Charges and Torsion

[EF & D. Hidalgo, CQG (2018), arXiv: 1703.10120]

[EF, D. Hidalgo & R. Oliveri, *in preparation*]

Ernesto Frodden

(a collaboration with Diego Hidalgo and Roberto Oliveri)

P. Universidad Católica de Chile // ICTP Visiting Scholar Programme (1/3)

06.05.2019, STARS, La Habana

Plan

- Towards Surface Charges
- (external) Motivation for Torsion
- The Torsion Field
- Example: 2+1 Gravity with Torsion
- Example: 3+1 Gravity plus Spinors
- Ending Remarks

Charges in Gauge Theories
(wider context)

- Noether Theorems (First and Second)
- Surface Charges
- Surface Charges in Gravity

⋮

- Action Boundary Terms and 'Holographic Renormalization'?
- Asymptotic Symmetries and Associated Charges?
- Resolution of the Black Hole Information Paradox?

Noether and Beyond

- For gauge theories (first) Noether theorem FAILS to produce conserved currents
- The second Noether theorem gives OFF-SHELL identities for the equations of motion

Stop computing and think! ...We need to do better...

People developed methods to compute charges for gauge theories:

1. Surface integrals (Regge-Teitelboim, Hamiltonian)
2. Abbott-Deser-Tekin (for asymptotics with $\pm\Lambda$)
3. Ashtekar-Magnon (conformal symmetry)

⋮

n. Surface charges (covariant symplectic)

It comes in two versions:

A. à la Iyer-Wald [Iyer V and Wald R M (1994) Phys. Rev. D 50 846]
B. à la Anderson-Torre-Barnich-Brandt
[Anderson I M and Torre C G 1996 PRL 77 20]
[Barnich G and Brandt F 2002 Nucl. Phys. B 633 3]

... both are ⋆equivalent⋆

Emmy Noether
1882-1935

Noether and Beyond

- **For gauge theories (first) Noether theorem FAILS to produce conserved currents**
- **The second Noether theorem gives OFF-SHELL identities for the equations of motion**

Stop computing and think! ...We need to do better...

People developed methods to compute charges for gauge theories:

1 Surface integrals (Regge-Teitelboim, Hamiltonian)

2 Abbott-Deser-Tekin (for asymptotics with $\pm\Lambda$)

3 Ashtekar-Magnon (conformal symmetry)

⋮

n Surface charges (covariant symplectic)

It comes in two versions:

A à la Iyer-Wald [Iyer V and Wald R M (1994) Phys. Rev. D 50 846]

B à la Anderson-Torre-Barnich-Brandt

[Anderson I M and Torre C G 1996 PRL 77 20]

[Barnich G and Brandt F 2002 Nucl. Phys. B 633 3]

... both are ⋆equivalent⋆

Emmy Noether
1882-1935

Noether and Beyond

- **For gauge theories (first) Noether theorem FAILS to produce conserved currents**
- **The second Noether theorem gives OFF-SHELL identities for the equations of motion**

Stop computing and think! ...We need to do better...

People developed methods to compute charges for gauge theories:

1. Surface integrals (Regge-Teitelboim, Hamiltonian)
2. Abbott-Deser-Tekin (for asymptotics with $\pm\Lambda$)
3. Ashtekar-Magnon (conformal symmetry)

⋮

n. Surface charges (covariant symplectic)

It comes in two versions:

A. à la Iyer-Wald [Iyer V and Wald R M (1994) Phys. Rev. D 50 846]
B. à la Anderson-Torre-Barnich-Brandt [Anderson I M and Torre C G 1996 PRL 77 20] [Barnich G and Brandt F 2002 Nucl. Phys. B 633 3]

... both are ⋆equivalent⋆

Emmy Noether
1882-1935

Torsion as an Aside

Current context:
In preparation a **Surface Charge Toolkit** (EF,DH,RO): Check of several gravity theories

(scalar fields, Maxwell, Lovelock theories, Skyrme, Einstein-Cartan, Dirac spinors, Chern-Simons, Yang-Mills, and BF theories)

Motivation:

1. Colleagues asking about **the role of torsion for charges?**

   L Avilés: Can *torsional boundary terms of the action* have an influence on charges?

   S Riquelme: How the *first law of black hole thermodynamics* change with torsion?

2. Simultaneously, and appealing paper appeared
   ["Contorsion Spacetime Thermodynamics" S Speziale et al (2018)]

   Extension of Jacobson's argument to torsional, Riemann-Cartan, spacetimes
   [Jacobson T, PRL 75, 1260 (1995) arXiv:9504004]

In locally Rindler patches	*Thermodynamics*	$\rightarrow$	Einstein equations
	$\delta E = T_U \delta S$	$\rightarrow$	$G_{\mu\nu} = 8\pi T_{\mu\nu}$

What is the role of torsion in surface charges?

A fast answer to first question: None action boundary term affects the surface charges formula

Torsion as an Aside

Current context:
In preparation a **Surface Charge Toolkit** (EF,DH,RO): Check of several gravity theories

(scalar fields, Maxwell, Lovelock theories, Skyrme, Einstein-Cartan, Dirac spinors, Chern-Simons, Yang-Mills, and BF theories)

Motivation:

1 Colleagues asking about **the role of torsion for charges?**

L Avilés: Can *torsional boundary terms of the action* have an influence on charges?

S Riquelme: How the *first law of black hole thermodynamics* change with torsion?

2 Simultaneously, and appealing paper appeared

["Contorsion Spacetime Thermodynamics" S Speziale et al (2018)]

Extension of Jacobson's argument to torsional, Riemann-Cartan, spacetimes

[Jacobson T, PRL 75, 1260 (1995) arXiv:9504004]

In locally Rindler patches	*Thermodynamics*	$\rightarrow$	Einstein equations
	$\delta E = T_U \delta S$	$\rightarrow$	$G_{\mu\nu} = 8\pi T_{\mu\nu}$

What is the role of torsion in surface charges?

A fast answer to first question: None action boundary term affects the surface charges formula

Torsion as an Aside

Current context:
In preparation a **Surface Charge Toolkit** (EF,DH,RO): Check of several gravity theories

(scalar fields, Maxwell, Lovelock theories, Skyrme, Einstein-Cartan, Dirac spinors, Chern-Simons, Yang-Mills, and BF theories)

Motivation:

1 Colleagues asking about **the role of torsion for charges?**

L Avilés: Can *torsional boundary terms of the action* have an influence on charges?

S Riquelme: How the *first law of black hole thermodynamics* change with torsion?

2 Simultaneously, and appealing paper appeared
["Contorsion Spacetime Thermodynamics" S Speziale et al (2018)]

Extension of Jacobson's argument to torsional, Riemann-Cartan, spacetimes
[Jacobson T, PRL 75, 1260 (1995) arXiv:9504004]

In locally Rindler patches	*Thermodynamics*	$\rightarrow$	Einstein equations
	$\delta E = T_U \delta S$	$\rightarrow$	$G_{\mu\nu} = 8\pi T_{\mu\nu}$

What is the role of torsion in surface charges?

A fast answer to first question: None action boundary term affects the surface charges formula

Torsion as an Aside

Current context:
In preparation a **Surface Charge Toolkit** (EF,DH,RO): Check of several gravity theories

(scalar fields, Maxwell, Lovelock theories, Skyrme, Einstein-Cartan, Dirac spinors, Chern-Simons, Yang-Mills, and BF theories)

Motivation:

1 Colleagues asking about **the role of torsion for charges?**

L Avilés: Can *torsional boundary terms of the action* have an influence on charges?

S Riquelme: How the *first law of black hole thermodynamics* change with torsion?

2 Simultaneously, and appealing paper appeared
["Contorsion Spacetime Thermodynamics" S Speziale et al (2018)]

Extension of Jacobson's argument to torsional, Riemann-Cartan, spacetimes
[Jacobson T, PRL 75, 1260 (1995) arXiv:9504004]

In locally Rindler patches	*Thermodynamics*	$\rightarrow$	Einstein equations
	$\delta E = T_U \delta S$	$\rightarrow$	$G_{\mu\nu} = 8\pi T_{\mu\nu}$

What is the role of torsion in surface charges?

A fast answer to first question: None action boundary term affects the surface charges formula

Torsion as an Aside

Current context:
In preparation a **Surface Charge Toolkit** (EF,DH,RO): Check of several gravity theories

(scalar fields, Maxwell, Lovelock theories, Skyrme, Einstein-Cartan, Dirac spinors, Chern-Simons, Yang-Mills, and BF theories)

Motivation:

1. Colleagues asking about **the role of torsion for charges?**

   L Avilés: Can *torsional boundary terms of the action* have an influence on charges?

   S Riquelme: How the *first law of black hole thermodynamics* change with torsion?

2. Simultaneously, and appealing paper appeared

   ["Contorsion Spacetime Thermodynamics" S Speziale et al (2018)]

   Extension of Jacobson's argument to torsional, Riemann-Cartan, spacetimes

   [Jacobson T, PRL 75, 1260 (1995) arXiv:9504004]

In locally Rindler patches	*Thermodynamics*	$\rightarrow$	Einstein equations
	$\delta E = T_U \delta S$	$\rightarrow$	$G_{\mu\nu} = 8\pi T_{\mu\nu}$

What is the role of torsion in surface charges?

A fast answer to first question: None action boundary term affects the surface charges formula

What is Torsion?

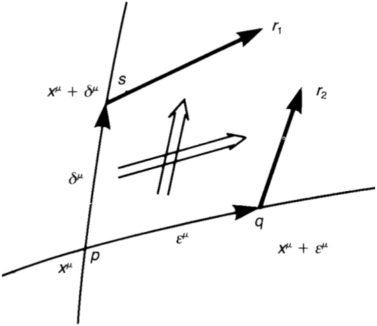

Figura: Nakahara's textbook

$$r_1 r_2 = pr_2 - pr_1 = \left(\Gamma^{\mu}{}_{\nu\lambda} - \Gamma^{\mu}{}_{\lambda\nu}\right)\varepsilon^{\lambda}\delta^{\nu} = T^{\mu}{}_{\nu\lambda}\,\varepsilon^{\lambda}\delta^{\nu}$$

[In tetrad-connection formalism $T^a \equiv De^a = de^a + \omega^a{}_b \wedge e^b = T^a{}_{bc}\, e^b \wedge e^c$]

General Relativity in 2+1 plus a Torsion Term

(**J Zanelli:** pick up this simple example)

[Warning: We use first order formalism (tetrad and spin connection) in form language]

$$L[e,\omega] = \varepsilon_{abc} e^a R^{bc}(\omega) + \frac{1}{3\ell^2} \varepsilon_{abc} e^a e^b e^c + \alpha e_a T^a$$

For an exact symmetry

$$\begin{aligned} \delta_\epsilon e^a &= D(\xi \lrcorner e^a) + \xi \lrcorner (De^a) + \lambda^a{}_b e^b = 0 \\ \delta_\epsilon \omega^{ab} &= \xi \lrcorner R^{ab} - D\lambda^{ab} = 0 \end{aligned}$$

The **surface charge density** is

$$\boxed{k_\epsilon = -\varepsilon_{abc}(\lambda^{ab}_\omega \delta e^c - \delta\omega^{ab} \xi \lrcorner e^c) + 2\alpha \xi \lrcorner e^a \delta e_a}$$

Split the connection in torsionless and torsionful parts $\omega = \widetilde{\omega}(e) + \overline{\omega}$.
Everything splits ($\delta\omega = \delta\widetilde{\omega} + \delta\overline{\omega}$ and $\lambda_\omega = \lambda_{\widetilde{\omega}} + \lambda_{\overline{\omega}}$) and exactly cancels the extra α-term

$$\boxed{k_\epsilon = -\varepsilon_{abc}(\lambda^{ab}_{\widetilde{\omega}} \delta e^c - \delta\widetilde{\omega}^{ab} \xi \lrcorner e^c)}$$

Torsion disappears from the surface charge density!
Or, in this example,
The surface charge density is transparent to spacetime torsion!

General Relativity in 2+1 plus a Torsion Term

(**J Zanelli:** pick up this simple example)

[Warning: We use first order formalism (tetrad and spin connection) in form language]

$$L[e,\omega] = \varepsilon_{abc} e^a R^{bc}(\omega) + \frac{1}{3\ell^2}\varepsilon_{abc} e^a e^b e^c + \alpha e_a T^a$$

For an exact symmetry

$$\begin{aligned} \delta_\epsilon e^a &= D(\xi \lrcorner e^a) + \xi \lrcorner (De^a) + \lambda^a{}_b e^b = 0 \\ \delta_\epsilon \omega^{ab} &= \xi \lrcorner R^{ab} - D\lambda^{ab} = 0 \end{aligned}$$

The **surface charge density** is

$$\boxed{k_\epsilon = -\varepsilon_{abc}(\lambda^{ab}_\omega \delta e^c - \delta\omega^{ab} \xi \lrcorner e^c) + 2\alpha \xi \lrcorner e^a \delta e_a}$$

Split the connection in torsionless and torsionful parts $\omega = \widetilde{\omega}(e) + \overline{\omega}$.
Everything splits ($\delta\omega = \delta\widetilde{\omega} + \delta\overline{\omega}$ and $\lambda_\omega = \lambda_{\widetilde{\omega}} + \lambda_{\overline{\omega}}$) and exactly cancels the extra α-term

$$\boxed{k_\epsilon = -\varepsilon_{abc}(\lambda^{ab}_{\widetilde{\omega}} \delta e^c - \delta\widetilde{\omega}^{ab} \xi \lrcorner e^c)}$$

Torsion disappears from the surface charge density!

Or, in this example,

The surface charge density is transparent to spacetime torsion!

General Relativity in 2+1 plus a Torsion Term

(**J Zanelli:** pick up this simple example)

[Warning: We use first order formalism (tetrad and spin connection) in form language]

$$L[e,\omega] = \varepsilon_{abc} e^a R^{bc}(\omega) + \frac{1}{3\ell^2}\varepsilon_{abc} e^a e^b e^c + \alpha e_a T^a$$

For an exact symmetry

$$\begin{aligned} \delta_\epsilon e^a &= D(\xi \lrcorner e^a) + \xi \lrcorner (De^a) + \lambda^a{}_b e^b = 0 \\ \delta_\epsilon \omega^{ab} &= \xi \lrcorner R^{ab} - D\lambda^{ab} = 0 \end{aligned}$$

The **surface charge density** is

$$\boxed{k_\epsilon = -\varepsilon_{abc}(\lambda_\omega^{ab}\delta e^c - \delta\omega^{ab}\xi \lrcorner e^c) + 2\alpha \xi \lrcorner e^a \delta e_a}$$

Split the connection in torsionless and torsionful parts $\omega = \widetilde{\omega}(e) + \overline{\omega}$.
Everything splits ($\delta\omega = \delta\widetilde{\omega} + \delta\overline{\omega}$ and $\lambda_\omega = \lambda_{\widetilde{\omega}} + \lambda_{\overline{\omega}}$) and exactly cancels the extra α-term

$$\boxed{k_\epsilon = -\varepsilon_{abc}(\lambda_{\widetilde{\omega}}^{ab}\delta e^c - \delta\widetilde{\omega}^{ab}\xi \lrcorner e^c)}$$

Torsion disappears from the surface charge density!
Or, in this example,
The surface charge density is transparent to spacetime torsion!

2+1 Black Hole with Torsion

"BTZ with torsion" [García A A, Hehl F W, Heinicke C, and Macías A, 2003, arXiv: 0302097]

The model has a black hole solution: A rotating BTZ with torsion

$$e^a = \left[f(r)\,\mathbf{dt}\,,\ \frac{1}{f(r)}\,\mathbf{dr}\,,\ r\left(\mathbf{d}\phi - \frac{J}{2r^2}\,\mathbf{dt}\right)\right]; \quad f^2(r) = \left(\frac{J}{2r}\right)^2 - M + \frac{\mathcal{T}^2+1}{\ell^2} r^2; \quad \alpha = -2\frac{\mathcal{T}}{\ell}$$

$$\omega^0{}_1 = \left(\frac{\mathcal{T}^2+1}{\ell^2} r - \frac{J\mathcal{T}}{2r\ell}\right)\mathbf{dt} + \left(\frac{r\mathcal{T}}{\ell} - \frac{J}{2r}\right)\mathbf{d}\phi; \quad \omega^0{}_2 = -\frac{1}{f(r)}\left(\frac{J}{2r^2} + \frac{\mathcal{T}}{\ell}\right)\mathbf{dr}; \quad \omega^1{}_2 = -f(r)\left(\frac{\mathcal{T}}{\ell}\mathbf{dt} + \mathbf{d}\phi\right)$$

equations of motion

$$T^a = De^a = -\frac{\mathcal{T}}{\ell}\varepsilon^a{}_{bc} e^c \wedge e^b; \qquad R^{ab} = -\frac{1}{\ell^2} e^a \wedge e^b$$

It has two exact symmetries: Time ∂_t and axial ∂_ϕ. The **surface charges densities** ($dk_\xi = 0$)

$$\boxed{k_t = \delta M \mathbf{d}\phi - \frac{\mathcal{T}^2+1}{\ell^2}\delta J \mathbf{dt}} \qquad \boxed{k_\phi = -\delta J \mathbf{d}\phi - \delta M \mathbf{dt}}$$

Charges identified as: **Mass** and **Angular Momentum**

$$\delta M = \frac{1}{2\pi}\oint k_t; \qquad \delta J = \frac{1}{2\pi}\oint k_{-\phi}$$

As expected: **Torsion does not contribute to the charges.** [A difference with the publication above]

2+1 Black Hole with Torsion

"BTZ with torsion" [García A A, Hehl F W, Heinicke C, and Macías A, 2003, arXiv: 0302097]

The model has a black hole solution: A rotating BTZ with torsion

$$e^a = \left[f(r)\,\mathbf{dt}\,,\ \frac{1}{f(r)}\,\mathbf{dr}\,,\ r\left(\mathbf{d}\phi - \frac{J}{2r^2}\,\mathbf{dt}\right)\right]; \quad f^2(r) = \left(\frac{J}{2r}\right)^2 - M + \frac{\mathcal{T}^2+1}{\ell^2} r^2; \quad \alpha = -2\frac{\mathcal{T}}{\ell}$$

$$\omega^0{}_1 = \left(\frac{\mathcal{T}^2+1}{\ell^2} r - \frac{J\mathcal{T}}{2r\ell}\right)\mathbf{dt} + \left(\frac{r\mathcal{T}}{\ell} - \frac{J}{2r}\right)\mathbf{d}\phi; \quad \omega^0{}_2 = -\frac{1}{f(r)}\left(\frac{J}{2r^2} + \frac{\mathcal{T}}{\ell}\right)\mathbf{dr}; \quad \omega^1{}_2 = -f(r)\left(\frac{\mathcal{T}}{\ell}\mathbf{dt} + \mathbf{d}\phi\right)$$

equations of motion

$$T^a = De^a = -\frac{\mathcal{T}}{\ell}\varepsilon^a{}_{bc} e^c \wedge e^b; \qquad R^{ab} = -\frac{1}{\ell^2} e^a \wedge e^b$$

It has two exact symmetries: Time ∂_t and axial ∂_ϕ. The **surface charges densities** ($dk_\xi = 0$)

$$\boxed{k_t = \delta M \mathbf{d}\phi - \frac{\mathcal{T}^2+1}{\ell^2}\delta J \mathbf{dt}} \qquad \boxed{k_\phi = -\delta J \mathbf{d}\phi - \delta M \mathbf{dt}}$$

Charges identified as: **Mass** and **Angular Momentum**

$$\delta M = \frac{1}{2\pi}\oint k_t; \qquad \delta J = \frac{1}{2\pi}\oint k_{-\phi}$$

As expected: **Torsion does not contribute to the charges.** [A difference with the publication above]

2+1 Black Hole with Torsion

"BTZ with torsion" [García A A, Hehl F W, Heinicke C, and Macías A, 2003, arXiv: 0302097]

The model has a black hole solution: A rotating BTZ with torsion

$$e^a = \left[f(r)\mathbf{dt},\ \frac{1}{f(r)}\mathbf{dr},\ r\left(\mathbf{d}\phi - \frac{J}{2r^2}\mathbf{dt}\right)\right]; \quad f^2(r) = \left(\frac{J}{2r}\right)^2 - M + \frac{\mathcal{T}^2+1}{\ell^2}r^2; \quad \alpha = -2\frac{\mathcal{T}}{\ell}$$

$$\omega^0{}_1 = \left(\frac{\mathcal{T}^2+1}{\ell^2}r - \frac{J\mathcal{T}}{2r\ell}\right)\mathbf{dt} + \left(\frac{r\mathcal{T}}{\ell} - \frac{J}{2r}\right)\mathbf{d}\phi; \quad \omega^0{}_2 = -\frac{1}{f(r)}\left(\frac{J}{2r^2} + \frac{\mathcal{T}}{\ell}\right)\mathbf{dr}; \quad \omega^1{}_2 = -f(r)\left(\frac{\mathcal{T}}{\ell}\mathbf{dt} + \mathbf{d}\phi\right)$$

equations of motion

$$T^a = De^a = -\frac{\mathcal{T}}{\ell}\varepsilon^a{}_{bc}e^c \wedge e^b; \qquad R^{ab} = -\frac{1}{\ell^2}e^a \wedge e^b$$

It has two exact symmetries: Time ∂_t and axial ∂_ϕ. The **surface charges densities** ($dk_\xi = 0$)

$$\boxed{k_t = \delta M \mathbf{d}\phi - \frac{\mathcal{T}^2+1}{\ell^2}\delta J \mathbf{dt}} \qquad \boxed{k_\phi = -\delta J \mathbf{d}\phi - \delta M \mathbf{dt}}$$

Charges identified as: **Mass** and **Angular Momentum**

$$\delta M = \frac{1}{2\pi}\oint k_t; \qquad \delta J = \frac{1}{2\pi}\oint k_{-\phi}$$

As expected: **Torsion does not contribute to the charges.** [A difference with the publication above]

Einstein-Cartan-Dirac Theory

(A more complicated example with a true matter source for torsion)

$$L[e,\omega,\psi] = \epsilon_{abcd}e^a e^b R^{cd} + \frac{i}{3}\alpha\epsilon_{abcd}e^b e^c e^d\left(\overline{\psi}\gamma^a\gamma_5 D\psi + \overline{D\psi}\gamma^a\gamma_5\psi\right)$$

Surface charge density

$$\boxed{k_\epsilon = -2\epsilon_{abcd}(\lambda^{ab}_{\omega}e^c\delta e^d - \delta\omega^{ab}e^c\xi\lrcorner e^d) + \frac{i}{3}\alpha\epsilon_{abcd}e^b e^c\xi\lrcorner e^d(\overline{\psi}\gamma^a\gamma_5\delta\psi + \delta\overline{\psi}\gamma^a\gamma_5\psi)}$$

Work in progress: To check that

$$k_\epsilon \to -2\epsilon_{abcd}(\lambda^{ab}_{\widetilde{\omega}}e^c\delta e^d - \delta\widetilde{\omega}^{ab}e^c\xi\lrcorner e^d)$$

Preliminary calculations: The torsion vanish directly from surface charge formula.

Ending Remarks

A couple of new things

- General formulae for charges in a) 2+1 GR with torsion and b) Einstein-Cartan-Dirac
- Conserved charges for rotating BTZ with torsion

Then we **conjecture** that this is always the case, when surface charges are correctly treated:

» Torsional contribution to surface charges identically vanishes «
(conjecture)

Tentative consequences:

- Jacobson and torsional-Jacobson arguments are the very same.
- Torsion does not enter in charges and therefore can not be measured with them.

For the future

- To finish the Surface Charge Toolkit project
- To find a general prove that torsion does not enters in surface charges

Surface charges are a powerful tool to decide what charges are physically meaningful.. use it!

Ending Remarks

A couple of new things

- General formulae for charges in a) 2+1 GR with torsion and b) Einstein-Cartan-Dirac
- Conserved charges for rotating BTZ with torsion

Then we **conjecture** that this is always the case, when surface charges are correctly treated:

» Torsional contribution to surface charges identically vanishes «
(conjecture)

Tentative consequences:

- Jacobson and torsional-Jacobson arguments are the very same.
- Torsion does not enter in charges and therefore can not be measured with them.

For the future

- To finish the Surface Charge Toolkit project
- To find a general prove that torsion does not enters in surface charges

Surface charges are a powerful tool to decide what charges are physically meaningful.. use it!

Ending Remarks

A couple of new things

- General formulae for charges in a) 2+1 GR with torsion and b) Einstein-Cartan-Dirac
- Conserved charges for rotating BTZ with torsion

Then we **conjecture** that this is always the case, when surface charges are correctly treated:

» Torsional contribution to surface charges identically vanishes «

(conjecture)

Tentative consequences:

- Jacobson and torsional-Jacobson arguments are the very same.
- Torsion does not enter in charges and therefore can not be measured with them.

For the future

- To finish the Surface Charge Toolkit project
- To find a general prove that torsion does not enters in surface charges

Surface charges are a powerful tool to decide what charges are physically meaningful.. use it!

¡Gracias!