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Gravitational Residual Symmetries as Lie-Bäcklund Transformations

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Generalized Geometrical Criteria to find Residual Symmetries.

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The line element.

$$\mathbf{ds}^2 = g_{\alpha\beta}(x^\mu, U^J(x^\nu))\mathbf{dx}^\alpha \mathbf{dx}^\beta \tag{1}$$

Residual Symmetries

They Are diffeormorphisms that need to be compensated by redefinitions of structural functions and derivatives of structural functions in order to satisfy the following two condition:

(i) **Preserve form invariant of the metric ansatz:** the dependence of the gravitational potentials $g_{\alpha\beta}$ in terms of the new quantities must be exactly the same Structural functions in the original independent variables.

(ii) **Complementary Conditions:** The transformer structural functions must have the same dependencies in new independent coordinates as the old structural functions in old independent coordinates .
[Ayon-Beato and Velázquez Rodríguez 2016]

Example

The Non-diverging Kundt's class space-times ([Stephani2003], [Griffiths2009]) are spaces whose solutions admit a null geodesic congruence, generated by a vector field k, which is shear-free, twist-free and expansion-free. The line element is,

Non-diverging Kuntz's Class.

$$\boldsymbol{ds}^2 = 2P^{-2}\boldsymbol{dzd\bar{z}} - 2\boldsymbol{du}\left(\boldsymbol{dv} + W\boldsymbol{dz} + \overline{W}\boldsymbol{d\bar{z}} + H\boldsymbol{du}\right), \quad (2)$$

where the only restriction on structural functions is that $P_v = 0$.

Residual Symmetries for Non-diverging Kuntz's Class

z-Reparametrization

$$
\begin{aligned}
&\tilde{u} = u, \quad \tilde{v} = v, \quad \tilde{z} = \tilde{z}(z),\\
&\widetilde{P}^2 = P^2 \left(\frac{\partial \tilde{z}}{\partial z}\right) \overline{\left(\frac{\partial \tilde{z}}{\partial z}\right)},\\
&\widetilde{W} = W \left(\frac{\partial \tilde{z}}{\partial z}\right)^{-1} + \frac{\overline{\left(\frac{\partial \tilde{z}}{\partial u}\right)}}{P^2 \left(\frac{\partial \tilde{z}}{\partial z}\right) \overline{\left(\frac{\partial \tilde{z}}{\partial z}\right)}},\\
&\widetilde{H} = H - W \left(\frac{\partial \tilde{z}}{\partial u}\right) \left(\frac{\partial \tilde{z}}{\partial z}\right)^{-1} - \overline{W \left(\frac{\partial \tilde{z}}{\partial u}\right) \left(\frac{\partial \tilde{z}}{\partial z}\right)^{-1}} + \\
&\qquad - \frac{1}{P^2} \frac{\overline{\left(\frac{\partial \tilde{z}}{\partial u}\right)} \left(\frac{\partial \tilde{z}}{\partial u}\right)}{\left(\frac{\partial \tilde{z}}{\partial z}\right) \overline{\left(\frac{\partial \tilde{z}}{\partial z}\right)}}.
\end{aligned}
\tag{3}
$$

Non-diverging Kundtz's Class in new quantities

Transformed ansatz looks the same

$$\boldsymbol{d}\tilde{\boldsymbol{s}}^2 = 2\tilde{P}^{-2}\boldsymbol{d}\tilde{\boldsymbol{z}}\boldsymbol{d}\bar{\tilde{\boldsymbol{z}}} - 2\boldsymbol{d}\tilde{\boldsymbol{u}}\left(\boldsymbol{d}\tilde{\boldsymbol{v}} + W\boldsymbol{d}\tilde{\boldsymbol{z}} + \overline{\tilde{W}}\boldsymbol{d}\bar{\tilde{\boldsymbol{z}}} + \tilde{H}\boldsymbol{d}\tilde{\boldsymbol{u}}\right), \tag{4}$$

Lie-Point Criteria Developed by Ayón Beato and Velázquez Rodríguez

The question of how to find the continuous transformations

Continuos Transformations?

$$x^\alpha \longmapsto \tilde{x}^\alpha = \tilde{x}^\alpha(x^\mu, U^J; \epsilon) \quad ; \quad U^I \longmapsto \tilde{U}^I = \tilde{U}^I(x^\mu, U^J; \epsilon)$$

Which are Residual Symmetries

- $$\begin{aligned} \mathbf{ds}^2 &= g_{\alpha\beta}(x^\mu, U^J(x^\nu))\mathbf{dx}^\alpha \mathbf{dx}^\beta \\ &= g_{\alpha\beta}(\tilde{x}^\mu, \tilde{U}^J(\tilde{x}^\nu))\mathbf{d\tilde{x}}^\alpha \mathbf{d\tilde{x}}^\beta = \mathbf{d\tilde{s}}^2 \end{aligned}$$
- Complementary conditions.

that preserve the Was generally addressed by two of the authors in [Ayon-Beato and Velázquez Rodríguez 2016].

Description of the method already developed

Infinitesimal version of the one-parameter Lie-point transformations:

$$\tilde{x}^\alpha = x^\alpha + \varepsilon \xi^\alpha(x^\mu, u^J) + \cdots , \tag{5a}$$

$$\tilde{u}^I = u^I + \varepsilon \eta^I(x^\mu, u^J) + \cdots , \tag{5b}$$

which give rise to the generator

$$\boldsymbol{X} = \xi^\alpha(x^\mu, u^J)\boldsymbol{\partial}_\alpha + \eta^I(x^\mu, u^J)\boldsymbol{\partial}_I, \tag{6}$$

with components defined as usual,

$$\xi^\alpha(x^\mu, u^I) \equiv \left.\frac{\partial \tilde{x}^\alpha}{\partial \varepsilon}\right|_{\varepsilon=0}, \quad \eta^I(x^\mu, u^J) \equiv \left.\frac{\partial \tilde{u}^I}{\partial \varepsilon}\right|_{\varepsilon=0}. \tag{7a}$$

- Form invariance for the metric ansatz:

$$\begin{aligned}\mathbf{d}\tilde{s}^2 &= g_{\alpha\beta}(x^\mu + \varepsilon\xi^\mu + \cdots, u^J + \varepsilon\eta^J + \cdots) \\ &\quad \times \mathbf{d}(x^\alpha + \varepsilon\xi^\alpha + \cdots)\mathbf{d}(x^\beta + \varepsilon\xi^\beta + \cdots) \\ &= \mathbf{d}s^2 + \varepsilon\Big[\Big(\xi^\mu\partial_\mu g_{\alpha\beta} + 2g_{\mu\alpha}\partial_\beta\xi^\mu + \eta^I\partial_I g_{\alpha\beta}\Big)\mathbf{d}x^\alpha\mathbf{d}x^\beta \\ &\quad + 2g_{\alpha\beta}\partial_I\xi^\beta\mathbf{d}x^\alpha\mathbf{d}u^I\Big] + \cdots, \end{aligned} \tag{8}$$

- If happen that some structural function $U^{\bar{I}}$ satisfy that $U^{\bar{I}}_{\hat{\alpha}} = 0$ and we required keep this condition in new coordinates, that is $\tilde{U}^{\bar{I}}_{\hat{\alpha}} = 0$, it necessary imply that $0 = \eta^{\bar{I}}_{\hat{\alpha}}$. Then from (**??**),

$$0 = \eta^{\bar{I}}_{\hat{\alpha}} = D_{\hat{\alpha}}\eta^{\bar{I}} - U^{\bar{I}}_\beta D_{\hat{\alpha}}\xi^\beta. \tag{9}$$

Infinitesimal Lie-Point T. + C. Invariancia + C. Conditions

$$\xi^\alpha \partial_\alpha g_{\mu\nu} + 2 g_{\alpha(\mu} \partial_{\nu)} \xi^\alpha + \eta^I \partial_I g_{\mu\nu} = 0$$

$$\partial_I \xi^\alpha = 0$$

$$\partial_{\hat{\alpha}} \eta^{\bar{I}} = \partial_{\hat{J}} \eta^{\bar{I}} = \partial_{\hat{\alpha}} \xi^{\bar{\beta}} = 0,$$

[Ayon-Beato and Velázquez Rodríguez 2016]

Only apply to metrics that do not depend on derivatives.

Lie-Point criteria is not always is enought

If we apply the lie-point criteria to the Kerr-Golberg ansatz,

$$\mathbf{d}\boldsymbol{s}^2 = \mathbf{d}\boldsymbol{x}^2 + \mathbf{d}\boldsymbol{y}^2 + 2\mathbf{d}\boldsymbol{u}\mathbf{d}\boldsymbol{v} + 2\rho\mathbf{d}\boldsymbol{u}\mathbf{d}\boldsymbol{x} + (\omega - v\rho_{,x})\,\mathbf{d}\boldsymbol{u}^2, \quad (10)$$

where ρ and ω are functions independent of v, the residual transformations obtained are, [Velazquez2016],

$$\tilde{x} = x + \xi^x(u), \quad \tilde{y} = y + \varepsilon_1, \quad \tilde{u} = \lambda u + \varepsilon_2, \quad \tilde{v} = \lambda^{-1} v,$$
$$\tilde{\rho} = \lambda^{-1}(\rho - \partial_u \xi^x), \quad \tilde{\omega} = \lambda^{-2}(\omega - \partial_u \xi^x(2\rho - \partial_u \xi^x)). \quad (11)$$

Although (11) are residual symmetries they are not the most general residual symmetries. For example the transformation,

$$\tilde{v} = v + C(u), \quad \tilde{w} = w + C(u)\rho_x - 2\dot{C},$$

is also a residual transformation of the line element (10).

Our Goal

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Our Goal

We want to generalize the geometrical criteria developed by Ayón Beato and Velázquez Rodríguez in order to give a completely general Geometrical method to find Residual Symmetries of any metric ansatz.

Metric ansatz with dependencies in derivatives

General Dependencies

$$ds^2 = g_{\alpha\beta}(x^\mu, U^I(x^\mu), U^I_\alpha(x^\mu), U^I_{\alpha\beta}(x^\mu), \dots)dx^\alpha dx^\beta$$

Non Local Transformations

Infinitesimal Lie-Bäcklund Transformations

$$\tilde{x}^{\alpha} = x^{\alpha} + \epsilon\xi^{\alpha}(x^{\mu}, U^{J}, U^{J}_{\alpha}, U^{J}_{\alpha\beta}, \ldots) + \cdots \quad (12a)$$

$$\tilde{U}^{I} = U^{I} + \epsilon\eta^{I}(x^{\mu}, U^{J}, U^{J}_{\alpha}, U^{J}_{\alpha\beta}, \ldots) + \cdots \quad (12b)$$

$$\tilde{U}^{I}_{\alpha} = U^{I}_{\alpha} + \epsilon\eta^{I}_{\alpha}(x^{\mu}, U^{J}, U^{J}_{\alpha}, U^{J}_{\alpha\beta}, \ldots) + \cdots \quad (12c)$$

.

$$\tilde{U}^{I}_{\alpha,\ldots,\omega} = U^{I}_{\alpha\beta,\ldots,\omega} + \epsilon\eta^{I}_{\alpha,\ldots,\omega}(x^{\mu}, U^{J}, U^{J}_{\alpha}, U^{J}_{\alpha\beta}, \ldots) + \cdots,$$

.

Where prolongations $\eta^{I}_{\alpha,\ldots,n}$ are defined according to ,

$$\eta^{I}_{\alpha\beta,\ldots,\omega} = \frac{D^{\omega}}{Dx^{\alpha}Dx^{\beta}\cdots Dx^{\omega}}\left(\eta^{I} - U^{I}_{\mu}\xi^{\mu}\right) + \xi^{\mu}U^{I}_{\mu\alpha\beta,\ldots,\omega} \quad (13)$$

Form Invariance Condition

$$\begin{aligned}\mathbf{d\tilde{s}}^2 &= g_{\alpha\beta}(\tilde{x}^\mu, \tilde{U}^J(\tilde{x}^\nu), \tilde{U}^J_\alpha(\tilde{x}^\nu), \dots)\mathbf{d\tilde{x}}^\alpha \mathbf{d\tilde{x}}^\beta \\ &= g_{\alpha\beta}(x^\mu, U^J(x^\nu), U^J_\alpha(x^\nu), \dots)\mathbf{dx}^\alpha \mathbf{dx}^\beta + \epsilon(\cdots) \\ &= \mathbf{ds}^2\end{aligned} \tag{14}$$

Invariance Conditons

$$\mathbf{X}(g_{\alpha\beta}) + 2g_{\mu(\beta}D_{\alpha)}\xi^\mu = 0, \tag{15}$$

donde

$$\mathbf{X} = \xi^\mu\partial_\mu + \eta^I\partial_I + \eta^I_\alpha\frac{\partial}{\partial U^I_\alpha} + \cdots + \tag{16}$$

$$\frac{D}{Dx^\alpha} = \partial_\alpha + U^J_\alpha\partial_J + U^J_{\alpha\beta}\frac{\partial}{\partial U^J_\beta} + \cdots + \tag{17}$$

Complementary Conditions

If happen that $U^{\bar{I}}$ does not depend on $x^{\hat{\alpha}}$, then $U^{\bar{I}}_{\hat{\alpha}} = 0$. The complementary conditions means to keep the same conditions on the transformed functions , $\tilde{U}^{\bar{I}}_{\hat{\alpha}} = 0$,

$$\tilde{U}^{\bar{I}}_{\hat{\alpha}} \equiv \frac{\partial \tilde{U}^{\bar{I}}}{\partial \tilde{x}^{\hat{\alpha}}} = U^{\bar{I}}_{\hat{\alpha}} + \epsilon \eta^{\bar{I}}_{\hat{\alpha}} + \cdots .$$

This necessarily imply that $0 = \eta^{\bar{I}}_{\hat{\alpha}}$, then from (13),

$$0 = \eta^{\bar{I}}_{\hat{\alpha}} = \frac{D\eta^{\bar{I}}}{Dx^{\hat{\alpha}}} - U^{\bar{I}}_{\beta} \frac{D\xi^{\beta}}{Dx^{\hat{\alpha}}} .$$

Complete Criteria

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Form invariance

$$\mathbf{X}(g_{\alpha\beta}) + 2g_{\mu(_\beta}D_{\alpha)}\xi^{\mu} = 0, \tag{18}$$

Complementary Conditions

$$0 = \eta^{\bar{I}}_{\hat{\alpha}} = \frac{D\eta^{\bar{I}}}{Dx^{\hat{\alpha}}} - U^{\bar{I}}_{\bar{\beta}}\frac{D\xi^{\bar{\beta}}}{Dx^{\hat{\alpha}}}. \tag{19}$$

Kerr-Golberg Ansatz

Example (Kerr-Golberg, [Kerr-Globerg 61])

$$\mathbf{ds}^2 = \mathbf{dx}^2 + \mathbf{dy}^2 + 2\mathbf{dudv} + 2\rho\mathbf{dudx} + (\omega - v\rho_x)\,\mathbf{du}^2,$$

where $\omega_{\mathbf{v}} = \rho_{\mathbf{v}} = 0$.

Generator's Components

$$\xi^x = \xi^x(u) \tag{20a}$$

$$\xi^y = \xi^y(u) \tag{20b}$$

$$\xi^u = \xi^u(u) \tag{20c}$$

$$\xi^v = -\dot{\xi}^y y - \dot{\xi}^u v + \ddot{\xi}^u x^2 + A(u)x + B(u) \tag{20d}$$

$$\eta^\rho = -\dot{\xi}^x - \rho\dot{\xi}^u - 2x\ddot{\xi}^u - A(u) \tag{20e}$$

$$\eta^\omega = \left(x^2\ddot{\xi}^u + A(u)x + B(u) - \dot{\xi}^y y\right)\rho_x - 2\omega\dot{\xi}^u \tag{20f}$$

$$- 2\rho\dot{\xi}^x - 2\left(\dddot{\xi}^u x^2 + \dot{A}(u)x + \dot{B}(u)\right) + 2y\ddot{\xi}^y \tag{20g}$$

Generators

$$\begin{aligned}
\mathbf{X^u}[\xi^\mathbf{u}(\mathbf{u})] &= \xi^u\partial_\mathbf{u} - \dot{\xi}^u(v\partial_\mathbf{v} + \rho\partial_\rho + 2\omega\partial_\omega + \rho_x\partial_{\rho_\mathbf{x}}) \\
&+ \ddot{\xi}^u(x^2\partial_\mathbf{v} - 2x\partial_\rho + x^2\rho_x\partial_\omega - 2\partial_{\rho_x}) \\
&- 2\dddot{\xi}^u x^2\partial_\omega, && (21a) \\
\mathbf{X^x}[\xi^\mathbf{x}(\mathbf{u})] &= \xi^x\partial_\mathbf{x} - \dot{\xi}^x\left(\partial_\rho + 2\rho\partial_\omega\right), && (21b) \\
\mathbf{X^y}[\xi^\mathbf{y}(\mathbf{u})] &= \xi^y\partial_\mathbf{y} - \dot{\xi}^y y(\partial_\mathbf{v} + \rho_x\partial_\omega) \\
&+ 2\ddot{\xi}^y y\partial_\omega, && (21c) \\
\mathbf{X}^\rho[\eta^\rho(\mathbf{u})] &= \eta^\rho(-x\partial_\mathbf{v} + \partial_\rho - x\rho_x\partial_\omega) \\
&+ 2x\dot{\eta}^\rho\partial_\omega. && (21d) \\
\mathbf{X^v}[\xi^\mathbf{v}(\mathbf{u})] &= \xi^v(\partial_\mathbf{v} + \rho_x\partial_\omega) - 2\dot{\xi}^v\partial_\omega. && (21e)
\end{aligned}$$

$\mathbf{X}^{\mathbf{y}}[\xi^{\mathbf{y}}(\mathbf{u})]$

$$\tilde{y} = y + b(u, \epsilon), \quad (22a)$$

$$\tilde{v} = v - \dot{b}y, \quad (22b)$$

$$\tilde{w} = w - \dot{b}y\rho_x + 2\ddot{b}y - \dot{b}^2. \quad (22c)$$

$\mathbf{X}^{\mathbf{x}}[\xi^{\mathbf{x}}(\mathbf{u})] + \mathbf{X}^{\rho}[\dot{\xi}^{\mathbf{x}}(\mathbf{u})]$

$$\tilde{x} = x + a(u, \epsilon), \quad (23a)$$

$$\tilde{v} = v - \dot{a}x, \quad (23b)$$

$$\tilde{w} = -\rho_x\dot{a}x - \dot{a}^2 + 2\ddot{a}x - 2\rho\dot{a}. \quad (23c)$$

$\mathbf{X^u}[\xi^{\mathbf{u}}(\mathbf{u})])$

$$\tilde{u} = \phi(u), \tag{24a}$$

$$\tilde{v} = \frac{1}{\dot{\phi}}(v + x^2\ddot{\phi}), \tag{24b}$$

$$\tilde{\rho} = \frac{1}{\dot{\phi}}(\rho - 2x\ddot{\phi}). \tag{24c}$$

$$\tilde{w} = \frac{1}{\dot{\phi}^2}\left(w + x^2\ddot{\phi}\rho_x - 2v\frac{\ddot{\phi}}{\dot{\phi}}(\dot{\phi} - 1) - 2x^2\frac{\ddot{\phi}^2}{\dot{\phi}}(\dot{\phi} - 1) - 2x^2\dddot{\phi}\right). \tag{24d}$$

$\mathbf{X^v}[\xi^{\mathbf{v}}(\mathbf{u})] + \eta^{\rho}_{\mathbf{x}}\partial_{\rho_{\mathbf{x}}}$

$$\tilde{v} = \tilde{v} + C(u), \tag{25a}$$
$$\tilde{w} = w + C(u)\rho_x - 2\dot{C}, \tag{25b}$$

$\mathbf{X}^{\rho}[\eta^{\rho}(\mathbf{u})]$

$$\tilde{v} = \tilde{v} - D(u), \tag{26a}$$
$$\tilde{\rho} = \rho + D(u), \tag{26b}$$
$$\tilde{w} = w - D(u)x\rho_x + 2\dot{D}x, \tag{26c}$$

The conmutators

$$[\mathbf{X^u}[\xi^{\mathbf{u}}],\mathbf{X^x}[\xi^{\mathbf{x}}]] = \mathbf{X^x}[\xi^u\dot{\xi}^x] + 2\mathbf{X}^\rho[\xi^x\ddot{\xi}^u]$$

$$[\mathbf{X^u}[\xi^{\mathbf{u}}],\mathbf{X}^\rho[\eta^\rho]] = \mathbf{X}^\rho[\frac{d}{du}(\xi^u\eta^\rho)]$$

$$[\mathbf{X^u}[\xi^{\mathbf{u}}],\mathbf{X^v}[\xi^{\mathbf{v}}]] = \mathbf{X}^v[\frac{d}{du}(\xi^u\xi^v)]$$

$$[\mathbf{X^u}[\xi^{\mathbf{u_1}}],\mathbf{X^u}[\xi^{\mathbf{u_2}}]] = \mathbf{X^u}[(\xi^{u_1})^2\frac{d}{du}(\frac{\xi^{u_2}}{\xi^{u_1}})]$$

$$[\mathbf{X^x}[\xi^{\mathbf{x}}],\mathbf{X}^\rho[\eta^\rho]] = -\mathbf{X^v}[\xi^x\eta^\rho]$$

$$[\mathbf{X^y}[\xi^{\mathbf{y_1}}],\mathbf{X^y}[\xi^{\mathbf{y_2}}]] = -\mathbf{X^v}[(\xi^{y_1})^2\frac{d}{du}(\frac{\xi^{y_2}}{\xi^{y_1}})]$$

Mode Expansion

$$\mathbf{X^u}[\xi^{\mathbf{u}}(\mathbf{u})] = \xi^u\partial_{\mathbf{u}} - \dot{\xi}^u(v\partial_{\mathbf{v}} + \rho\partial_\rho + 2\omega\partial_\omega + \rho_x\partial_{\rho_{\mathbf{x}}}) + \ddot{\xi}^u(x^2\partial_{\mathbf{v}} - 2x\partial_\rho + x^2\rho_x\partial_\omega - 2\partial_{\rho_x}) - 2\dddot{\xi}^u x^2\partial_\omega,$$

$$\xi^u(u) = \sum_{-\infty}^{\infty} a_n\left(-ie^{-inu}\right)$$

$$\mathbf{X^u}[\xi^{\mathbf{u}}(\mathbf{u})] = \sum_{-\infty}^{\infty} a_n L_n$$

$$L_n = -ie^{-inu}[\partial_{\mathbf{u}} + in(v\partial_{\mathbf{v}} + \rho\partial_\rho + 2\omega\partial_\omega + \rho_x\partial_{\rho_x}) - n^2(x^2\partial_{\mathbf{v}} - 2x\partial_\rho + x^2\rho_x\partial_\omega - 2\partial_{\rho_x}) + 2in^3x^2\partial_\omega],$$

Similarly we can do the same for all generators, we have obtained,

$$
\begin{aligned}
[L^u_n, L^u_m] &= (n-m)L^u_{n+m}\\
[L^u_n, L^x_m] &= mL^x_{m+n} - 2in^2 L^\rho_{n+m}\\
[L^u_n, L^y_m] &= -mL^y_{m+n}\\
[L^u_n, L^\rho_m] &= -(m+n)L^\rho_{m+n}\\
[L^u_n, L^v_m] &= -(m+n)L^v_{m+n}\\
[L^x_n, L^\rho_m] &= iL^v_{m+n}\\
[L^y_n, L^y_m] &= (m-n)L^v_{m+n}
\end{aligned}
$$

Conclusions

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- We have generalized method developed in [Ayon-Beato and Velázquez Rodríguez 2016] to find residual symetries.
- If we impose standard diffeomorphisms Lie-Backlund Residual Symmetries no always are reduced to Lie-Point Residual Symmetries.

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Thanks