Partial Compositeness and The Fermion/Sfermion Mass Hierarchy

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Outline

Motivations for the Model

Fermion/Sfermion Mass Hierarchy

Gaugino and Gravitino Masses

Parameter Space for Fermion/Sfermion Mass Hierarchy

Motivations for the Model

Minimal Supersymmetric Standard Model

$\frac{m_t}{m_e} \simeq 3.5 \times 10^5$ however $\frac{m_{\tilde{t}}}{m_{\tilde{e}}} = ?$

Experimental constraints $\Longrightarrow$ $\frac{m_{\tilde{t}}}{m_{\tilde{e}}} \sim 10^{-2}$

Fermion/Sfermion Mass Hierarchy

Supersymmetric UV scale Lagrangian:

$$\mathcal{L}_\Phi = \left[\Phi^\dagger\Phi\right]_D + \frac{1}{\Lambda_{\rm UV}^{\delta-1}}\left(\left[\Phi\mathcal{O}^c\right]_F + h.c.\right)$$

G. Cacciapaglia et al. (2009)

Renormalized supersymmetric Lagrangian at IR scale:

$$\begin{aligned}\mathcal{L}_\Phi = &\left[\Phi^\dagger\Phi\right]_D + \left[\Phi^{c(1)\dagger}\Phi^{c(1)}\right]_D + \left[\Phi^{(1)\dagger}\Phi^{(1)}\right]_D \\ &+\varepsilon_\Phi\Lambda_{\rm IR}\left(\left[\Phi\Phi^{c(1)}\right]_F + h.c.\right) + m^{(1)}\left(\left[\Phi^{(1)}\Phi^{c(1)}\right]_F + h.c.\right)\end{aligned}$$

Fermion/Sfermion Mass Hierarchy

Wave function renormalization:

$$\mu \frac{d\widetilde{\varepsilon}_\Phi}{d\mu} = (\delta - 1)\widetilde{\varepsilon}_\Phi + \zeta_\Phi \frac{N}{16\pi^2}\widetilde{\varepsilon}_\Phi^3$$

Elementary field - composite operator mixing:

$$\varepsilon_\Phi \equiv \widetilde{\varepsilon}_\Phi(\Lambda_{\rm IR})\frac{\sqrt{N}}{4\pi} = \frac{1}{\sqrt{Z_\Phi}}\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{\delta-1}\frac{\sqrt{N}}{4\pi} \simeq \frac{1}{\sqrt{\zeta_\Phi}}\sqrt{\frac{\delta - 1}{\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{2(1-\delta)} - 1}}$$

Fermion Mass Hierarchy

Fermionic part of the Lagrangian at IR scale:

$$\mathcal{L}_{fermion} = i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + i\psi^{\dagger(1)}\bar{\sigma}^\mu\partial_\mu\psi^{(1)} + i\psi^{\dagger c(1)}\bar{\sigma}^\mu\partial_\mu\psi^{c(1)}$$
$$-\varepsilon_\Phi\Lambda_{\rm IR}\left(\psi\psi^{c(1)} + h.c.\right) - m^{(1)}\left(\psi^{(1)}\psi^{c(1)} + h.c.\right)$$

R. Contino and A. Pomarol (2004)

Diagonalizing the mass matrix, the massless state:

$$|\psi_0\rangle \simeq \mathcal{N}_\Phi\left\{|\psi\rangle - \frac{1}{g_\Phi^{(1)}\sqrt{\zeta_\Phi}}\sqrt{\frac{\delta-1}{\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{2(1-\delta)}-1}}\,|\psi^{(1)}\rangle\right\}$$

Fermion Mass Hierarchy

Electroweak breaking term with an elementary Higgs:

$$\lambda \psi_L \psi_R H$$

Fermion mass:

$$m_\psi \simeq \frac{\lambda}{\sqrt{Z_L Z_R}} \frac{v}{\sqrt{2}} \mathcal{N}_\Phi^2 \simeq \begin{cases} \frac{\lambda}{\zeta_\Phi}(\delta - 1)\frac{16\pi^2}{N}\frac{v}{\sqrt{2}} & \delta \geq 1, \\ \frac{\lambda}{\zeta_\Phi}(1-\delta)\frac{16\pi^2}{N}\frac{v}{\sqrt{2}}\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{2(1-\delta)} & 0 \leq \delta < 1, \end{cases}$$

Sfermion Mass Hierarchy

Solving for F terms, the scalar part of the Lagrangian at IR scale:

$$\mathcal{L}_{scalar} = -\partial_\mu \phi^\dagger \partial^\mu \phi - \varepsilon_\Phi^2 \Lambda_{\rm IR}^2 \phi^\dagger \phi - \varepsilon_\Phi g_\Phi^{(1)} \Lambda_{\rm IR}^2 (\phi^\dagger \phi^{(1)} + h.c.)$$
$$-\partial_\mu \phi^{(1)\dagger} \partial^\mu \phi^{(1)} - m^{(1)2} \phi^{(1)\dagger} \phi^{(1)} - \partial_\mu \phi^{c(1)\dagger} \partial^\mu \phi^{c(1)} - \left(g_\Phi^{(1)2} + \varepsilon_\Phi^2 \right) \Lambda_{\rm IR}^2 \phi^{c(1)\dagger} \phi^{c(1)}$$

Diagonalizing the mass matrix, the massless state:

$$|\phi_0\rangle \simeq \mathcal{N}_\Phi \left\{ |\phi\rangle - \frac{1}{g_\Phi^{(1)} \sqrt{\zeta_\Phi}} \sqrt{\frac{\delta - 1}{\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{2(1-\delta)} - 1}} |\phi^{(1)}\rangle \right\}$$

Sfermion Mass Hierarchy

SUSY breaking term with a composite X:

$$\xi_4 \frac{g_\Phi^{(1)2}}{\Lambda_{\rm IR}^2}\left[\mathcal{X}^\dagger \mathcal{X} \Phi^{(1)\dagger}\Phi^{(1)}\right]_D \quad \Longrightarrow \quad \xi_4 g_\Phi^{(1)2} \frac{|F_\mathcal{X}|^2}{\Lambda_{\rm IR}^2} \phi^{(1)\dagger}\phi^{(1)}$$

Sfermion mass:

$$\widetilde{m}^2 \simeq \mathcal{N}_\Phi^2 \varepsilon_\Phi^2 \xi_4 \frac{|F_\mathcal{X}|^2}{\Lambda_{\rm IR}^2} \simeq \begin{cases} \frac{(\delta-1)}{\zeta_\Phi}\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{2(\delta-1)} \xi_4 \frac{|F_\mathcal{X}|^2}{\Lambda_{\rm IR}^2} & \delta \geq 1 \,, \\ \frac{(1-\delta)}{\zeta_\Phi} \xi_4 \frac{|F_\mathcal{X}|^2}{\Lambda_{\rm IR}^2} & 0 \leq \delta < 1 \,, \end{cases}$$

Gaugino Mass

Supersymmetric UV scale Lagrangian:

$$\mathcal{L}_V = \left(\frac{1}{4}\left[W^\alpha W_\alpha\right]_F + h.c.\right) + 2\tilde{\varepsilon}_V\left[V\mathcal{J}\right]_D$$

Renormalized supersymmetric Lagrangian at IR scale:

$$\mathcal{L}_V = \left(\frac{1}{4}\left[W^\alpha W_\alpha\right]_F + \frac{1}{4}\left[W^{(1)\alpha} W^{(1)}_\alpha\right]_F + h.c.\right) + \Lambda_{\mathrm{IR}}^2\left[\left(\varepsilon_V V + g_V^{(1)} V^{(1)} + \frac{\Phi_V^{(1)} + \Phi_V^{(1)\dagger}}{\sqrt{2}\Lambda_{IR}}\right)^2\right]_D$$

Gaugino Mass

Diagonalizing the mass matrix, the massless state:

$$|\lambda_0\rangle \simeq \mathcal{N}_V \left\{ |\lambda\rangle - \frac{1}{g_V^{(1)}\sqrt{2\zeta_V \log\left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)}} |\lambda^{(1)}\rangle \right\}$$

Gaugino mass:

$$\frac{\xi_3}{2}\frac{g_V^{(1)2}}{\Lambda_{IR}}\left([\mathcal{X}W^{\alpha(1)}W_\alpha^{(1)}]_F + h.c.\right) \quad \Longrightarrow \quad M_\lambda = \mathcal{N}_V^2 \varepsilon_V^2 \xi_3 \frac{F_\mathcal{X}}{\Lambda_{IR}} \simeq g^2 \xi_3 \frac{F_\mathcal{X}}{\Lambda_{IR}}$$

Gravitino Mass

Supersymmetric UV scale Lagrangian:

$$\mathcal{L}_H = \frac{4}{3}\left[H_\mu E^\mu\right]_D + \frac{2\tilde{\varepsilon}_H}{\Lambda_{\rm UV}}\left[H_\mu \Theta^\mu\right]_D$$

Renormalized supersymmetric Lagrangian at IR scale:

$$\mathcal{L}_H = \frac{4}{3}\left[H_\mu E^\mu\right]_D + \frac{4}{3}\left[H_\mu^{(1)} E^{(1)\mu}\right]_D + 2\Lambda_{IR}^2\left[\left(\varepsilon_H H^\mu + g_H^{(1)} H_\mu^{(1)}\right)^2\right]_D$$

Gravitino Mass

Diagonalizing the mass matrix, massless state:

$$|\psi_{\mu 0}\rangle \simeq \mathcal{N}_H \left\{ |\psi_\mu\rangle - \frac{1}{g_H^{(1)}\sqrt{\zeta_H}} \frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}} \, |\psi_\mu^{(1)}\rangle \right\}$$

Gravitino mass:

$$-\frac{1}{4}\frac{W}{M_P^2}\psi_\rho \left[\sigma^\mu, \bar{\sigma}^\rho\right] \psi_\mu + h.c. \quad \Longrightarrow \quad m_{3/2} \simeq \xi_3 \frac{F_\chi}{\sqrt{3}M_P}$$

Fermion/Sfermion Mass Hierarchy

$$\frac{y_e}{y_t} \simeq \frac{1-\delta_e}{\delta_t - 1}\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{2(1-\delta_e)}$$

$$\frac{m^2_{\tilde{e}}}{m^2_{\tilde{t}}} = \frac{1-\delta_e}{\delta_t - 1}\left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{2(1-\delta_t)} \qquad \delta_t < \delta_t^* ,$$

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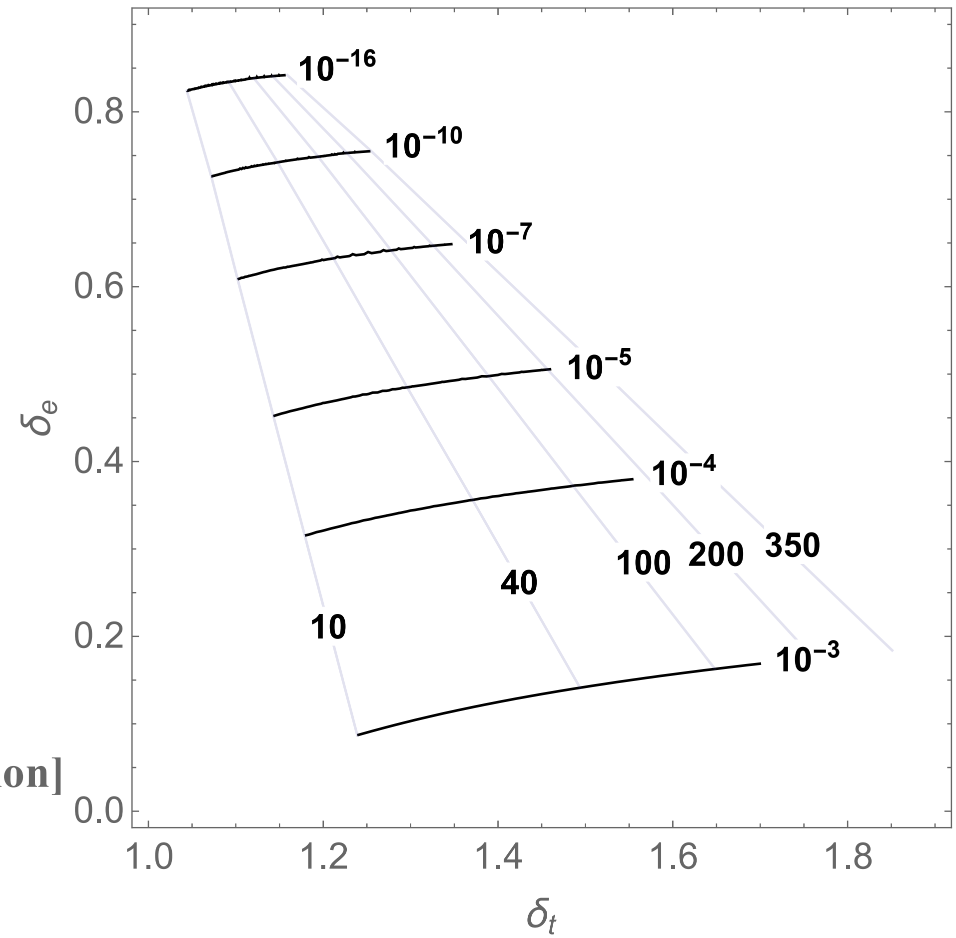

Conclusion

The model explains the fermion/sfermion mass hierarchies ✓

The results match the 5D results as conjectured by AdS/CFT correspondence ✓