The Mitchell Conference at Texas A&M University, May 21 2018

Loop-induced Single Top Partner Production and Its Exotic Decays

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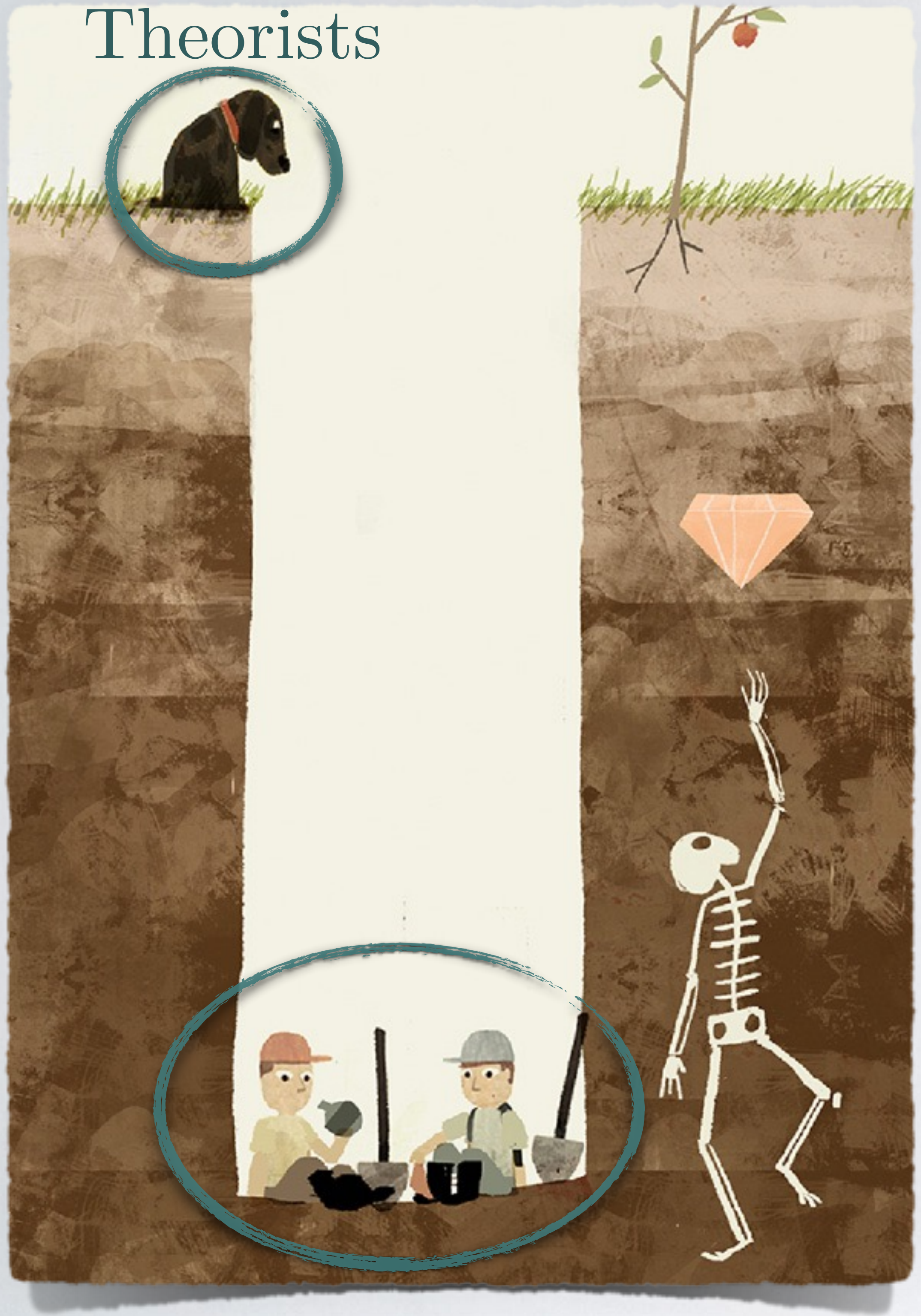
J. H. Kim and Ian M. Lewis [arXiv:1803.06351]

Haider Alhazmi, **J. H. Kim**, K. C. Kong, Ian M. Lewis [To appear soon]

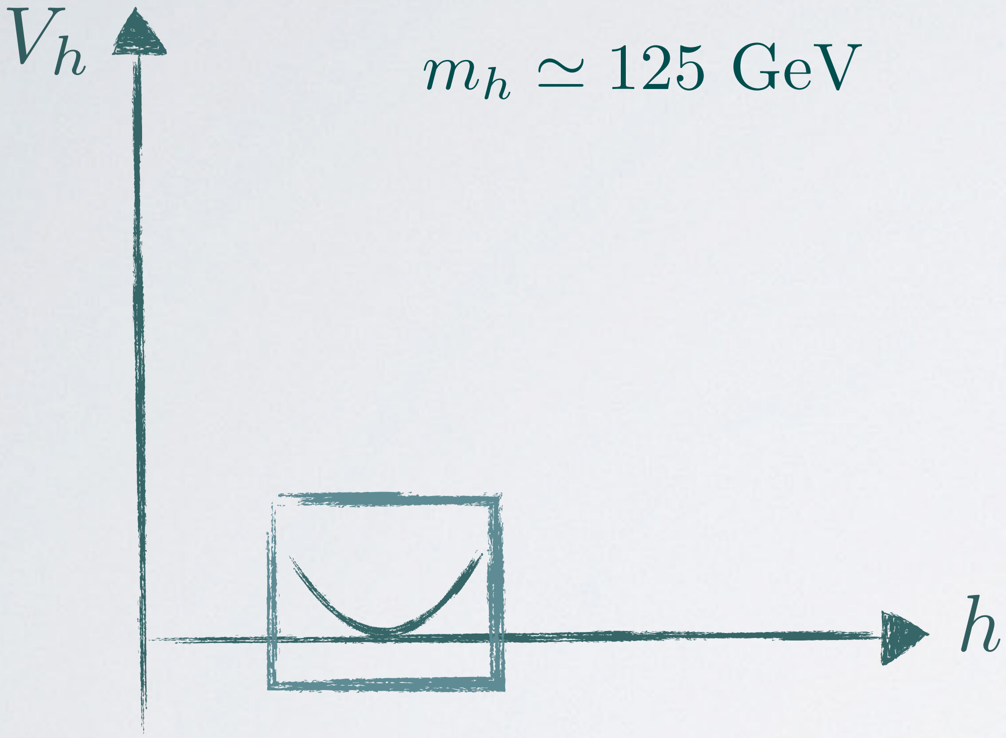
Outline


SAM & DAVE DIG A HOLE, Mac Barnett & Jon Klassen

- Motivation
- Previous studies & Constraints
- The model
- New decays and productions of top partners.
- New search strategies
- Conclusions

Motivation

$$V_h = \boxed{\frac{m_h^2}{2} h^2}$$



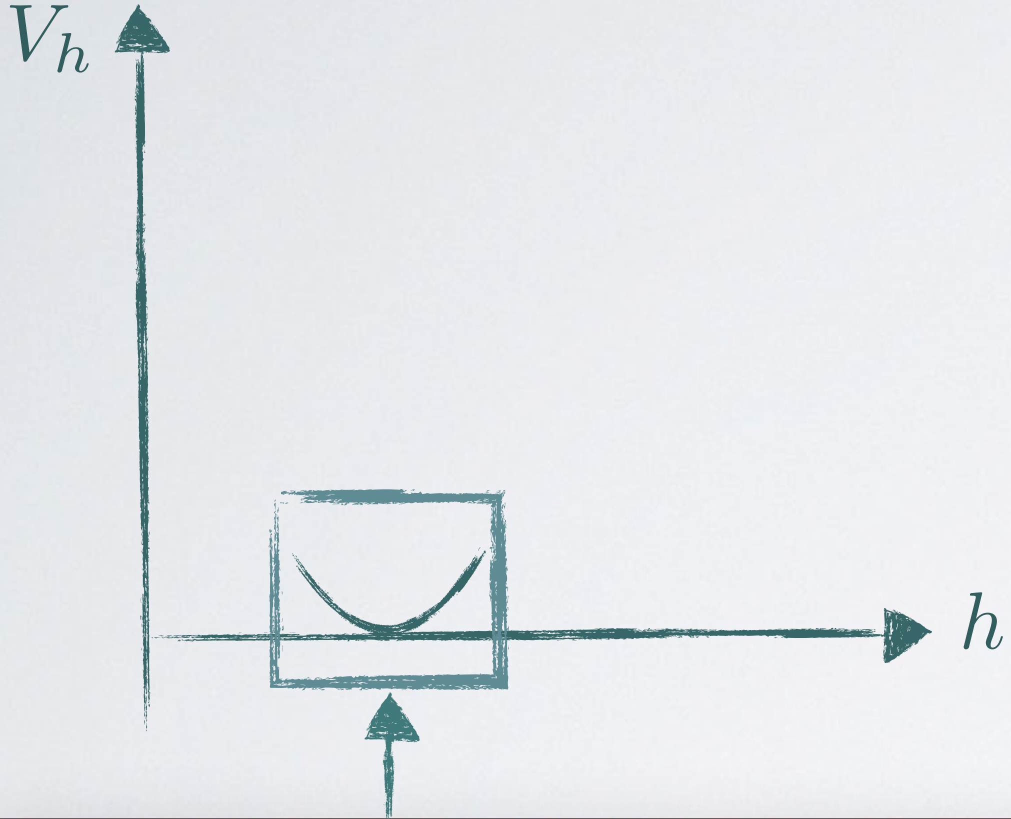
$m_h \simeq 125$ GeV

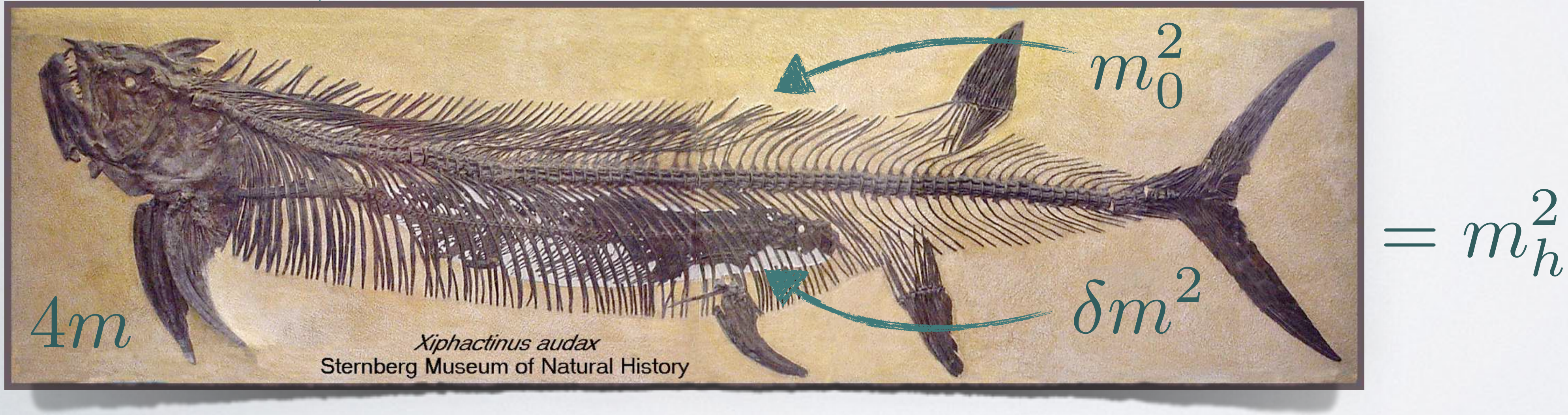
In a bottom-up approach

- One of the big pictures that we want to understand is the origin of the EWSB.
- The Higgs potential provides a backbone to access to the EWSB.
- And so far only the first term has been probed in a bottom-up approach.

Motivation

$$V_h = \boxed{\frac{m_h^2}{2} h^2}$$

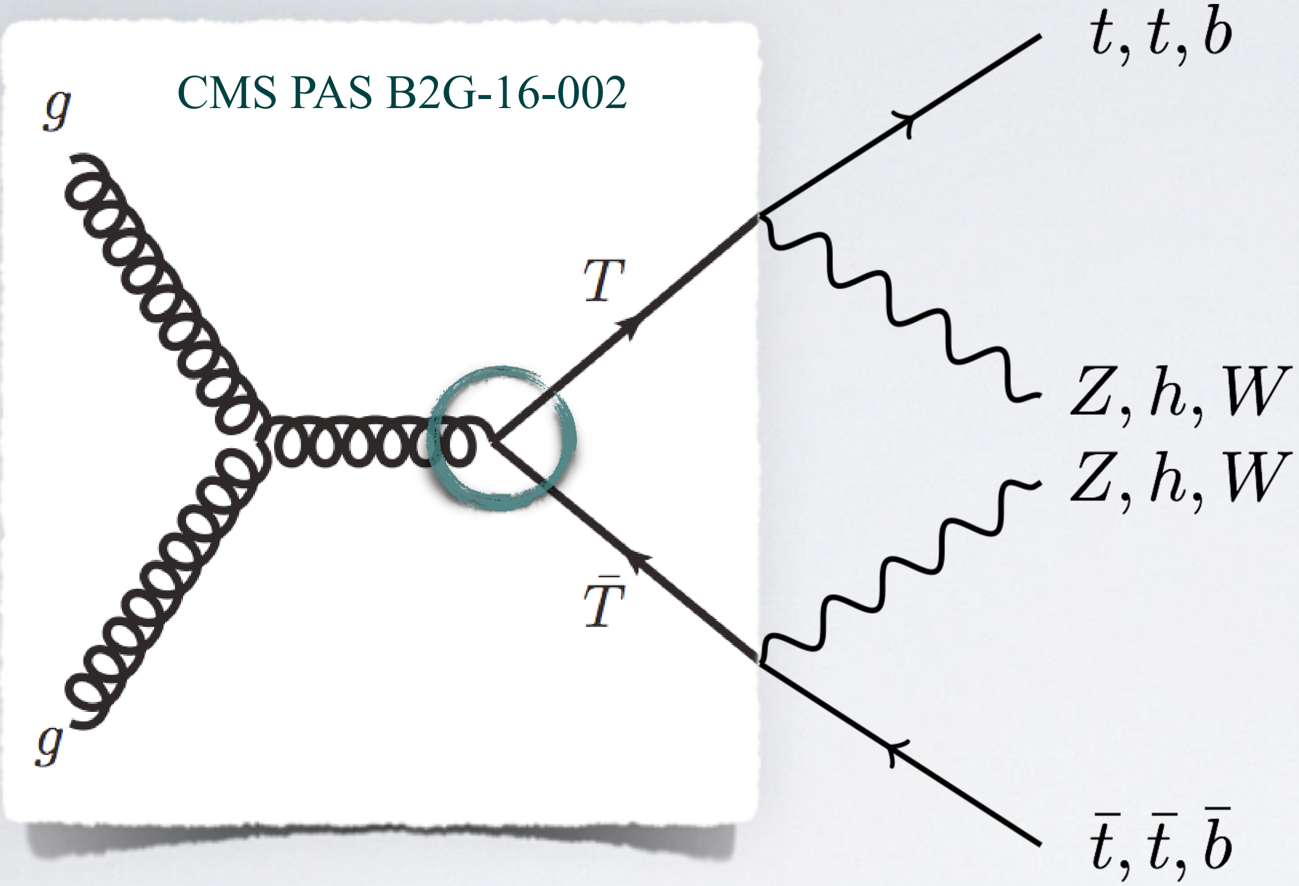


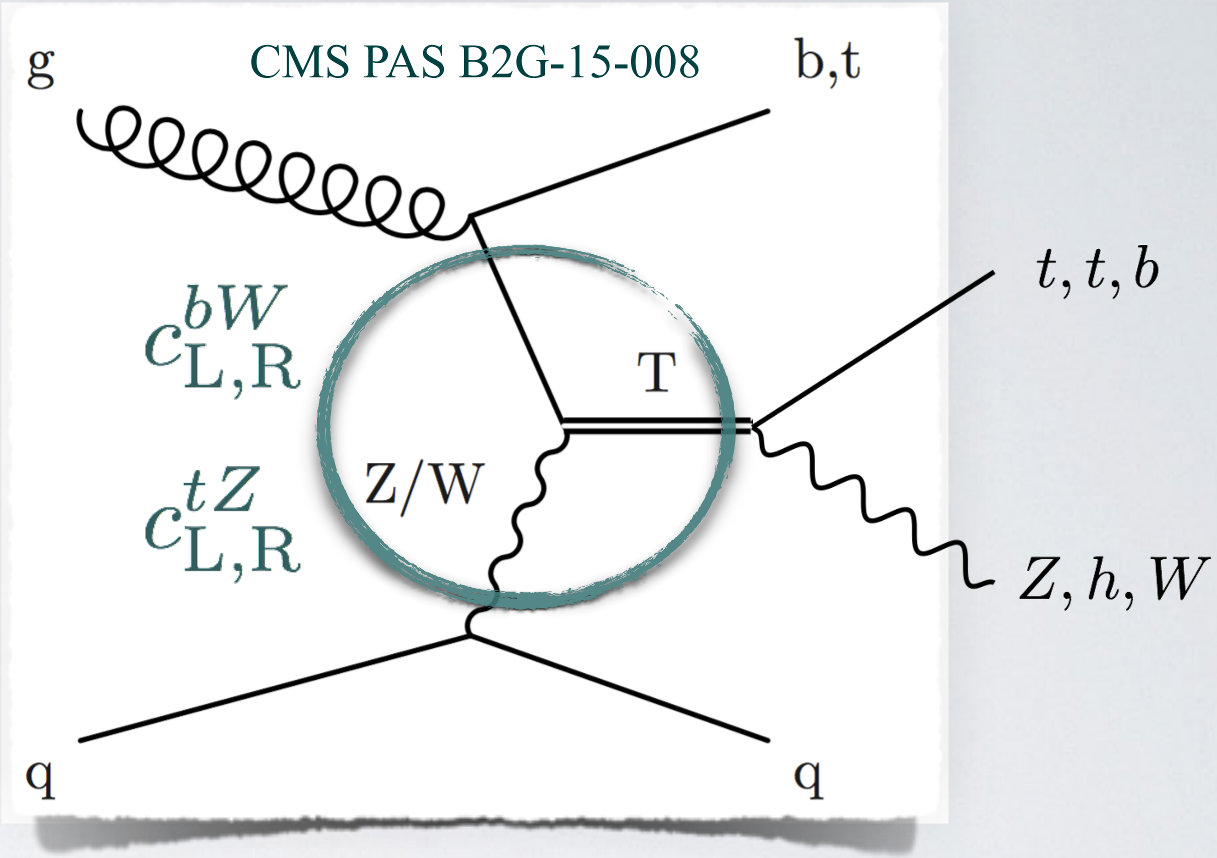


Sternberg Museum in Kansas

- If the SM is a fundamental theory all the way up to the planck scale, then m_h^2 suffers for a radiative instability.
- Historically, this fine-tuning problem has been thought of as a key corridor to understand the next layer of new physics.
- Vector-like top partners (T) are postulated to exist on course to solve the problem (e.g. composite Higgs and Little Higgs models).

Current direct searches for *T*



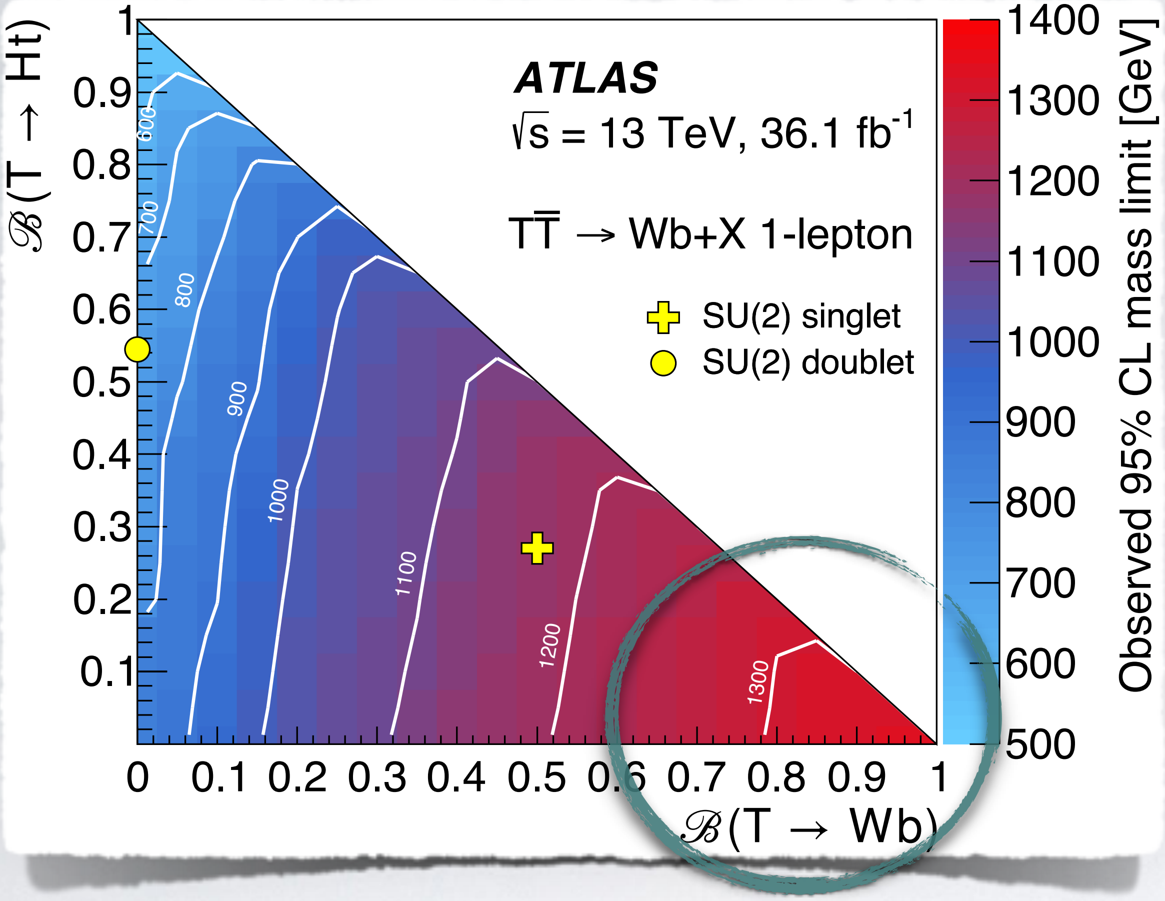


- Typically the *T* can be produced in pair or in single.
- The vertex responsible to create *T* in pair is the strong coupling.
- The single production is induced by EW coupings.
- Searches are restricted to *T* decays to *tZ*, *th* and *bW*.

Current Bounds on T

ATLAS, arXiv:1707.03347

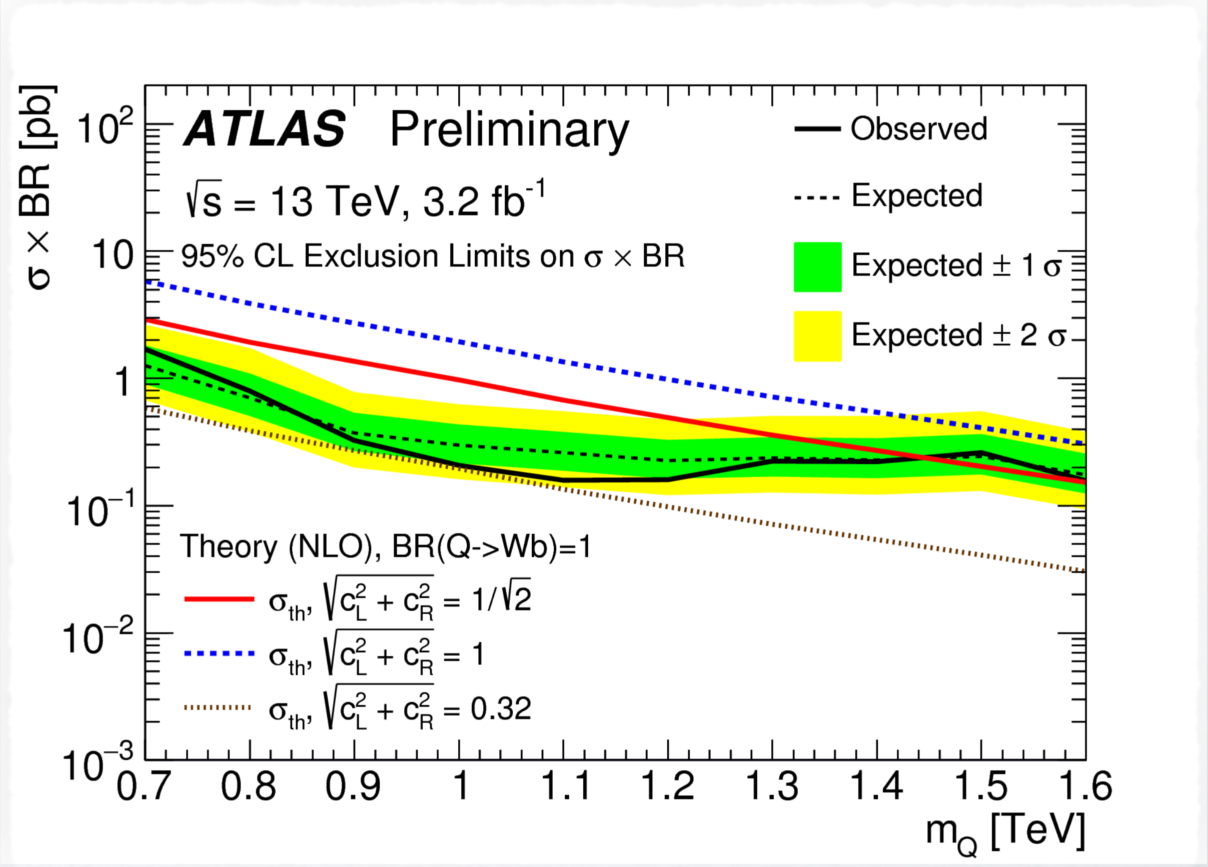
$T\overline{T} \to W^+bW^-\bar{b}$



$m_T \gtrsim 1.3$ TeV

$T \to W^+b$

ATLAS-CONF-2016-072



$m_T > 1 \sim 1.8$ TeV

- Recent bounds on T from the pair and single productions.
- The bounds keep increasing every year.
- A considerable amount of searches are still going on for various T decays to tZ, th and bW.

Absolutely nothing

Theorists?

SAM & DAVE DIG A HOLE
[Mac Barnett & Jon Klassen]

Experimentalists

- But, we found absolutely nothing in standard channels.
- Maybe our ideas do not apprear to be the way that a nature works.
- What are we going to do about it?
- How are we going to make a progress to understand the next layer of physics?

Exotic Productions & Decays

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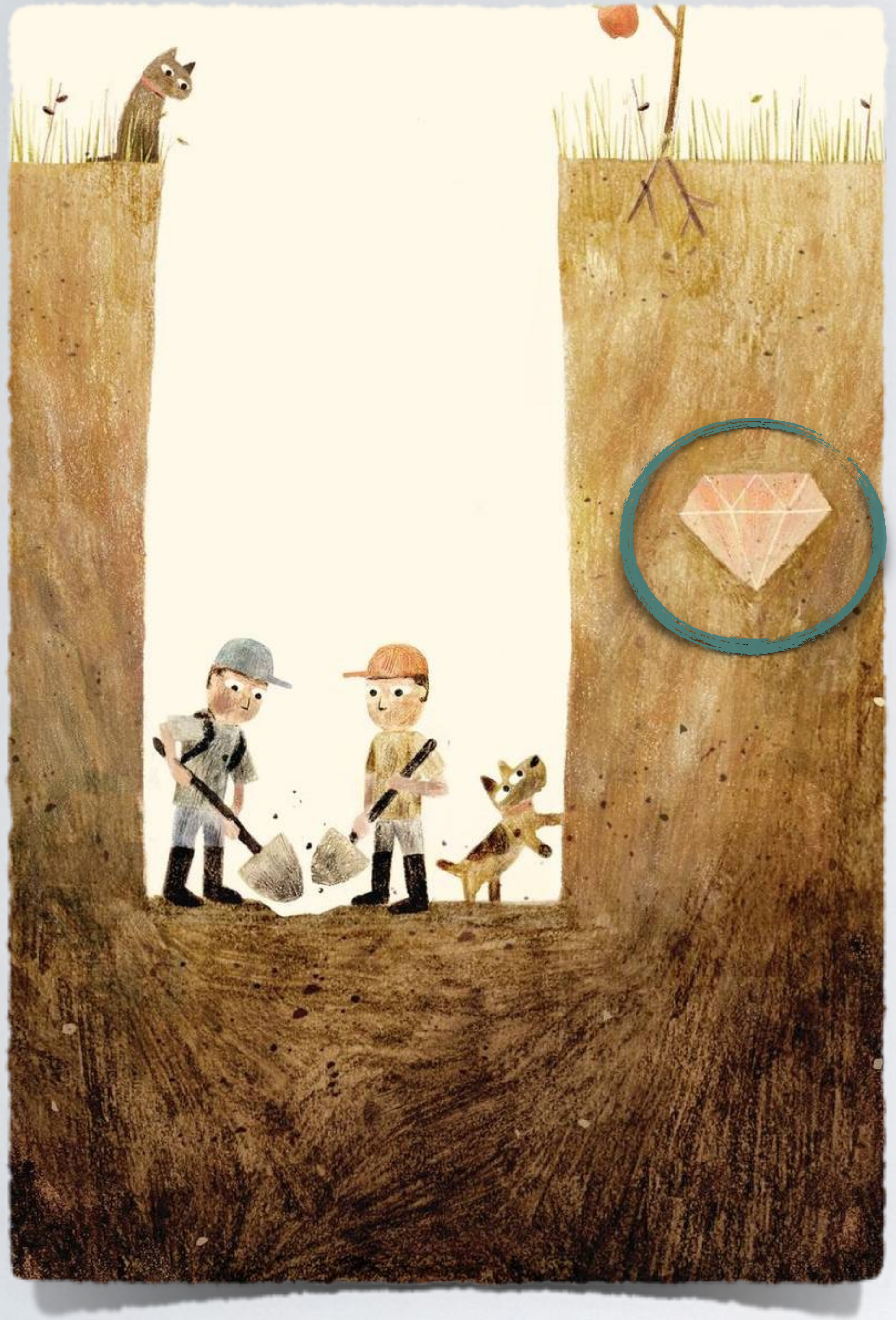


- The null results leaves open possibilities, since T can be hiding in exotic places.
- Maybe there are new decay modes we haven’t considered.
- Maybe there are new production channels we haven’t searched for.
- All of these new questions will chart new phenomenologies for T .

A minimal Lagrangian

- We consider a simplified Lagrangian with a SU(2)$_{\mathrm{L}}$ singlet T and an additional gauge singlet scalar S.

- A minimal set of interactions we identify consists of :

$$\mathcal{L}_{\mathrm{NP}} = \bar{T} i \not{D} T - m_2 \bar{T} T + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2} m_S^2 S^2 + \left(-\lambda_2 S \bar{T}_L T_R + \mathrm{h.c.} \right)$$

$$\mathcal{L}_{\mathrm{mix}} = -\left(\boxed{\lambda_t \bar{Q}_L \tilde{\Phi} T_R} + \lambda_1 S \bar{T}_L t_R + \mathrm{h.c.} \right)$$

Mixing

$$\mathcal{L}_m = -\begin{bmatrix} \bar{t}_L & \bar{T}_L \end{bmatrix} \begin{bmatrix} \frac{y_t v}{\sqrt{2}} & \boxed{\frac{\lambda_t}{\sqrt{2}} v} \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} t_R \\ T_R \end{bmatrix} + \mathrm{h.c.}$$

$$\not{D} = \not{\partial} - i g' \frac{2}{3} \not{B} - i g_s \not{G}$$

$$\Phi = \begin{pmatrix} -iG_p \\ \frac{1}{\sqrt{2}}(v + h + iG_0) \end{pmatrix}$$

$$Q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

- Allowing T to mix with a top quark.

Working in the mass eigenbasis

- The amount of mixing is dictated by sin θ_L after diagonalizing the mass matrix.

$$\begin{bmatrix} t_L \\ T_L \end{bmatrix} \longrightarrow \begin{bmatrix} \cos\theta_L & -\sin\theta_L \\ \sin\theta_L & \cos\theta_L \end{bmatrix} \begin{bmatrix} t_L \\ T_L \end{bmatrix}$$

$$\begin{bmatrix} t_R \\ T_R \end{bmatrix} \longrightarrow \begin{bmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{bmatrix} \begin{bmatrix} t_R \\ T_R \end{bmatrix}$$

$$M_D = \begin{bmatrix} m_t & 0 \\ 0 & m_T \end{bmatrix}$$

173GeV

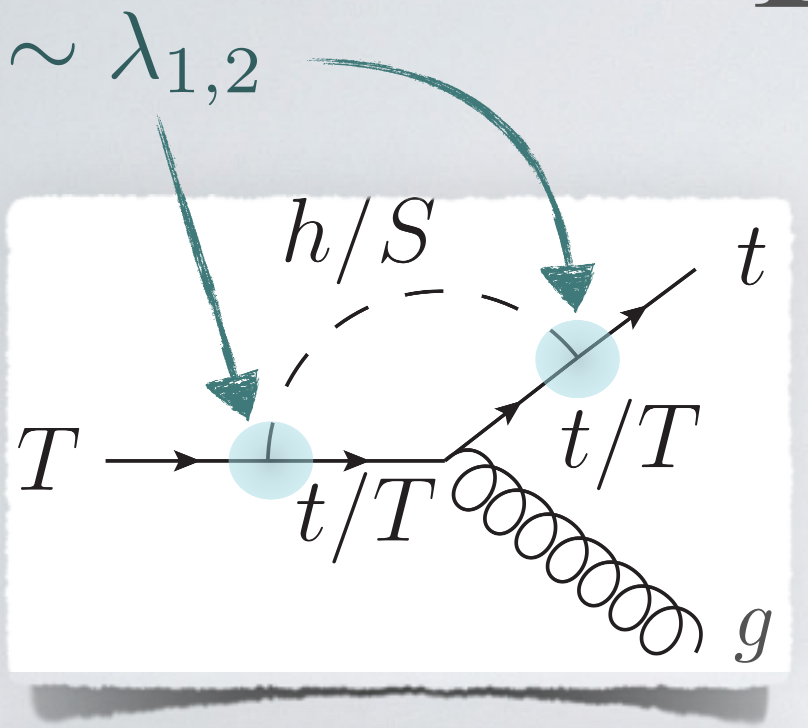
- The *S* and Higgs can mix as well, but for simplicity we will not introduce a mixing angle for the scalar sector.

- In this simplified model, we have 5 independent parameters.

$$\boxed{\lambda_1 \quad \lambda_2 \quad \sin\theta_L \quad m_T \quad m_S}$$

Exotic T decays

J. H. Kim, I. M. Lewis [2018]

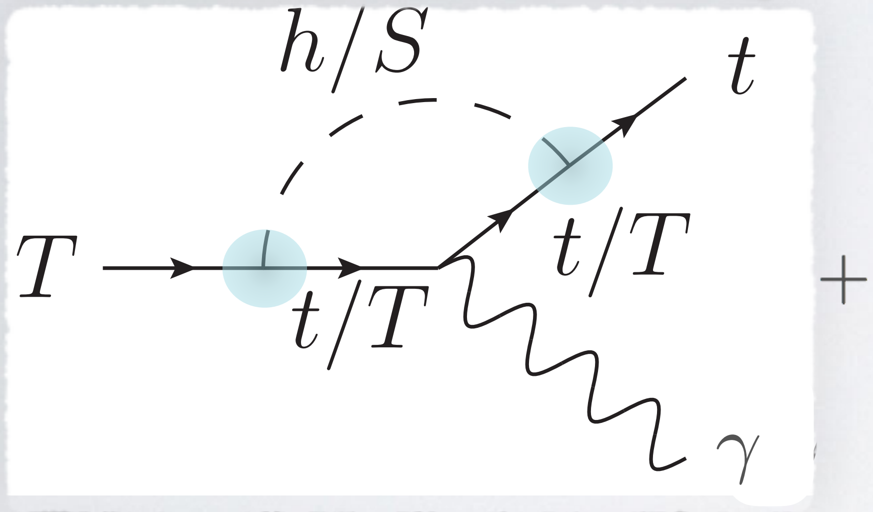


$+$ Counter terms

External self-energies

W and Z bosons loops

Goldstone bosons loops



$+$ Counter terms

External self-energies

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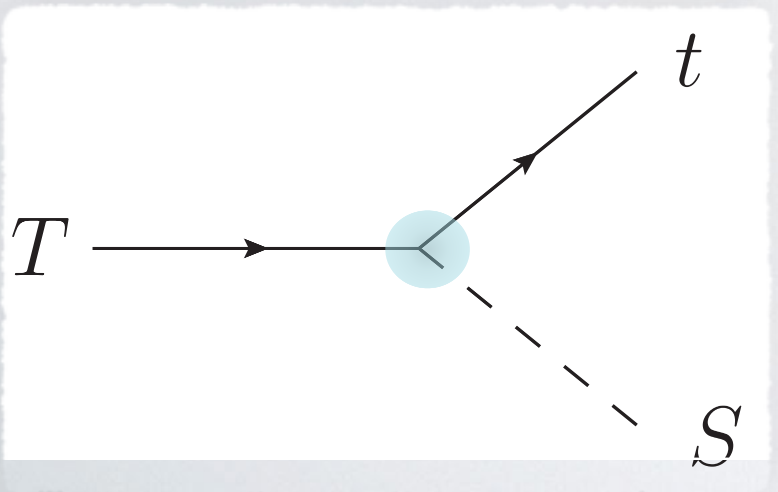


If $m_T > m_S + m_t$

- The scalar S mediates loop-level decays of T:

$$T \to t\, g$$

$$T \to t\, \gamma$$

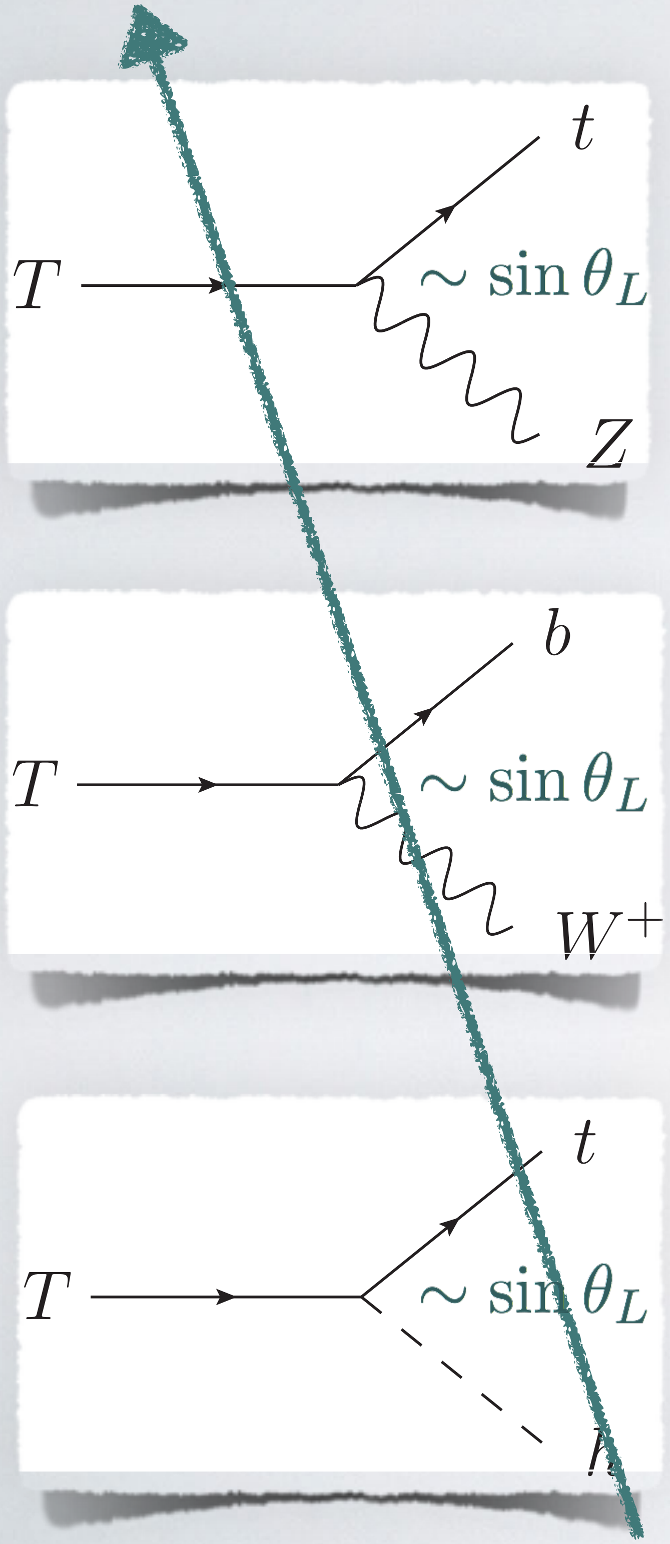
- These decays are allowed when $\sin\theta_L = 0$, because we can freely dial up and down the couplings $\lambda_{1,2}$.
- If the scalar mass is light, there is a new tree-level decay of T:

$$T \to t\, S$$

Classic T decay modes

J. H. Kim, I. M. Lewis [2018]

$0 \quad (\text{as } \sin\theta_L \to 0)$



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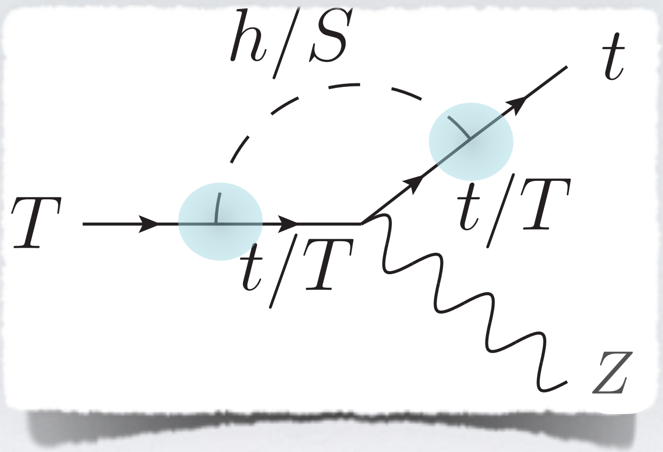

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Counter terms

External self-energies

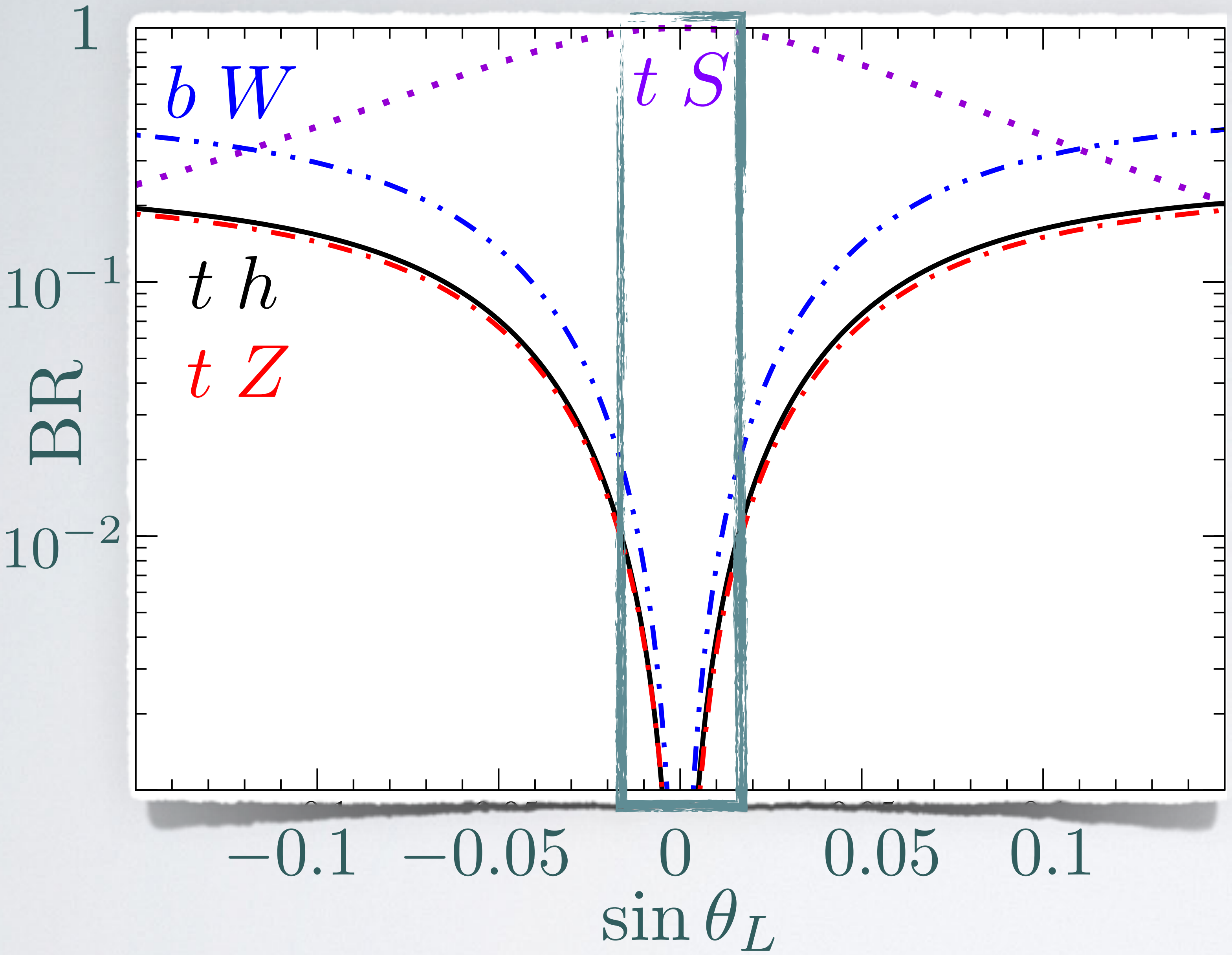
W and Z bosons loops

Goldstone bosons loops

(survives in the zero-mixing limit)

- All classic tree-level decay modes are controlled by $\sin\theta_L$.
- They all vanish in the limit of $\sin\theta_L \longrightarrow 0$.
- Except for the loop-level decay $T \longrightarrow t\,Z$.

Branching ratios ($m_T > m_S + m_t$)



J. H. Kim, I. M. Lewis [2018]

$m_T = 1.5$ TeV

$m_S = 200$ GeV

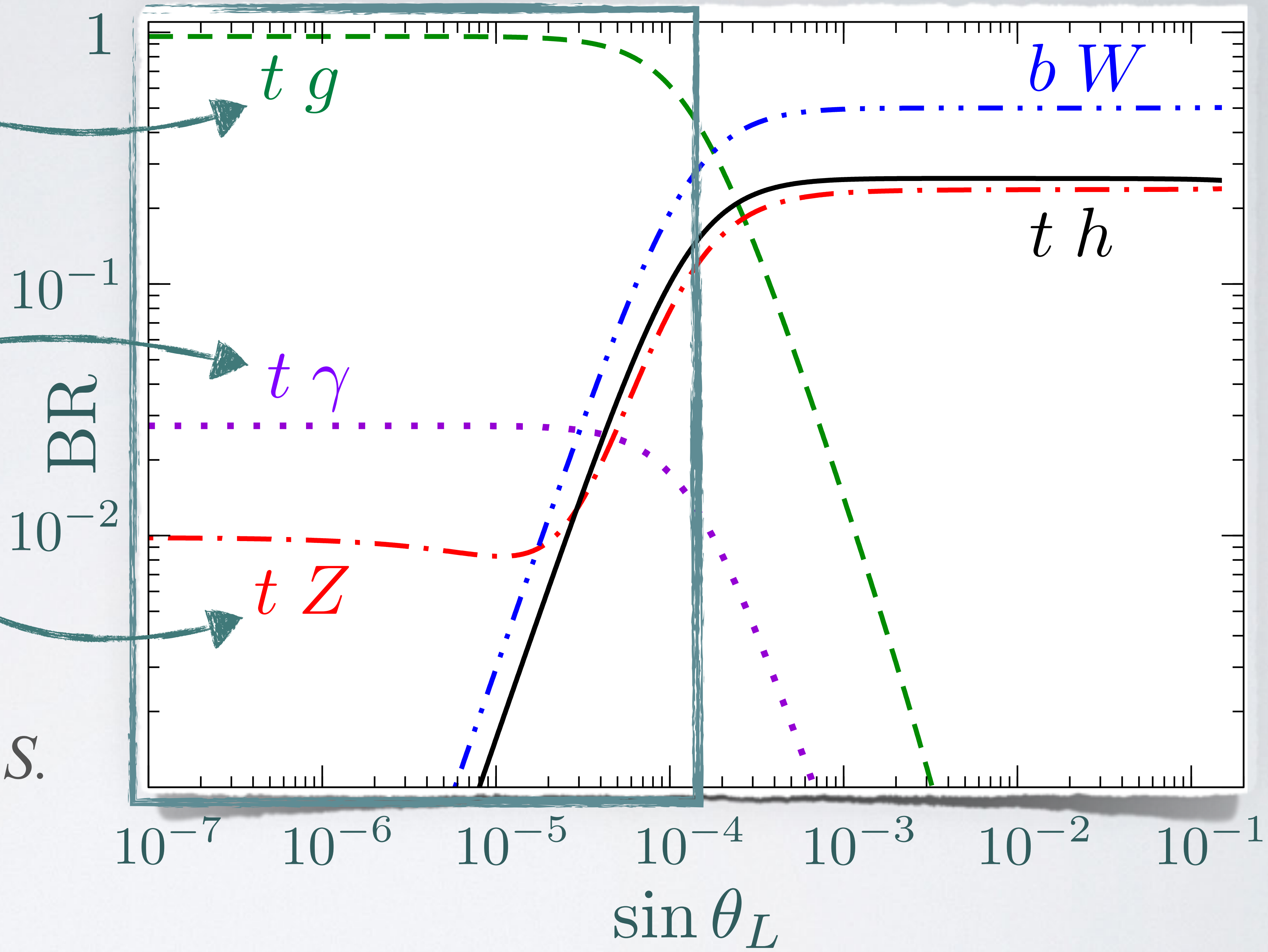
$\lambda_1 = \lambda_2 = 1$

- When T is heavier than S.

- $T \rightarrow t\ S$ decay is nearly 100% in the small mixing regime ($\sin\theta_L < 0.01$), since other classic tree-level decay modes simply vanish.
- The classic decay modes can be match-fit only if the mixing angle is sizeable.

Branching ratios ($m_T > m_S + m_t$)

J. H. Kim, I. M. Lewis [2018]



$\mathrm{BR}(T \to tg) \sim 96\%$

$\mathrm{BR}(T \to t\gamma) \sim 3\%$

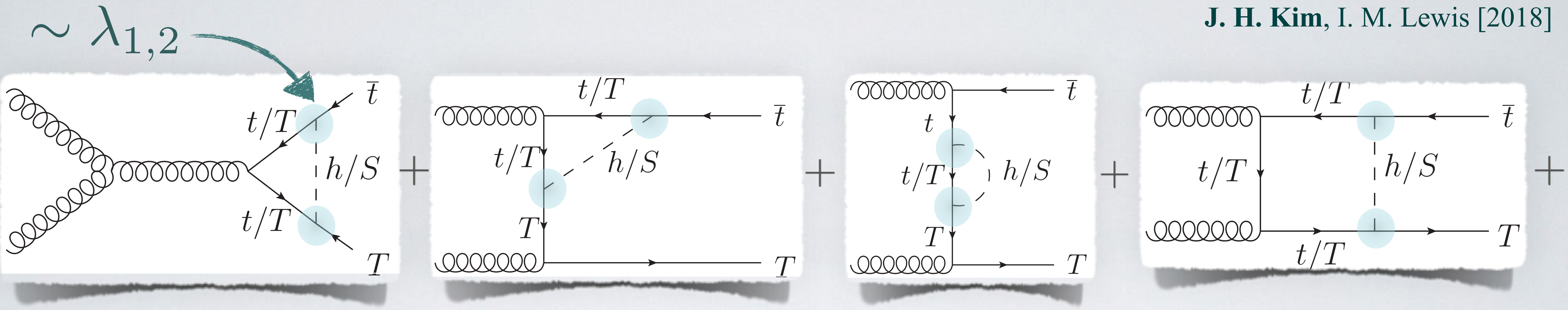
$\mathrm{BR}(T \to tZ) \sim 1\%$

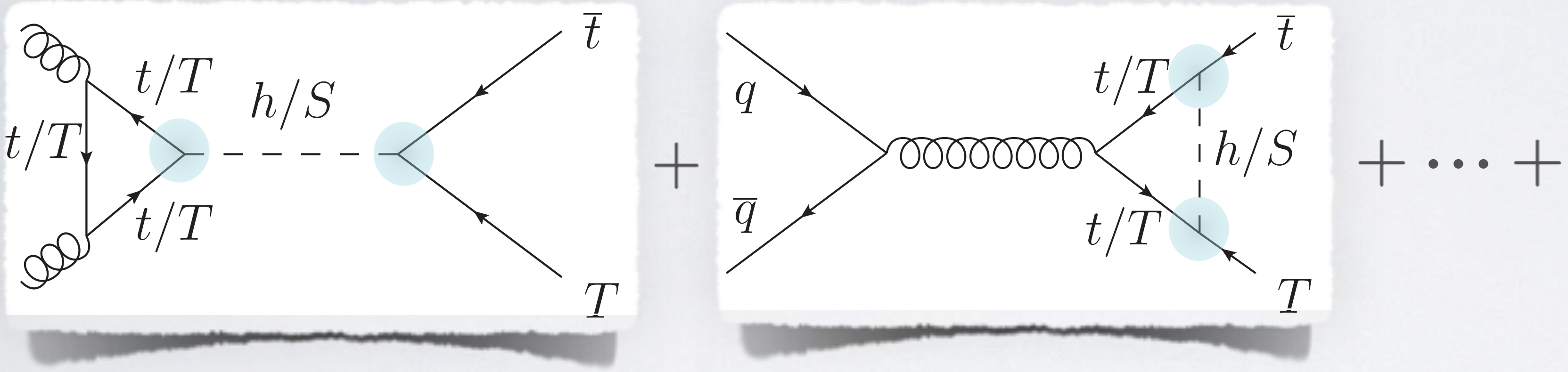
- When *T* is lighter than *S*.

- In the zero-mixing limit, all classic decays are suppressed & vanishing.
- All loop decays $T \to t\,g$, $T \to t\,\gamma$, $T \to t\,Z$ dominate.
- This strongly indicate that *T* phenomenology will substantially change.

Loop-induced $T\,t$ productions

J. H. Kim, I. M. Lewis [2018]





Counter terms

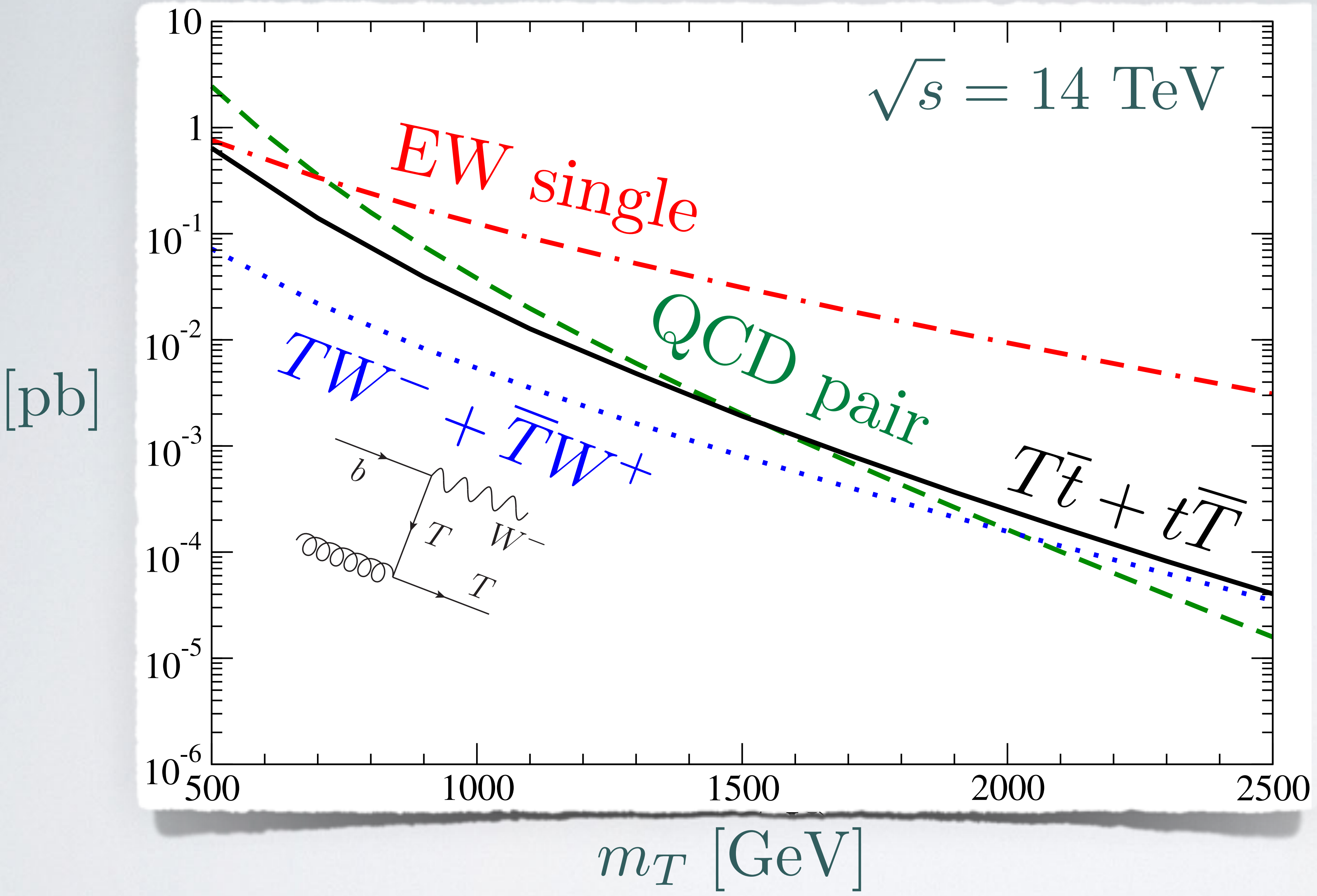
External self-energies

W and Z bosons loops

Goldstone bosons loops

- The scalar S can mediate loop-induced $g\,g \rightarrow T\,t$ productions.
- There are also loop-induced $q\,\bar{q} \rightarrow T\;t$ productions.
- Even they are loop-suppressed, we can freely dial up and down the couplings $\lambda_{1,2}$ to control a total cross section.

Production cross sections



J. H. Kim, I. M. Lewis [2018]

$m_S = 200$ GeV

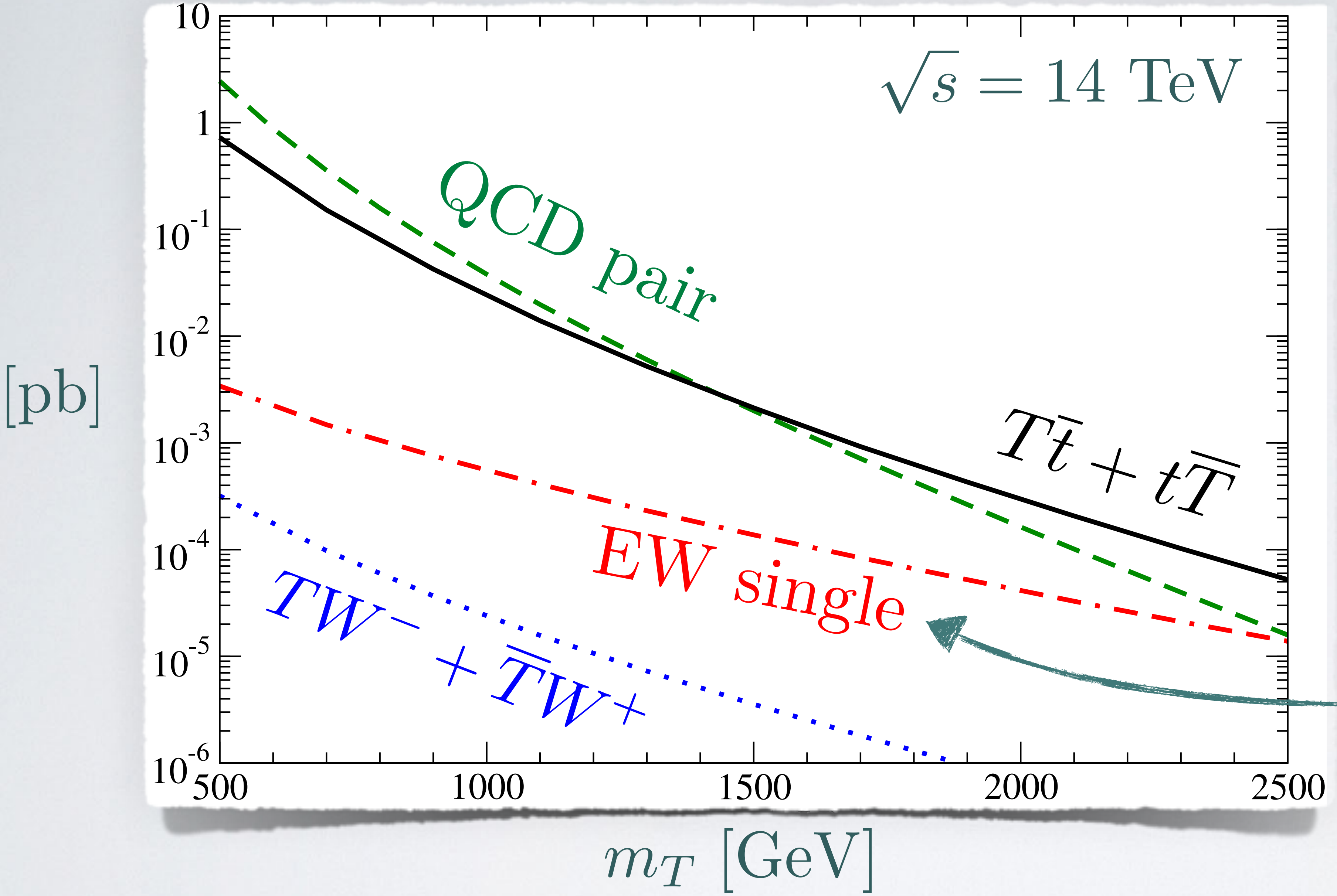
$\lambda_1 = \lambda_2 = 3$

$\boxed{\sin\theta_L = 0.15}$

- Production cross sections as a function of *T* mass.

- The EW single *T* production dominates if the mixing angle is large.
- The loop-induced *T t* productions stay way below.

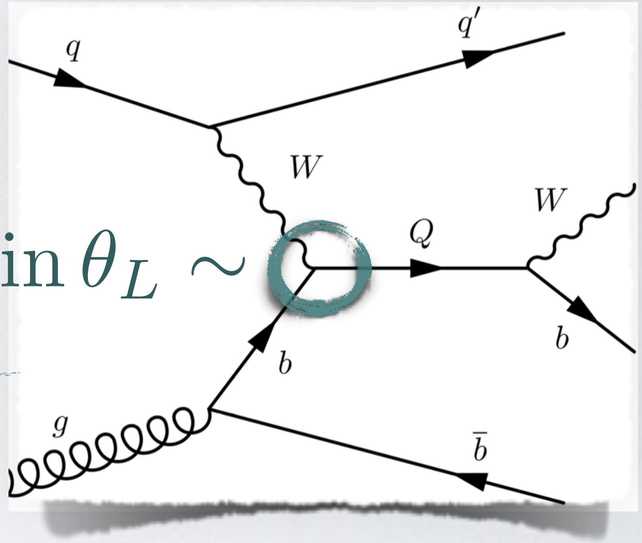
Production cross sections



$m_S = 200$ GeV

$\lambda_1 = \lambda_2 = 3$

$\boxed{\sin\theta_L = 0.01}$



$\sin\theta_L \sim$

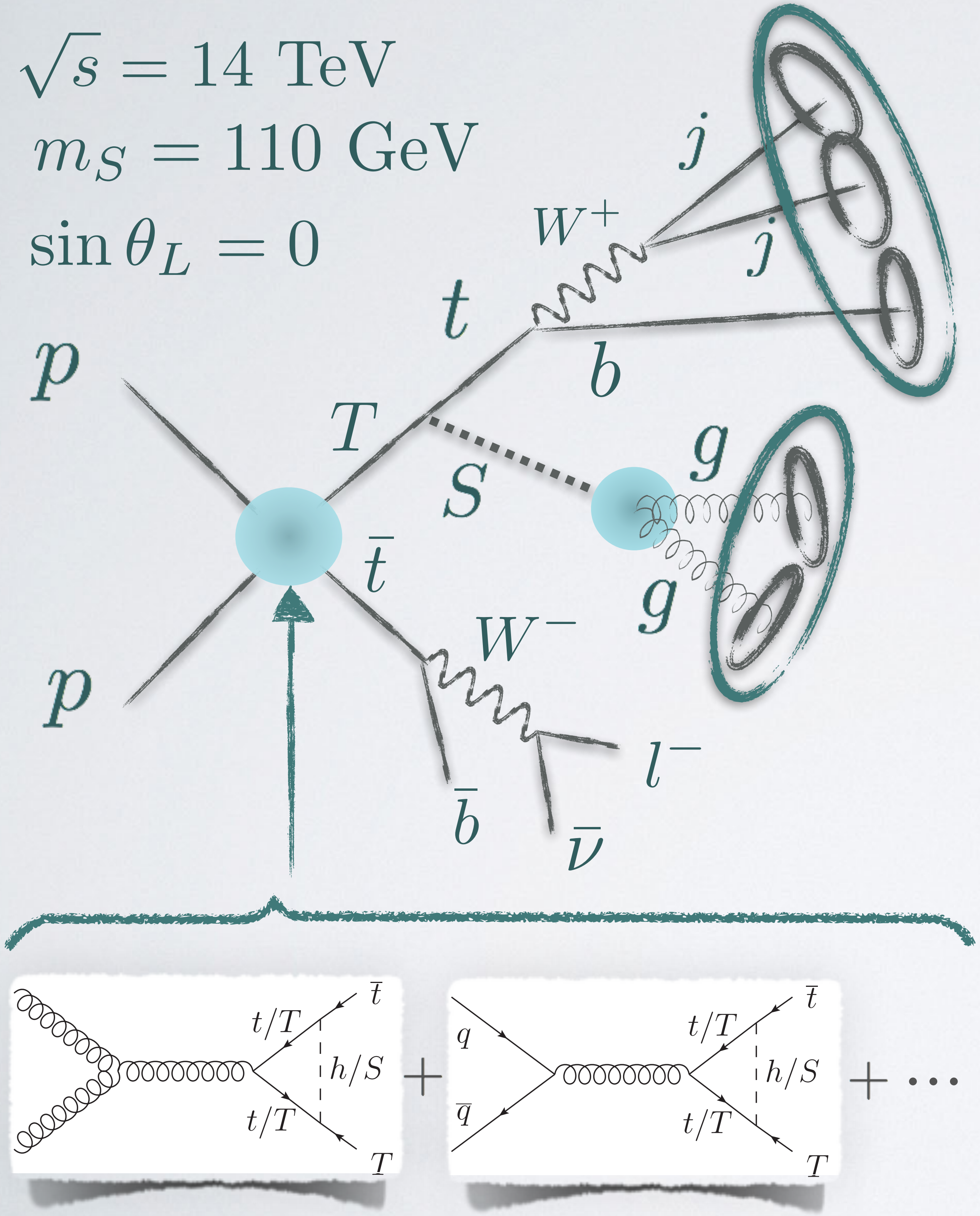
- But the tide changes in the small-mixing regime where the EW single *T* production loses its dominance.
- The loop-induced *T t* productions become more important.

T searches in the $T\bar{t}+t\bar{T}$ channel

$\sqrt{s}=14$ TeV

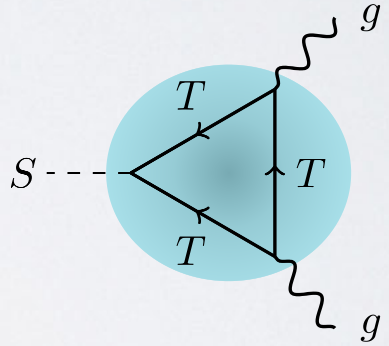
$m_S=110$ GeV

$\sin\theta_L=0$



J. H. Kim, I. M. Lewis [2018]

- Now we talk about a sensitivity of the $T\,t$ production at the LHC.
- T decays to $t\,S$ nearly 100 % in the zero mixing case ($\sin\theta_L=0$).
- S exclusively decays into gg nearly 100 %

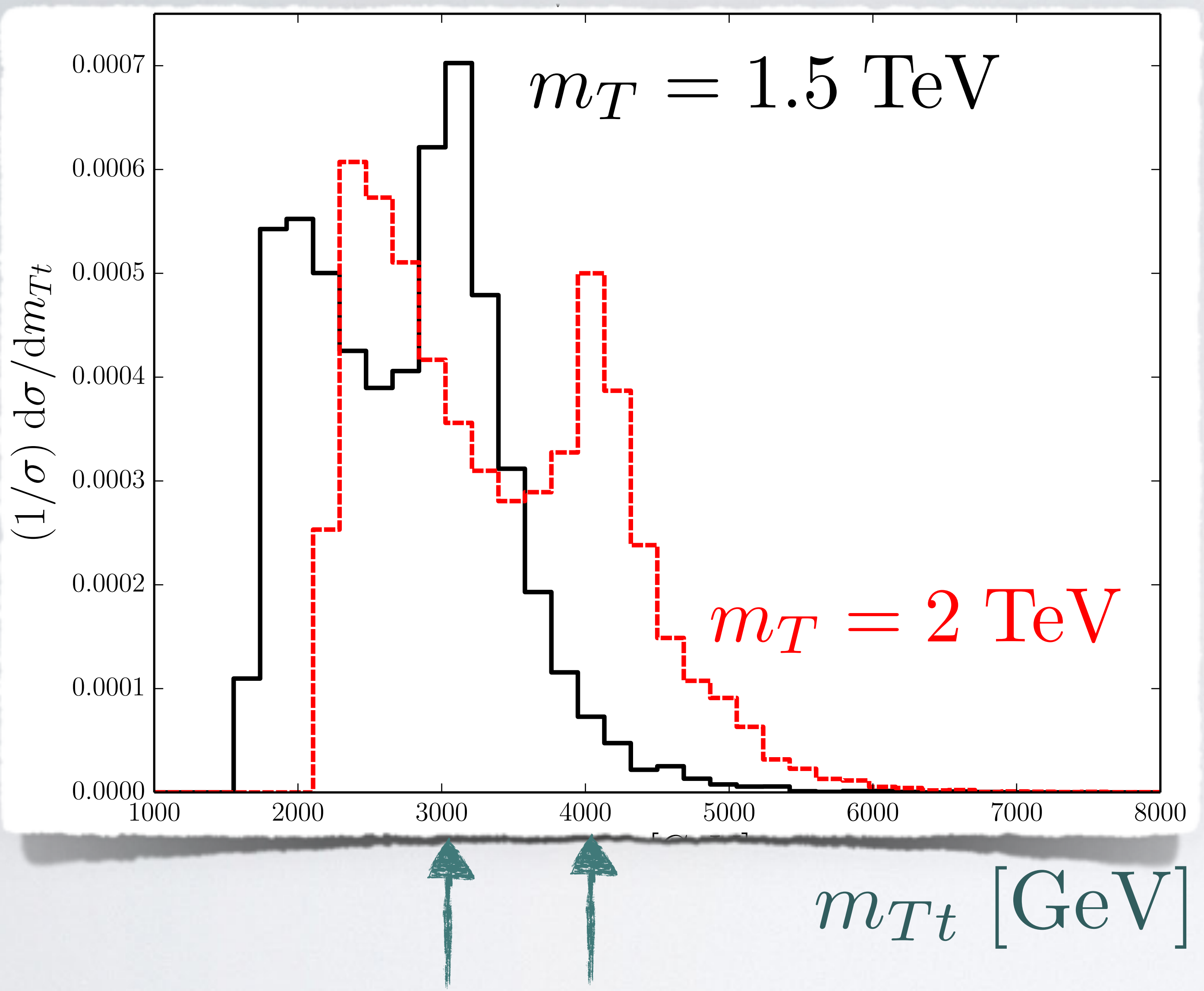


- Both tops decay semi-leptonically.
- The production vertex includes all loop contributions.

What's in the loops?

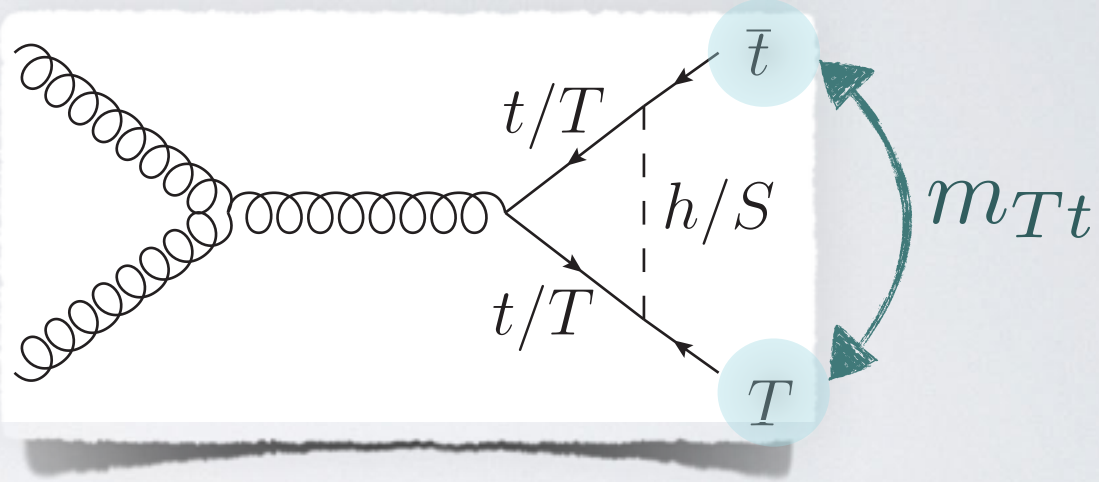
$\sqrt{s} = 14$ TeV parton-level

J. H. Kim, I. M. Lewis [2018]



m_{Tt} 3 TeV m_{Tt} 4 TeV

- When $m_{Tt} \sim 2m_T$, the internal top partners in the loop can go on-shell.



- It gives rise to the peaks in m_{Tt} distributions.
- It can significantly alter the final state kinematic distributions (e.g p_T , ΔR …)

Summary cut-flow table

$m_T = 1.5$ TeV

$\lambda_{1,2} = 2$

5.8% 0.08% 0.06% 0.0036% 0.0028%

Backgrounds

Significance

$m_T = 1.5$ TeV, $\lambda_{1,2} = 2$	Signal [fb]	$t\bar{t}$ [fb]	Single t [fb]	W [fb]	VV [fb]	σ
Basic cuts	0.055	1.3×10^3	2.8×10^3	2.7×10^3	88	0.036
$N^{1.5}_{t_{had}} = N^{1.5}_S = 1$	3.2×10^{-3}	1.11	1.6	0.098	2.5×10^{-3}	0.11
Reconstructed t_{lep}	1.2×10^{-3}	0.073	0.070	4.7×10^{-4}	$\ll \mathcal{O}(10^{-5})$	0.17
1400 GeV $< m^{reco}_T <$ 1550 GeV	9.2×10^{-4}	0.015	9.4×10^{-3}	$\ll \mathcal{O}(10^{-5})$	$\ll \mathcal{O}(10^{-5})$	0.32
2865 GeV $< m^{reco}_{Tt}$	6.3×10^{-4}	1.5×10^{-3}	7.2×10^{-5}	$\ll \mathcal{O}(10^{-5})$	$\ll \mathcal{O}(10^{-5})$	0.81
2050 GeV $< H^{reco}_T$ $\Delta R^{reco}_{t_{had}S} < 3.41$ $1.63 < \Delta R^{reco}_{t_{lep}S}$	5.8×10^{-4}	$\ll \mathcal{O}(10^{-5})$	$\ll \mathcal{O}(10^{-5})$	$\ll \mathcal{O}(10^{-5})$	$\ll \mathcal{O}(10^{-5})$	5.0

$\times 3.1$

$$N_{sig} = 1.7 \quad (\text{for } L = 3\ \text{ab}^{-1})$$

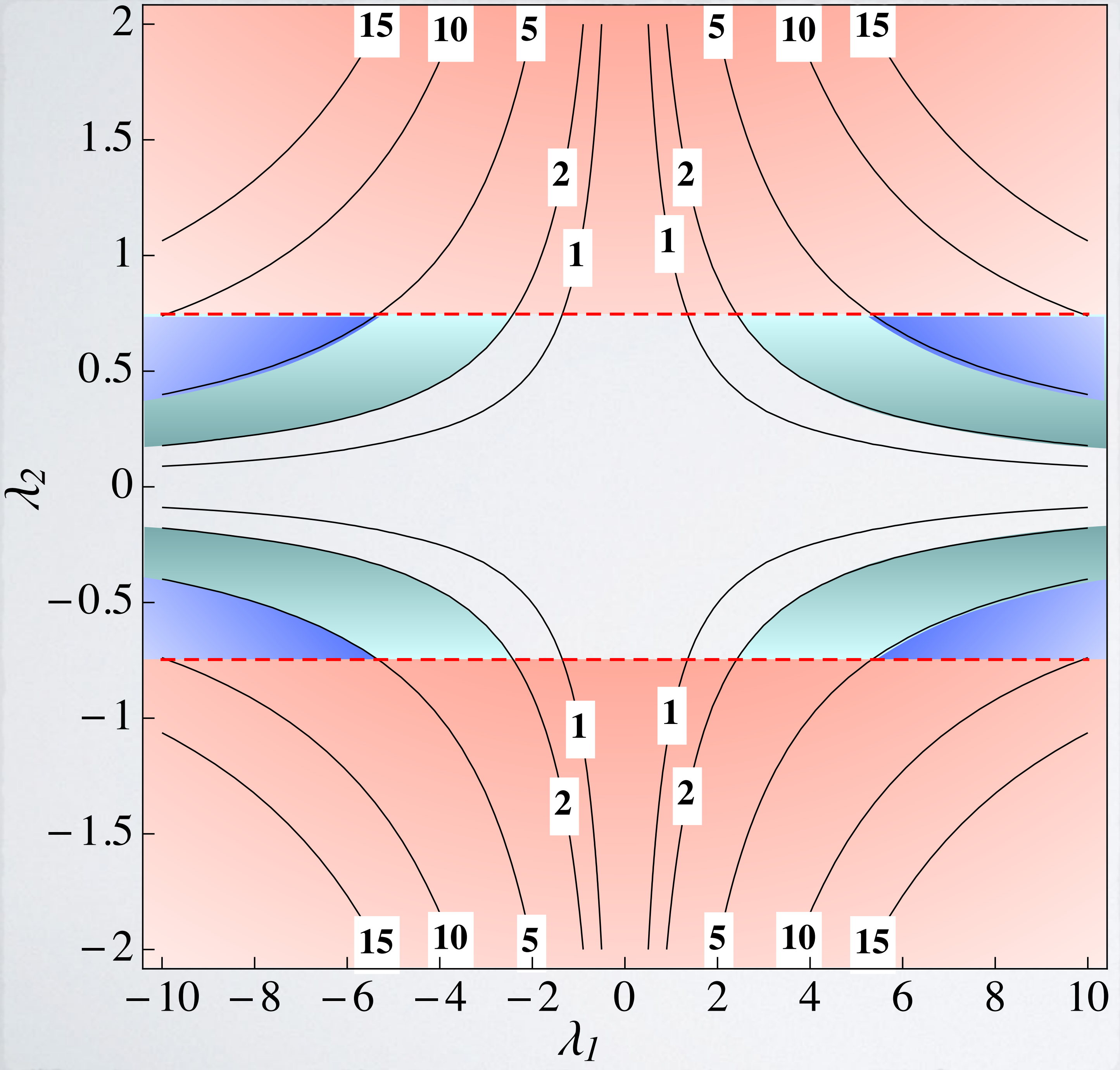
- A cut-flow table showing cross sections of each stage in fb.
- It shows that jet substructure analysis can effectively reduce the overall size of backgrounds.
- 5σ significance is achievable for a luminosity of 3 ab^{-1} .

Contours of constant significance

$\sqrt{s} = 14$ TeV $\quad \boxed{m_T = 1.5 \text{ TeV}} \quad \sin\theta_L = 0$

$L = 3 \text{ ab}^{-1} \quad m_S = 110$ GeV

J. H. Kim, I. M. Lewis [2018]



- Significances are calculated for a luminosity of 3 ab^{-1}.

Constraint from the scalar resonant search

$> 5\sigma$ observation

$> 2\sigma$ observation

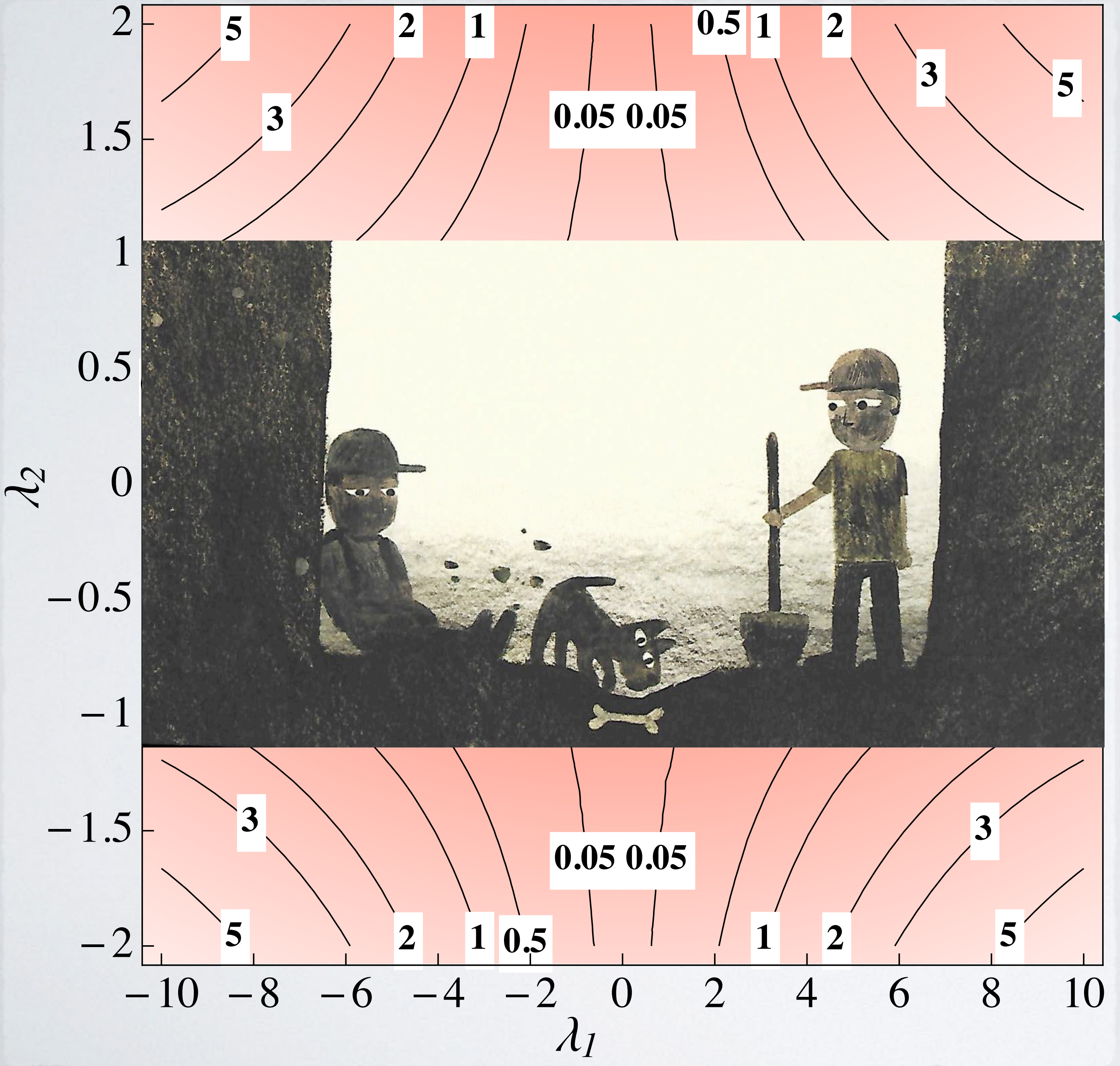
- We have a good fraction of parameter space that can be probed by the collider search.

Contours of constant significance

$\sqrt{s} = 14$ TeV $\boxed{m_T = 2.0 \text{ TeV}}$ $\sin\theta_L = 0$

$L = 3$ ab^{-1} $m_S = 110$ GeV

J. H. Kim, I. M. Lewis [2018]



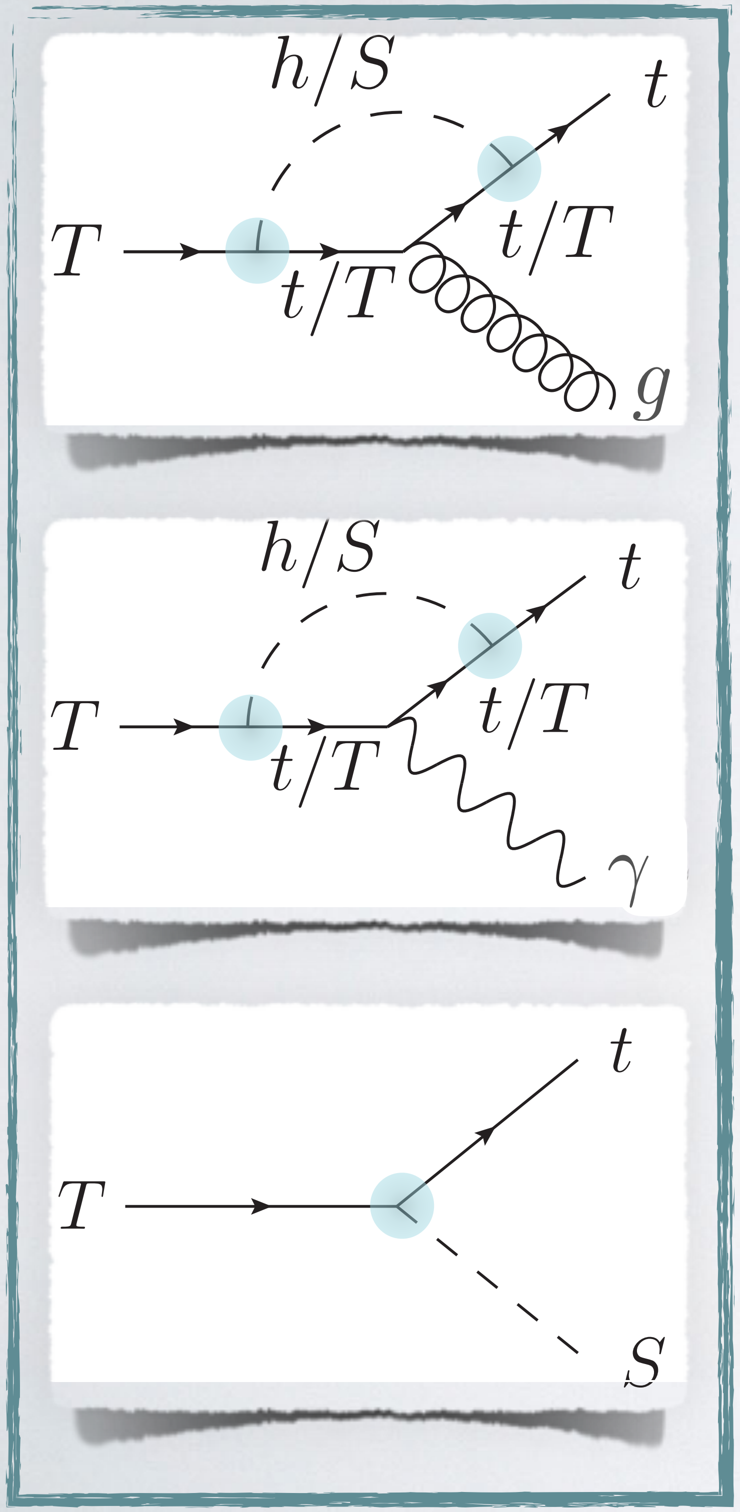
Constraint from the scalar resonant search

$> 1\sigma$ observation

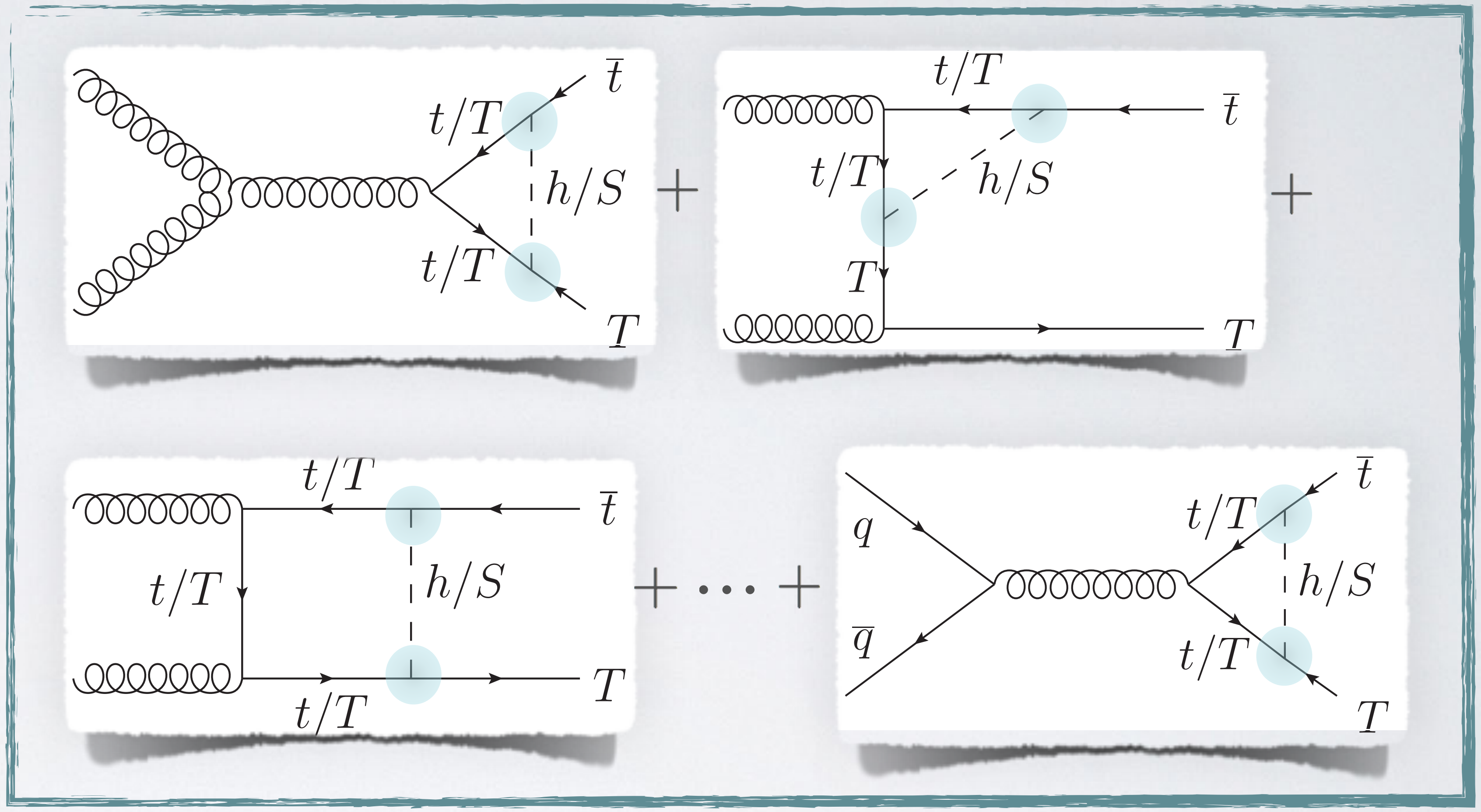
- Probing top partner masses beyond 2 TeV will be challenging.
- We might need a high energy collider with a decent amount of luminosity.

Summary

New decays



New productions



Thank you for listening!

Back-up

The SM fine tuning problem.

$$m^2_{\text{higgs}} = m^2_{\text{bare}} + \boxed{\Delta m^2_{\text{higgs}}} \longleftarrow \Delta m^2_{\text{higgs}} = -\frac{|\lambda|^2}{8\pi^2}\Lambda^2 + \text{finite}$$

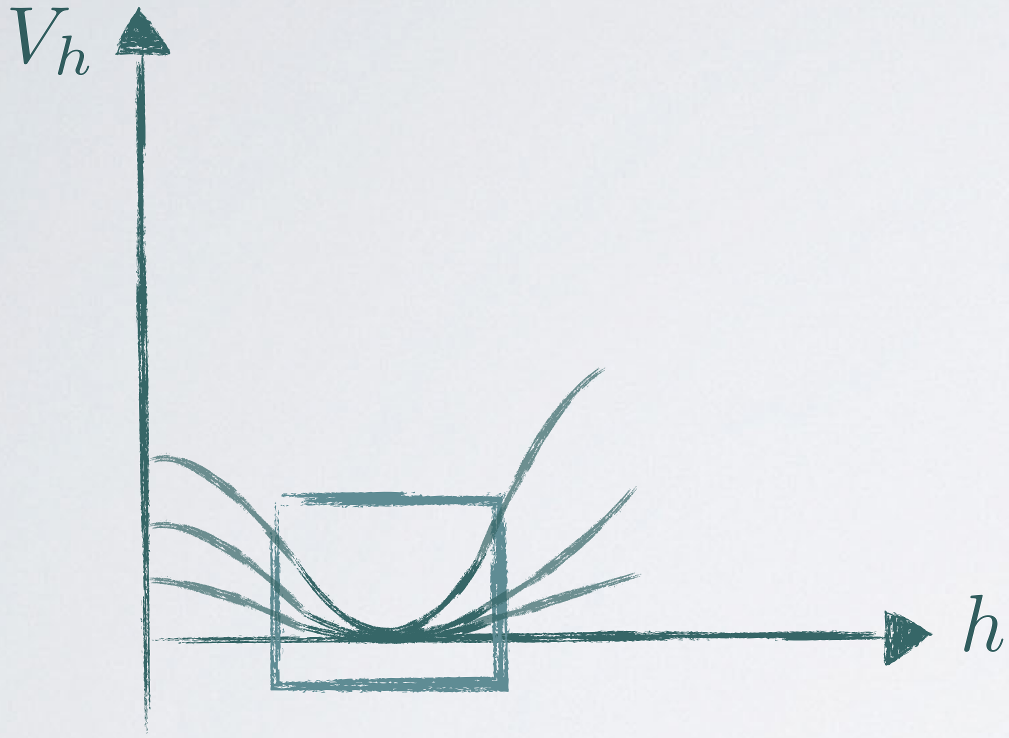
$$\Big\downarrow$$

$\mathcal{O}(10^{38}) - \mathcal{O}(10^{38}) \sim 10^4$!?

- We should expect to see new physics in the scope of the naturalness paradigm.

e.g. Composite Higgs models

$$V_h = \boxed{\frac{m_h^2}{2} h^2}$$



- e.g. In composite Higgs models, the Higgs potential is radiatively generated by a top and T' loops. K. Agashe, R. Contino, A. Pomarol [2005]

$$V(h) = \alpha \cos \frac{h}{f} - \beta \sin^2 \frac{h}{f}$$

- The role of T' is to cut off the quadratic divergence to the Higgs mass in the loop.

$$m_h^2 \sim \frac{N_C}{4\pi^2} m_t^2 \frac{\enclose{circle}{m_{T'}^2}}{f^2}$$

How Composite Higgs Models address the hierarchy problem.

- The Higgs potential is **radiatively** generated by the top quark loop in the SM

$$\delta m_h^2 = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \boxed{\Lambda_f^2}$$

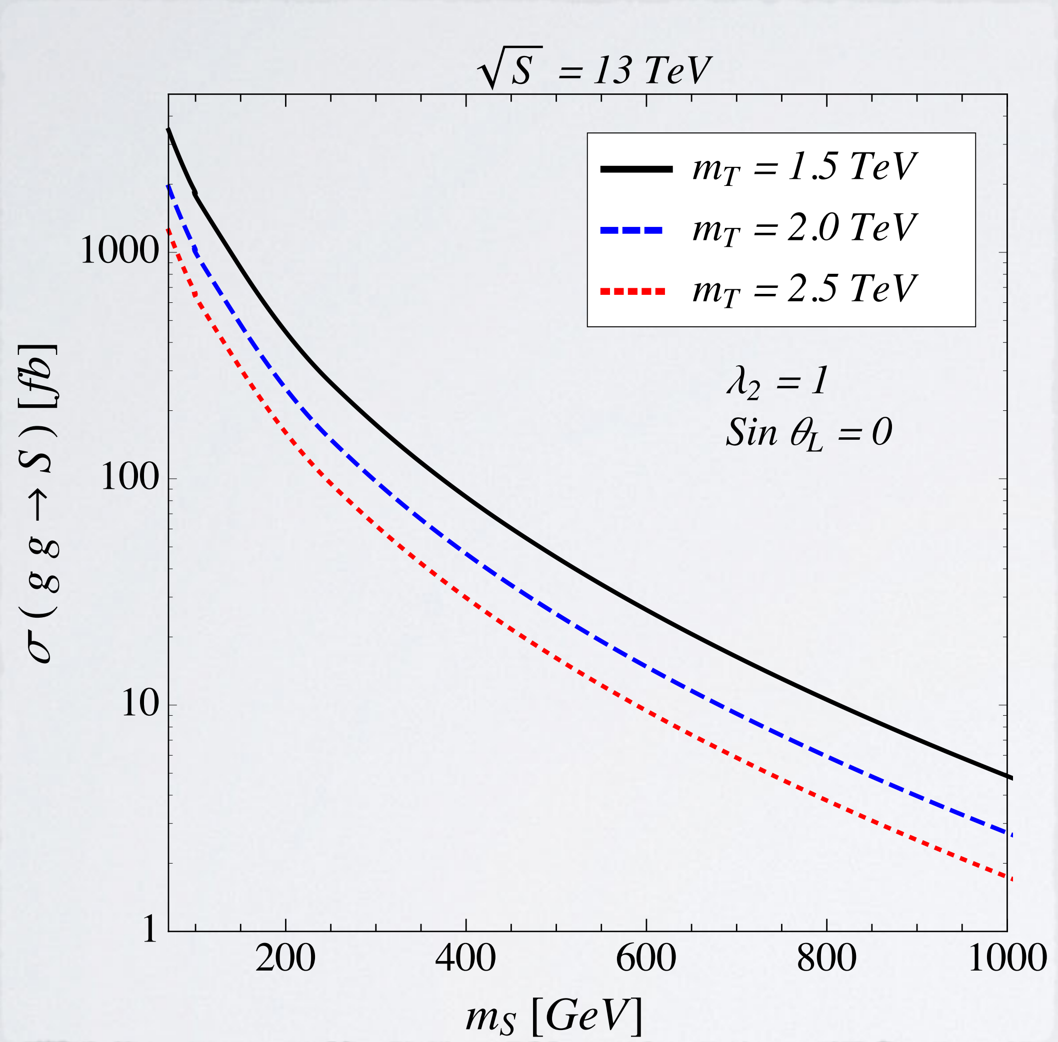
mass scale of resonances

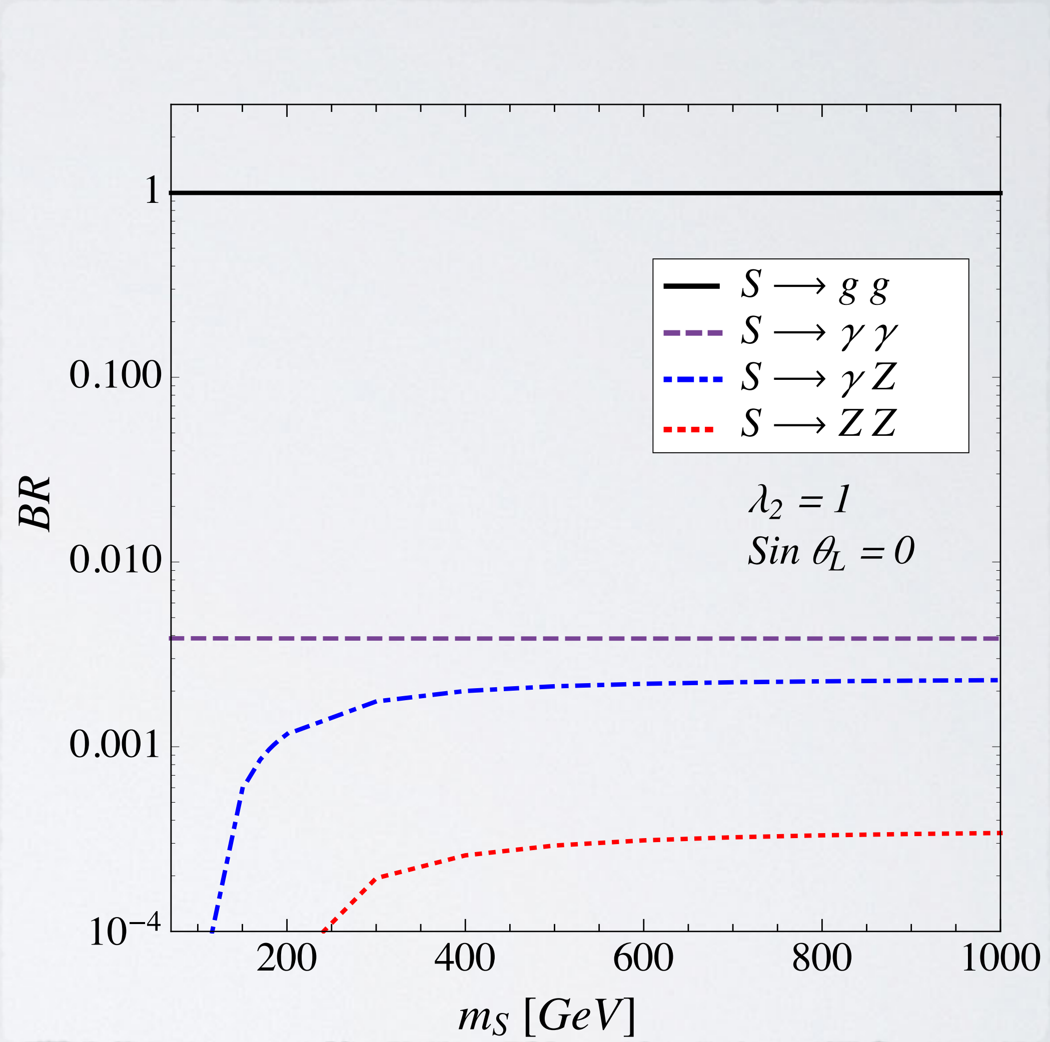
$$\Downarrow$$

$$\Delta = \frac{\delta m_h^2}{m_h^2} \simeq \Big(\frac{\Lambda_f}{400\mathrm{GeV}}\Big)^2 \Big(\frac{125\mathrm{GeV}}{m_h}\Big)^2$$

- $\Lambda_f \sim$ 1TeV gives a mild tuning $\Delta \simeq 10$

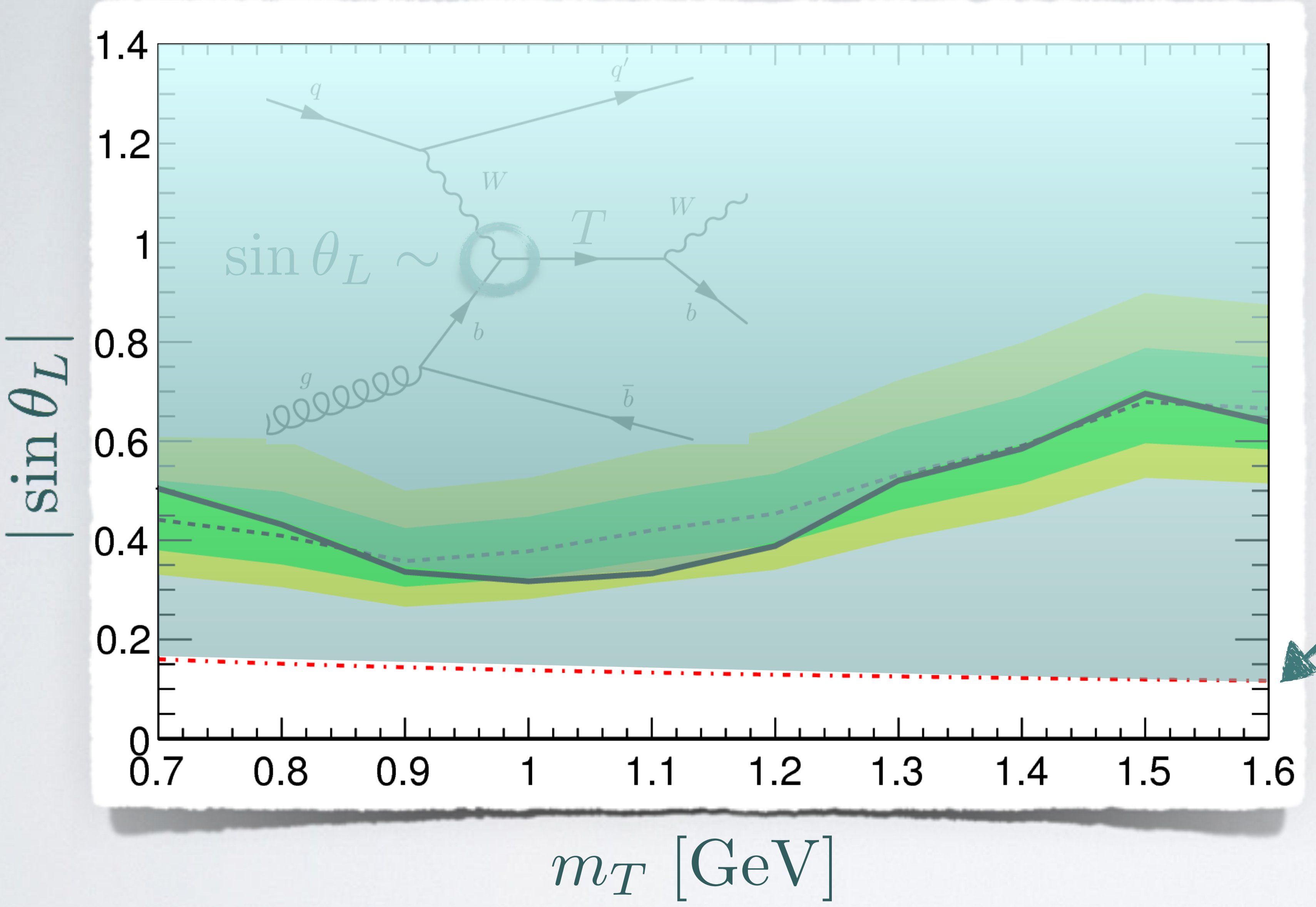
The scalar resonance production and decays





Limits on the mixing angle

ATLAS-CONF-2016-072 $\sqrt{s} = 13$ TeV 3.2 fb^{-1} singlet T



- Collider bounds are weak.

  $\sin\theta_L < 0.3 \sim 0.65$

  (for $m_T < 1 \sim 1.6$ TeV)

by oblique parameters

Chien-Yi Chen, S. Dawson, I. M. Lewis [2014]

J. A. A. Saavedra, R. Benbrik, S. Heinemeyer, M. P. Victoria [2013]

S. Dawson, E. Furlan [2012]

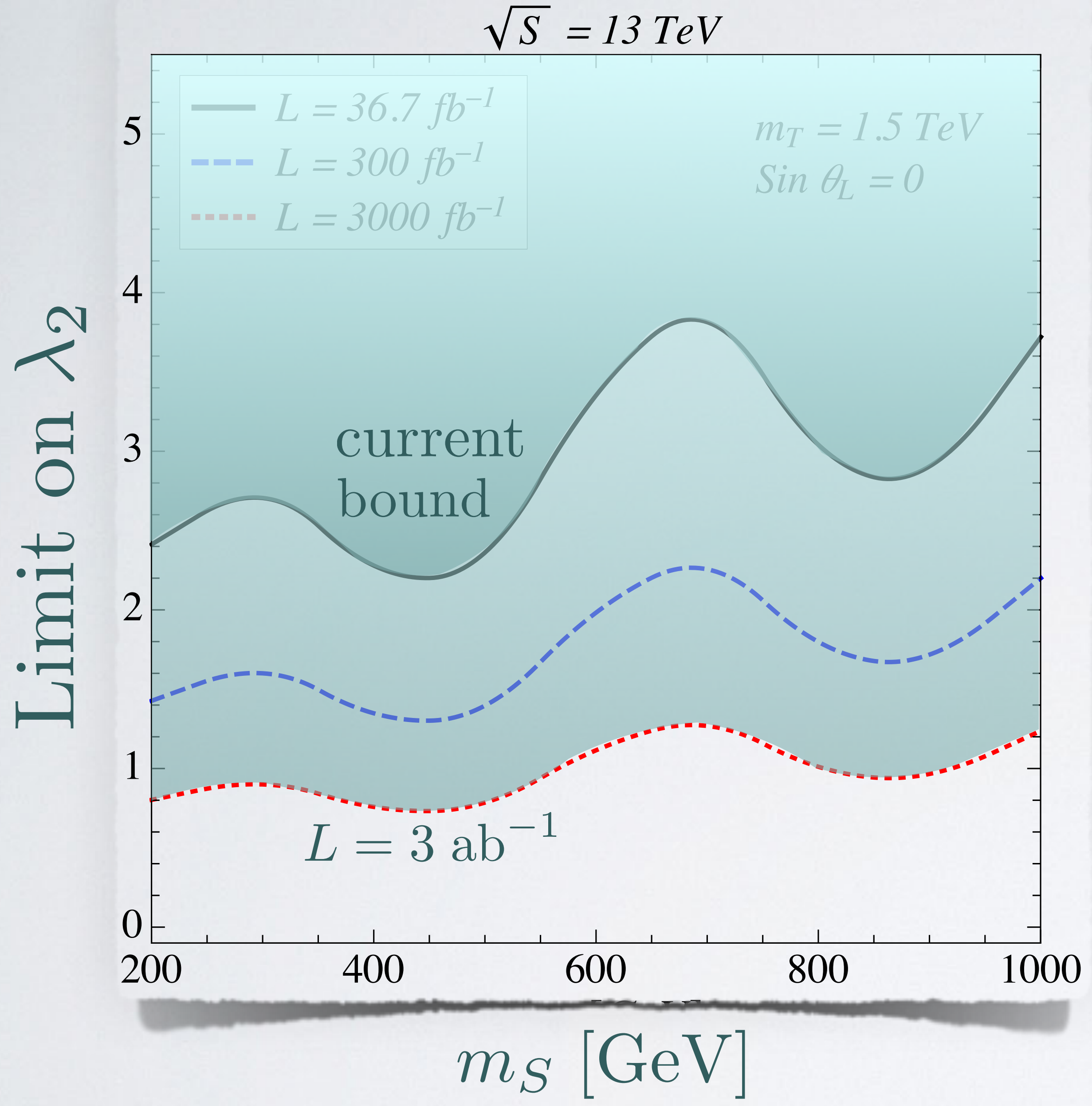
H. J. He, N. Polonsky, S.F. Su [2001]

- The strongest limits are obtained by oblique parameters.

  $\sin\theta_L < 0.11 \sim 0.16$ (for $m_T < 1 \sim 2$ TeV)

Constraints on the Scalar *S*

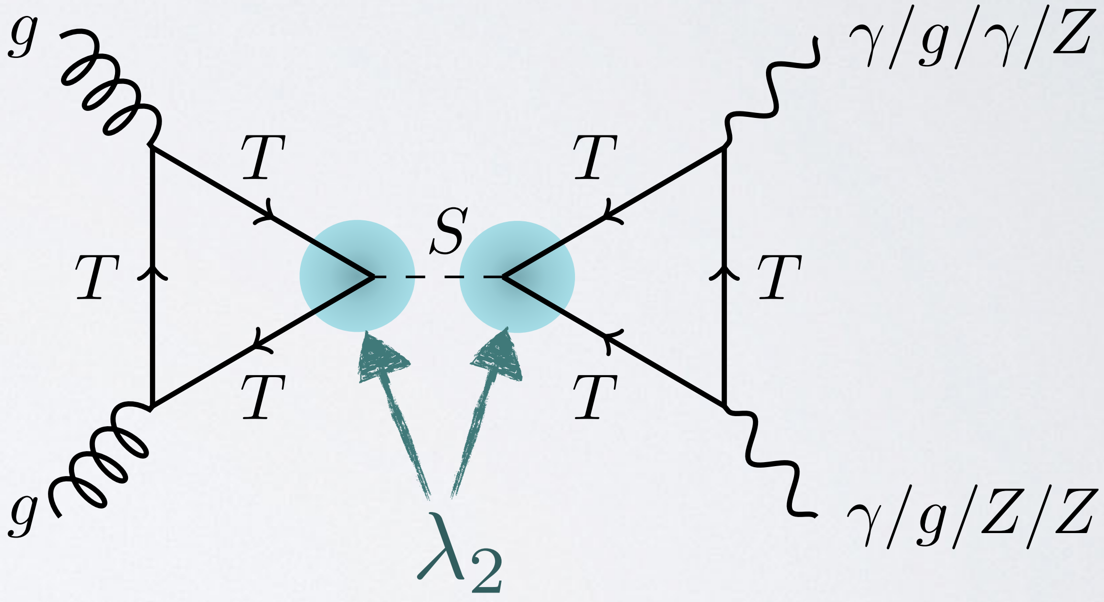
J. H. Kim, I. M. Lewis [2018]



high mass region

CMS-PAS-HIG-17-013

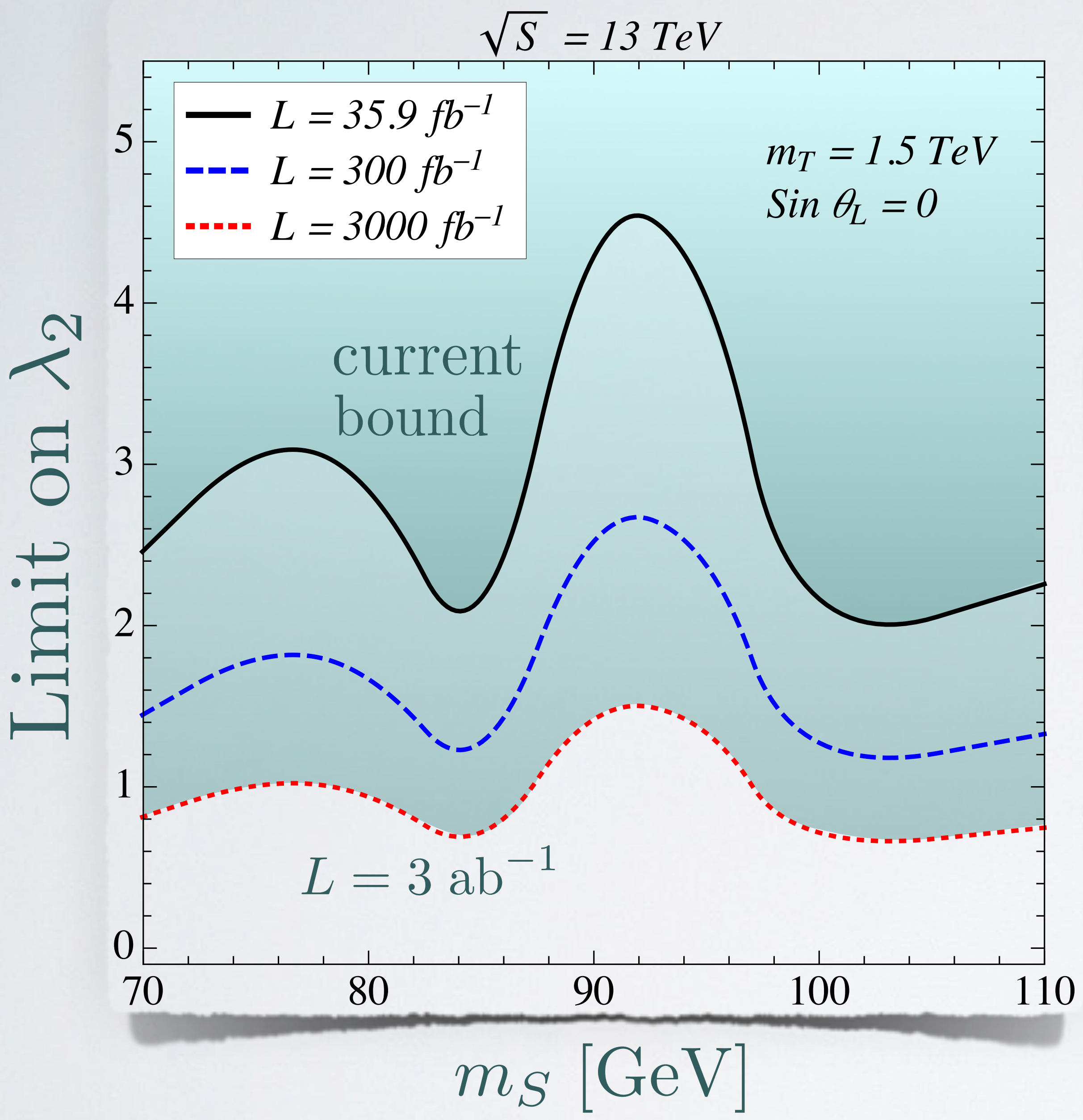
- Scalar resonant searches can put significant constraints on λ_2 and m_S.
- *S* can decay into $\gamma\gamma$, gg, γZ and ZZ in the $\sin\theta_L \rightarrow 0$ limit.



- Diphoton searches set the most stringent limit on *S*.

Constraints on the Scalar S

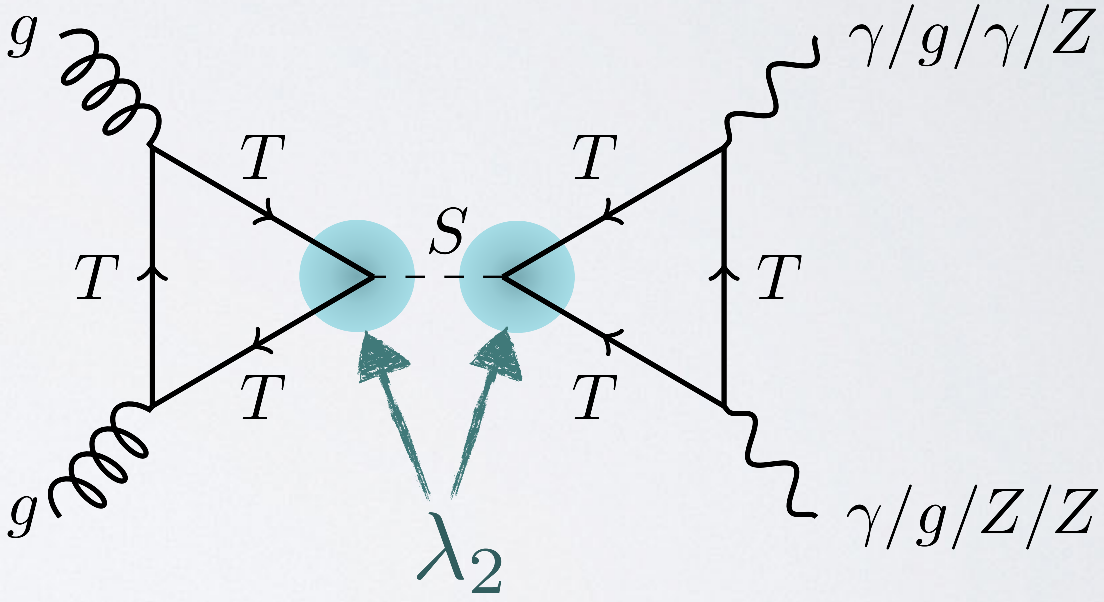
J. H. Kim, I. M. Lewis [2018]



low mass region

ATLAS collab. [1707.04147]

- Scalar resonant searches can put significant constraints on λ_2 and m_S.
- S can decay into $\gamma\gamma$, gg, γZ and ZZ in the sin $\theta_L \to 0$ limit.

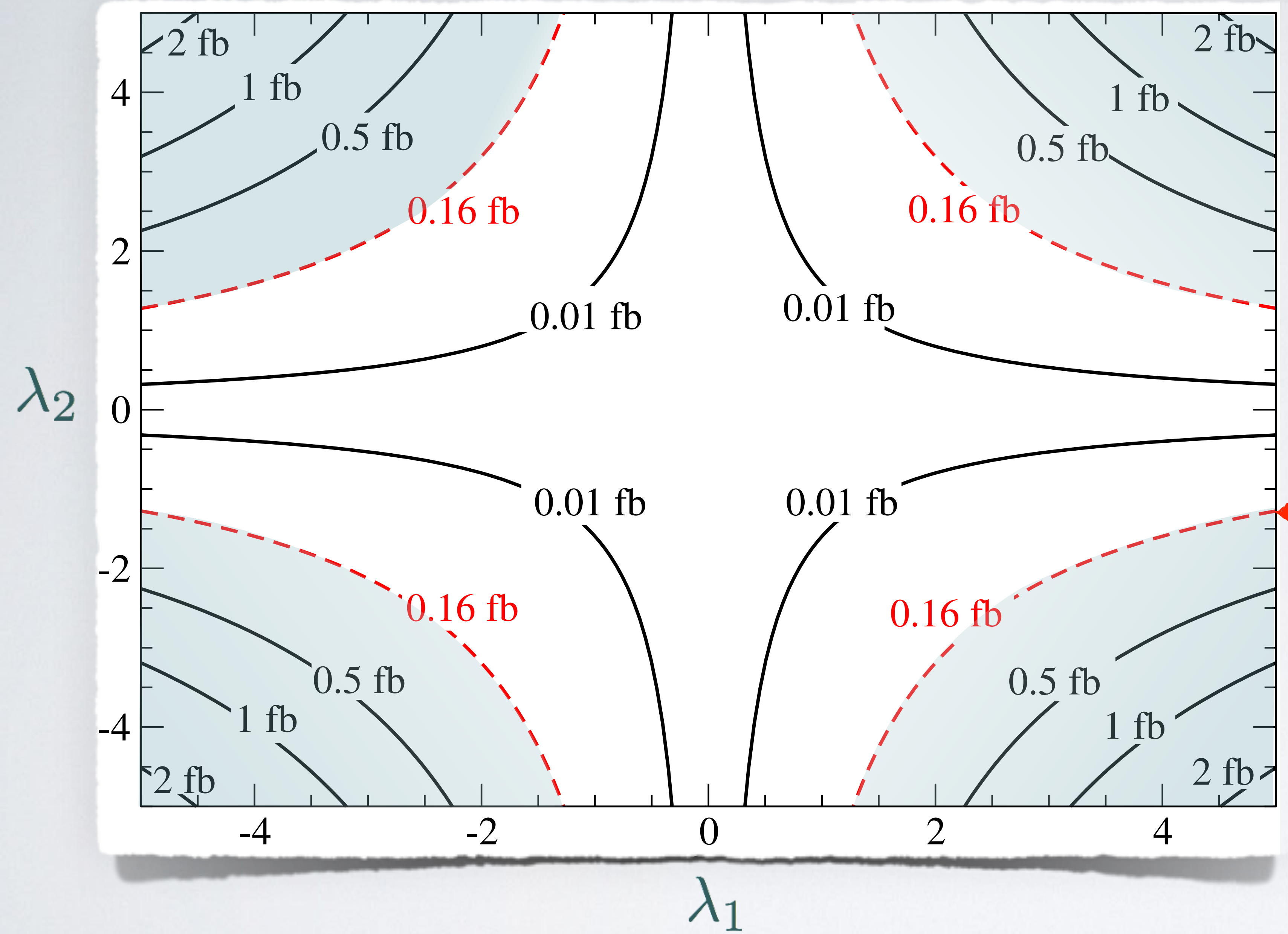


- Diphoton searches set the most stringent limit on S.

$T\bar{t} + t\overline{T}$ vs QCD pair

$\sigma(pp \to T\bar{t} + t\overline{T})$ [fb] $\sqrt{s} = 14$ TeV

J. H. Kim, I. M. Lewis [2018]



$m_T = 2$ TeV

$m_S = 200$ GeV

$\boxed{\sin\theta_L = 0}$

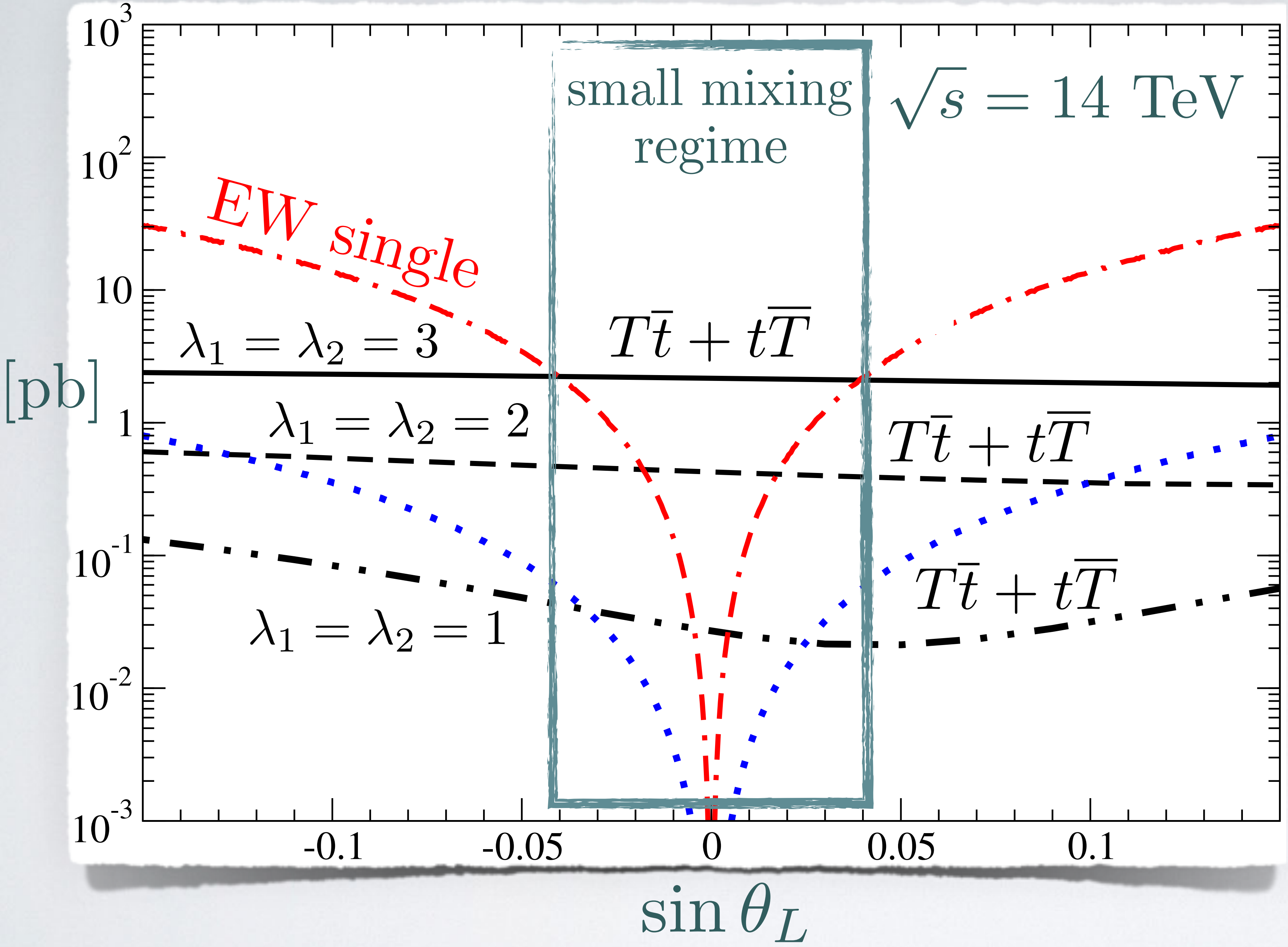
$\sigma(pp \to T\bar{t} + t\overline{T}) \propto \lambda_1^2\lambda_2^2$

$\sigma^{\text{pair}}(pp \to T\overline{T})$ [fb]

- At higher top partner masses, the *T t* productions are kinematically favorable, and can easily beat the QCD pair production in much wider space.

$T\bar{t} + t\bar{T}$ vs EW single

J. H. Kim, I. M. Lewis [2018]



$m_S = 200$ GeV

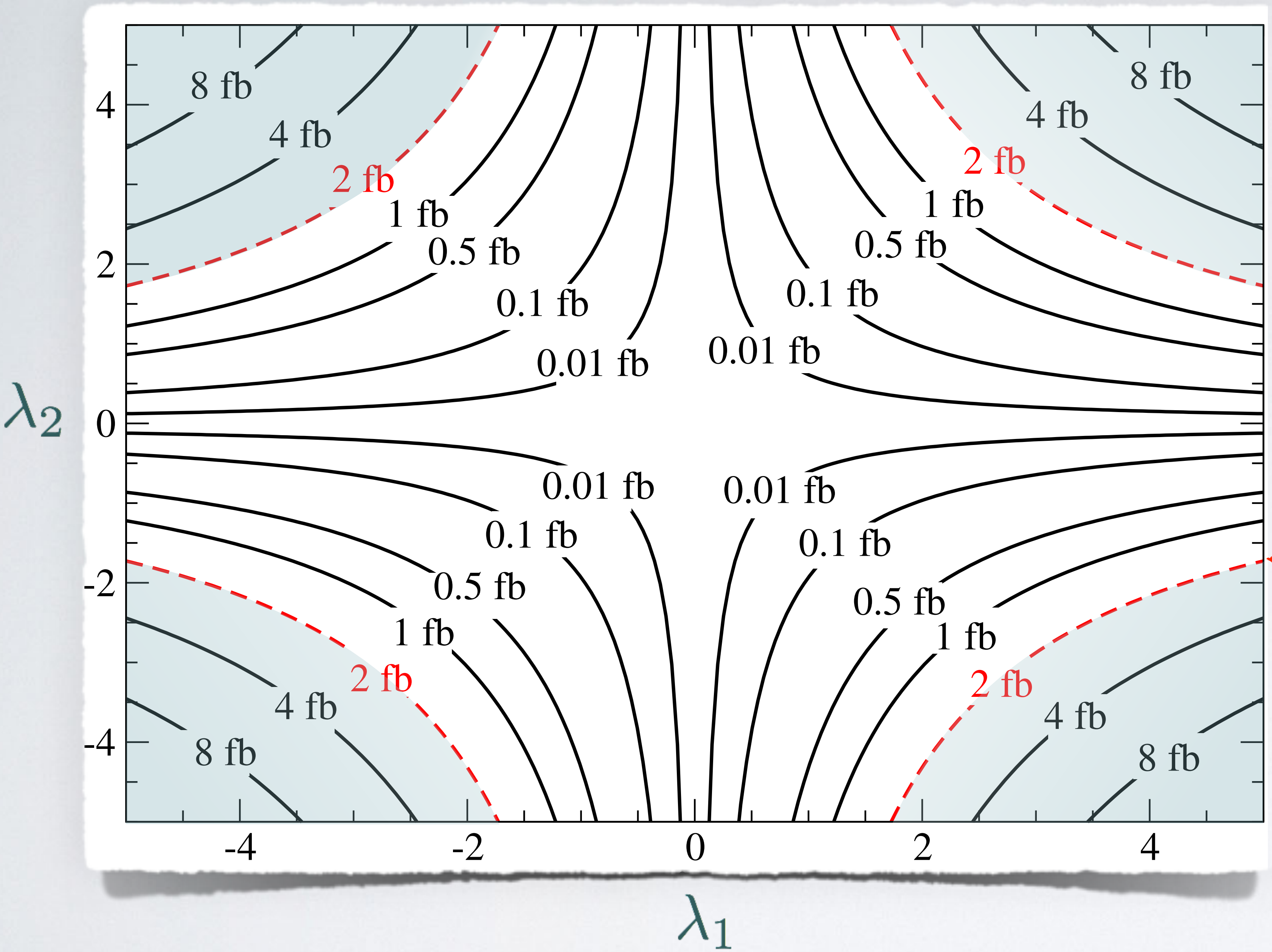
$m_T = 1.5$ TeV

- Comparisons with the EW single production as a function of $\sin\theta_L$.
- For $\sin\theta_L < 0.04$, the $T\,t$ productions are main production modes.

$T\bar{t} + t\bar{T}$ vs QCD pair

$\sigma(pp \to T\bar{t} + t\bar{T})$ [fb] $\sqrt{s} = 14$ TeV

J. H. Kim, I. M. Lewis [2018]



$m_T = 1.5$ TeV

$m_S = 200$ GeV

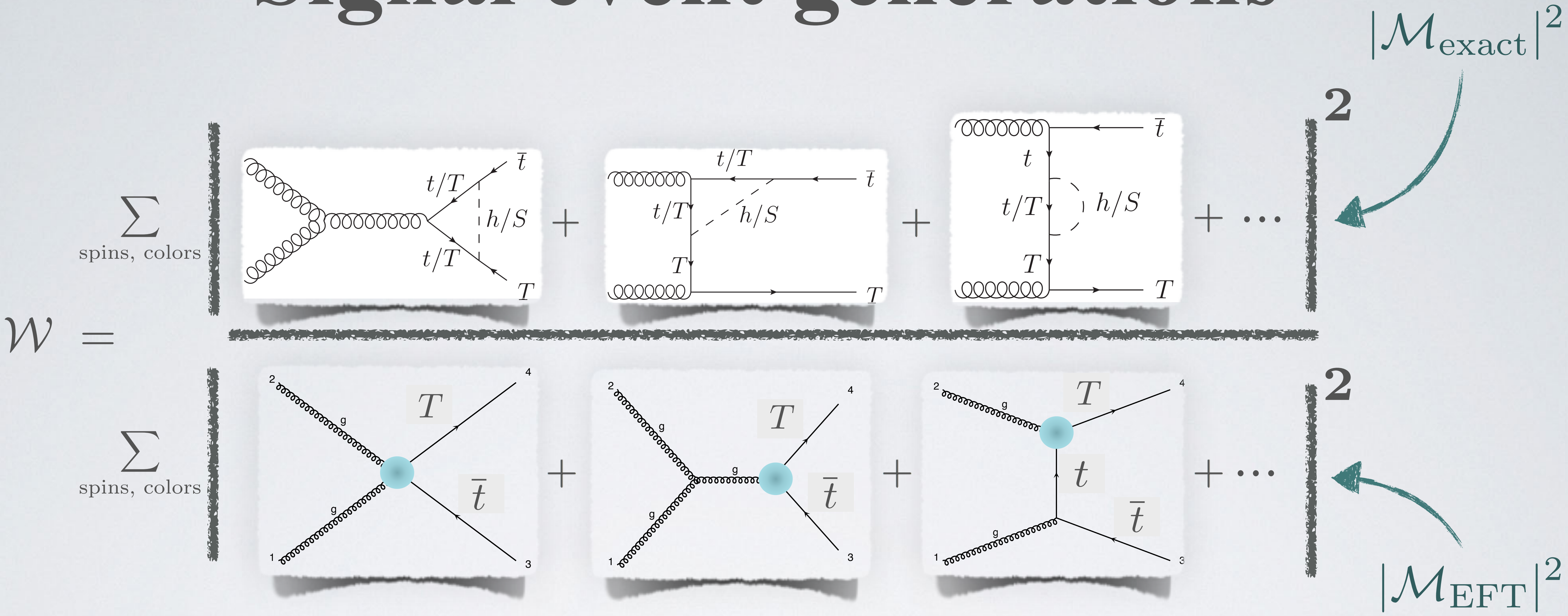
$\boxed{\sin\theta_L = 0}$

$\sigma(pp \to T\bar{t} + t\bar{T}) \propto \lambda_1^2 \lambda_2^2$

$\sigma^{\text{pair}}(pp \to T\overline{T})$ [fb]

- Comparisons with the QCD pair production in the (λ₁, λ₂) plane.
- There is a large parameter space where the *T t* productions outperform the QCD pair production.

Signal event generations



- We first generate events based on EFT-type contact interactions using MadGraph.
- We reweight the $|\mathscr{M}|^2$ of the EFT by $|\mathscr{M}|^2$ of the exact loop calculation of the theory on an event-by-event basis.
- The reweighted events are showered and hadronized by Pythia.

Background simulations

$\sqrt{s} = 14$ TeV

Abbreviations	Backgrounds	Matching	$\sigma \cdot \mathrm{BR}(\mathrm{fb})$
$t\bar{t}$	$t\bar{t} + \mathrm{jets}$	4-flavor	2.91×10^3 fb
Single t	$tW + \mathrm{jets}$	5-flavor	4.15×10^3 fb
	$tq + \mathrm{jets}$	4-flavor	77.2 fb
W	$W + \mathrm{jets}$	5-flavor	4.96×10^3 fb
VV	$WW + \mathrm{jets}$	4-flavor	111 fb
	$WZ + \mathrm{jets}$	4-flavor	43.5 fb

dominant backgrounds

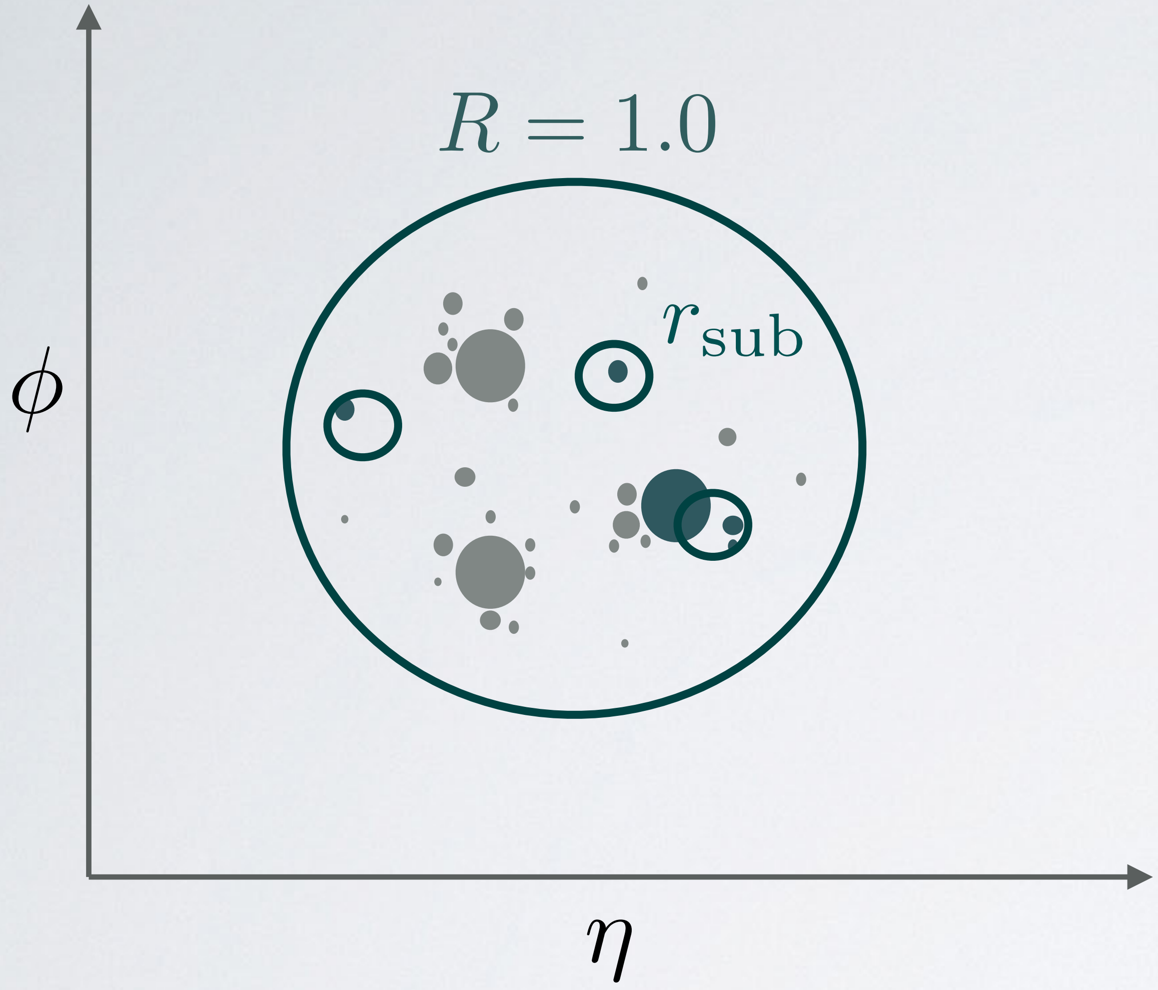
- We performed full background simulations, with generation-level cuts

$p_T > 30$ GeV and $|\eta| < 5$ (for partons)

$p_T^\ell > 30$ GeV and $|\eta^\ell| < 2.5$ (for leptons)

$H_T > 700$ GeV (scalar sum of p_T of all partons)

Template Overlap Method (TOM)

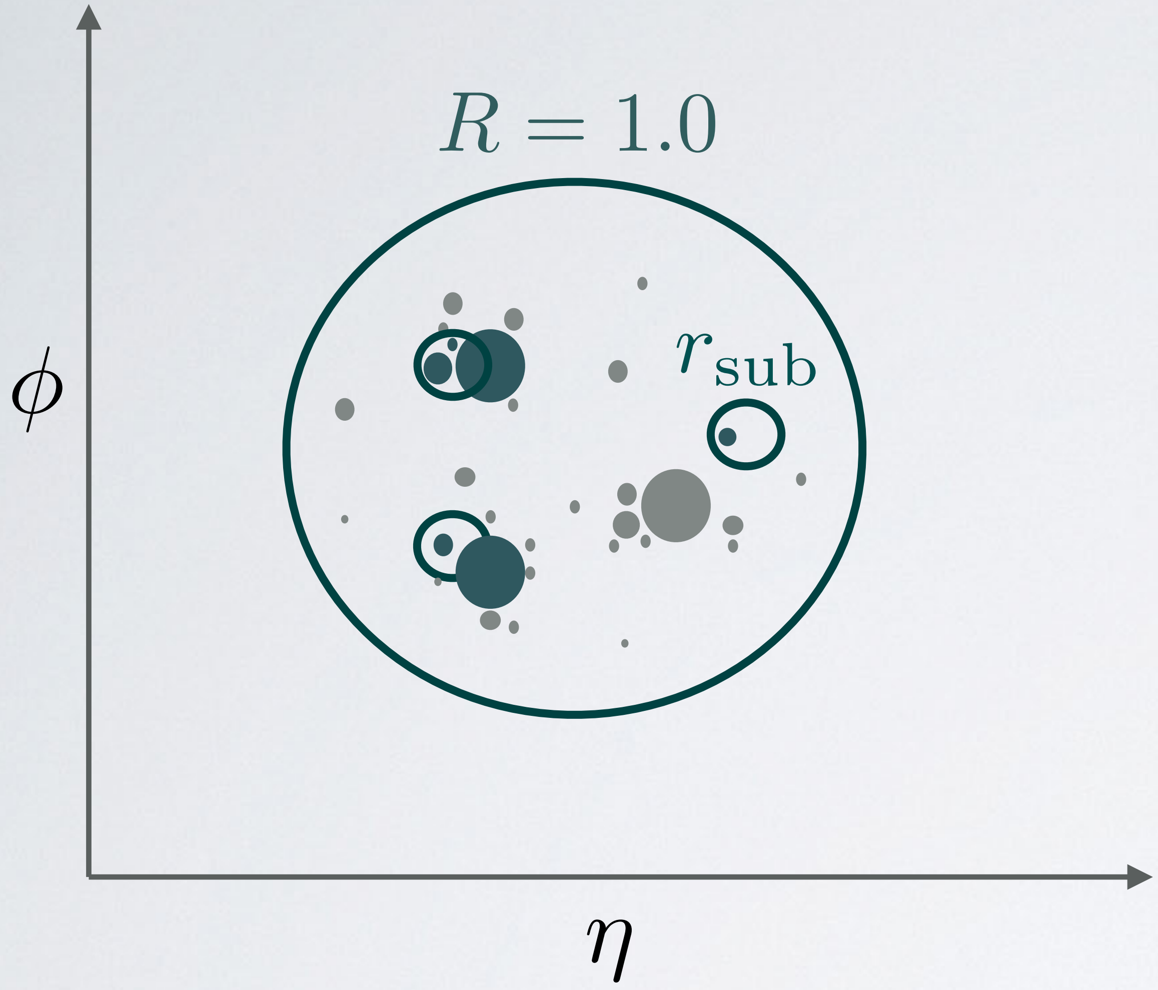


- **TOM** utilises kinematically constrained three **templates** within a top fat jet.
- Template partons are matched to jet energy distribution.
- Once found a good match it gives ``**Ov**`` score as an output variable.

$$\boxed{Ov} = max\ exp\left[-\sum_f \frac{1}{2\sigma_f^2}\left(\sum_j (\boxed{E_j} - \boxed{E_f}) F(f,j)\right)^2\right]$$

Probability

Template Overlap Method (TOM)



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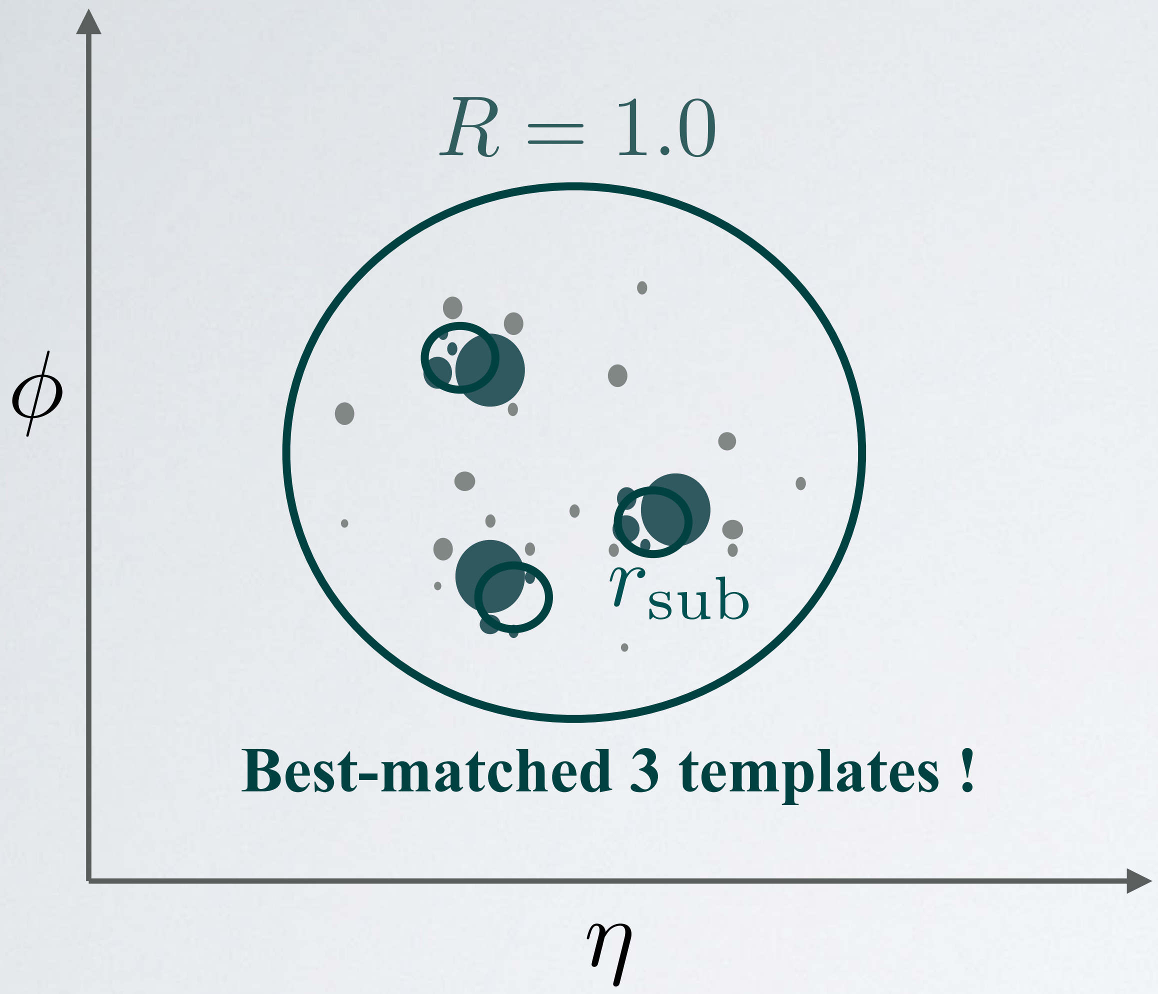
Probability

● ○ r_{sub}

Template Overlap Method (TOM)



Best-matched 3 templates !

- **TOM** utilises kinematically constrained three **templates** within a top fat jet.
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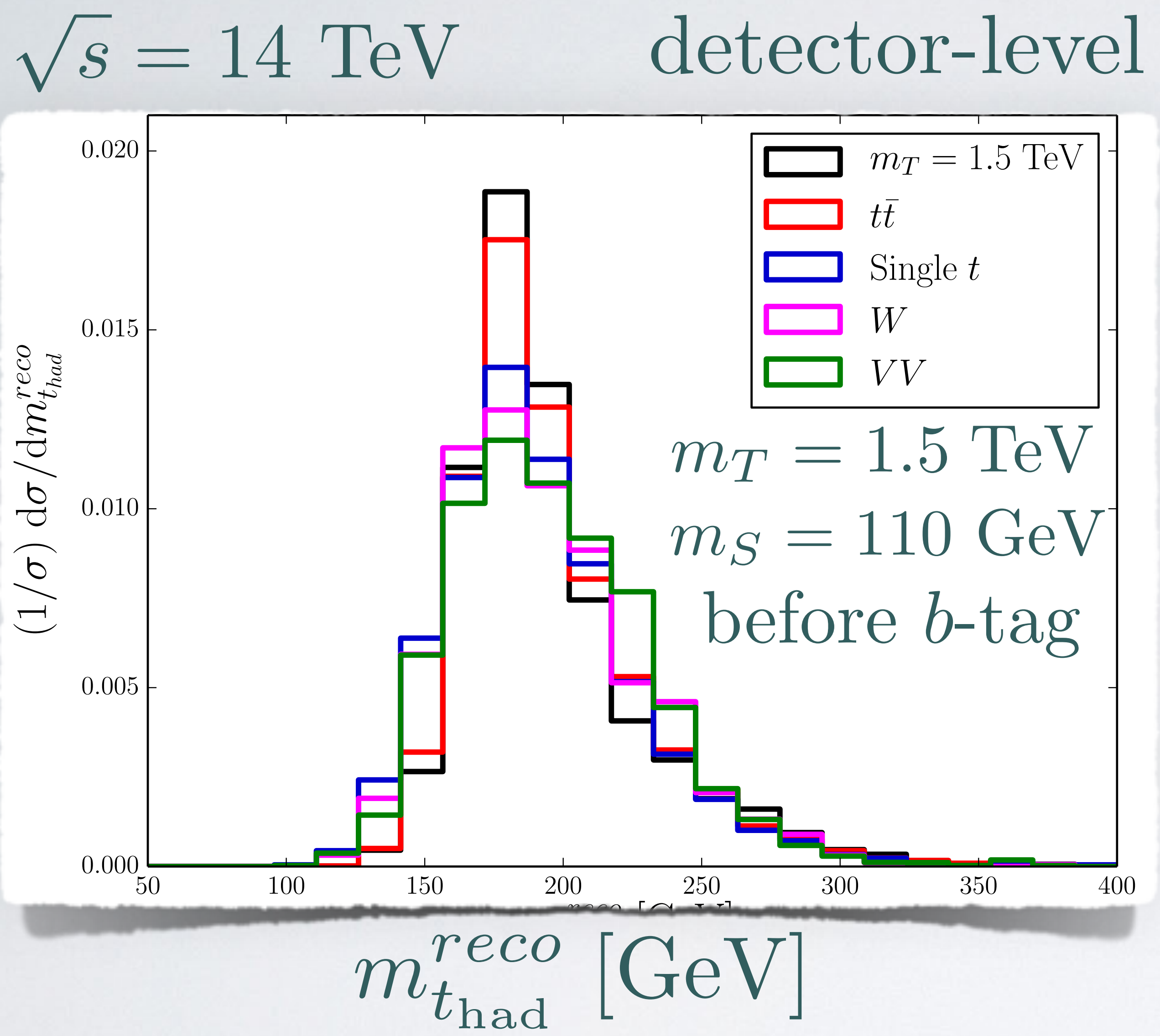
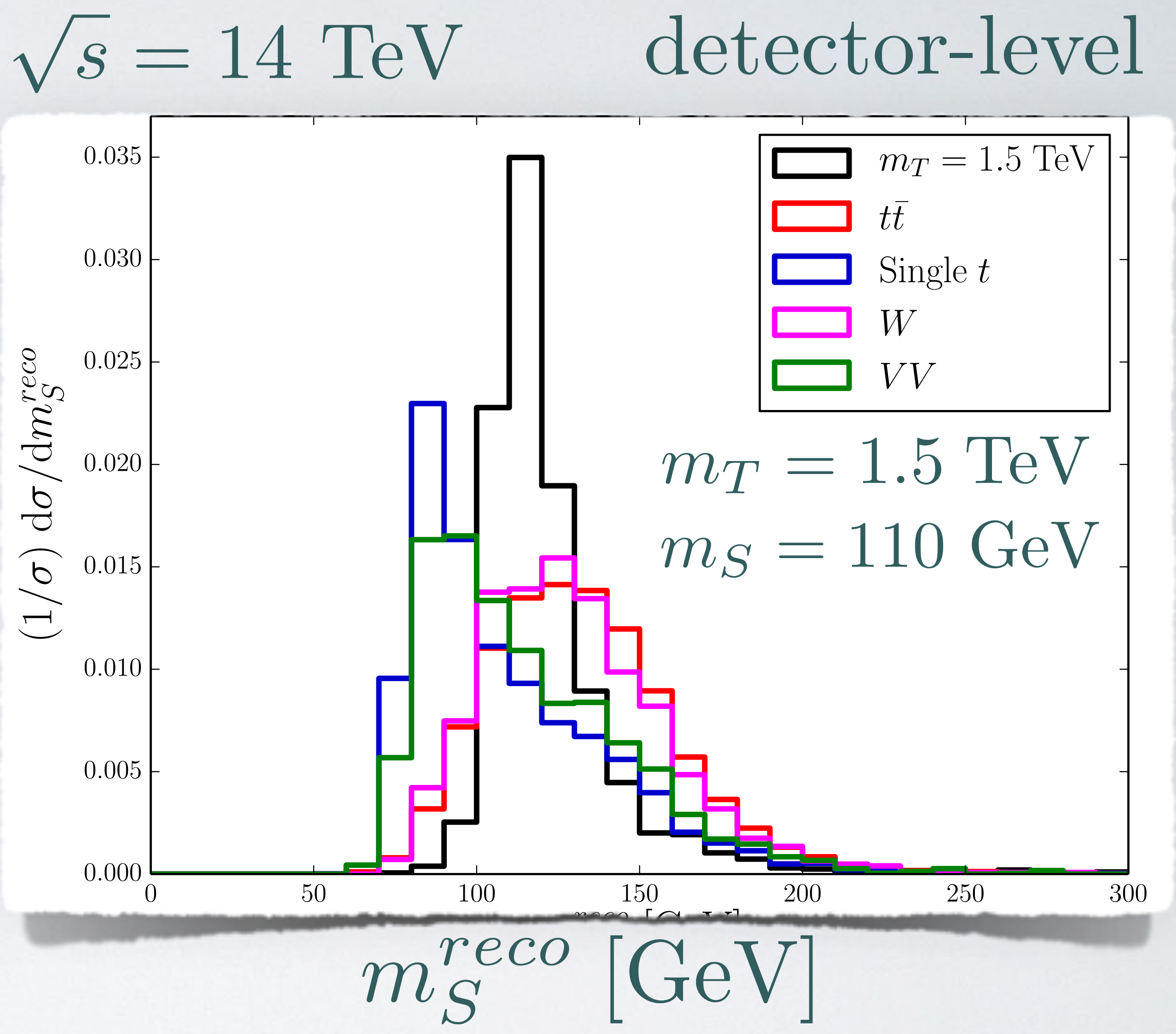
Probability

● ○ r_{sub}

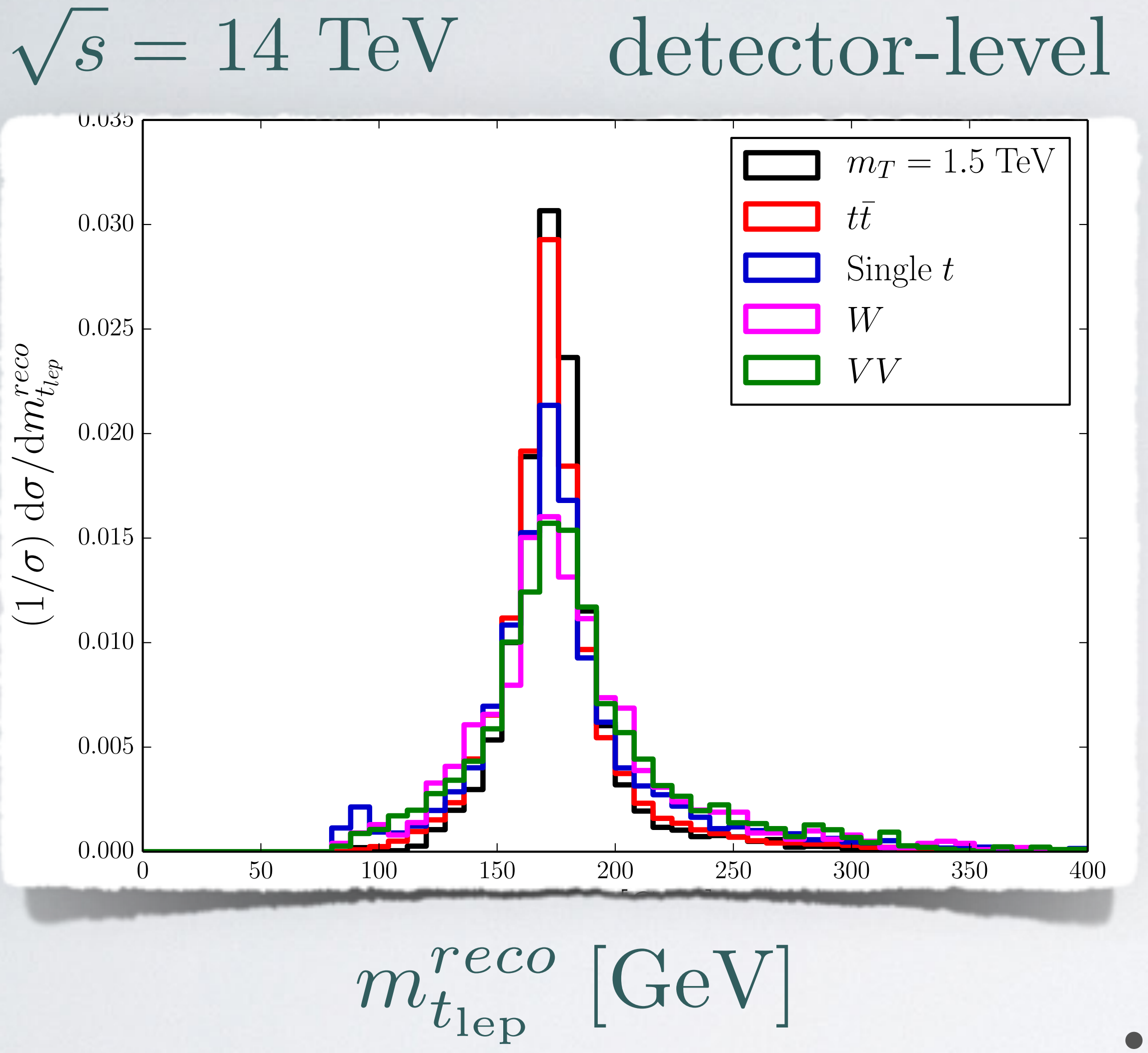
Reconstructed t_{had} and S





- Reconstructed invariant mass distribution of top-tagged fat jet.
- Reconstructed invariant mass distribution of scalar-tagged fat jet.

t_{lep} Reconstruction



- We solve a quadratic equation:

$$m_{\ell\nu}^2 = m_W^2 \quad (\text{ with } p_\nu^2 = 0 \text{ })$$

- To get two possible solutions for the neutrino longitudinal momentum:

$$p_L^\nu = \frac{1}{2\,(p_T^\ell)^2}\left(A\ p_L^\ell \pm |\vec{p}_\ell|\ \sqrt{A^2 - 4\ \left(p_T^\ell\right)^2\ \not{E}_T^2}\right)$$

$$(\text{where } A = m_W^2 + 2\vec{p}_T^{\,\ell} \cdot \vec{\not{E}}_T)$$

- To break the two fold-ambiguity, we choose the one which minimizes the quantity:

$$|m_{b\ell\nu}^2 - m_t^2|$$

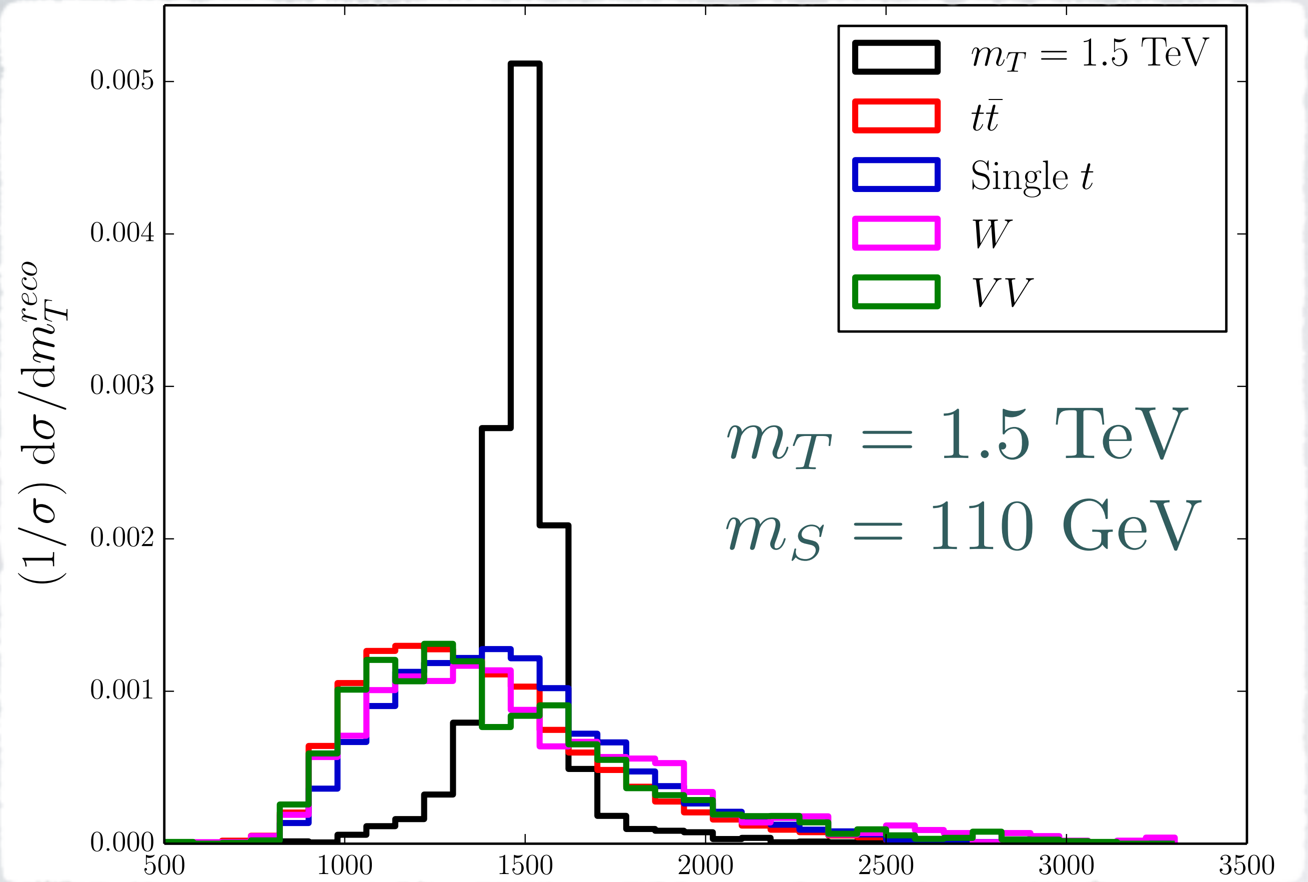
Based on

V. Barger, T. Han, D. G. E. Walker [2008]

S. Gopalakrishna, T. Han, I. M. Lewis,
Z. g. Si, Y. F. Zhou [2010]

Reconstructed *T* and *T t*-system

$\sqrt{s} = 14$ TeV detector-level

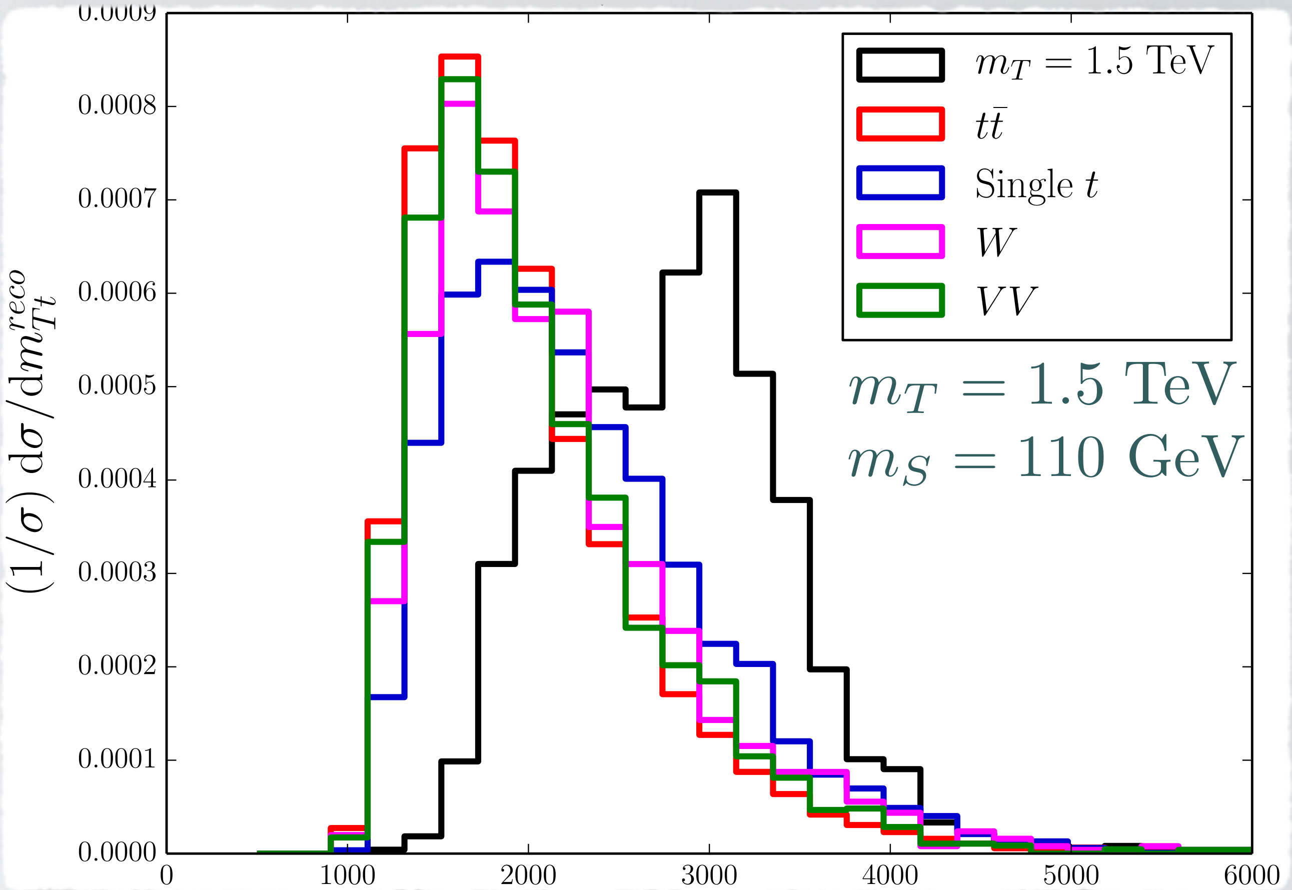


$m_T = 1.5$ TeV

$m_S = 110$ GeV

m_T^{reco} [GeV]

$\sqrt{s} = 14$ TeV detector-level



$m_T = 1.5$ TeV

$m_S = 110$ GeV

m_{Tt} [GeV]

- Reconstructed invariant mass distribution of *T* displays a sharp peak.
- When $m_{Tt} \sim 2m_T$, the internal top partners can go on-shell, giving rise to the peak in the m_{Tt} distribution. This can be clearly seen even at the detector level, helping to suppress the backgrounds.

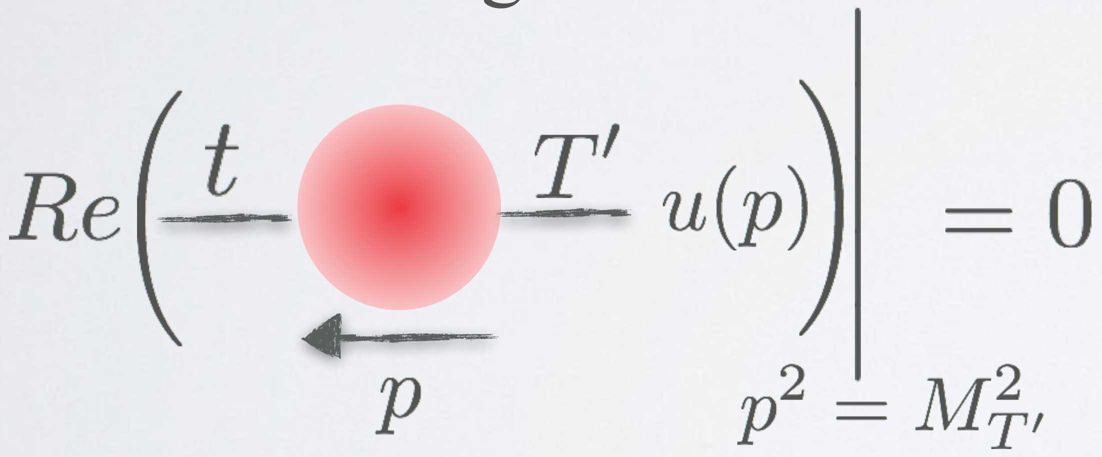
Renormalizing the Lagrangian

- Wave function renormalization constants (w.f.c.) for fermions

$$\begin{bmatrix} t_{L0} \\ T'_{L0} \end{bmatrix} \simeq \begin{bmatrix} 1+\frac{1}{2}\delta Z^L_{tt} & \frac{1}{2}\delta Z^L_{tT} \\ \frac{1}{2}\delta Z^L_{Tt} & 1+\frac{1}{2}\delta Z^L_{TT} \end{bmatrix} \begin{bmatrix} t_L \\ T'_L \end{bmatrix} \quad , \quad \begin{bmatrix} t_{R0} \\ T'_{R0} \end{bmatrix} \simeq \begin{bmatrix} 1+\frac{1}{2}\delta Z^R_{tt} & \frac{1}{2}\delta Z^R_{tT} \\ \frac{1}{2}\delta Z^R_{Tt} & 1+\frac{1}{2}\delta Z^R_{TT} \end{bmatrix} \begin{bmatrix} t_R \\ T'_R \end{bmatrix}$$

$$\begin{bmatrix} \bar{t}_{L0} \\ \bar{T}'_{L0} \end{bmatrix} \simeq \begin{bmatrix} 1+\frac{1}{2}\delta \bar{Z}^L_{tt} & \frac{1}{2}\delta \bar{Z}^L_{tT} \\ \frac{1}{2}\delta \bar{Z}^L_{Tt} & 1+\frac{1}{2}\delta \bar{Z}^L_{TT} \end{bmatrix} \begin{bmatrix} \bar{t}_L \\ \bar{T}'_L \end{bmatrix} \quad , \quad \begin{bmatrix} \bar{t}_{R0} \\ \bar{T}'_{R0} \end{bmatrix} \simeq \begin{bmatrix} 1+\frac{1}{2}\delta \bar{Z}^R_{tt} & \frac{1}{2}\delta \bar{Z}^R_{tT} \\ \frac{1}{2}\delta \bar{Z}^R_{Tt} & 1+\frac{1}{2}\delta \bar{Z}^R_{TT} \end{bmatrix} \begin{bmatrix} \bar{t}_R \\ \bar{T}'_R \end{bmatrix}$$

- For off-diagonal w.f.c. we use on-shell renormalization conditions:



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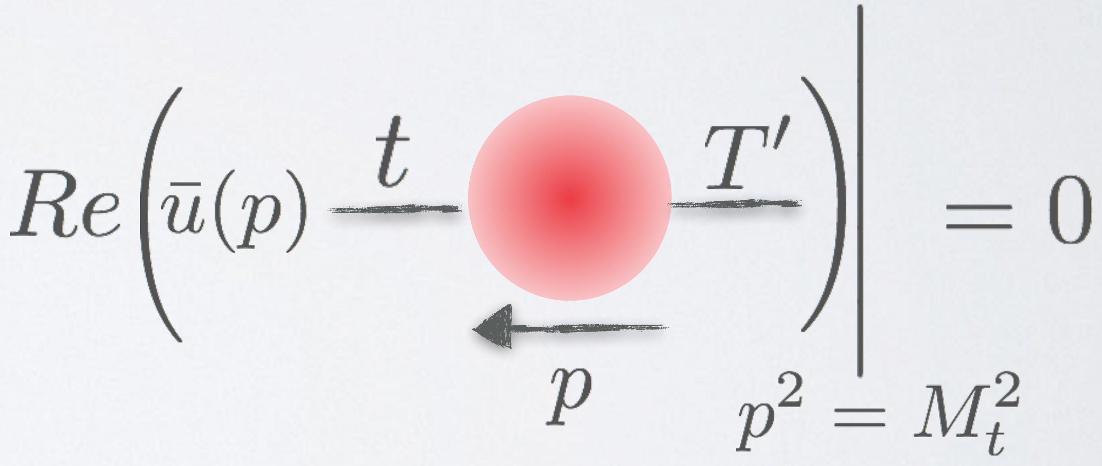
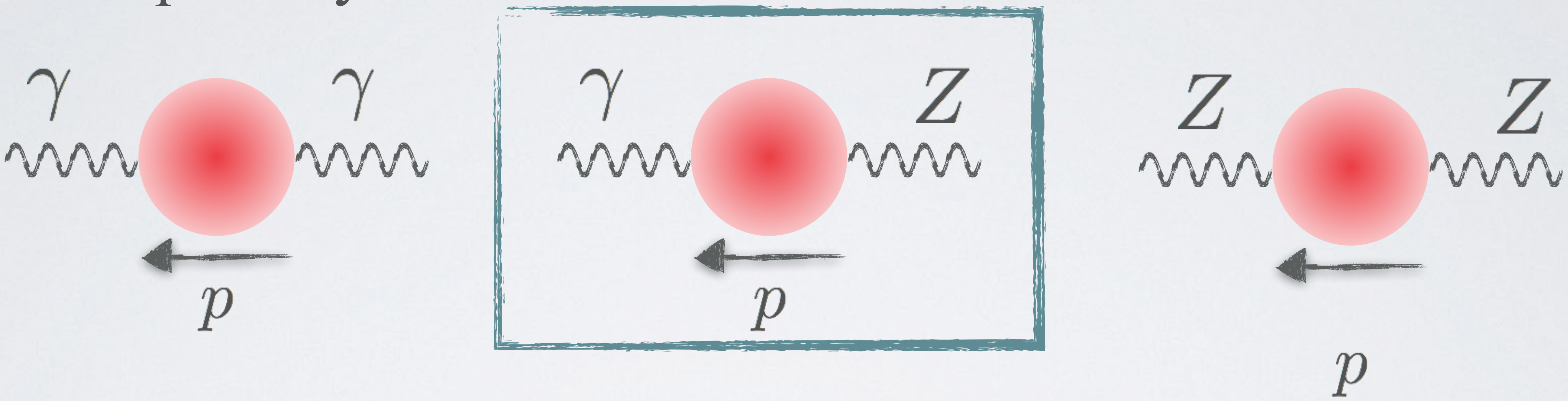


- For diagonal w.f.c. use the mass pole and unite residue conditions.

Renormalizing the Lagrangian

- Due to mixing, we can't renormalize the photon and *Z* boson fields separately.



- Wave function renormalization constants (w.f.c.) for *A* and *Z* fields

$$\begin{bmatrix} A_0 \\ Z_0 \end{bmatrix} \simeq \begin{bmatrix} \sqrt{Z_\gamma} & -\Delta_Z - \Delta_0 \\ \Delta_0 & \sqrt{Z_Z} \end{bmatrix} \begin{bmatrix} A \\ Z \end{bmatrix}$$

$$\Delta_0 = \frac{\Pi_{\gamma Z}(0)}{M_Z^2} \qquad \Delta_Z = \frac{Re[\Pi_{\gamma Z}(M_Z^2)] - \Pi_{\gamma Z}(0)}{M_Z^2}$$

$T \rightarrow t\ g$