ONE-LOOP DIVERGENCES IN 7D EINSTEIN AND 6D CONFORMAL GRAVITIES

"Einstein meets Weyl and Lanczos again"

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- Revisit two one-loop quantum gravity computations in the light of the AdS/CFT correspondence

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- Report recent progress on “listening to the CFT trace anomaly with gravity”

Plan

- Maldacena's AdS = CFT
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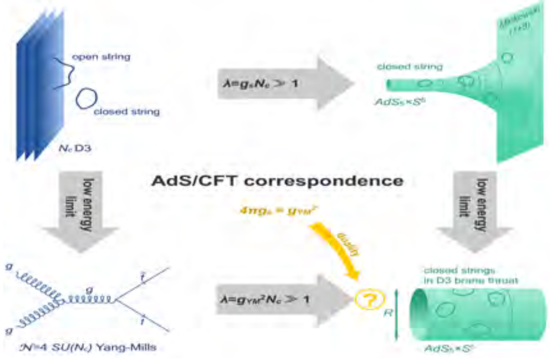

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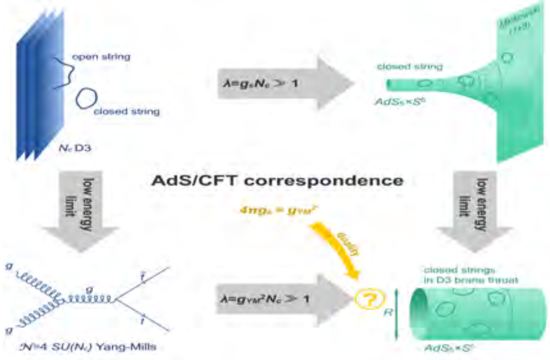

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Realization of two deeply-rooted ideas in physics:
- the holographic principle [G.'t Hooft / L.Susskind]
- the string of the large-N gauge theory [G.'t Hooft]

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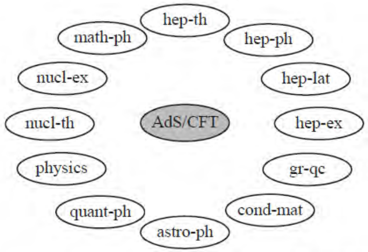

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[AdS/CFT Duality User Guide, Makoto Natsuume]

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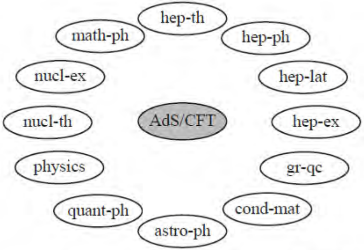

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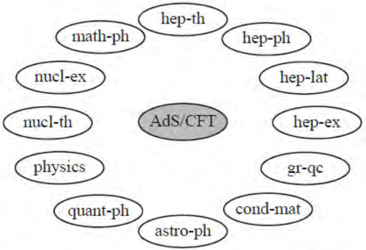

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A word of caution: ‘solution in search of a problem’ [arXiv:1211.0004 [physics.pop-ph]]

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$$Tr\, e^{\Delta\, t} = \sum_{eigenvalues} e^{-\lambda\, t} \sim \frac{A}{4\pi t} - \frac{L}{8\sqrt{\pi t}} + \frac{1}{6}(1-r)$$

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- The ubiquitous '**c**': if we put a CFT on a manifold of Euler character χ and linear size L, the free energy has the asymptotic form as $L \uparrow \infty$ [J.Cardy+I.Peschel'88]

$$F \sim \#L^2 + \#L - \frac{c}{6}\chi \ln L$$

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$$\langle T\rangle_{2D} = c\,E_2$$

$$\langle T\rangle_{4D} = -a\,E_4 + c\,W^2$$

$$\langle T\rangle_{6D} = -a\,E_6 + c_1\,I_1 + c_2\,I_2 + c_3\,I_3$$

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Brief chronology of loop computations in quantum gravity

- One loop divergencies in the theory of gravitation [G.'t Hooft + M.Veltman'74]
- Renormalization of higher derivative quantum gravity [K.Stelle'77]
- Quantizing Gravity With A Cosmological Constant [S.Christensen+M.Duff'80]
- The Ultraviolet Behavior of Einstein Gravity [M.Goroff+A.Sagnotti'86]
- ...
- One-loop divergences in 6D conformal gravity [Y.Pang'12]
- One-loop gravity divergences in D > 4 cannot all be removed [S.Deser'16]

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- Quadratic fluctuations of the 6D conformal gravity action

$$S_{CG} = \int d^6x \sqrt{g_E} \left[Ric \nabla^2 Ric - \frac{3}{10} R \nabla^2 R - 2 Riem Ric^2 - R Ric^2 + \frac{3}{25} R^3 \right]$$

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- Expansion + York decomposition + Jacobians + gauge fixing + ghost determinants + factorization ...

$$Z^{1-loop}_{Weyl} = \left[\frac{\det_{\perp}\left\{\Delta_L^{(1)} - \frac{1}{3}R\right\}\cdot\det\left\{\Delta_L^{(0)} - \frac{1}{5}R\right\}}{\det_{\perp\top}\left\{\Delta_L^{(2)} - \frac{1}{3}R\right\}\left\{\Delta_L^{(2)} - \frac{1}{5}R\right\}\left\{\Delta_L^{(2)} - \frac{2}{15}R\right\}}\right]^{1/2}$$

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- Restriction to S^6 [A.Tseytlin'13], symmetric Einstein S^2xS^4, S^3xS^3, $S^2xS^2xS^2$ [Y.Pang'12], and Ricci-flat metrics [M.Beccaria+A.Tseytlin'17] produce

$$a = \frac{377}{20160}\,,\; c_1 = \frac{1507}{45}\,,\; c_2 = \frac{635}{126}\,,\; c_3 = -\frac{1639}{420}$$

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Holographic dictionary: IR/UV connection $\Lambda_{IR} = \Lambda_{UV}$ [L.Susskind+E.Witten'98]

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One-loop divergences of Einstein gravity on a 7D Poincaré-Einstein metric

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- Poincaré-Einstein bulk metric with an Einstein metric on the conformal boundary [A.Besse'02, C.Fefferman+R.Graham'12]

$$\hat{g}_{PE} = \frac{dx^2 + (1 - R\,x^2)^2 g_E}{x^2}$$

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- Now two miracles occur with this particular PE/E metric
  - 1) Simple holographic recipe to read off the type-B coeffs. [F.Bugini+DD'16]
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- Constraining anomaly coefficients for SUSY multiplets and connection with Superconformal Index [M.Beccaria+A.Tseytlin'18, J.Liu+B.McPeak'18].

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- Conformal anomalies and (shear viscosity) -to- (entropy density) ratio of the quark-gluon plasma/soup (RHIC, LHC)
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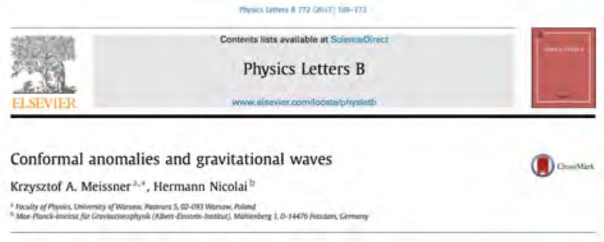
- Conformal anomalies and (shear viscosity) -to- (entropy density) ratio of the quark-gluon plasma/soup (RHIC, LHC) [Y.Kats+P.Petrov’07]

- Conformal anomalies and gravitational waves (LIGO) [K.Meissner+H.Nicolai’17]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Conformal anomalies and gravitational waves

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References and further details: JHEP 02 (2019) 188 [arXiv:1811.10380 [hep-th]]

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MANY THANKS FOR YOUR ATTENTION !