Modeling anisotropic magnetized compact stars with γ-metric: The white dwarfs picture

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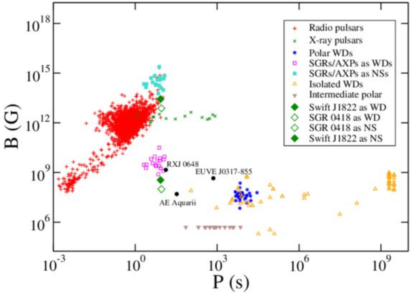
Outline

Motivation

- Presence of magnetic fields in compact stars and its effects in the EoS and the structure.



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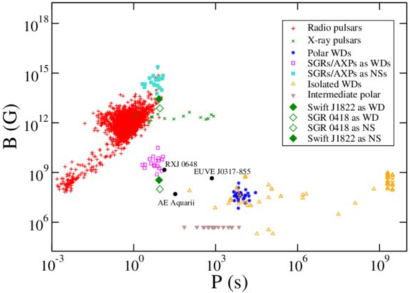
- Presence of magnetic fields in compact stars and its effects in the EoS and the structure.
- A fermion system under the action of an external magnetic field $\mathbf{B} = B\hat{z}$ features anisotropic EoS.



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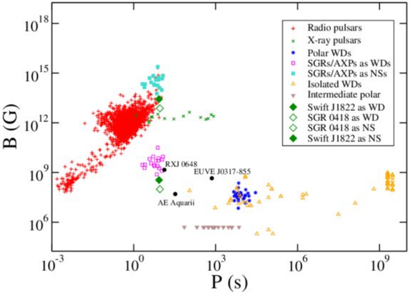

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- Modeling in the literature:
  - Structure with TOV neglecting the anisotropy.
  - Numerical relativity schemes: magnetic fields added through Maxwell equation (not included in the EoS).

γ metric (or Zipoy-Voorhees)

- Static, axisymmetric and asymptotically flat family of solutions to the Einstein equations in spherical coordinates,

$$ds^2 = -\Delta^{\gamma} dt^2 + \Delta^{\gamma^2-\gamma-1}\Sigma^{1-\gamma^2} dr^2 + r^2\Delta^{1-\gamma}\Sigma^{1-\gamma^2} d\theta^2 + r^2 \sin^2\theta \Delta^{\gamma^2-\gamma} d\phi^2,$$

$$\Delta = \left(1 - \frac{2m}{r}\right),$$

$$\Sigma = \left(1 - \frac{2m}{r} + \frac{m^2}{r^2}\sin^2\theta\right)$$

- Parameters:
  - m: related to the gravitational mass $M = \gamma m$.
  - γ: related to the shape of the object, with the quadrupolar moment $Q = m^3\gamma(1-\gamma^2)/3$

$\gamma \to 0$: Minkowski ($M = Q = 0$)

$\gamma \to 1$: Schwarszchild ($Q = 0$)

Modeling magnetized compact stars with γ-metric

- For small deformations ($\gamma \simeq 1$)

$$ds^2 = -\left[1 - 2m(r)/r\right]^{\gamma} dt^2 + \left[1 - 2m(r)/r\right]^{-\gamma} dr^2 + r^2 \sin\theta d\phi^2 + r^2 d\theta^2$$

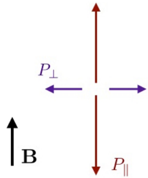
  - Isotropic energy momentum-tensor:

$$\frac{dP}{dr} = -\frac{(E+P)\left[\frac{r}{2} + 4\pi r^3 P - \frac{r}{2}\left(1 - \frac{2M}{r}\right)^{\gamma}\right]}{r^2\left(1 - \frac{2M}{r}\right)^{\gamma}}$$

radius R defined at $P(R) = 0$

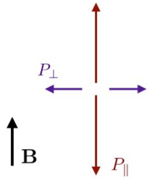
Modeling magnetized compact stars with γ-metric

- Small deformations ($\gamma \simeq 1$)
- Given the pressure anisotropy in the magnetized EoS:



Modeling magnetized compact stars with γ-metric

- Small deformations ($\gamma \simeq 1$)
- Given the pressure anisotropy in the magnetized EoS:
  - Spheroidal objects, parametrization $z = \gamma r$
  - $P_\perp(r)$ and $P_\parallel(z(r))$



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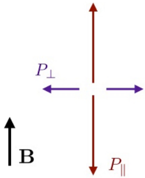
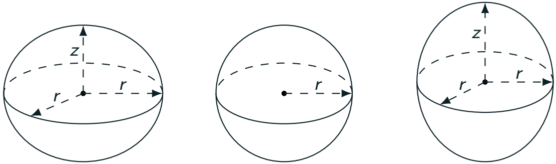
  - $\gamma = \frac{P_\parallel(r)}{P_\perp(r)} \approx \frac{P_{\parallel 0}}{P_{\perp 0}}$



$\gamma < 1,\ P_\parallel < P_\perp$	$\gamma = 1,\ P_\parallel = P_\perp$	$\gamma > 1,\ P_\parallel > P_\perp$
oblate spheroid	sphere	prolate spheroid



Modeling magnetized compact stars with γ-metric: structure equations

$$\frac{dM}{dr} = 4\pi r^2 \frac{(E_\parallel + E_\perp)}{2}\gamma,$$

$$\frac{dP_\perp}{dr} = -\frac{(E_\perp + P_\perp)[\frac{r}{2} + 4\pi r^3 P_\perp - \frac{r}{2}(1-\frac{2M}{r})^\gamma]}{r^2(1-\frac{2M}{r})^\gamma},$$

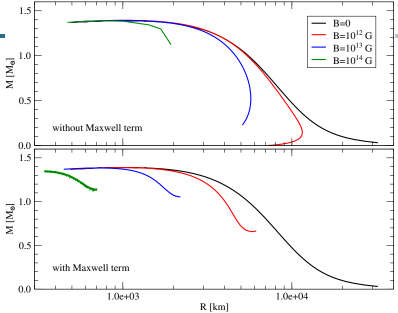
$$\frac{dP_\parallel}{dz} = \frac{1}{\gamma}\frac{dP_\parallel}{dr}$$

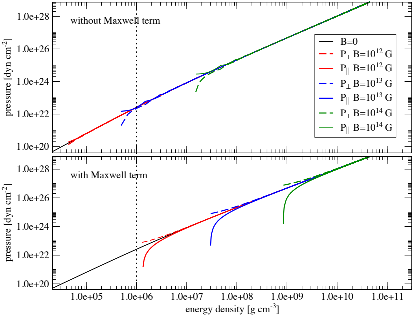
$$= -\frac{(E_\parallel + P_\parallel)[\frac{r}{2} + 4\pi r^3 P_\parallel - \frac{r}{2}(1-\frac{2M}{r})^\gamma]}{\gamma r^2(1-\frac{2M}{r})^\gamma},$$

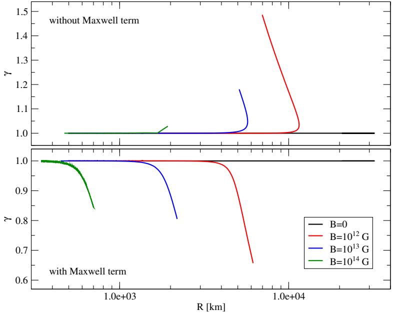
$$E_\parallel = E(P_\parallel), E_\perp = E(P_\perp)$$

Reduces to TOV equations at $B = 0$: $\gamma = 1$, $P_\parallel = P_\perp$

Results: White dwarfs (WDs)







Results: WDs. Stability and super-Chandrasekhar masses

- Small deformations:
  $|E_\perp(r) - E_\parallel(r)|/E_0 \lesssim 10^{-3} \Rightarrow P_\parallel(r)/P_\perp(r) \approx P_{\parallel 0}/P_{\perp 0}$.

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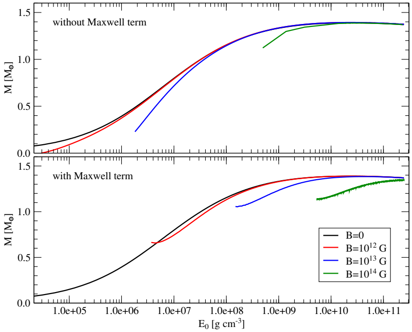
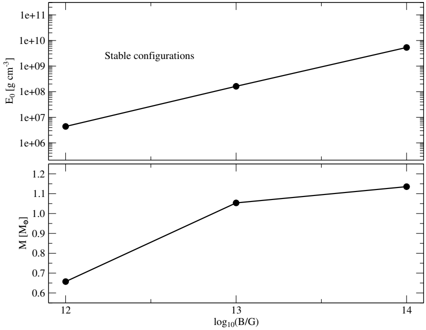
- With Maxwell term: M vs E_0 minimum defines an instability onset.





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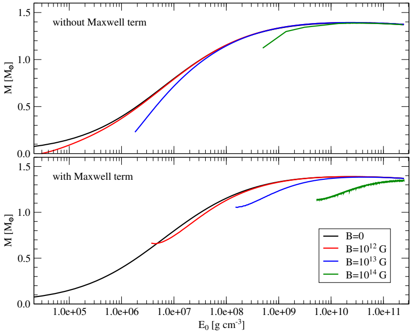

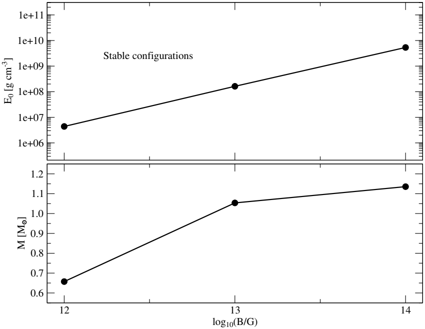

- $B > 10^{14}$ G: densities that yield stable objects are beyond WDs density range. *No super-Chandrasekhar masses.*

Summary

- We propose a model to describe the structure of an axially symmetric deformed compact star, provided it is spheroidal.
- The ansatz for the γ parameter allows to relate the magnetic anisotropy on the EoS with the geometric deformation. Reasonable results for small deformations and TOV solutions when $\gamma = 1$.
- An improvement of the ansatz is necessary if extending to highly deformed objects.
- WDs:
  - Magnetic field effects are relevant at *low and intermediate* density regime with respect to B.
  - Prolate (oblate) deformation without (with) Maxwell term for stable compact stars with respect to the corresponding central densities solutions at $B = 0$. Maximum masses not affected.

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Thank you for your attention.

Table I. Values of γ, central energy density, equatorial and polar radii and mass for the most deformed configurations, which determine the onset of instability at 10^{12} G, 10^{13} G and 10^{14} G, in all cases ignoring and considering Maxwell term.

B [G]	Maxwell term	γ	E_0 [g cm^{-3}]	R [km]	Z [km]	M [$M_\odot$]
10^{12}	without	1.0033	1.03519×10^6	9988.1	10021.0	0.379
		1.4864	2.79605×10^4	6973.6	10365.3	0.017
	with	0.7267	4.40397×10^6	5778.9	4199.4	0.657
10^{13}	without	1.1802	1.80308×10^6	5096.4	6014.7	0.248
	with	0.8259	1.62578×10^8	2141.1	1768.3	1.054
10^{14}	without	1.0289	4.97109×10^8	1928.5	1984.2	1.122
	with	0.8458	5.34925×10^9	699.8	591.9	1.135