# A Forward Branching Phase Space Generator for Hadron Colliders

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# Introduction

https://arxiv.org/abs/1806.09678

**A Forward Branching Phase Space Generator for Hadron colliders**

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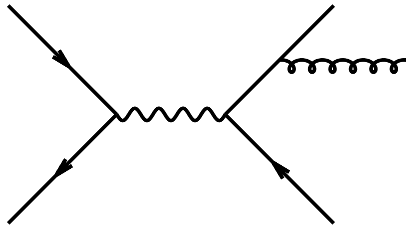
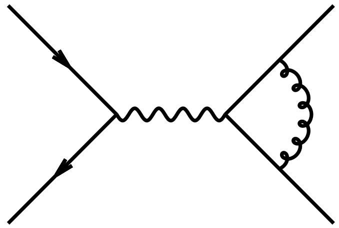
# Next-to-Leading Order Calculations



real emission contributions
$m + 1$ parton kinematics



virtual corrections
$m$ parton kinematics

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} d\boldsymbol{\sigma}^{R} + \int_{m} d\boldsymbol{\sigma}^{V}$$

IR divergent

☞regularize in $d = 4 - 2\varepsilon$ dim

# Next-to-Leading Order Calculations

introduce local counterterm $d\boldsymbol{\sigma}^A$ with same singularity structure as $d\boldsymbol{\sigma}^R$:

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} \underbrace{\left[d\boldsymbol{\sigma}^R - d\boldsymbol{\sigma}^A\right]}_{\text{finite}} + \int_{m+1} d\boldsymbol{\sigma}^A + \int_m d\boldsymbol{\sigma}^V$$

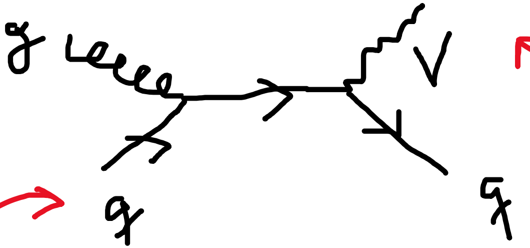
finite

↓

can safely set $\varepsilon \longrightarrow 0$

perform integral numerically in four dimension

# Born Phase Space (V+1 parton kinematics)




$$\hat{p}_a = \frac{1}{2}(0, 0, \hat{x}_a, \hat{x}_a)$$

$$\hat{p}_b = \frac{1}{2}(0, 0, -\hat{x}_b, \hat{x}_b)$$

$$\hat{p}_\mu^{(i)} = \hat{p}_T^{(i)}\left(\sin\hat{\phi}_i, \cos\hat{\phi}_i, \sinh\hat{\eta}_i, \cosh\hat{\eta}_i\right)$$

$$\hat{Q}_\mu = \left(\vec{\hat{q}}_T, \hat{\alpha}_T \sinh\hat{\eta}_q, \hat{\alpha}_T \cosh\hat{\eta}_q\right) \text{ where } \hat{\alpha}_T = \sqrt{\hat{q}_T^2 + M_V^2},$$



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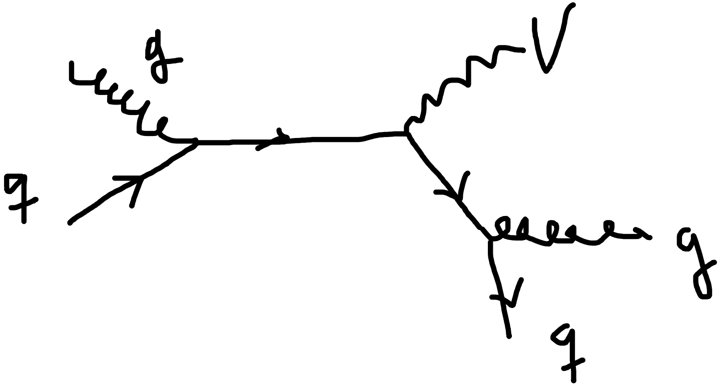
# Born Phase Space (V+1 parton kinematics)

$$d\hat{x}_a d\hat{x}_b d\Phi(\hat{p}_a \hat{p}_b; \hat{Q}, \{\hat{p}\}_n) = \frac{1}{(16\pi^3)^{n+1}} \frac{2}{S} \left( \prod_{i=1}^{n} d\hat{p}_T^{(i)} d\hat{\eta}_i d\hat{\phi}_i \times \hat{p}_T^{(i)} \right) \times d\hat{\eta}_q \times \Theta(1-\hat{x}_1)\Theta(1-\hat{x}_2) \,, \tag{2.5}$$

with

$$\begin{aligned} \hat{\vec{q}}_T &= -\sum_{i=1}^{n} \hat{\vec{p}}_T^{(i)} \\ \hat{x}_a &= \frac{1}{\sqrt{S}} \left( \hat{\alpha}_T e^{\eta_q} + \sum_{i=1}^{n} \hat{p}_T^{(i)} e^{\hat{\eta}_i} \right) \\ \hat{x}_b &= \frac{1}{\sqrt{S}} \left( \hat{\alpha}_T e^{-\eta_q} + \sum_{i=1}^{n} \hat{p}_T^{(i)} e^{-\hat{\eta}_i} \right) \,, \end{aligned} \tag{2.6}$$

# Final State Forward Branching Phase Space




$$\hat{Q} = Q,\ \hat{p}_{ab} = \hat{p}_a + \hat{p}_b = p_a + p_b + \alpha\, p_{12}^L,$$

$$\hat{p}_J = p_{12} + \alpha\, p_{12}^L,\ p_{12}^L = (p_1 + p_2)_L$$

Rapidity and transverse momentum is left invariant.

$$d\Phi_3^{\rm FINAL}(p_a, p_b; Q, p_1, p_2) = d\Phi_2(\hat{p}_a, \hat{p}_b; \hat{Q}, \hat{p}_J) \times \left[\frac{d\,p_1}{(2\pi)^3}\ \delta(p_1^2)\right] \times J(\hat{p}_J, p_1)\ .$$



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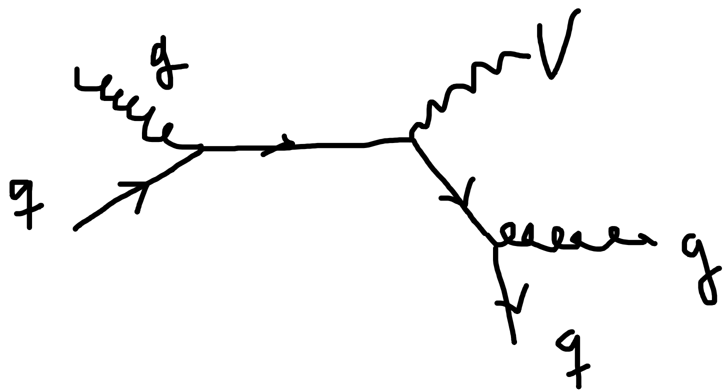
# Final State Forward Branching Phase Space




$$\hat{Q} = Q,\ \hat{p}_{ab} = \hat{p}_a + \hat{p}_b = p_a + p_b + \alpha\, p_{12}^L,$$

$$\hat{p}_J = p_{12} + \alpha\, p_{12}^L,\ p_{12}^L = (p_1 + p_2)_L$$

Rapidity and transverse momentum is left invariant.

$$J(\hat{p}_J, p_1) = \left|\frac{2}{1-\beta_-/\beta_+}\right| \times \sqrt{\frac{(\hat{p}_J^T)^2}{(\hat{p}_J^L)^2}} = \left|\frac{2}{1-\beta_-/\beta_+}\right|,$$

$$\beta_\pm = \frac{(\hat{p}_J^L \cdot p_1^L) \pm \sqrt{(\hat{p}_J^T)^4 + 2(\hat{p}_J^L)^2(\hat{p}_J^T \cdot p_1^T) + (\hat{p}_J^L \cdot p_1^L)^2}}{(\hat{p}_J^L)^2}.$$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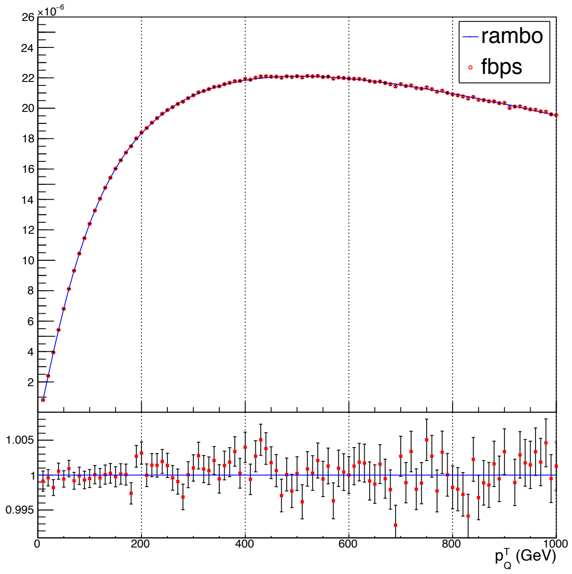
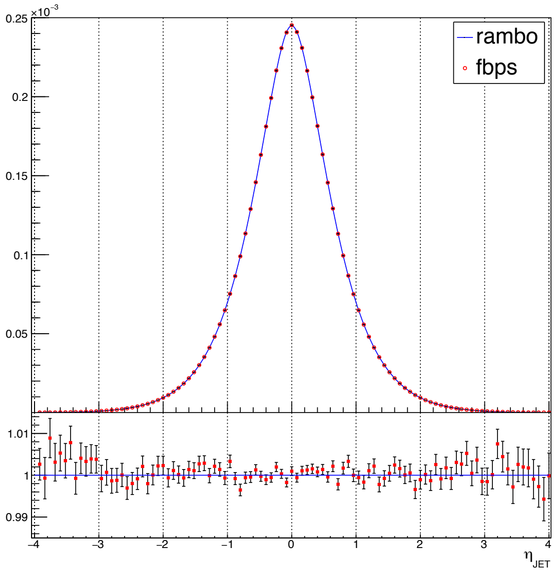
# Final State Forward Branching Phase Space



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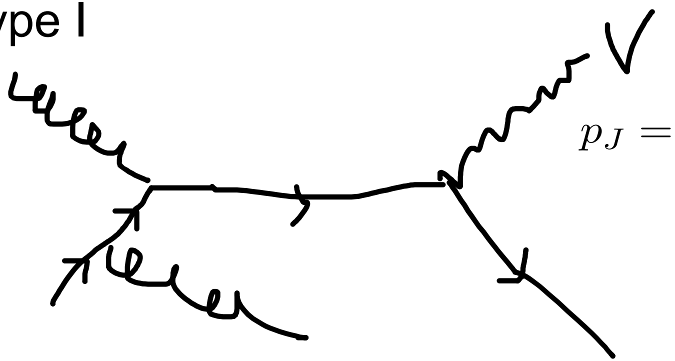
# Initial State Forward Branching Phase Space – Type I

Type I




$p_J = \hat{p}_J,\ p_{ab} = \hat{p}_{ab} - \alpha p_1^L,\ Q = \hat{Q} - \hat{p}_1^T - (1+\alpha) p_1^L$

Jet momentum is invariant.

$$d\Phi_3^{\mathrm{INIT,I}}(p_a, p_b; Q, p_J, p_1) = d\Phi_2(\hat{p}_a, \hat{p}_b; \hat{Q}, \hat{p}_J) \times \left[ \frac{d\,p_1}{(2\pi)^3}\ \delta(p_1^2) \right] \times J(\hat{Q}, p_1)\ .$$



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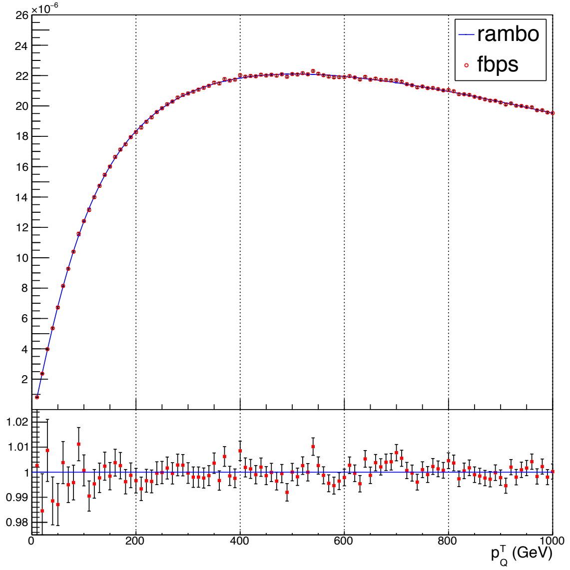
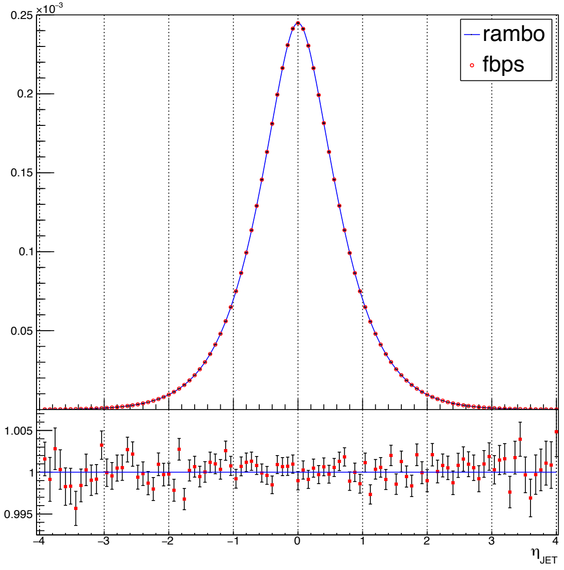
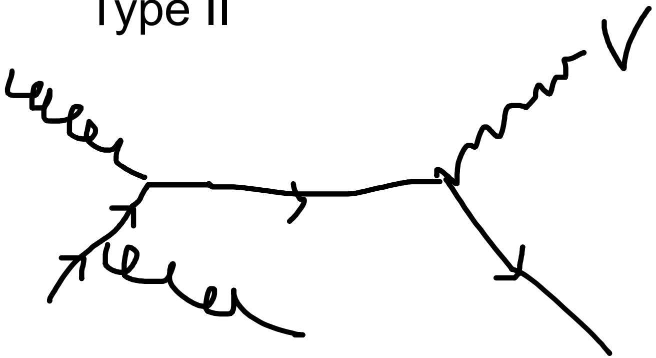
# Initial State Forward Branching Phase Space – Type I



# Initial State Forward Branching Phase Space - Type II

Type II

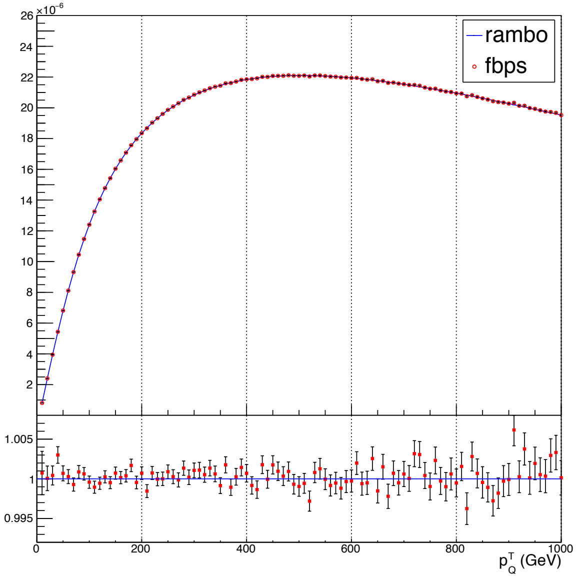


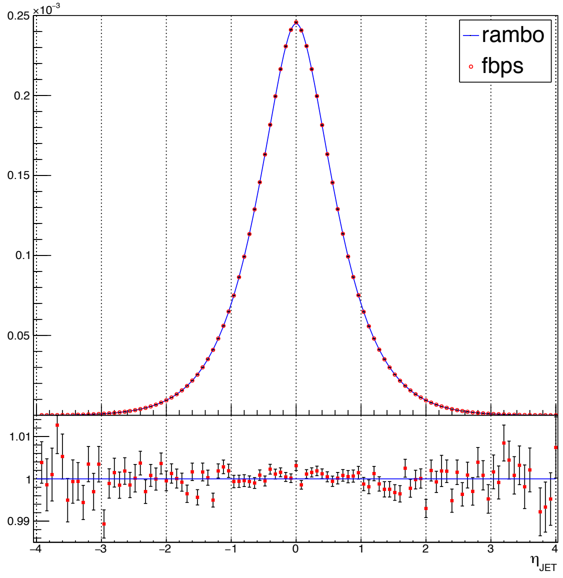

$$Q = \hat{Q},\ p_{ab} = \hat{p}_{ab} - \alpha p_1^L,\ p_J = \hat{p}_J - p_1^T - (1+\alpha) p_1^L$$

Vector boson momentum is invariant.

$$d\Phi_3^{\mathrm{INIT,I}}(p_a, p_b; Q, p_J, p_1) = d\Phi_2(\hat{p}_a, \hat{p}_b; \hat{Q}, \hat{p}_J) \times \left[\frac{d\,p_1}{(2\pi)^3}\ \delta(p_1^2)\right] \times J(\hat{Q}, p_1)\ .$$

# Initial State Forward Branching Phase Space - Type II



# Phase Space Generator for V+1 jet

$$
\begin{aligned}
d\Phi_3^{\text{exclusive}}(p_a, p_b; Q, p_1, p_2) &= d\Phi_3(p_a, p_b; Q, p_1, p_2) \\
&\times \left[\Theta(R-\Delta_{12}) + \Theta(\Delta_{12}-R)\left(\Theta(p^T_{\text{min}} - p^T_1)\Theta(p^T_2 - p^T_{\text{min}}) + \Theta(p^T_{\text{min}} - p^T_2)\Theta(p^T_1 - p^T_{\text{min}})\right)\right] \\
&= d\Phi_2(\hat{p}_a, \hat{p}_b; \hat{Q}, \hat{p}_J) \times \left[\frac{d\,p_1}{(2\pi)^3}\ \delta(p_1^2)\right] \\
&\times \Big[\Theta(R-\Delta_{12}) J^{\text{FINAL}}(\hat{p}_J, p_1)\delta(M^{\text{FINAL}}(\{\hat{p}\}_2 \to \{p\}_2)) \\
&\quad +\Theta(\Delta_{12}-R)\Big(\Theta(p^T_1 < p^T_{\text{MIN}})\Theta(p^T_2 > p^T_{\text{MIN}}) + (1 \leftrightarrow 2)\Big) J^{\text{INIT}}(\hat{Q}, p_1)\delta(M^{\text{INIT}}(\{\hat{p}\}_2 \to \{p\}_2))\Big]\ .
\end{aligned}
$$



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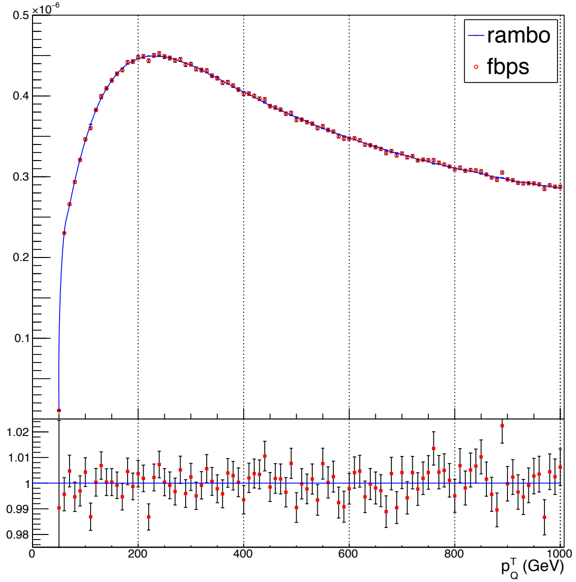
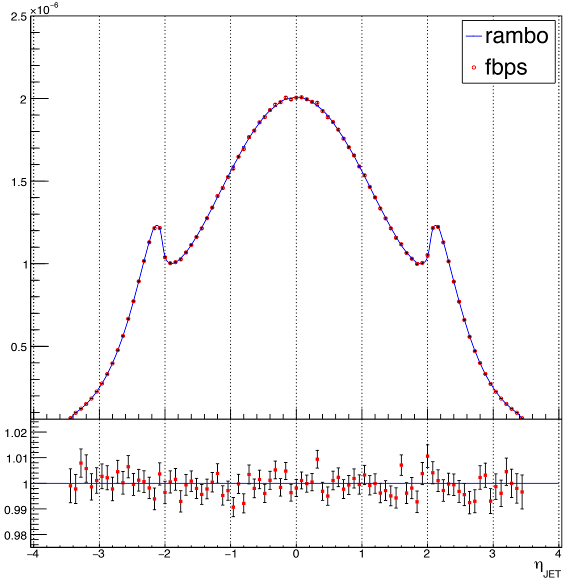
# Phase Space Generator for V+1 jet




For the FBPS, the observables are born momentum, while the born phase space is
re-weighted by the bremsstrahlung event(s).



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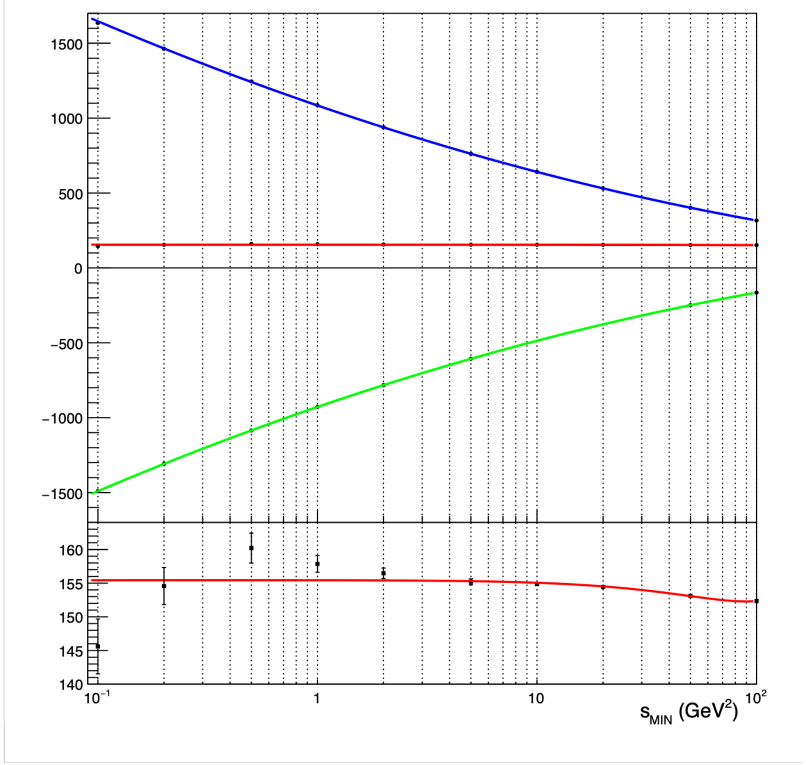
# V+1 jet at NLO using FBPS (This is preliminary!)

Inclusive XS




$\sqrt{S}$=14 TeV

$p_T^{\text{jet}} > 50$ GEV

$|\eta_{\text{jet}}| < 3, \mu_R = \mu_F = M_Z$

CT14nlo PDFs



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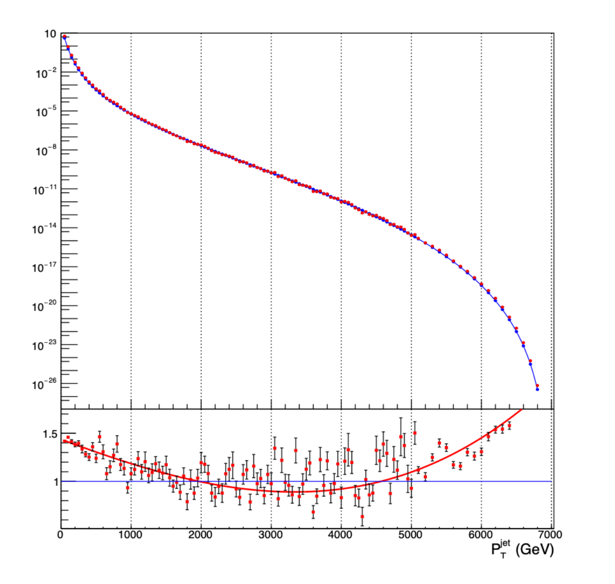
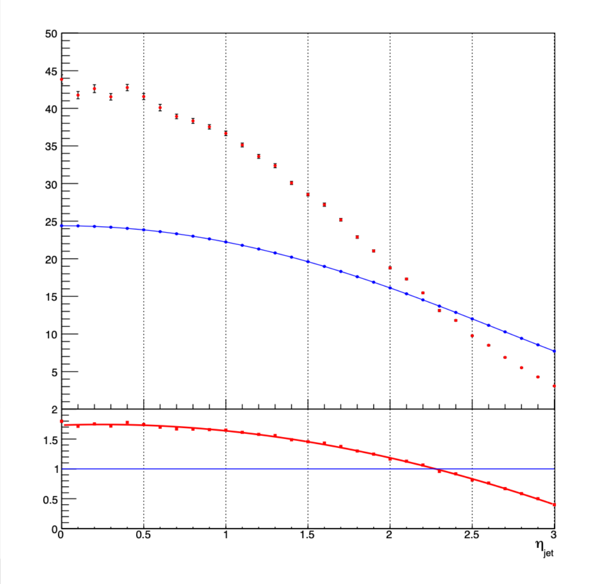
# V+1 jet at NLO using FBPS (This is preliminary!)

Jet distributions




$\sqrt{S}$=14 TeV

CT14nlo PDFs



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# NLO Results by reweighting Born events

$$d\sigma^{\mathrm{LO}}(\{(p_T,\eta,\phi)_i\}) = \sum_{a,b} \frac{f_a(x_1) f_b(x_2)}{2 s_{12}} \mathcal{M}^{(0)}_{ab}(\{(p_T,\eta,\phi)_i\})$$

$$d\sigma^{\mathrm{NLO}}(\{(p_T,\eta,\phi)_i\}) = \sum_{a,b} \frac{f_a(x_1) f_b(x_2)}{2 s_{12}} \mathcal{M}^{(0)}_{ab}(\{(p_T,\eta,\phi)_i\})$$
$$\times \left(1 + V(\{(p_T,\eta,\phi)_i\}) + \int dk_B \frac{s_{12}}{\hat{s}_{12}} \frac{f_a(\hat{x}_1) f_b(\hat{x}_2)}{f_a(x_1) f_b(x_2)} \frac{\mathcal{M}^{(1)}_{ab}(\{(p_T,\eta,\phi)_i\}, k_B)}{\mathcal{M}^{(0)}_{ab}(\{(p_T,\eta,\phi)_i\})}\right)$$

K-factors at the level of Born events.

# Outlook

- The FBPS generator allows for the re-weighting of Born events and generation of n exclusive jets.
- Issues of missed binning in histograms is absent.
- Generalizability is a concern. However, FKS subtraction might be the answer.
- Finish the implementation of V+1 jets using the FBPS generator.