Thermodynamic properties of a magnetized neutral vector bosons gas at any temperature.

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Motivation

The thermodynamics properties of relativistic magnetized boson gas has been studied by several authors in the low temperature limit $T \ll m$.

- ✓ H. Rojas,Physics Letters B379, 148 (1996).
- ✓ V. R. Khalilov,Theoretical and Mathematical Physics 129, 1357 (2001).
- ✓ Gretel Quintero Angulo, Int. J. Mod. Phys. Conf. Ser 45, 1760047 (2017)
- ✓ G. Quintero Angulo, A. Pérez Martínez and H. Pérez Rojas. Phys. Rev. C 96, 045810 (2017)

Goals:
To extend these works **for any temperature T**, in oder to obtained equations of state (EoS) that allow more realistic desciptions of magnetic neutron stars.

Thermodynamical properties

1. Non-relativistic case. ➡ $\varepsilon_{NR}(p,B) = \frac{p^2}{2m} - skB$

2. Relativistic case.

   $B = 0$ ➡ $\varepsilon_R(p) = \sqrt{p^2 + m^2}$

$\vec{B} = (0,0,B)$

$s = -1, 0, 1$

$\hbar = c = k_B = 1$

Density of state:

$$g(\epsilon) = \sum_{s=-1,0,1} \sum_{\vec{p}} \delta[\varepsilon - \varepsilon(p,s,B)]$$

Thermodynamical potential:

$$\Omega(\mu,T,B) = -T\int_{-\infty}^{\infty} d\varepsilon\, g(\epsilon) \ln[f_{BE}(\varepsilon\,,\mu)]$$

Thermodynamic magnitudes:

$$\boldsymbol{\rho} = -\partial\Omega/\partial\mu,$$

$$\boldsymbol{M} = -\partial\Omega/\partial B$$

$$\boldsymbol{Cv} = \partial E/\partial T, \quad \boldsymbol{\chi} = \partial M/\partial B$$

$$\boldsymbol{P}_{\parallel} = -\,\Omega, \quad \boldsymbol{P}_{\perp} = -\Omega + B\,\partial\Omega/\partial B,$$

,

Non-relativistic magnetized bosons gas

Energy spectrum:

$$\varepsilon_{\mathrm{NR}}(p, s, B) = \frac{p^2}{2m} - skB$$

$$\vec{B} = (0,0,B)$$

$$s = -1, 0, 1$$

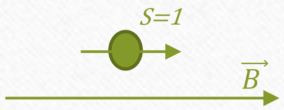
Thermodynamical potential:

$$\Omega_{NR}(\mu, T, B) = -(m/2\pi)^{3/2} T^{5/2} \left\{ g_{5/2}\left(e^{\beta(\mu - ikB)}\right) + g_{5/2}\left(e^{\beta\mu}\right) + g_{5/2}\left(e^{\beta(\mu + ikB)}\right) \right\}$$

Condition for BEC:

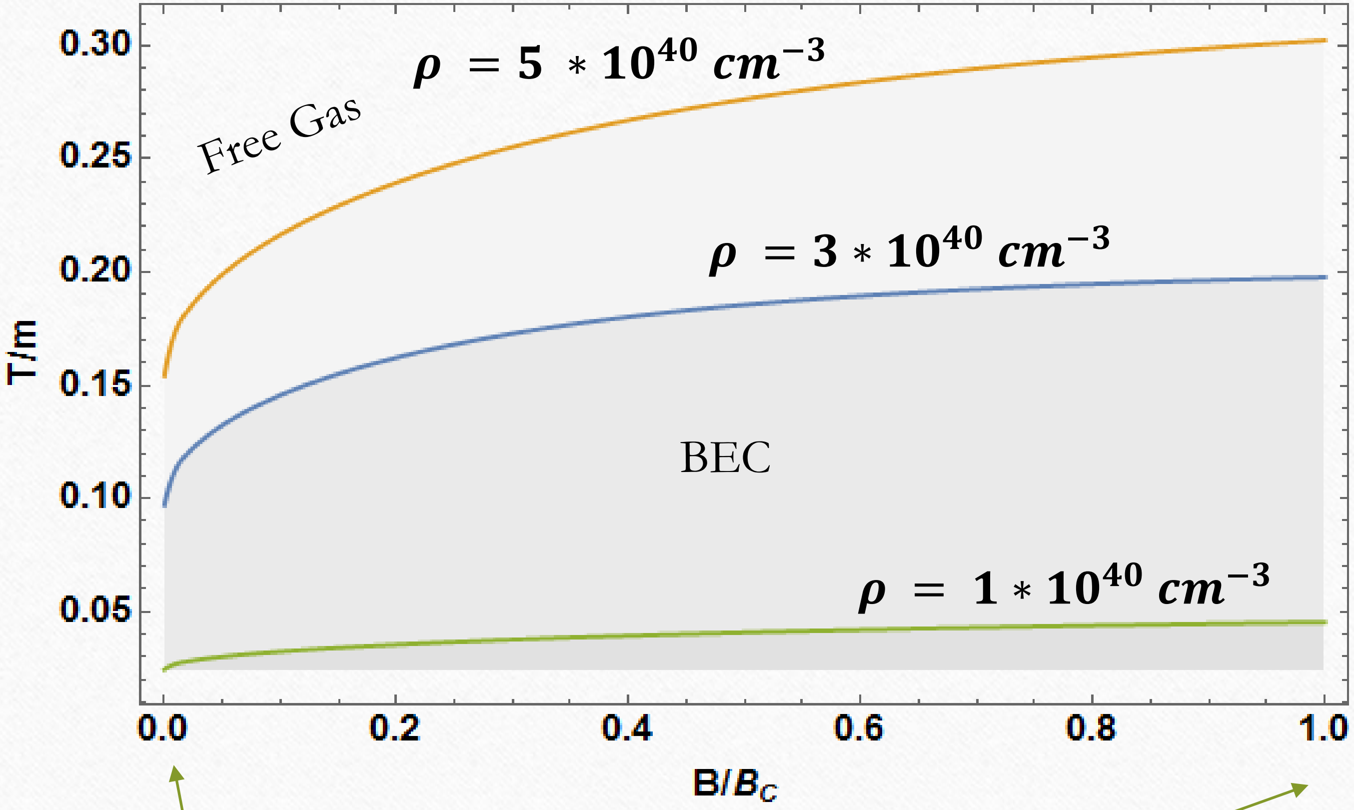
$$\mu = \varepsilon_{\min} = -kB$$

Ground state



BEC

$$T_c = T_c(B,\rho)$$



$$\frac{T_\infty}{T_0} = \sqrt[3]{9}$$

Kenji Yamada, Progressof Theoretical Physics,Vol. 67, No.2 (1982)

The condensate region grows when the density of particle increase.

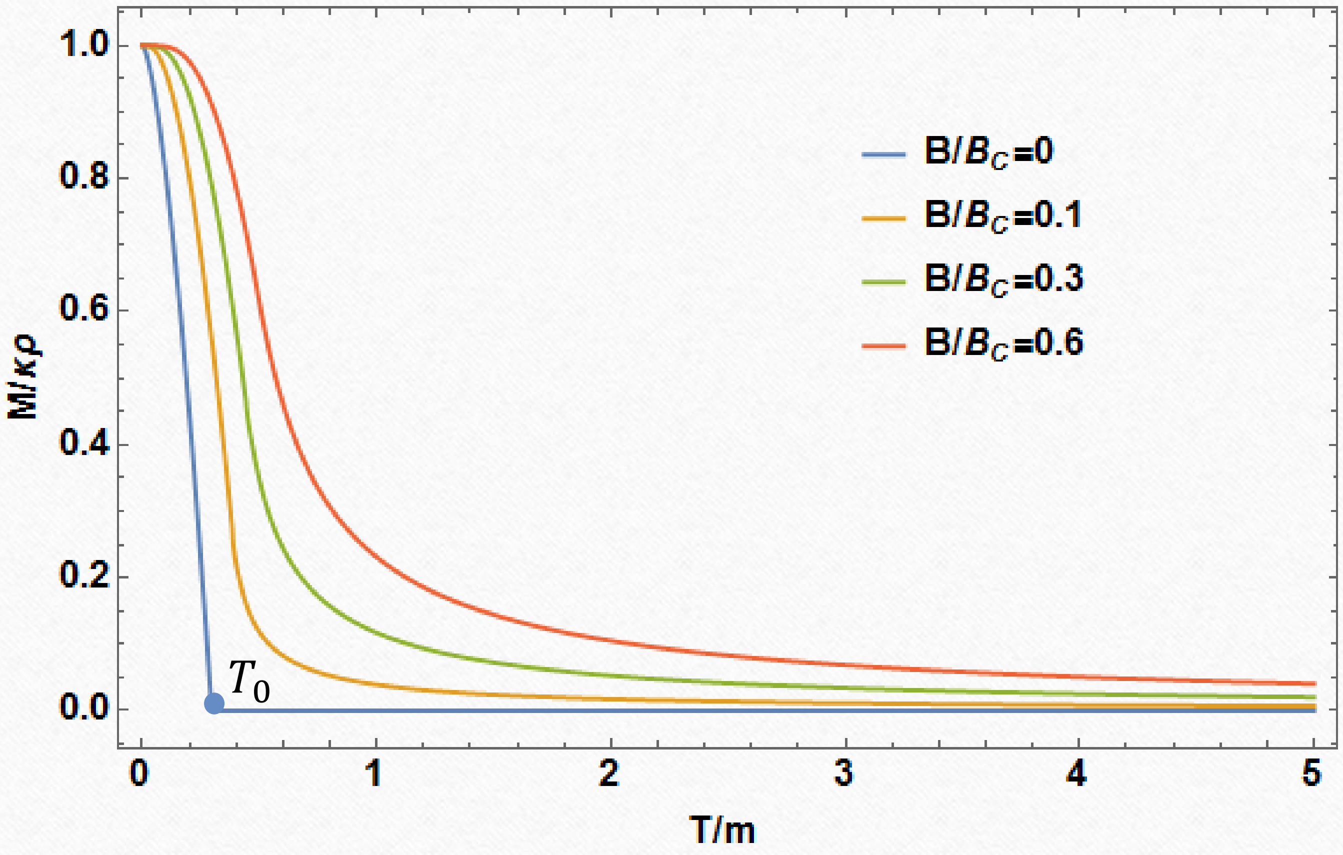
$$T_c(0,\rho) = T_0 = \frac{2\pi}{m}\left[\frac{\rho}{3\, g_{3/2}(1)}\right]^{2/3}$$

$$T_c(\infty,\rho) = T_\infty = \frac{2\pi}{m}\left[\frac{\rho}{g_{3/2}(1)}\right]^{2/3}$$

$T/m = 1 \sim 10^{13} K$ $\quad$ $Bc = m/2k \sim 10^{20} G$

Magnetic propeties

$\rho = 10^{40}\ cm^{-3}$



Magnetización:

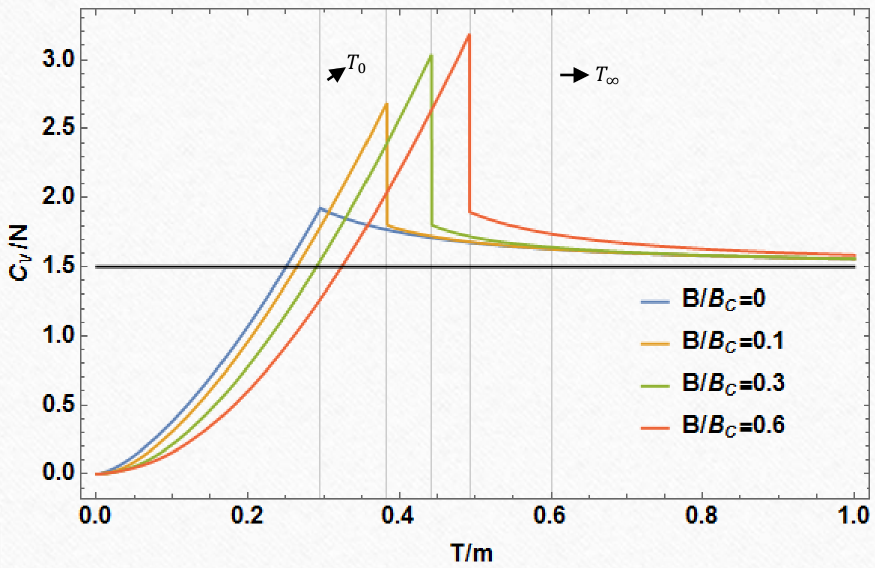
$$\mathcal{M} = k(\rho_{gs} + \rho_{+} - \rho_{-})$$

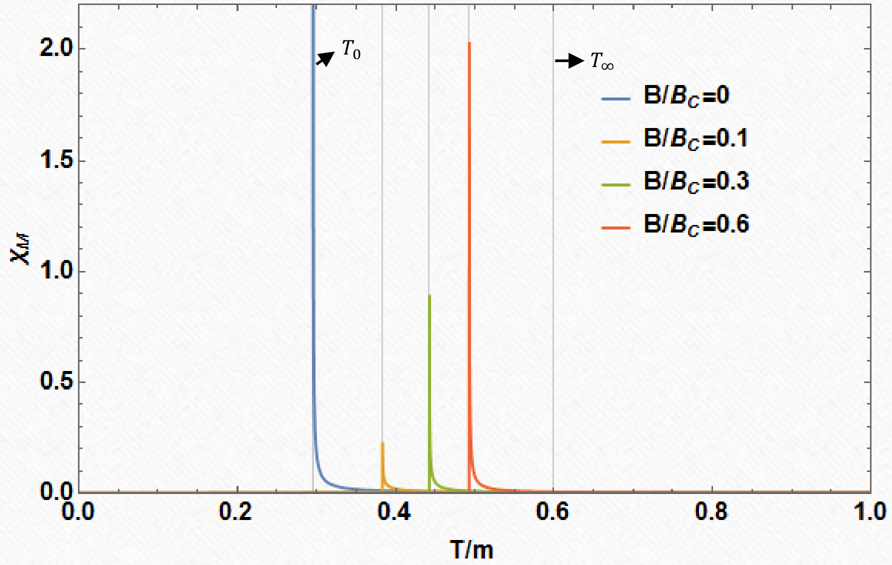
$\boldsymbol{T \gg m}$: $\boldsymbol{\mathcal{M} \rightarrow 0}$, the thermal agitation is greater than the ordering effect of the magnetic field.

$\boldsymbol{T = 0}$: $\boldsymbol{\mathcal{M} = \mathcal{M}_{max} = k\rho}$, the spins of all particles are aligned with $\vec{B}$

$$\boxed{M(B = 0) = \kappa\rho_{gs}}$$ → Bose Einstein Ferromagnetism

Heat capacity and magnetic suceptivility





$\boldsymbol{\rho = 10^{40}\ cm^{-3}}$

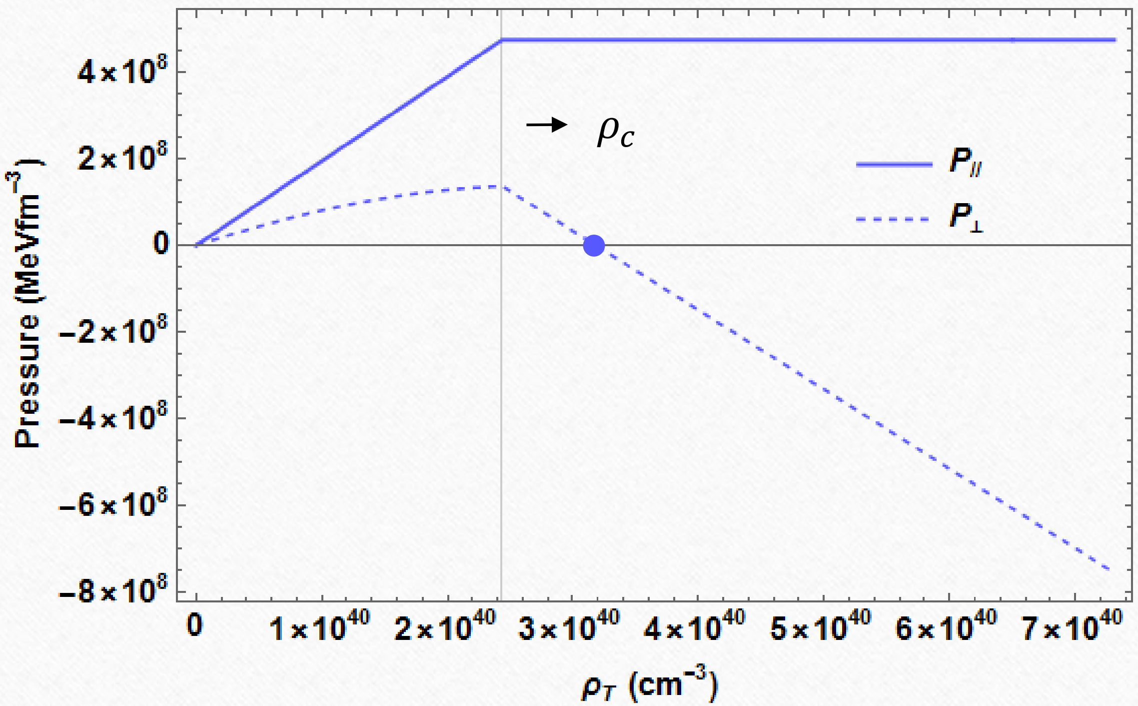
- The peaks in both susceptibility occur at the same T indicating the phase transition to the condensate state.

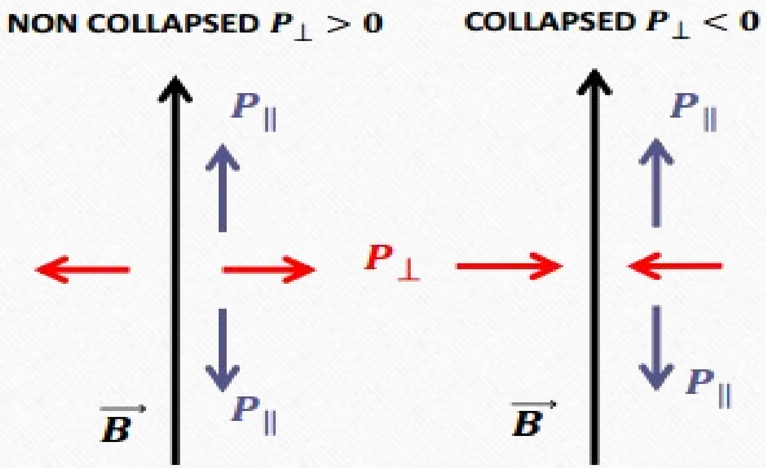
$$\chi(T,0) = \begin{cases} \dfrac{2k^2}{\lambda^3 T} g_{1/2}(z), & \textit{non-condensate} \\ \infty, & \textit{condensate} \end{cases}$$

Kenji Yamada, Progress of Theoretical Physics, Vol. 67, No.2 (1982)

Anisotropic pressure. Tranversel magnetic collapse

$T = 5 * 10^{12}\ K,\ \ B/B_C = 0,5.$





- $P_\perp \leq 0$ the perpendicular pressure becomes cero and even take negative values, this can be interpreted as the system becoming unstable.
- This imposes an upper limit to the densities of magnetized bosons gas.

Relativistic bosons gas B=0

Energy spectrum:

$$\epsilon_{\mathrm{R}}(p) = \sqrt{p^2 + m^2}$$

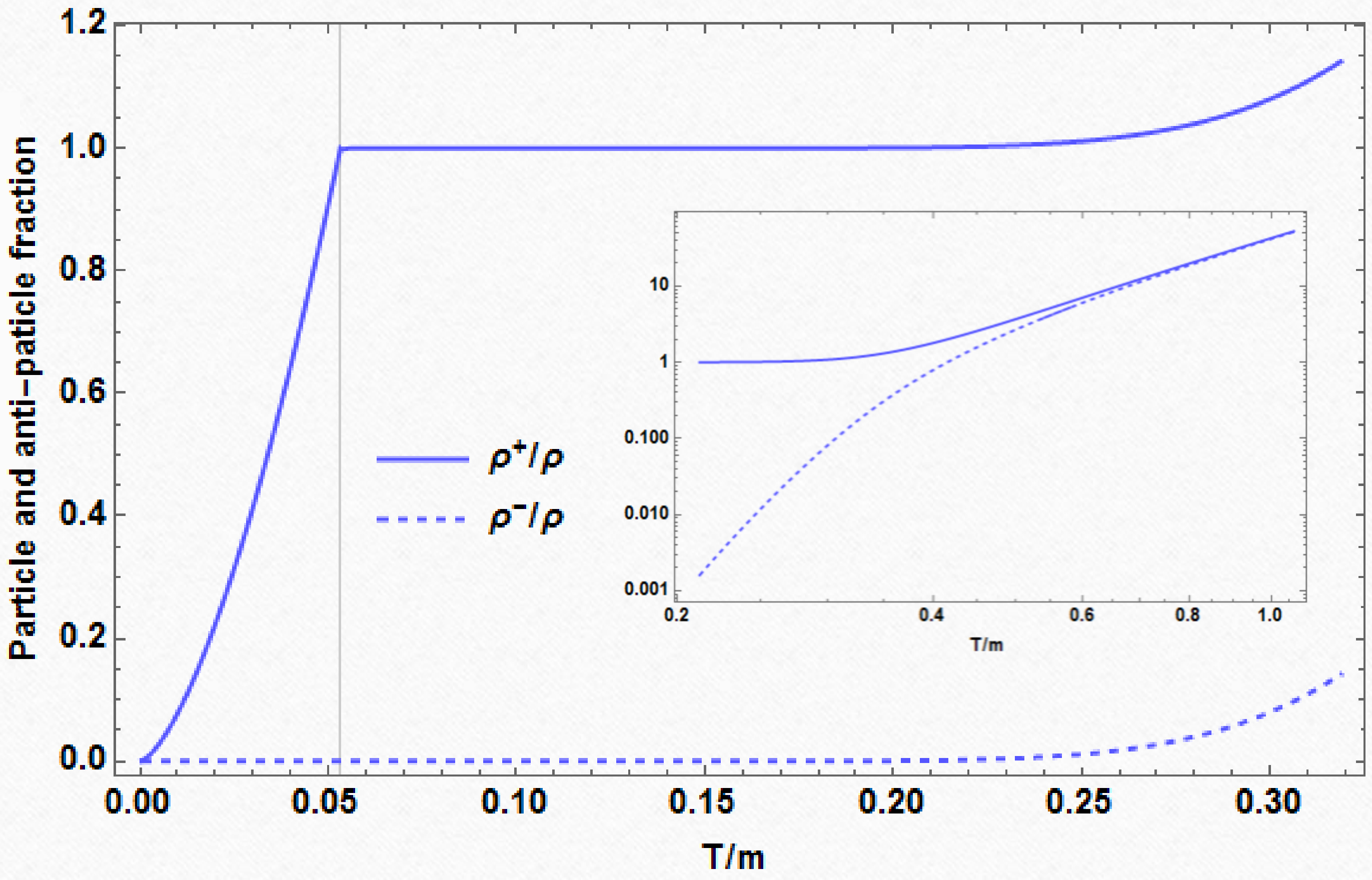
Thermodynamical potiential:

$$\Omega_R^{\pm}(\mu, T) = -\frac{3m^2T^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{e^{n\mu/T} + e^{-n\mu/T}}{n^2}K_2(nm/T)$$

Condition for the condensate:

$$\mu = m$$

Particle fraction



- $\mathbf{0.25 > \frac{T}{m} > 1}$ The anti-particles become relevat and even have the same oder as the particles.

- Near to the low temperature limit the anti-particle must be taken into acount since the can contribute to the termodynamic properties

Summary

❖ We have studied some thermodynamic properties of a magnetized neutral vector bosons gas at any temperature starting from a non-relativistic spectrum and a relativistic one without magnetic field.

❖ non-relativistic case:

1. The critical temperature of the BEC depends of: ρ, T, B.
2. The gas shows the Bose-Einstein ferromagnetism.
3. Under certain conditions, the perpendicular pressure might be negative and the system becomes unstable.

❖ Relativistic case:

1. Near to the low temperatures limit the antiparticle must be taken into account since the can contribute to the thermodynamic properties

Next Step

Relativistic case $B \neq 0$

$$\varepsilon_{RB}(p_\perp, p_3, B) = \sqrt{p_3^2 + p_\perp^2 + m^2 - 2skB\sqrt{p_\perp^2 + m^2}}$$