Lattice field theory: challenges and opportunities

In Celebration of Al Mueller's 70th Birthday

October 23, 2009

Norman H. Christ

Columbia University

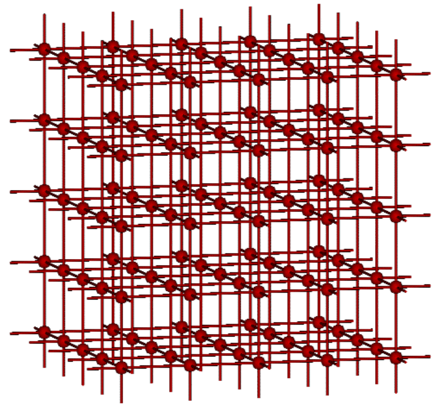
Outline

- Introduction
- Lattice QCD (RBC/UKQCD)
  - Extrapolation to physics masses
- SU(3) color with 8 and 12 flavors

  (Xiao-Yong Jin & Bob Mawhinney)
- Conclusions

Introduction

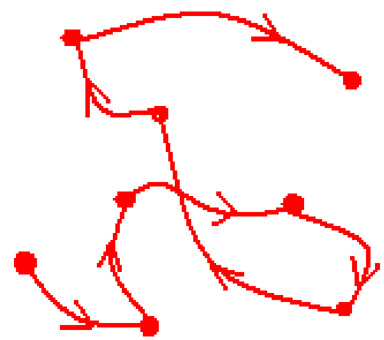
Lattice QCD



- Regulate using a space-time lattice.
- Evaluate Euclidean Feynman path integral numerically.
  - **Precise non-perturbative formulation.**
  - **Potential numerical errors.**

$$\sum_n \langle n| e^{-Ht} \mathcal{O} |n\rangle = \int d[U_\mu(n)] e^{-\mathcal{A}[U]_{\text{gauge}}} \det(D+m) \mathcal{O}[U]$$

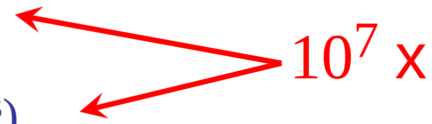
$$\det(D+m) = \int d[\phi] d[\phi^*] e^{-\phi^\dagger \frac{1}{(D+m)} \phi}$$



- Evaluate using Monte Carlo methods with hybrid molecular dynamics + Langevin evolution.

Lattice methods

- Introduced by Wilson in 1973
- 1st numerical evaluation by Creutz 1979.
- Driven by spectacular technological progress:
  - VAX 780 (1984)   1 Mflops (10^6)
  - BG/P (2007)   20 Tflops ($2\ 10^{13}$)



- Matching algorithmic innovation
  - RHMC/Hasenbusch methods (2006)
    > 10 x speedup

Ab initio method ! ?

- In standard theory one can often distinguish:
  - 1^{st} principles derivation
  - Justification based on familiar examples
  - *Ad hoc* assumption
- Lattice results can be even less transparent
  - Stop the simulation when the result looks good?
  - Adjust the fitting function to improve χ^2 ?
  - Change the action to remove visible errors?
- Consumer beware!

Lattice QCD

RBC Collaboration

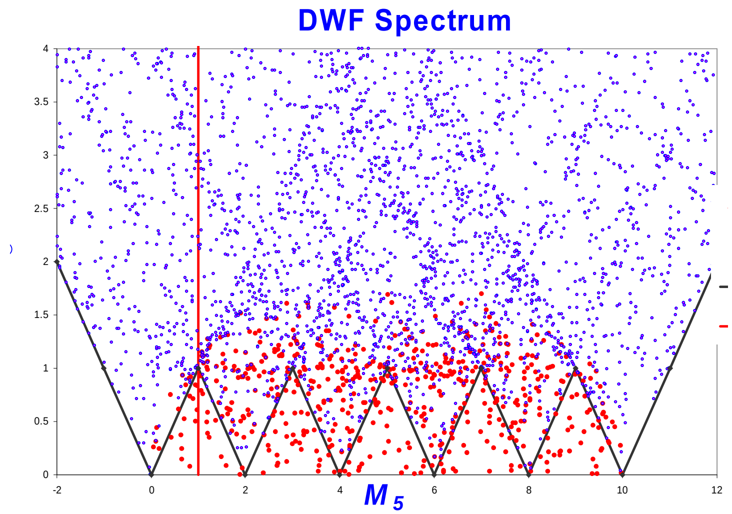
- RBRC
  - Yasumichi Aoki
  - Tom Blum (Connecticut)
  - Saumitra Chowdhury
  - Chris Dawson (Virginia)
  - Tomomi Ishikawa
  - Taku Izubuchi (BNL)
  - Shigemi Ohta (KEK)
  - Ran Zhou
- BNL
  - Michael Creutz
  - Shinji Ejiri
  - Prasad Hegde
  - Chulwoo Jung
  - Frithjof Karsch
  - Swagato Mukherjee
  - Chuan Miao
  - Peter Petreczky
  - Amarjit Soni
  - Ruth Van de Water
  - Alexander Velytsky
  - Oliver Witzel
- Columbia
  - Norman Christ
  - Michael Endres
  - Xiao-Yong Jin
  - Changhoan Kim
  - Matthew Lightman
  - Meifeng Lin (MIT)
  - Qi Liu
  - Robert Mawhinney
  - Hao Peng
  - Dwight Renfrew
  - Shinji Takeda

UKQCD Collaboration

- Edinburgh
  - Peter Boyle
  - Luigi del Debbio
  - Alistair Hart
  - Chris Kelly
  - Tony Kennedy
  - Richard Kenway
  - Chris Maynard
  - Brian Pendleton
  - Jan Wennekers
  - James Zanotti
- Southampton
  - Dirk Brommel
  - Jonathan Flynn
  - Patrick Fritzsch
  - Elaine Goode
  - Chris Sachrajda

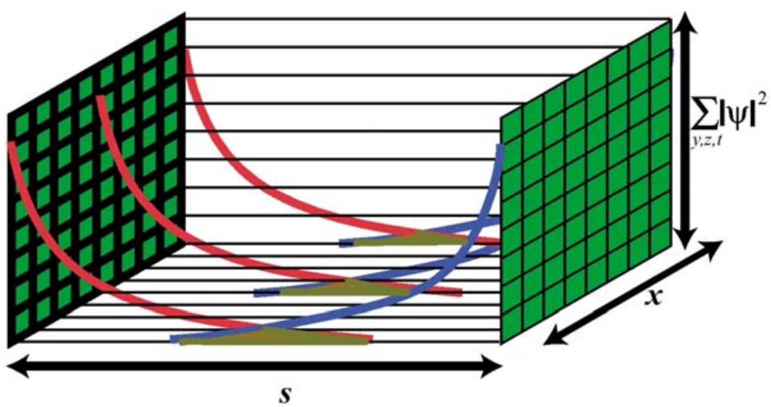
Domain Wall Fermions

- 5-D theory with 4-D, chiral surface states.
- Typical 5-D extent of 16.
- "Revolution" in the lattice treatment of fermions.



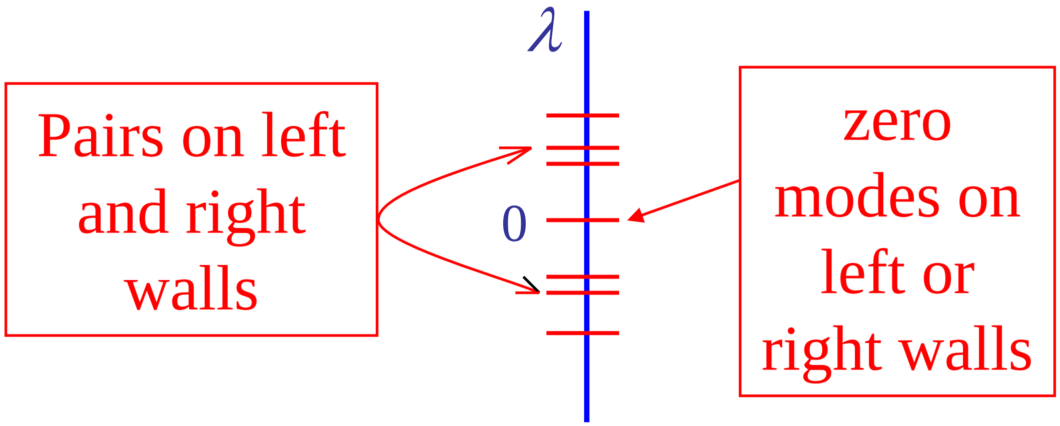
5-D mass

Simulations run at $M_5 = 1.8$



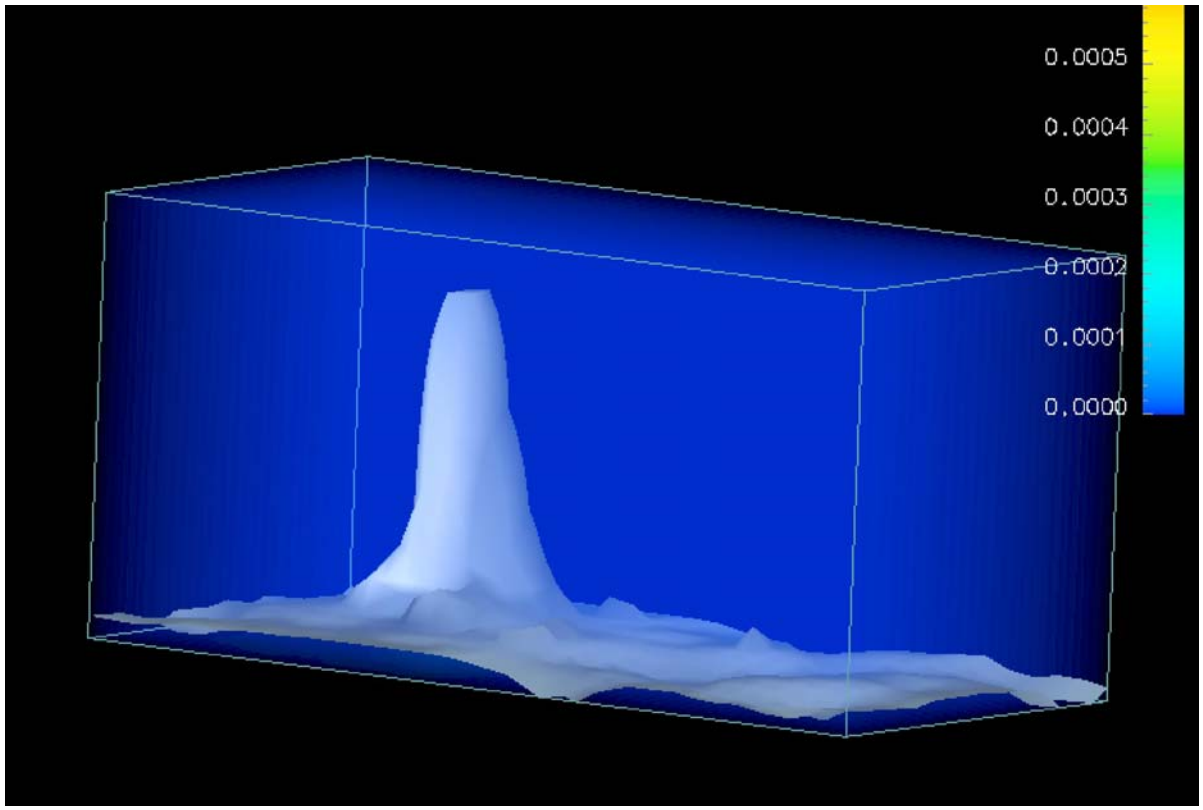
The Ghost of Doubling Problem

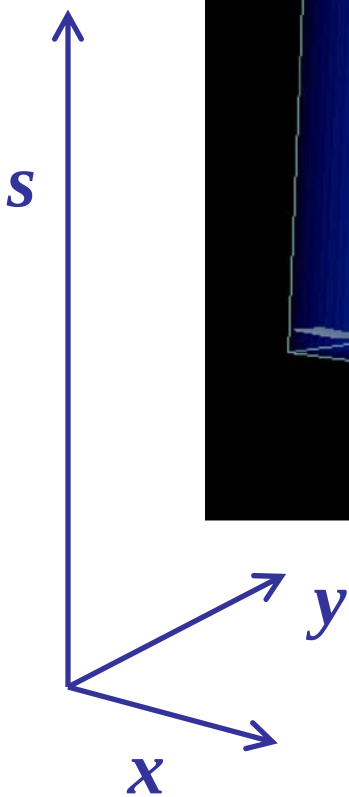
- For the Dirac operator, eigenvalues are paired except for zero modes:



- If the Pontryagin index changes, all modes must mix between left and right walls.
- Tearing gauge fields implies violating chirality.

Local chirality violation





Peter Boyle

Lattice Chiral Symmetry Breaking

- For $L_s < \infty$ the right and left states can mix.
- Gives "residual" mass, $m_{\rm res}$, plus higher dimension operators:

$$\mathcal{L}_{\rm eff} = \overline{\psi}\{D^\mu \gamma^\mu + m\}\psi + m_{\rm res}\overline{\psi}\psi + c_{\rm SW}\overline{\psi}\sigma^{\mu\nu}\psi F^{\mu\nu}$$

- Both $m_{\rm res}$ and $c_{\rm SW}$ decrease rapidly as L_s grows or as $g^2 \rightarrow 0$:

$$m_{\rm res}(L_s) = c_1 \frac{e^{-\lambda_c L_s}}{L_s} + c_2 \frac{1}{L_s}$$

Standard 5-D states with λ ~ lattice cutoff.

Localized states created by changing topology.

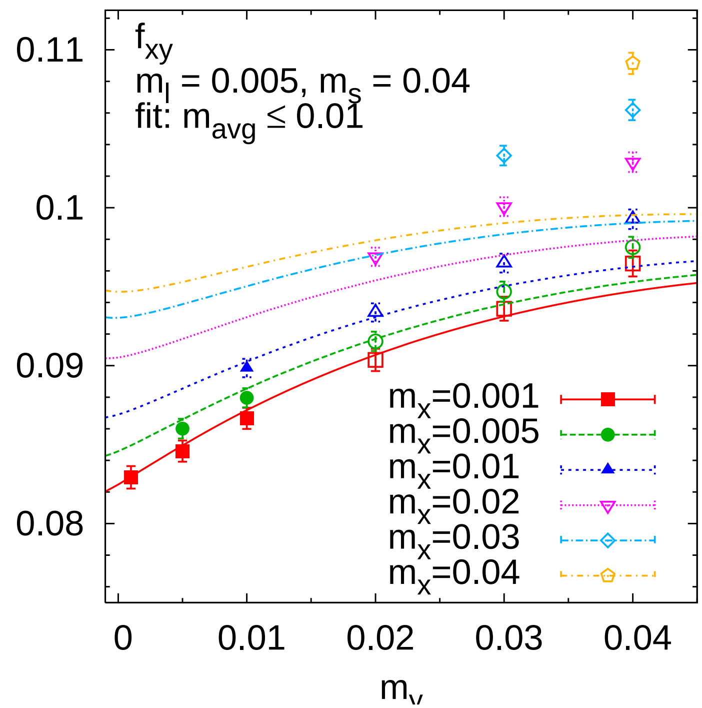
Kaon and Pion Physics

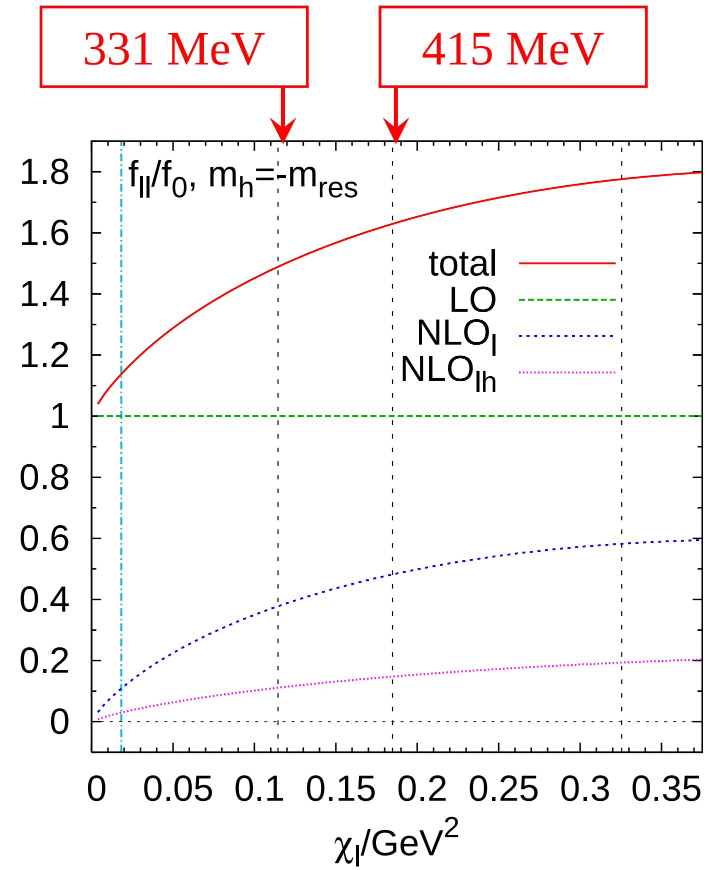
- RBC/UKQCD gauge ensembles:

Volume	*1/a*	*L*	m_π	Time units	$m_{\text{quark}}a$
24^3 x 64	1.73 GeV	2.7 fm	315 MeV	9000	0.005+0.0032
			402 MeV	9000	0.01+0.0032
32^3 x 64	2.32 GeV	2.7 fm	300 MeV	7000	0.004+0.0006
			350 MeV	8000	0.006+0.0006
			410 MeV	6000	0.008+0.0006

Pseudo scalar decay constant

- Calculate f_π on the coarse 1/a = 173 GeV ensemble
- SU(3) x SU(3) ChPT fails for m_{PS} ~ 420 MeV



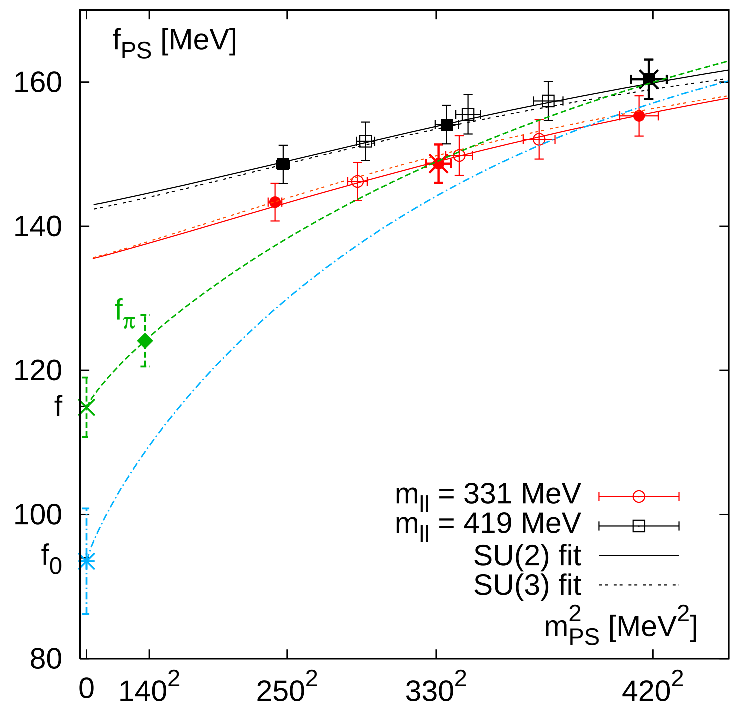


Pseudo scalar decay constant

- SU(2) x SU(2) ChPT yields f_π = 124.1 (3.6)(6.9) MeV

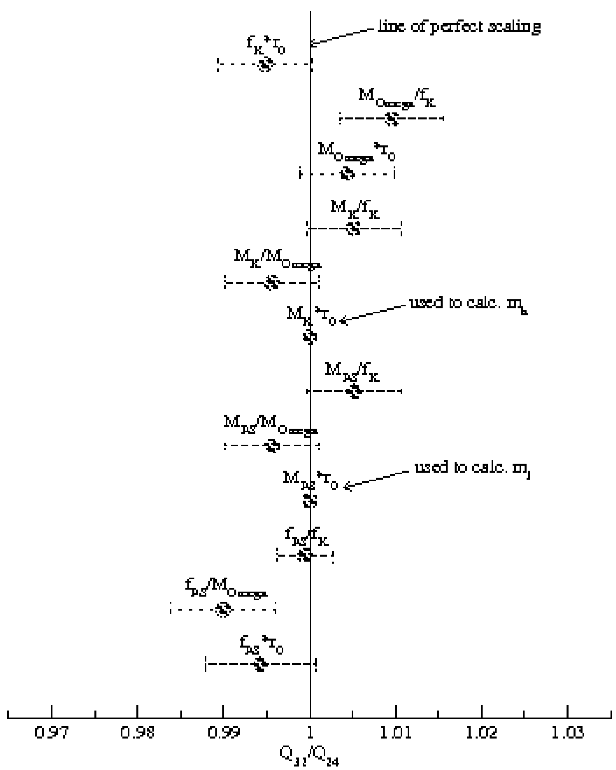
  Experiment: f_π = 130.7(4)

- Discrepancy comes from $O(a^2)$ errors?

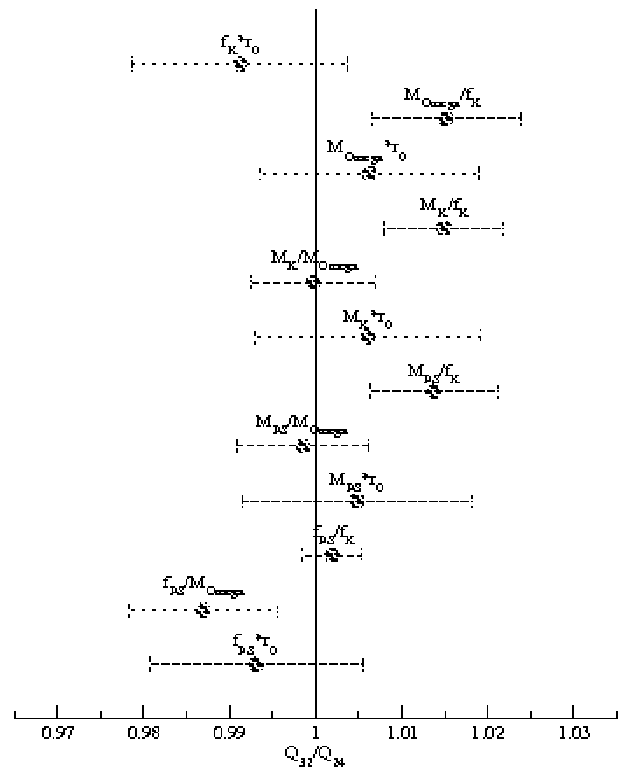


Pseudo scalar decay constant

- New results from 1/*a* = 2.32 GeV ensemble.
- Discover $O(a^2)$ error ~ 2-3% !



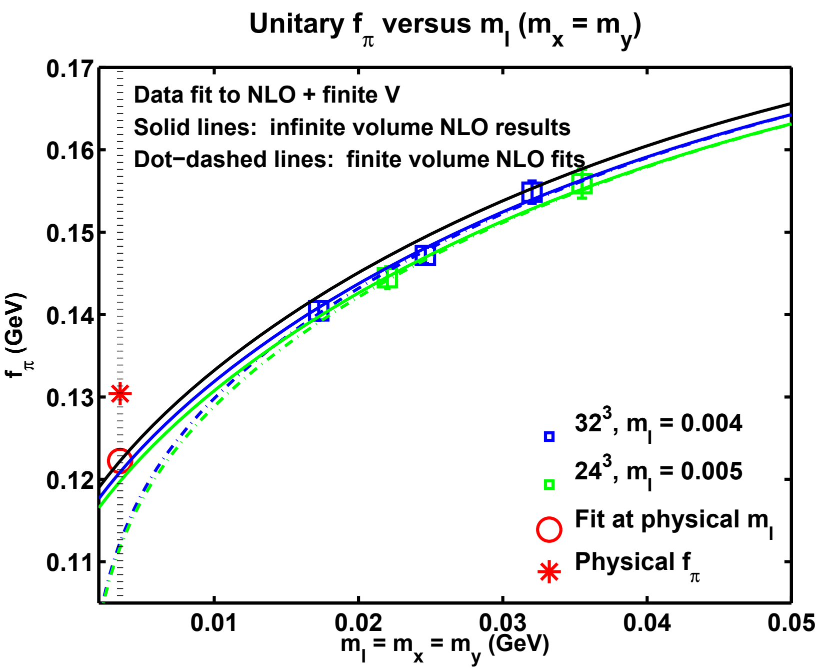
(c) $m_l^{24} = 0.005,\ m_h^{24} = 0.04$

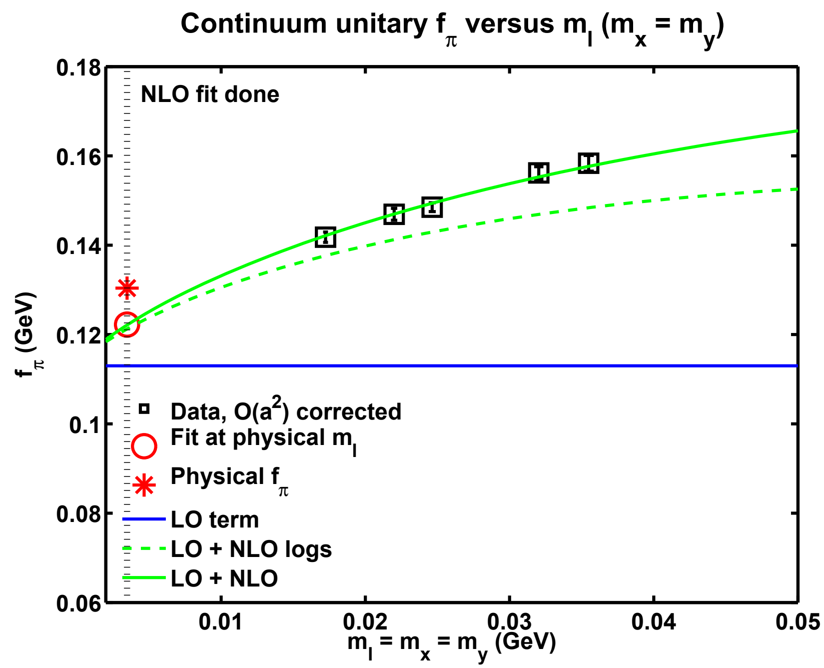


(d) $m_l^{24} = 0.01,\ m_h^{24} = 0.04$

Pseudo scalar decay constant

- Now include continuum and NLO SU(2) x SU(2) ChPT

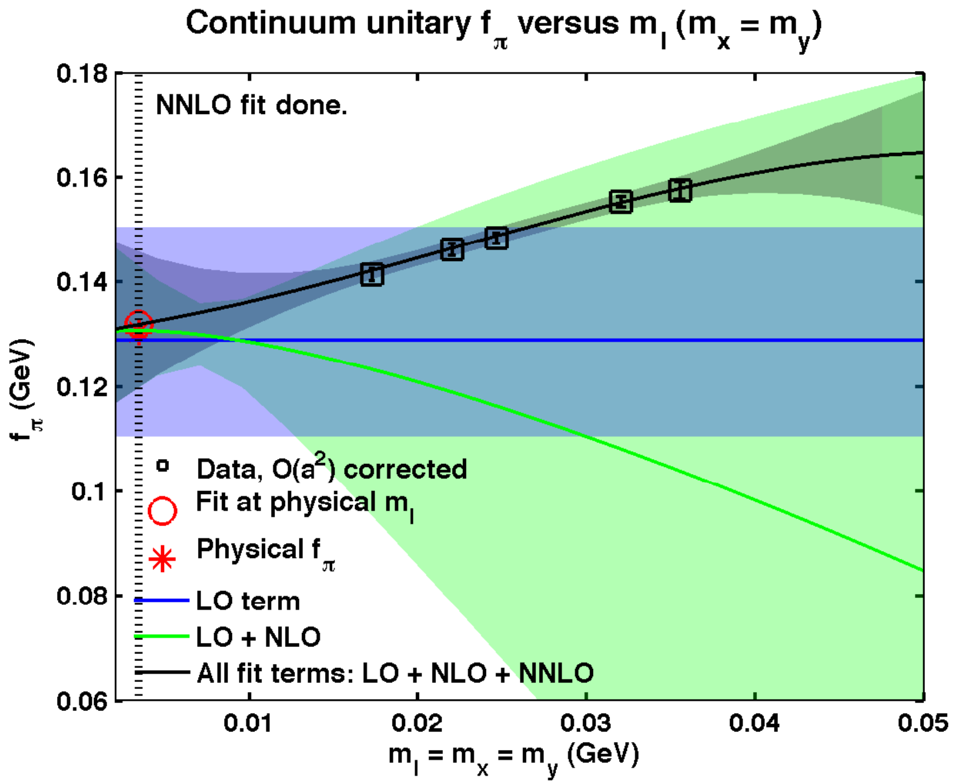




- Now f_π = 122.2 (3.4) MeV
- NLO term ~ 20-30% of LO → NNLO ~ 5-10%?

Pseudo scalar decay constant

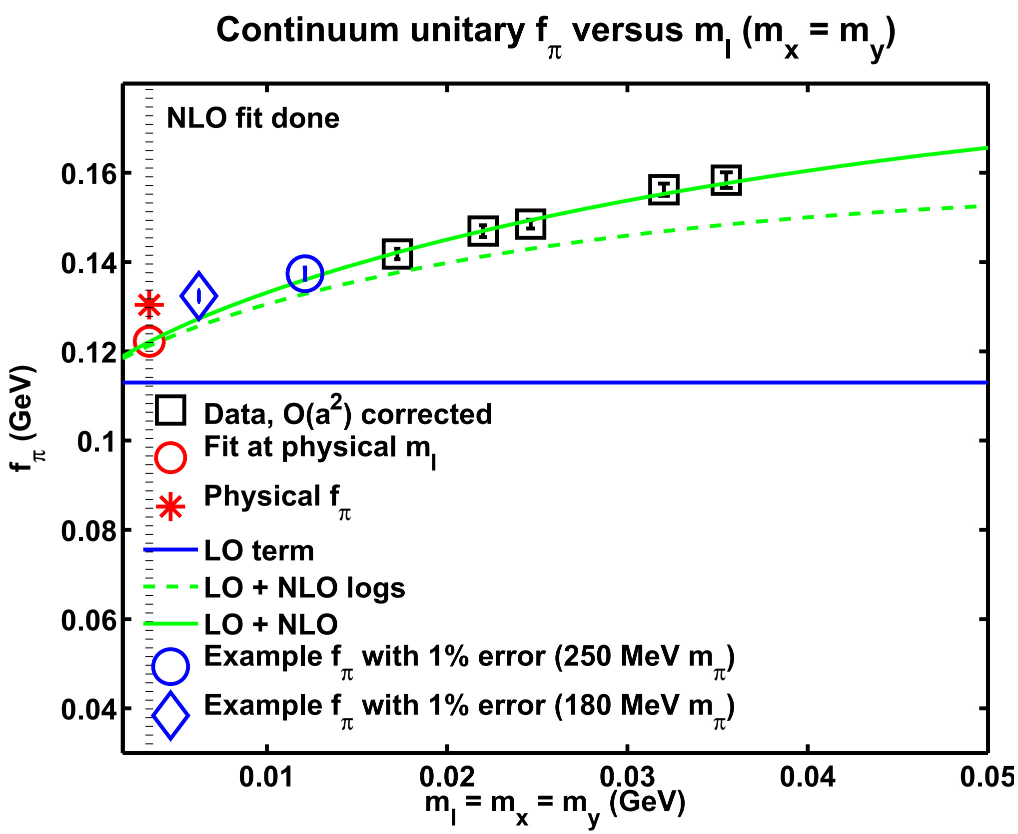
- Dealing with NNLO effects:



- f_π = 122.2 (3.4) → 133 (13) MeV
- NNLO (with 15 extra parameters) ill determined for 220 MeV ≤ m_π ≤ 430 MeV

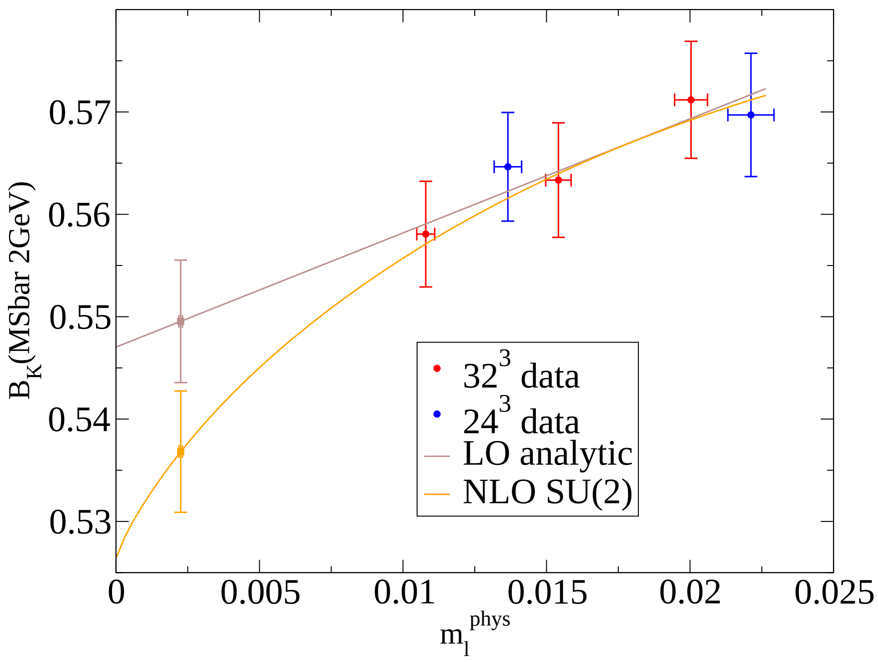
Pseudo scalar decay constant

- Smaller quark masses are needed!



Continuum results

- DWF results from two lattice spacings
  - Small, 1-2% , $O(a^2)$ errors.
  - $\boldsymbol{B_K}$ **= 0.524(30)** [PRL, 2008] → $\boldsymbol{B_K}$ **= 0.537(19)** [preliminary, 2009].
  - $m_{ud}^{\mathrm{MS}}(2\ \mathrm{GeV}) = 3.47 \pm 0.10_{\mathrm{stat}} \pm 0.17_{\mathrm{NRP}}$ MeV
  - $m_s^{\mathrm{MS}}(2\ \mathrm{GeV}) = 94.3 \pm 3.4_{\mathrm{stat}} \pm 4.5_{\mathrm{NRP}}$ MeV



“QCD” with many light flavors: $2 < N_f \leq 16.5$

Motivation

- Classification the long-distance behavior of each gauge theory with group G and fermion Rep R provided $\beta(g) < 0$ for small g.
- Bank's–Zaks argument suggests infrared fixed point and possible conformal long-distance behavior as $N_f \rightarrow 16.5$.
- Construct "walking technicolor" model with sufficiently large confinement scale.
- Reliable results require lattice methods!
- Slowly running coupling → large distance scale must be explored.

Recent Work

- Appelquist, Fleming and Neil [PRL 100, 171607 (2008)] present evidence for a infrared fixed point with N_f=12: $\lim_{L \to \infty} g^{SF}(L) = g_{\infty}$.
- Deuzeman, Lombardo, and Pallante [arXiv:0904.4662 (hep-ph)] conclude that N_f=12 shows conformal behavior.
- Recent work by Xiao Yong Jin and Bob Mawhinney: Carefully study an array of standard observables: chiral condensate, static quark potential, m_π, m_ρ, f_π and temperature dependence.

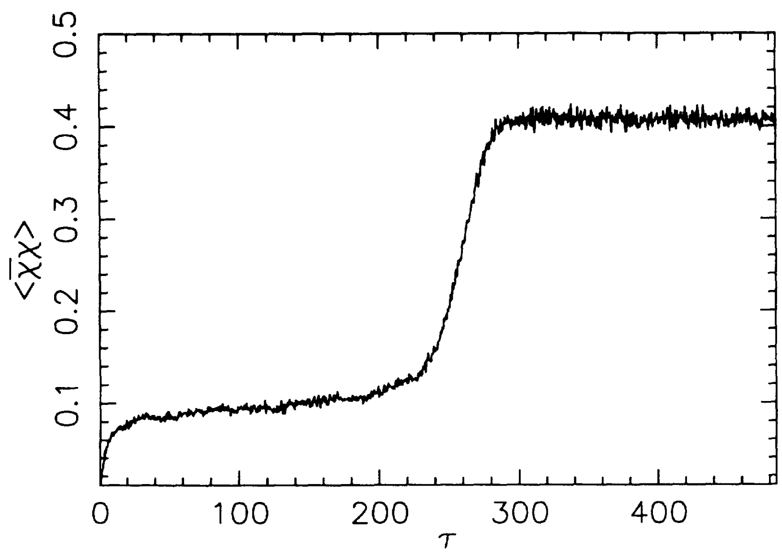
Computational strategy

- Use staggered fermions to reduce cost.
- Recall one single-component staggered field χ_n describes 4 flavors or "tastes" of spin-1/2 particle.
- Using 1, 2 or 3 such fields gives 4, 8 and 12 flavors.
- No *rooting* but $SU_L(12) \times SU_R(12)$ symmetry is broken by $O(a^2)$ effects (reduced with DBW2 action).
- There is an exact $SU_L(3) \times SU_R(3)$ subgroup.

Recall N_f = 4 and 8

- Both $N_f = 4$ and 8 show confinement and vacuum chiral symmetry breaking of QCD.

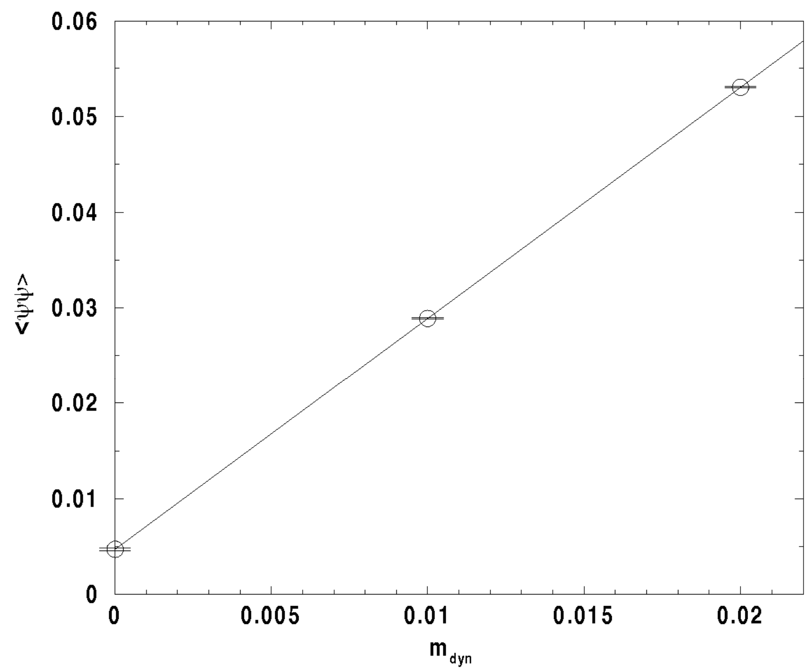
- $N_f = 8$, (Wilson action) has a treacherous bulk transition.



- Requires $g^2 < g_{\mathrm{crit}}^2$ where lattice scale shrinks by 3x .

Behavior of chiral condensate

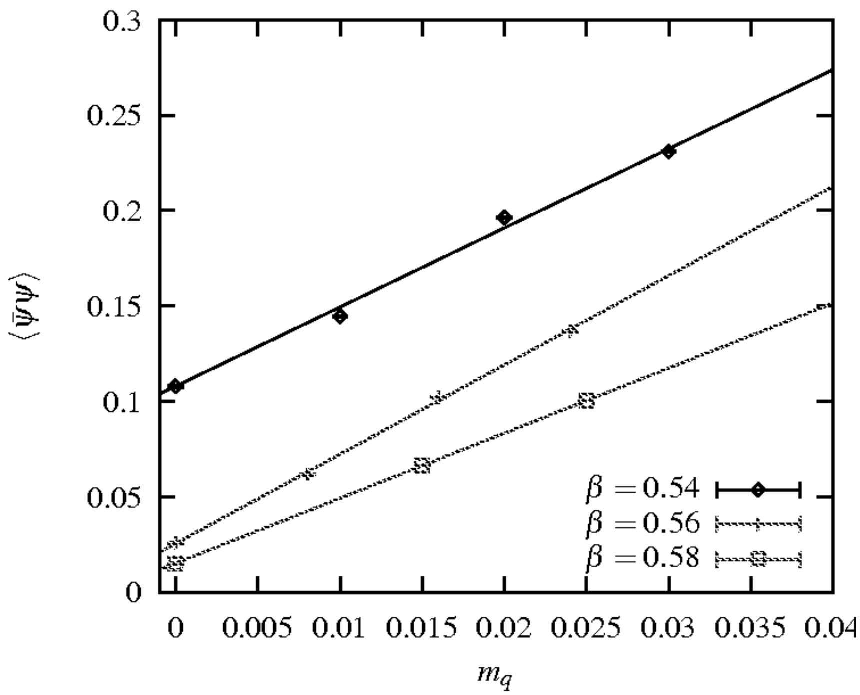
- Exact staggered chiral symmetry spontaneously broken by $< \overline{\psi}\, \psi > \neq 0$



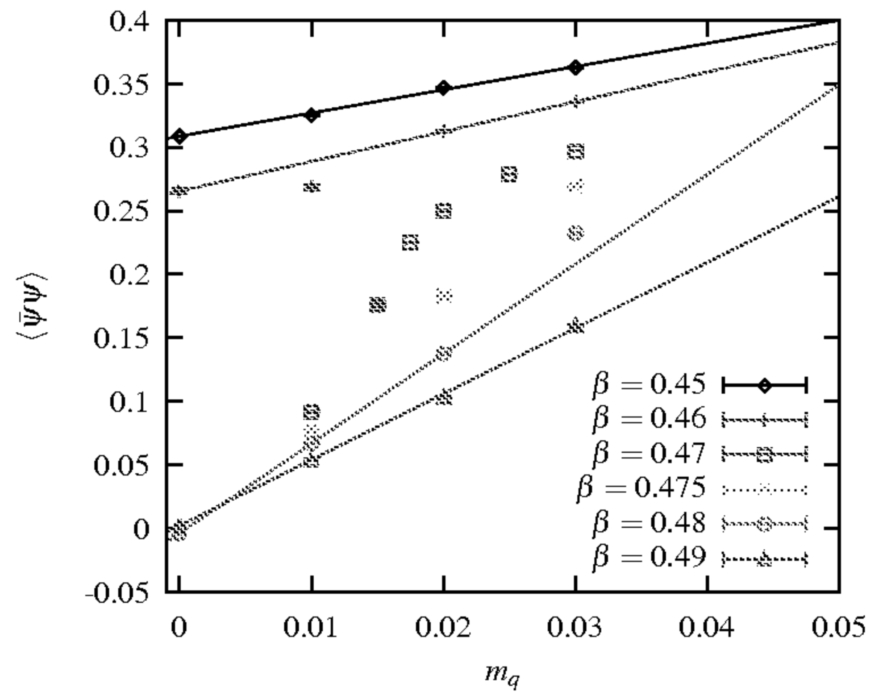
$N_f = 4$

Behavior of chiral condensate

- Compare $N_f = 8$ and 12 as β increases.



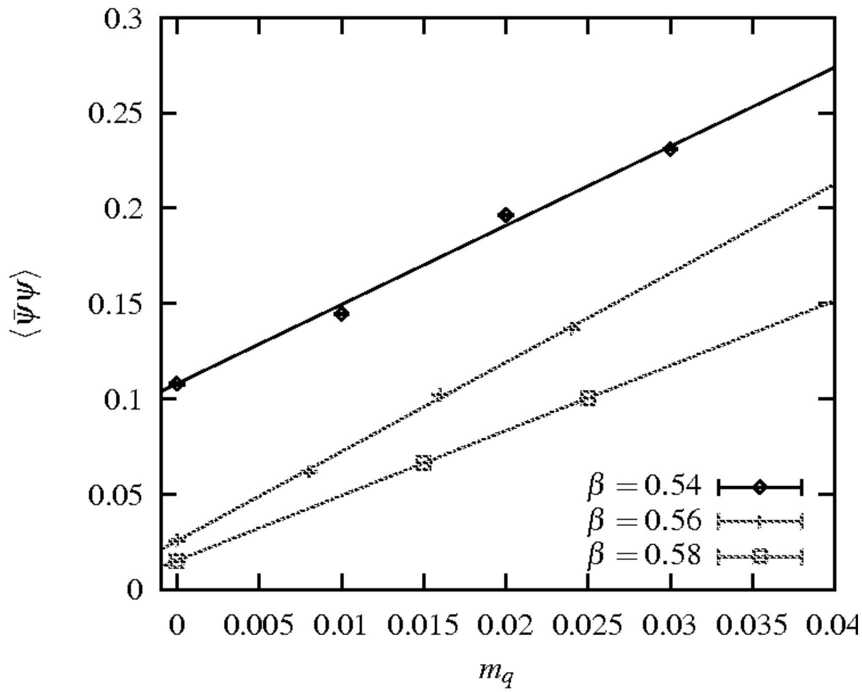
$N_f = 8$



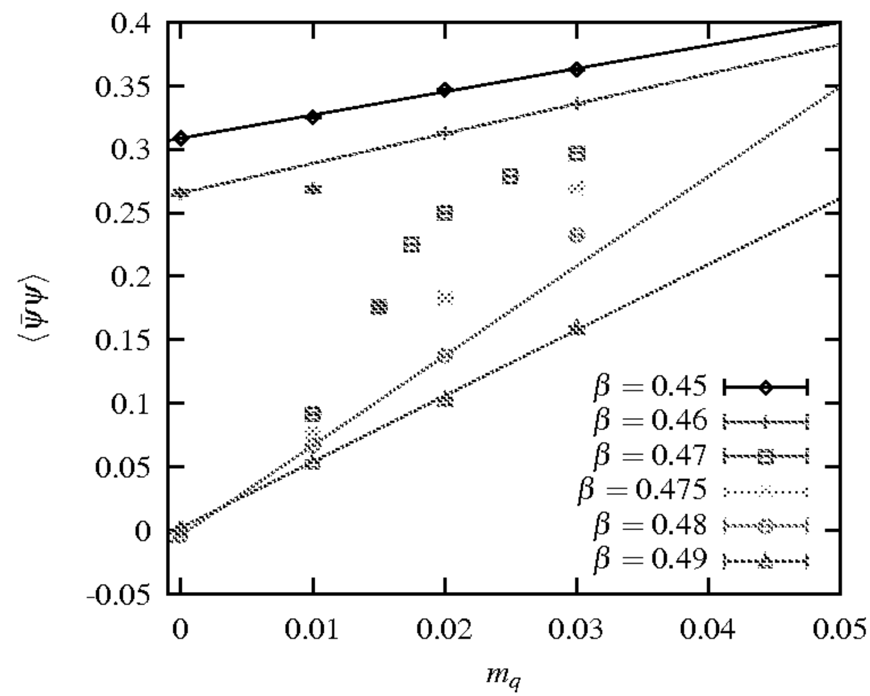
$N_f = 12$

Behavior of chiral condensate

- Scale change from strong to weak coupling:
  - $N_f = \ \ 8$: f_π falls 2x, m_ρ falls 2x
  - $N_f = 12$: f_π falls 10x, m_ρ falls 6x



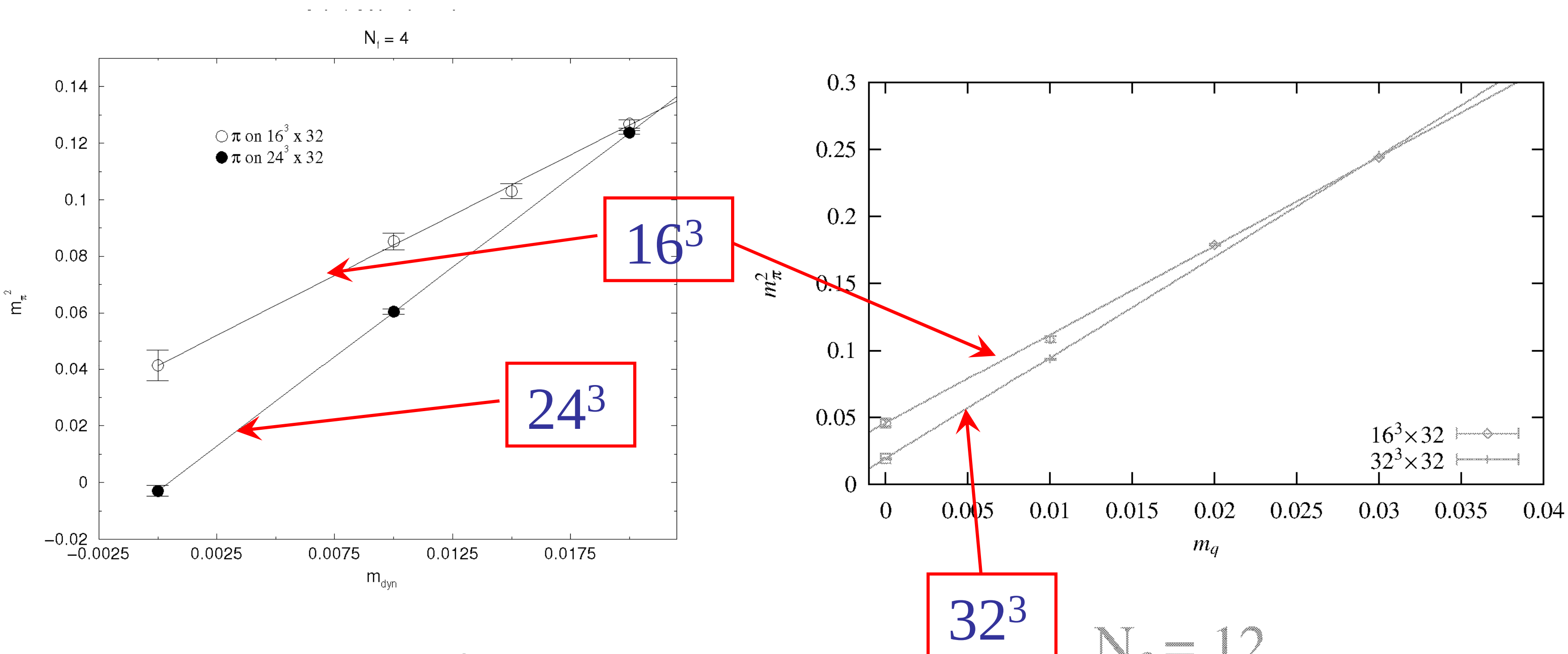
$\mathrm{N_f = 8}$



$\mathrm{N_f = 12}$

Is there a Goldstone pion?

- m_π small but non-zero
- Goldstone finite volume sensitivity
- Nf=12 shows familiar QCD-like behavior!



$N_f = 4$

$N_f = 12$

Lattice Field Theory

- Hard when you don’t know the answer in advance!
- Very much a theorist’s subject: refined command of field theory and phenomenology required.
- Objectivity and care of experimental work absolutely required: opportunities for self-delusion are rampant.

A new direction for Al!

arXiv:0906.3175v1 [hep-ph] 17 Jun 2009

A lattice test of strong coupling behaviour in QCD at finite temperature

E. Iancu

Institut de Physique Théorique de Saclay, F-91191 Gif-sur-Yvette, France
E-mail: Edmond.Iancu@cea.fr

A. H. Mueller

Department of Physics, Columbia University, New York, NY 10027, USA
E-mail: amh@phys.columbia.edu

ABSTRACT: We propose a set of lattice measurements which could test whether the deconfined, quark–gluon plasma, phase of QCD shows strong coupling aspects at temperatures a few times the critical temperature for deconfinement, in the region where the conformal anomaly becomes unimportant. The measurements refer to twist–two operators which are not protected by symmetries and which in a strong–coupling scenario would develop large, negative, anomalous dimensions, resulting in a strong suppression of the respective lattice expectation values in the continuum limit. Special emphasis is put on the respective operator with lowest spin (the spin–2 operator orthogonal to the energy–momentum tensor within the renormalization flow) and on the case of quenched QCD, where this operator is known for arbitrary values of the coupling: this is the quark energy–momentum tensor. The proposed lattice measurements could also test whether the plasma constituents are pointlike (as expected at weak coupling), or not.