On the phase diagram of the Nambu Jona-Lasinio Lagrangian

in collaboration with
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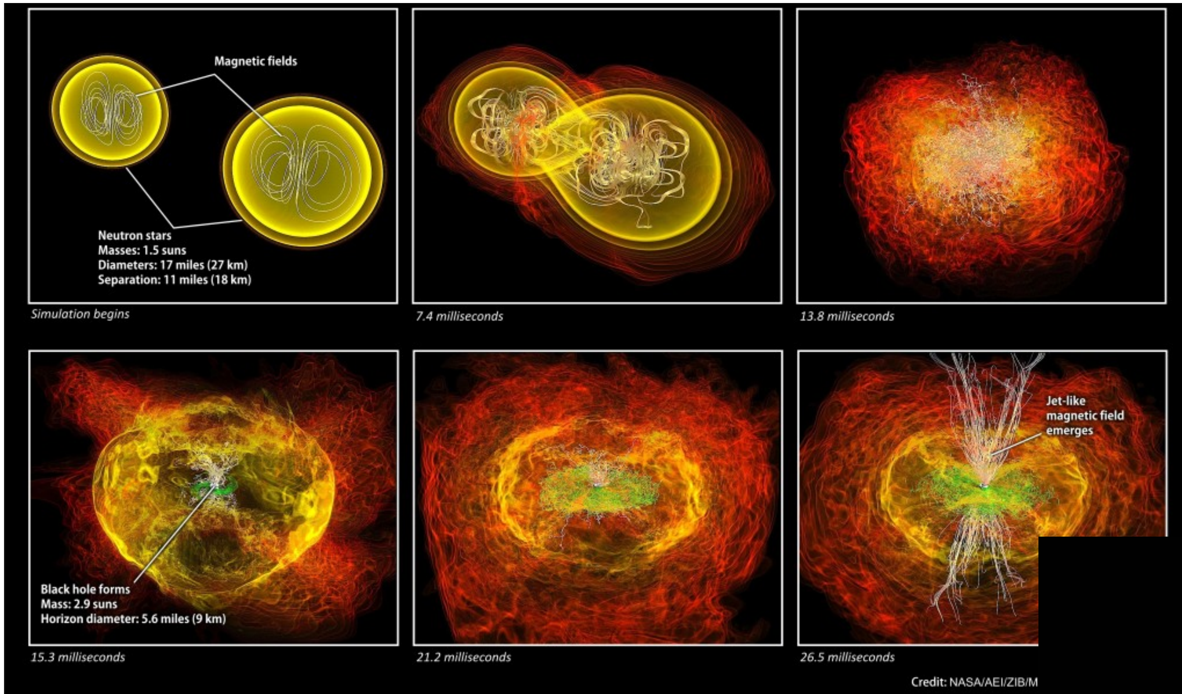
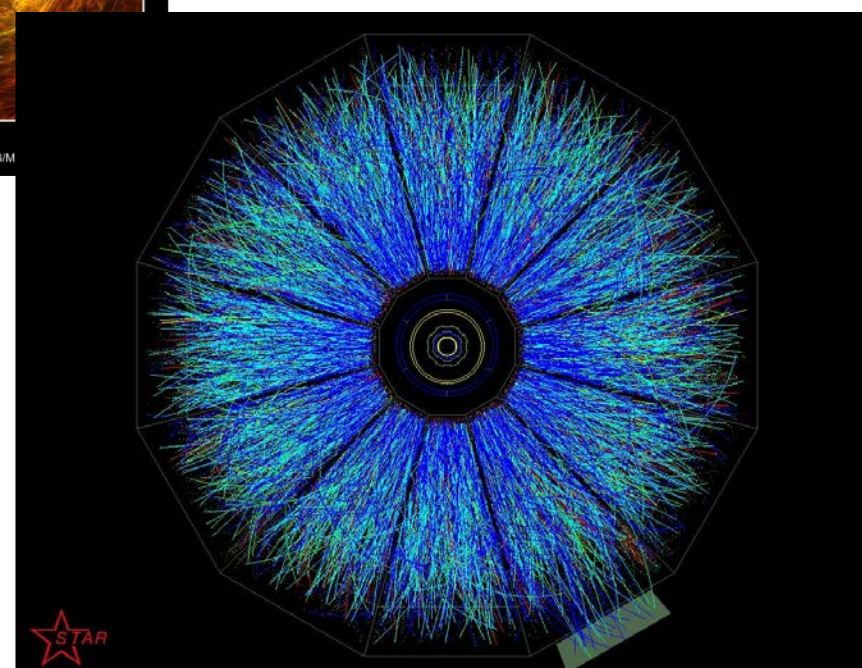
Simulations of Neutron Stars, Neutron Star Collisions and Heavy Ion Collisions

need the same input

PHASE DIAGRAMM OF STRONGLY INTERACING MATTER s(T,μ), ε(T,μ)

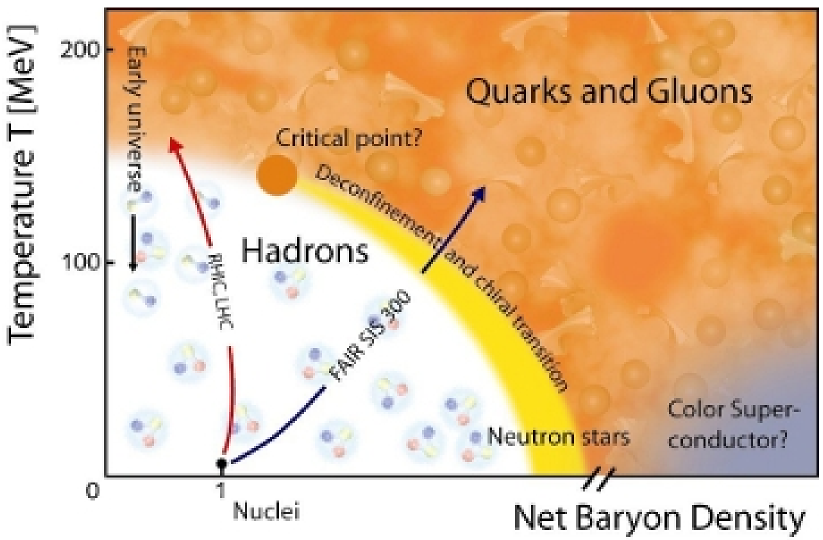


Heavy ion collision:
symmetric nuclear matter
$d = u$
$0 < \rho < 4\rho_0$



Neutron Star collisions
asymmetric matter
$d > u$
$0 < \rho < 8\rho_0$

What are the problems?



Why not calculate simply?
Quantumchromodynamics (QCD) can be calculated on a lattice

but only for μ=0 (same number of quarks and antiquarks)

Taylor expansion allows for calculations for μ/T << 1

Neutron Stars as well as Heavy Ion collisions need calculations at

finite chemical potential

- ❑ either assumptions about continuation to finite μ
- ❑ or effective theories which allow for such an extension intrinsically

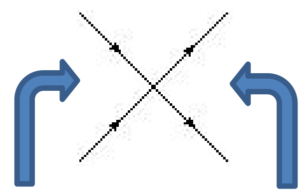
Effective Lagrangian
to study phase phase diagram and phase transitions
at finite chemical potential
(NICA,FAIR, neutron stars)

The Nambu Jona Lasinio Lagrangian is such an effective field theory

Nambu

- allows for predictions for finite T and μ
- needs as input only vacuum values + YM Polyakov loop
- shares the symmetries with the QCD Lagrangian
- can be « derived » from QCD Lagrangian

Jona-Lasinio

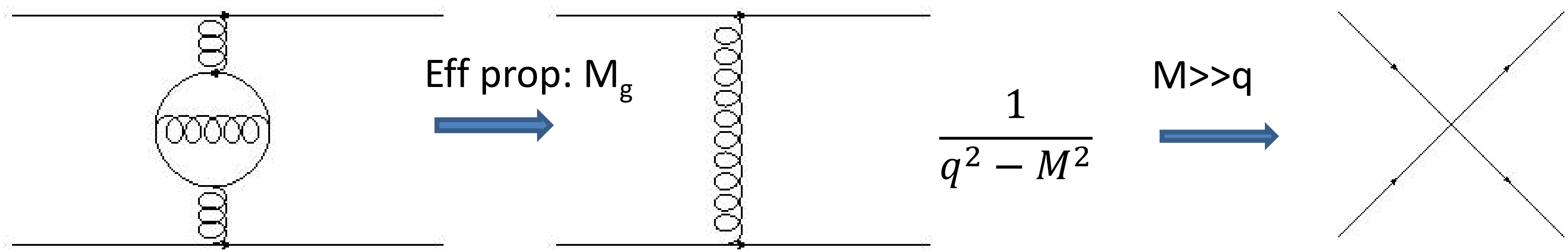


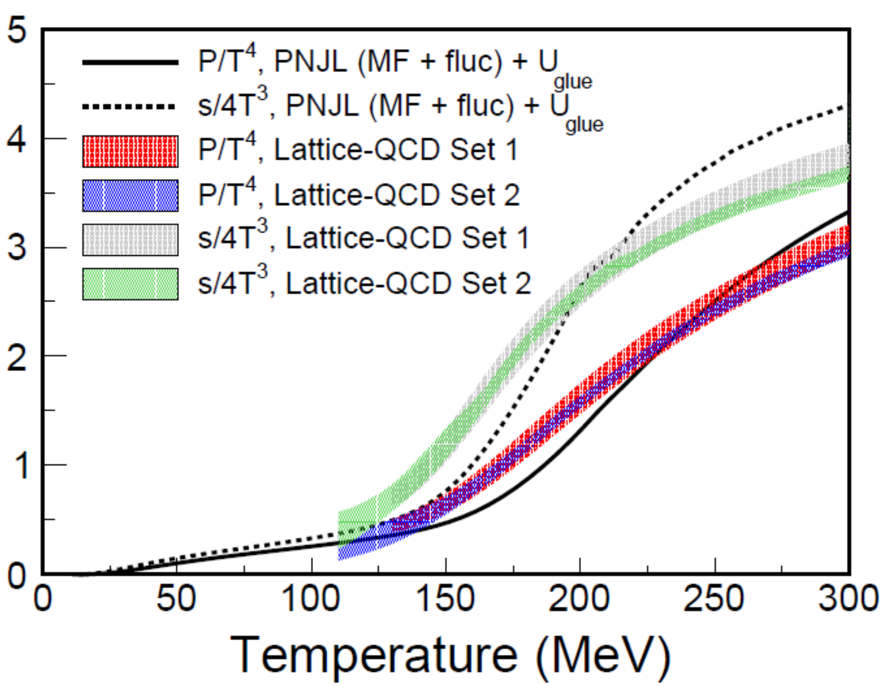
$$\mathscr{L}_{NJL} = \bar{\Psi}_i(i\gamma_\mu\partial^\mu - \hat{M}_0)\Psi_i - G_c^2 \; [\bar{\Psi}_i\gamma^\mu T^a\delta_{ij}\Psi_j] \; [\bar{\Psi}_k\gamma_\mu T^a\delta_{kl}\Psi_l]$$

$$+ \; H \det_{ij} [\bar{\Psi}_i(1-\gamma_5)\Psi_j] - H \det_{ij} [\bar{\psi}_i(1+\gamma_5)\psi_j] + \sum_{ij} \bar{\psi}_i\mu_{ij}\gamma_0\psi_j$$

NJL Lagrangian

⇒ An *effective Lagrangian* with the same symmetries for the quark degrees of freedom as QCD can be obtained by discarding the gluon dynamics completely.





Renewed interest because

Going beyond leading order in N_c + including a gluon mean field potential brings PNJL energy density and entropy density closer to lattice results

Phys.Rev. C96, 045205

Polyakov NJL: gluons on a static level

Eur.Phys.J. C49 (2007) 213-217

It is not possible to introduce gluons as dynamical degrees of freedom without spoiling the simplicity of the NJL Lagrangian which allows for real calculations

but

one can introduce gluons through an effective potential for the Polyakov loop

$$\frac{\mathbf{U(T,\Phi,\bar{\Phi})}}{\mathbf{T^4}} = -\frac{\mathbf{b_2(T)}}{\mathbf{2}}\bar{\Phi}\Phi - \frac{\mathbf{b_3}}{\mathbf{6}}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{\mathbf{b_4}}{\mathbf{4}}(\bar{\Phi}\Phi)^3$$

First: Parameter from Yang Mills   Now: including interaction between quarks and gluons

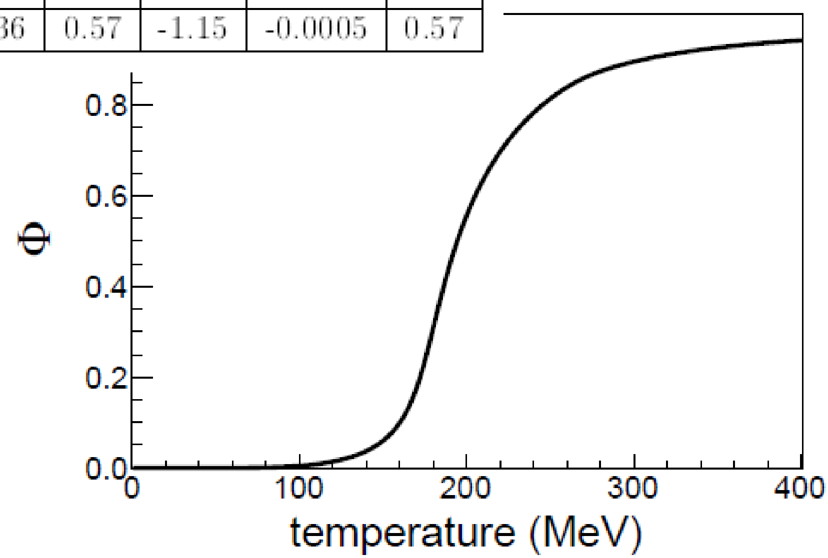
$$b_2(T) = a_0 + \left(\frac{a_1}{1+\tau}\right) + \frac{a_2}{(1+\tau)^2} + \frac{a_3}{1+\tau)^3} \qquad \tau = f\frac{T - T_{glue}}{T_{glue}} \qquad T_{glue} = a + bT + cT^2 + dT^3 + e\frac{1}{T}$$

a_0	a_1	a_2	a_3	b_3	b_4	a	b	c	d	e	f
6.75	-1.95	2.625	-7.44	0.75	7.5	0.086	0.36	0.57	-1.15	-0.0005	0.57

Parameters-> right pressure in the SB limit

Φ is the order parameter of the deconfinement transition

$$\mathbf{\Phi = \frac{1}{N_c}\ \mathrm{Tr}_c\ \langle P \exp\left(-\int_0^\beta d\tau A_0(x,\tau)\right)\rangle}$$

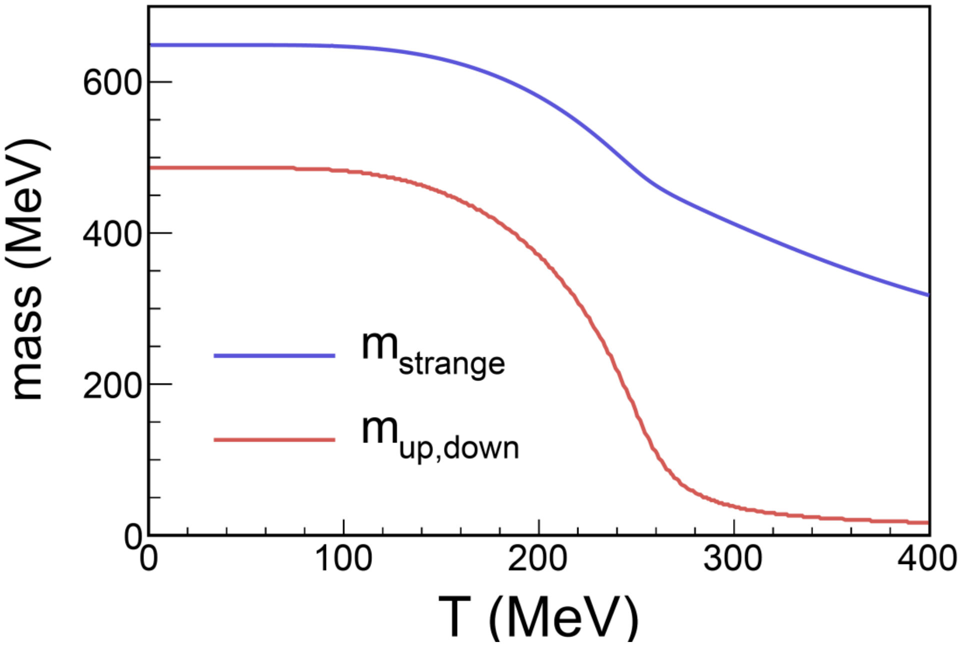


Quark Masses in NJL and PNJL

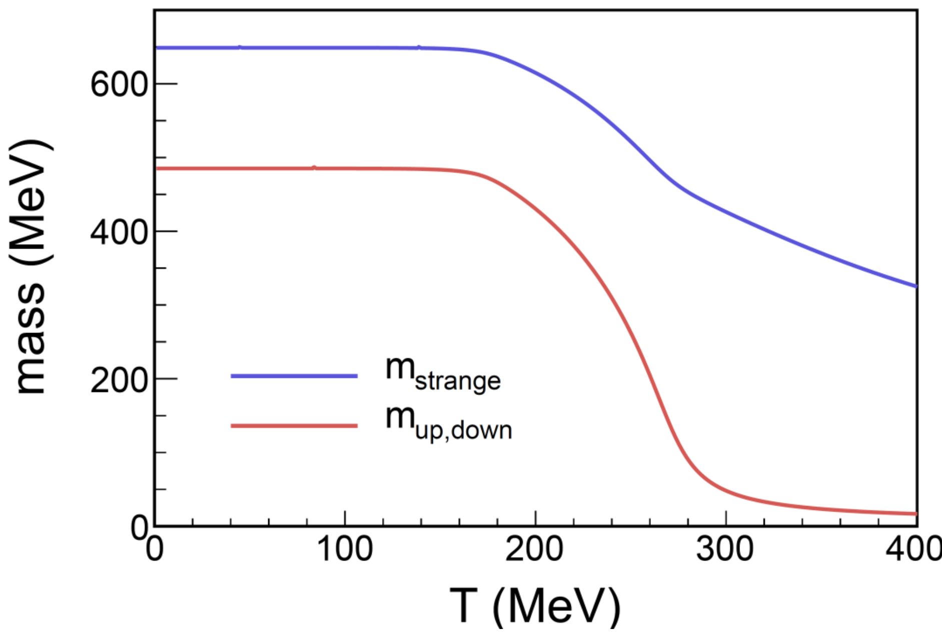
Quark masses are obtained by minimizing the grand canonical potential

$$\mathbf{M} = \hat{\mathbf{M}}_0 - 4\mathbf{G} < \bar{\psi}\psi > +2\mathbf{H} < \bar{\psi}'\psi' >< \bar{\psi}''\psi'' >$$

NJL



PNJL



In PNJL the transition is steeper than in NJL

How can we get mesons?

Quarks are the degrees of freedom of the Lagrangian

To study the phase transition we need mesons

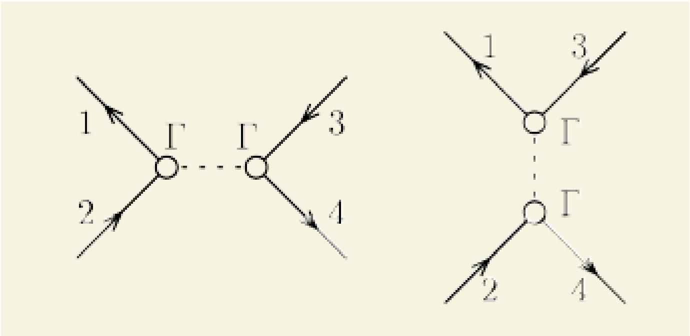
Use a Trick : **Fierz transformation** of the original Lagrangian

Fierz Transformation allows for a reordering of the field operators in 4 point contact interactions. It is simultaneously applied in Dirac, color and flavor space

Example in Dirac space:

$$(\bar{\chi}\gamma^{\mu}\psi)(\bar{\psi}\gamma_{\mu}\chi) = (\bar{\chi}\chi)(\bar{\psi}\psi) - \frac{1}{2}(\bar{\chi}\gamma^{\mu}\chi)(\bar{\psi}\gamma_{\mu}\psi) - \frac{1}{2}(\bar{\chi}\gamma^{\mu}\gamma_5\chi)(\bar{\psi}\gamma_{\mu}\gamma_5\psi) - (\bar{\chi}\gamma_5\chi)(\bar{\psi}\gamma_5\psi)$$

Scalar vector peudovector pseudoscalar



How can we get mesons? II

$$\mathscr{L}_{int} = -\mathbf{G_c^2} \ [\bar{\Psi}_i \gamma^\mu \mathbf{T^a} \delta_{ij} \Psi_j] \ [\bar{\Psi}_k \gamma_\mu \mathbf{T^a} \delta_{kl} \Psi_l]$$

Fierz transformation transforms original Lagrangian to one for mesons

$$\mathcal{L}_{\rm Pseudo\ scalar} = \mathbf{G} \ (\Psi_i \ \tau^a_{il} \ 1\!\!1_c i\gamma_5 \ \Psi_l) \ (\Psi_k \ \tau^a_{kj} \ 1\!\!1_c i\gamma_5 \ \Psi_j) \ ; \qquad \mathbf{G} = \frac{\mathbf{N_c^2} - 1}{\mathbf{N_c^2}} \mathbf{G_c}$$

$\mathscr{K}$

Singulet in color mixing of flavour

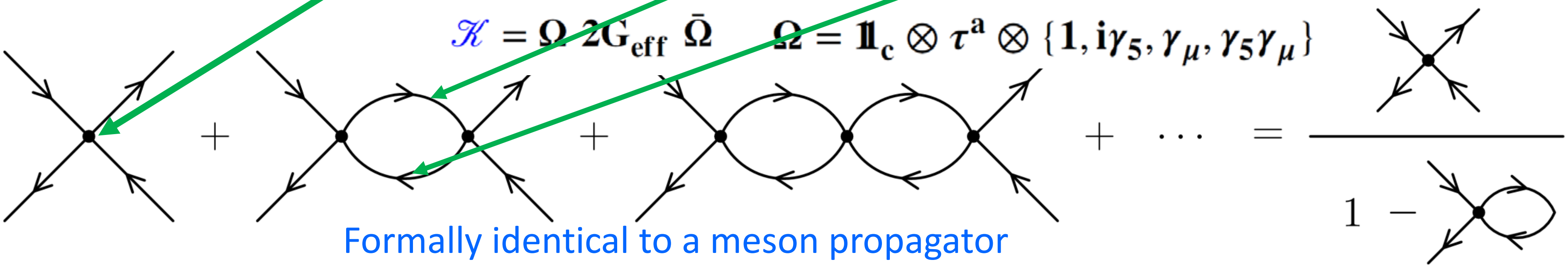
Similar terms can be obtained for
Vector mesons γ_μ
Scalar Mesons 1
Pseudovector mesons $\gamma_\mu \gamma_5$

How can we get mesons? III

We use $\mathscr{K}$ as a kernel for a Bethe-Salpeter equation (relativistic Lippmann-Schwinger eq.)

$$T(p) = \mathscr{K} + i\int \frac{d^4k}{(2\pi)^4}\, \mathscr{K}\, S\left(k+\frac{p}{2}\right) S\left(k-\frac{p}{2}\right) T(p)$$

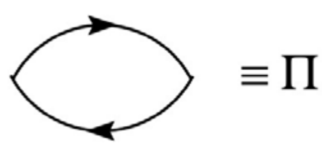
$$\mathscr{K} = \Omega\, 2G_{eff}\, \bar{\Omega} \qquad \Omega = \mathbb{1}_c \otimes \tau^a \otimes \{1, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu\}$$



Formally identical to a meson propagator

In (P)NJL one can sum up this series analytically:

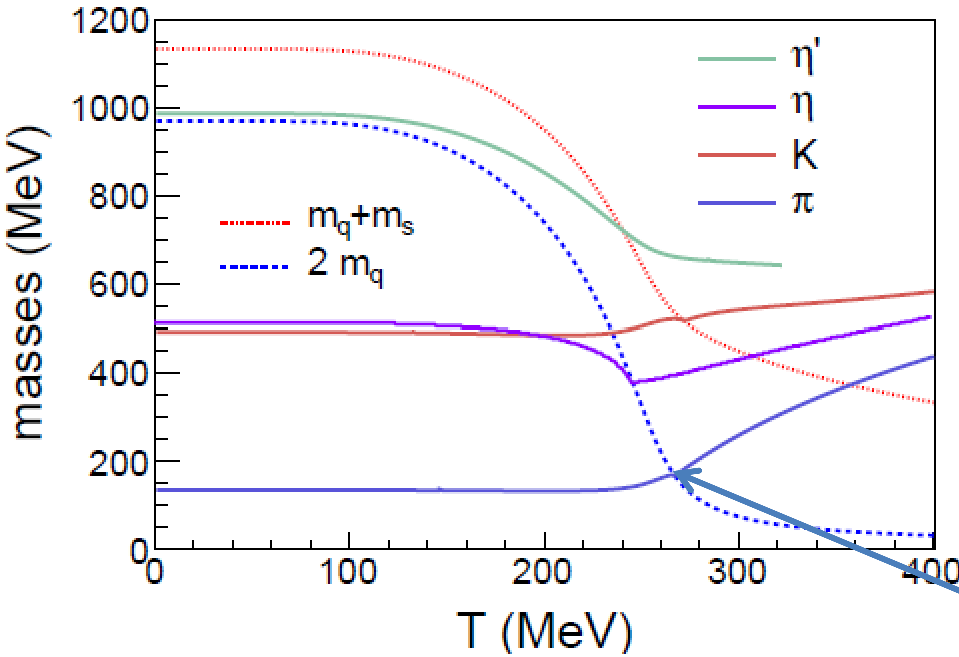
$$T(p) = \frac{2G_{eff}}{1 - 2G_{eff}\,\Pi(p)}\,, \qquad \Pi(p_0, p) = -\frac{1}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\,\Omega\, S\left(k+\frac{p}{2}\right)\Omega\, S\left(k-\frac{p}{2}\right)$$



How to get mesons? IV

The meson pole mass and the width one obtains by solving:

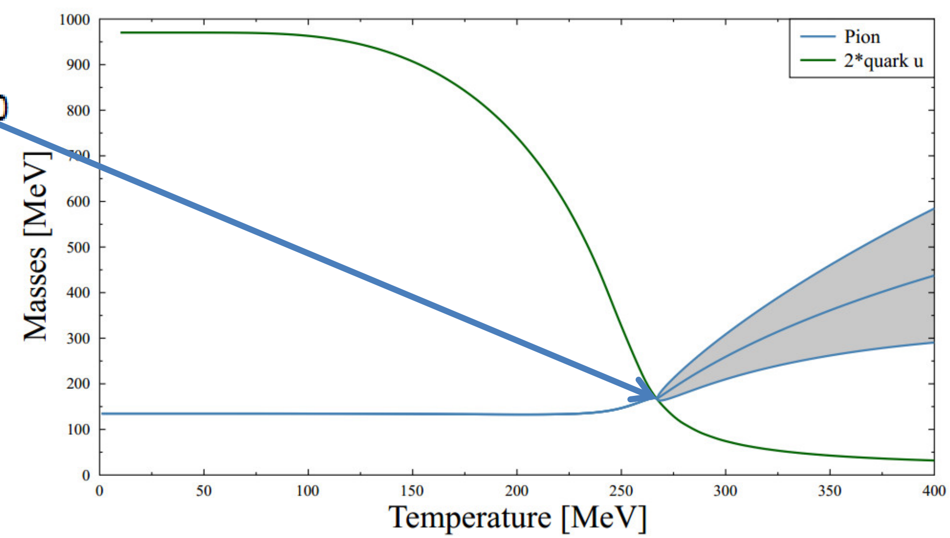
$$1 - 2G_{eff}\ \Pi(p_0 = M_{meson} - i\Gamma_{meson}/2, p = 0) = 0$$



masses of pseudoscalar mesons and of quarks at μ = 0

At T=0 physical and calculated mass agree quite well

When mesons become unstable they develop a width



Looking back

We have seen that the (P)NJL model describes quite well meson and baryon properties
For this one has to fix the 5 parameters of the model

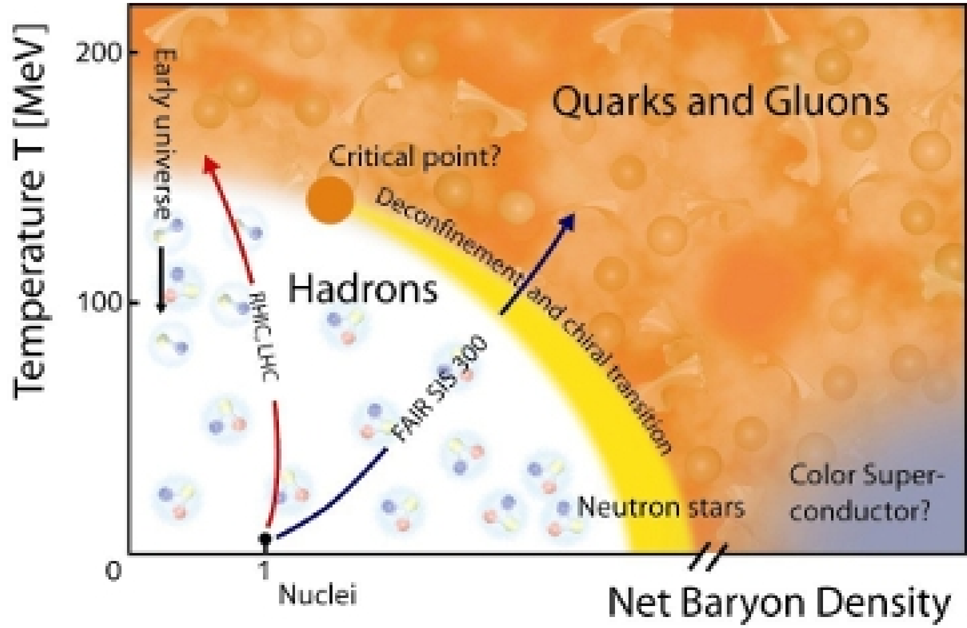
Λ = upper cut off of the internal momentum loops
G_c = coupling constant
M_0 = bare mass of u,d and s quarks
H= coupling constant 't Hooft term

These parameters have been adjusted to reproduce

Masses of π and K in the vacuum , as well as the η-η' mass splitting
π decay constant, $q\bar{q}$ condensate (-241 MeV)^3

Therefore:
All masses, cross sections etc. at finite μ and T
follow without any new parameters from ground state observables.

The Phase diagram of PNJL in T and μ



To obtain the phase diagram
one starts from the partition function

$$Z = Tr[\exp -\beta(H-\mu N)] = \exp(-\beta\Omega)$$

and obtains in order N_c , $(1/N_C)^{-1}$, ,the number of colors:

$$\begin{aligned} &\Omega_q^{(-1)}(T,\mu_i;\langle\bar\psi_i\psi_i\rangle,\Phi,\bar\Phi) \\ = \quad & \ln(Tr[\exp(-\beta\int dx^3(-\bar\psi(i\partial\!\!\!/ - m)\psi - \mu\bar\psi\psi))]) \\ + \quad & 2G\sum_k < \bar\psi_k\psi_k >^2 -4K\prod_i < \bar\psi_k\psi_k > +U_{PNJL} \end{aligned}$$

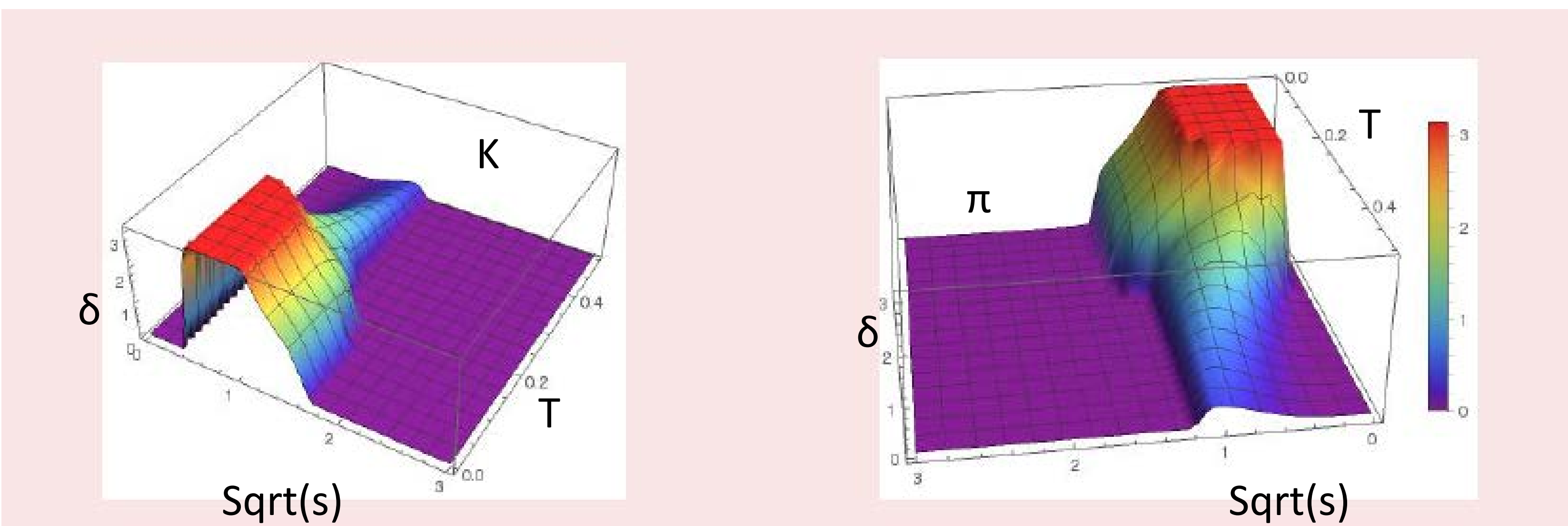
In this order the lattice data cannot be reproduced

In order to improve one has to go to the order $O(N_c = 0)$. In this order meson loops contribute and on obtains a mesonic contribution to the grand potential

$$\Omega_q^{(0)}(T,\mu_i) = \sum_{M \in J^\pi = \{0^+,0^-\}} \Omega_M^{(0)}(T,\mu_M(\mu_i))$$

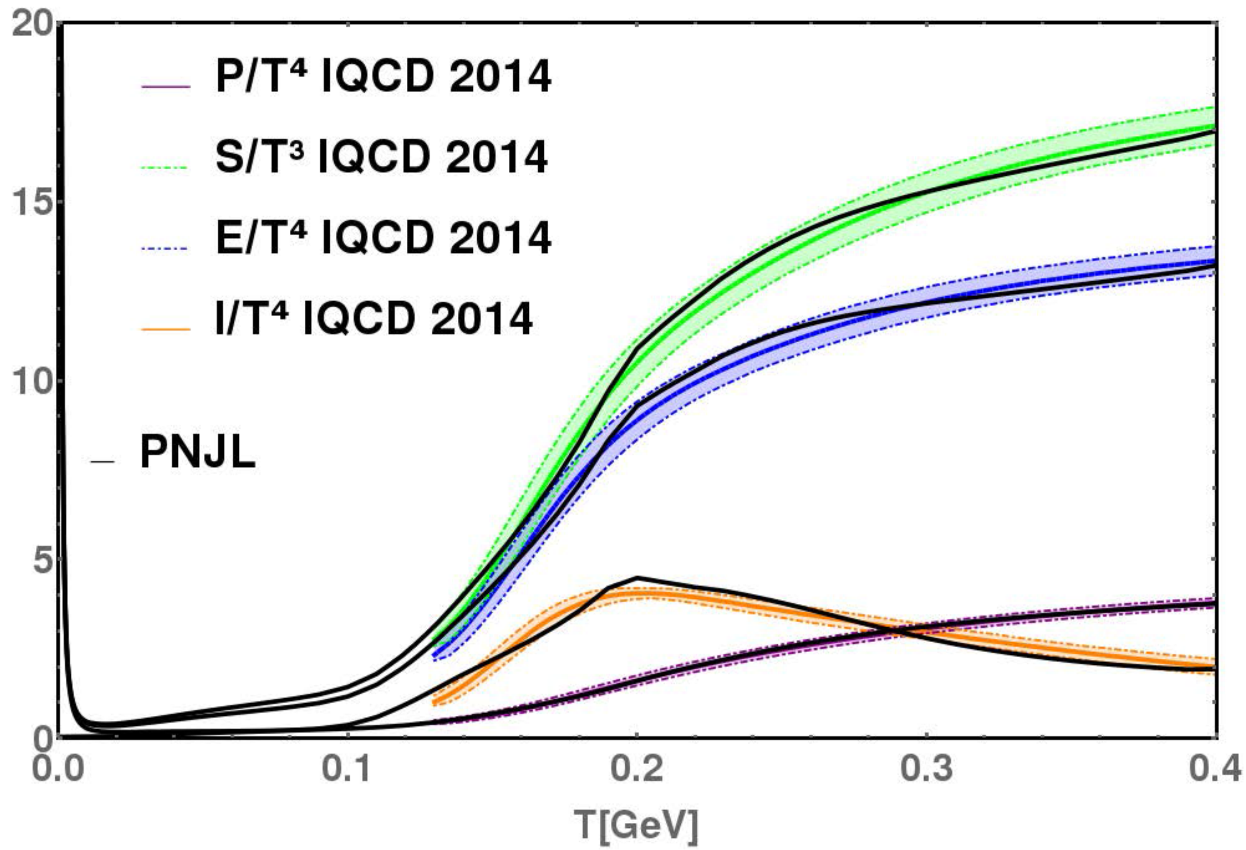
$$\Omega_M^{(0)}(T,\mu_M) = -\frac{g_M}{2\pi}\int \frac{d^3p}{(2\pi)^3}\int_0^{+\infty} d\omega \left[\frac{1}{e^{\beta(\omega-\mu_M)}-1} + \frac{1}{e^{\beta(\omega+\mu_M)}-1}\right] \delta(\omega,\mathbf{p};T,\mu_M)$$

with the phase shifts δ



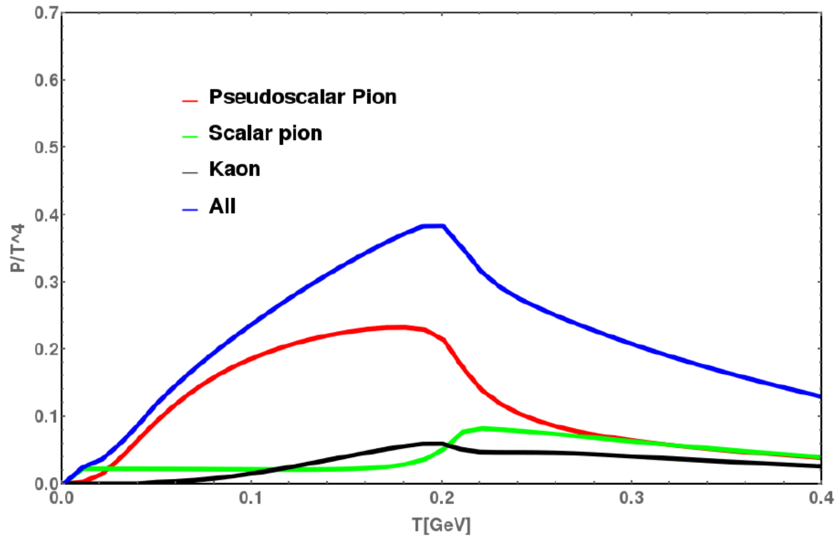
Comparison with lattice results for μ =0

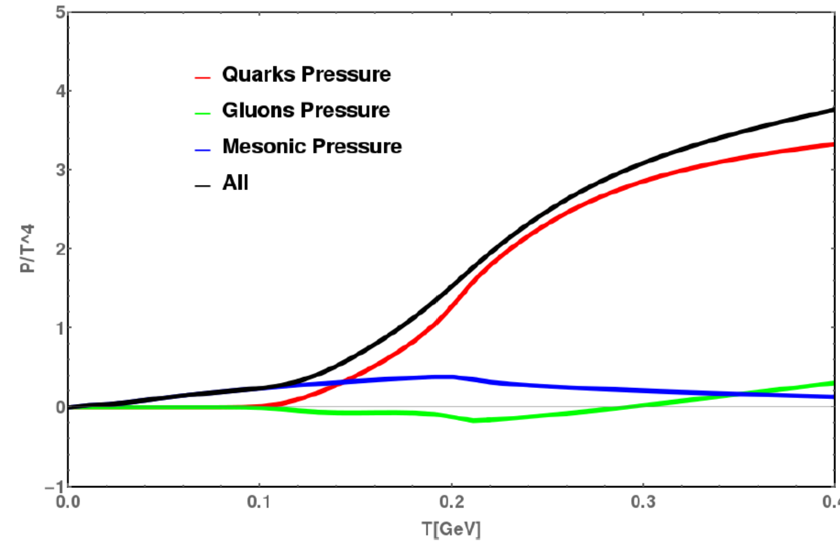
In the order O(N_c =0) and including the g-q iteraction
we can reproduce Pressure P, entropy density s, energy density E and interaction measure I of the lattice calculations at μ=0

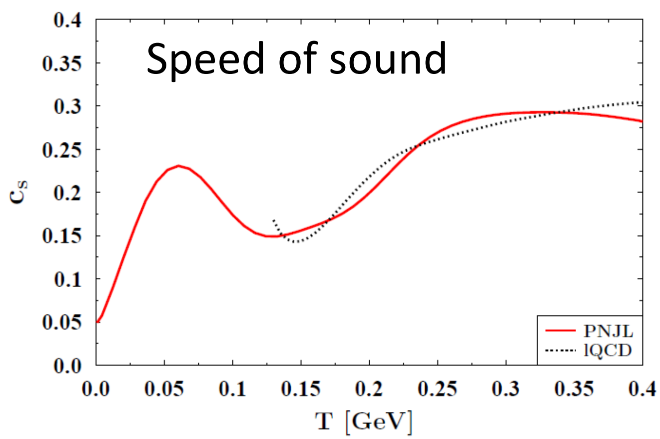


This allows to explore the phase diagram in the whole T,μ plane

Where does the pressure come from?





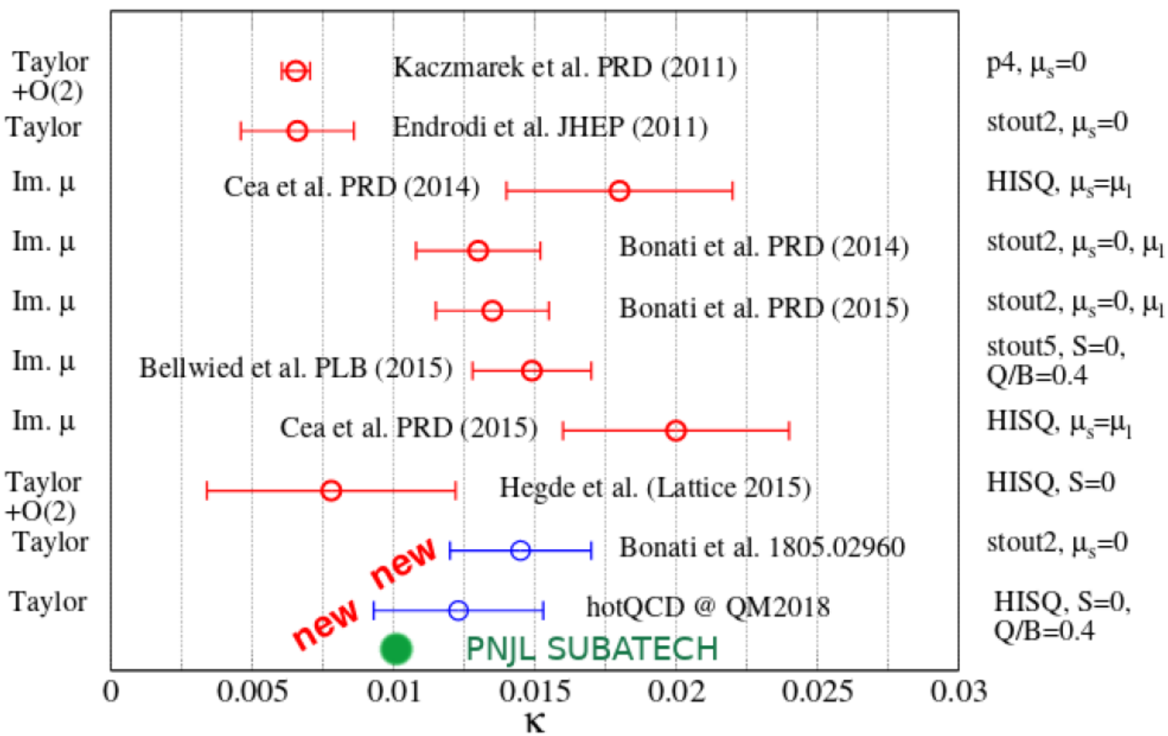


Also the speed of sound comes close to the lattice results.

Extension to small but finite chemical potential

Like in lattice calculation: Taylor expansion around μ = 0

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \dots \qquad \kappa = \left. \frac{\partial^2 \frac{T_c(\mu_B)}{T_c(0)}}{\partial \mu_B^2} \right|_{\mu_B = 0}$$



We find

T_c = 138 MeV
κ = 0.01

Very similar to lattice calculations

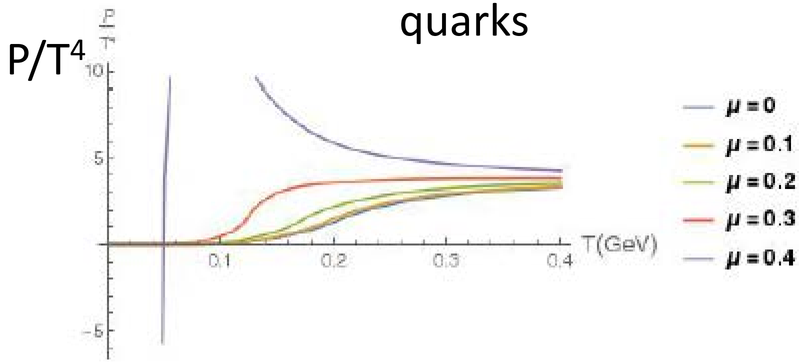
Also for small but finite μ we reproduce the lattice results: confidence for larger μ

The PNJL equation of state for finite μ

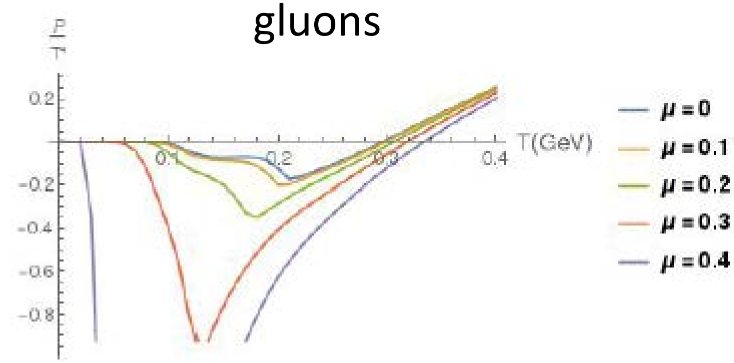
Calculation of thermal quantities at finite μ is straight forward in PNJL

Contribution to the pressure of the different particles

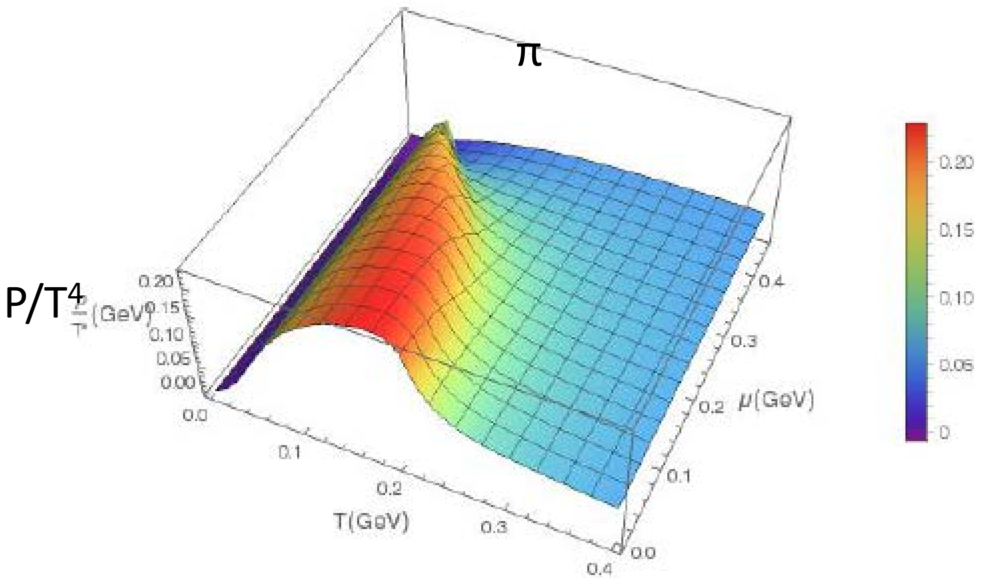
quarks



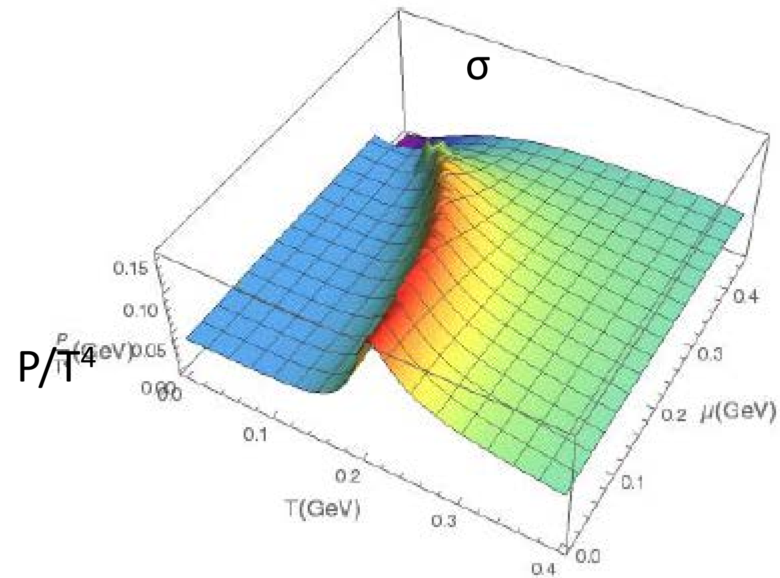
gluons



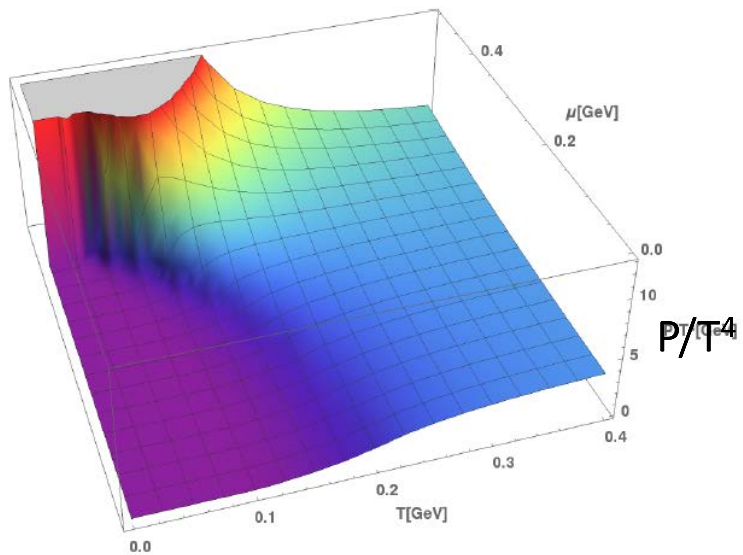
π



σ

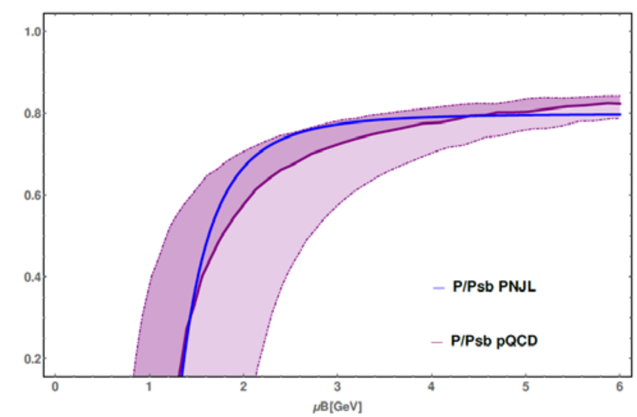


Total P(T,μ)

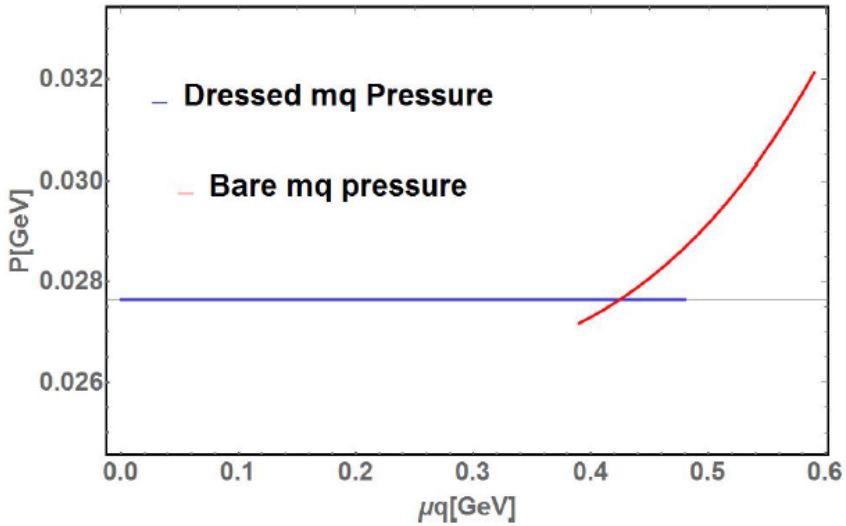


for large μ contact with pQCD

PNJL in the error bars of pQCD



For small temperatures the equation of state shows a first order phase transition



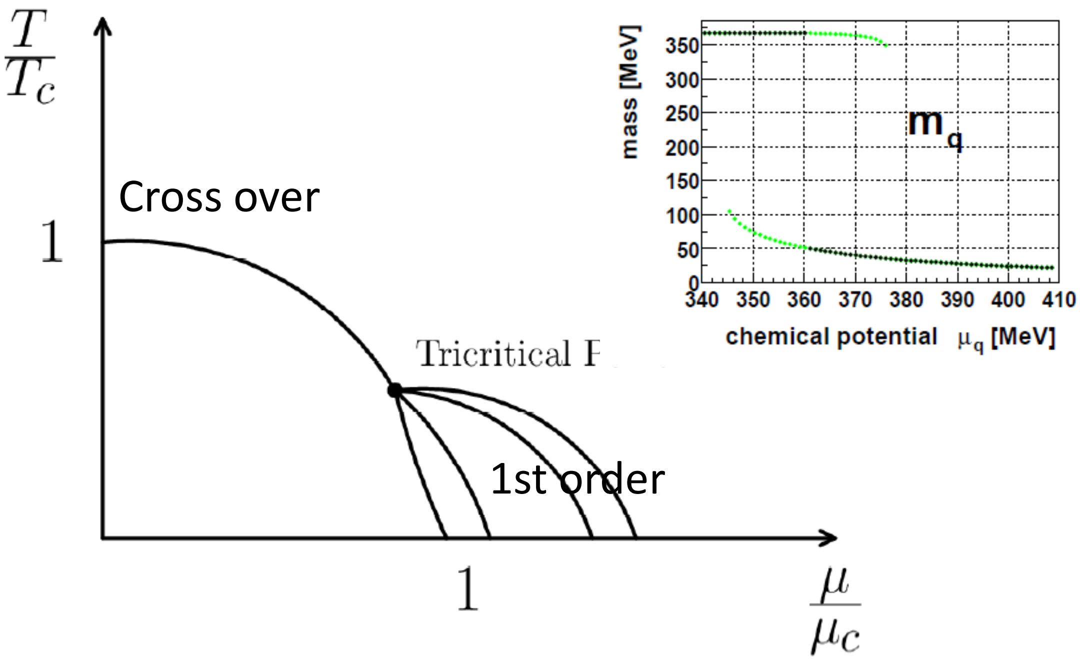
Masses close to the tricritical point

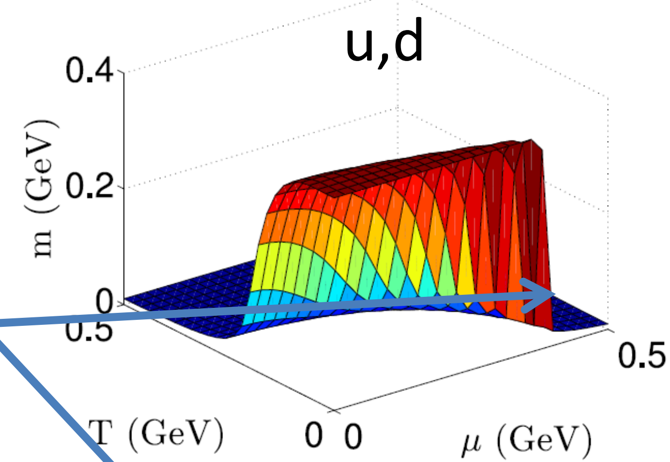
PNJL Lagrangian:
transition between quarks and hadrons

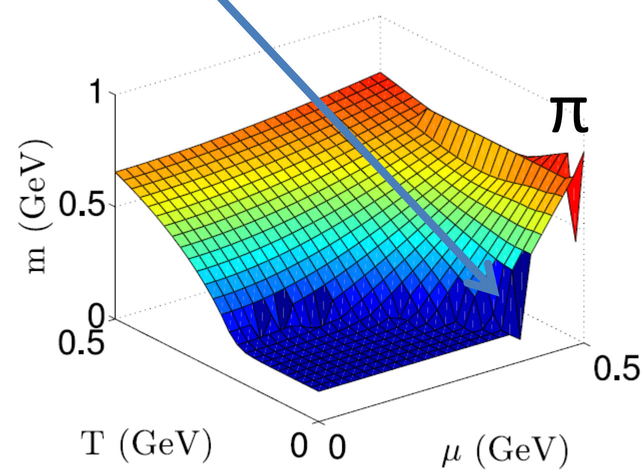
Cross over at $\mu = 0$

1st order transition $\mu >> 0$

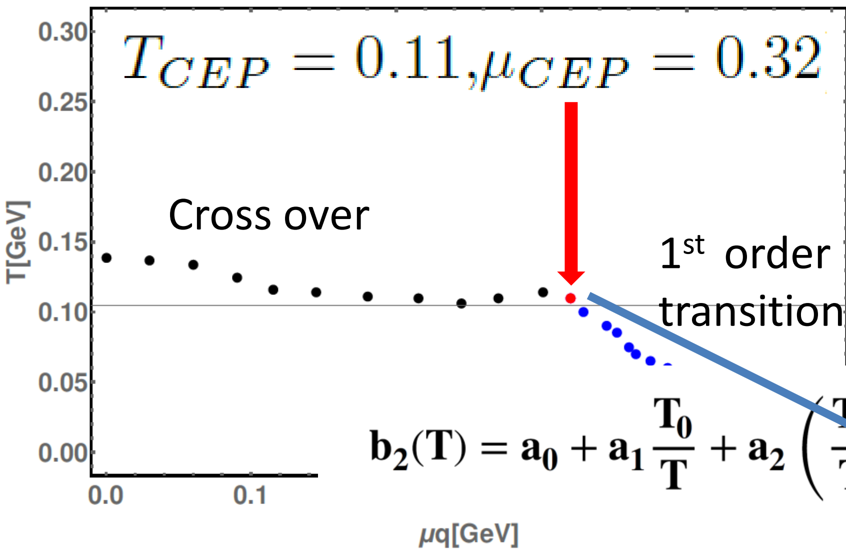
sudden change of q and meson mass







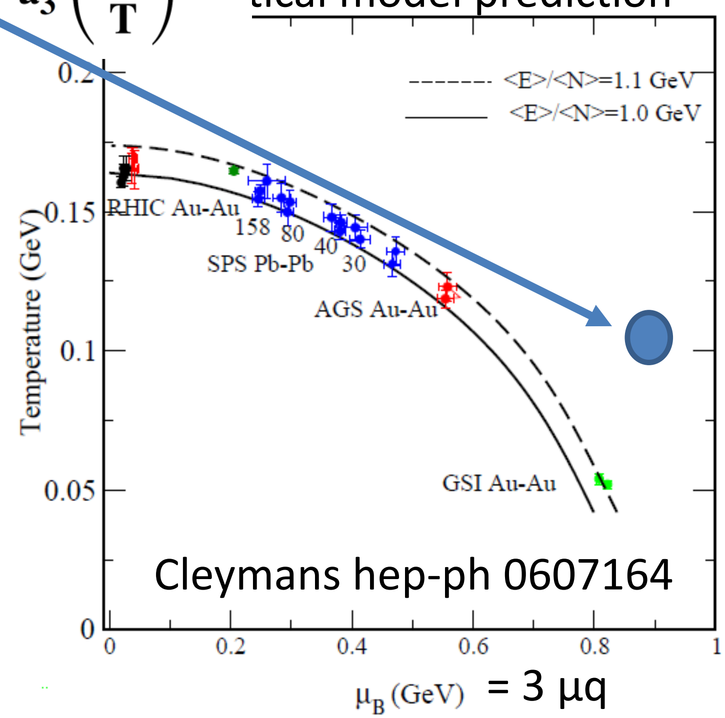
Borderline between quark gluon plasma and hadrons



$$b_2(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3$$

Freeze out condition

tical model prediction



Statistical model predicitions:

Phase transition point reachable in experiments at NICA and FAIR

Summary of our long way

Starting point: NJL Lagrangian which shares the symmetries with QCD

Fierz transformation -> color less meson channel and qq channels -> baryons

Bethe Salpeter equation in $q\bar{q}$ → mesons as pole masses

All masses described (10% precision) by 7 parameters fitted to ground state properties
(PNJL needs additional parameters to fix the Polyakov loop)

good description of lattice data at μ=0 and for expansion coeff for finite μ
Extension of all masses to finite T and μ without any new parameter

Allows to access to describe equation of state and phase diagram at finite μ
necessary for neutron star, neutron star collisions and heavy ion physics

We find a first order phase transition for finite μ.
Open the way to explore the consequences:
Transport approaches for heavy ion collisions

Baryons

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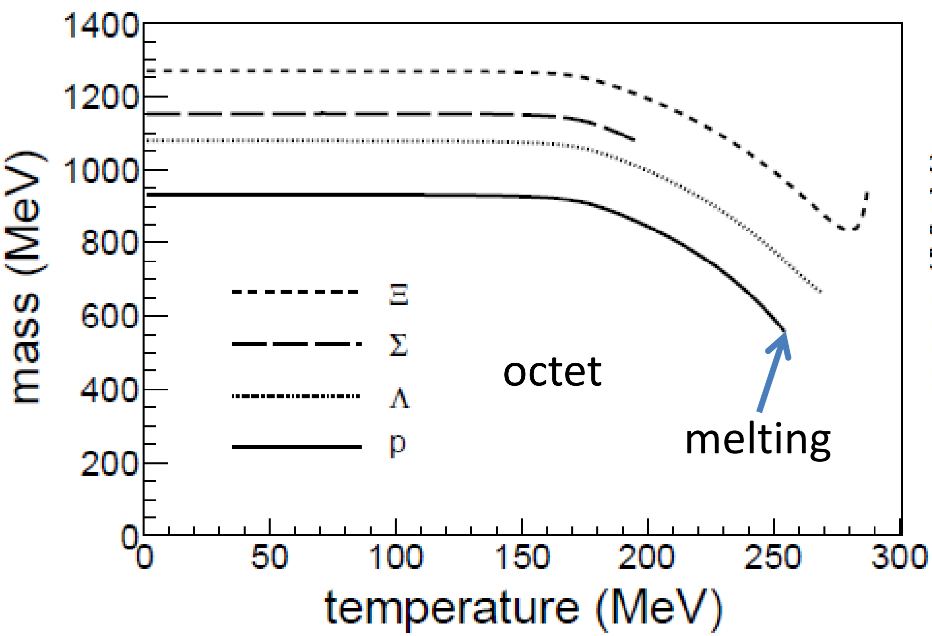
Omitting Dirac and flavor structure :

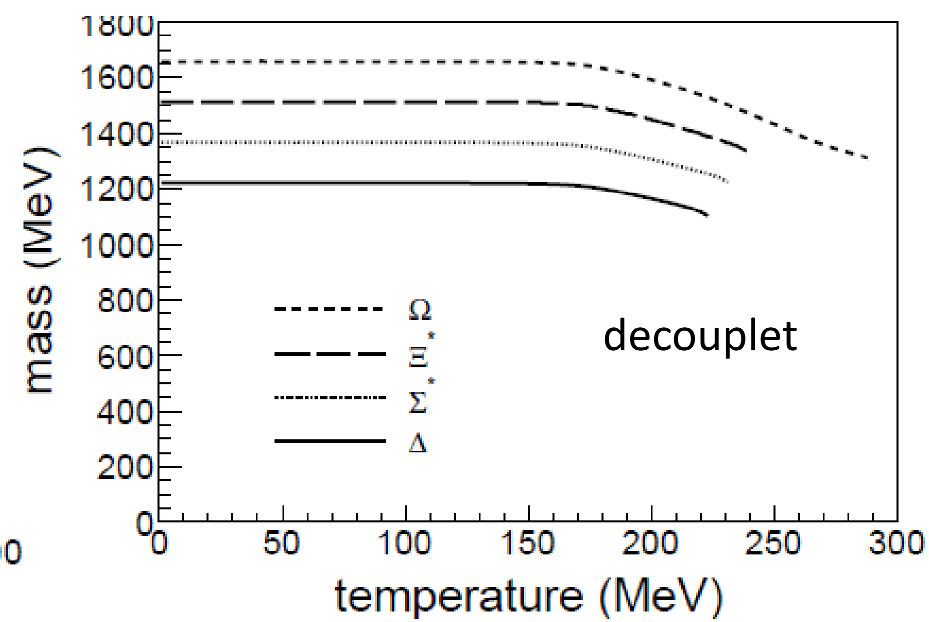
$$\left[1 - \frac{2}{m_{quark}} \frac{1}{\beta} \sum_n \int \frac{d^3q}{(2\pi)^3} S_q(i\omega_n, q)\; t_D(i\nu_l - i\omega_n, -q)\right]\Bigg|_{i\nu_l \to P_0 + i\epsilon = M_{Baryon}} = 0$$

where we approximated the quark propagator for the exchanged quark by:

$$S_q(q) = \frac{1}{\not q - m_{quark}} \to -\frac{\mathbb{1}_{Dirac}}{m_{quark}}$$

5% error (Buck et al. (92))





The more strange quarks the higher the melting temperature