Ferromagnetic vacuum phase transition?

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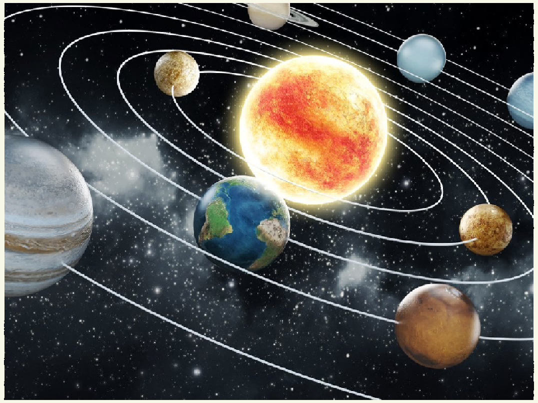
Motivation



Magnetized $e^- - e^+$ vacuum pressure

- In paper [H. Perez Rojas and E.Rodriguez Querts,International Journal of Modern Physics A Vol. **21**, No. 18 (2006)], we calculated the energy-momentum tensor for vacuum in a strong magnetic field B and found the arising of a negative pressure orthogonal it which acts at each point with rotational symmetry around any straight line parallel to **B**.

- It was found that for $B \ll B_c$ (where $B_c = m^2c^3/e\hbar \sim 4.41 \times 10^{13}$ G) the perpendicular pressure can be written as $P_\perp \approx -\frac{\alpha B^4}{120\pi^2 B_c^2}$, and for fields $B \sim 4.5 \times 10^5$ G one obtain the pressure $P_\perp \sim -1.35 \times 10^{-9} dyn\ cm^{-2}$, which may be larger than Casimir pressure. For instance for a distance between plates $d = 0.1 cm$, it gives $P_C \sim -10^{-14} dyn\ cm^{-2}$.

- Along B the pressure it is three times smaller in modulus and is positive $P_\| \approx \frac{\alpha B^4}{360\pi^2 B_c^2}$, leading to $P_\| \simeq 4.6 \times 10^{-10} dyn\ cm^{-2}$.

- The negative pressure is understood since the quantity corresponding to the classical orbit radius in the quantum case $r_0 = \sqrt{\hbar c/eB}$ decreases with increasing B. The spread of the charged particle wave function in the plane orthogonal to **B** contains a factor $e^{-\xi^2}$, with $\xi = (x_1 + x_0)/r_0$, where x_0 is the x coordinate of the orbit's centre. For large B the wave function becomes a peaked curve proportional to $\delta(x_1 + x_0)$, and the wave function amplitude in the perpendicular plane is confined to a region of area $\sim r_0^2 = \hbar c/eB$, whereas the particles move freely in the direction parallel to **B**. That behaviour is valid for virtual particles, and it leads to quantum vacuum to become unable to transmit momentum in the plane orthogonal to the field.

- We conclude that the pressure exerted on a body by magnetized quantum vacuum stretches it along the field and contracts perpendicularly to it. This effect is the analog of an electron-positron gas. The stretching-squeezing effect obviously increases with increasing B and we understand it as due to its interaction with the virtual electron positron quanta of vacuum. We may state that *magnetized quantum vacuum would drain momentum orthogonal to the external field to any incoming particle.*

Photon propagation and withdraw of momentum orthogonal to B

The withdraw of momentum orthogonal to the magnetic field is easily seen from the dispersion equation for a photon propagating in magnetized vacuum.

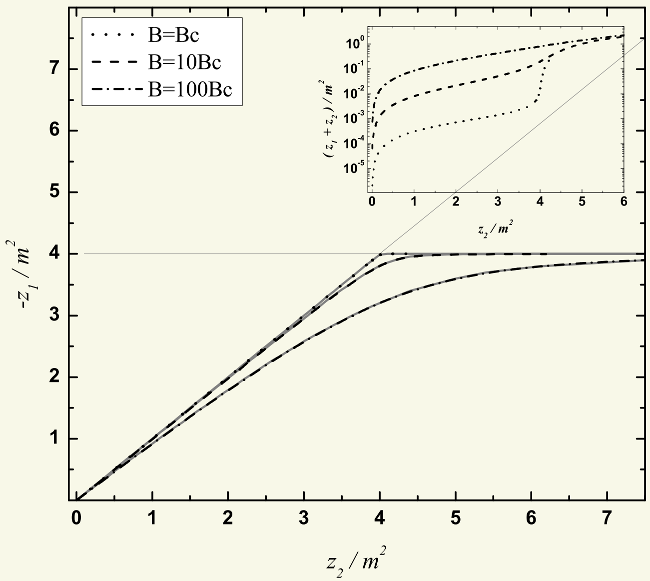


Figure 1: Dispersion Eq. for the second mode $(z_1 = k_\parallel^2 - \omega^2,\ z_2 = k_\perp^2)$

- In the previously considered limit $B \ll B_c$,

$$\omega^{i2} - k_\parallel^2 = k_\perp^2 \left(1 - \frac{C^i \alpha b^2}{45\pi}\right), \; b = B/B_c, \quad (1)$$

  $C^i = 7, 4$ for $i = 2, 3$, corresponding respectively to photon plane polarization along and orthogonal to B.

- The last expression is the dispersion equation in presence of the magnetic field for an incoming photon which far from the magnetized region, satisfies the usual light cone equation $\omega_0^2 = k_\parallel^2 + k_\perp^2$.

- In other words, the effect of the magnetic field is to decrease the incoming transverse momentum $k_\perp$ to a value $k'_\perp = (1 - C^i \alpha b^2/45\pi)k_\perp < k_\perp$, and in consequence, the initial photon squared energy (frequency) is also decreased.

Photon propagation and withdraw of momentum orthogonal to B

- If the photon moves perpendicular to B,

$$k_{||} = 0, \qquad \text{and} \qquad \omega^{2\prime} = k_{\perp}^{2\prime}. \tag{2}$$

- Interestingly, if it is reflected perpendicularly by a half-silvered mirror, keeping its plane polarization, its dispersion equation is the light cone one $\omega' = k'_{\perp}$.

- We have, by calling $\Delta\omega^i = \omega^{i\prime} - \omega^i$

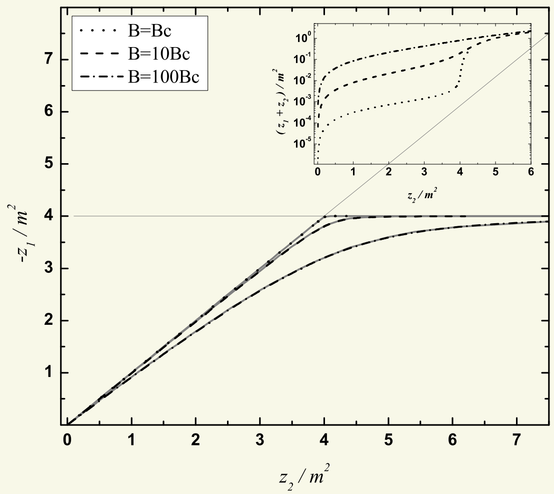
$$\Delta\omega^i = -\frac{C^i \alpha b^2}{90\pi}\omega^i \tag{3}$$

- Assuming $B \sim 4.5 \times 10^5$ G, we get for $i = 2$, $\Delta\omega^i \sim -10^{-20}\omega^i$.

The pendant task is to design an appropriate experiment to detect this effect.
We consider that interferometry could be used successfully to test it.

$B \sim B_c, \quad \omega \simeq 2m, \quad k_\parallel < \omega$:

$$z_1 = (\mathbf{k}\cdot\mathbf{B})^2/\mathbf{B}^2 - \omega^2 = k_\parallel^2 - \omega^2,$$
$$z_2 = (\mathbf{B}\times\mathbf{k})^2/\mathbf{B}^2 = k_\perp^2, \quad z_1 + z_2 = k_\mu k^\mu = k^2,$$



The dispersion equation for the second mode (having plane polarization parallel to B) may be written

$$z_1 + z_2 = \frac{2\alpha e B m e^{-z_2/2eB}}{\sqrt{z_1 + 4m^2}}. \tag{4}$$

This Eq. is valid in a neighborhood of $z_1 \lesssim -4m^2$.

- Its limit for $\mathbf{k} \to \mathbf{0}$ is $\omega \neq 0$. Actually, it describes a massive vector boson particle closely related to the electron-positron pair (see below).
- One can estimate its behavior very near $z_1 = -4m^2$, by assuming $z_1 = -4m^2 + \epsilon$ and $z_2 = 4m^2 - \epsilon$, where ϵ is a small quantity. One can obtain the solution approximately as $(z_1 + 4m^2)^{3/2} = 2\alpha e B m e^{-z_2/2eB}$, from which,

$$\omega^2 = \sqrt{k_\parallel^2 + 4m^2 - (2\alpha e B m e^{-z_2/eB})^{2/3}}. \tag{5}$$

- Thus, the transverse momentum of the original photon ($z_2 \sim 2m$) is trapped by the magnetized medium, the resulting quasi-particle being deviated to move along the field as a longitudinally polarized vector boson of mass

$$m_0 \simeq \sqrt{4m^2 - m^2(2\alpha b e^{-2/b})^{2/3}} \tag{6}$$

- Notice that from the dispersion equation we observe that it and its solutions) become complex for $z_1 < -4m^2$, or equivalently $\omega^2 > k^2 + 4m^2$.

- We shall assume that the Coulomb potential among virtual is pairs negligibly small due to screening as compared to the pair interaction with the external magnetic field through the pair magnetic moment $\mu = e\hbar/mc$. If we deal with the two-particle Green function, it is expected to get a bound state problem of a system with zero net charge, but a nonzero magnetic moment. We assume phenomenologically what is expected from such a system in its ground state, with mass around $2m$ and total spin $S = 1$. Its components are $S_z = 1, 0, -1$, the ground state being $\psi_e(\frac{1}{2}, -\frac{1}{2})\psi_p(\frac{1}{2}, \frac{1}{2})$, which contributes with a positive magnetic moment $\mu = \frac{e}{m}$, chiral non-invariant (electron L and positron R), and correspond to the projection $S_z = 0$ interacting with the external field B. We take the coupled pair mass $m_0 = 2m$ for simplicity. Due to the coupling with the external field B the effective mass of the coupled pair is expected to be

$$m_0' = m_0\sqrt{1 - B/2B_c} < 2m. \tag{7}$$

- Thus, for $B \simeq 2B_c$ the photon energy required for pair creation is decreased, and we assume bounded virtual pairs from vacuum as having effective mass m_0 in the region of transparency close to the threshold. We conclude that similarly to the modification of the photon spectrum in the magnetized region due to the photon interaction with virtual $e^{\pm}$ pairs, the latter may couple leading to a neutral virtual vector boson having a coupling with B through its magnetic moment. This leads to an effective mass $m'(B)$ which decreases much faster with B than in the photon case. This means to reduce the threshold frequencies from the gamma region to the very low frequency region. The spectrum of the bound pair system we assume to be

$$E = \sqrt{k_{\parallel}^2 + 4m^2 - 2m\mu B} \tag{8}$$

Bosonized vacuum and ferromagnetic quantum vacuum phase transition

- We shall assume that $e^{\pm}$ virtual pairs of opposite spin couple leading to bound states (a sort of virtual positronium, or neutral Cooper pairs), and we take them as described as said above, by a neutral vector boson with nonzero magnetic moment $\mu = eB/mc$. As pointed out in [G. Quintero Angulo, A. Perez Martínez, H. Perez Rojas, Phys. Rev. C 96, 045810 (2017)] this instability is avoided in a magnetized neutral vector boson gas by self-magnetization at field intensities lower than the critical field $2B_c$ [M. Chaichian, S. Masood, C. Montonen, A. Perez Martinez, H. Perez Rojas, Phys. Rev. Lett. 84, 5261 (2000)].
- Magnetized vacuum may decay at fields of order $2B_c$, since its effective mass is $4m^2 - 2m\mu B_c \simeq 0$. The quantum vacuum would become unstable at such field intensities and conditions for self-magnetization of vacuum (and matter plus vacuum) can be found to prevent the instability. A true ferromagnetic phase transition occurs for quantum vacuum. We use the virtual vector boson propagator term with $S = 0$ as $D^{-1} = [k_{\parallel}^2 + k_4^2 + 4m^2 - 2eB_c]$. After a Wick rotation leading to $-\omega^2 \to k_4^2$ and $z_1^2 \to \sqrt{k_3^2 + k_4^2}$. We shall write

$$\begin{aligned}\Omega &= \frac{1}{4\pi^2}\int_0^e \frac{de'}{e'}\int dk_4 dk_3 \frac{2B}{k_{\parallel}^2 + k_4^2 + 4m^2 - 2e'B_c} \\ &= \frac{1}{2\pi}\int_0^\infty \frac{dy}{y^2}[e^{-(4m^2-2eB)y} - e^{-4m^2 y}] \\ &= \frac{eB}{\pi}\int_0^\infty \frac{dx}{x^2} e^{-(2B_c/B)x}[e^x - x - 1]\end{aligned} \tag{9}$$

where in the last line we made the change of variables $x = 2eBy$ and regularized the divergence.

The magnetization is

$$\mathcal{M} = -\frac{\Omega}{B} + \frac{eB_c}{2\pi B}\int_0^\infty \frac{dx}{x} e^{-(2B_c/B)x}[e^x - x - 1] \tag{10}$$

The magnetization diverges as $B \to B_c$ The vacuum pressures in this case are $p_3 = -\Omega$, $p_\perp = -\Omega - B\mathcal{M}$, so that

$$p_\perp = -\frac{eB_c}{2\pi}\int_0^\infty \frac{dx}{x} e^{-(2B_c/B)x}[e^x - x - 1], \tag{11}$$

Thus, the transverse pressure is a negative quantity which diverges for $B > 2B_c$. The divergence of Ω suggests the occurrence of the phase transition from paramagnetic to ferromagnetic vacuum.
The drain of momentum orthogonal to the magnetic field done by vacuum, if verified experimentally by the shift of frequency, would give a basis in support of the suggested phase transition at very large fields of order $2B_c$ and high frequencies. Thinking on its application as a model for astrophysical jets, notice that extremely large magnetic fields confined around the jet axis may be surrounded by matter showing very small field intensities, if the original fields are generated by helicoidal huge currents. The problem is equivalent to the field created by a solenoid. The large magnetic fields inside the ”coil” induce vacuum (or vacuum plus electrically charged particles) to produce negative pressures which suck matter around it which in turn, by describing helicoidal paths, increase the central field up to reach a self-magnetization regime, leading a process propagating linearly.