D-instantons in N=4 SYM and multiparticle production

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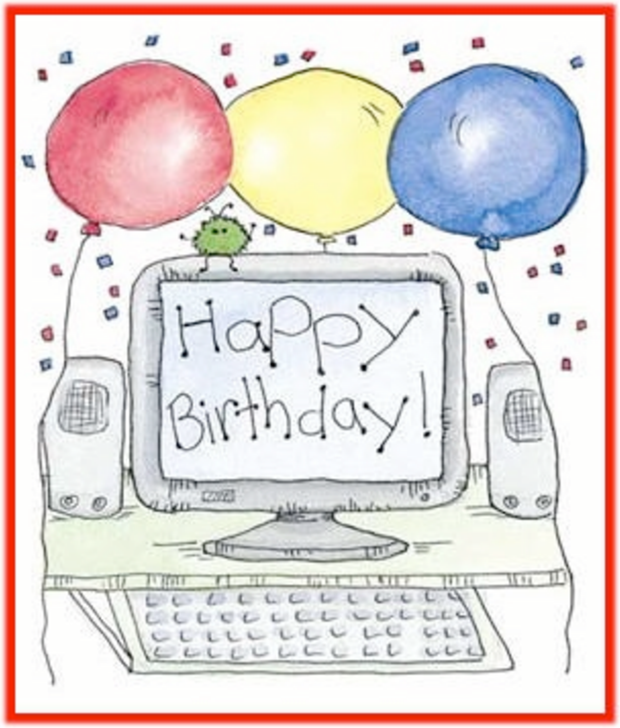
D. Kharzeev and E.L.: 0910.3355[hep-ph]

Happy birthday to you, Al



Hopes and problems

N=4 SYM	QCD
fixed coupling	running coupling
no confinement	confinement ???
small coupling:	
BFKL Pomeron, OPE,	BFKL Pomeron, OPE,
multiparticle production	multiparticle production
large coupling:	
classical weak gravity	multiparticle production
no multiparticle production	saturation, geometrical scale
exact solution	no solution
	in confinemet region

The question

N=4 SYM can give an educated guide for a description of multiparticle systems, which we believe do not affect by confinement of quark and gluons and, perhaps, by running coupling,

BUT

Can such system be produced in N=4SYM ?

$$\lambda = 4\,\pi\,N_c\,g_s\,; \quad g_s = \frac{g^2_{YM}}{4\pi} = \alpha_{YM}$$

$$R \ = \alpha^{1/2}\,\lambda^{1/4}$$

$$\lambda \gg 1 \ \text{ but } \ g_s \ll 1;$$

CDF/AdS correspondence:

- **BFKL Pomeron** $\longrightarrow$ **Reggeized graviton, with** $\Delta = 2 - 2/\sqrt{\lambda}$

$$2\,Im A \ = \ |A|^2 \ + \ \mathcal{O}\left(\frac{2}{\sqrt{\lambda}}\right)$$ $\leftarrow$ **Diffractive production**

(Brower, Polchinski, Strassler & C.I.Tan (2006), Kotikov & Lipatov (2006), Hatta, Iancu & Mueller(2007))

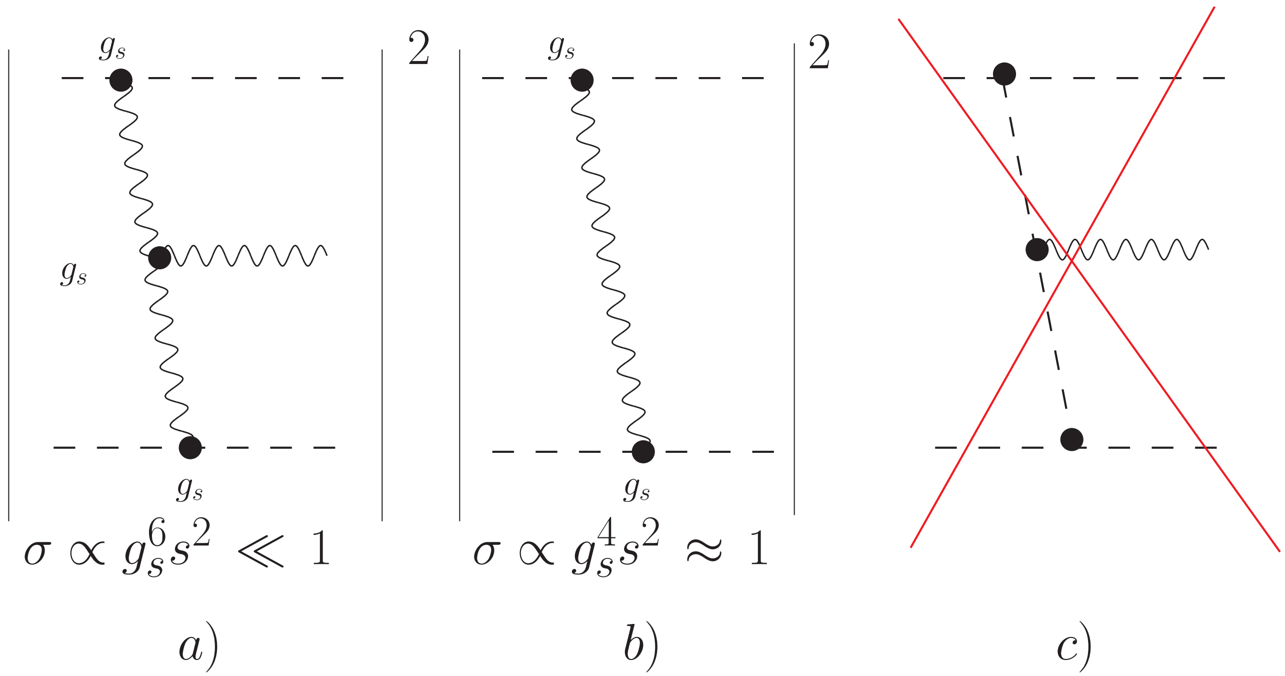
A dilemma:

- *To find a new mechanism for the inelastic production in the framework of N=4 SYM other than reggeized graviton interaction,*

or

- *To accept that N=4 SYM is irrelevant to any experimental data that have been measured before LHC era, with a chance that even at the LHC it will be responsible only for a quarter (or less) of the total cross section*

New mechanism for MPP



- $-S_{\mathbf{E}}^{\mathbf{boson}} = \int d^{10}x \sqrt{g} \{\mathcal{R} - (\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2} e^{2\phi} (\partial_\mu a)(\partial^\mu a)\}$

Are we doomed to have small MPP?

NO for large number of produced particles $(N \gg 1)$

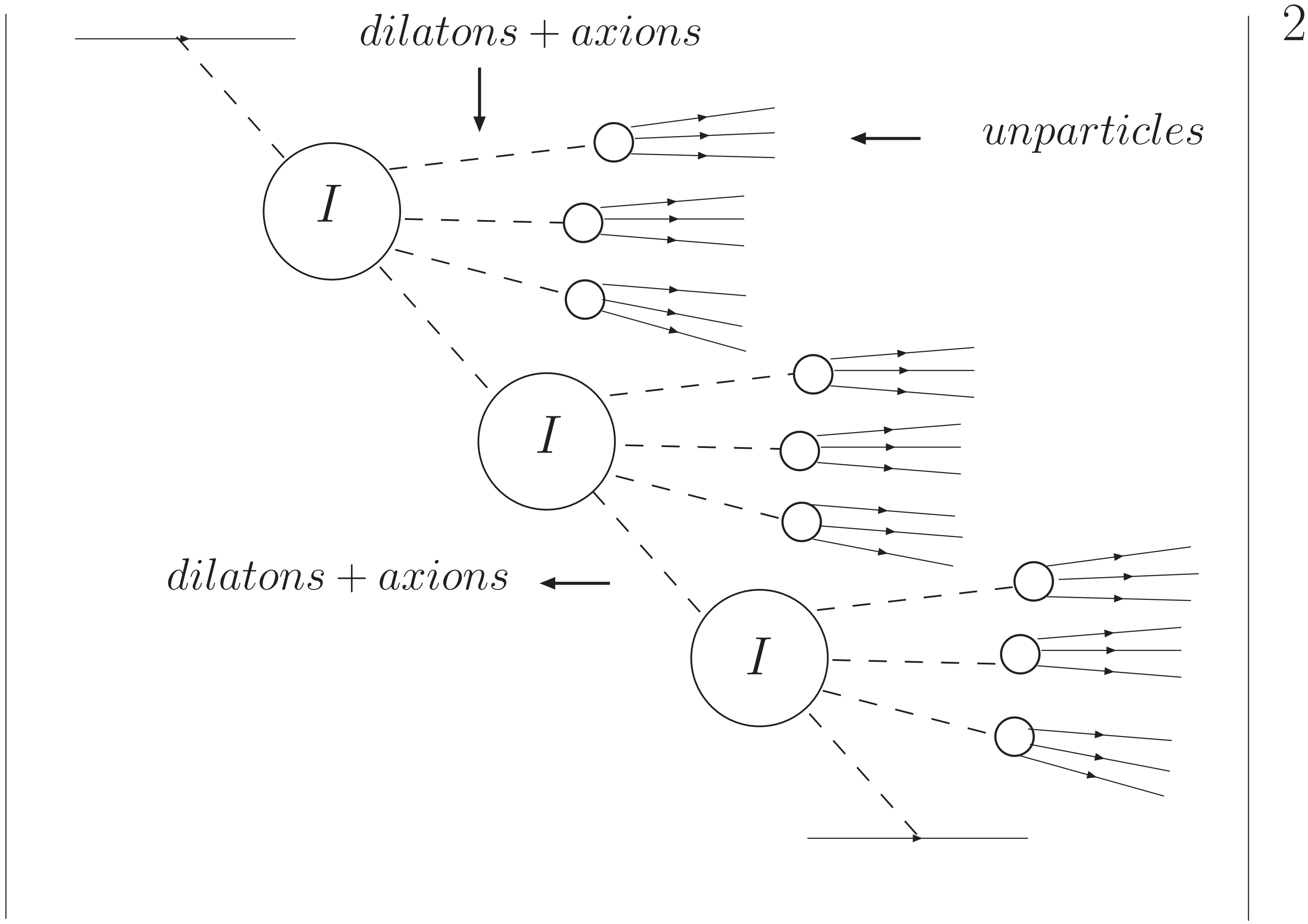
$$\Delta n \, \Delta \phi \sim 1 \text{ or } \phi \propto 1/n \ll 1$$

$\Longrightarrow$ *classical fields*;

$\Longrightarrow$ *instantons with coupling* $\propto 1/\alpha_S$;

(Dyson (1952), Lipatov (1977) + . . .)

Main idea:



D-instanton

Equation of motion for S_E^{boson}:

$$\mathcal{R}_{\mu\nu} = \frac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}e^{2\phi}(\partial_\mu a)(\partial_\nu a);$$

$$\nabla_\mu\left(e^{2\phi}\,\partial^\mu a\right) = 0; \qquad \nabla^2\phi + e^{2\phi}(\partial_\mu a)(\partial^\mu a) = 0$$

Instanton solution at $R_{\mu\nu} = 0$

- **Axion :** $a - a_\infty = \pm\left(e^{-\phi} - e^{-\phi_\infty}\right)$
- **Dilaton:** $g^{\mu\nu}\,\nabla_\mu\partial_\nu\left(\sqrt{g}g^{\mu\nu}\,\partial_\nu e^{\phi}\right) = 0$

$$e^{\phi} = e^{\phi_\infty} + G(x_0, z_0; x, z) = g_s + G(x_0, z_0; x, z)$$

x_0 = **position of the instanton;** z_0 = **its size.**

- **Instanton distribution function:**

$$n(z) \;\propto\; \frac{1}{z^5};$$

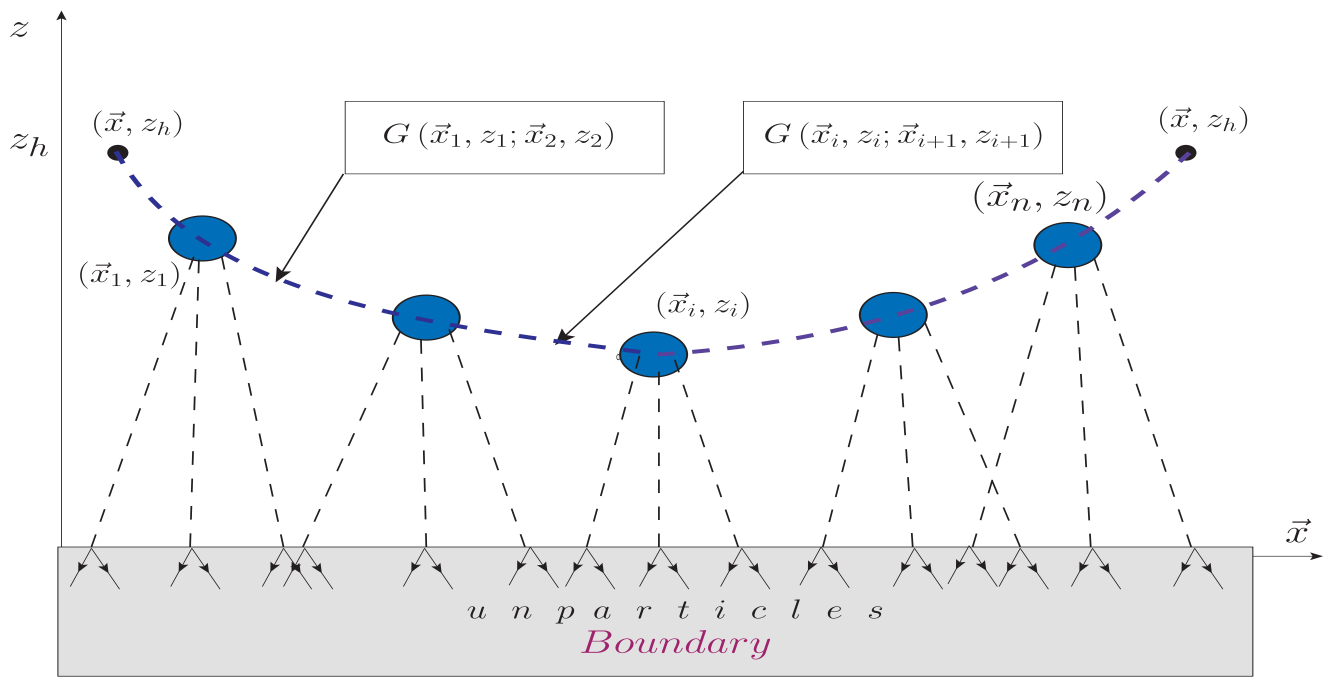
- **Contribution of the instanton to the action:**

$$S_{\text{D-inst}} \;=\; \frac{2\pi}{g_s} \;=\; E_{\text{sph}}\, t \;\equiv\; \kappa;$$

- **The estimate of the sphaleron energy ($t = z$):**

$$E_{\text{sph}} \;=\; \frac{2\pi}{g_s\, z} \;=\; \frac{\kappa}{z};$$

General form of multiperipheral diagram



$$A(\vec{x}, z_h; \vec{x}_{n+1}, z_h) \;=$$

$$\int G(\vec{x}, z_h; \vec{x}_1, z_1) \prod_{i=1}^{n} \boxed{\frac{dz_i}{z_i^5}\, e^{-\frac{2\pi}{g_s}}}\, d^4 x_i\, \Gamma(2 \to n_i)\, G(\vec{x}_i, z_i; \vec{x}_{i+1}, z_{i+1})$$

$\Gamma(2 \to n_i)$

- $G\left(\vec{x}_k, z_k | k_1, \dots, k_{n_i}\right) \xrightarrow{Fourier\ \ image}$

$$\left\langle \phi_{inst}(\vec{x}_k, z_k|\vec{x}, z)\, \phi_{inst}(\vec{x}_k, z_k|\vec{x}', z') \prod_{i=1} \phi_{inst}(\vec{x}_k, z_k|\vec{r}_i, z \to 0) \right\rangle$$

- $\Gamma(2 \to n_i) = \left\{ \prod G\left(\vec{x}_k, z_k | k_1, \dots, k_{n_i}\right) \; G^{-1}\left(k_i^2\right) \right\}_{k_i^2 \to 0}$

- $\sigma \propto A\left(\vec{x}, z_h; \vec{x}_{n+1}, z_h\right) \times A^*\left(\vec{x}, z_h; \vec{x}_{n+1}, z_h\right)$

$$\propto \prod_{i=1} \langle 0 | \phi_{inst}(\vec{x}_k, z_k|\vec{r}_i, z \to 0)\, \phi_{inst}(\vec{\bar{x}}_k, \bar{z}_k|\vec{\bar{r}}_i, z \to 0) | 0 \rangle$$

- $\phi_{inst}(\vec{x}_k, z_k|\vec{r}_i, z \to 0) = \dfrac{2\pi}{g_s} \dfrac{\Gamma(4)}{\pi^2 \Gamma(2)} \dfrac{z_k^4}{\left(z_k^2 + (\vec{r}_i - \vec{x}_k)^2\right)^4}$

$$\xrightarrow{r_i \gg x_k} \frac{2\pi}{g_s} \frac{\Gamma(4)}{\pi^2 \Gamma(2)} \frac{z_k^4}{r_i^8}$$

- $\langle 0|\phi_{inst}\left(\vec{r}_i\right)\ \phi_{inst}\left(\vec{\bar{r}}_i|0\right)\rangle\ =\ \int e^{i\vec{k}\cdot(\vec{r}_i-\vec{\bar{r}}_i)}\,|\langle 0|\phi|k\rangle|^2\,\rho(k)\frac{d^4k}{(2\pi)^4}$

- $|\langle 0|\phi|k\rangle|^2\,\rho(k)\ =\ A^2_{d=4}\Theta(k_0)\,\Theta(k^2)\,(k^2)^{d-2}\longleftarrow$ **scale invariance**

- $D_{unparticle}(k^2)\ =\ k^4\ \ln\left(-k^2\right)$

$$A_4\ =\ \frac{1}{D_{unparticle}(k^2)}\int d^2r e^{i\vec{k}\cdot\vec{r}}\,\phi_{inst}\left(z;r\right)\ =\ \frac{2\pi}{g_s}\,\frac{z^4}{4\,4!}$$

(Georgi (2007))

Sum over produced unparticles

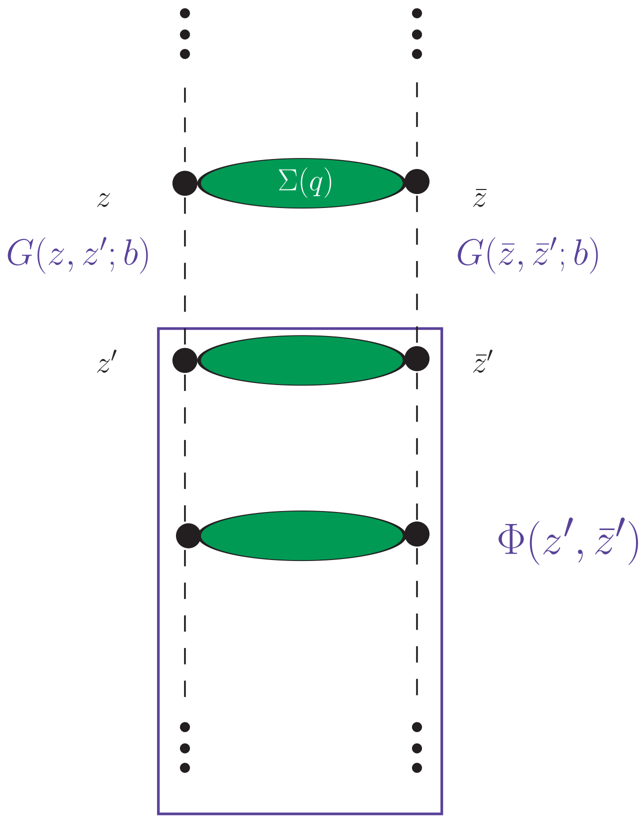
- $$\Sigma(q^2) = \sum_n \Sigma_n(q^2); \quad \Sigma_n = \prod_i^n \Gamma^2\,(2 \to n)\,\frac{d^4 k_i}{(2\pi)^4};$$

- $$\int^{E^2_{sph}} dq^2\, \Sigma(q^2) = z_k\, \bar{z}_k\, \tilde{\Sigma}(\kappa) =$$

$$\pi^{5/2}\, z_k\, \bar{z}_k \sum_{n=1}^{\infty} \frac{\Gamma\,(n+1)\,\Gamma\,(4\,n - 3/2)}{\Gamma\,(4\,n)\ \ (4n-1)\ \Gamma\,(8\,n-3)} \left(\frac{1}{48\,\pi^3}\right)^n \kappa^{10\,n-2}$$

with $\kappa = 2\pi/g_s$

Equation



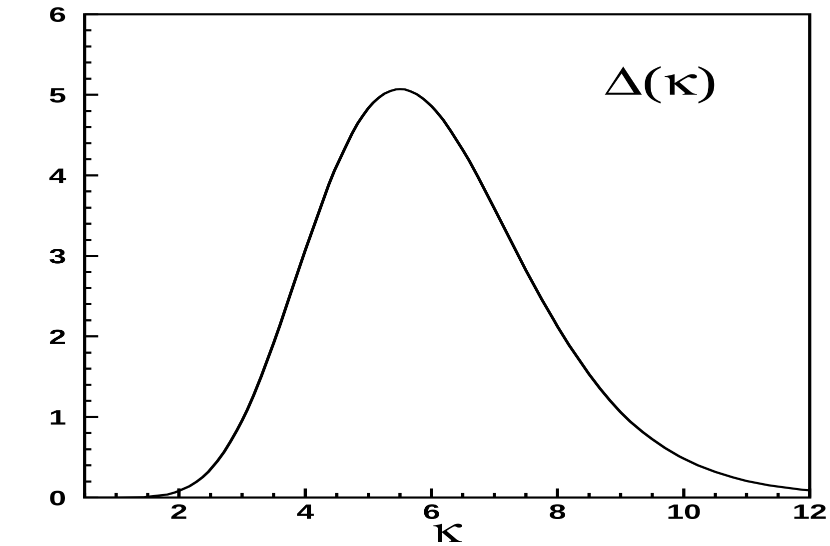
with $u \;=\; \frac{(z_k - z_{k-1})^2 + b^2}{2\, z_k\, z_{k-1}}$

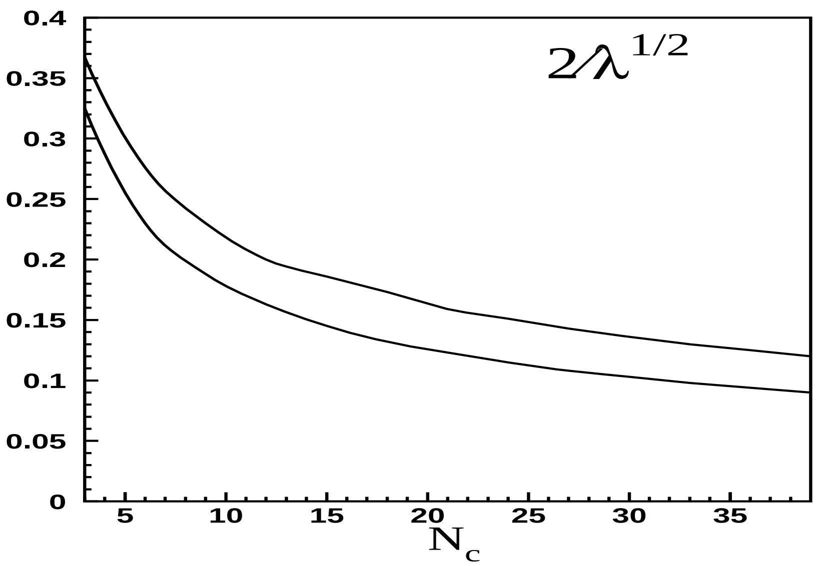
- $$\frac{d\,\Phi\,(z, \bar{z})}{d \ln s} = \kappa^4\, e^{-2\,\kappa} \int \frac{dz'}{z'^5}\, \frac{d\bar{z}'}{\bar{z}'^5}\; K\,(z, \bar{z}; z', \bar{z}')\;\; \Phi\,(z', \bar{z}')$$

- $$K\,(z, \bar{z}; z', \bar{z}') = \int d^2 b\, G\,(z, z'; b)\;\; G\,(\bar{z}, \bar{z}'; b)\;\; \int dq^2\;\; \Sigma(q)$$

- $$G\,(z_k, x_k; z_{k-1}, x_{k-1}) \;=\; G_3\,(u) \;=\; \frac{1}{4\pi}\, \frac{1}{\left\{1 + u + \sqrt{u(u+2)}\right\}^2\;\sqrt{u(u+2)}}$$

Solution





- $\Phi = \phi(z_h \bar{z}_h)\, s^{\Delta}$

$$\sigma_{inelastic} \propto s^{\Delta - 2}$$

- $\Delta = \frac{\pi}{16\,12} \frac{\alpha'^4}{z_h^2\, \bar{z}_h^2} \kappa^4 e^{-2\kappa}\, \tilde{\Sigma}\,(\kappa)$

- $N_c = 3, \quad \lambda = 35$
- $z_h \simeq 0.2 fm$ from $\alpha'_{eff} \approx \alpha'/2$ (closed string)

Conclusions

1. We found the new mechanism for multiparticle production in the framework of N=4 SYM which is different from the reggeized graviton interaction;

2. The value of the cross section depends crucially on the behaviour of the hadronic wave function at small z. We argue that the size of the typical D-instanton $z_h \simeq$ the size of QCD instanton from lattice simulation;

3. Our D-instanton-based approach suggests an important role for topological effects in high energy collisions.