Neutron Star Equation of State after the GW170817 event

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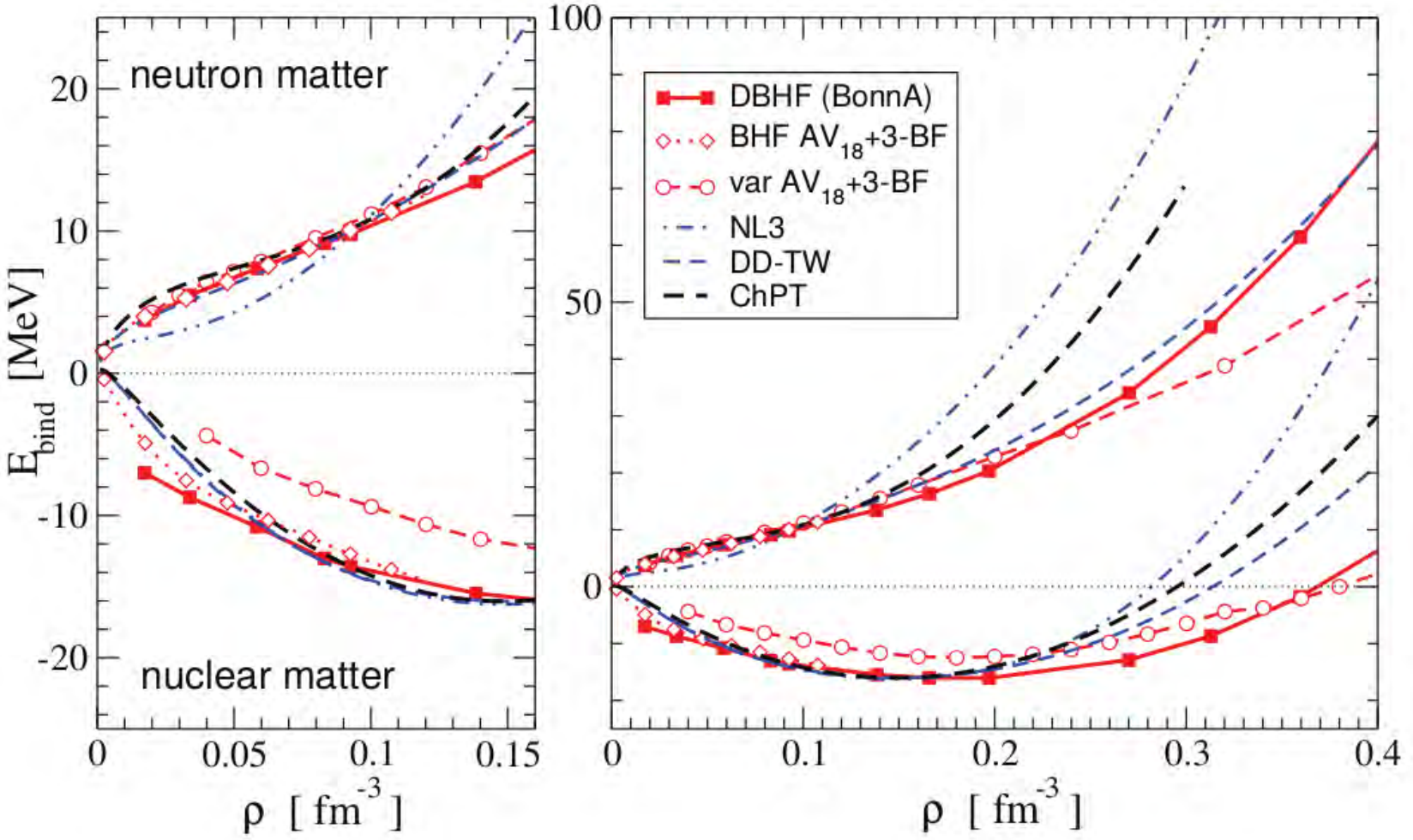
May 7, 2019

Outline

- A brief introduction to the neutron star equation of state.
- The interplay of the symmetry energy and DUrca cooling.
- Astrophysics measurements of compact stars: multi-messenger astronomy & the GW170817 event.
- The compact star mass twins case.
- Astrophysical implications and perspectives.

Nuclear Matter



C. Fuchs, H.H. Wolter, EPJA 30(2006)5

Flow Constraint

Klaehn et al. PhysRev C74 (2006)
P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002)

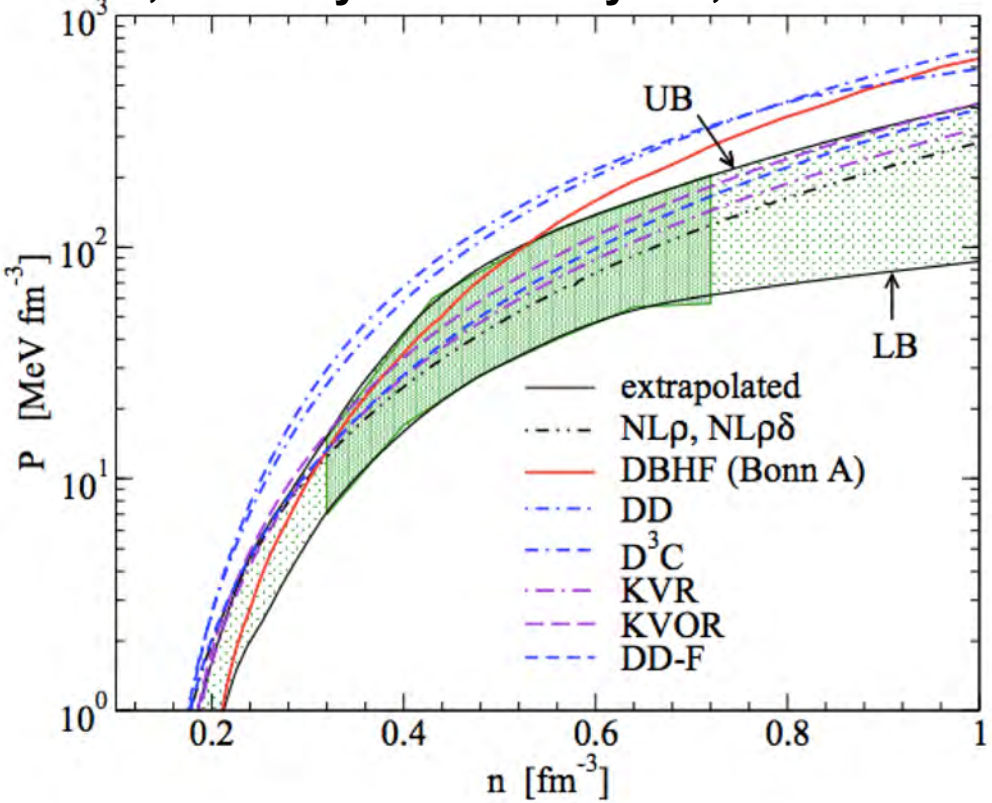
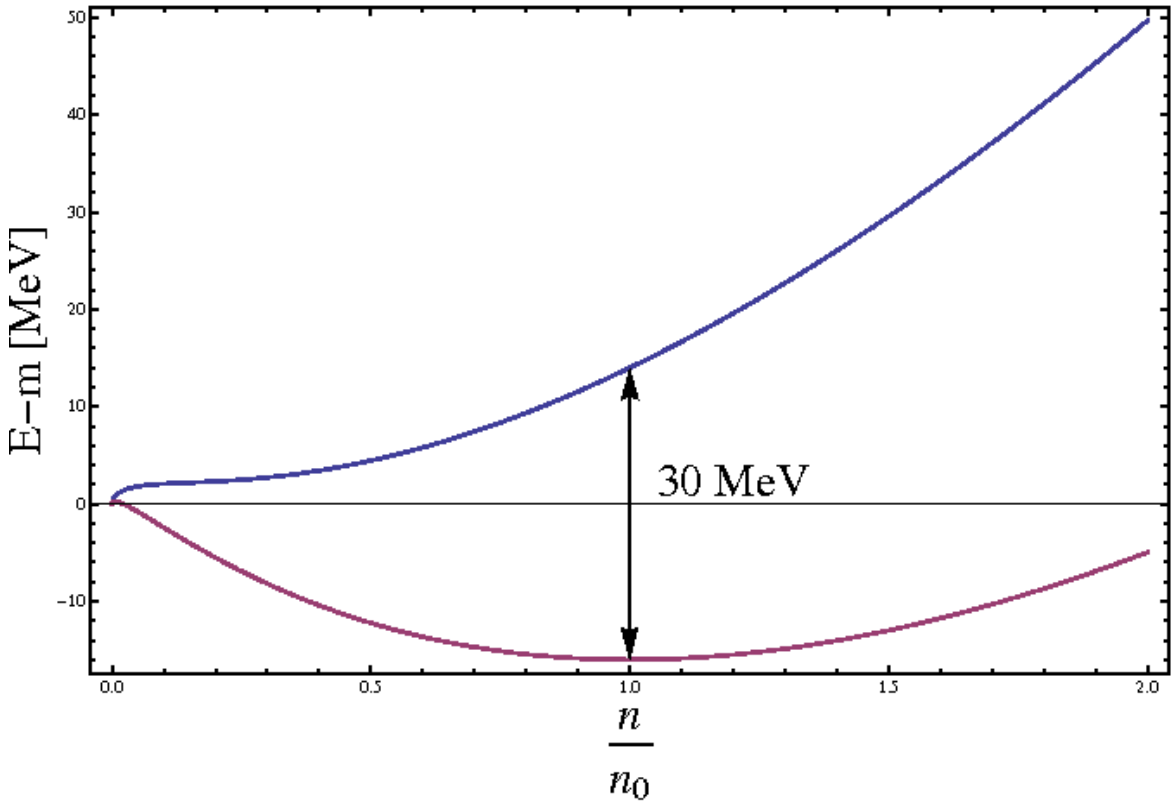


FIG. 6: Pressure region consistent with experimental flow data in SNM (dark shaded region). The light shaded region extrapolates this region to higher densities within an upper (UB) and lower border (LB).

Nuclear Symmetry Energy

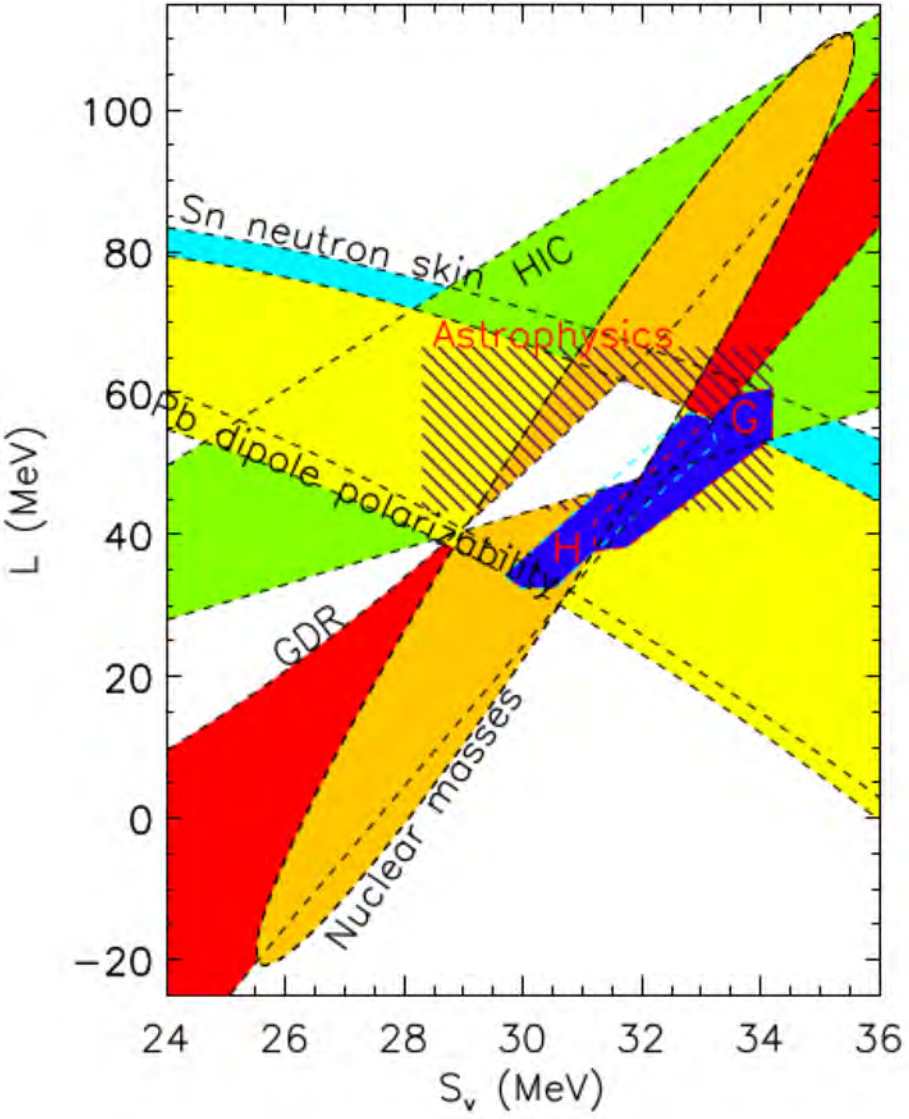


is the difference between symmetric nuclear matter and pure neutron matter:

$$E(n,x) = E(n,x=1/2) + E_s(n) * \alpha^2(x) + E_q(n) * \alpha^4(x) + O(\alpha^6(x))$$

where $\alpha = 1-2x$

Measuring the symmetry energy



Lattimer and Lim
(2013) ApJ 771 51

Neutron Star Equation of State

The energy per nucleon in neutron star core matter is given by:

$$\begin{aligned} E_{\rm tot}(n,\{x_i\}) &= E_{\rm b}(n,x_p) + E_{\rm lep}(n,x_e,x_\mu)\ , \\ E_{\rm b}(n,x_p) &= E_0(n) + S(n,x_p) \\ E_{\rm lep}(n,x_e,x_\mu) &= E_e(n,x_e) + E_\mu(n,x_\mu)\ , \end{aligned}$$

where $n = n_p + n_n$ is the total baryon density and $x_i = n_i/n$, $i = p, e, \mu$ are the fractions of protons, electrons and muons, respectively. The baryonic part is very well described by the parabolic approximation w.r.t. the asymmetry

$$\alpha = \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,$$

resulting in $S(n,x_p) = (1-2x_p)^2 E_s(n)$. The leptonic contribution is a sum of the Fermi gas expressions for the contributing leptons $l = e, \mu$

$$E_l(n,x_l) = \frac{1}{n}\frac{p_{F,l}^4}{4\pi^2}\left[\sqrt{1+z_l^2}\left(1+\frac{z_l^2}{2}\right) - \frac{z_l^4}{2}\mathrm{Arsinh}\left(\frac{1}{z_l}\right)\right]\ ,$$

where $z_l = m_l/p_{F,l}$. For massless leptons ($z_l \to 0$), this expression goes over to

$$E_l(n,x_l)\big|_{m_l=0} = \frac{1}{n}\frac{p_{F,l}^4}{4\pi^2} = \frac{3}{4}\left(3\pi^2 n\right)^{1/3} x_l^{4/3}\ .$$

Charge neutrality and β-equillibrium

Under neutron star conditions charge neutrality holds,

$$x_p = x_e + x_\mu \ .$$

The $\beta-$ equilibrium with respect to the weak interaction processes $n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$ (and similar for muons), for cold neutron stars (temperature T below the neutrino opacity criterion $T < T_\nu \sim 1$ MeV) implies

$$\mu_n - \mu_p = \mu_e = \mu_\mu \ .$$

The chemical potentials are defined as

$$\mu_i = \frac{\partial \varepsilon_i}{\partial n_i} = \frac{\partial}{\partial x_i} E_i(n, \{x_j\}) \ , \quad i, j = n, p, e, \mu \ ,$$

where $\varepsilon_i = n \ E_i(n, \{x_j\})$ is the partial energy density of species i in the system. From the above equations:

$$\mu_e = 4(1 - 2x) E_s(n) \ .$$

Since electrons in neutron star interiors are ultrarelativistic,

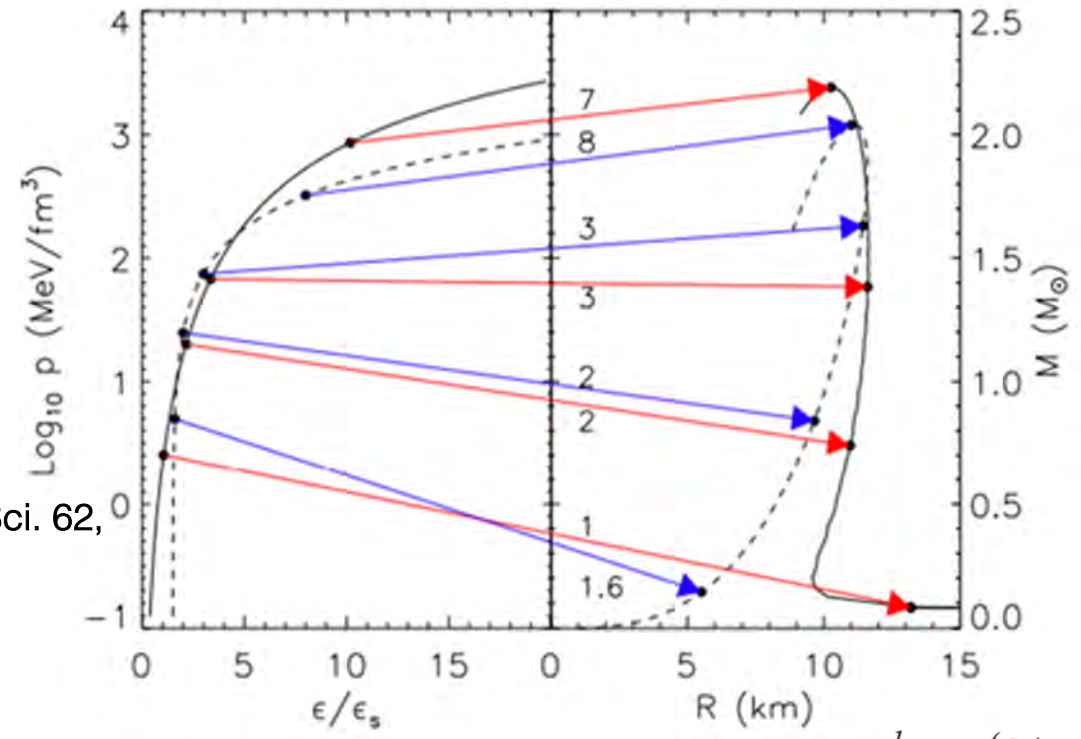
$$\mu_e = \sqrt{p_{F,e}^2 + m_e^2} \approx p_{F,e}, \text{ and } p_{F,e} = (3\pi^2 n_e)^{1/3} = (3\pi^2 n)^{1/3} (x - x_\mu)^{1/3} \ ,$$

$$\frac{x - x_\mu}{(1 - 2x)^3} = \frac{64 E_s^3(n)}{3\pi^2 n} \ , \qquad (x - x_\mu)^{2/3} - x_\mu^{2/3} = \frac{m_\mu^2}{(3\pi^2 n)^{2/3}} \ .$$

The total pressure is then given as $P(n) = n^2 \left(\frac{\partial E_{\rm tot}}{\partial n} \right)$.

Compact Star Sequences

(M-R ⇔ EoS)



Lattimer,
Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)
arXiv: 1305.3510

- TOV Equations

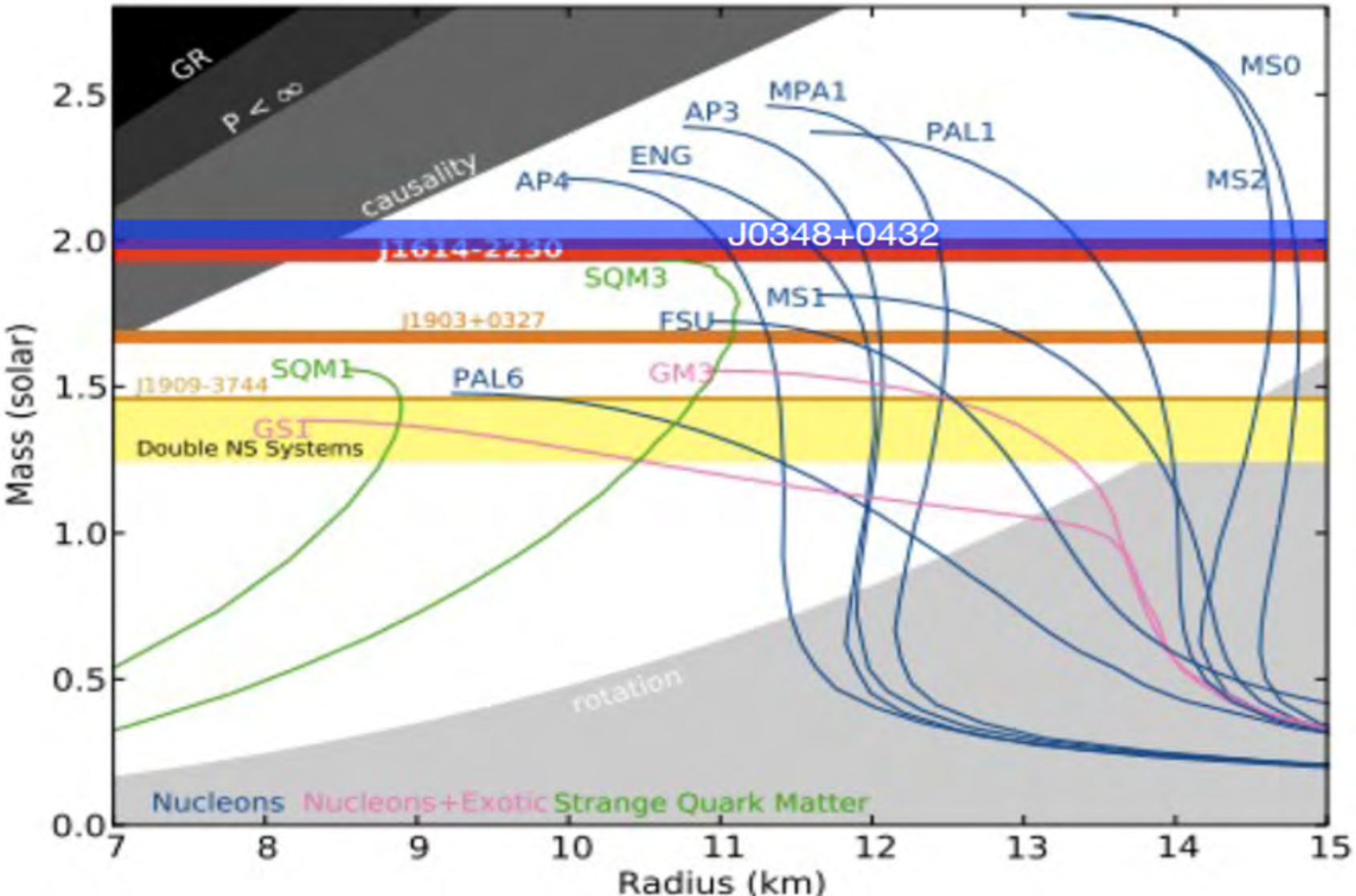
$$\frac{dp}{dr} = -\frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

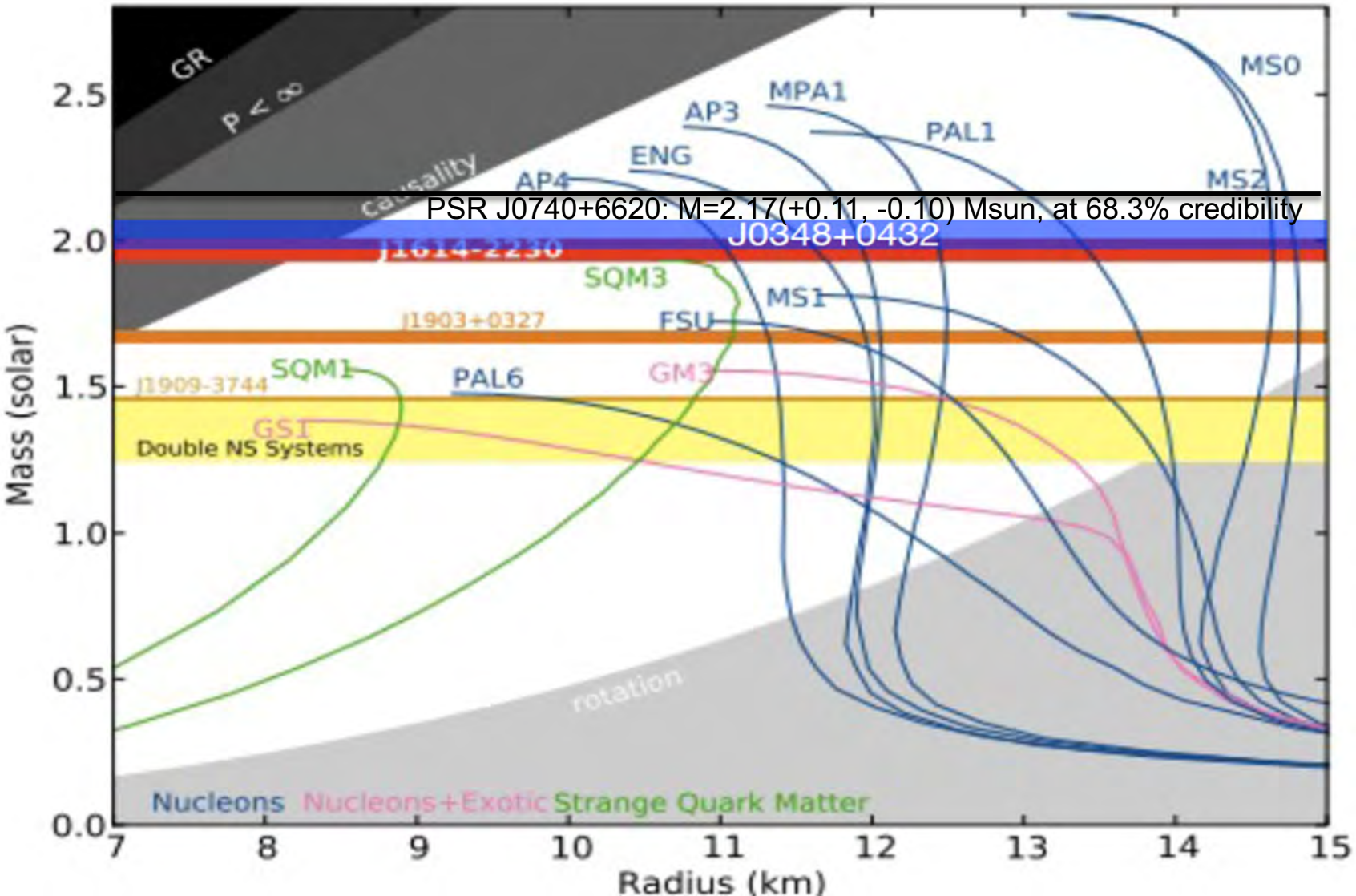
- Equation of State (EoS)

$$p(\varepsilon)$$

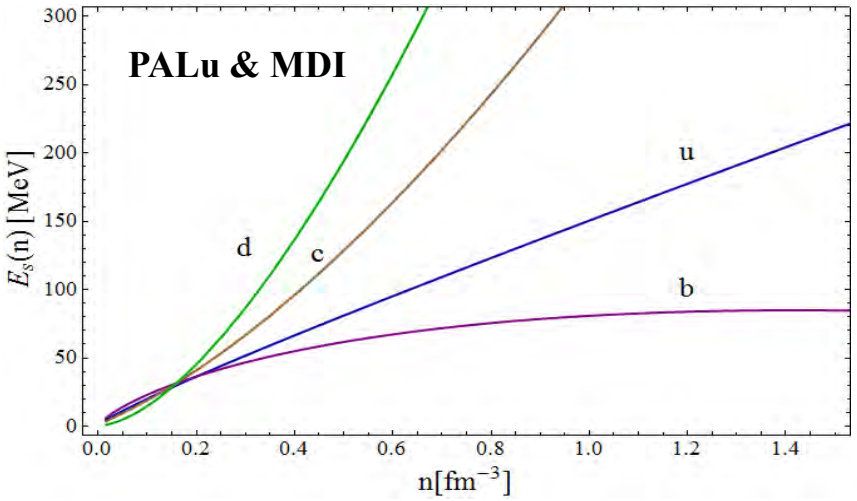
Massive Neutron Stars

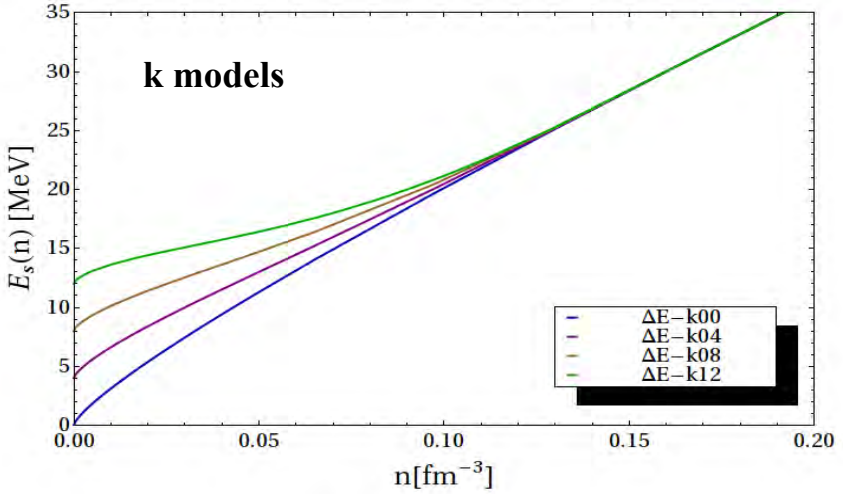


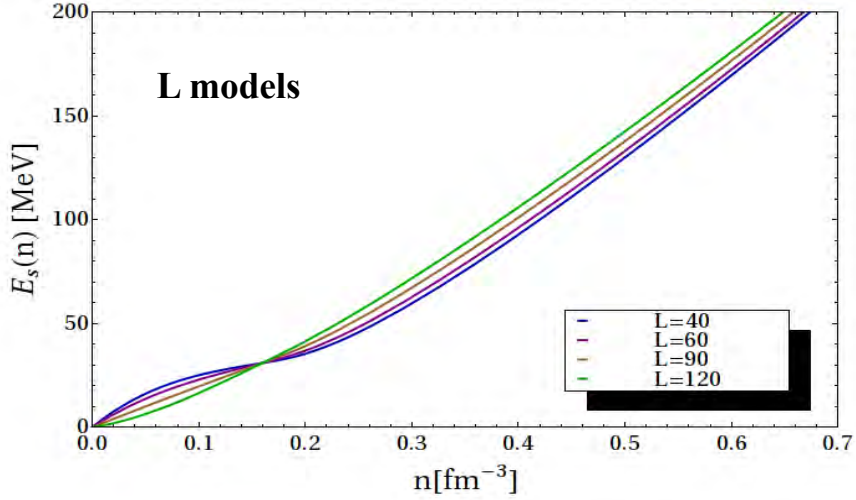
Massive Neutron Stars

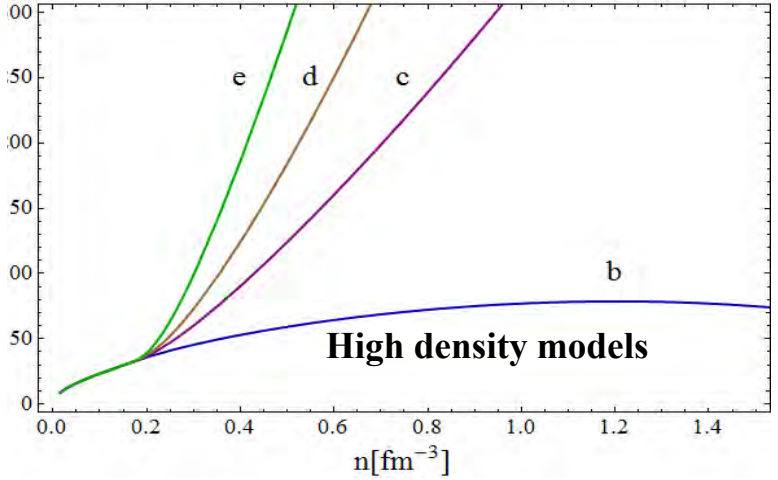


Symmetry energy effects



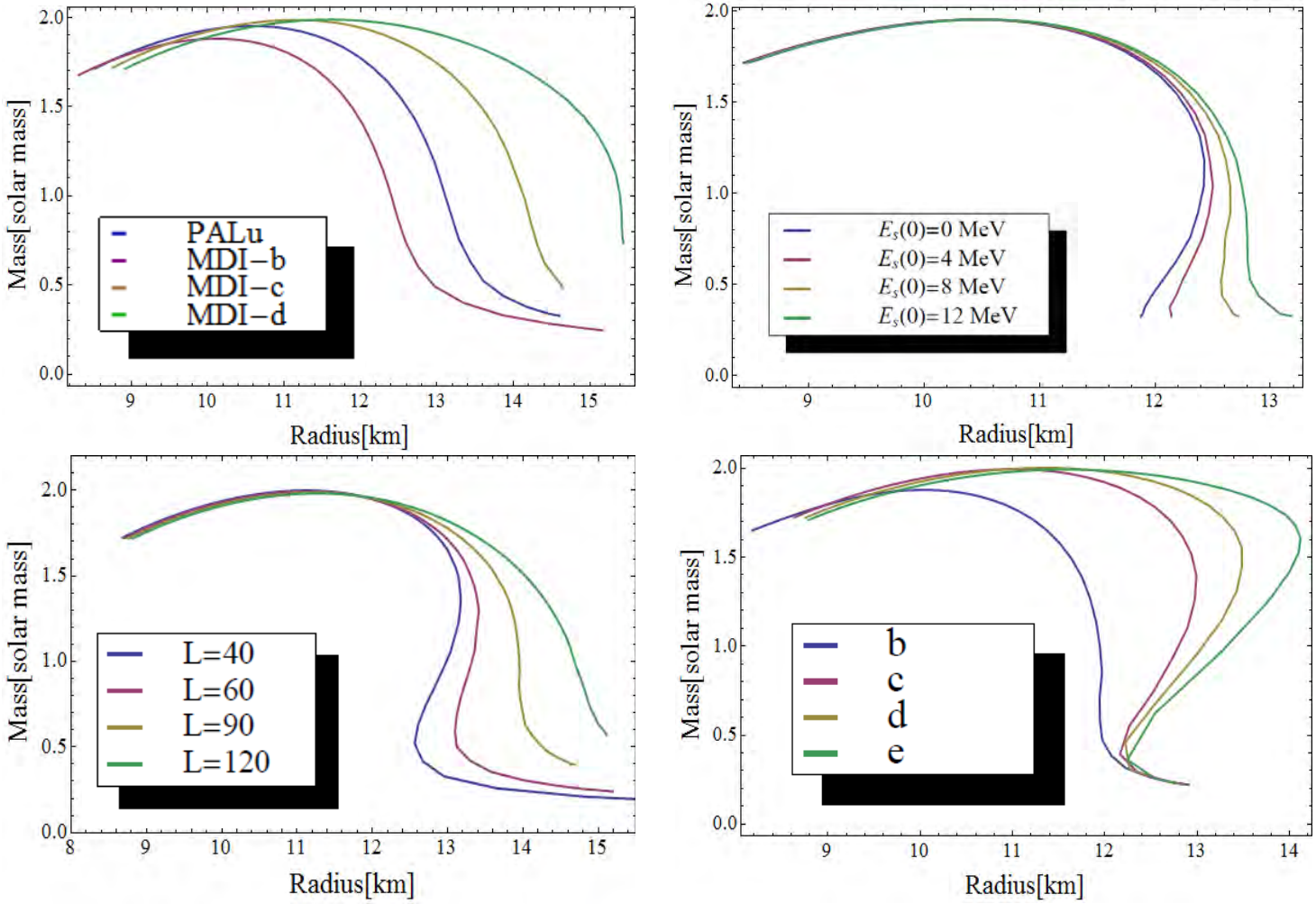






S. Kubis and D. E. Alvarez-Castillo - arXiv:1205.6368

Symmetry energy effects



S. Kubis and D. E. Alvarez-Castillo - arXiv:1205.6368

Neutron Star Cooling Processes

Process Name	Process	Emissivity Q_ν (erg cm^{-3} s^{-1})	Reference
Bremsstrahlung	$n+n \to n+n+\nu_e+\bar{\nu}_e$ $n+p \to n+p+\nu_e+\bar{\nu}_e$ $p+p \to p+p+\nu_e+\bar{\nu}_e$	$\simeq 10^{19} T_9^8$	Page, Geppert and Weber [92]
Modified Urca	$n+n \to n+p+e^-+\bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\simeq 10^{20} T_9^8$	Friman and Maxwell [93]
Direct Urca	$n \to p+e^-+\bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\simeq 10^{27} T_9^6$	Lattimer et al. [94]
Quark Urca	$d \to u+e^-+\bar{\nu}_e$ $u+e^- \to d+\nu_e$	$\simeq 10^{26} \alpha_c T_9^6$	Iwamoto [95]
Kaon Condensate	$n+K^- \to n+e^-+\bar{\nu}_e$ $n+e^- \to n+K^-+\nu_e$	$\simeq 10^{24} T_9^6$	Brown et al. [96]
Pion Condensate	$n+\pi^- \to n+e^-+\bar{\nu}_e$ $n+e^- \to n+\pi^-+\nu_e$	$\simeq 10^{26} T_9^6$	Maxwell et al. [97]

Direct Urca is the fastest cooling process.
Threshold for onset: $p_{F,n} < p_{F,p} + p_{F,e}$. For electrons only then $x_{DU}=1/9$.

DUrca Process Constraint

E_s plays an important role in determination of the activation of the direct Urca (DU) cooling process

$$n \rightarrow p + e + \bar{\nu}_e.$$

If the central density in a neutron star exceeds the critical value which allows the DU process to operate then this process triggers a dramatic drop of the core temperature due to rapid energy loss by neutrino emission. This process can therefore not be operative in typical neutron stars as we do observe cooling neutron stars much older than the typical transport timescale ($\sim$ 1000 years) with surface temperatures that are not compatible with DU cooling.

The DU threshold condition is derived from the triangle inequality for the Fermi momenta of neutron, proton and electron (neutrinos are neglected):

$$n_n^{1/3} < n_p^{1/3} + n_e^{1/3} \ ,$$

which can be formulated in terms of proton and muon fractions as

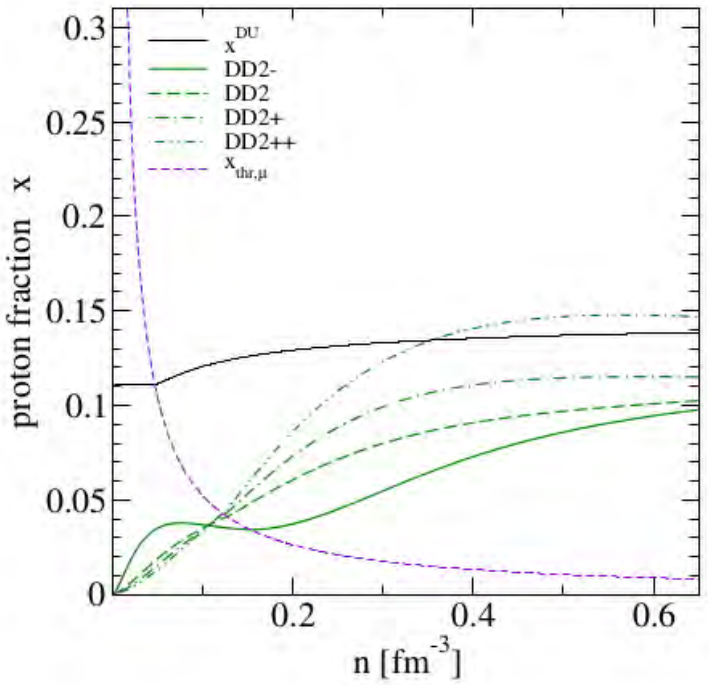
$$(1-x)^{1/3} < x^{1/3} + (x - x_\mu)^{1/3} \ .$$

Below the muon threshold $x^{\rm DU} = 1/9 = 11.1$.

D. E. Alvarez-Castillo, D. Blaschke and T. Klahn. (2016)
arXiv: 1604.08575

DUrca Process Constraint

E_s	$n_{\rm DU}[{\rm fm}^{-3}]$	$n_c[{\rm fm}^{-3}]$				
		1.25	1.40	1.60	1.80	2.00
DD2-	-	0.331	0.352	0.385	0.423	0.472
DD2	-	0.331	0.354	0.387	0.426	0.478
DD2+	-	0.325	0.349	0.384	0.425	0.479
DD2++	0.354	0.314	0.339	**0.375**	**0.416**	**0.469**



D. E. Alvarez-Castillo, D. Blaschke and T. Klahn. (2016)
arXiv: 1604.08575

Symmetry energy Conjecture

Klaehn et al. PhysRev C74 (2006)

PHYSICAL REVIEW C **74**, 035802 (2006)

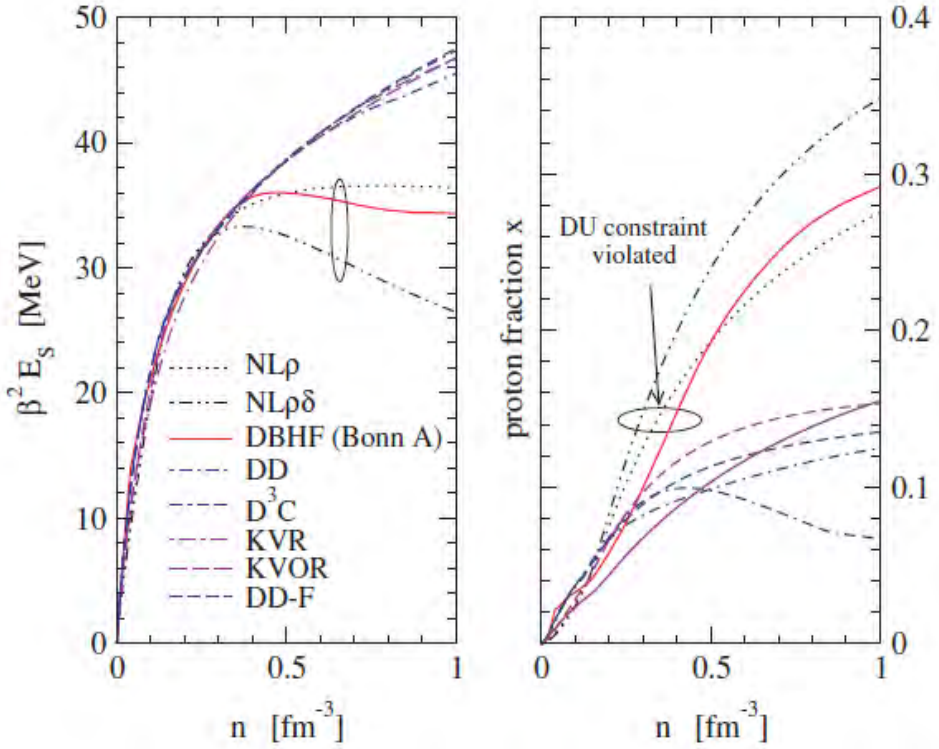
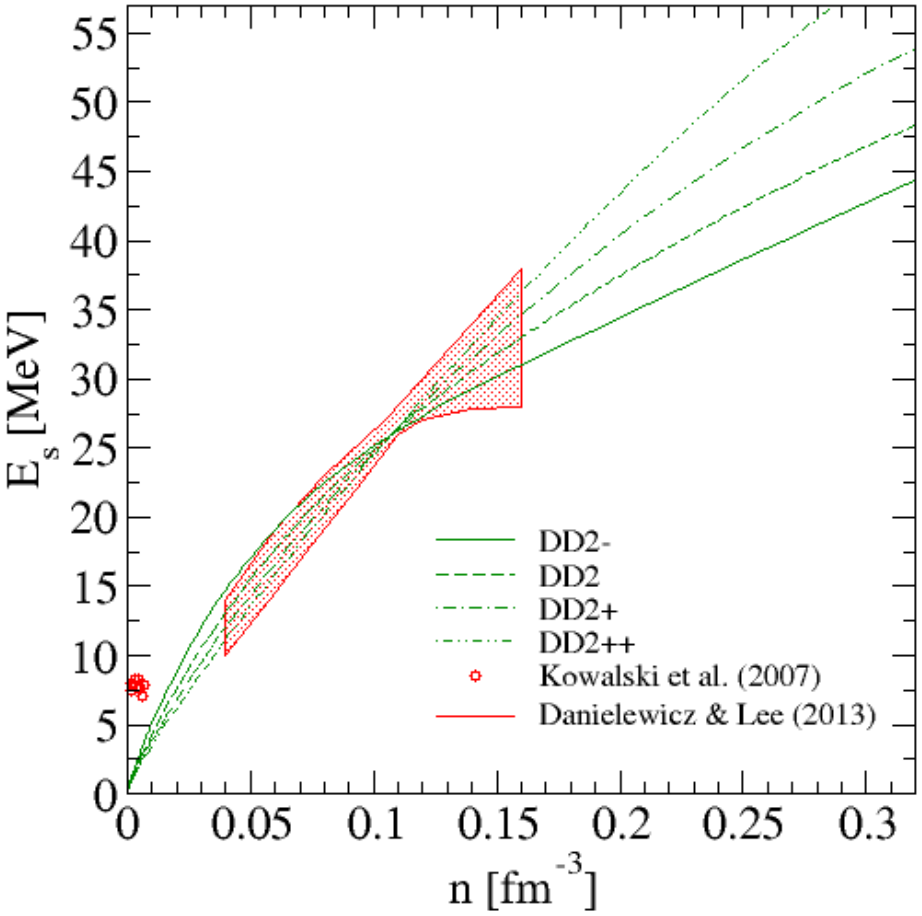


FIG. 7. (Color online) Density dependence of the asymmetry contribution to the energy per particle (left panel) and of the proton fraction (right panel) in NSM. Encircled curves correspond to EoSs that violate the DU-constraint.

Nuclear Symmetry Energy

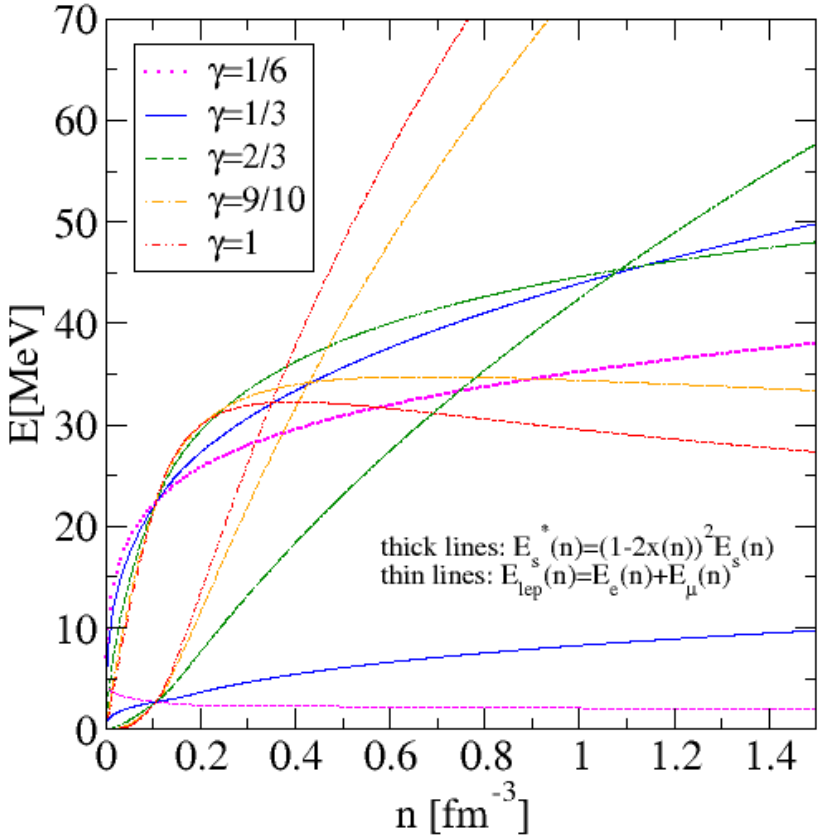


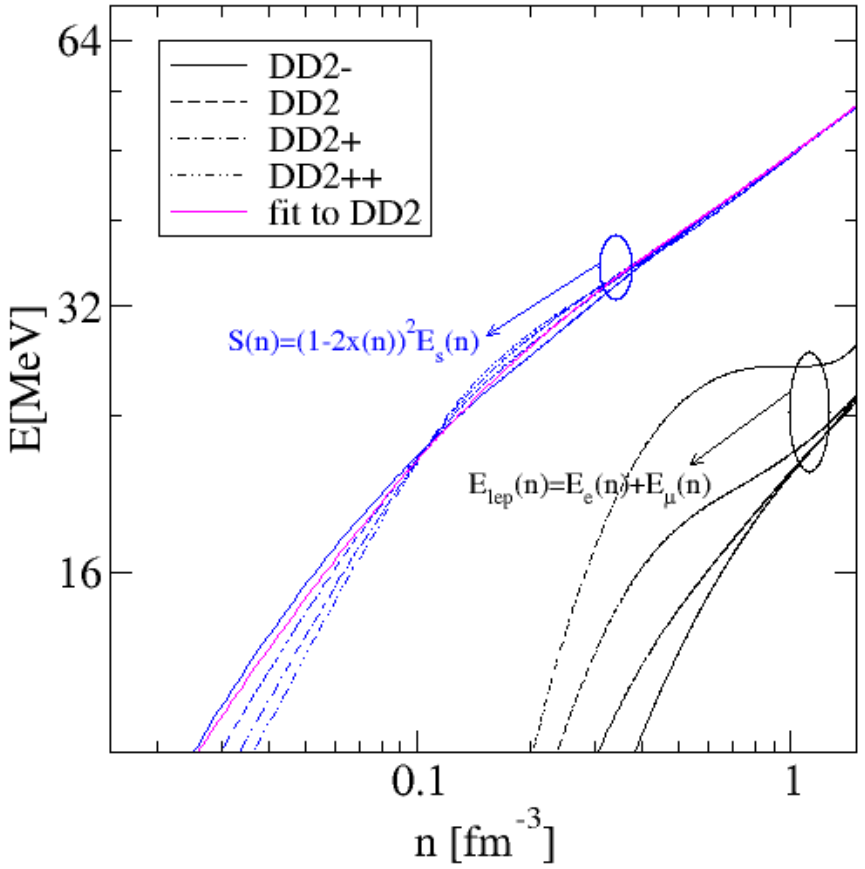
$E_s(n)$	Parametrization		$\Gamma_\rho(n_{\rm sat})$	a_ρ
Stiff	DD2+	DD2F+	3.806504	0.342181
Medium	DD2	DD2F	3.626940	0.518903
Soft	DD2−	DD2F−	3.398486	0.742082

S. Typel, Phys. Rev. C 89, 064321 (2014)

Universal symmetry energy contribution

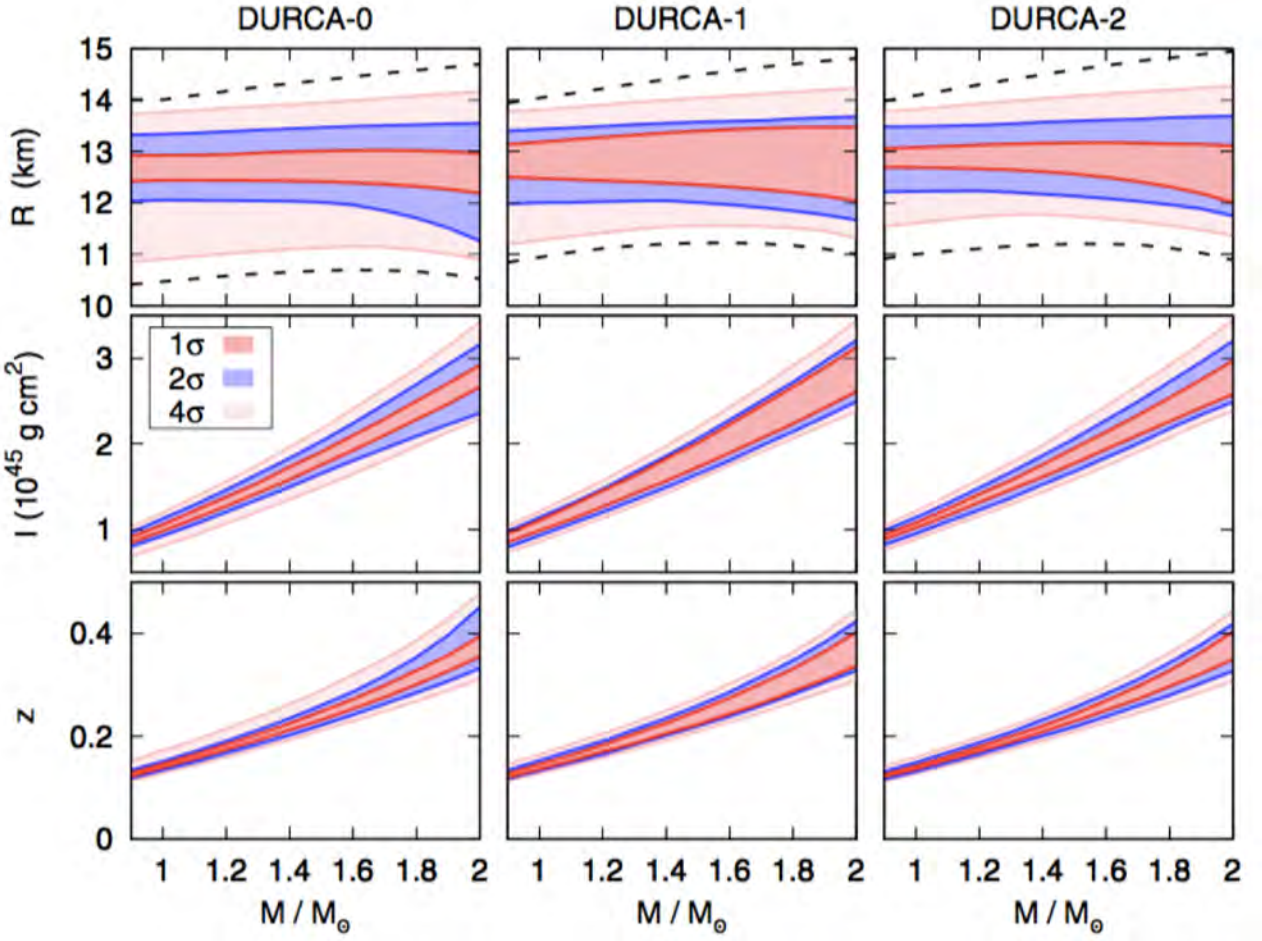
D. E. Alvarez-Castillo, D. Blaschke and T. Klahn. (2016)
arXiv: 1604.08575





The symmetry energy contribution to the neutron star EoS behaves universal only then E_{sym} and therefore the proton fraction x is bounded (right panel)!

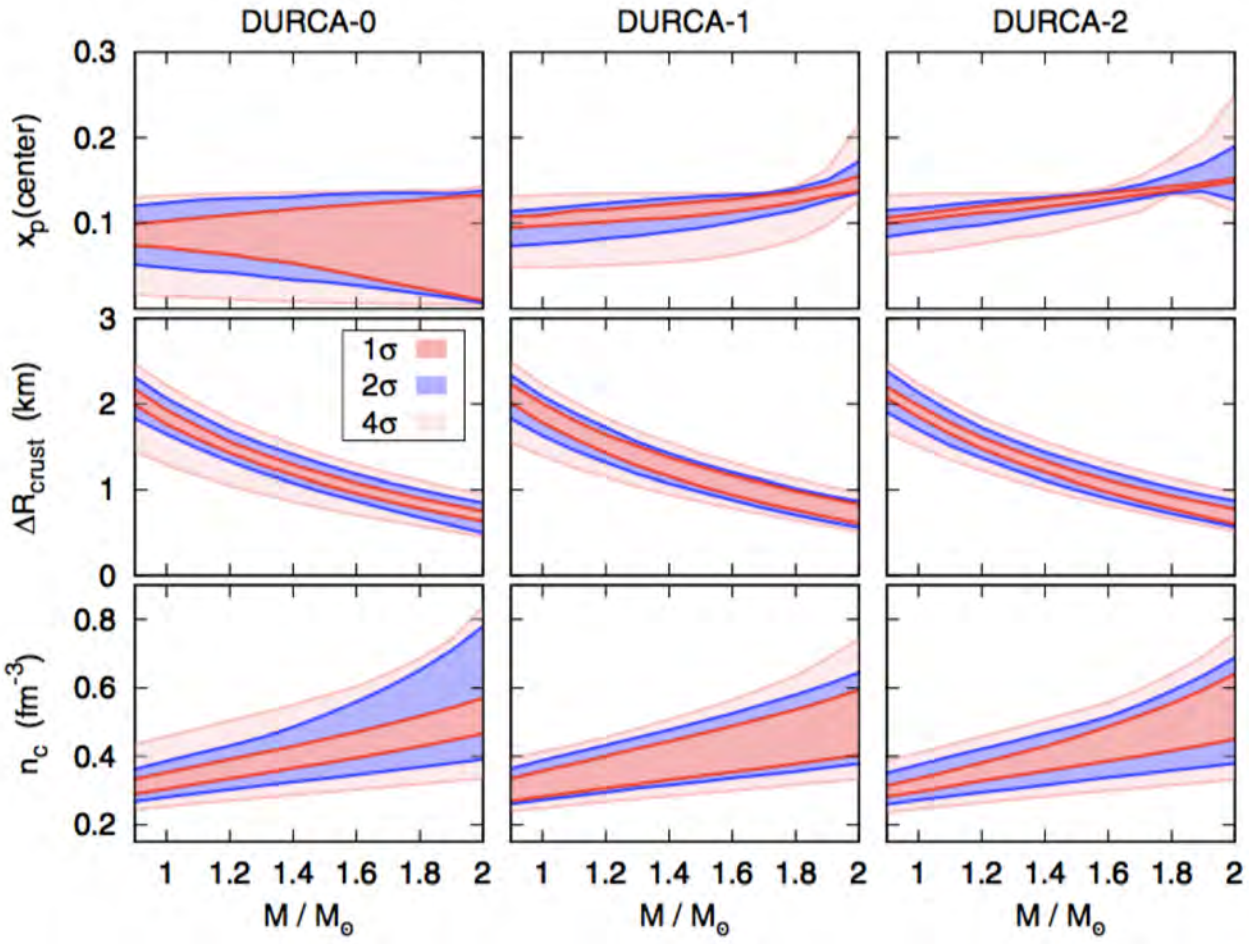
Predictions for neutron stars properties



If composed exclusively of nucleons and leptons, our prediction is that neutron stars have a radius of 12.7 ± 0.4 km for masses between 1 and 2$M_\odot$

J. Margueron, R. Hoffmann Casali, F. Gulminelli - Phys. Rev. C 97, 025806 (2018)

Predictions for neutron stars properties



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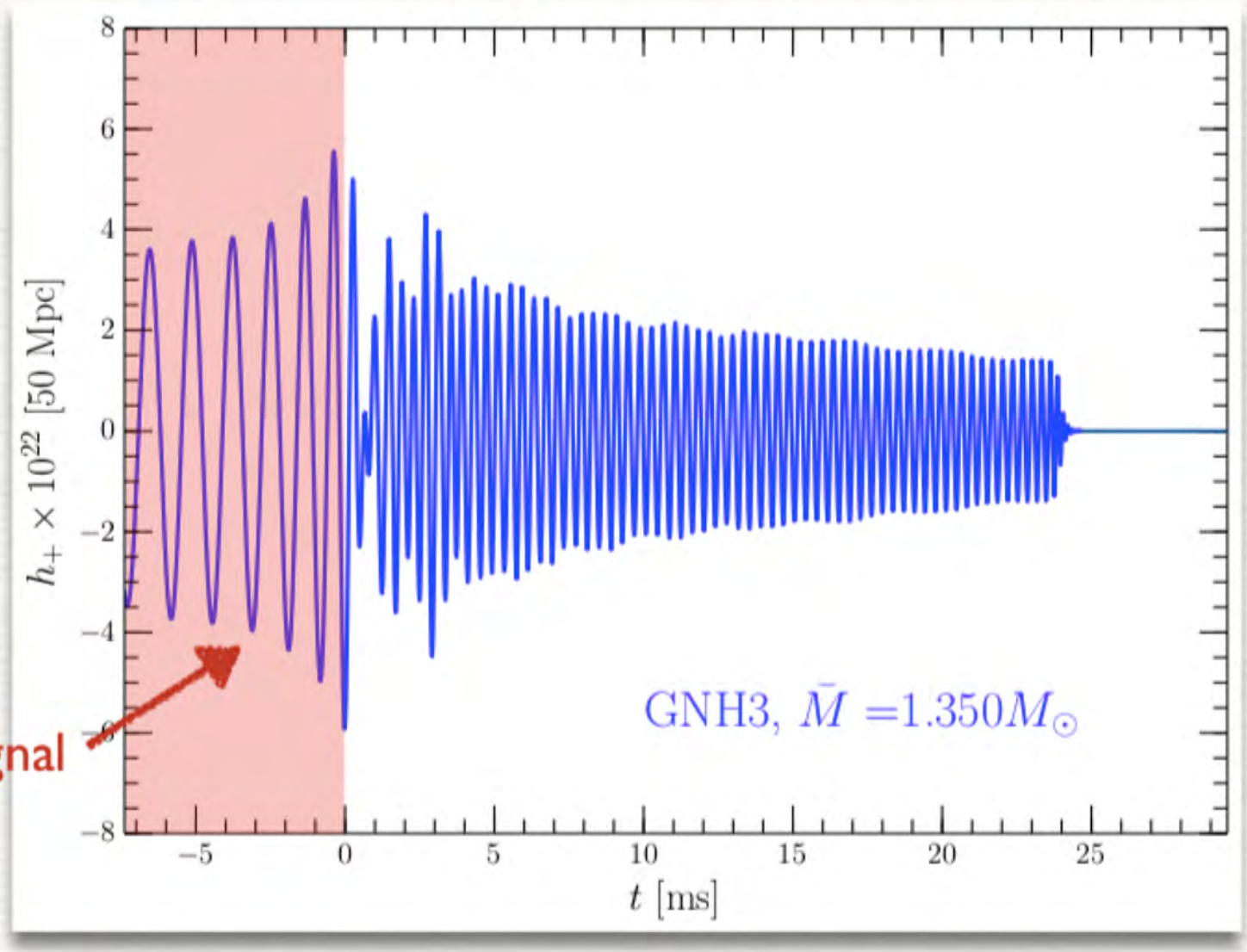
J. Margueron, R. Hoffmann Casali, F. Gulminelli - Phys. Rev. C 97, 025806 (2018)

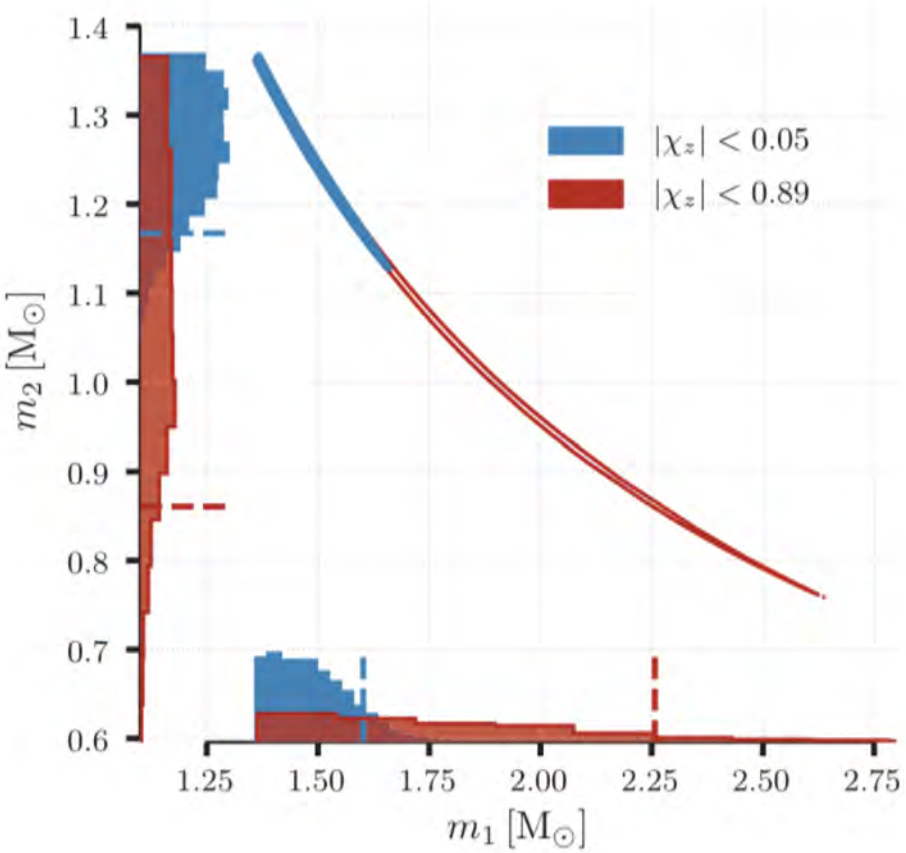
GW170817
and
Tidal deformability

Anatomy of the GW signal



Chirp signal

Implications from GW170817



GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral
B.⊠P. Abbott et al. arXiv:1712.00451

What can we learn from the inspiral II

- Waveforms incl. finite-size effects are described by tidal deformability (how a star reacts on an external tidal field)
- Offer possibility to constrain EoS because tidal deformability depends on EoS

$$\Lambda \equiv \frac{2}{3} k_2 \left(\frac{R}{M} \right)^5$$

- Corresponding to ~10 % error in radius R for nearby events (<100Mpc) (e.g. Read et al. 2013)
- Note: faithful templates to be constructed

R/M compactness (EoS dependent)

k_2 tidal love number (EoS dependent)

Computing the love number/tidal deformability

Extension of a standard TOV solver (i.e. numerically an integration of coupled ODEs):

Ansatz for the metric including a l=2 perturbation

$$\begin{aligned} ds^2 &= -e^{2\Phi(r)}\left[1+H(r)Y_{20}(\theta,\varphi)\right]dt^2 \\ &\quad +e^{2\Lambda(r)}\left[1-H(r)Y_{20}(\theta,\varphi)\right]dr^2 \\ &\quad +r^2\left[1-K(r)Y_{20}(\theta,\varphi)\right]\left(d\theta^2+\sin^2\theta d\varphi^2\right) \end{aligned}$$

Following Hinderer et al. 2010

Integrate standard TOV system:

$$\begin{aligned} e^{2\Lambda} &= \left(1-\frac{2m_r}{r}\right)^{-1}, \\ \frac{d\Phi}{dr} &= -\frac{1}{\epsilon+p}\frac{dp}{dr}, \\ \frac{dp}{dr} &= -(\epsilon+p)\frac{m_r+4\pi r^3 p}{r(r-2m_r)}, \\ \frac{dm_r}{dr} &= 4\pi r^2\epsilon. \end{aligned}$$

And additional eqs. for perturbations:

$$\begin{aligned} \frac{dH}{dr} &= \beta \qquad (11) \\ \frac{d\beta}{dr} &= 2\left(1-2\frac{m_r}{r}\right)^{-1} H\left\{-2\pi\left[5\epsilon+9p+f(\epsilon+p)\right]\right. \\ &\quad \left.+\frac{3}{r^2}+2\left(1-2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2}+4\pi r p\right)^2\right\} \\ &\quad +\frac{2\beta}{r}\left(1-2\frac{m_r}{r}\right)^{-1}\left\{-1+\frac{m_r}{r}+2\pi r^2(\epsilon-p)\right\}. \end{aligned}$$

EoS to be provided ε(p)

(K(r) given by H(r))

Note: Although multidimensional problem – computation in 1D since absorbed in Y20

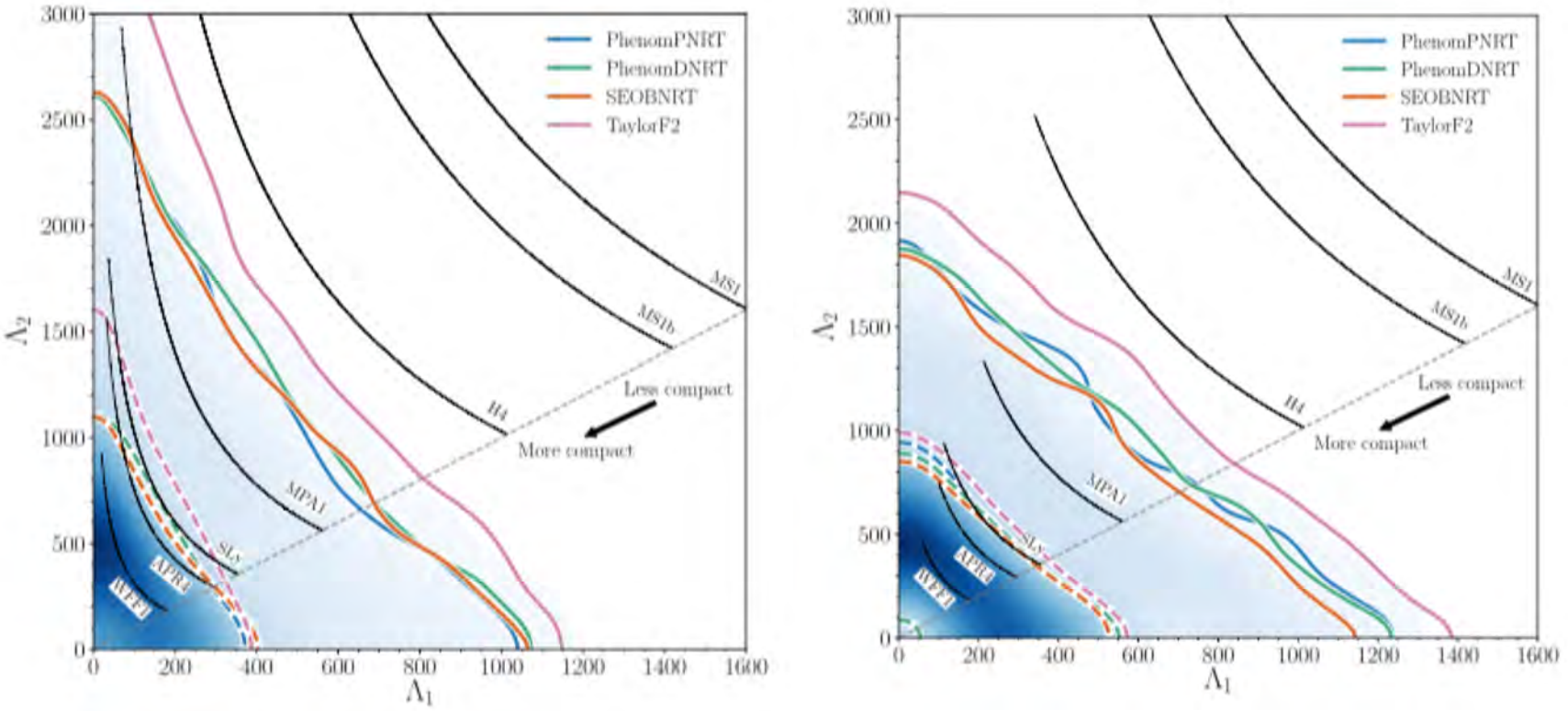
Love number

$$y = \frac{R\beta(R)}{H(R)}$$

$$\begin{aligned} k_2 = {} & \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \\ & \times\Big\{2C[6-3y+3C(5y-8)] \\ & +4C^3[13-11y+C(3y-2)+2C^2(1+y)] \\ & +3(1-2C)^2[2-y+2C(y-1)]\ln(1-2C)\Big\}^{-1} \end{aligned}$$

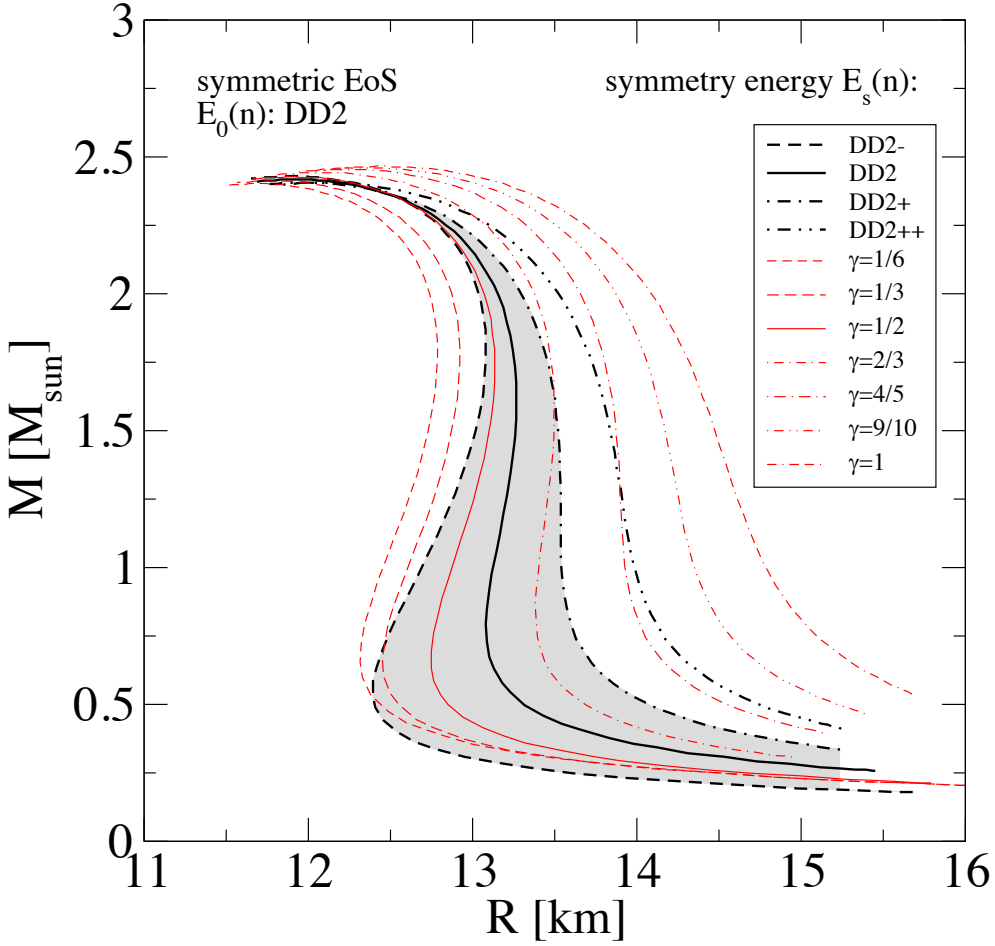
where $C = M/R$ is the compactness of the star.

Implications from GW170817

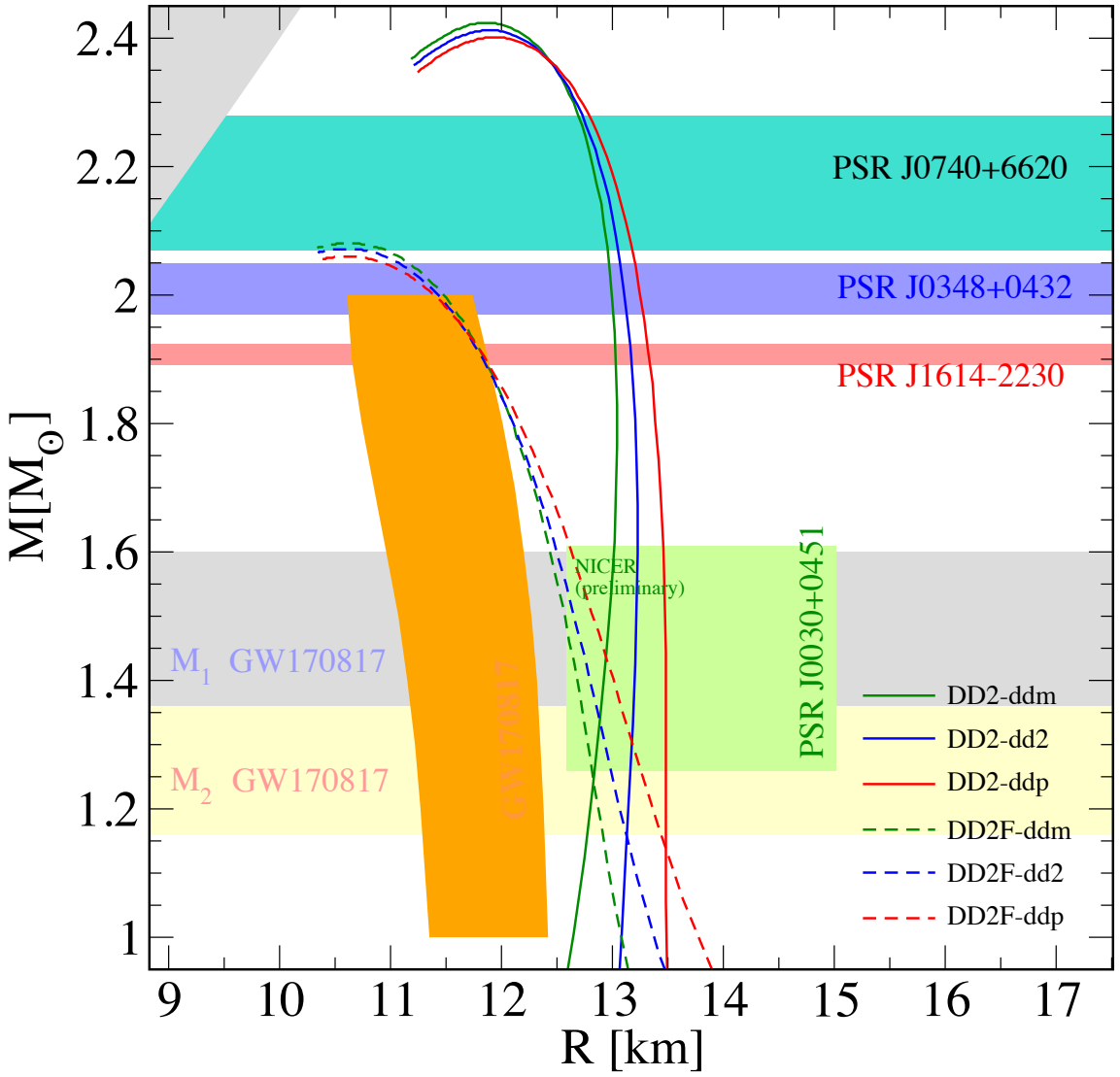


Properties of the Binary Star Merger GW170817
B. P. Abbott et al., Phys. Rev. X 9, 011001 (2019)

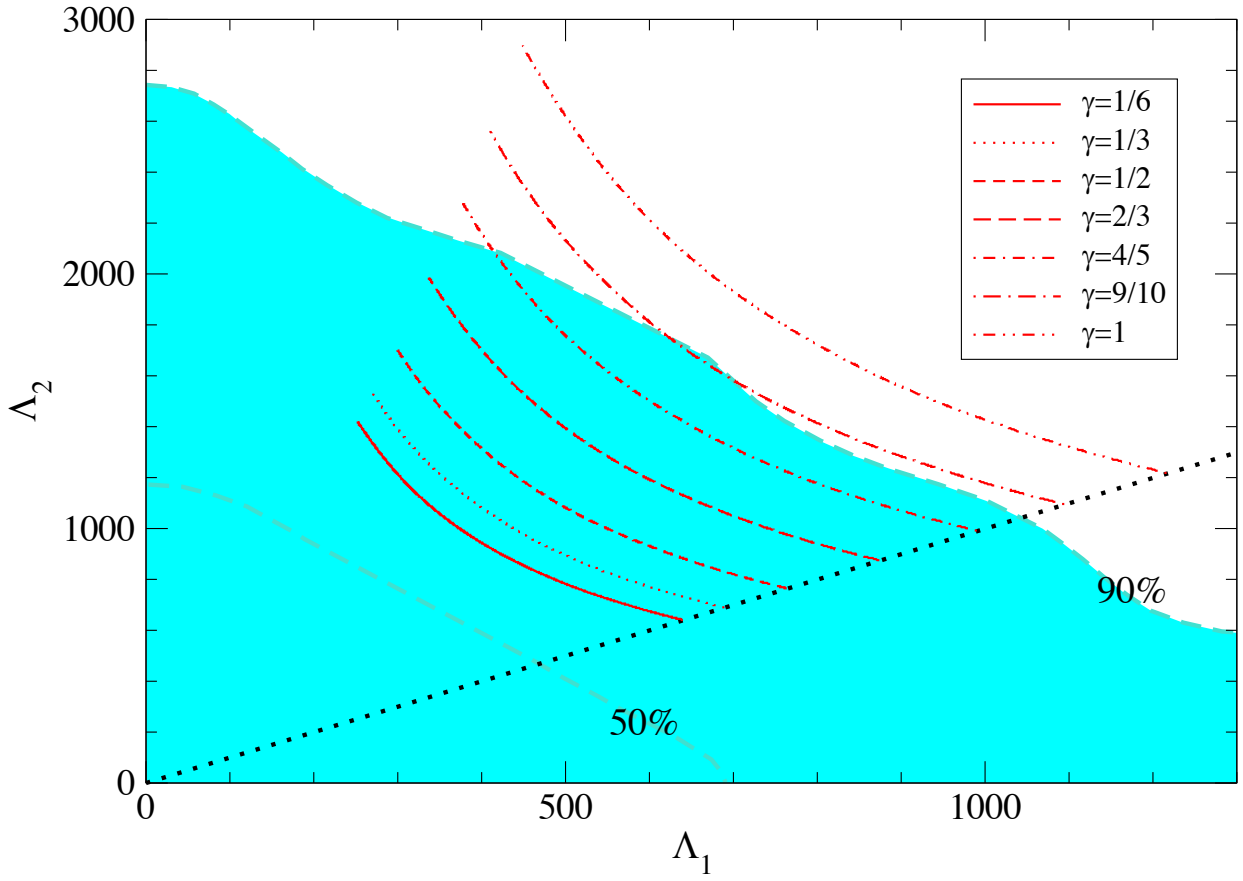
Symmetry energy effects



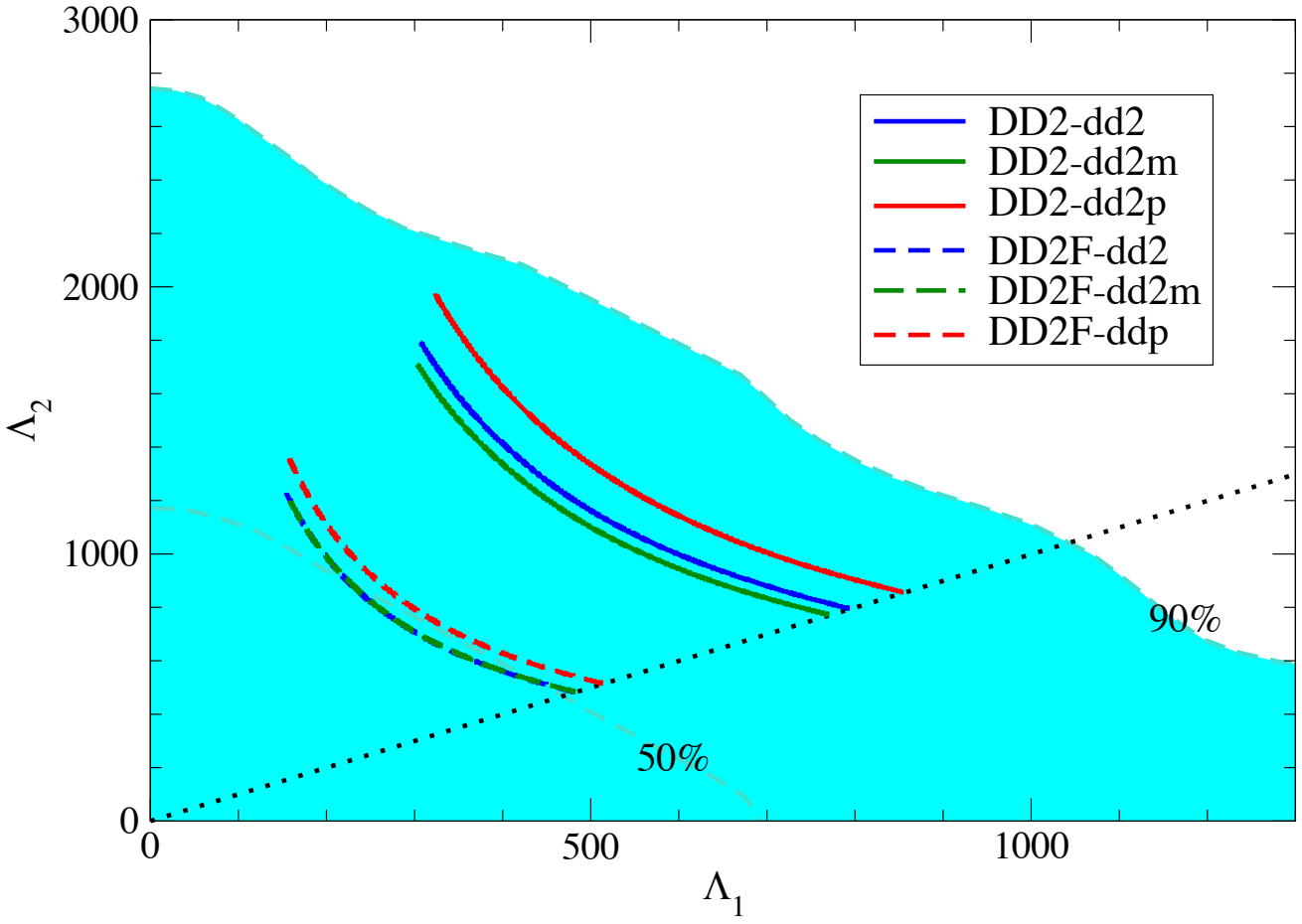
Implications from GW170817



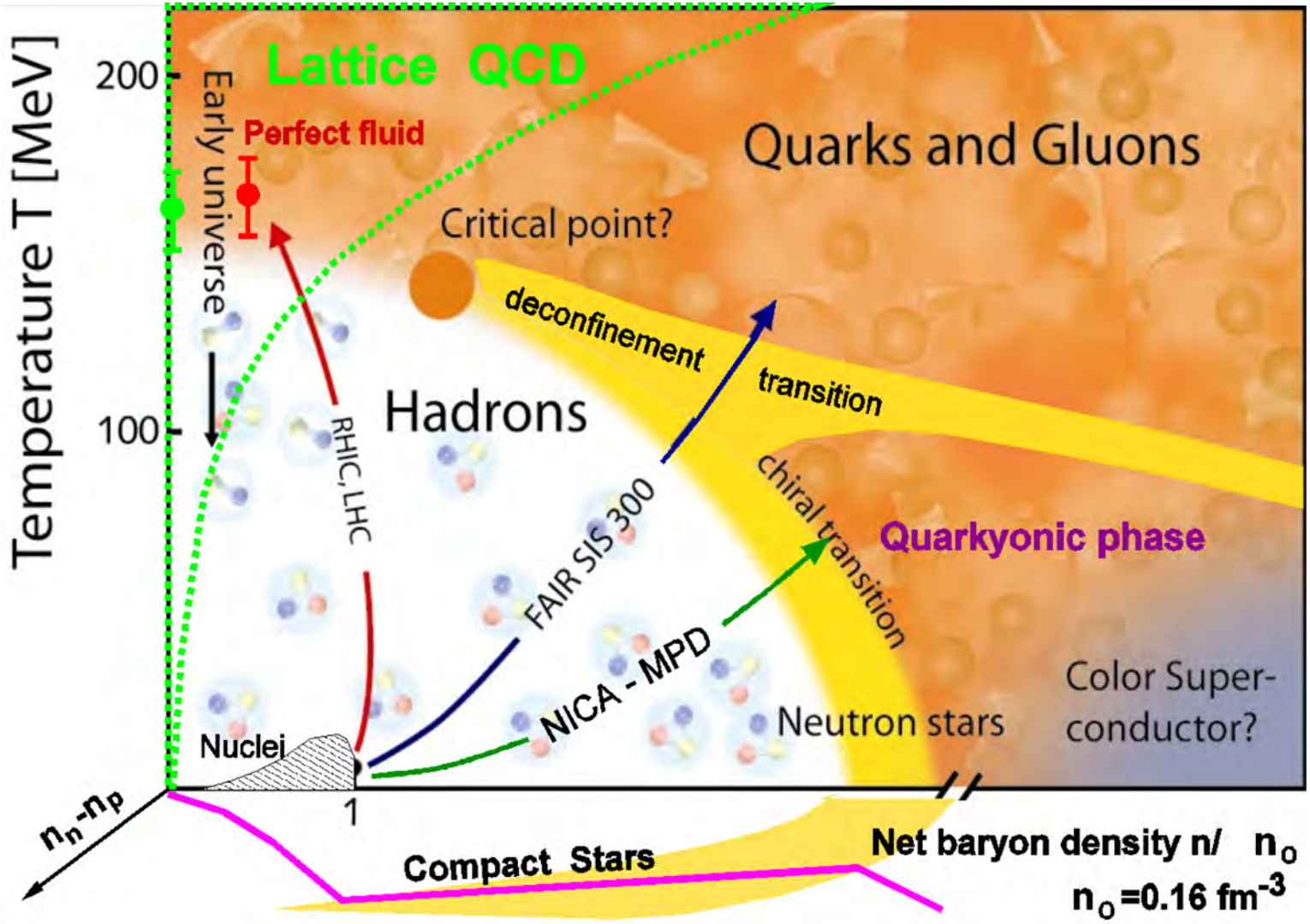
Implications from GW170817



Implications from GW170817



Critical Endpoint in QCD

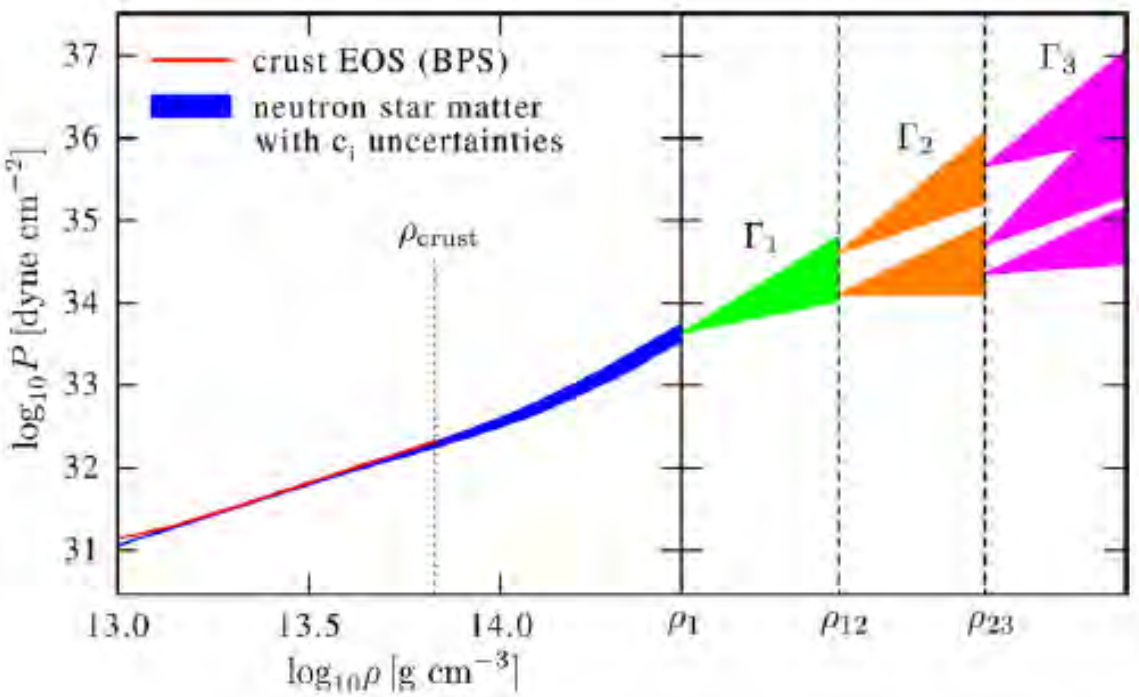


Piecewise polytrope EoS

Hebeler et al., ApJ 773, 11 (2013)

$$P_i(n) = \kappa_i n^{\Gamma_i}$$

$$i = 1 \ : \ n_1 \le n \le n_{12}$$
$$i = 2 \ : \ n_{12} \le n \le n_{23}$$
$$i = 3 \ : \ n \ge n_{23} \ ,$$



Here, 1st order PT in region 2:

$$\Gamma_2 = 0 \text{ and } P_2 = \kappa_2 = P_{\rm crit}$$

$$P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn},$$

$$\varepsilon(n)/n = \int dn \, \frac{P(n)}{n^2} = \int dn \, \kappa n^{\Gamma-2} = \frac{\kappa \, n^{\Gamma-1}}{\Gamma - 1} + C,$$

$$\mu(n) = \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa \, \Gamma}{\Gamma - 1} n^{\Gamma-1} + m_0,$$

Seidov criterion for instability: $\dfrac{\Delta\varepsilon}{\varepsilon_{\rm crit}} \ge \dfrac{1}{2} + \dfrac{3}{3}\dfrac{P_{\rm crit}}{\varepsilon_{\rm crit}}$

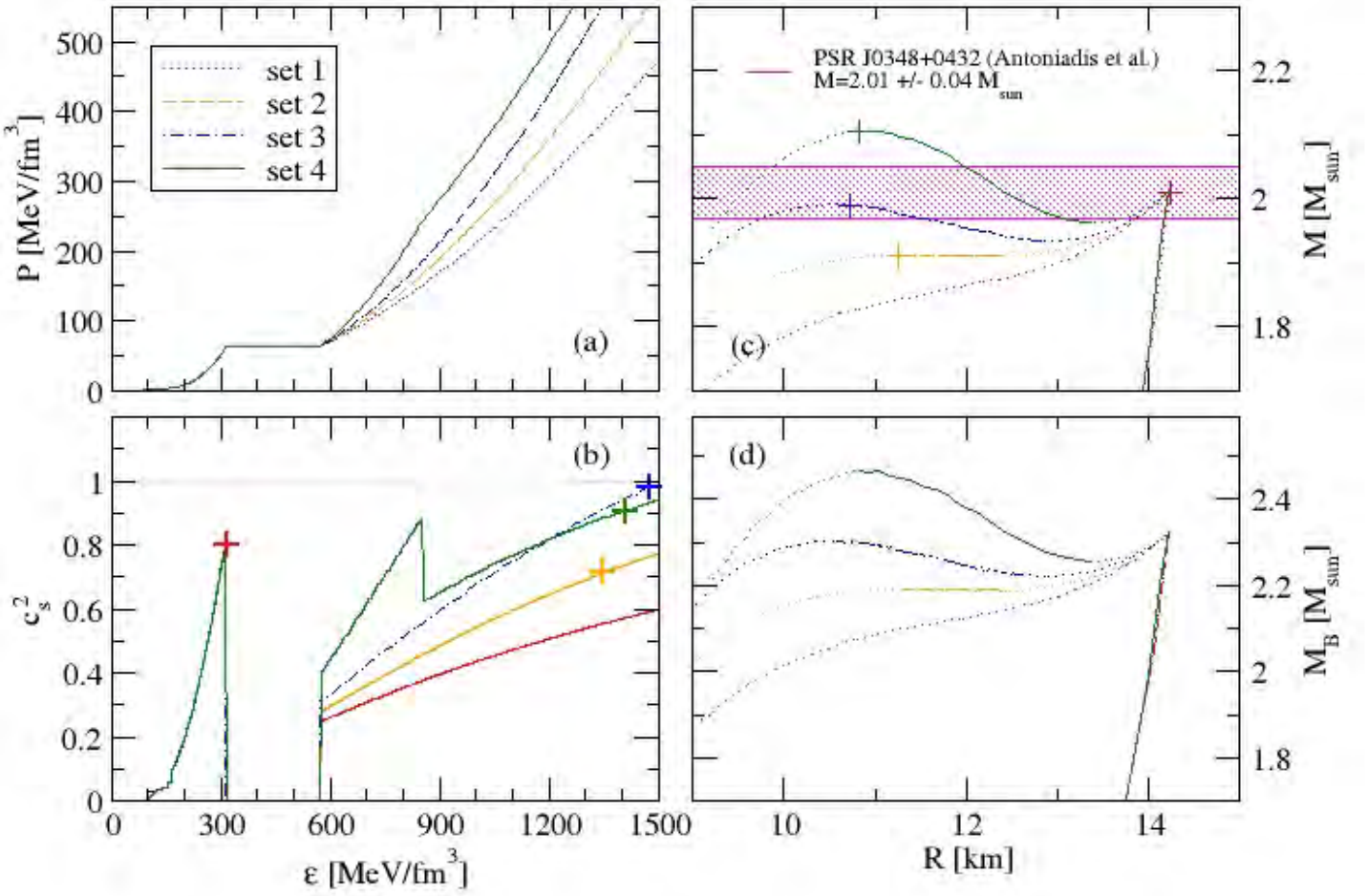
$$n(\mu) = \left[(\mu - m_0)\frac{\Gamma - 1}{\kappa\Gamma}\right]^{1/(\Gamma-1)}$$

$$P(\mu) = \kappa \left[(\mu - m_0)\frac{\Gamma - 1}{\kappa\Gamma}\right]^{\Gamma/(\Gamma-1)}$$

Maxwell construction:

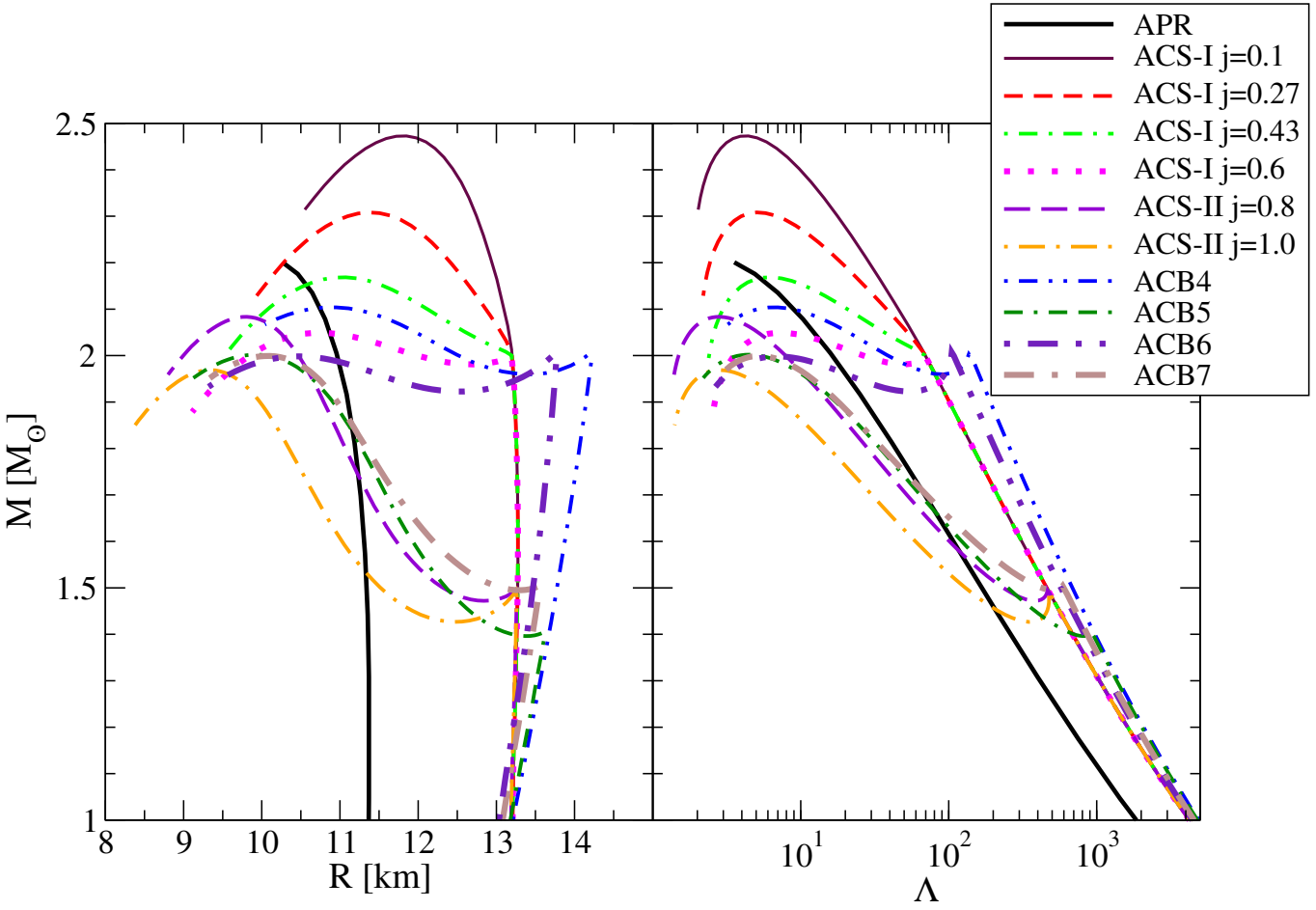
$$P_1(\mu_{\rm crit}) = P_3(\mu_{\rm crit}) = P_{\rm crit}$$
$$\mu_{\rm crit} = \mu_1(n_{12}) = \mu_3(n_{23})$$

Compact Star Twins



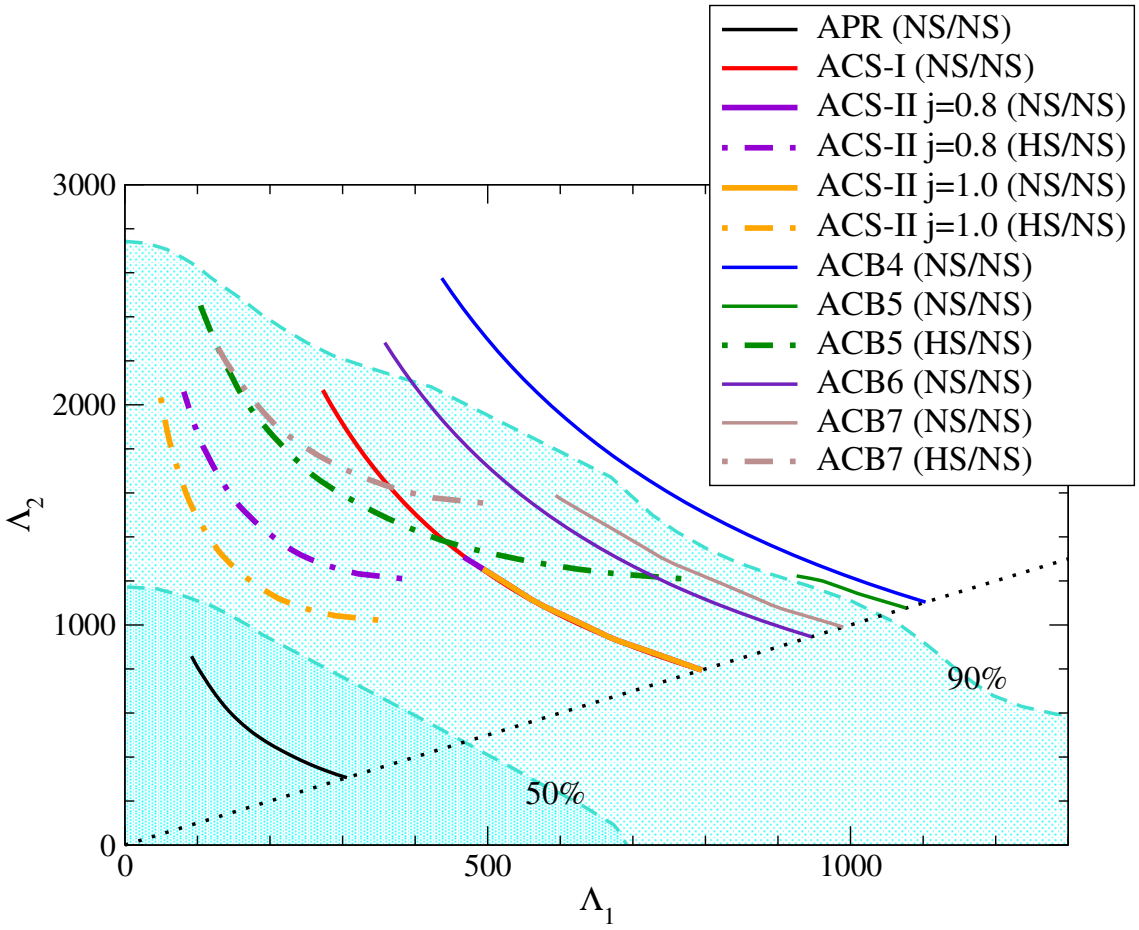
Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
arXiv: 1703.02681v2, Phys. Rev. C 96, 045809 (2017)

Implications from GW170817



Vasileios Paschalidis, Kent Yagi, David Alvarez-Castillo,
David B. Blaschke, Armen Sedrakian
Phys. Rev. D 97, 084038 (2018), arXiv:1712.00451

Implications from GW170817



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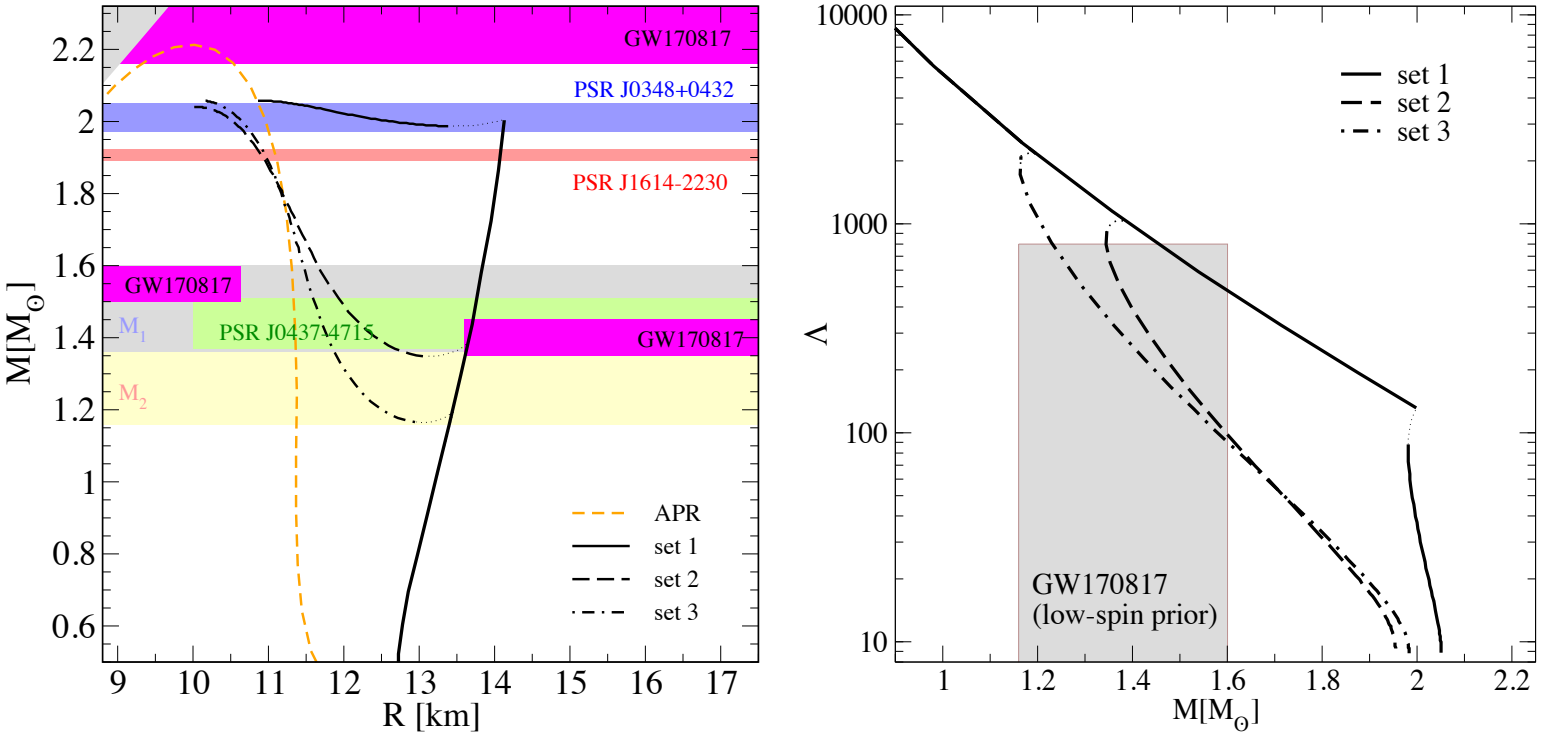
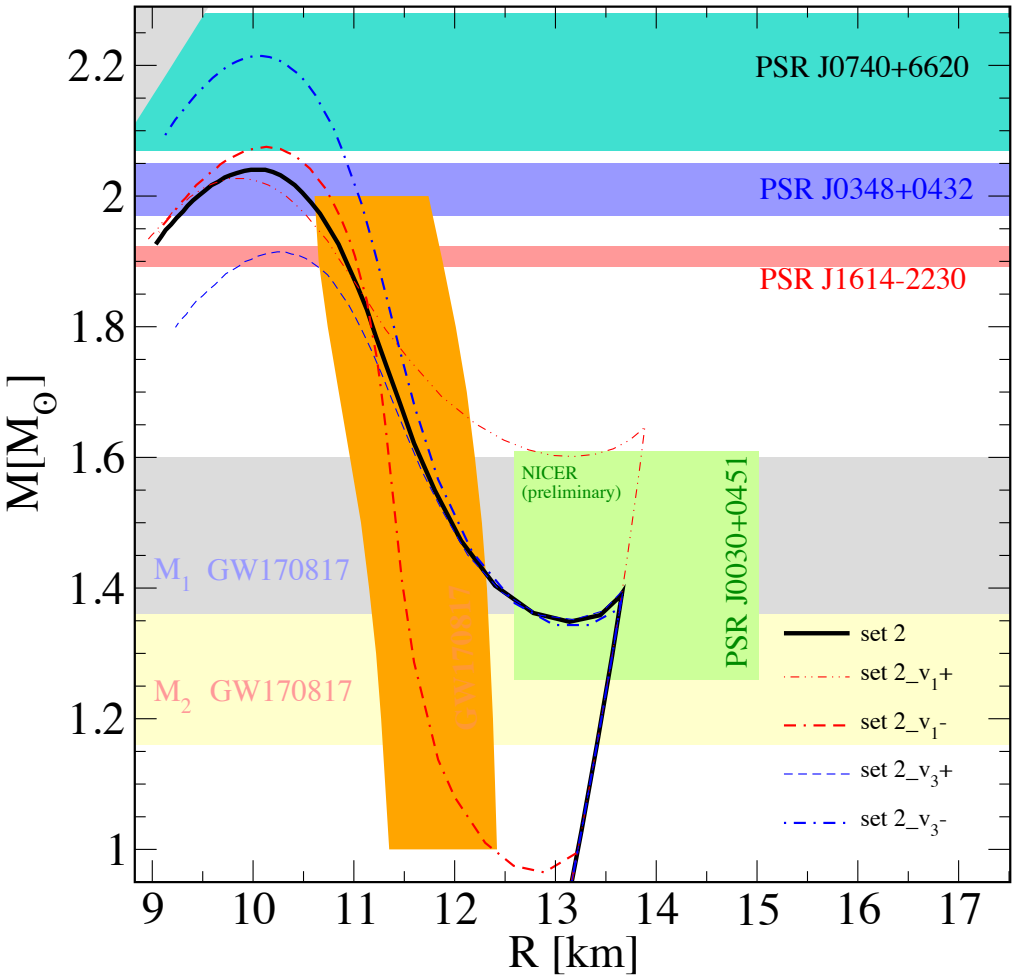
Implications from GW170817 Nonlocal NJL



D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

Implications from GW170817 Nonlocal NJL



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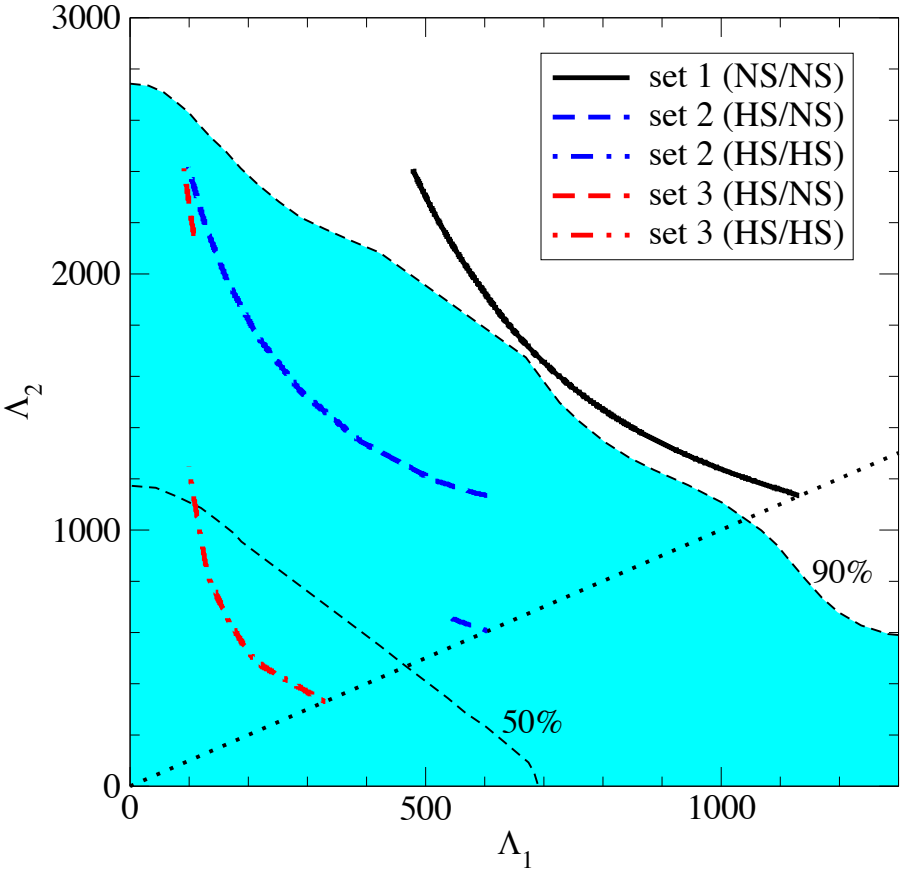
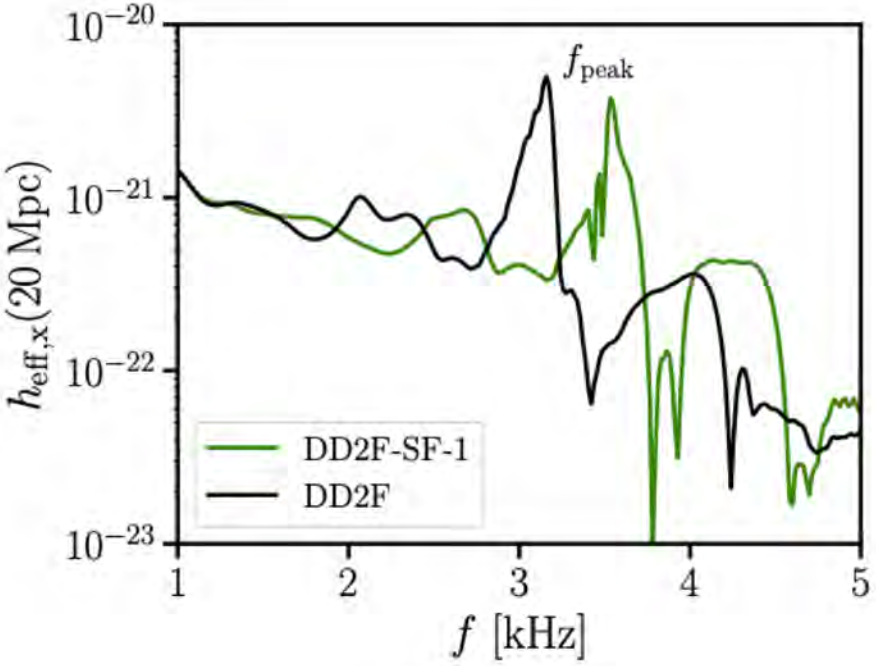
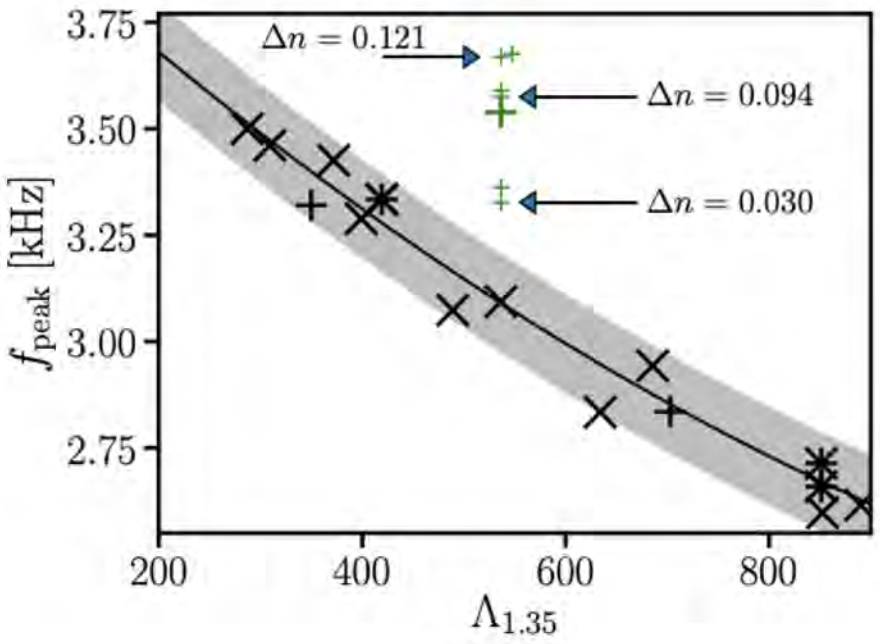
Nonlocal NJL



D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

Gravitational Wave Signals
First Order Phase Transitions





A. Bauswein et al. - arXiv: 1904.01306, PRL 122 (2019) 061102

Conclusions

- USEC conjecture has been corroborated and E_s related quantities found to be correlated with the NS radius.
- GW170817 favours softer EoS and together with the Durca constraint DD2F-like EoS are favoured. Hybrid stars with strong first order transition are also favoured.
- Future GW observations, NICER and SKA will soon result into stronger NS EoS constraints.
- Many possible astrophysical scenarios for mass twins could be confirmed implying a CEP in QCD.

Gracias