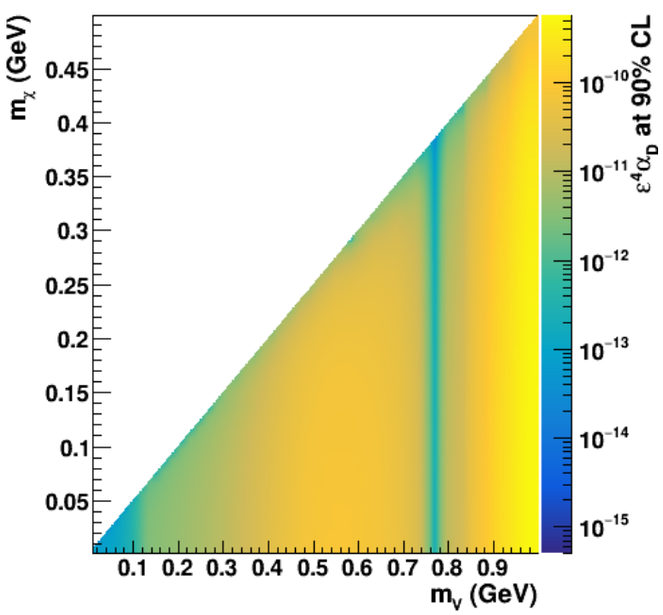


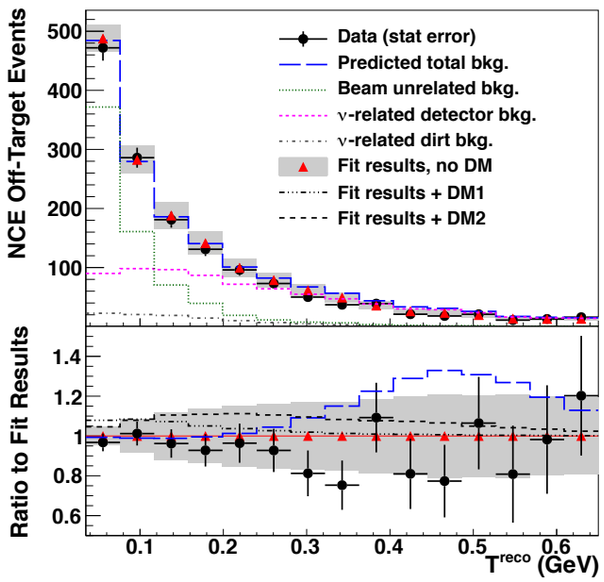
TABLE I. Number of selected data events with predicted backgrounds.

background source	events
beam-unrelated **(cosmic)**	697 ± 11
beam-related, detector **(CCQE)**	775 ±454
beam-related, dirt **(nu induced neutrons)**	107 ± 81
total estimated background	1579 ±529
constrained-fit background	1548 ±198
data events	1465 ± 38

- **Data consistent with bkg. only**
- **Systematics dominated.**

MiniBooNE DM results

[1702.02688]



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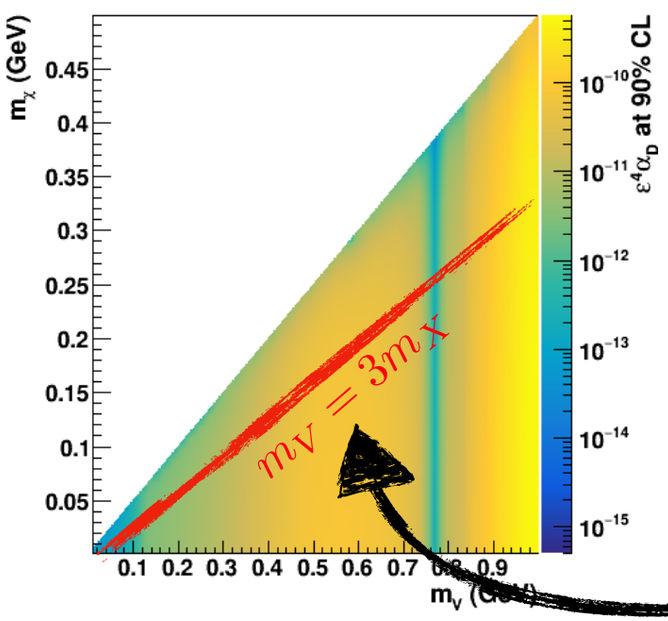


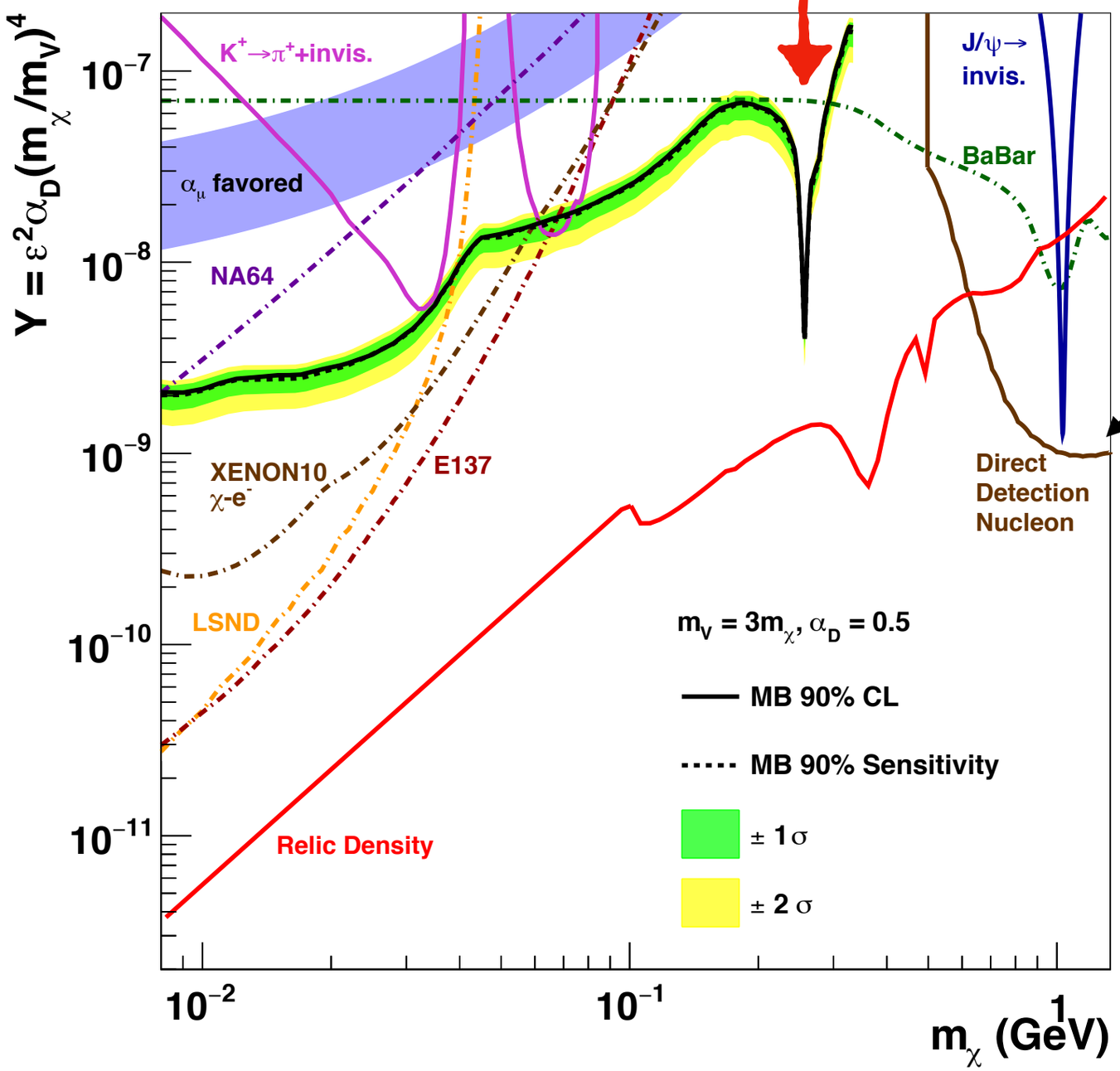
- **Data consistent with bkg. only**
- **Systematics dominated.**

Take a slice of parameter space to compare to other searches.

Dark Matter Search in a Proton Beam Dump with MiniBooNE

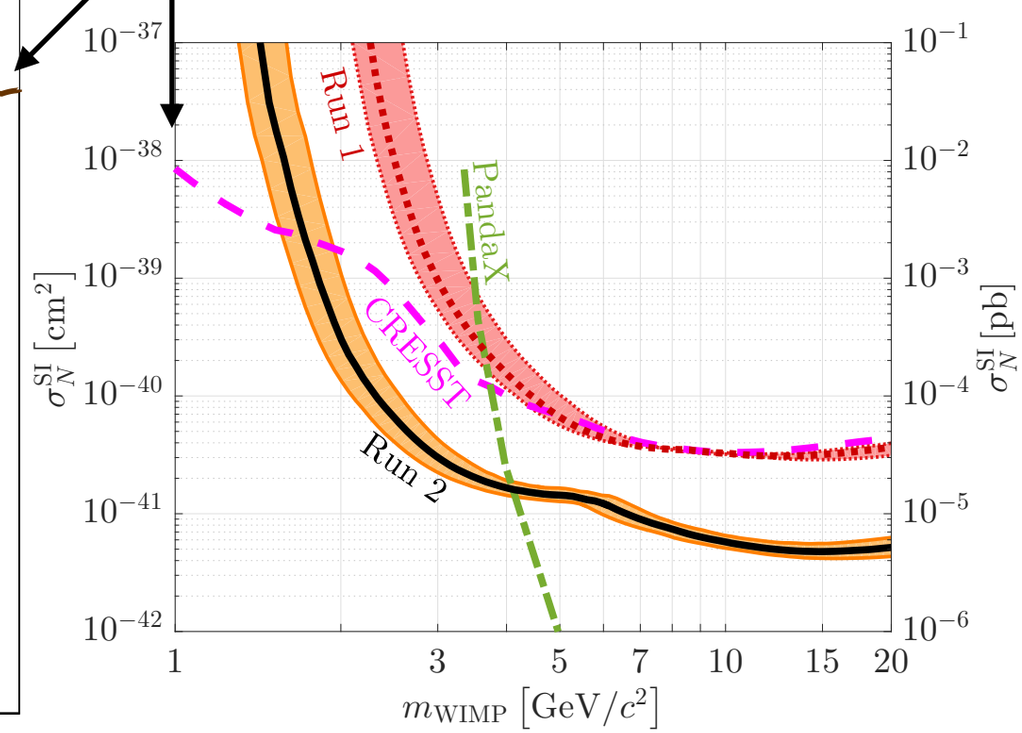
ρ, ω

[1702.02688]



Success at extending cross section bounds to sub-GeV regime.

join here



Future improvements

Timing cuts: DM is slower than neutrinos

Timing cut (nsec)	Background Reduction (%)	WIMP Velocity β	WIMP Mass (MeV)
3.0	90	0.9984	85
4.6	99	0.9974	108
5.9	99.9	0.9967	122

Table 3: WIMP velocity for various WIMP masses, assuming a WIMP momentum of 1.5 GeV. Also shown are the timing delay (cut) and background reduction levels achieved for a specific WIMP velocity.

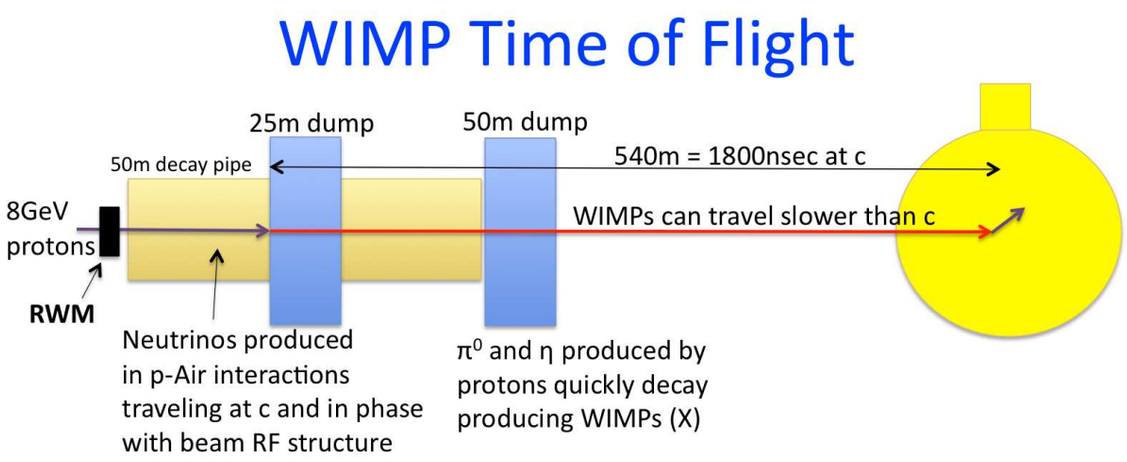
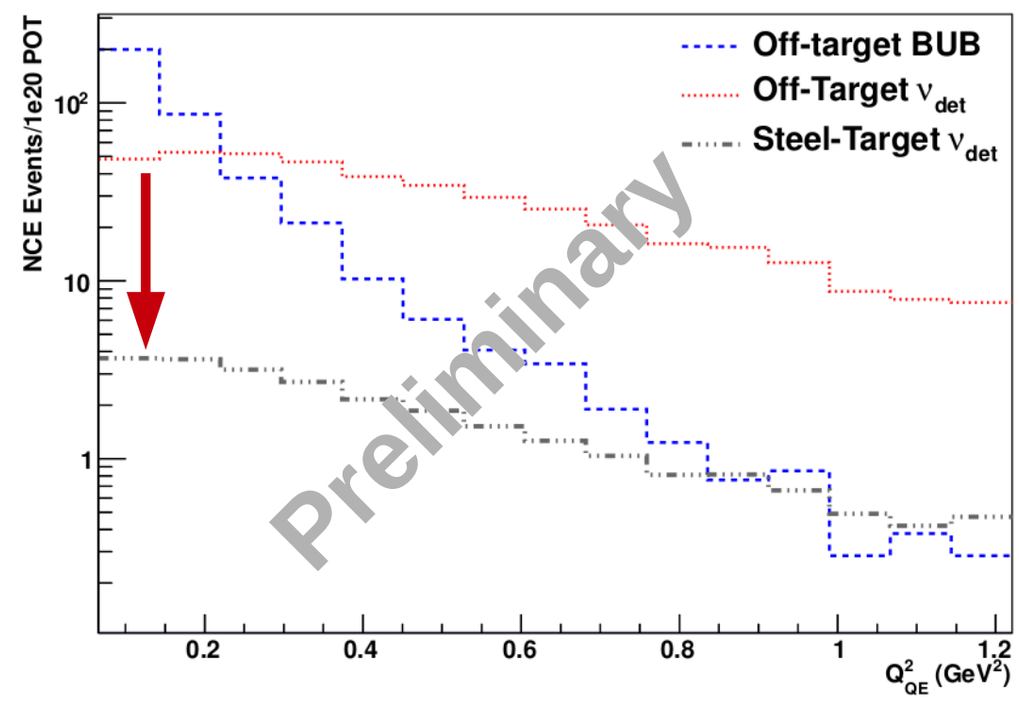


Figure 11: Simple timing drawing showing the production and reconstruction of events.

[1211.2258]

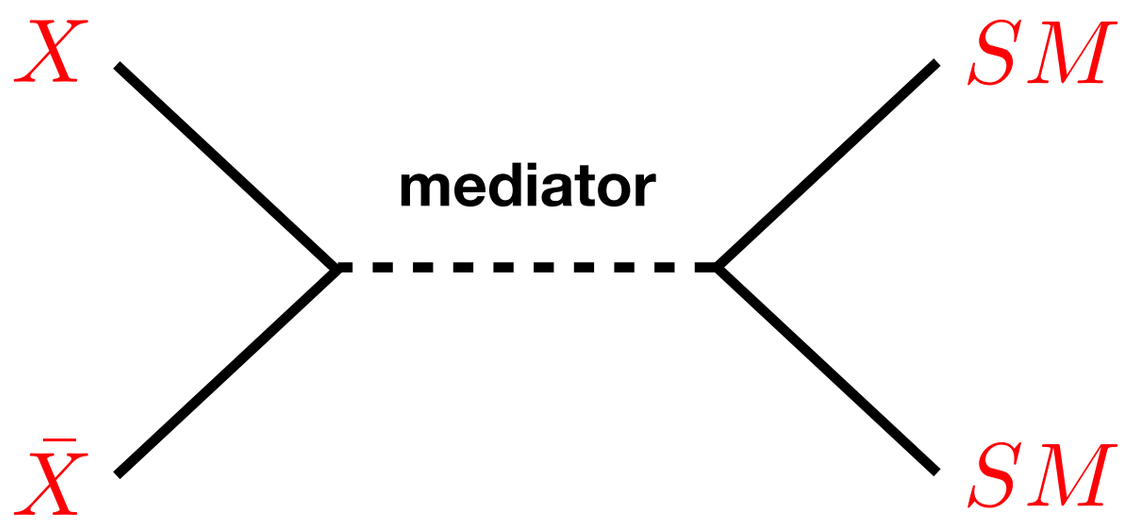
Dedicated steel beam dump



[Alexis A. AguilarArévalo TAUP 2017 talk]

A caveat

- **Analyses focus on predictive regime where relic density controlled by**



$\sim g_X^2 g_{SM}^2$

-Same couplings in experiments.
-Provides a target.

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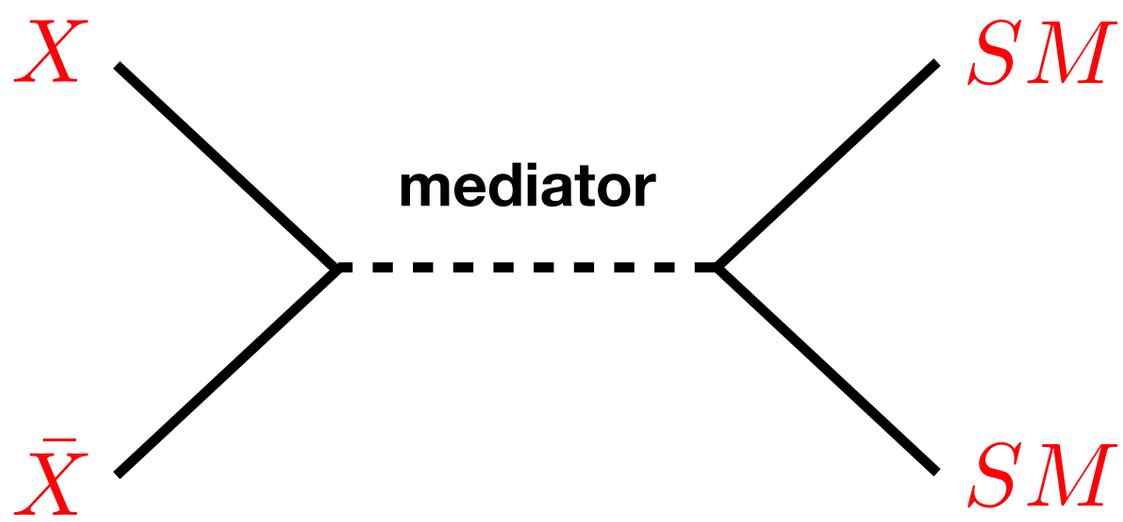

$\sim g_X^2 g_{SM}^2$

-Same couplings in experiments.
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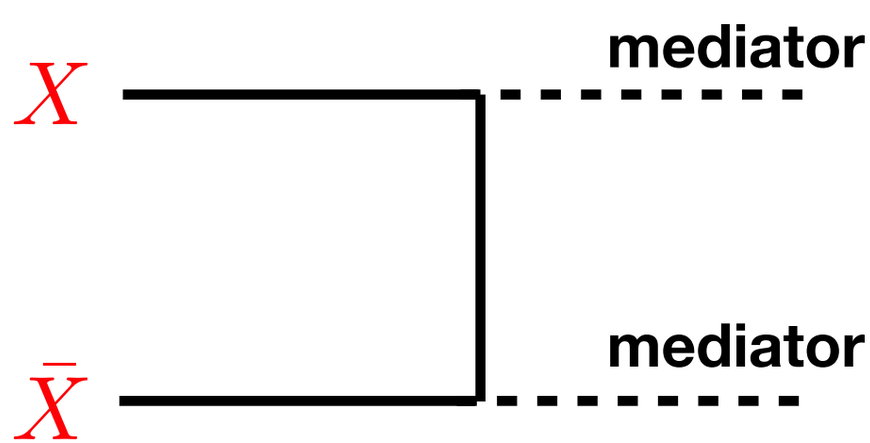
- **"Worst case":** $m_X < m_{med}$



$\sim g_X^4$

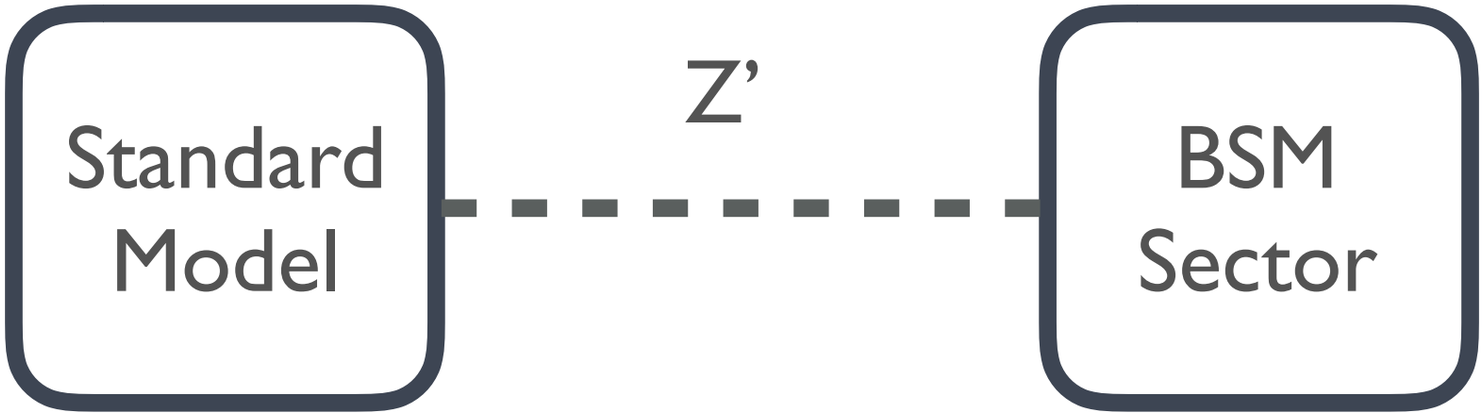
Need a DM collider…

SM coupling has no target

Beyond Dark Photons

Gauged Baryon Number $U(1)_B$

[Rajpoot (1989), Foot, Joshi, Lew (1989), …Carone, Murayama (1995), Perez, Wise (2010), …Graesser, Shoemaker, Vecchi (2011, Batell, deNiverville, McKeen, Pospelov, Ritz (2014), Dobrescu , Frugiuele (2014), Coloma, Dobrescu, Frugiuele, Harnik (2014)]



$$\mathcal{L}_q = \frac{g_z}{2} Z'_\mu \left(\frac{1}{3} \sum_q \bar{q}\gamma^\mu q + z_\chi \bar{\chi}\gamma^\mu \chi \right)$$

gauge coupling

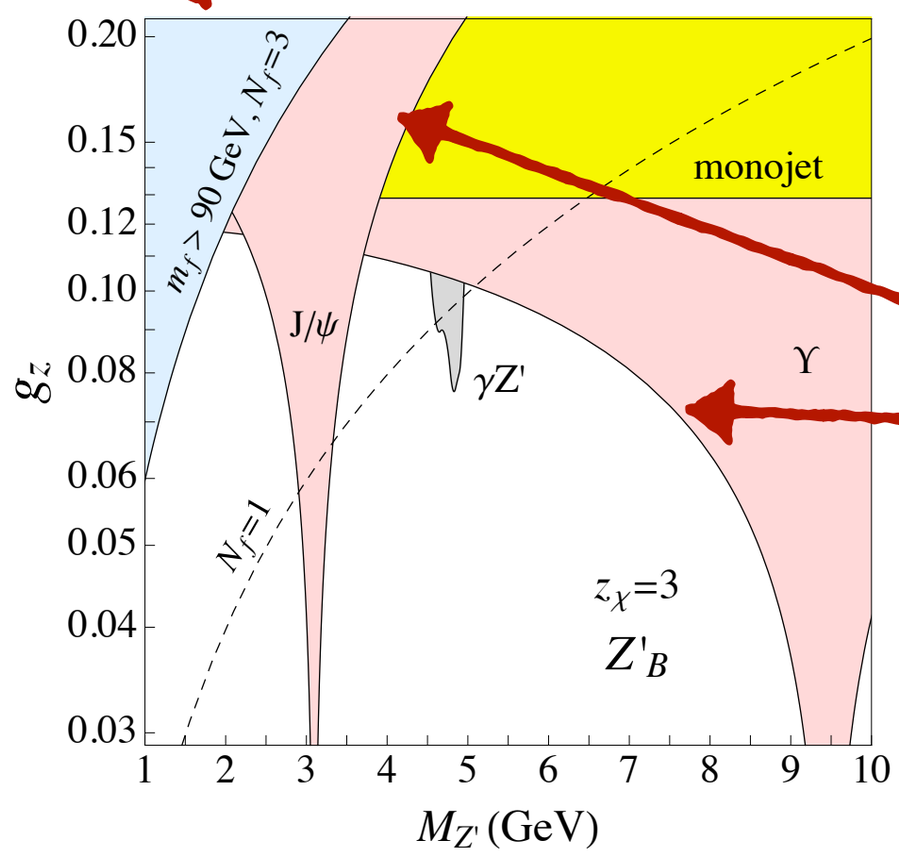
DM's U(1)$_B$ charge

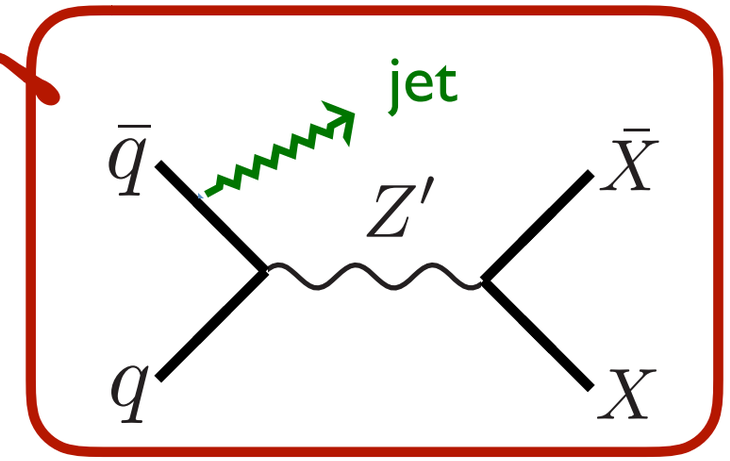
Existing Bounds

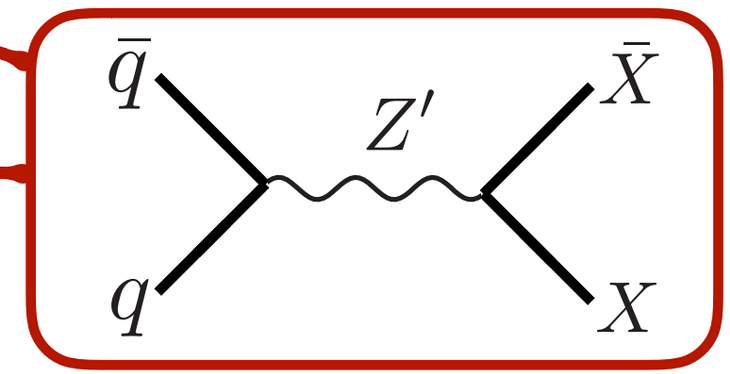
[Graesser , Shoemaker, Vecchi 1410.1566]

[Dobrescu , Frugiuele, 1410.1566]

anomalies



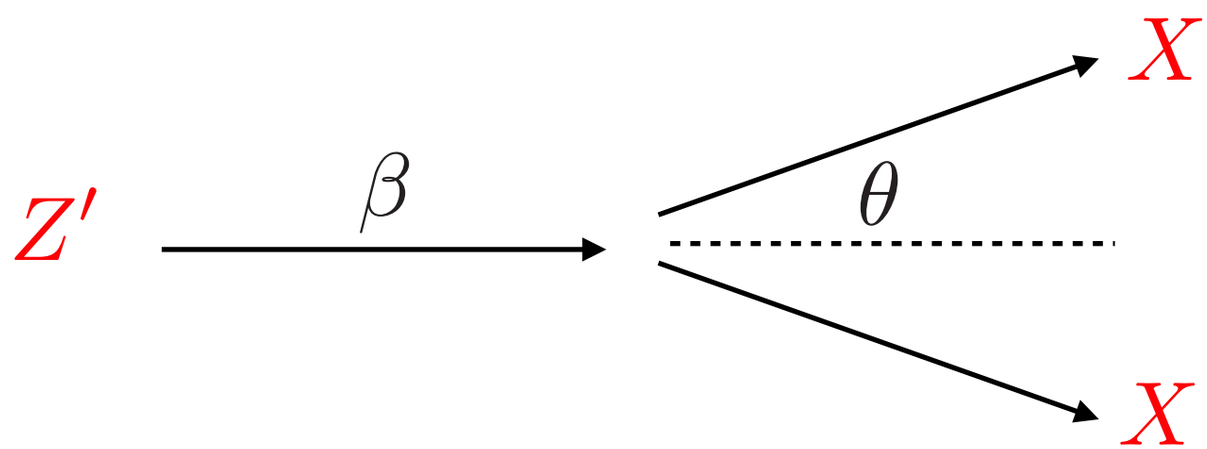




Dark Kinematics

[Coloma, Dobrescu, Frugiuele, Harnik, 1512.03852,]

- Consider a Z' produced in the target with a given energy.



- Two-body kinematics dictates the DM energy (neglect small DM mass):

$$E_\chi = \frac{M_{Z'}^2}{2E_{Z'}(1-\beta\cos\theta)}$$

Z' dist. very forward peaked

- Similar expression for neutrino bkg. but with Z' -> Meson parent.

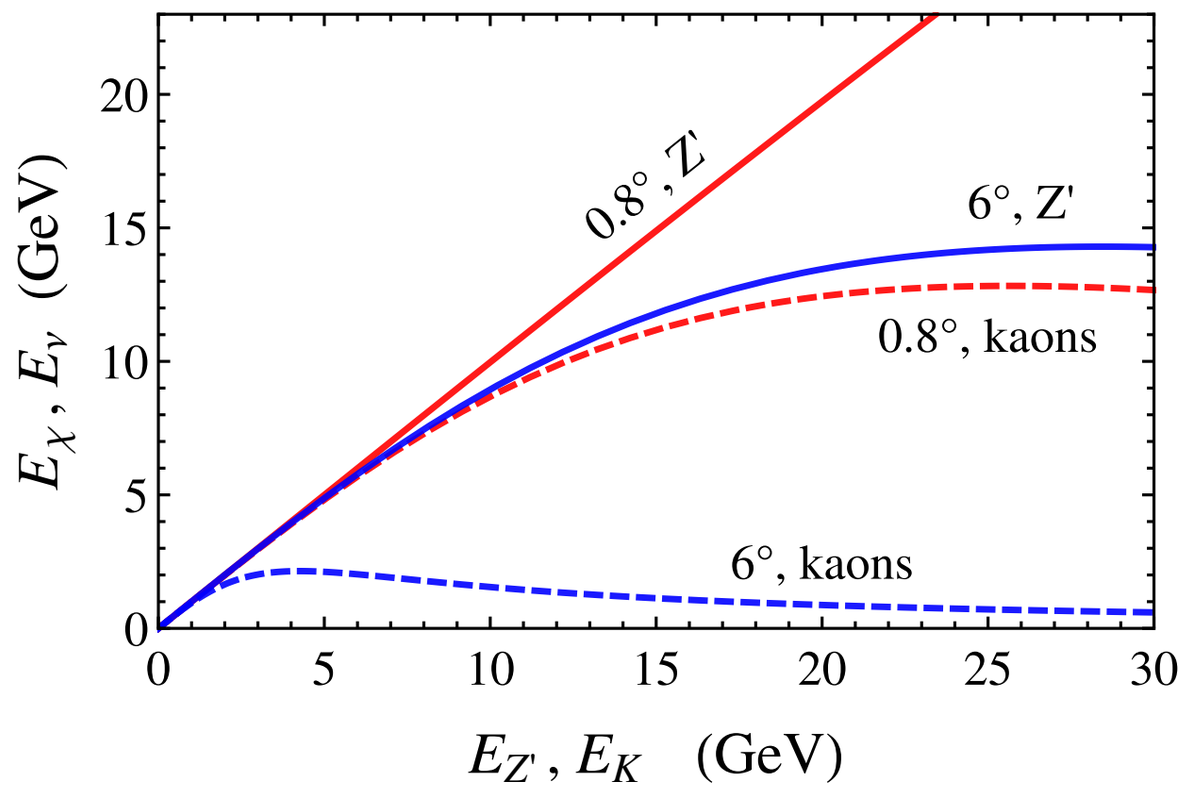
For pion parents: hard to get high energy & large angles.

=>For > 2 degree off-axis almost no pions!

Z' vs. Kaons

[Coloma, Dobrescu, Frugiuele, Harnik, 1512.03852,]

daughter energy



0.8 = NOvA

6 = MiniBooNE

dashed = neutrinos

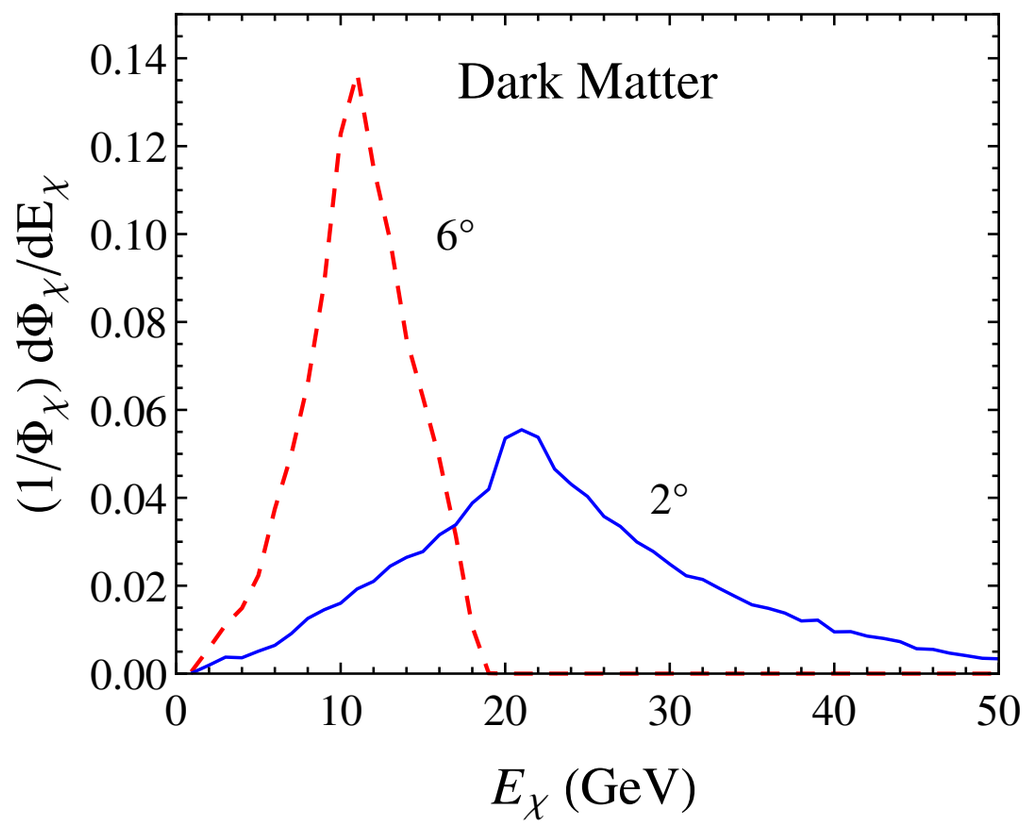
solid = DM

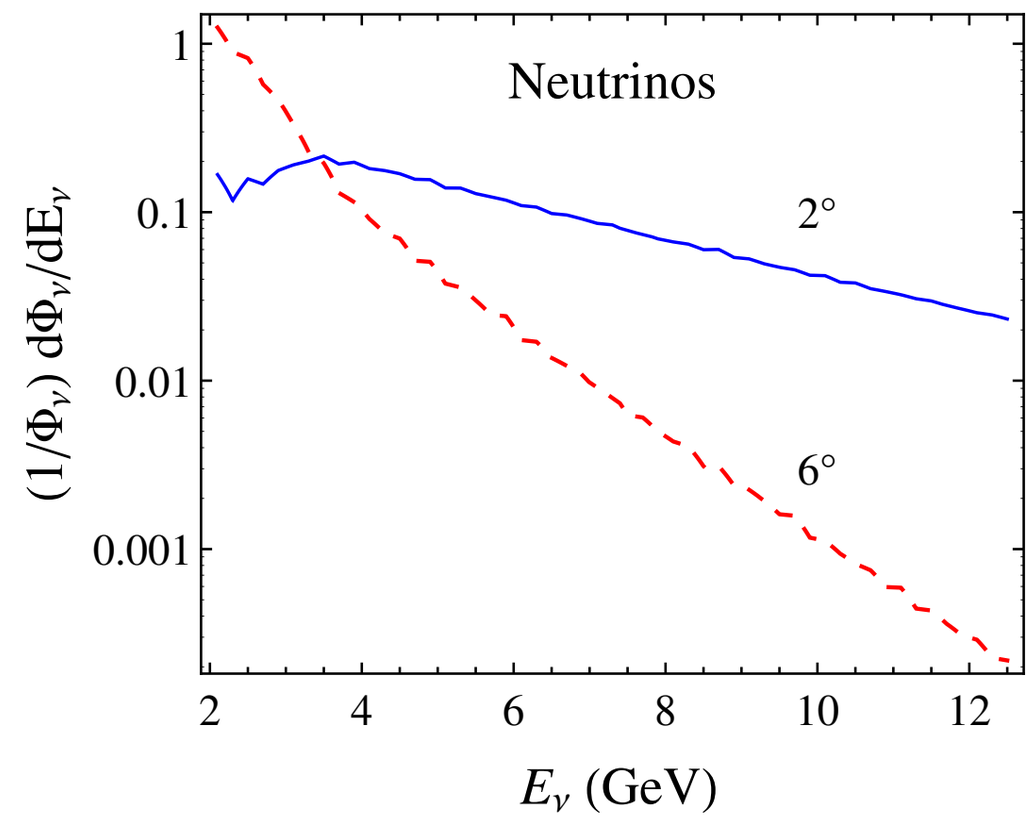
parent energy

It pays to go a few degrees off-axis.

Differential Fluxes

[Coloma, Dobrescu, Frugiuele, Harnik, 1512.03852,]





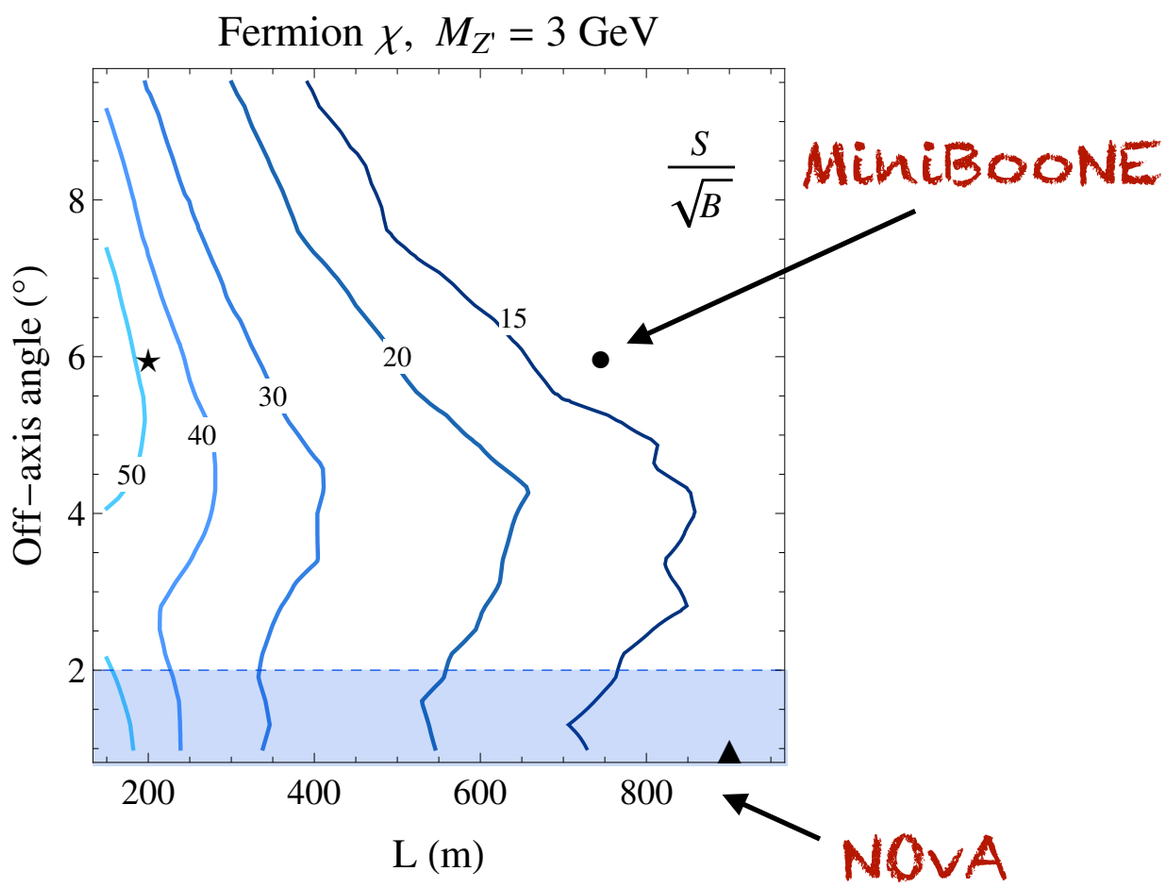
Difference in kaon/Z' mass offers a good handle for distinguishing DM/neutrinos.

Scan over Possible Detectors

[Coloma, Dobrescu, Frugiuele, Harnik, 1512.03852]

$$\mathcal{L}_q = \frac{g_z}{2} Z'_\mu \left(\frac{1}{3} \sum_q \bar{q}\gamma^\mu q + z_\chi \bar{\chi}\gamma^\mu \chi \right)$$

Benchmark: $g_z = 0.1,\ z_\chi = 3$ and $m_\chi = 750$ MeV



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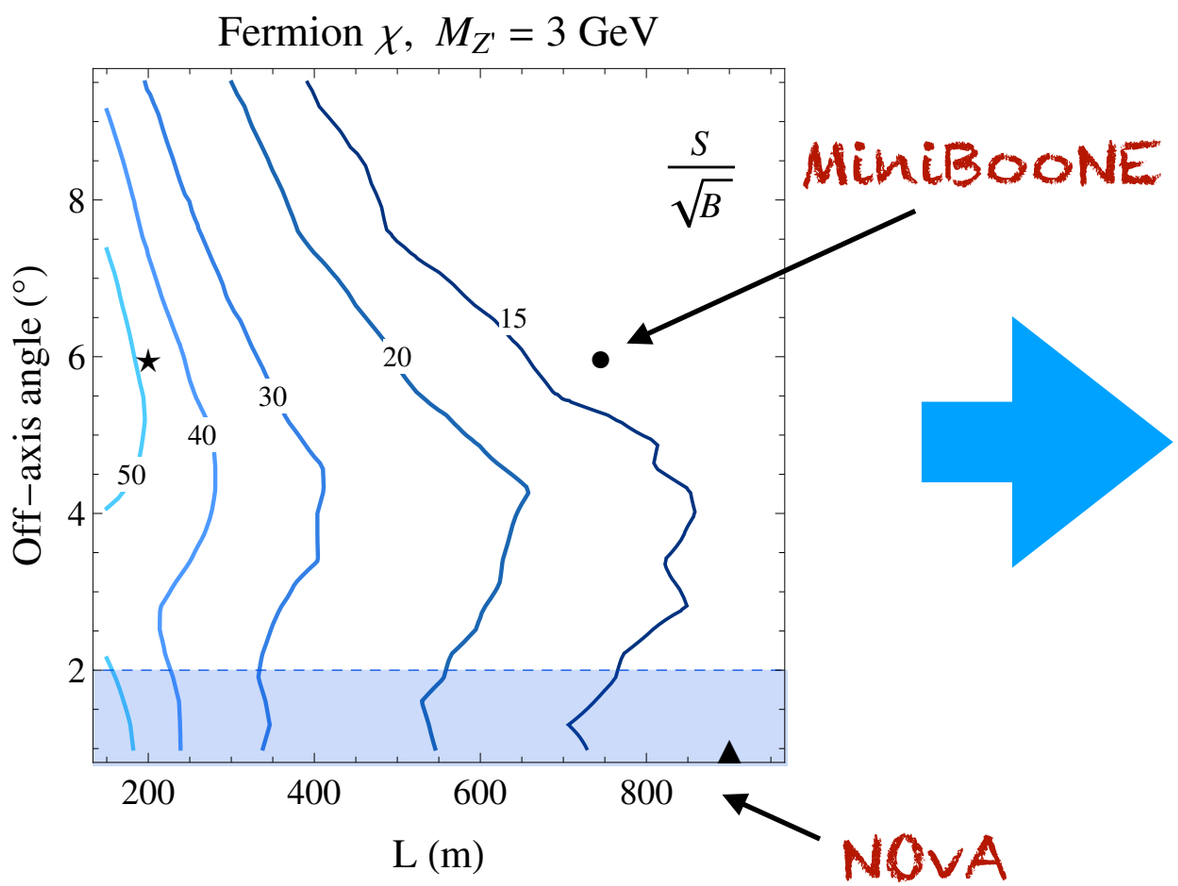
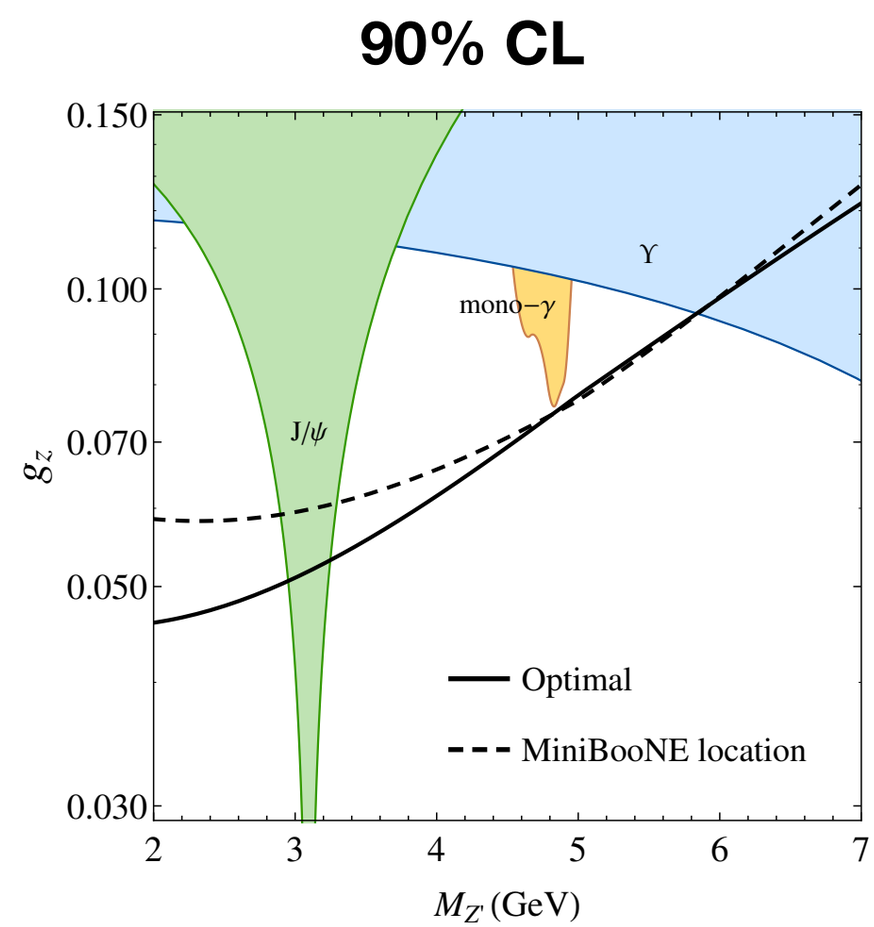
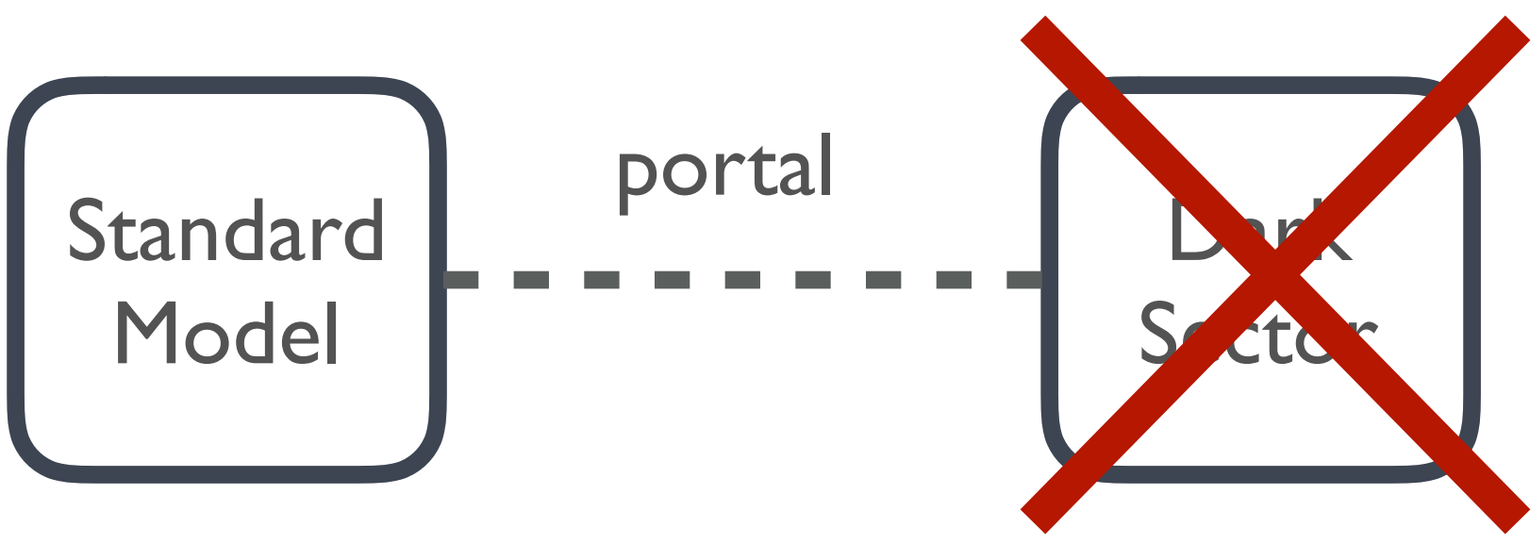




New Gauge Symmetries sans DM



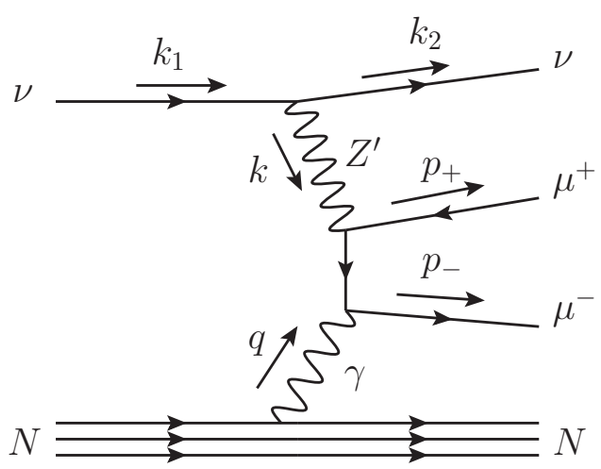
Gauged Lepton Numbers

[Altmannshofer, Gori, Pospelov, Yavin, 1406.2332]

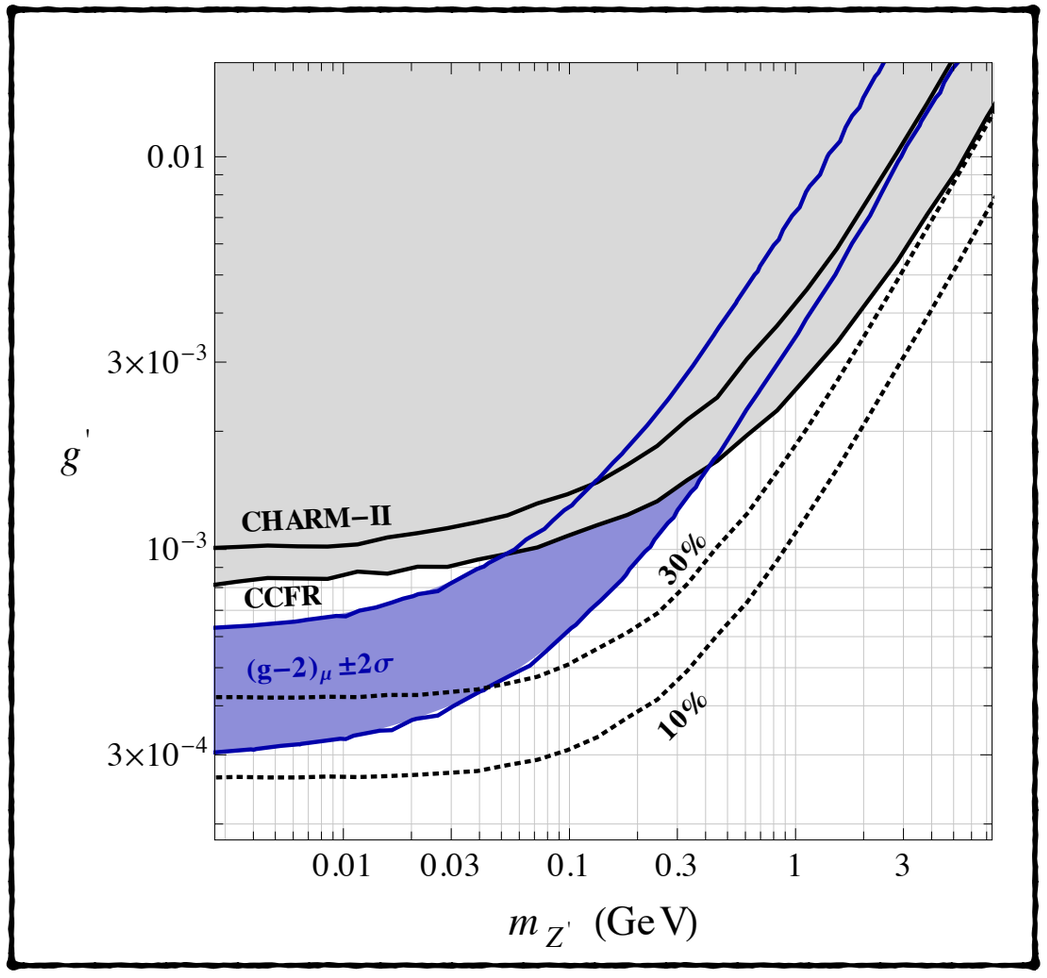
Promote a global symmetry to a gauge symmetry

=> SM fields charged under a new force!

$$U(1)_{L_\mu - L_\tau}$$



Nu trident cross section @ 10-30% accuracy in Ar w/5 GeV neutrinos



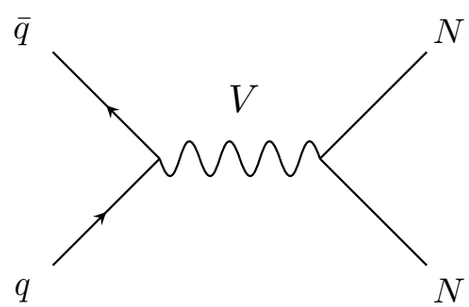
A Non-Sterile 4th Neutrino

Batell, Pospelov, Shuve [1604.06099]

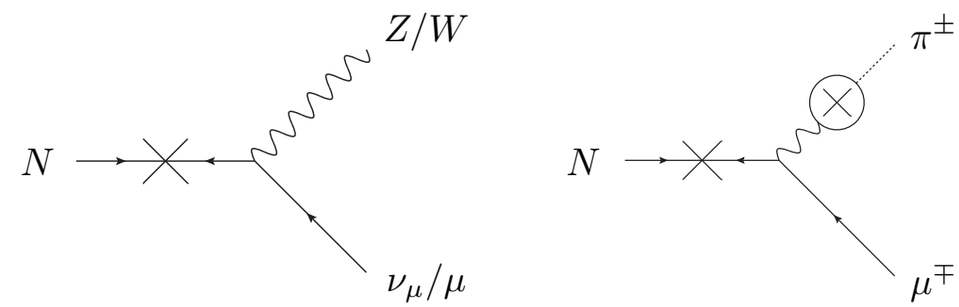
$$U(1)_{B-L}$$

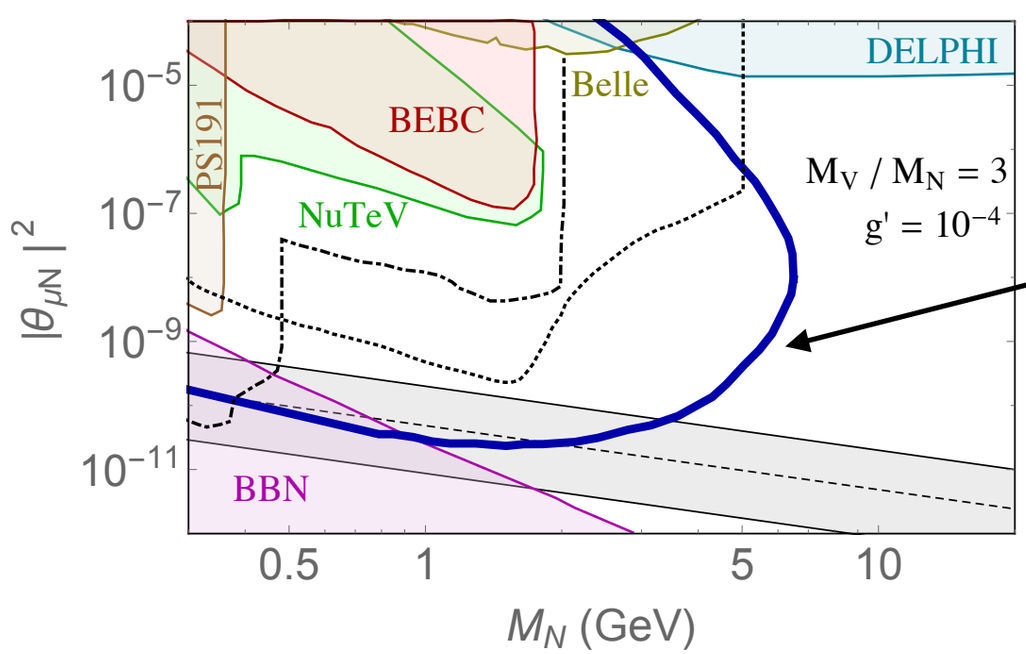
- **If B-L is gauged, need 3 RH neutrinos for anomaly cancellation.**
- **Go look for them!**

Production via new force



decay via EW force





SHiP sensitivity

Conclusions

- An array of interesting, well-motivated physics to search for with neutrino near detectors.

- Cast a wide net!

    - We need to simultaneously expand the theoretical terrain and to widen the experimental search strategies if we are going to uncover the New Standard Model.

NSI vs. NuTeV

[Coloma, Denton, Gonzalez-Garcia, Maltoni, Schwetz, 1701.04828]

NSI generalizes 4-fermion interactions:

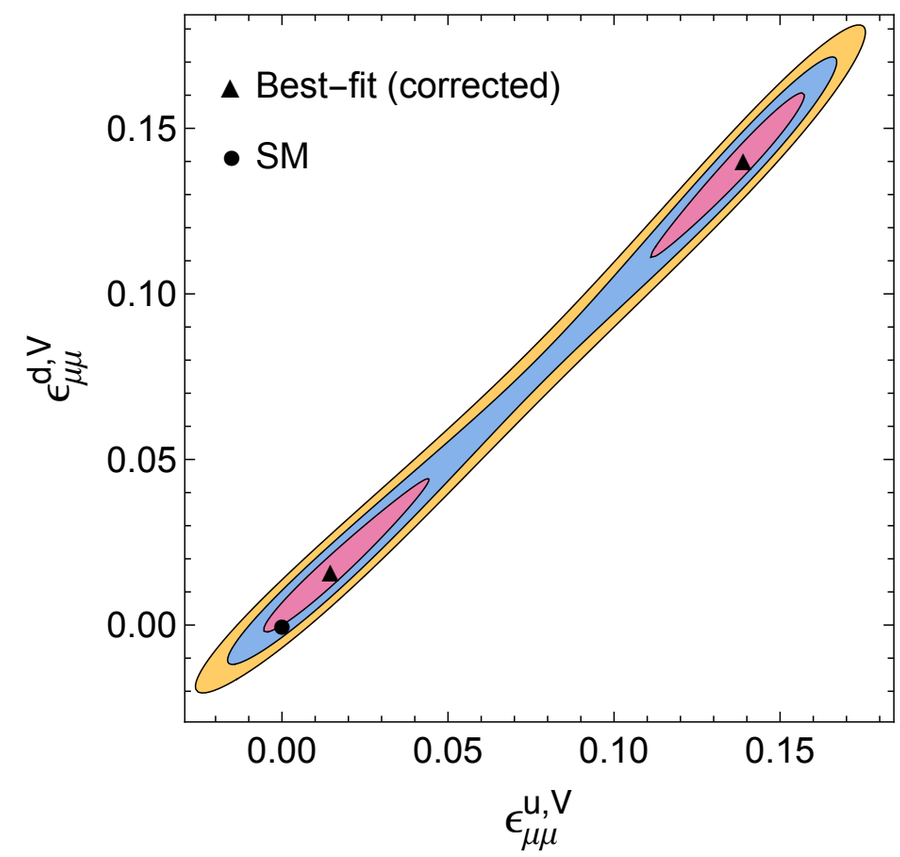
$$\mathcal{L}_{\rm NSI} = -\sqrt{2} G_F \epsilon^{fV}_{\alpha\beta} \left[\overline{\nu}_{\alpha L}\gamma^\rho \nu_{\beta L}\right]\left[\bar{f}\gamma_\rho f\right]$$

$$R^\nu_\mu = \frac{\sigma_{\rm NC}(\nu_\mu)}{\sigma_{\rm CC}(\nu_\mu)} = (\tilde{g}^L_\mu)^2 + r(\tilde{g}^R_\mu)^2\,,$$

$$R^{\bar{\nu}}_\mu = \frac{\sigma_{\rm NC}(\bar{\nu}_\mu)}{\sigma_{\rm CC}(\bar{\nu}_\mu)} = (\tilde{g}^L_\mu)^2 + \frac{1}{r}(\tilde{g}^R_\mu)^2\,,$$

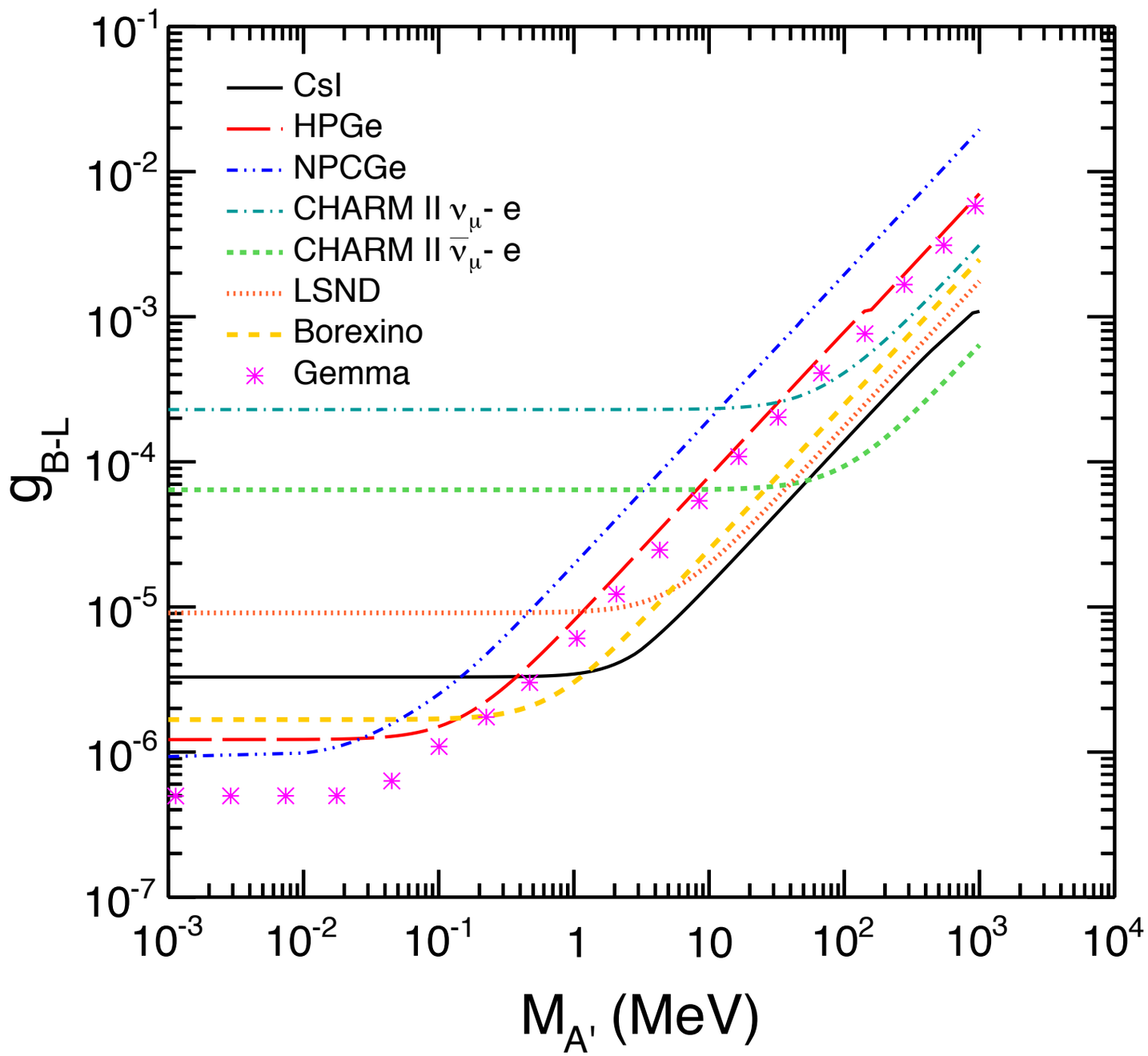
$$g^P_{\rm eff} \equiv (\tilde{g}^P_\mu)^2 = \sum_{q=u,d}\left[(g^P_q + \epsilon^{q,P}_{\mu\mu})^2 + \sum_{\alpha\neq\mu}|\epsilon^{q,P}_{\mu\alpha}|^2\right]\,,$$

NuTeV



Bounds on Neutrino Scattering

$$U(1)_{B-L}$$



Existing Baryonic Z' Bounds: Monojets

Shoemaker, Vecchi, [1112.5457]