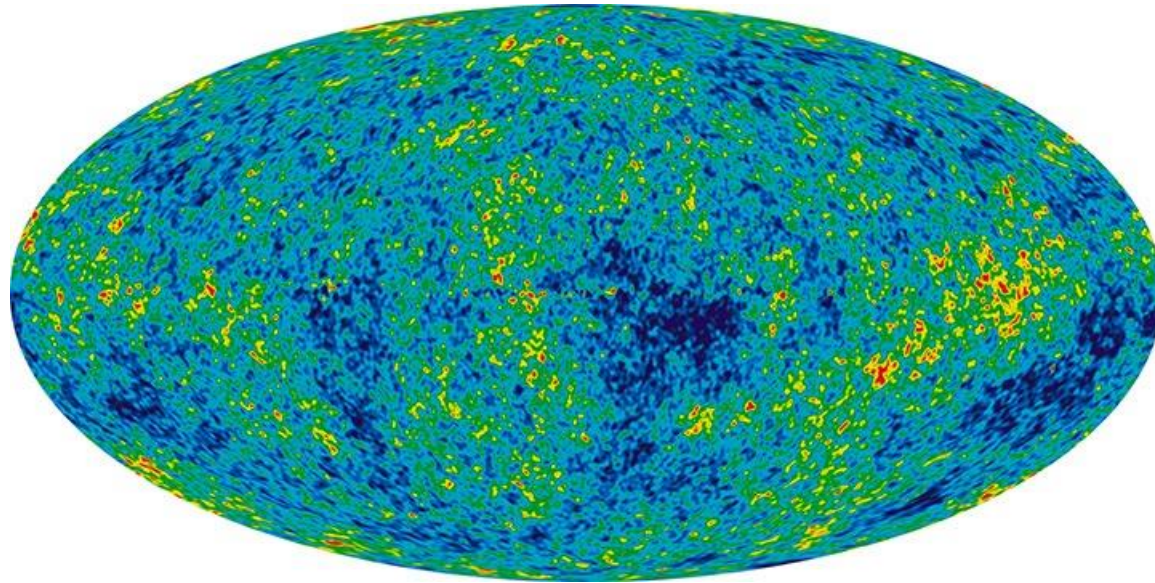


Dark Energy Constraints from Expansion Rate and Density Fluctuation Data

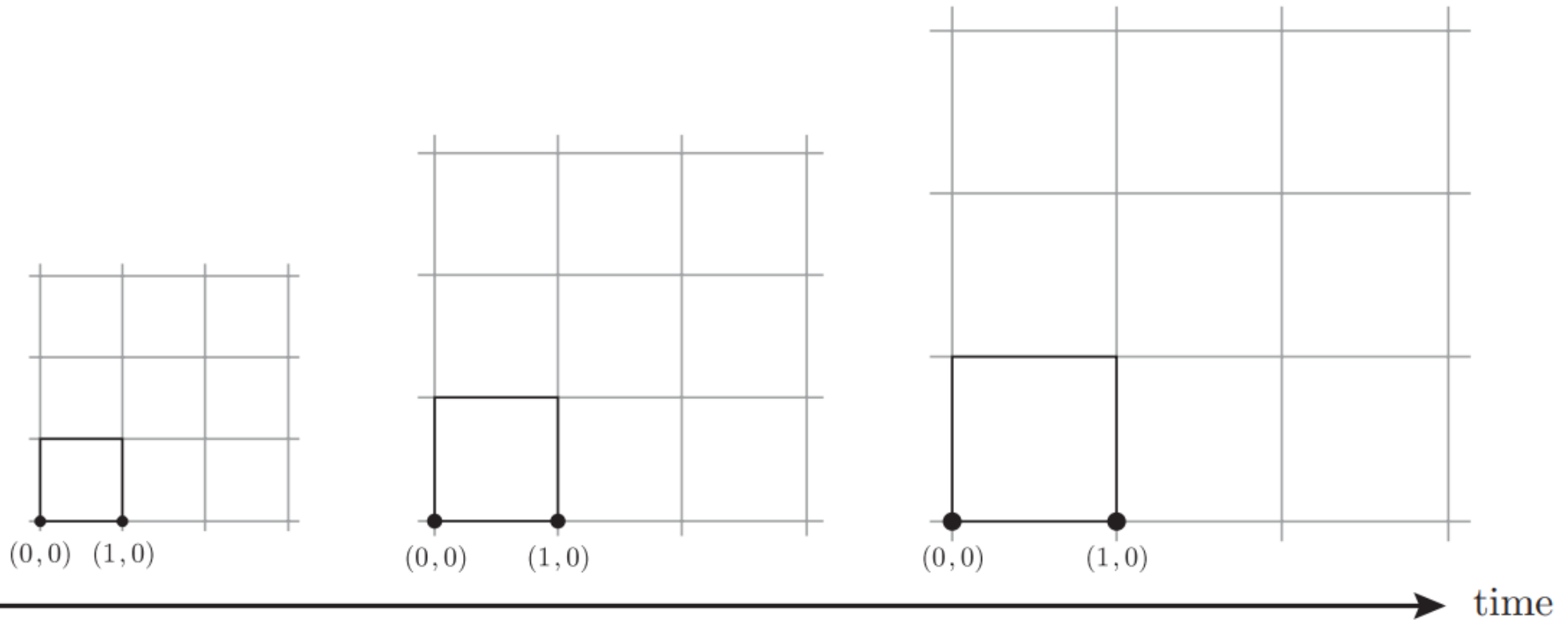


Joseph Ryan, Sanket Doshi, and Bharat Ratra

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The Expanding Universe (1/2)

.The universe is expanding.



Source: *Cosmology*, Daniel Baumann

The Expanding Universe (2/2)

•Physical distance x_p changes with time:

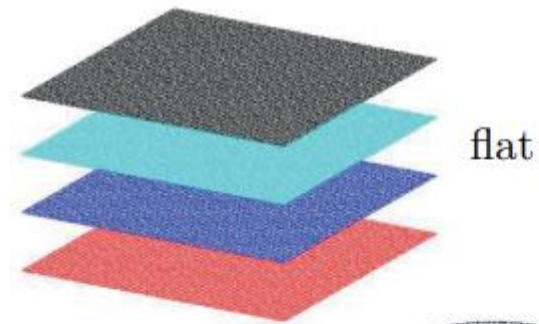
$$x_p = a(t)x$$

• $a(t)$ is the scale factor:

$$\frac{a(t)}{a_0} = \frac{1}{1+z}$$

The Curved Universe

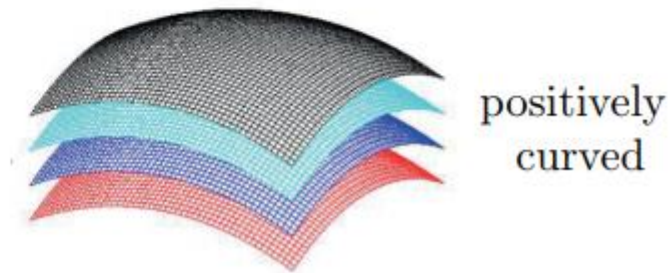
• $k = 0$



• $k = -1$

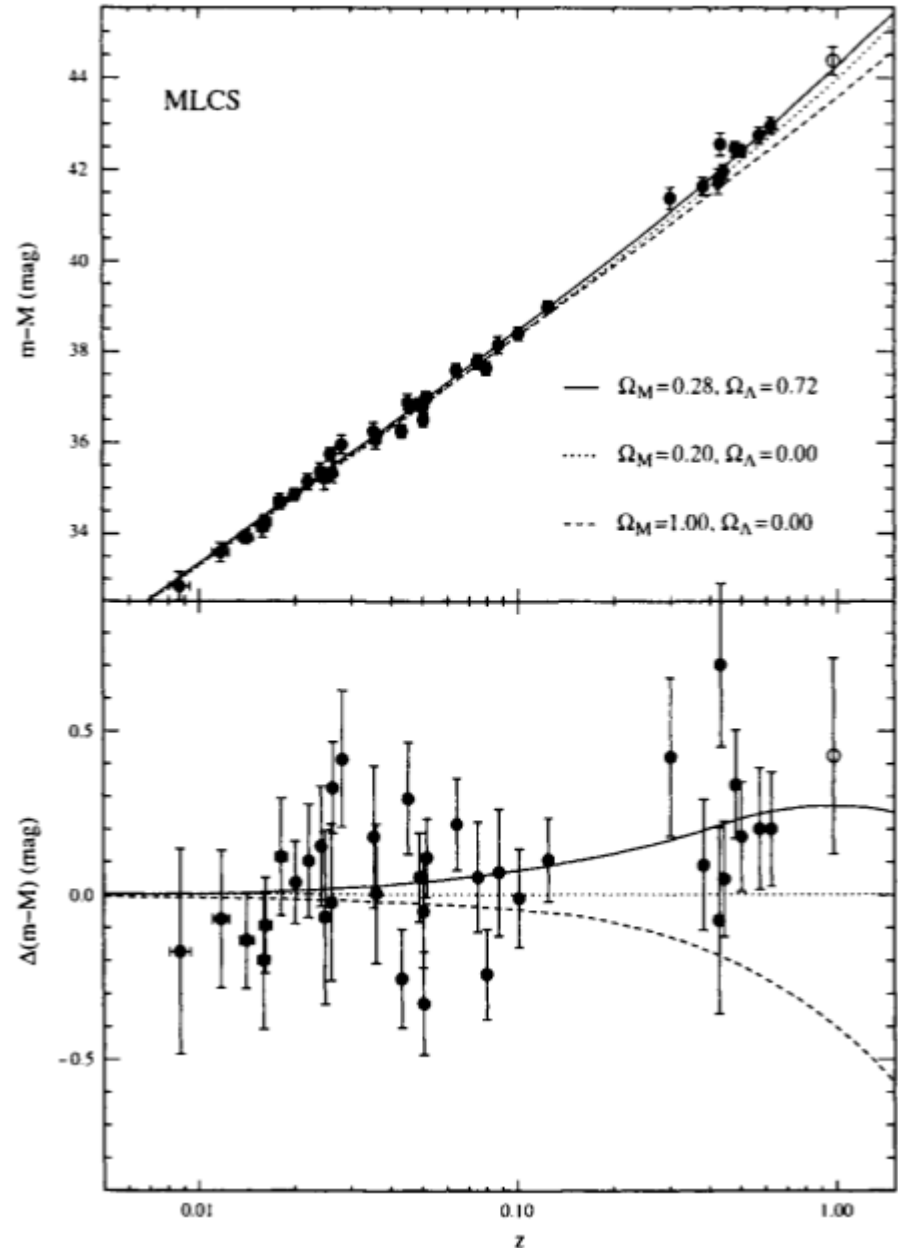


• $k = +1$



The Accelerating Universe

- This expansion is accelerating today.
- Why?



The Friedmann equations

•The scale factor evolves by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad ; \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

•Where

$$H(t) \equiv \frac{\dot{a}}{a} \quad ; \quad \rho = \rho_m + \rho_\gamma + \rho_{DE}$$

Three Dark Energy Models (1/2)

.Define:

$$\frac{p}{\rho} = w$$

. Λ CDM (constant; has $w = -1$)

.XCDM (time-evolving; has $w \neq -1$)

. ϕ CDM (time-evolving scalar field with $w = w(t)$)

ϕ CDM (1/2)

• Assume a homogeneous scalar field

$$\phi = \phi(t)$$

• with energy density

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V$$

• and potential

$$V(\phi) \propto \phi^{-\alpha}$$

ϕ CDM (2/2)

•The field evolves by

$$\ddot{\phi} + \frac{3\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$

•and the scale factor evolves by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} (\rho_m + \rho_\phi) - \frac{k}{a^2}$$

Three Dark Energy Models (2/2)

• Λ CDM:

$$E(z) = \sqrt{\Omega_{m0} (1+z)^3 + \Omega_{k0} (1+z)^2 + \Omega_{\Lambda}}$$

•XCDM:

$$E(z) = \sqrt{\Omega_{m0} (1+z)^3 + \Omega_{k0} (1+z)^2 + \Omega_{X0} (1+z)^{3(1+w_x)}}$$

• ϕ CDM:

$$E(z) = \sqrt{\Omega_{m0} (1+z)^3 + \Omega_{k0} (1+z)^2 + \Omega_{\phi}(z, \alpha)}$$

Data

.We collected 42 data points in total.

.31 $H(z)$ measurements from refs. [4] and [5]

.11 BAO measurements from refs. [6]-[11]

4.) Omer Farooq et al., "Hubble Parameter measurement constraints on the redshift of the deceleration-acceleration transition, dynamical dark energy, and space curvature", 15 November 2016. arXiv:1607.03537v2.

5.) A.L. Ratsimbazafy et al., "Age-dating luminous red galaxies observed with the Southern African Large Telescope", *MNRAS*, **467** (3), 1 June 2017.

6.) Shadab Alam et al., "The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample", 11 July 2016. arXiv: 1607.03155v1.

7.) Ashley J. Ross et al., "The Clustering of the SDSS DR7 Main Galaxy Sample I: A 4 per cent Distance Measure at $z = 0.15$ ", 21 January 2015. arXiv:1409.3242v2.

8.) Metin Ata et al., "The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: First measurement of Baryon Acoustic Oscillations between redshift 0.8 and 2.2", 21 May 2017.

9.) Julian E. Bautista et al., "Measurement of BAO correlations at $z = 2.3$ with SDSS DR12 Ly α -Forests", 27 March 2017. arXiv:1702.00176v2.

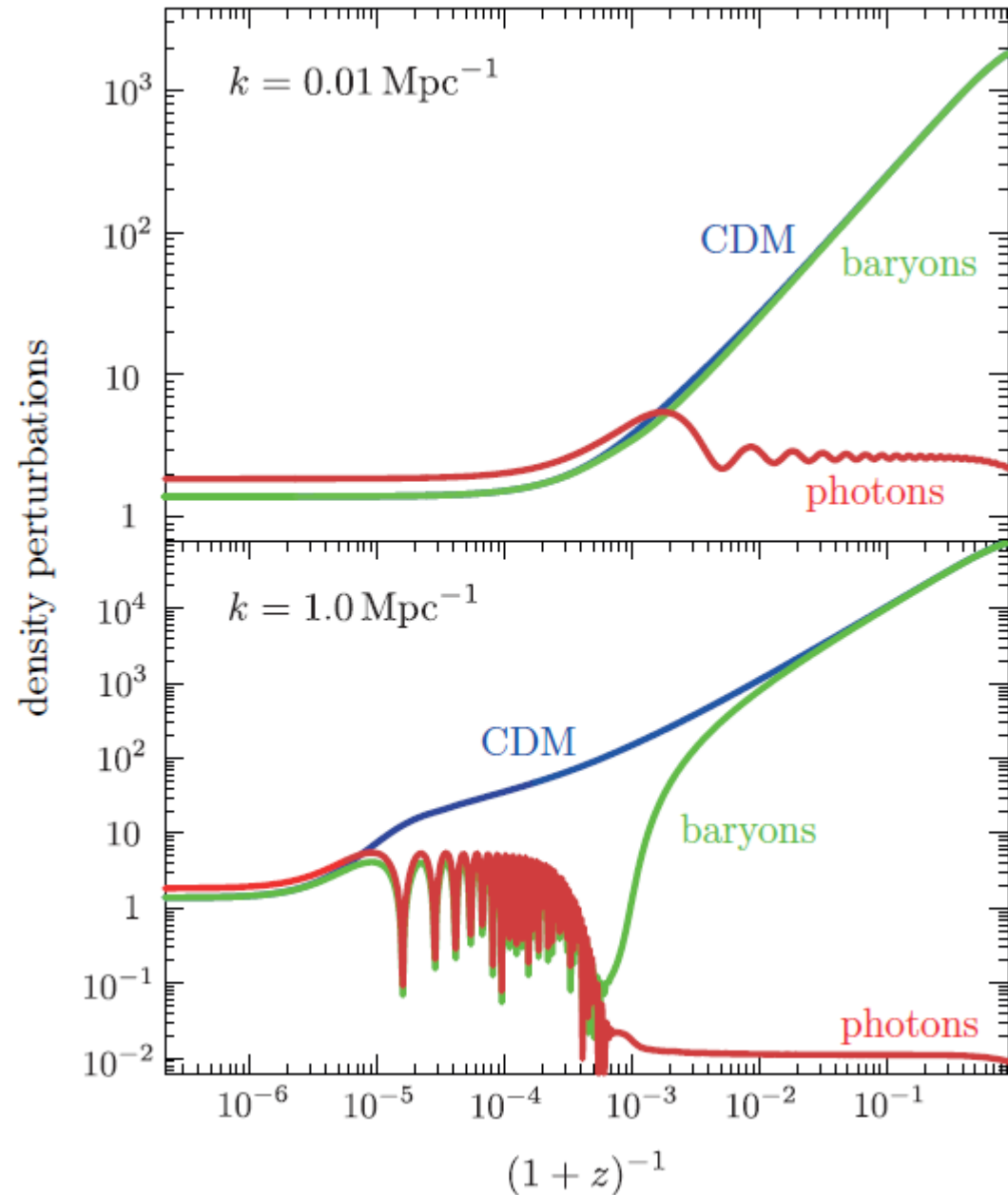
10.) Florian Beutler, et al., "The 6dF Galaxy Survey: Baryon Acoustic Oscillations and the Local Hubble Constant", 16 June 2011. arXiv:1106.3366v1.

11.) Andreu Font-Ribera, et al., "Quasar-Lyman α Forest Cross-Correlation from BOSS DR11: Baryon Acoustic Oscillations", 20 May 2014. arXiv:1311.1767v2.

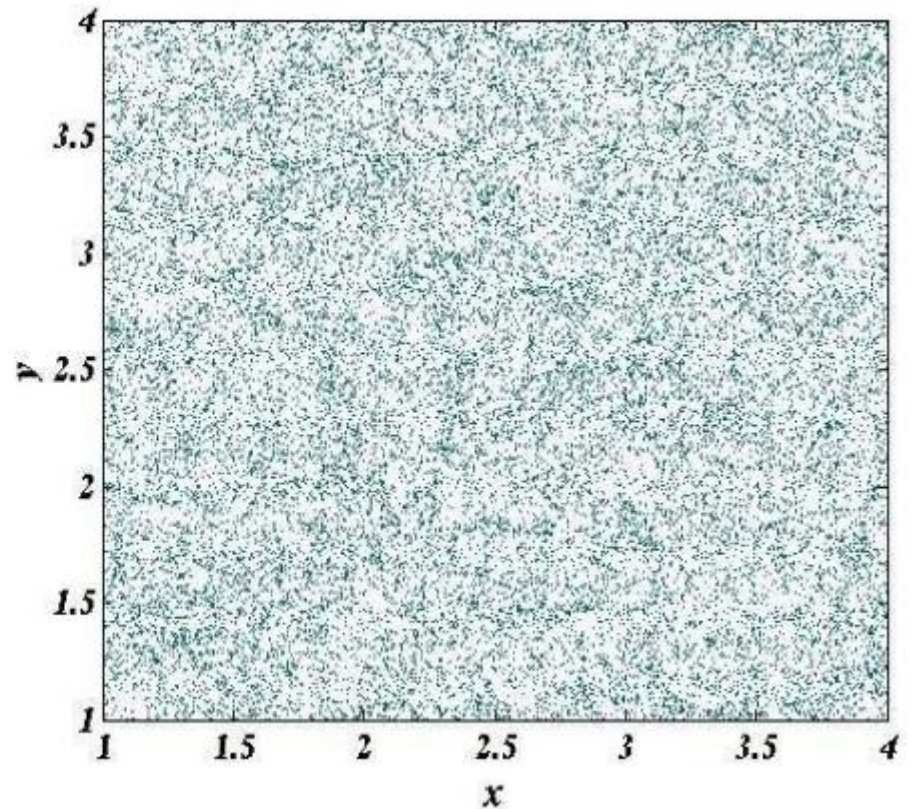
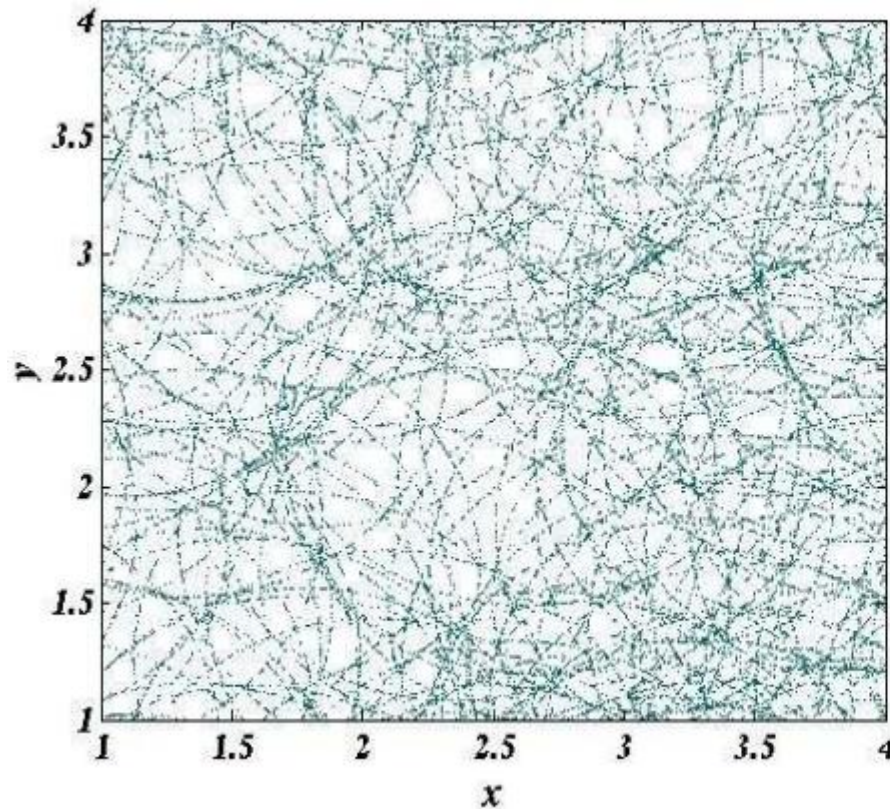
BAO (theory)

•Acoustic oscillations in coupled photon-baryon fluid

•Apparent in CMB and matter distribution



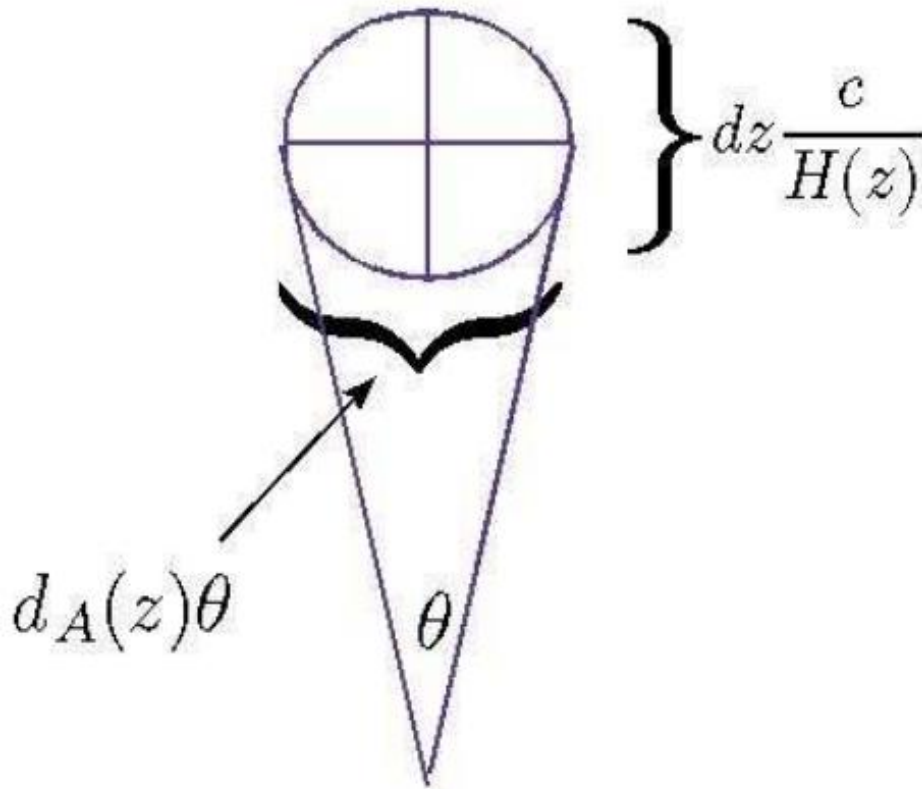
BAO in galaxy distribution (schematic)



Source: “Baryon Acoustic Oscillations”, arXiv: 0910.5224v1

Distances from BAO

.Can use D_A and $1/H(z)$ to calculate other distances: D_M , D_V , D_H



BAO (data)

• r_s = sound horizon distance

$$r_s(t) = \int_0^t c_s(\tau) d\tau$$

• $r_{s, \text{fid}}$ = fiducial sound horizon distance

z	Data point	Value	Uncertainty
0.38	$D_M \times (r_{s, \text{fid}}/r_s)$	1518	22
0.51	$D_M \times (r_{s, \text{fid}}/r_s)$	1977	27
0.61	$D_M \times (r_{s, \text{fid}}/r_s)$	2283	32
0.38	$H(z) \times (r_s/r_{s, \text{fid}})$	81.5	1.9
0.51	$H(z) \times (r_s/r_{s, \text{fid}})$	90.4	1.9
0.61	$H(z) \times (r_s/r_{s, \text{fid}})$	97.3	2.1
0.106	r_s/D_V	0.336	0.015
0.15	$D_V \times (r_{s, \text{fid}}/r_s)$	664	25
1.52	$D_V \times (r_{s, \text{fid}}/r_s)$	3855	170
2.33	$\frac{(D_H)^{0.7}(D_M)^{0.3}}{r_s}$	13.94	0.35
2.36	$(1/H(z)) \times (c/r_s)$	9.0	0.3

H(z)

•Recall:

$$H(z) = H_0 E(z)$$

•E(z) can be calculated;
H₀ is marginalized

z	$H(z)$	σ_H
0.070	69	19.6
0.090	69	12
0.120	68.6	26.2
0.170	83	8
0.179	75	4
0.199	75	5
0.200	72.9	29.6
0.270	77	14
0.280	88.8	36.6
0.352	83	14
0.3802	83	13.5
0.400	95	17
0.4004	77	10.2
0.4247	87.1	11.2
0.4497	92.8	12.9
0.470	89	50

z	$H(z)$	σ_H
0.4783	80.9	9
0.480	97	62
0.593	104	13
0.680	92	8
0.781	105	12
0.875	125	17
0.880	90	40
0.900	117	23
1.037	154	20
1.300	168	17
1.363	160	33.6
1.430	177	18
1.530	140	14
1.750	202	40
1.965	186.5	50.4

Data Analysis (preliminaries, 1/2)

• χ^2

• Uncorrelated:

$$\chi_A^2(\vec{p}) = \sum_{i=1}^N \frac{[A_{th}(\vec{p}) - A_{obs}(\vec{p})]^2}{\sigma_i^2}$$

• Correlated:

$$\chi_A^2(\vec{p}) = [\vec{A}_{th}(\vec{p}) - \vec{A}_{obs}(\vec{p})]^T \mathcal{C}^{-1} [\vec{A}_{th}(\vec{p}) - \vec{A}_{obs}(\vec{p})]$$

Data Analysis (preliminaries, 2/2)

•Likelihood function:

$$\mathcal{L}(\vec{p}) = e^{-\chi^2(\vec{p})/2}$$

•Marginalized likelihood:

$$\mathcal{L}(\vec{p}) = \int \mathcal{L}(\vec{p}, \nu) \pi(\nu) d\nu$$

Data Analysis (priors)

- Gaussian prior assumed for H_0 .

- Two cases (from ref. [4]):

- $H_0 = 68 \pm 2.8$ km/s/Mpc (1 pc = 3.26 ly)

- $H_0 = 73.24 \pm 1.74$ km/s/Mpc

$$\pi(H_0) = \frac{1}{\sqrt{2\pi\sigma_{H_0}^2}} \exp \left[\frac{-(H_0 - \bar{H}_0)^2}{2\sigma_{H_0}^2} \right]$$

- For all other parameters, flat prior assumed.

Data Analysis (Best Fit)

• Record parameters p_i that minimize:

$$\chi^2 = -2\ln \left[\int \mathcal{L}(\vec{p}, \nu) \pi(\nu) d\nu \right]$$

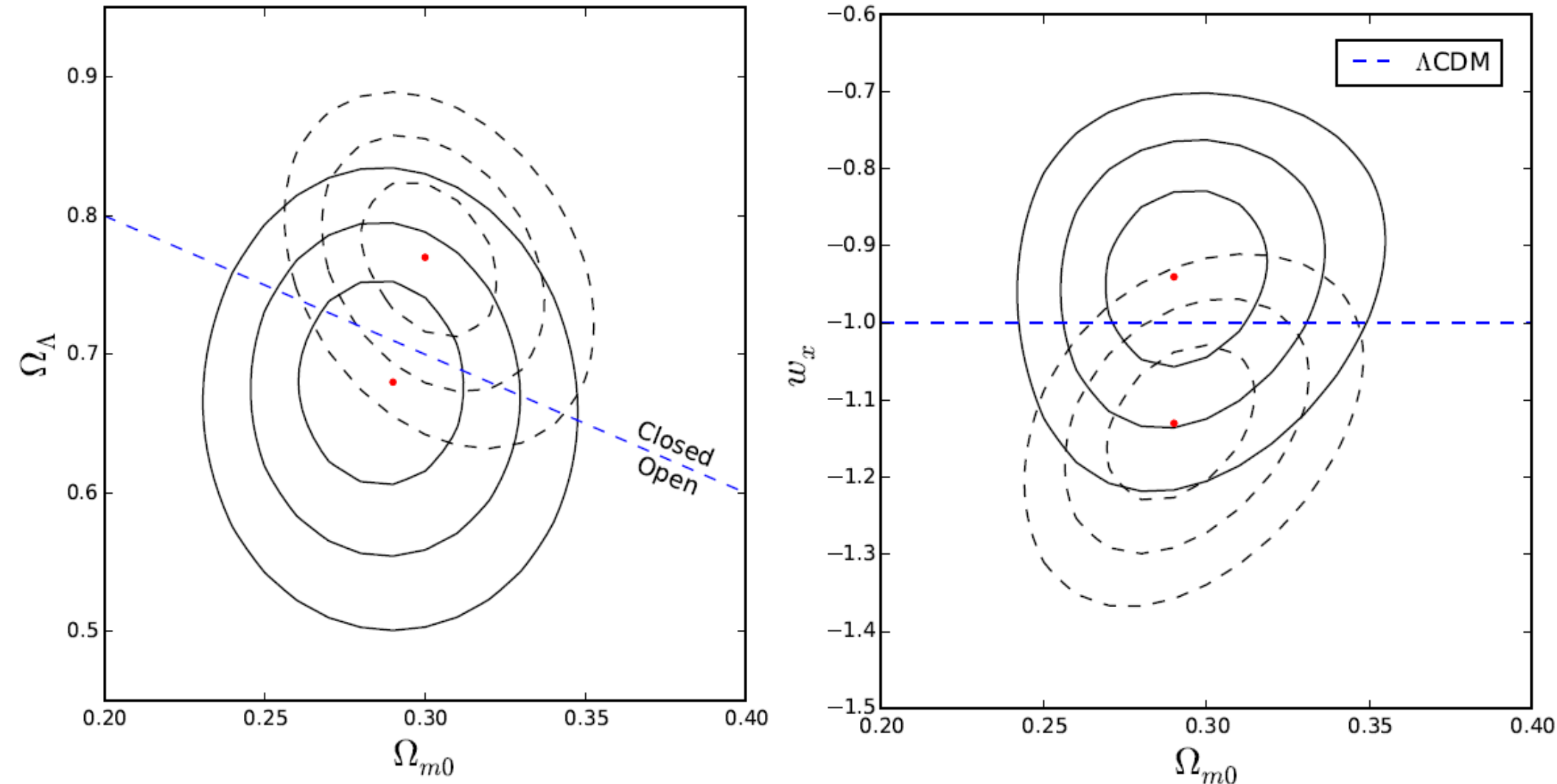
• these are the best fit parameters.

• If two models have different numbers of parameters and/or data points, use

$$AIC = \chi^2 + 2k \quad ; \quad BIC = \chi^2 + k \ln N$$

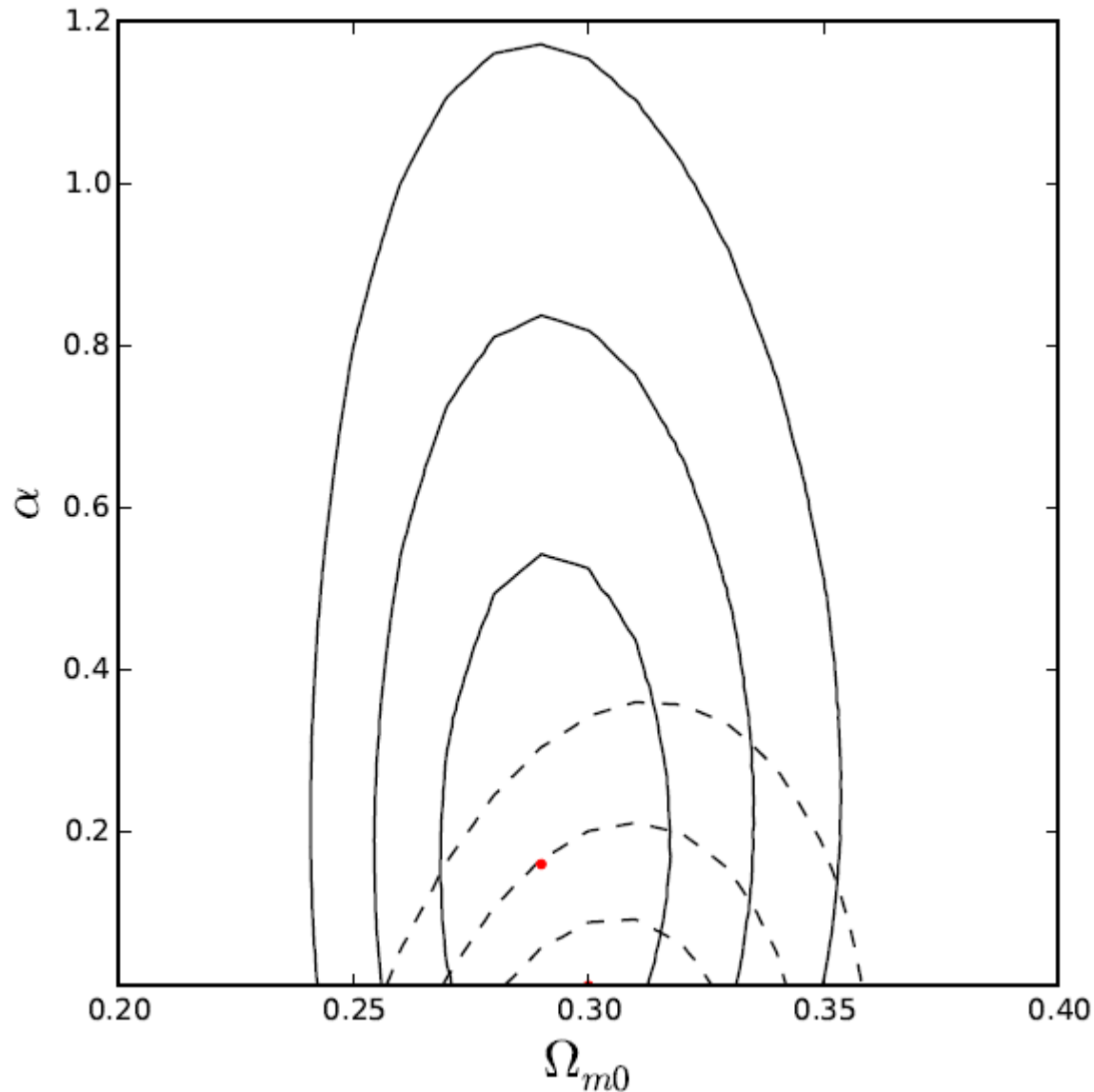
• $k = \#$ of parameters, $N = \#$ of data points.

Two-parameter Models (1/2)



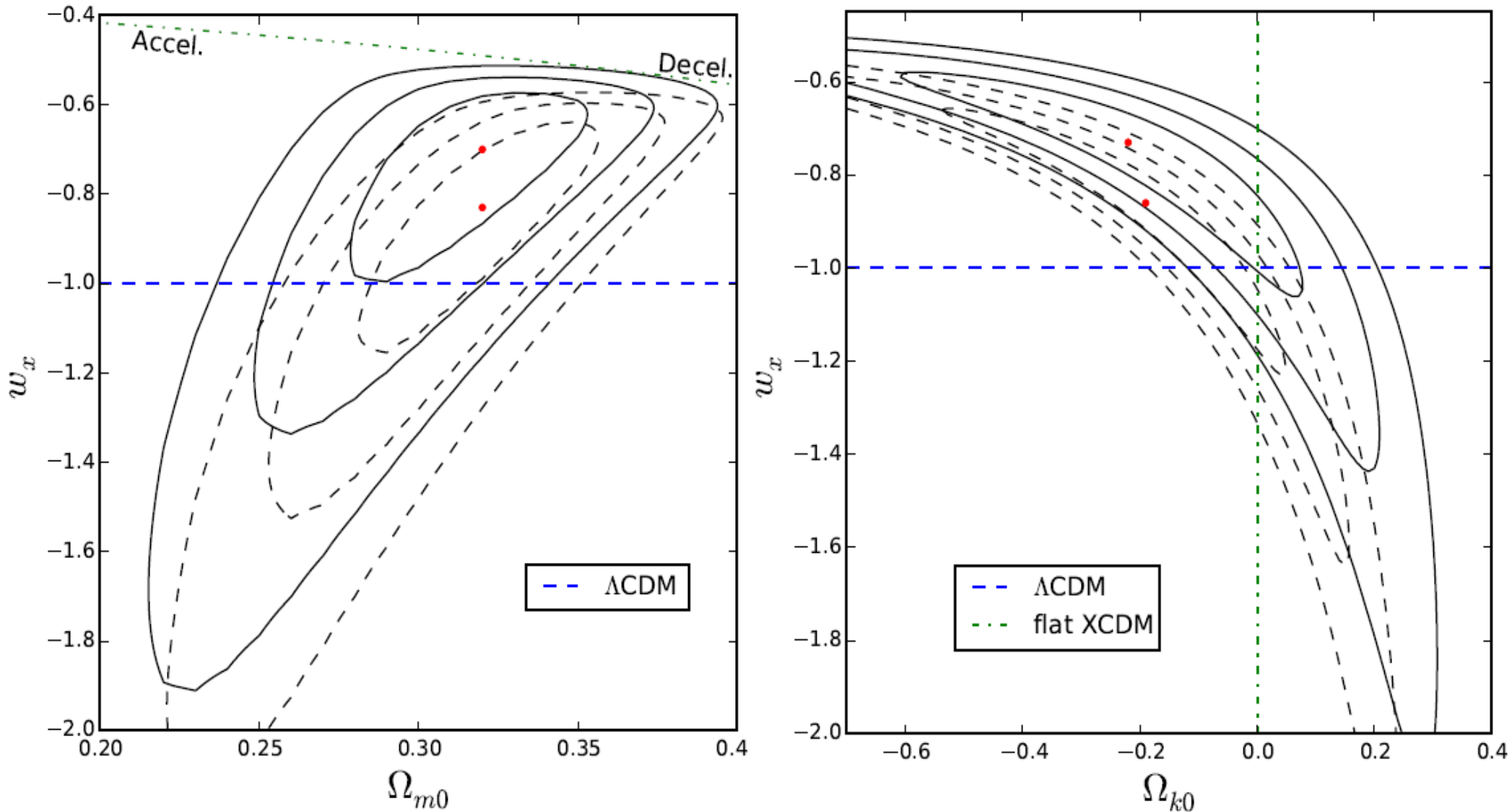
• Solid black: $H_0 = 68$ km/s/Mpc; Dashed black: $H_0 = 73.24$ km/s/Mpc

Two-parameter models (2/2)



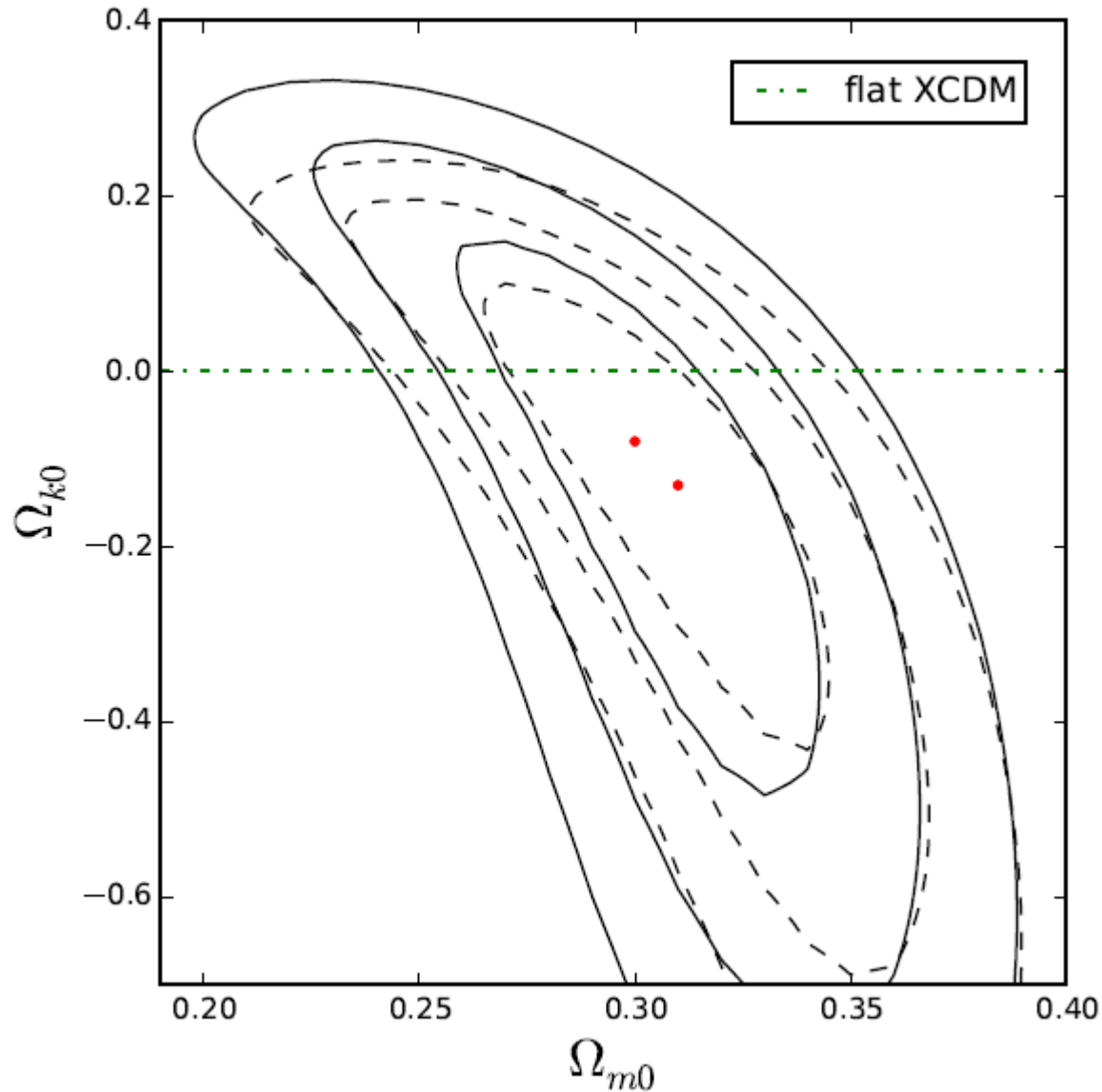
• Solid black: $H_0 = 68$ km/s/Mpc; Dashed black: $H_0 = 73.24$ km/s/Mpc

Three-parameter XCDM (1/2)



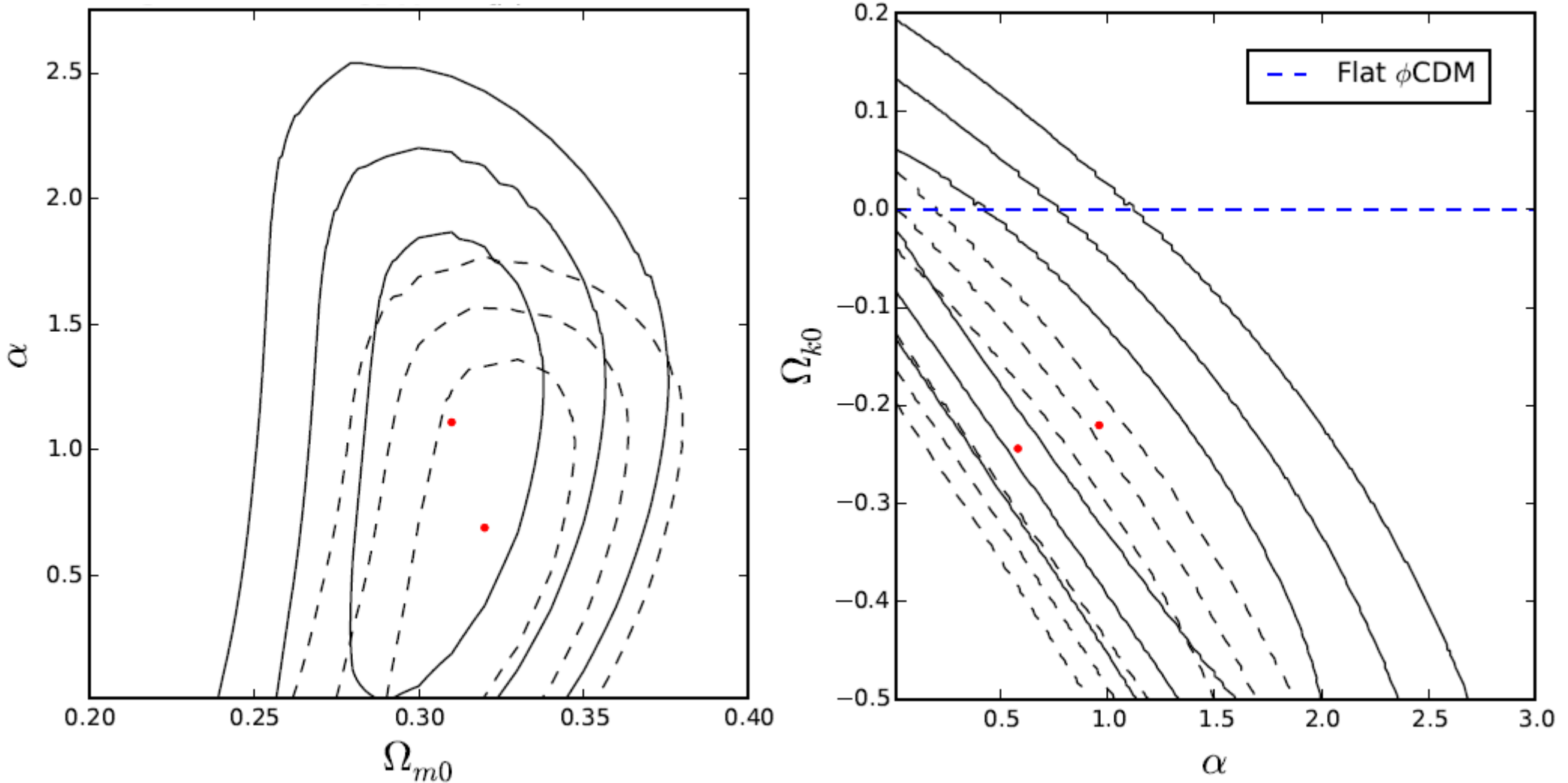
• Solid black: $H_0 = 68$ km/s/Mpc; Dashed black: $H_0 = 73.24$ km/s/Mpc

Three-parameter Λ CDM (2/2)



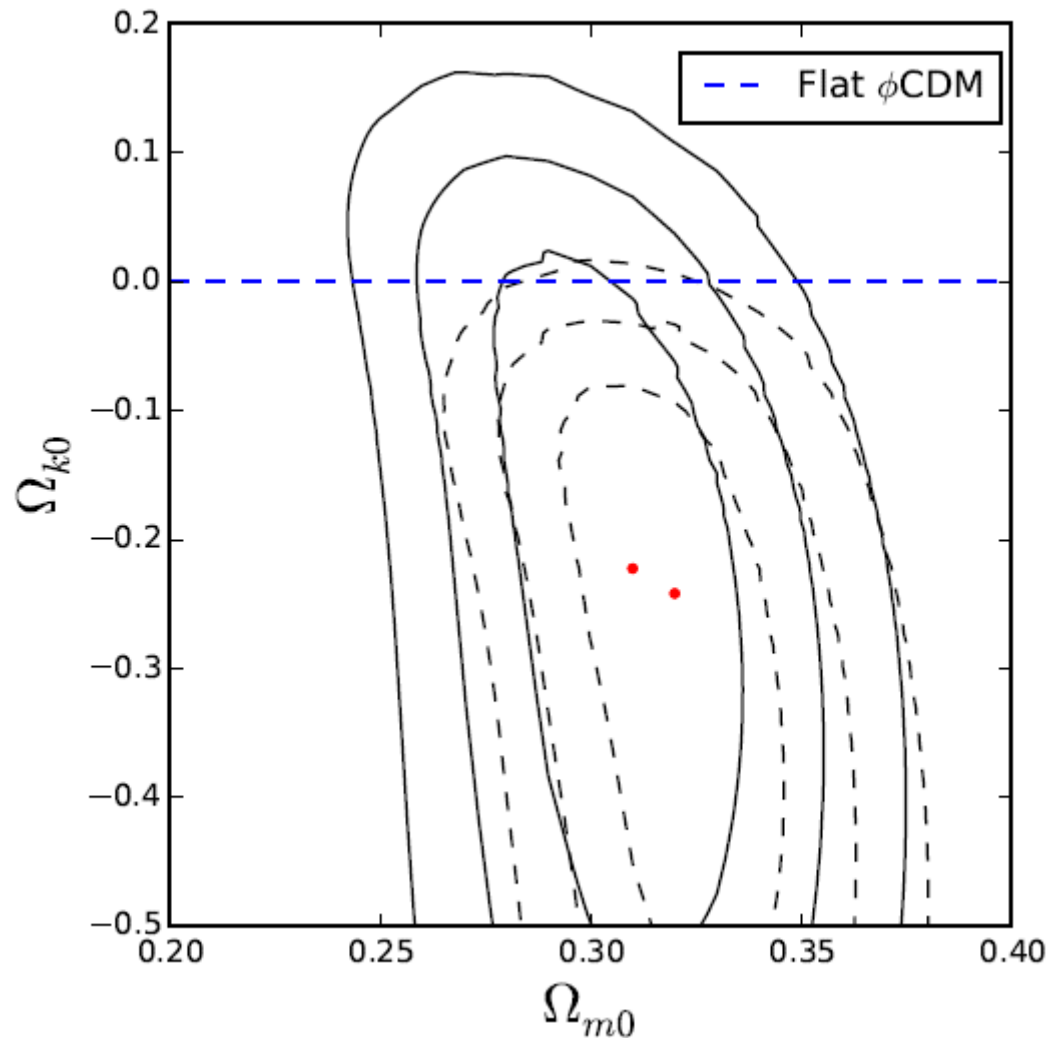
• Solid black: $H_0 = 68$ km/s/Mpc; Dashed black: $H_0 = 73.24$ km/s/Mpc

Three-parameter ϕ CDM (1/2)



• Solid black: $H_0 = 68$ km/s/Mpc; Dashed black: $H_0 = 73.24$ km/s/Mpc

Three-parameter ϕ CDM (2/2)



• Solid black: $H_0 = 68$ km/s/Mpc; Dashed black: $H_0 = 73.24$ km/s/Mpc

Best Fit Parameters

H_0 prior ($\text{km s}^{-1}\text{Mpc}^{-1}$)	Model	Ω_{m0}	$\Omega_{\Lambda 0}$	w_X	α	χ^2	$\Delta\chi^2$	AIC	BIC
68 ± 2.8	ΛCDM	0.29	0.68	–	–	25.35	0.00	29.35	32.83
	Flat XCDM	0.29	–	–0.94	–	25.04	–0.31	29.04	32.52
	Flat ϕCDM	0.29	–	–	0.16	25.05	–0.30	29.05	32.53
73.24 ± 1.74	ΛCDM	0.30	0.77	–	–	26.92	0.00	30.92	34.40
	Flat XCDM	0.29	–	–1.13	–	28.26	1.34	32.26	35.74
	Flat ϕCDM	0.30	–	–	0.01	32.62	5.70	36.62	40.10

H_0 prior ($\text{km s}^{-1}\text{Mpc}^{-1}$)	Model	Ω_{m0}	Ω_{k0}	w_X	α	χ^2	$\Delta\chi^2$	AIC	ΔAIC	BIC	ΔBIC
68 ± 2.8	XCDM	0.31	–0.18	–0.76	–	23.65	–1.70	29.65	0.30	34.86	2.03
	ϕCDM	0.31	–0.22	–	0.96	23.82	–1.53	29.82	0.47	35.03	2.20
73.24 ± 1.74	XCDM	0.32	–0.21	–0.84	–	26.48	–0.44	32.48	1.56	37.69	3.29
	ϕCDM	0.32	–0.26	–	0.62	26.30	0.95	32.30	1.38	37.51	3.11

Conclusion

- .The flat Λ CDM model is consistent with our data, although it does not always provide the best fit.
- .Our results are consistent with (but not conclusive proof of) mild dark energy dynamics and non-flat spatial hypersurfaces.

Acknowledgments

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.Thanks to Lado Samushia (KSU) and Omer Farooq (ERAU) for useful discussions. Thanks also to Tyler Mitchell (KSU) for help with computational resources.

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- 2.) *Cosmology*, Daniel Baumann.
- 3.) *Modern Cosmology*, Scott Dodelson.
- 4.) Omer Farooq et al., "Hubble Parameter measurement constraints on the redshift of the deceleration-acceleration transition, dynamical dark energy, and space curvature", 15 November 2016. arXiv:1607.03537v2.

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