Can the symmetry breaking in the SM be determined by the “second minimum” of the Higgs potential?

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Abstract

The possibility that the spontaneous symmetry breaking in the Standard Model (SM) may be generated by the Top-Higgs Yukawa interaction (which determines the so called “second minimum” in the SM) is investigated. A former analysis about a QCD action only including the Yukawa interaction of a single quark with a scalar field is here extended. We repeat the calculation done in that study of the two loop effective action for the scalar field of the mentioned model. A correction of the former evaluation allowed to select a strong coupling $a(\mu, \Lambda_{QCD})$ = 0.2254 GeV at an intermediate scale μ = 11.63 GeV, in order to fix the minimum of the potential at a scalar mean field determining 175 GeV for the single quark mass. Further, a scalar field mass m = 44 GeV is evaluated, which is also of the order of the experimental Higgs mass. The work is also considering the effects of employing a running with momenta strong coupling. For this purpose, the finite part of the two loop potential contribution determined by the strong coupling, was represented as a momentum integral. Next, substituting in this integral the experimental values of the running coupling, the minimum of the potential curve as a function of the mean field was again fixed to the top quark mass by reducing the scale to the value μ = 4.95 GeV. The consideration of the running coupling also deepened the potential value at the minimum and slightly increased the mass of the scalar field up to 53.58 GeV. These results rested in assuming that the low momentum dependence of the coupling is “saturated” to a constant value being close to its experimental value at the lowest momentum measured.

Based in: ***Letters in High Energy Physics*, 2**, 15 (2019)

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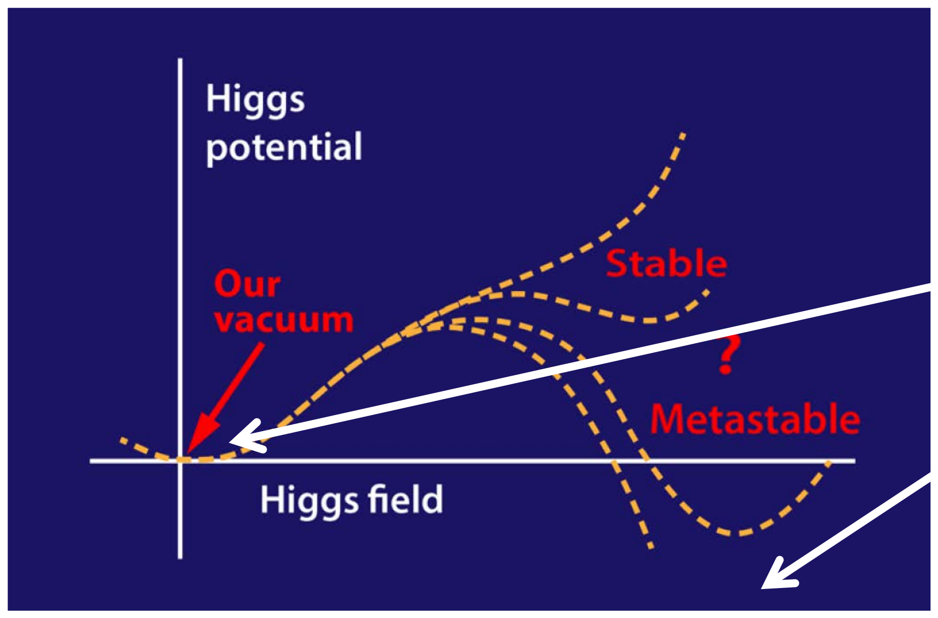
1. Motivation



The work is motivated by an existing circumstance in connection with the Standard Model: The effective potential for the scalar Higgs field shows two minima.

The first one, which is associated with the vacuum in which our world lives, and a second one which is determined by the Yukawa interaction between the Higgs field and the Top quark one, as illustrated in the figure at the left.

Moreover, some previous evaluations had indicated that the Universe could be metastable. That is, the second minimum showing lower energy that the first one.

But, the second minimum had been identified only after the SM calcualtions had increased in precison by arriving up to the second loop approximation (second order in the expansion in the Planck constant) . Thus, it comes to the mind the following question:
What possibilities can exist for employing this minimum for generating the breaking of the gauge symmetry in the SM?

To explore this possibility is the general aim of the prersented work, which is motivated by the possibility of eliminating the metastability and other current difficulties with the SM.

2. Form of the model Lagrangian for exploring the Yukawa interaction between the Top quark and scalar Higgs field within the more general framework of the SM.

$$Z[j,\eta,\overline{\eta},\xi,\overline{\xi},\rho] = \frac{1}{\mathcal{N}}\int \mathcal{D}[A,\overline{\Psi},\Psi,\overline{c},c,\phi]\times$$
$$\times \exp[i\, S[A,\overline{\Psi},\Psi,\overline{c},c,\phi]].$$

The Feynman expansion of the model is defined by the generating functional at the left

$$S = \int dx(\mathcal{L}_0+\mathcal{L}_1),$$
$$\mathcal{L}_0 = \mathcal{L}^g + \mathcal{L}^{gh} + \mathcal{L}^q + \mathcal{L}^\phi,$$
$$\mathcal{L}^g = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{a,\nu} - \partial^\nu A^{a,\mu}) -$$
$$\frac{1}{2\alpha}(\partial_\mu A^{\mu,a})(\partial^\nu A_\nu^a),$$
$$\mathcal{L}^{gh} = (\partial^\mu \chi^{*a})\partial_\mu \chi^a,$$
$$\mathcal{L}^q = \overline{\Psi}\, i\gamma^\mu \partial_\mu\, \Psi,$$
$$\mathcal{L}^\phi = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi,$$

There is one fermion field representing the Top quark , a gauge SU(3) gluon field of QCD and a singlet scalar field as a simple modelling of the Higgs doublet in the SM. The ghost fields of the gauge theory are also included.

At the left: the free terms of the Lagrangian. At right: the interaction terms.

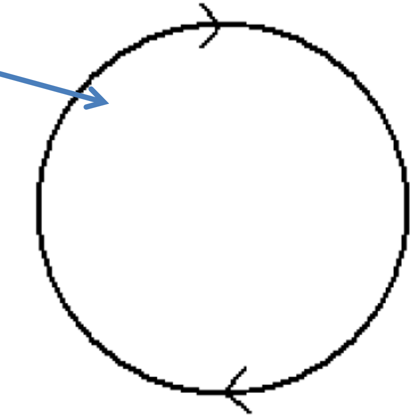
$$\mathcal{L}_1 = -\frac{g}{2} f^{abc}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) A^{b,\mu} A^{c,\nu} -$$
$$g^2 f^{abe} f^{cde} A_\mu^a A_\nu^b A^{c,\mu} A^{d,\nu} -$$
$$g f^{abc}(\partial^\mu \chi^{*a})\chi^b A_\mu^c +$$
$$g\overline{\Psi} T^a \gamma^\mu \Psi A_\mu^a + y\, \overline{\Psi}\Psi\, \phi.$$

3. Calculation of the sum of the one and two loops effective potential for the scalar Higgs field to discuss the symmetry breaking due to Yukawa interaction.

$$\Gamma^{(1)}[\phi] = -V^{(D)} N \int \frac{dp^D}{i\,(2\pi)^D} \times Log[Det\,(G^{(0)rr'}_{ii'}(\phi,p))],$$

$$D = 4 - 2\epsilon,$$

The expression for the one loop contribution to effective action for the scalar field in terms of the quark propagator, which mass is linear in the scalar field. Dimensional regularization was employed



$$G^{(0)rr'}_{ii'}(\phi,p) = \delta^{ii'}\left(\frac{1}{-p_\mu\gamma^\mu+\phi}\right)^{rr'} = -\frac{\delta^{ii'}}{p^2-\phi^2}(p_\mu\gamma^\mu+\phi)^{rr'}$$

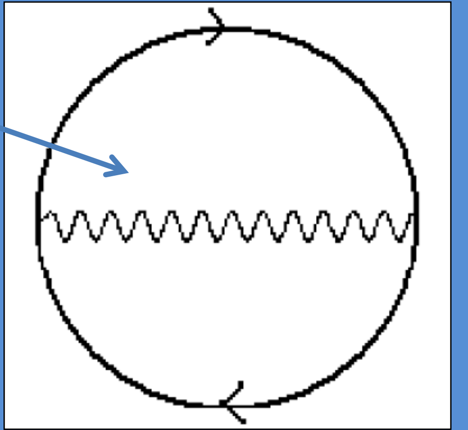
The integral can be explictily evaluated in terms of the assumed homogeneous scalar field as a function of the dimension D. Then, after deleting the pole terms in D-4 (Minimal Substraction) , leads to a finite formula for the action . Considering that the effective potential is minus the effective action, leads to the following result for the one loop effective potential:

$$v^{(1)}[\phi] = -\frac{3\phi^4}{32\pi^2}(-3+2\gamma-4\,\log(2)- 2\,\log(\pi)+2\,\log(\frac{\phi^2}{\mu^2})), = -\frac{3\phi^4}{32\pi^2}(-3+2\gamma+2\,\log(\frac{\phi^2}{4\pi\mu^2})).$$

Note that this potential is unbounded from below as function of the homogeneous scalar field value. Thus , in searching for a minimum the field would grow infinitely. This is the central term tending to induce a non vanishing mean scalar field.

$$\Gamma_g^{(2)}[\phi] = -V^{(D)} g^2 (N^2-1) \int \frac{dp^D dq^D}{i^2\,(2\pi)^{2D}} \times \frac{D\phi^2 - (D-2)p.(p+q)}{q^2(p^2-\phi^2)((p+q)^2-\phi^2)},$$

The two loop term of the effective action, determined by the quark gluon interactions.



$$g = g_0\,\mu^{2-\frac{D}{2}} = g_0\,\mu^{\epsilon},$$

The strong coupling in terms of the energy scale μ and the dimension.

The term can be decomposed in two integrals:

$$\Gamma_g^{(2)}[\phi] = \Gamma_g^{(2.1)}[\phi] + \Gamma_g^{(2,2)}[\phi],$$

$$\Gamma_g^{(2,1)}[\phi] = -V^{(D)} 2\phi^2 g^2 (N^2-1) \int \frac{dk_1^D dk_2^D}{i^2\,(2\pi)^{2D}} \times \frac{1}{k_1^2(k_2^2-\phi^2)((k_1+k_2)^2-\phi^2)}$$

$$= -V^{(D)} \frac{2\phi^2 g^2 (N^2-1)}{i^2\,(2\pi)^{2D}} J_{111}(0,\phi,\phi),$$

$$\Gamma_g^{(2,2)}[\phi] = -V^{(D)} \frac{(D-2) g^2 (N^2-1)}{2i^2(2\pi)^{2D}} \times \left(\int dk_1^D \frac{1}{k_1^2-\phi^2}\right)^2,$$

The first one can be evaluated by using the known expression of the function *J111*, in the form:

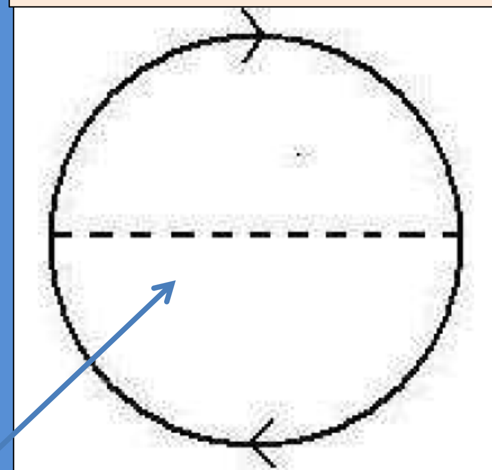
$$\Gamma_g^{(2,1)}[\phi] = -\frac{V^{(D)} 2 g_0^2 \mu^{2\epsilon}(N^2-1)}{(2\pi)^{8-4\epsilon}} \times \frac{A(\epsilon)\pi^{4-2\epsilon}}{\epsilon^2} \phi^4 (\phi^2)^{-2\epsilon},$$

After correcting an error in a former evaluation, the calculation of the second integral led to the corrected result:

$$\Gamma_g^{(2,2)}[\phi] = -V^{(D)} \frac{g_0^2 \mu^{2\epsilon}(N^2-1)2(1-\epsilon)}{2(2\pi)^{8-4\epsilon}} \times \pi^{4-2\epsilon}(\Gamma_g(\epsilon-1))^2 \phi^4 (\phi^2)^{-2\epsilon}.$$

$$\left[\gamma_g^{(2)}[\phi]\right]_{finite}^{\epsilon\to 0} = -\frac{g_0^2}{64\pi^4}\phi^4(30 - 28\gamma + 12\gamma^2 + \pi^2 + 56\log(2) - 48\gamma\log(2) + 48\log(2)^2 + 28\log(\pi) - 24\gamma\log(\pi) + 48\log(2)\log(\pi) + 12\log(\pi)^2 + (24\gamma - 28 - 48\log(2) - 48\log(\pi)) \times \log(\frac{\phi^2}{\mu^2}) + 12\,(\log(\frac{\phi^2}{\mu^2}))^2) = -v_g^{(2)}[\phi],$$

The result for the quar-gluon term of the effective action (minus the effective potential) as a function of the scalar field. The result is bounded form below and grows in absolute value more rapid than the one loop term diminishes at large scalar fields.



The contribution to the effective action of the term determined by the interaction of the fermion with the scalar has the form:

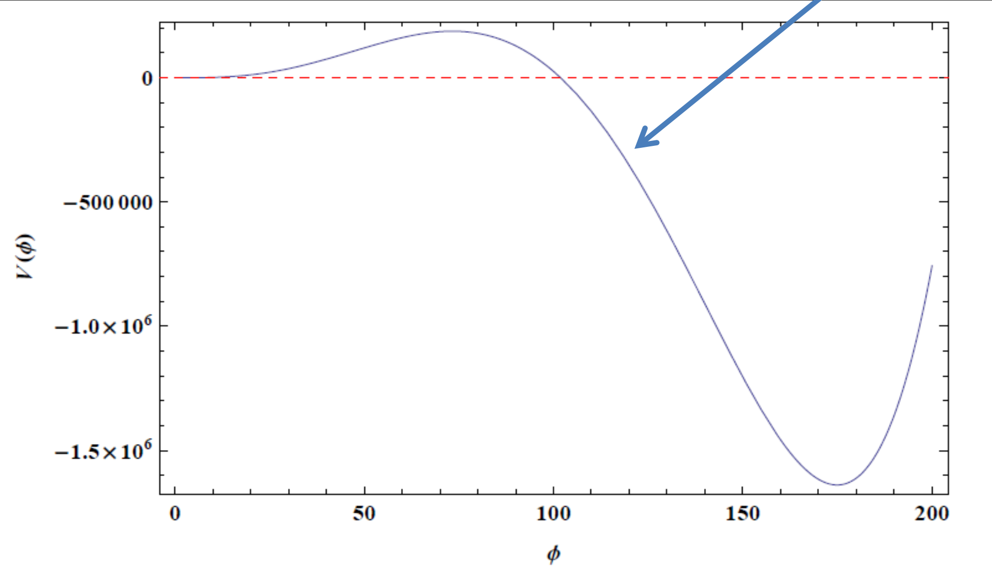
$$\Gamma_Y^{(2)}[\phi] = V^{(D)} 2N \int \frac{dp^D dq^D}{i^2\,(2\pi)^{2D}} \times \frac{p^2 - \frac{q^2}{4} + \phi^2}{q^2((p+\frac{q}{2})^2 - \phi^2)((p-\frac{q}{2})^2 - \phi^2)},$$

In a close way as the calcuation of quark- gluon term, the scalar–quark term in the effective potential was evaluated to the expression:

$$v_Y^{(2)}[\phi] = -\left[\gamma_Y^{(2)}[\phi]\right]_{finite}^{\epsilon\to 0} = -\frac{3}{512\pi^4}\phi^4(50 - 40\gamma + 12\gamma^2 + \pi^2 + 96\log(2) - 64\gamma\log(2) + 64\log(2)^2 + 48\log(\pi) - 32\gamma\log(\pi) + 64\log(2)\log(\pi) + 16\log(\pi)^2 - 8\log(4\pi) + 8\gamma\log(4\pi) - 4\log(4\pi)^2 + (24\gamma - 40 - 64\log(2) - 32\log(\pi) + 8\log(4\pi))\ \log(\frac{\phi^2}{\mu^2}) + 12\,(\log(\frac{\phi^2}{\mu^2}))^2).$$

4. The plot for the evaluated potential after fixing the mass of the fermion of the model to the observed Top quark mass of 175 GeV, indicating that the fixing can be done for an energy scale being in the measured region of the strong coupling.

After summing all the evaluated terms, the total effective potential is plotted in the figure below:



To construct the plot the scale μ was varied up to the point in which the mass of the fermion (defined by the mean value of the scalar field) became equal to the observed mass of the Top quark. The resulting values of μ and the other parameters were:

$$\mu = 11.63\ GeV,$$

$$g_0 = 1.68316 \quad \left(\alpha = \frac{(g_0)^2}{4\pi} = 0.225445\right),$$

$$\Lambda_{QCD} = 0.217\ GeV.$$

The strong coupling was expressed in terms of the energy scale μ through the known one loop renormalization group formula:

$$g_0(\mu, \Lambda_{QCD}) = 2\sqrt{\frac{2}{7}}\pi\sqrt{\frac{1}{\log(\frac{\mu}{\Lambda_{QCD}})}}.$$

where Λ_{QCD} takes the value: $0.217\ GeV$

For the mass of the scalar field:

$$\mathcal{L}^{\phi} = \frac{1}{2}\partial^{\mu}\delta\phi\partial_{\mu}\delta\phi - \frac{1}{2}\delta\phi V''[\phi]\delta\phi,$$

$$V''[\phi] = \frac{\partial^2}{\partial\phi^2}V[\phi+\delta\phi]\Big|_{\delta\phi=0}.$$

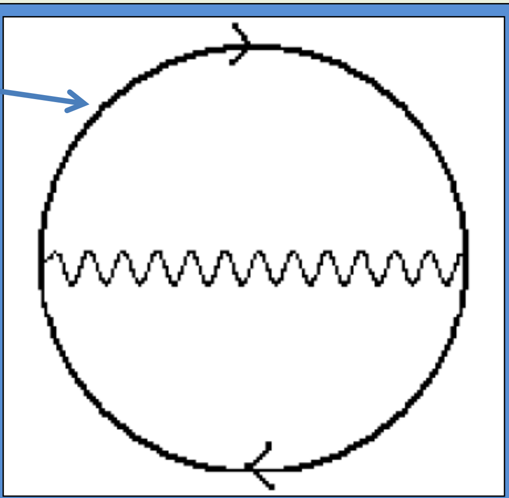
$$(\partial^{\mu}\partial_{\mu}\phi + V''[\phi])\delta\phi = (-p^2 + V''[\phi])\delta\phi = 0.$$

$$m_{\phi} = \sqrt{V''[\phi]}.$$

$$m_{\phi} = 45\ GeV$$

5. Improving the evaluation of the gluon two loop contribution to the effective potential by employing the running coupling in place of a fixed value one.

After integrating the momentum associated to the fermion loop quark-gluon term of the effective action, it can be expressed as a momentum integral :



$$\Gamma_g^{(2)}(\epsilon,g,\phi,\mu) = \int_0^\infty dq \int_0^1 dx\, 2^{-3+4\epsilon} g^2 q^{3-2\epsilon} \pi^{-4+2\epsilon}(-3+2\epsilon)\times \frac{\Gamma(\epsilon)(1-x)x(1-q^2(-1+x)x)^{-\epsilon}\phi^4(\frac{\phi}{\mu})^{-4\epsilon}}{\Gamma(2-\epsilon)} = \int_0^\infty dq\, L_g^{(2)}(q,\epsilon,g,\phi,\mu).$$

In which integrating the auxiliary variable *x* leads to the following expression:

$$L_g^{(2)}(q,\epsilon,g,\phi,\mu) = \frac{1}{3\Gamma(2-\epsilon)} g^2 q^{3-2\epsilon}\times 4^\epsilon (2\pi)^{-4+2\epsilon}(3-2\epsilon)\,\phi^4\,(\frac{\phi}{\mu})^{-4\epsilon}\,\Gamma(\epsilon)\times (-3\mathrm{AppelF}_1[2,\epsilon,\epsilon,3,u(q),v(q)] + -2\mathrm{AppelF}_1[3,\epsilon,\epsilon,4,u(q),v(q)],$$

$$u(q) = \frac{(-q^2+\sqrt{q^2(4+q^2)})}{2},$$

$$v(q) = \frac{(-q^2-\sqrt{q^2(4+q^2)})}{2},$$

in terms of the Appel functions series

$$\mathrm{AppelF}_1[a,b_1,b_2,c,x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{m+n}(b_1)_m(b_2)_n}{m!n!(c)_{m+n}}\times x^m y^n,$$

with the Pochhammer symbols defined as

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}.$$

$$S_{count}(\epsilon, g, \phi, \mu, \delta) = \int_0^\infty dq\ L_{count}(q, \epsilon, g, \phi, \mu, \delta)$$

The momentum integral giving the action term is divergent, but it became convergent after substracting a similar integral ***Scount*** in which the integrand was substituted by a specially chosen (as dependent of a new parameter δ) large momentum asymptotic *q* dependence ***Lcount*** .

$$\begin{aligned} S_{count}(\epsilon, g, \phi, \mu, \delta) &= \sum_{n=-\infty}^{\infty} S_{count}^{(n)}(g, \phi, \mu, \delta)\ \epsilon^n \\ &= -\frac{3g^2\phi^4}{32\pi^4\epsilon^2} + \frac{g^2\phi^4}{32\ \pi^4\epsilon} \times \\ &(-7 + 6\ \gamma - 6\log(4\pi) + 12\log(\frac{\phi}{\mu})) + \\ &\frac{g^2\phi^4}{64\ \pi^4} \times (-21 + \pi^2 + 10\gamma - \\ &48\log(2)^2 - 28\log(4\pi) + \\ &6(\gamma^2 + \gamma(3 - 2\gamma) - \log(\pi)\log(256\pi^2) + \\ &\gamma(-\gamma + \log(256\pi^4))) - \\ &18\log(\frac{1}{\delta^2}) + 6\log(\frac{1}{\delta^2})^2 + \\ &8(7 - 6\gamma + 6\log(4\pi) - 6\log(\frac{\phi}{\mu}))\log(\frac{\phi}{\mu})) \\ &+ O^{(1)}(\epsilon), \end{aligned}$$

But, the substracted integral can be evaluated and pole expanded in ε =2-D/2. The result is at the left.

Then, the delta parameter was chosen to make vanish the finite parte of the pole expansion. Since the O(1)(ε) vanish in the required limit ε->0 and the pole part exactly coincides with the usual one in the Minimal Substraction of the action, it follows that expression obtained by substracting ***Scount*** is the finite result for the effective action, but expressed as a momentum integral.

Therefore, the resulting finite formula for effective action, was expressed as a momentum integral. Thus, it allowed to explore the effects over the form of the effective potential of substituting the constant strong coupling by a momentum dependent one.

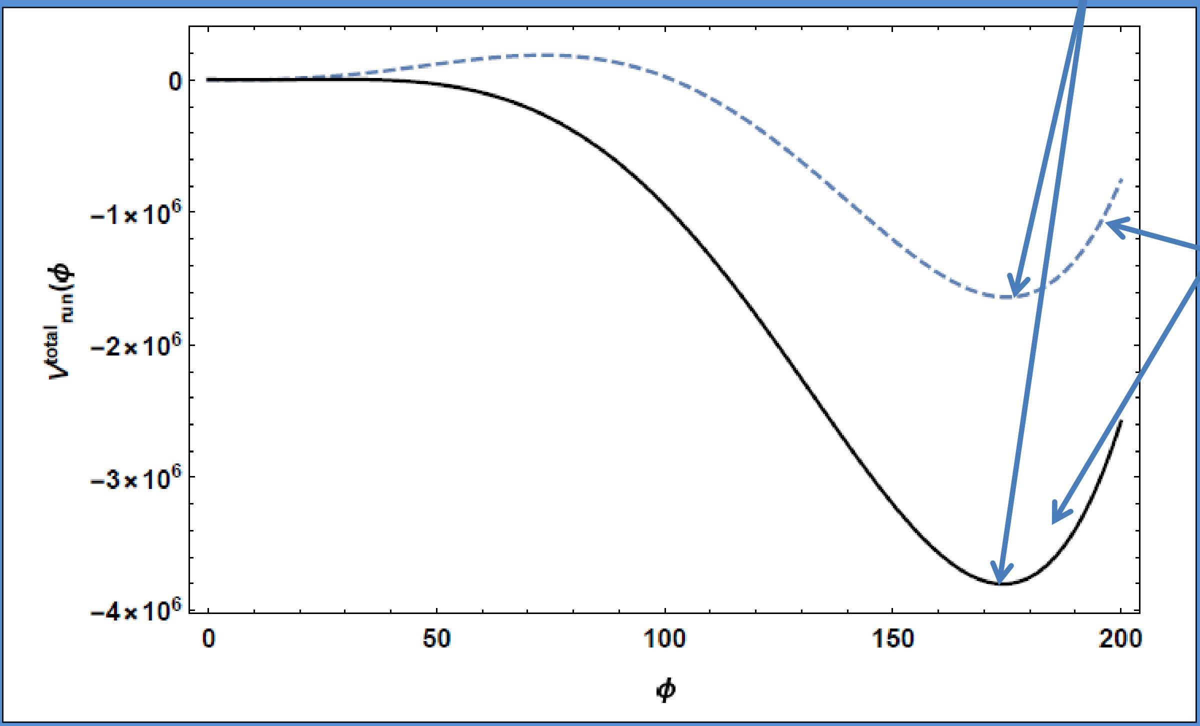
6. Plot of the potential after employing the momentum dependent coupling, for fixing again the measured Top quark mass for the fermion of the model.

$$g_{exp}(q) = \begin{cases} \frac{23.4193}{\log(32361.1672\,(\phi q)^2)}, & \phi q > 1.604 \\ g_{sat}, & \phi q < 1.604 \end{cases}$$

$$g_{sat} = 2.06.$$

Then, the running coupling employed for the evaluation of the potential was constructed as an interpolation of the data for the strong coupling in the Review of Particle Physics manual.

Again, the value of the energy scale μ was changed until the minimum of the curve became fixed at the scalar field value furnishing the observed Top quark mass of 175 GeV:



The ending value of the energy scale was reduced , although it remained in a region in which the strong coupling had been observed:

$$\mu = 4.95\ \text{GeV}.$$

The dashed line shows the potential evaluated for constant coupling. Note that the use of the running *g* deepened the potential.

For the mass of the scalar field it followed a slightly larger value:

$$m_\phi^{run} = \sqrt{V_{run}^{total\,\prime\prime}[0]} = 53.58\ \text{GeV}.$$

7. Conclusions

1. We have explored the possibility that the spontaneous symmetry breaking effect in the SM, could be implemented thanks to the Yukawa interaction of the Top quark with the Higgs field.

2. It was possible to fix the Top quark mass value at an intermediate energy scale μ = 11.63 GeV, and the scalar field mass resulted smaller than the observed Higgs mass.

3. These results suggest the possibility that in a modified SM not including the Mexican Hat potential and considering a positive mass squared term for the Higgs scalar, the observed Higgs mass of 126 GeV can be specified.

4. It was also obtained a formula for the finite effective two loop action in the Minimal Substraction scheme, which is still expressed as a momentum integral, and allows to use a running coupling.

5. By using the above result it was investigated the stability of the alternative spontaneously symmetry breaking pattern, after considering the effect of a running with momentum coupling in the calculation. It allowed to argue that is still possible to fix the Top quark mass at a slightly lower energy scale μ, which still is in the range in which the strong coupling is measured.

Possible extensions of the work

In a future, we plan to start from a Lagrangian being practically equivalent to the SM's one, in which all the Higgs field terms associated to the usual scalar doublet will be present, but in which only the negative mass squared term creating the Mexican Hat potential will not be considered. The idea will be to attempt using the various parameters in this slightly modified SM model, for implementing a symmetry breaking pattern being similar to the one discussed here. The successful definition of such a model could eliminate the current difficulties created by the second minimum in the formulation of the SM.

Thanks by the attention!