From Dipoles, to Waves and Strings

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Symposium for Al's 70th birthday
Columbia U., New-York, October 22-25 2009

- Dipoles,Waves,Strings

A Tribute to Al

- From Dipoles to Waves

Old and New problems

- From Dipoles to Strings

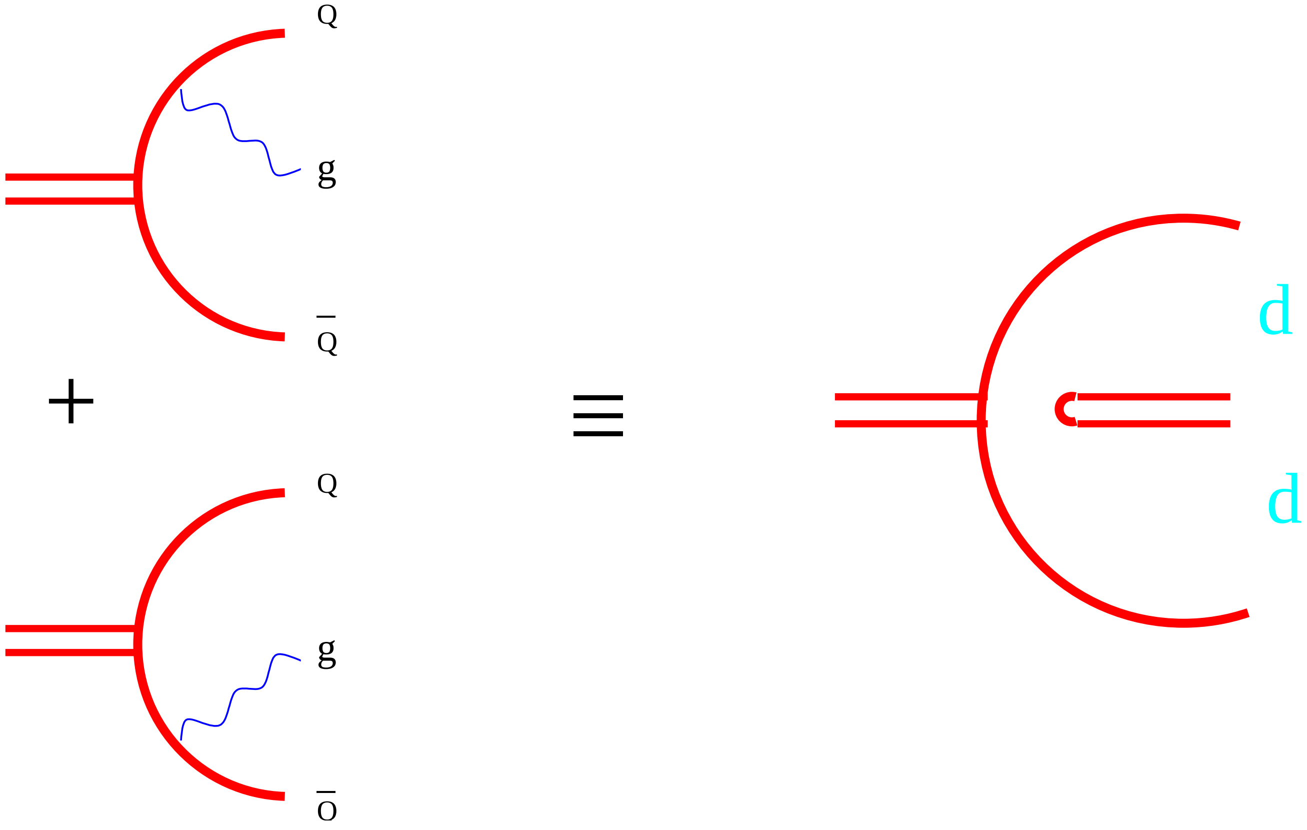
AdS/CFT, Hydrodynamics, and Beyond

The Dipole Paradigm

“Soft gluons in the infinite momentum wave function and the BFKL pomeron” Alfred H. Mueller, Cited 608 times.

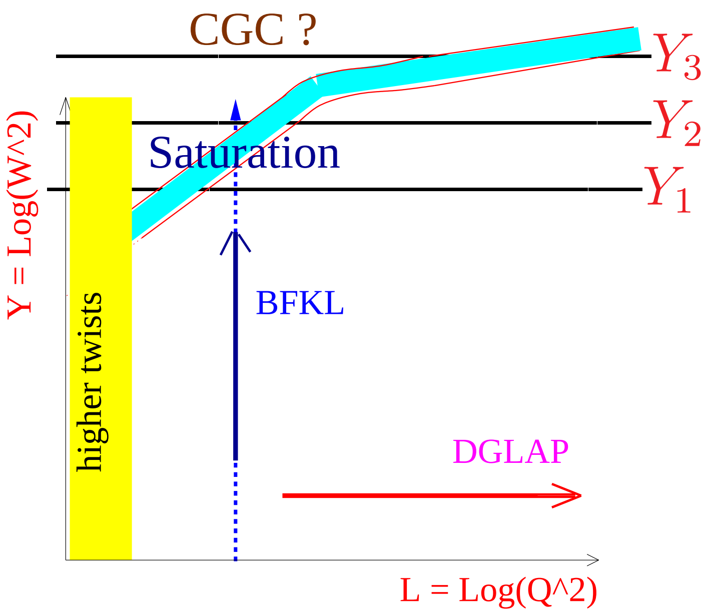
“Single and double BFKL pomeron exchange and a dipole picture of high-energy hard processes.” with Bimal Patel, Cited 407 times.

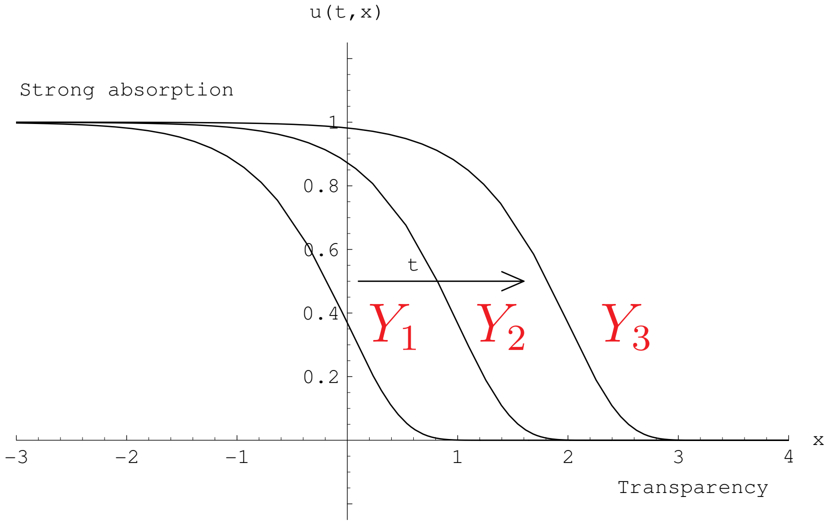
“Unitarity and the BFKL pomeron.” Alfred H. Mueller, Cited 332 times.



From Dipoles to Waves

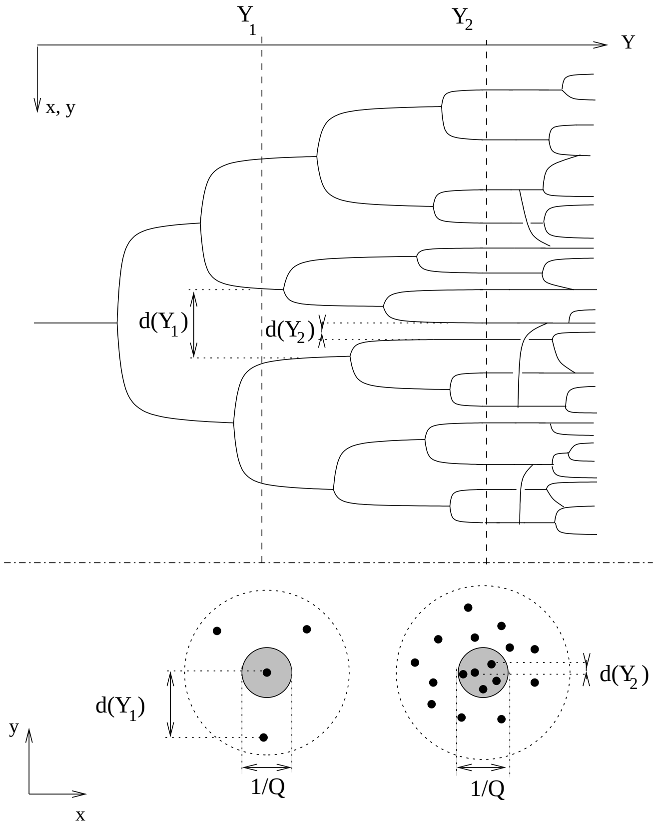
- Saturation and Traveling Waves





Dipoles as Branching

The Cascading Tree of Dipoles



Dilute Region : Exponential growth: BFKL

Transition Region : Transition to Saturation

BFKL → BK, JIMWLK, Fluctuations

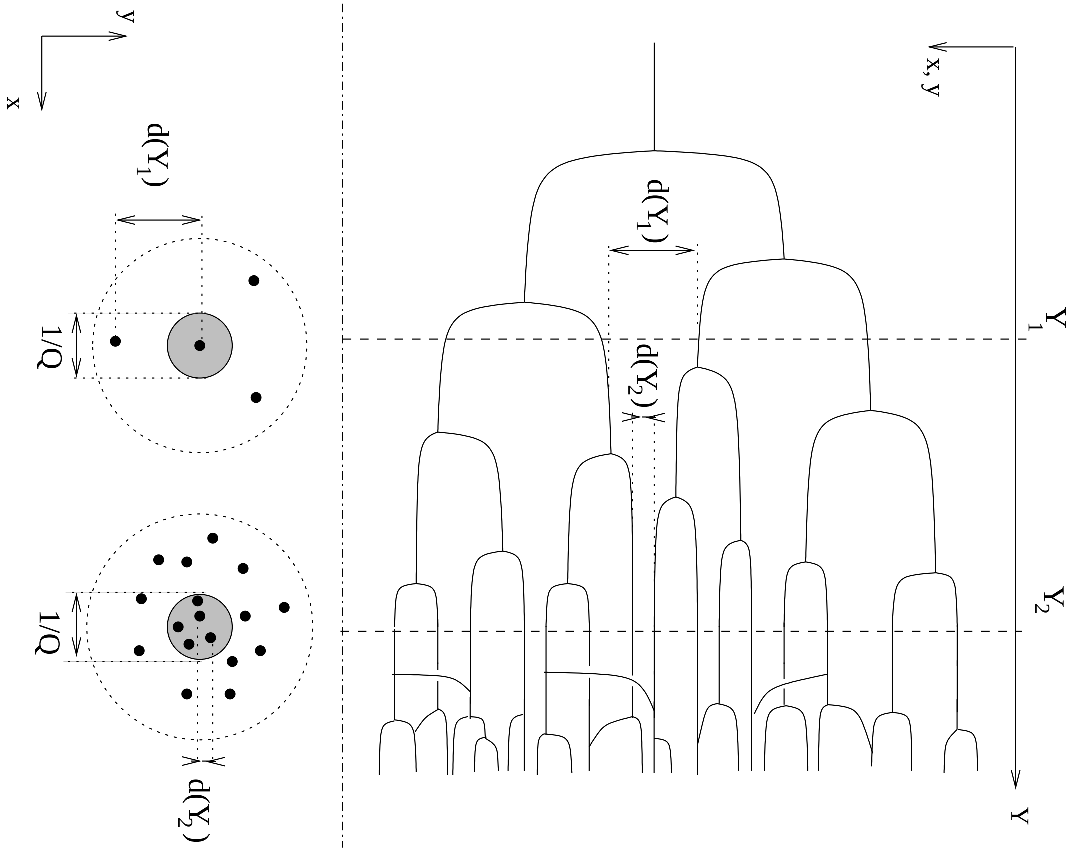
Dense Region : CGC?

Related Physics (*via* 90^o Rotation)

- Random Energy Models   Branching Polymers

  FKPP Traveling waves for $\left\langle e^{ix\mathcal{Z}} \right\rangle_t$; $\mathcal{Z} = \sum_i e^{\beta E_i}$

  Derrida, Spohn (1988)



- QCD Parton shower and Intermittency   with A.Bialas (1988)

$$\begin{aligned} \mathit{Time} &= (\ Y \rightarrow)\log Q^2 \\ \mathit{Space} &= (\log k^2 \rightarrow) y \end{aligned}$$

- Reggeon Field Theory   (2009)

$$\begin{aligned} \mathit{Time} &= Y \\ \mathit{Space} &= \vec{b} \end{aligned}$$

The "Generalized" BK equation

Work in progress

- The BFKL Evolution Operator

Diffusive Approximation :

$$\boxed{\chi(-\partial_L) \equiv 2\psi(1) - \psi(-\partial_L) - \psi(1+\partial_L) \sim A_0 + A_1\partial_L + A_2\partial_L^2 + \mathcal{O}(\partial_L^3)}$$

$(L \equiv \log k^2/\Lambda_{QCD}^2)$

A_2 : Diffusion term
A_1 : "Drift" term
A_0 : "Birth" (- "Merging") term
and κ : "Noise" ($\equiv$ "Splitting") term

- The "Generalized" BK Equation

$$\boxed{L^n\, \partial_Y N(L,Y) = \left\{A_2\partial_L^2 + A_1\partial_L\right\} N + A_0\left(N - N^2\right) + \sqrt{\kappa\ N}\ \nu(L,Y)}$$

- "Constant" BK: $n = 0, \kappa = 0$ ⇒ F-KPP

S.Munier, R.P., 2003,2004

- "Fluctuating" BK: $n = 0, \kappa \neq 0$ ⇒ sF-KPP

S.Munier; E.Iancu, A.Mueller, S.Munier 2004

- "Running" BK: $n = 1, \kappa = 0$ or $\neq 0$ ⇒ Problems !?

L. Albacete, Y. Kovchegov (2007)
A. Dumitru, E. Iancu, L. Portugal, , G. Soyez, D. Triantafyllopoulos (2007)
G.Beuf(2008)

- "Generalized" BK: $\forall\ n, \kappa$

Mapping to (s)FKPP “Universality Class”

- Change of Variables ⇒ Standard Diffusion

$$\boxed{L = X^{\beta} \; ; \quad \partial_L = \frac{X^{1-\beta}}{\beta}\,\partial_X \; ; \quad \beta = \frac{2}{n+2}\left(= \frac{2}{3}\ for\ n{=}1\right)}$$

- “radial sFKPP” Equation

$$\boxed{\frac{\partial N(X,Y)}{\partial Y} = \tilde{A}_2\,\partial_X^2 N + (\tilde{A}_d + \tilde{A}_1)\partial_X N + \tilde{A}_0\left(N - N^2\right) + \sqrt{\tilde{\kappa} N}\;\nu(X,Y)}$$

$\tilde{A}_2 = \frac{A_2}{\beta^2}$: Diffusion Constant

$\frac{\tilde{A}_d}{\tilde{A}_2} = (1-\beta)X^{-1}$: “Fractal Dimension” $2-\beta$

$\tilde{A}_1 = \frac{A_1}{\beta}X^{-(1-\beta)}$: X-decreasing “Drift”

$\tilde{A}_0 = X^{-2(1-\beta)}A_0$: X-decreasing “Birth + Merging”

$\tilde{\kappa} = \frac{\kappa}{\beta}X^{-3(1-\beta)}$: X-decreasing Noise ≡ Loop term

- Consequences

  i) Hierarchy at large X $\boxed{\tilde{A}_2 \gg \tilde{A}_1 \gg \tilde{A}_0 \gg \tilde{\kappa}}$

  ii) $(2-\beta)$-d *radial* sFKPP in a “negative gradient”
  (cf. Chemistry or Heat Eq. in Cylinder)

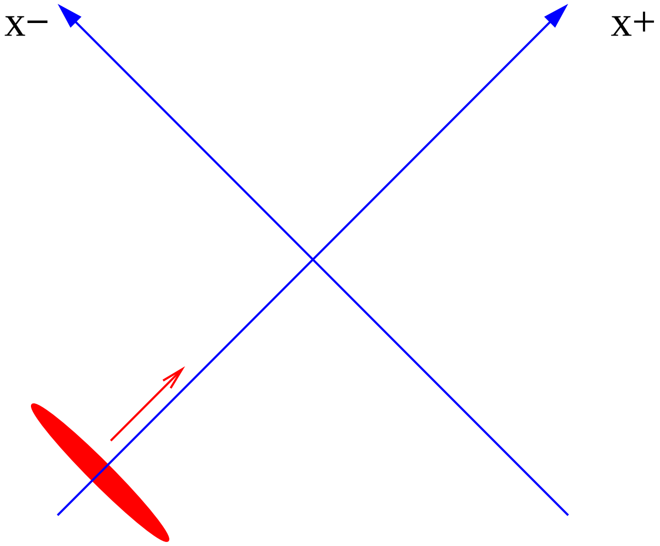
  iii) Intriguing $n \to \infty$ rescaling limit: 2-d sFKPP

$$\boxed{\partial_t U(X,T) = \partial_X^2 U + X^{-1}\partial_X U + \lambda X^{-2} U(1-U) + \sqrt{\varepsilon X^{-3} U}\,\eta(X,T)}$$

From Dipoles to Strings

with Romuald Janik (2005-$\cdots$)

- Saturation: Describing the Target ?



- The Dual Shock Wave

$$\boxed{ds^2 = \frac{-2dx^+dx^- + \mu_1 z^4 F(x^-)\{cf.\ \delta(x^-)\}dx^{-2} + d\mathbf{x}_\perp{}^2 + d}{z^2}}$$

Extension: $F(x^-) \to F(x^-, x_\perp, z)$ G.Beuf (2009)

- Dipole-Shock Wave Scattering

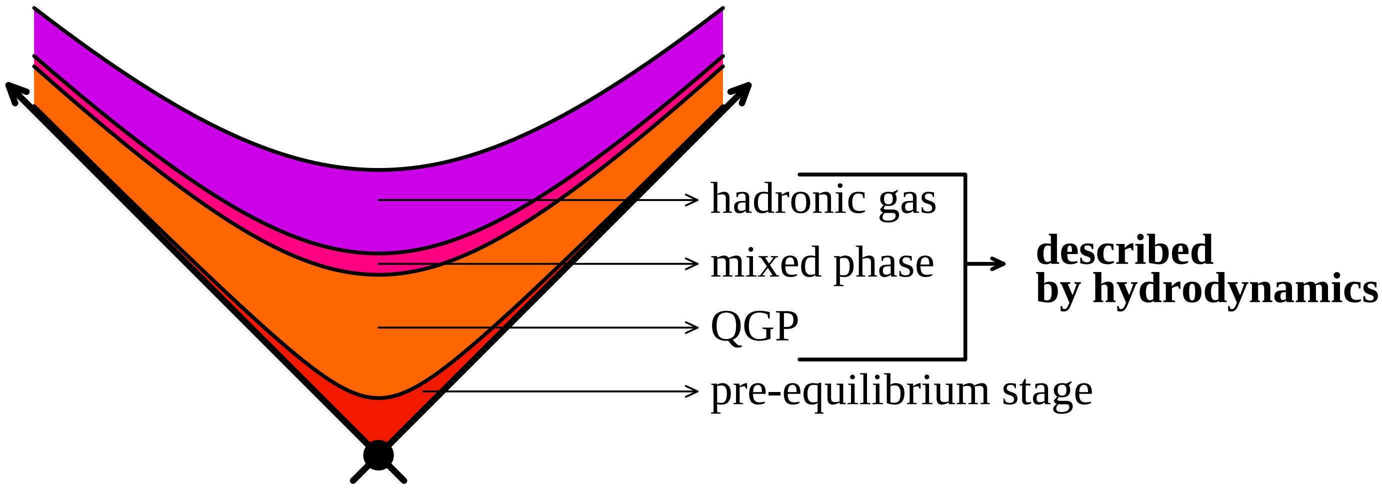
cf. Albacete,Kovchegov,Taliotis (2008)

- Shock-Wave Collisions

Grumiller,Romatschke ;Albacete,Kovchegov,Taliotis (2008)

- Boost-Invariant Approach

Gauge/Gravity and Boost-invariant Dynamics



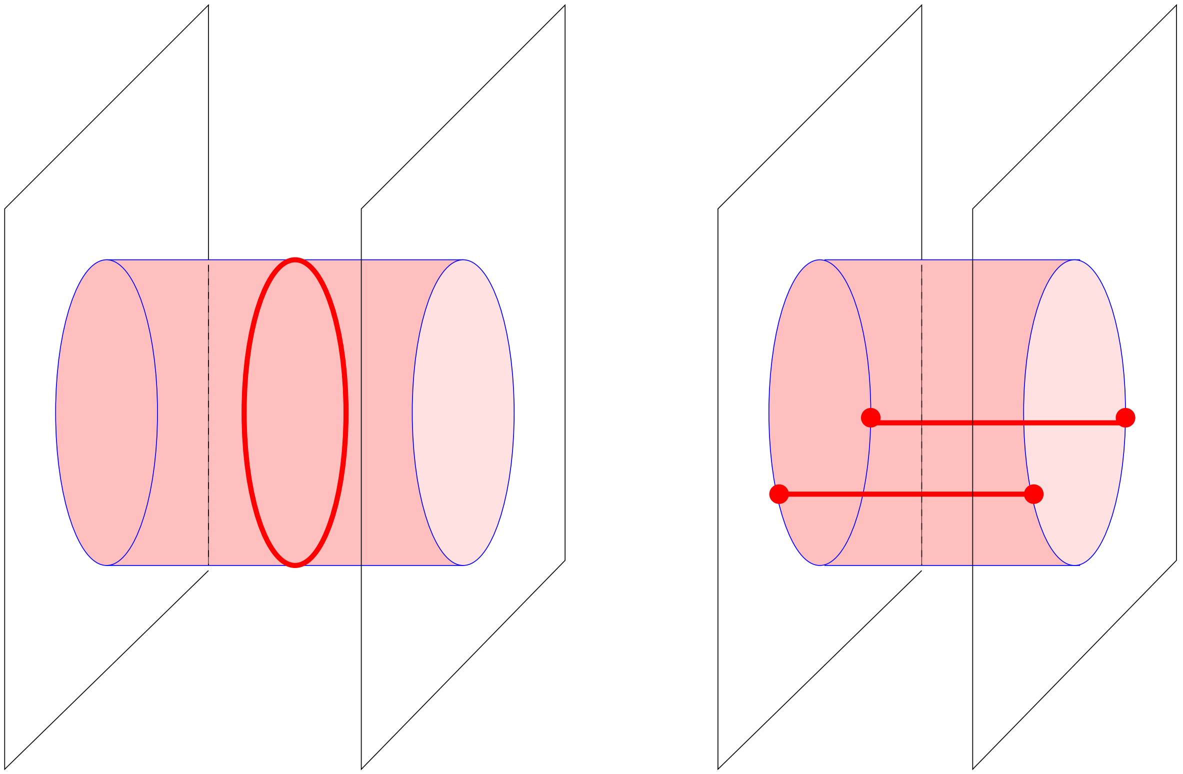
$$\boxed{\tau = \sqrt{x_0^2 - x_1^2}\ ; y = \frac{1}{2}\log\frac{x_0 + x_1}{x_0 - x_1}\ ;\ x_T = x_2, x_3}$$

Questions

- What is the Gravity Dual of a Flow ?
- QGP: (almost) Perfect fluid behaviour ?
- Why Hydro: η/s, Transport coefficients, Navier-Stokes ?
- Out-of-Equilibrium, Thermalization, Isotropization ?

I. The Gauge-Gravity Duality

Open String ⇔ Closed String



Schomerus, 2006

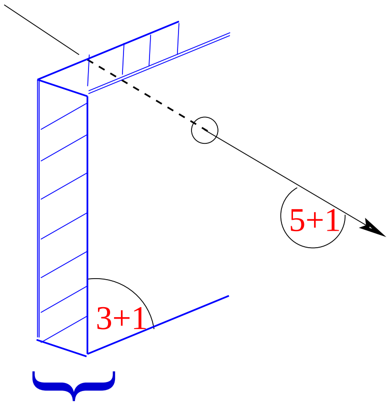
$$\begin{array}{rcl} Closed\ String & \Leftrightarrow & 1 - loop\ Open\ String \\ D{-}Brane\ \text{“}Universe\text{”} & \Rightarrow & Open\ String\ Ending \\ Gravity & \Leftrightarrow & Gauge \\ Large/Small\ Distance & \Rightarrow & Gravity/Gauge\ Correspondence \end{array}$$

AdS/CFT Correspondence

J.Maldacena, 1998

GRAVITY

AdS



Gravity Source

Macroscopic

$(g_s N)^{-1}$ -> 0

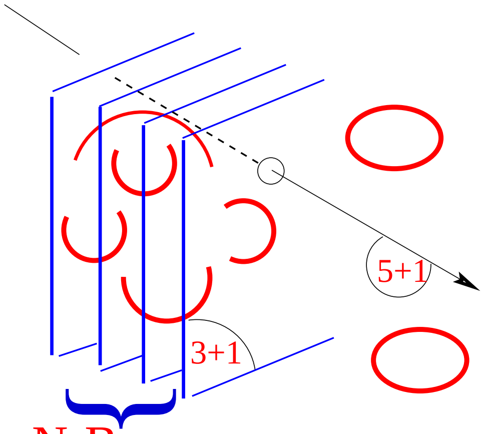
AdS_5 x S_5 Superstring

Weak Gravity

Strong Gravity

GAUGE

CFT



N-Branes

Microscopic

$g_s N$ -> 0

SU(N) Gauge Theory on the N-Branes

Strong Coupling

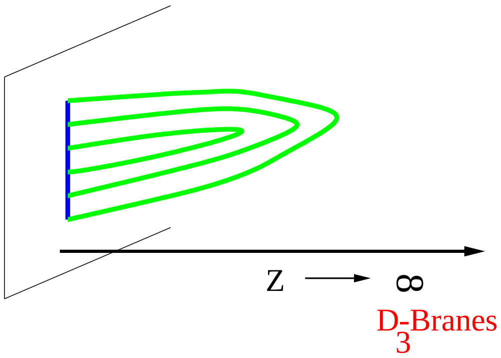
Weak Coupling

⟺

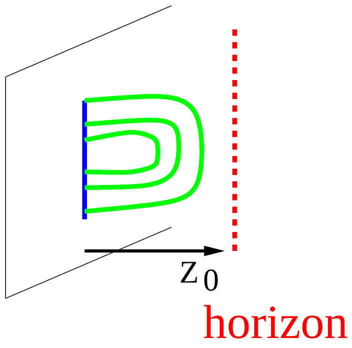
Duality

HOLOGRAPHY

- Holographic Principle: Brane/Bulk correspondence



$\boxed{AdS_5 \ \otimes S_5}$



$\boxed{Confining\ Geometry}$

- Brane $\longrightarrow$ Bulk: Holographic Renormalization

K.Skenderis, 2002

$$\boxed{ds^2 = \frac{g_{\mu\nu}(z)\ dx^\mu dx^\nu + dz^2}{z^2}}$$

$$\boxed{g_{\mu\nu}(z) = g^{(0)}_{\mu\nu}(=\eta_{\mu\nu}) + z^2 g^{(2)}_{\mu\nu}(=0) + z^4\,\langle T_{\mu\nu}\rangle \quad + z^6 \ldots +}$$

$+\,z^6 \ldots +$: from Einstein Eqs.

II. The late time flow

- Boost-invariant T^{μ}_{ν}

$$T_{\mu\nu}=\begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3\frac{d}{d\tau}f(\tau)-\tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau)+\frac{1}{2}\tau\frac{d}{d\tau}f(\tau) & 0 \\ 0 & 0 & 0 & \dots \end{pmatrix}$$

- Proper-time evolution

$f(\tau) \propto \tau^{-S}$: Family Index $\mathrm{T}_{\mu\nu}\mathrm{t}^{\mu}\mathrm{t}^{\nu} \geq 0 \Rightarrow 0 < \mathrm{s} < 4$
$f(\tau) \propto \tau^{-\frac{4}{3}}$: Perfect Fluid
$f(\tau) \propto \tau^{-1}$: Free streaming
$f(\tau) \propto \tau^{-0}$: Full Anisotropy $\epsilon = p_{\perp} = -p_L$

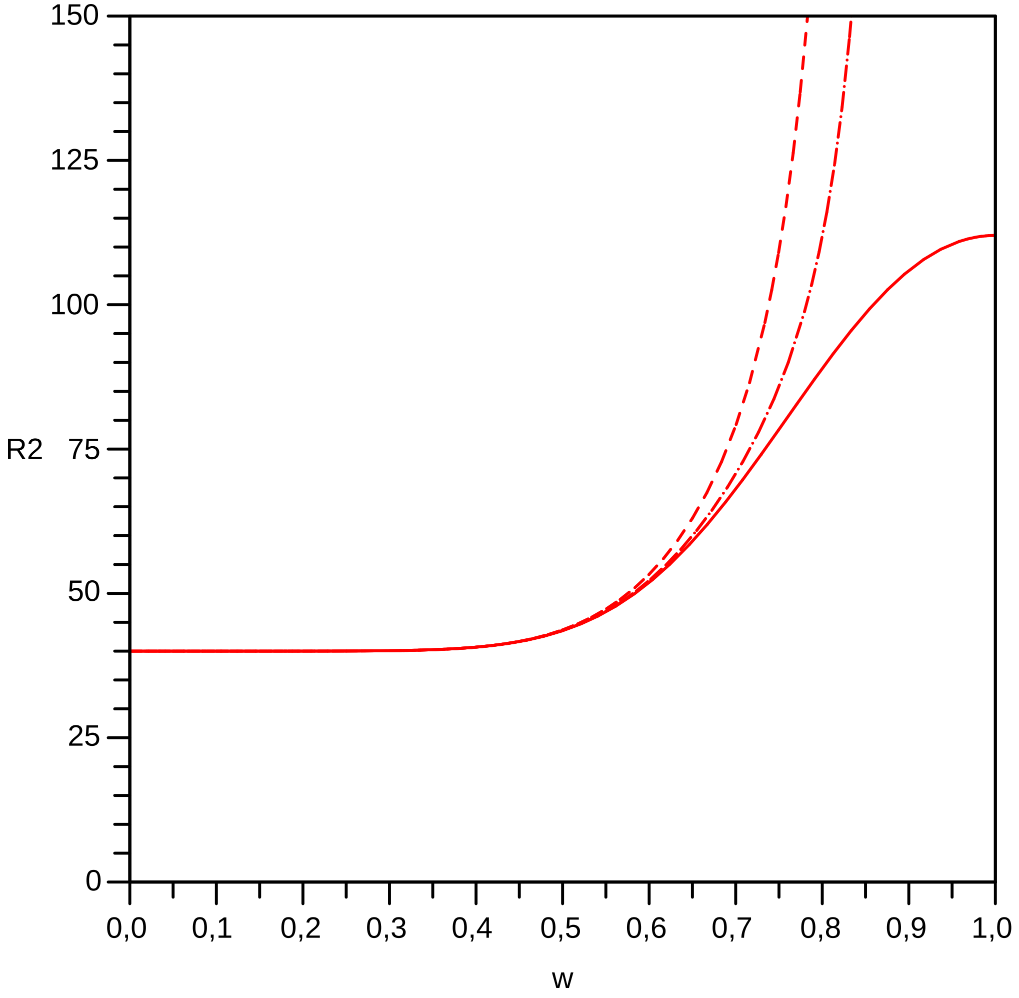
- Holographic renormalization: $\Rightarrow$

Holographic Scaling Variable v at large τ

$$\boxed{v = \frac{z}{\tau^{S/3}}}$$

AdS/CFT $\Rightarrow$ Perfect Fluid at large τ

Kreschtmann Scalar: $\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$



$s = \frac{4}{3} \pm .1$

“MicroCosmic” Censorship

A nonsingular background selects a moving Black Hole geometry dual to the perfect fluid at large proper-times

The Cooling Plasma/Moving Black Hole Duality

$$\boxed{v = \frac{z}{\tau^{1/3}}}$$

- Asymptotic metric

$$\boxed{ds^2 = \frac{1}{z^2}\left[-\frac{\left(1-\frac{e_0}{3}\ \frac{z^4}{\tau^{4/3}}\right)^2}{1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2+\left(1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)\left(\tau^2 dy^2+dx_\perp^2\right)\right]+\frac{dz^2}{z^2}}$$

- BH off in the 5th dimension ⇔ Hwa-Bjorken flow

$$Horizon:\ \ z_h = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \cdot \tau^{\frac{1}{3}}\ . \tag{1}$$

$$Temperature:\ \ T(\tau) \sim \frac{1}{z_h} \sim \tau^{-\frac{1}{3}} \tag{2}$$

$$Entropy:\ \ S(\tau) \sim Area \sim \tau \cdot \frac{1}{z_h^3} \sim const \tag{3}$$

Hydro beyond the Perfect fluid

- Going beyond perfect fluid

  In-flow Viscosity, Relaxation time, Transport Coeff., etc...

  Janik, Heller, Bak, Benincasa, Buchel, Nakamura, Sin,......
  Kinoshita, Mukoyama, Nakamura, Oda,Natsuume, Okamura,...

$$\boxed{\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{\eta}{\tau^2} + \cdots \Rightarrow \frac{\eta}{s} = \frac{1}{4\pi}}$$

- Going beyond boost-invariance

  Fluid/Gravity Duality

  Bhattacharyya, Hubeny, Minwalla, Ranganami, Loganayagam,...

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2T^{\mu\nu}_{2a} + 2T^{\mu\nu}_{2b} + (2-\log 2)\left(\frac{1}{3}T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- Going Beyond hydrodynamics? Beyond Equilibrium?

III. Early-time Boost-Invariant Flow

- General Boost-Invariant Fefferman-Graham metric:

$$\boxed{ds^2 = \frac{-e^{\,a(\tau,z)}\,d\tau^2 + \tau^2 e^{\,b(\tau,z)}\,dy^2 + e^{\,c(\tau,z)}\,dx_\perp^2}{z^2} + \frac{dz^2}{z^2}}$$

- Einstein Equation:

$$\boxed{R_{AB} + 4G_{AB} = 0}$$

- To be solved: $(\dot{a} = \partial_\tau a; a' = \partial_z a; \cdots)$

$$(\tau\tau) : \ddot{b}+2\ddot{c}-\frac{\dot{a}}{2}(\dot{b}+2\dot{c})+\frac{1}{2}(\dot{b}^2+2\dot{c}^2)-\frac{1}{\tau}(\dot{a}-2\dot{b}) = e^a\left\{a''-\frac{3a'}{z}+\left(\frac{a'}{2}-\frac{1}{z}\right)(a'+b'+2c')\right\}$$

$$(yy) : \ddot{b}-\dot{a}\dot{b}+\frac{1}{\tau}(\dot{b}-2\dot{a})+\frac{1}{2}(\dot{a}+\dot{b}+2\dot{c})\left(\dot{b}+\frac{2}{\tau}\right) = e^a\left\{b''-\frac{3b'}{z}+\left(\frac{b'}{2}-\frac{1}{z}\right)(a'+b'+2c')\right\}$$

$$(\perp\perp) : \ddot{c} - \dot{a}\dot{c} + \frac{\dot{c}}{2}\left(\dot{a} + \dot{b} + 2\dot{c} + \frac{2}{\tau}\right) = e^a\left\{c'' - \frac{3c'}{z} + \left(\frac{c'}{2} - \frac{1}{z}\right)(a' + b' + 2c')\right\}$$

$$(\tau z) : 2\dot{b}' + 4\dot{c}' + b'\left(\dot{b}+\frac{2}{\tau}\right) + 2\dot{c}c' - a'\left(\dot{b} - 2\dot{c} + \frac{2}{\tau}\right) = 0$$

$$(zz) : a'' + b'' + 2c\text{””} - \frac{1}{z}(a' + b' + 2c') + \frac{1}{2}(a'^2 + b'^2 + 2c'^2) = 0$$

Early-Time; General Features

G.Beuf, M.Heller, R.Janik, R.P., 2009

- Dependence on Initial Conditions

$$\boxed{a(\tau,z) = \ldots + z^8\ \{-1/16\,\tau^{-2\,s}s^2 - 1/6\,\tau^{-2\,s} + 1/6\,\tau^{-2\,s}s\} + \frac{z^4}{\tau^s}\left\{\frac{1}{96}\frac{z^4}{\tau^4}s^2 - \frac{1}{384}\frac{z^4}{\tau^4}s^4\right\} + \ldots}$$

scaling part $\{\ldots\}$ not dominant when s=0

- The metric is singular at all times (including $\tau = 0$!):

$$\text{Set}: \quad u(z^2) = \frac{1}{4z}a_0'(z) \qquad v(z^2) = \frac{1}{4z}b_0'(z) \qquad w(z^2) = \frac{1}{4z}c_0'(z)$$

$$\boxed{[u+v+w]_0^\infty \equiv \int_0^\infty (u'+v'+w')dz^2 = -2\int_0^\infty (u^2+v^2+w^2)z\,dz^2}$$

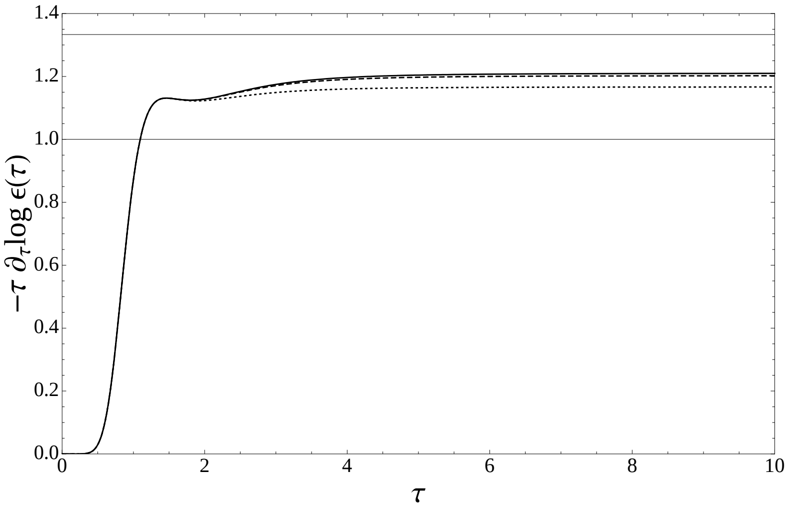
- "MicroCosmic Censorship":

$$\boxed{ds^2(z\sim z_{sing}) \sim \frac{1}{z^2}\left(1-\frac{z}{z_{sing}}\right)^2 d\tau^2 + \ldots + \frac{1}{z^2}dz^2}$$

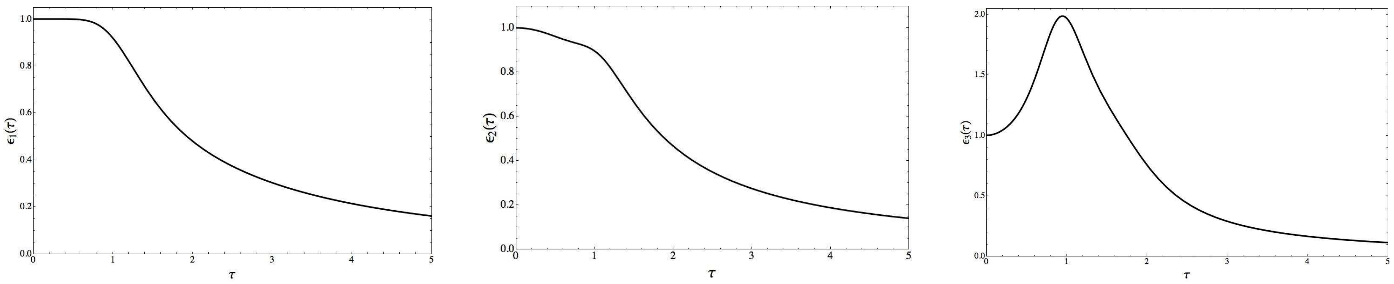
- To be satisfied: Initial Conditions + Constraints

Investigations on Thermalization

- “Family Index”: $s = -\tau\partial_\tau\epsilon(\tau)$



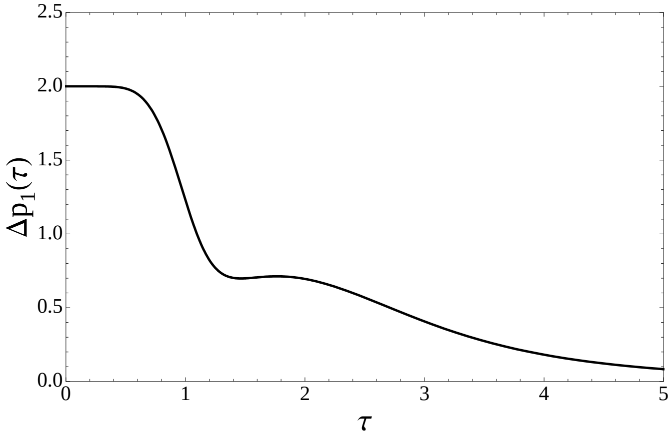
- Dependence on Initial Conditions



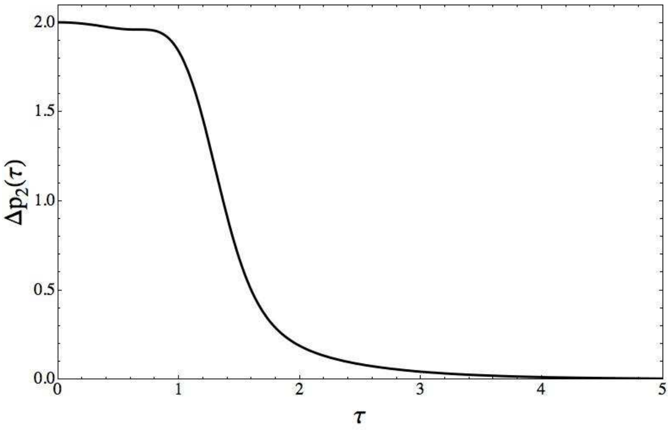
$[v{+}w{-}>]A)$: $\tanh(z^2)-\tan(z^2)$ $B)$: $\tanh(z^2+z^8/6)-\tan(z^2)$ $C)$: $2/3z^6(1+z^2/2)/(z^2-1)$

$C)$:Temporary violation of Positivity: $T_{\mu\nu}t^\mu t^\nu \geq 0 \Rightarrow \quad \frac{4\epsilon(\tau)}{\tau} \leq \epsilon'(\tau) \leq 0$

Isotropization of Pressure Density



$[v+w- >]A)$: $\tanh(z^2) - \tan(z^2)$



$B)$: $\tanh(z^2 + z^8/6) - \tan(z^2)$

$$\boxed{\Delta p\,(\tau) = 1 - \frac{p_{\parallel}\,(\tau)}{p_{\perp}\,(\tau)}}$$

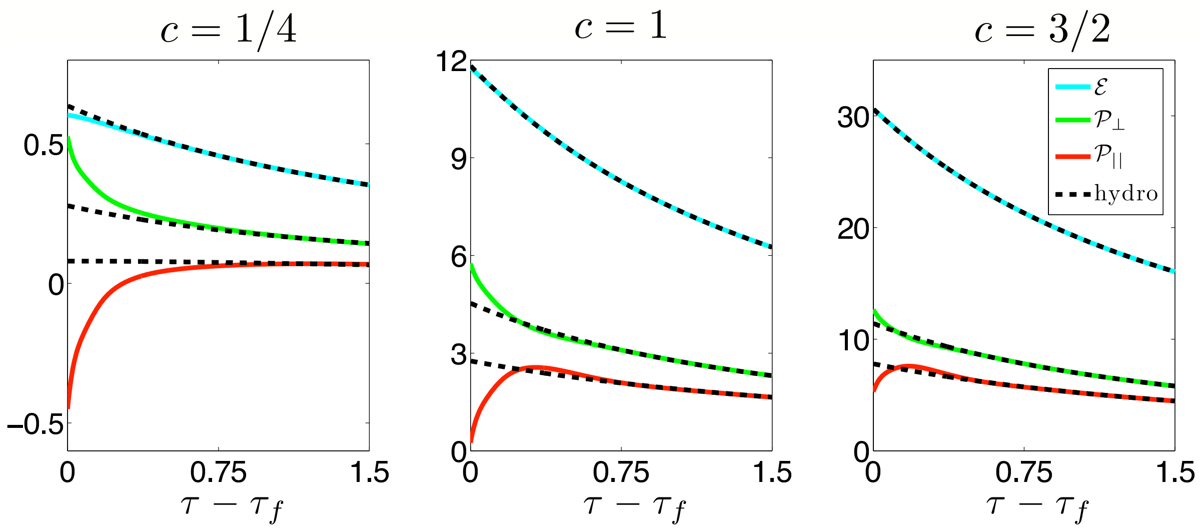
- “Fast” Road towards Isotropization
- Isotropization may stay “some time” Incomplete

Far-from-equilibrium Dynamics: Black hole formation

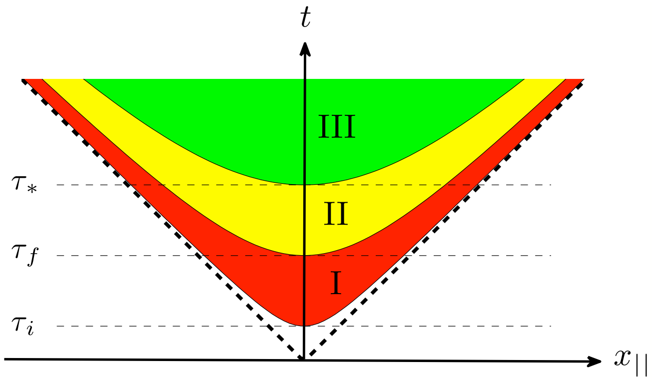
P.M.Chesler, L.G.Yaffe, 2009

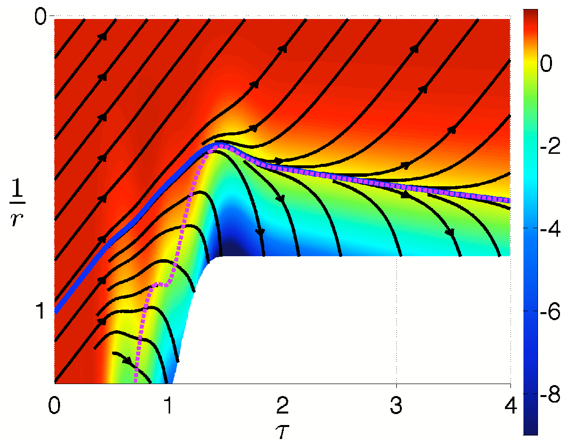
- 4d Time-dependent Shear

$$\boxed{ds^2 = -d\tau^2 + \tau^2 e^{-2\mathbf{c}\gamma(\tau)}\ dy^2 + e^{\mathbf{c}\gamma(\tau)}\ dx_\perp^2}$$



- 5d Black hole Formation





I : *4d Deformation* II : *Anisotropic Relaxation* III : *Hydro Regime*

THANK YOU, AL!

YOU GAVE, GIVE, WILL GIVE,

A LOT TO ALL OF US

EXTRA SLIDES

Conclusions and Prospects

Conclusions:

- Gauge-Gravity Correspondence
  A promising way to study Boost-Invariant Dynamics
- Late-time (Hydro)Dynamics
  Scaling, "almost-pefect" fluid, Einstein *vs.* Navier-Stokes
- Early-time Dynamics
  No scaling, singularity at all times, thermalization studies

Prospects

- More Numerical Work
  Classifying the thermalization solutions
- More "Translation" Work
  Relation with initial conditions
- More Theoretical Work
  Going beyond boost-invariance
- From S^4QCD to S^0QCD ?
  Approaching the "Gravity Dual" of QCD

Why Einstein Eqs. may govern the QGP?

Boost-Invariant Viscosity and Relaxation time

R.Janik, R.Janik and M.Heller;

- Shear Viscosity equation (first order)

$$\boxed{{}_{.}\tau\epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{\eta}{\tau^2}}$$

- Asymptotic Expansion of the Black Hole Solution

$$\boxed{a(\tau, z), b(\tau, z), c(\tau, z) \Rightarrow \sum_n \lambda_n^{a,b,c}(v)\ \tau^{-2n/3}}$$

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \Rightarrow \sum_n {\mathfrak{R}^2}_n\ \tau^{-2n/3}$$

- Results

$\boxed{\frac{\eta}{S} = \frac{1}{4\pi}}$ Universal viscosity (needs $n \rightarrow 2$)

$\boxed{\tau_{Rel} = (1 - \log 2)/2\pi T}$ Relaxation Time (needs $n \rightarrow 3$)

EMERGENCE of the 5d BLACK HOLE

Balasubramanian,de Boer,Minic; Myers; Janik,R.P.

- 4d Perfect Fluid “on the brane”

$$\boxed{\langle T_{\mu\nu}\rangle \propto g^{(4)}_{\mu\nu} = \begin{pmatrix} 3/z_0^4 = \epsilon & 0 & 0 & 0 \\ 0 & 1/z_0^4 = p_1 & 0 & 0 \\ 0 & 0 & 1/z_0^4 = p_2 & 0 \\ 0 & 0 & 0 & 1/z_0^4 = p_3 \end{pmatrix}}$$

- Holographic Renormalisation (Resummed)

$$ds^2 = -\frac{(1 - z^4/z_0^4)^2}{(1 + z^4/z_0^4)z^2}dt^2 + (1 + z^4/z_0^4)\frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

- $\Rightarrow 5d$ Black Brane with horizon at $z_0 \sim T_0^{-3}$

$$\boxed{ds^2 = -\frac{1 - \tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2}dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1 - \tilde{z}^4/\tilde{z}_0^4}\frac{d\tilde{z}^2}{\tilde{z}^2}}$$

$$z \longrightarrow \tilde{z} = \ z\Big/\sqrt{1+\frac{z^4}{z_0^4}}$$

AdS/CFT Background

- D_3-brane Solution of Supergravity: Horowitz,Strominger, 1991

$$\boxed{ds^2 = f^{-1/2}\left(-dt^2+\sum_1^3 dx_i^2\right)+f^{1/2}\left(dr^2+r^2 d\Omega_5\right)}$$

"Physical" Brane + Extra-Dimensions

$$f = 1+\frac{R^4}{r^4}\ ;\ R^4 = 4\pi\alpha\prime^2 g_{YM}^2 N_c$$

- "Maldacena limit":

$$\frac{\alpha'(\rightarrow 0)}{r(\rightarrow 0)} \rightarrow z\ ,\ R\ fixed\ \Rightarrow g_{YM}^2 N_c \rightarrow \infty$$

Strong coupling limit

$$\boxed{ds^2 = \frac{1}{R^2 z^2}\left(-dt^2+\sum_{1-3} dx_i^2+dz^2\right)+R^2 d\Omega_5}$$

Background Structure: AdS$_5$ × S_5 (same R^2)

Calculation of the Gravity Duals

- Boost-Invariant 5-d F-G metric:

$$\boxed{ds^2 = \frac{-e^{a(\tau,z)}d\tau^2 + \tau^2 e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2}{z^2} + \frac{dz^2}{z^2}}$$

- Scaling : $\boxed{v = \frac{z}{\tau^{S/4}}}$

$$[a(\tau,z), b(\tau,z), c(\tau,z)] = [a(v), b(v), c(v)] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)$$

$$v(2a'(v)c'(v)+a'(v)b'(v)+2b'(v)c'(v))-6a'(v)-6b'(v)-12c'(v)+vc'(v)^2 = 0$$

$$3vc'(v)^2 + vb'(v)^2 + 2vb''(v) + 4vc''(v) - 6b'(v) - 12c'(v) + 2vb'(v)c'(v) = 0$$

$$2vsb''(v) + 2sb'(v) + 8a'(v) - vsa'(v)b'(v) - 8b'(v) + vsb'(v)^2 +$$

$$+4vsc''(v) + 4sc'(v) - 2vsa'(v)c'(v) + 2vsc'(v)^2 = 0$$

- Asymptotic Solution

$$\begin{aligned} a(v) &= A(v) - 2m(v) \\ b(v) &= A(v) + (2s-2)m(v) \\ c(v) &= A(v) + (2-s)m(v) \end{aligned}$$

$A(v)=\frac{1}{2}\left(\log(1+\Delta(s)\,v^4)+\log(1-\Delta(s)\,v^4)\right)$ $m(v)=\frac{1}{4\Delta(s)}\left(\log(1+\Delta(s)\,v^4)-\log(1-\Delta(s)\,v^4)\right.$ $\Delta(s)=\sqrt{3s^2-8s+8/24}$

AdS/CFT: Selection of the Perfect Fluid

- Kreschtmann Scalar: $\boxed{\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}}$

$$\begin{aligned}\mathfrak{R}^2 = \frac{4}{\left(1-\Delta(s)^2 v^8\right)^4} \cdot \Big[& 10\,\Delta(s)^8 v^{32} - 88\,\Delta(s)^6 v^{24} + 42\,v^{24} s^2 \Delta(s)^4 + \\ & +112\,v^{24}\Delta(s)^4 - 112\,v^{24}\Delta(s)^4 s + 36\,v^{20} s^3 \Delta(s)^2 - 72\,v^{20} s^2 \Delta(s)^2 + \\ & +828\,\Delta(s)^4 v^{16} + 288\,v^{16}\Delta(s)^2 s - 288\,v^{16}\Delta(s)^2 - 108\,v^{16} s^2 \Delta(s)^2 + \\ & -136\,v^{16} s^3 + 27\,v^{16} s^4 - 320\,v^{16} s + 160\,v^{16} + 296\,v^{16} s^2 + 36\,v^{12} s^3 + \\ & -72\,v^{12} s^2 - 88\,\Delta(s)^2 v^8 + 42\,v^8 s^2 + 112\,v^8 - 112\,v^8 s + 10 \Big] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)\end{aligned}$$

- $\mathfrak{R}^2$ for $s = \frac{4}{3}$:

$$\boxed{{\mathfrak{R}^2}_{\text{perfect fluid}} = \frac{8(5w^{16} + 20w^{12} + 174w^8 + 20w^4 + 5)}{(1+w^4)^4}}$$

$$w = v/\Delta(\tfrac{4}{3})^{\frac{1}{4}} \equiv \sqrt[4]{3}\, v.$$

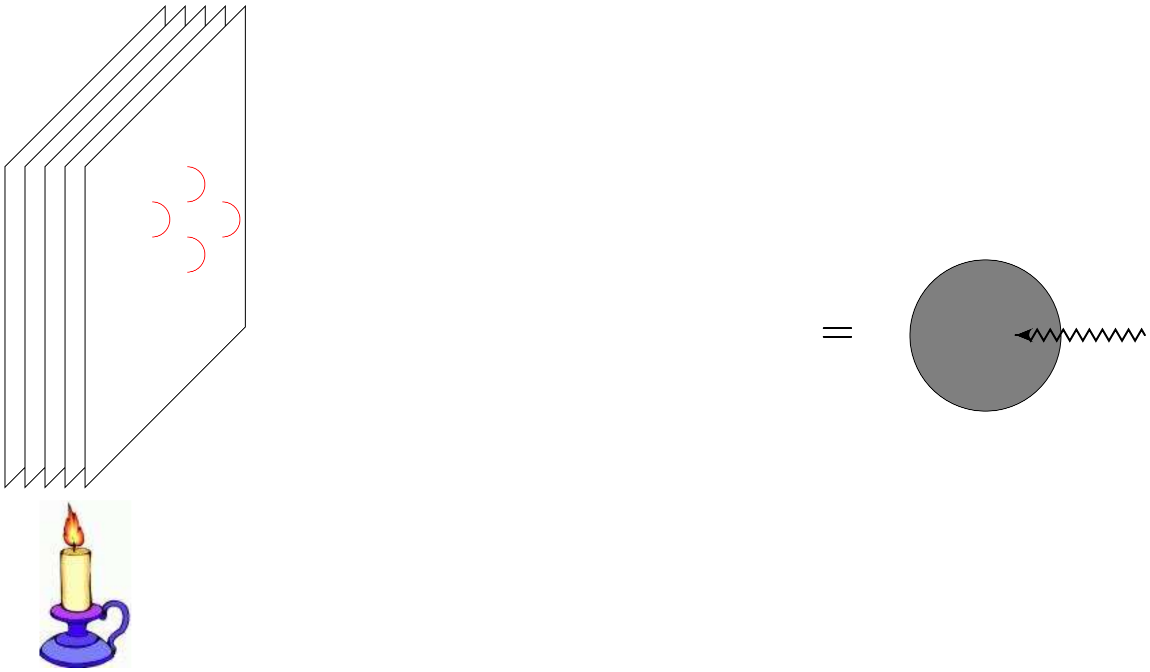
Hydro beyond the Perfect fluid

Static Case

Kovtun, Policastro, Son, Starinets (2001)

Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes
Will be absorbed by the D3 branes.
The process of absorption can be looked at from two different perspectives:



Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

$$\sigma_{abs}(\omega) \propto \int d^4x \, \frac{e^{i\omega t}}{\omega} \; \langle [T_{x_2x_3}(x), T_{x_2x_3}(0)] \rangle \Rightarrow \boxed{\frac{\eta}{s} \equiv \frac{\sigma_{abs}(0)/(16\pi \; G)}{A/(4 \; G)} = \frac{1}{4\pi}}$$

Preliminaries (1): Quasi-Normal Modes

R.Janik,R.P., 2006

- Scalar Excitation of a Moving Black Hole

$$\boxed{\Delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_n\left(\sqrt{-g}g^{ij}\partial_j\phi\right) = 0}$$

- Scalar “Quasi-Normal Modes”

$$\phi(\tau, v \equiv z/\tau^{1/3}) = f(\tau) \times \phi(v)$$

$$\boxed{f(\tau) = \sqrt{\tau}J_{\pm\frac{3}{4}}\left(\frac{3}{2}\omega\tau^{\frac{2}{3}}\right) \ \sim \ \tau^{\frac{1}{6}}\ e^{\frac{3}{2}i\omega\tau^{2/3}}}$$

- Short Excitation Decay

$$\boxed{\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667\, i \Rightarrow \tau \sim \frac{1}{8.3\ T}}$$

- e-folding conjecture

JJ Friess, SS Gubser, G. Michalogiorgakis,SS Pufu,

$$\boxed{\tau_{therm} \sim \ 4\tau_{e-fold} = 4 \times \frac{1}{8.3\ T_{peak}} \sim 4 \times .1 fermi}$$

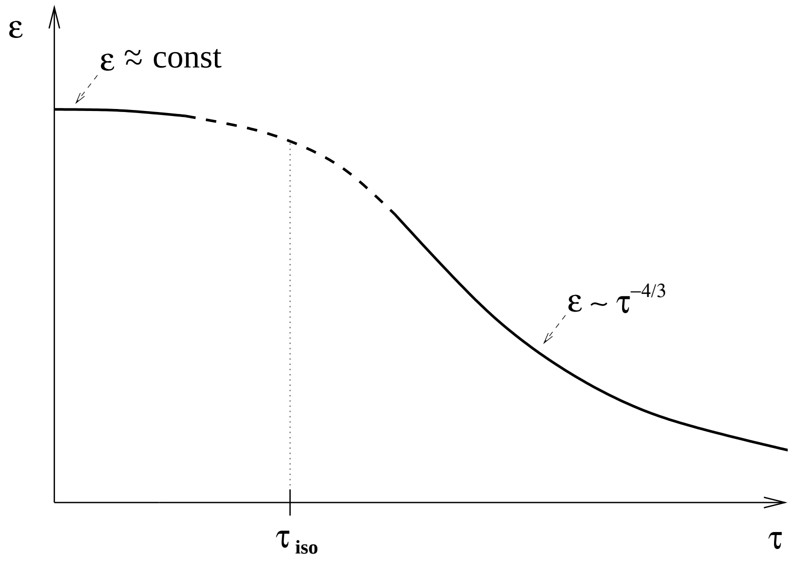
The Black Hole as an “Attractor”

Preliminaries (2): Scaling Solution

Kovchegov, Taliotis, 2007

- $\ast$ Evolution at small $(S = 0)$ *vs.* large $(S = 4/3)$ proper-ti

Assuming Monodromy $\in$ Regula



- $\ast$ Evaluation of The Isotropization/Thermalization time

$$Matching: \; z_h^{late}(\tau) = (3/e_0)^{\frac{1}{4}} \equiv z_h^{early}(\tau) = \tau$$

$$Isotropization: \; \tau_{iso} = \left(3N_c^2/2\pi^2 e_0\right)^{3/8}$$

$$Typical\ Scale: \epsilon(\tau) \; = \; e_0 \; \tau^{4/3}|_{\tau=.6} \sim 15 \; GeV fermi^{-3}$$

$$\Rightarrow \boxed{\tau_{iso} \sim .3 \; fermi}$$