A new look for (& at) good old Parton Dynamics

ALFEST

Yuri Dokshitzer
LPTHE Jussieu, Paris
and
PNPI, St Petersburg

"***Resummations***" - a world of a life-saving pQCD tool.

“***Resummations***” - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that “exponentiates” repetitive first (few low-) order elements.

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

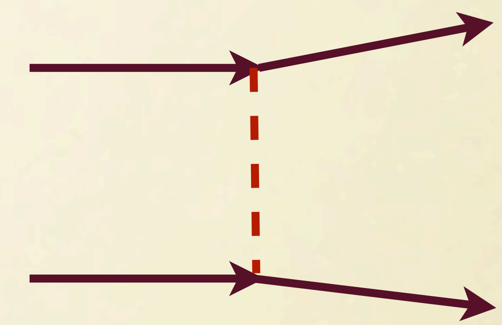
Simplest example of exponentiation - QCD form factors.

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

Simplest example of exponentiation - QCD form factors.

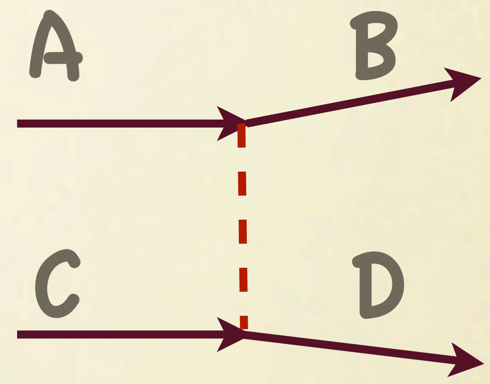


"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

Simplest example of exponentiation - QCD form factors.

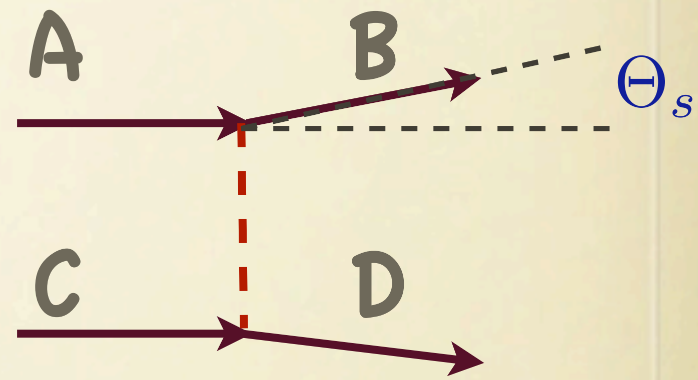


“***Resummations***” - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that “exponentiates” repetitive first (few low-) order elements.

This is how we come to speak of “***evolution***”
(of some observable in certain cleverly chosen variable - “*evolution time*”.)

Simplest example of exponentiation - QCD form factors.

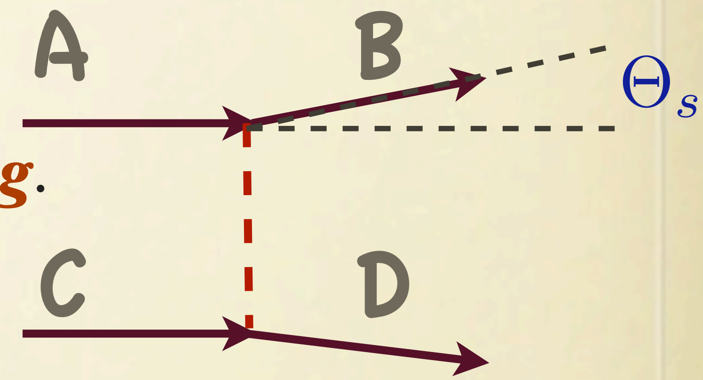


"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

Simplest example of exponentiation - QCD form factors.



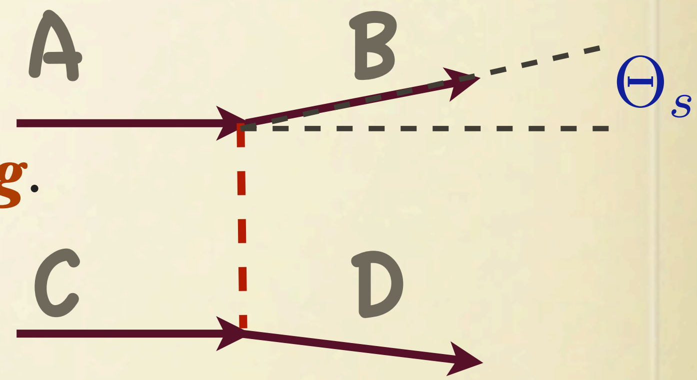
Any process involving "charged" particles causes ***bremsstrahlung***.

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

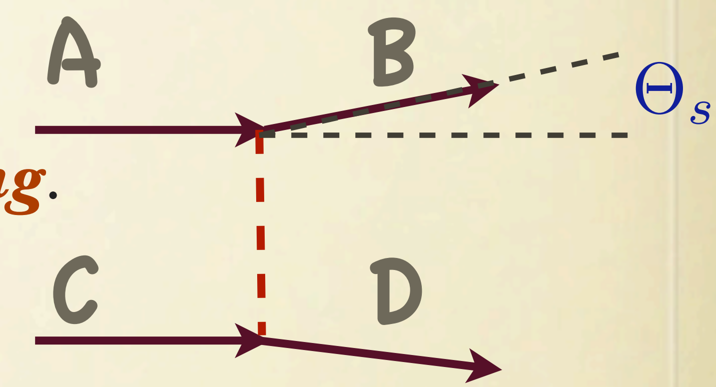
("*infrared catastrophe*")

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

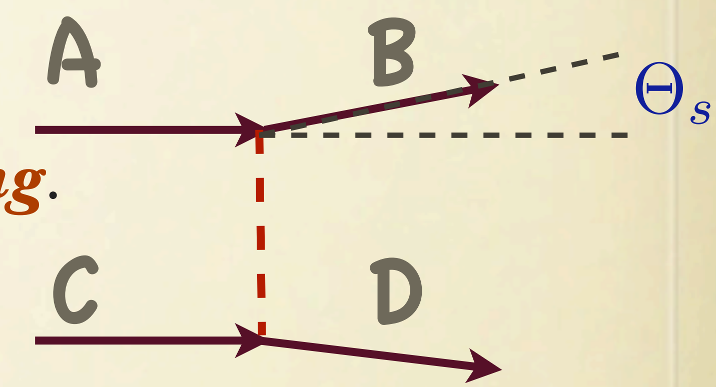
$$\mathcal{A} \propto \; F_A F_B F_C F_D$$

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto F_A F_B F_C F_D$$

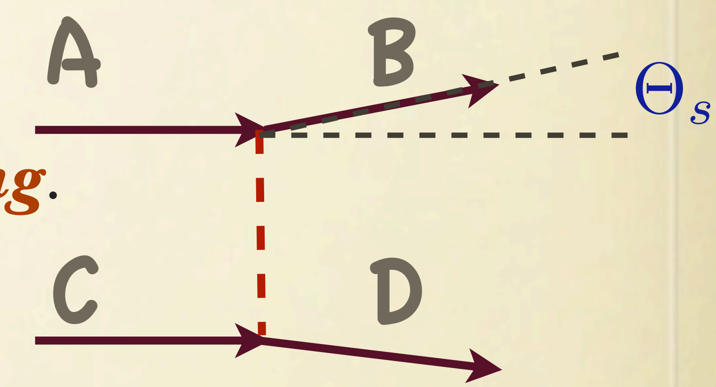
$$F_i = F_i(t)$$

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

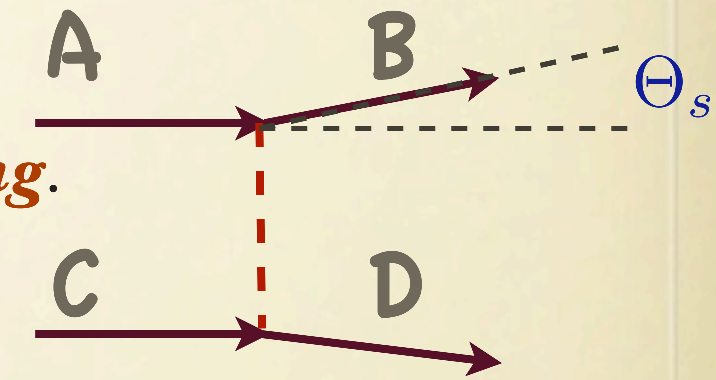
$$\mathcal{A} \propto F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad F_i = F_i(t)$$

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

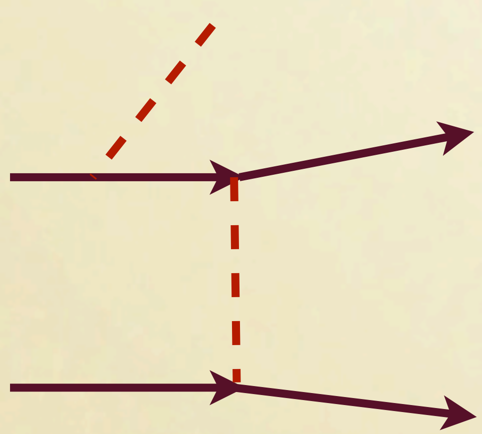
Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad F_i = F_i(t)$$

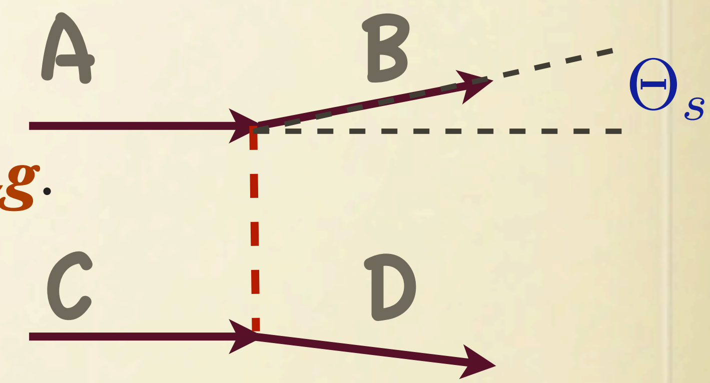


"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

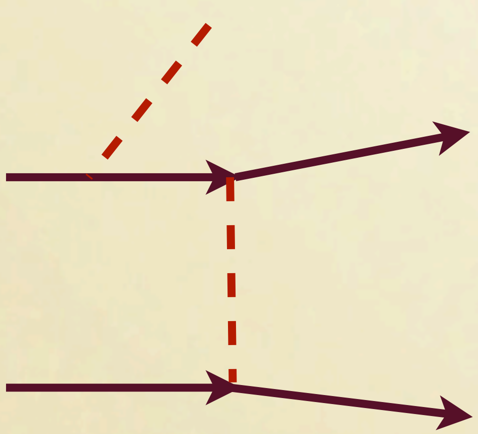
Simplest example of exponentiation - QCD form factors.

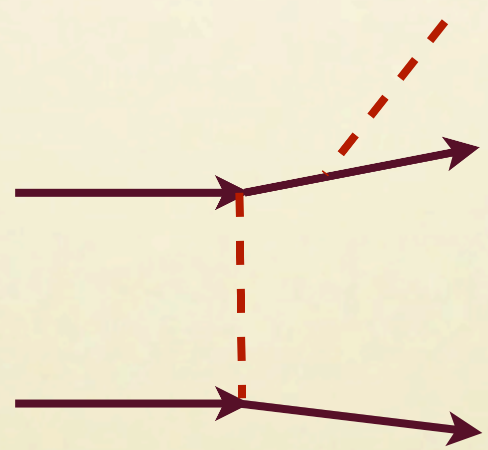


Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto \; F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad F_i = F_i(t)$$



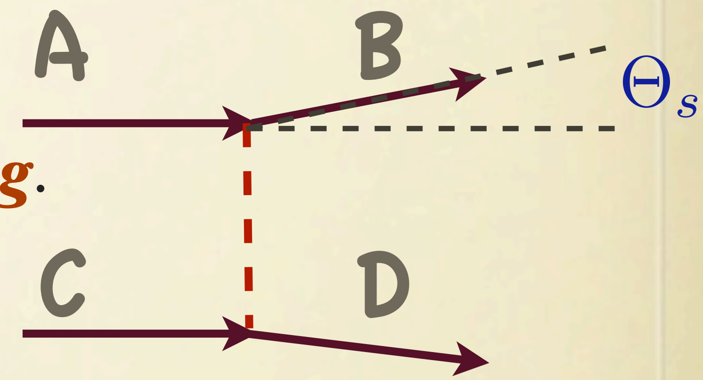


"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

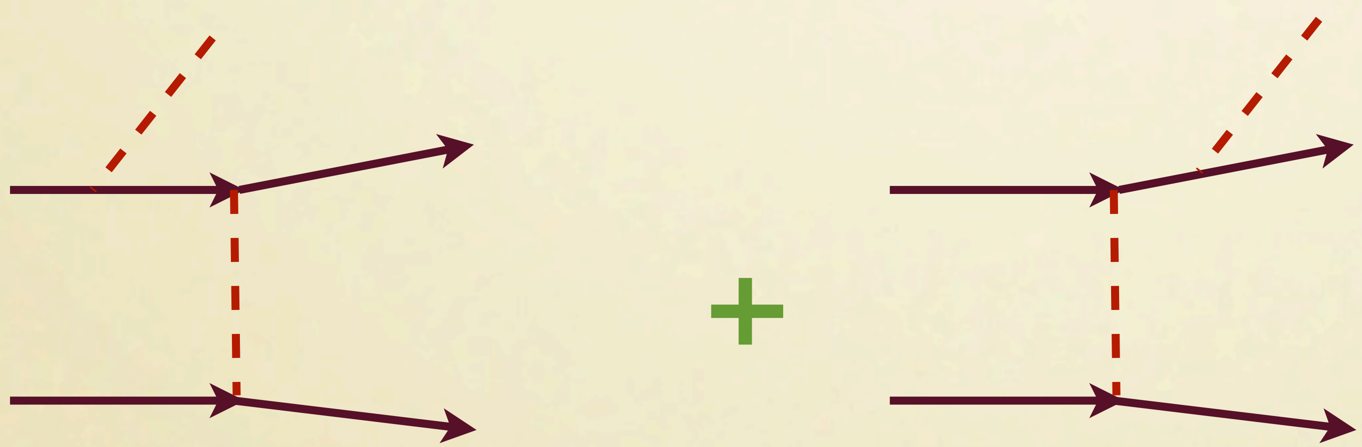
Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto \; F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad\qquad F_i = F_i(t)$$

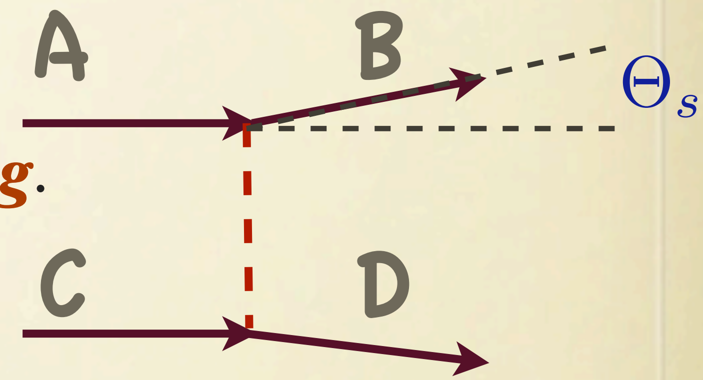


“***Resummations***” - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that “exponentiates” repetitive first (few low-) order elements.

This is how we come to speak of “***evolution***”
(of some observable in certain cleverly chosen variable - “*evolution time*”.)

Simplest example of exponentiation - QCD form factors.



Any process involving “charged” particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

(“*infrared catastrophe*”)

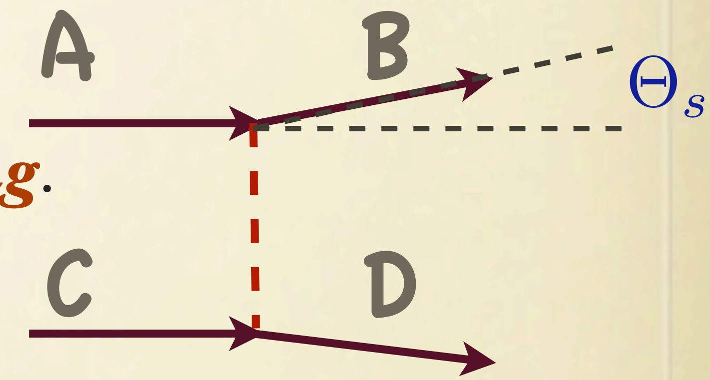
$$\mathcal{A} \propto \; F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad\qquad F_i = F_i(t)$$

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

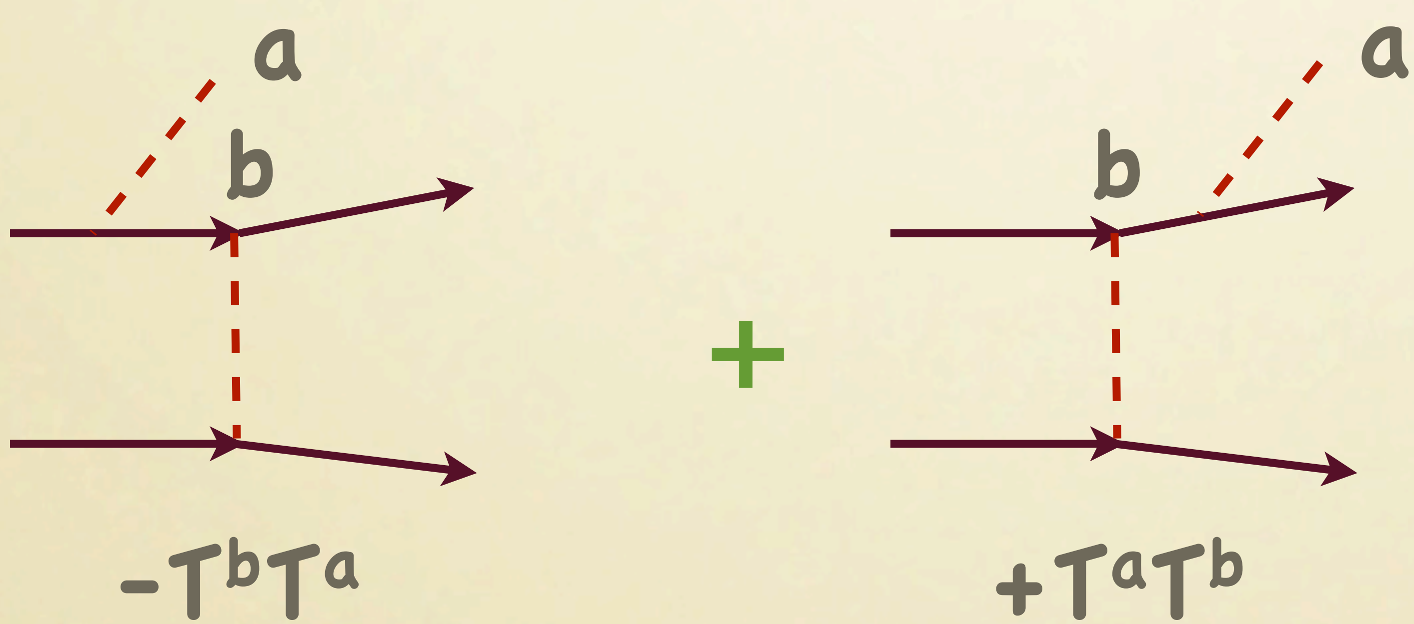
Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad F_i = F_i(t)$$

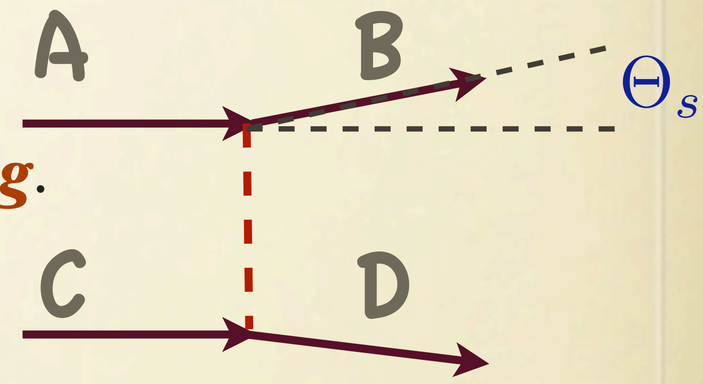


“***Resummations***” - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that “exponentiates” repetitive first (few low-) order elements.

This is how we come to speak of “***evolution***”
(of some observable in certain cleverly chosen variable - “*evolution time*”.)

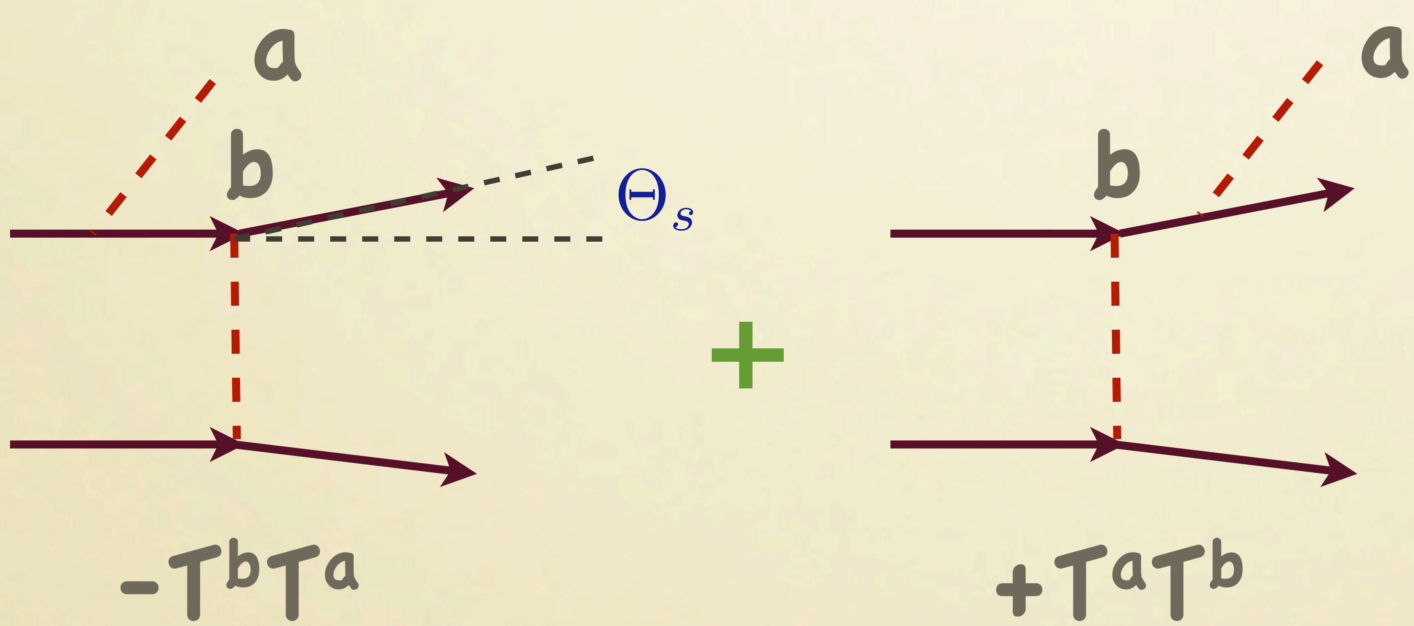
Simplest example of exponentiation - QCD form factors.



Any process involving “charged” particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

(“*infrared catastrophe*”)

$$\mathcal{A} \propto \; F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad\qquad F_i = F_i(t)$$

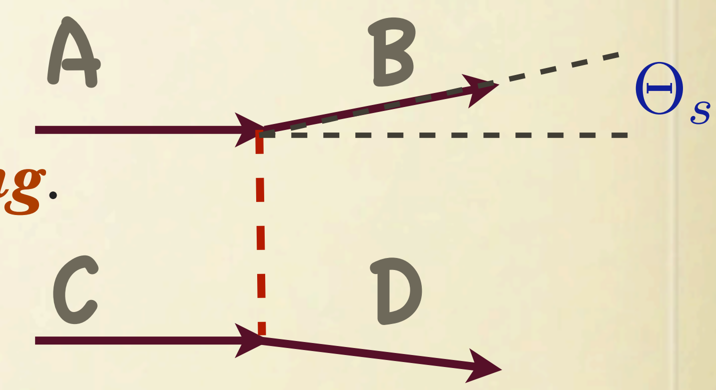


"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

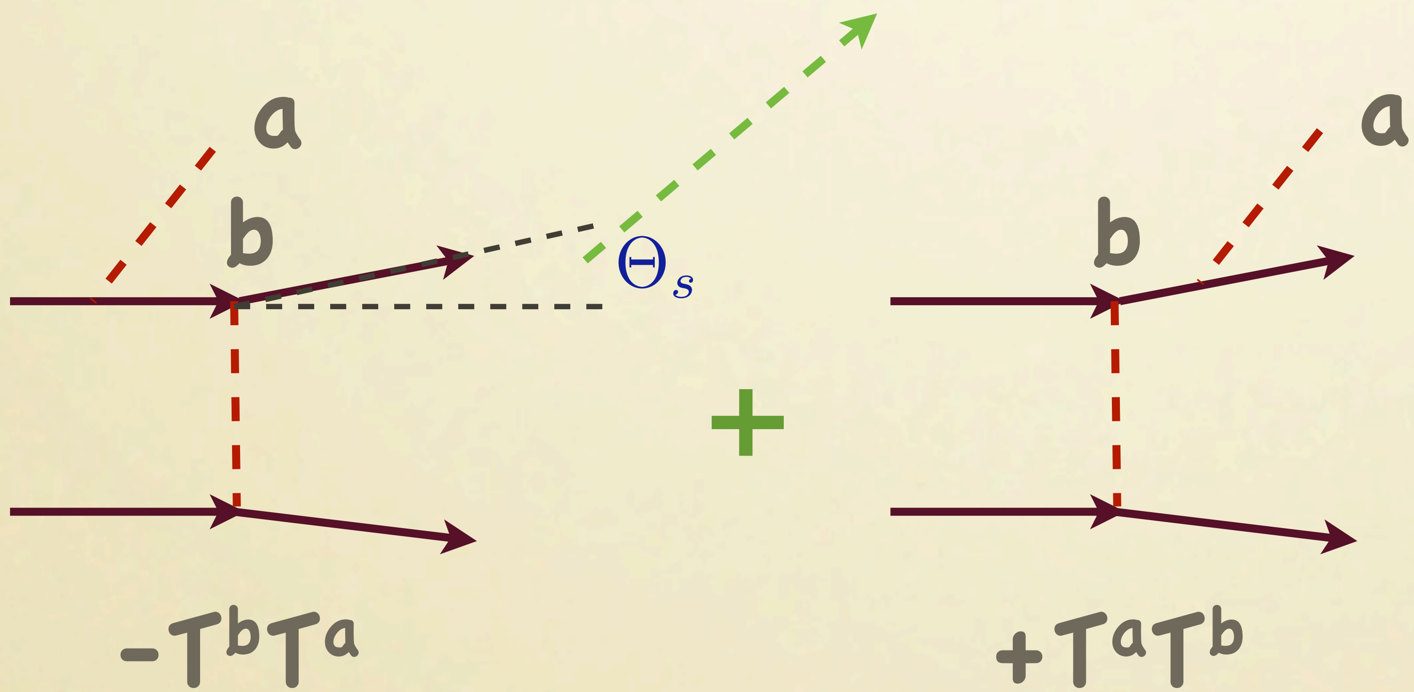
Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad F_i = F_i(t)$$

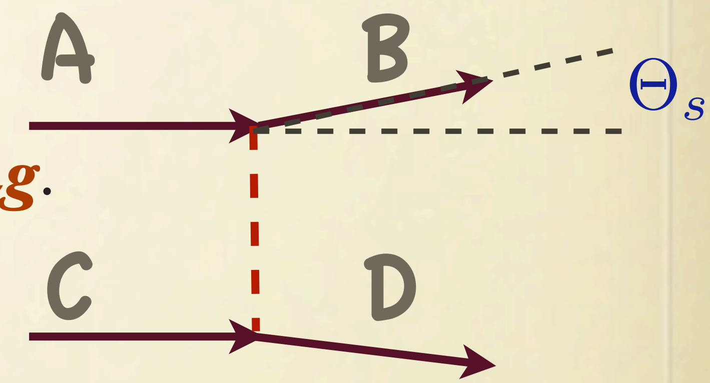


"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

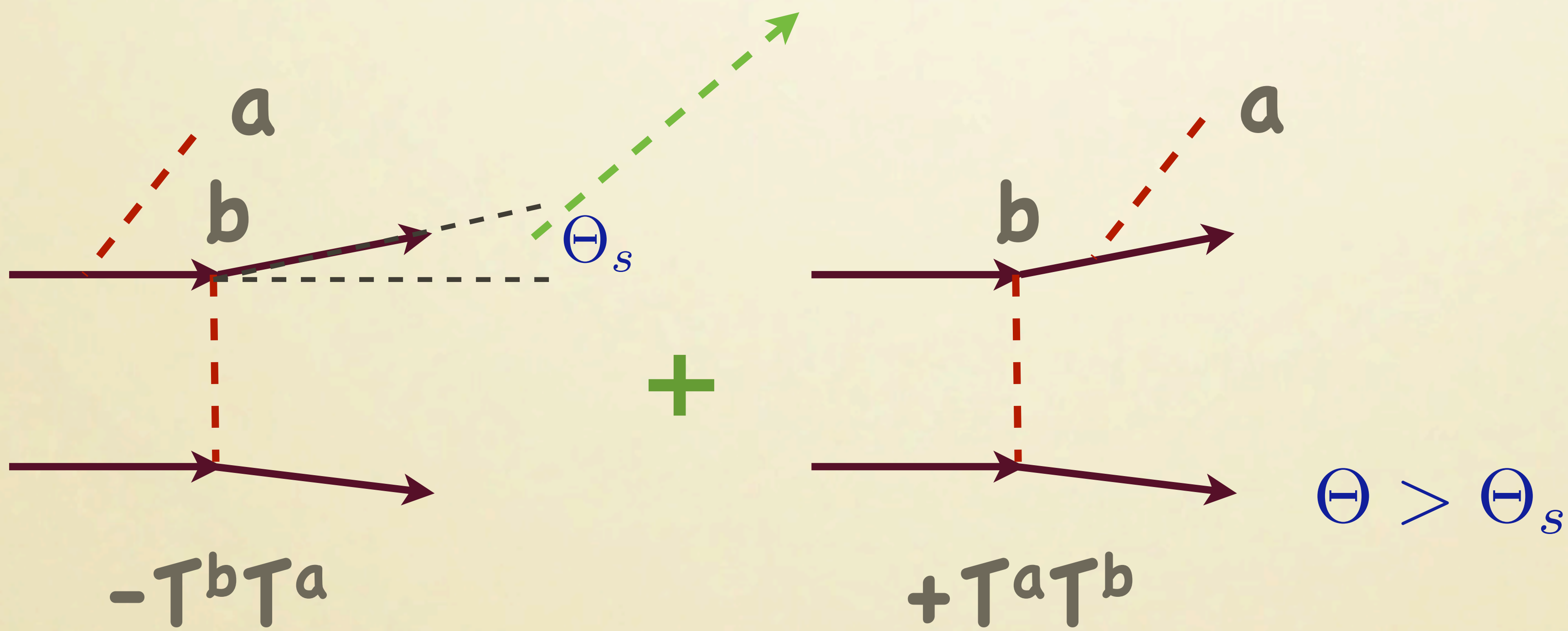
Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad F_i = F_i(t)$$

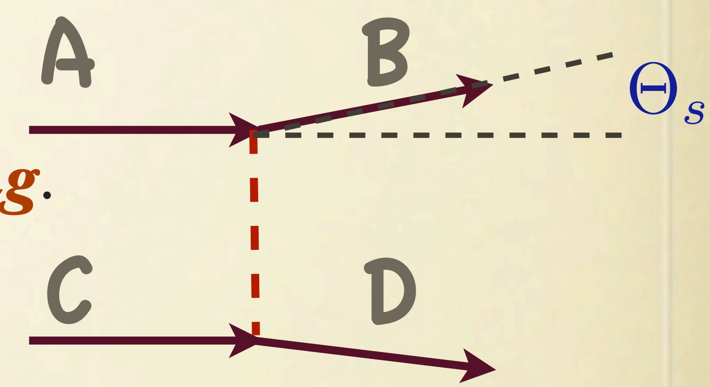


"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

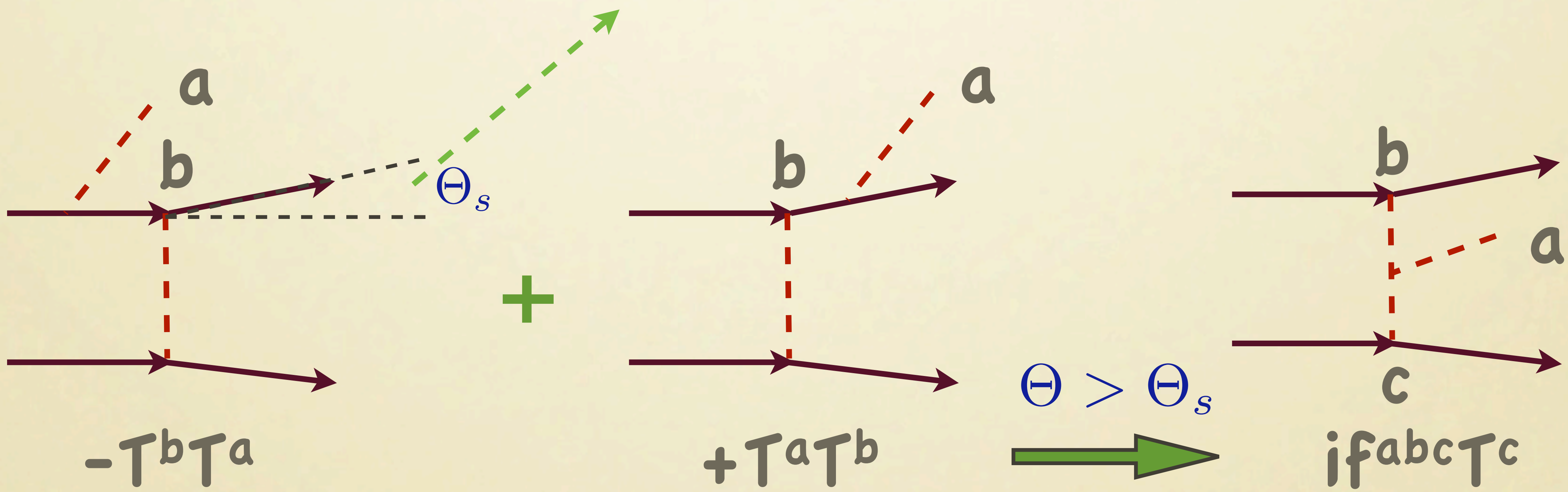
Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad F_i = F_i(t)$$

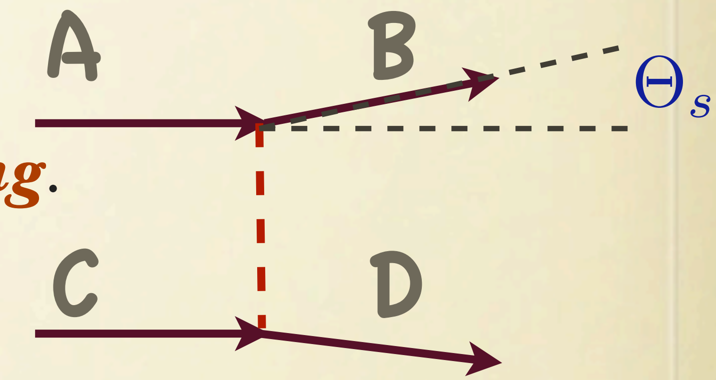


"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

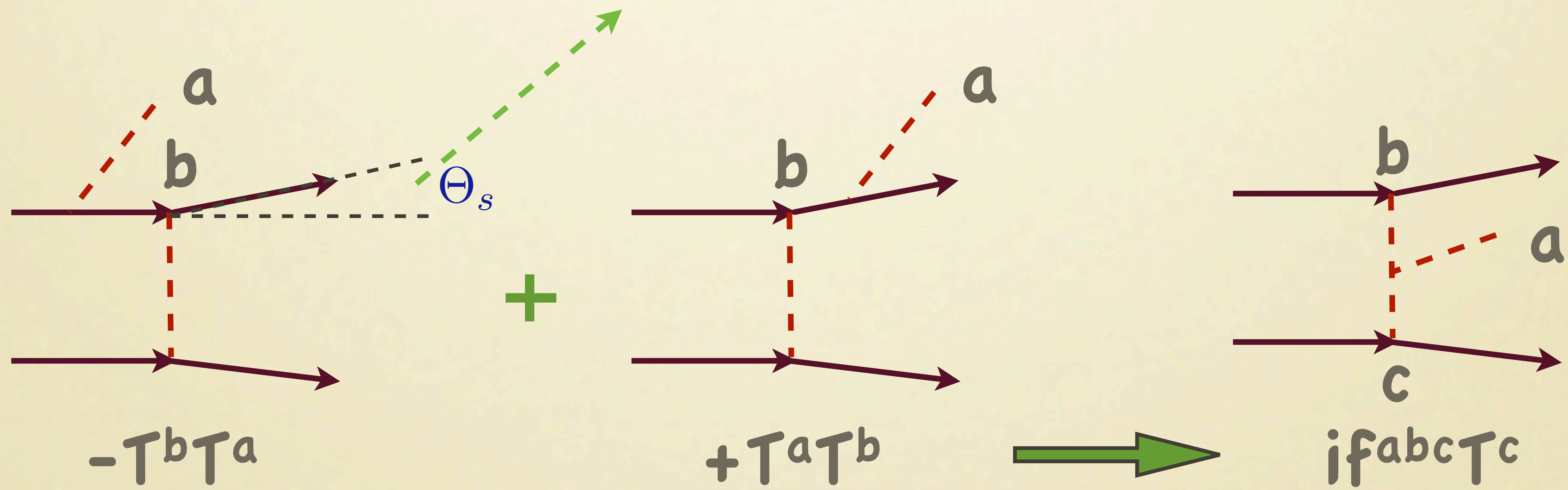
Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$\mathcal{A} \propto F_A F_B F_C F_D \cdot \mathcal{F}(s,t)$ $\qquad F_i = F_i(t)$



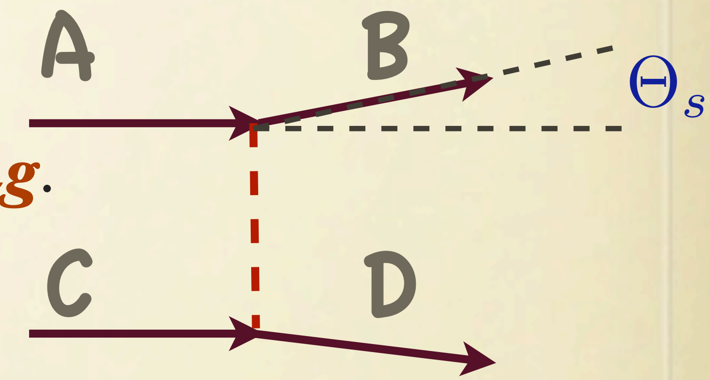
Soft radiation at angles $\Theta > \Theta_s$ is independent of the nature of scattering objects

"***Resummations***" - a world of a life-saving pQCD tool.

Instead of painstakingly calculating order-by-order perturbative contributions, write down the ***all-order*** answer that "exponentiates" repetitive first (few low-) order elements.

This is how we come to speak of "***evolution***"
(of some observable in certain cleverly chosen variable - "*evolution time*".)

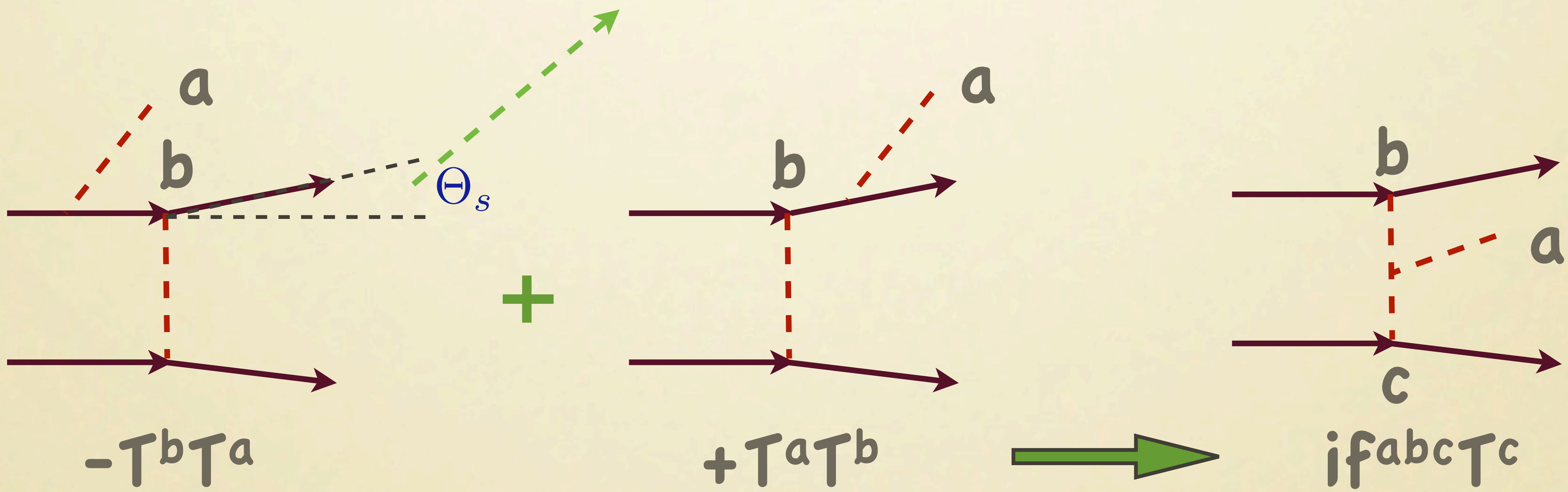
Simplest example of exponentiation - QCD form factors.



Any process involving "charged" particles causes ***bremsstrahlung***.
Probability of purely elastic scattering is (infinitely) suppressed.

("*infrared catastrophe*")

$$\mathcal{A} \propto \; F_A F_B F_C F_D \cdot \mathcal{F}(s,t) \qquad F_i = F_i(t)$$



Soft radiation at angles $\Theta > \Theta_s$ is independent of the nature of scattering objects

It "belongs" to the exchanged gluon **!**

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(q,k) \, \phi(y,k)$$

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial\phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2}\int d^2k\, K(q,k)\,\phi(y,k)$$

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$ rapidity-independent kernel !

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(q,k) \, \phi(y,k)$$

"*Lipatov vertex*" $\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

rapidity-independent kernel **!**

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(q,k) \, \phi(y,k)$$

"Lipatov vertex"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened?

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

"*Lipatov vertex*"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(q,k) \, \phi(y,k)$$

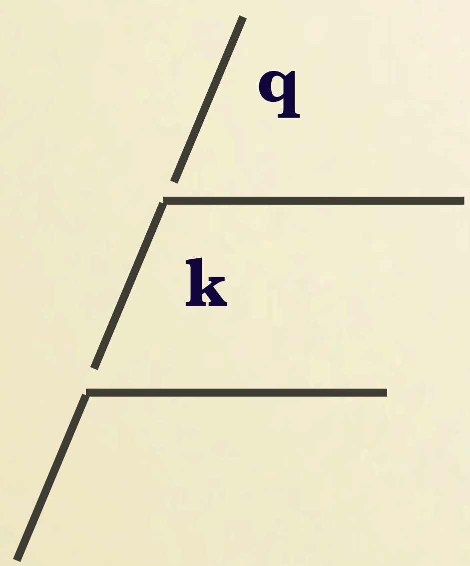
"*Lipatov vertex*"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(q,k) \, \phi(y,k)$$

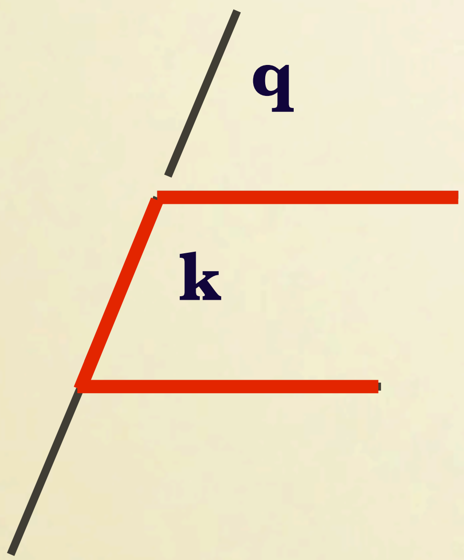
"*Lipatov vertex*"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

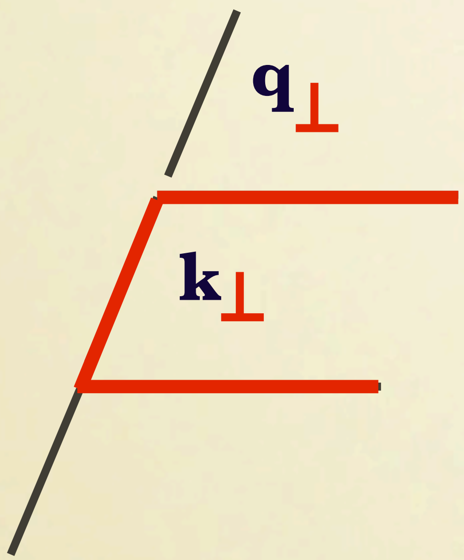
"*Lipatov vertex*"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

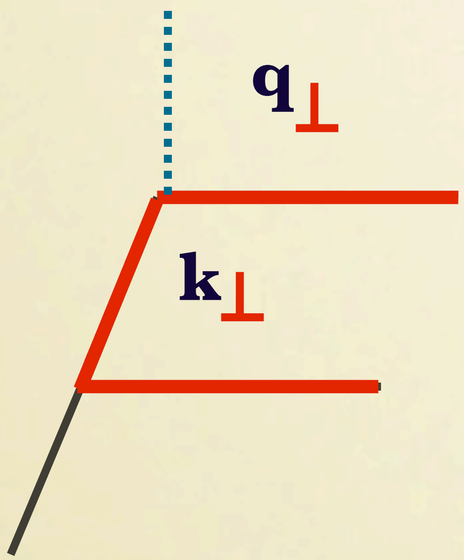
$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

"*Lipatov vertex*" $\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

rapidity-independent kernel **!**

How has it happened? Once again, quantum-mechanical coherence at work.



Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(q,k) \, \phi(y,k)$$

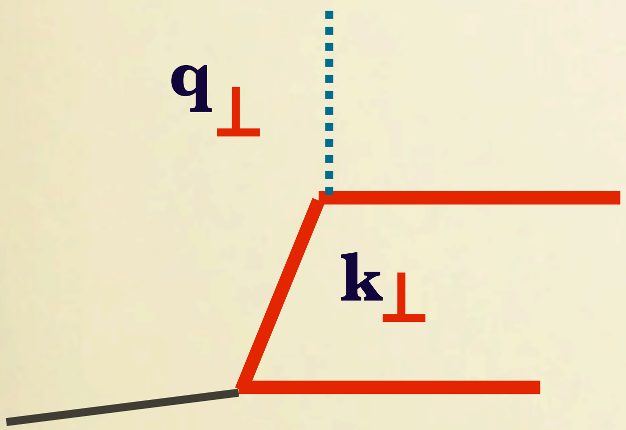
"Lipatov vertex"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

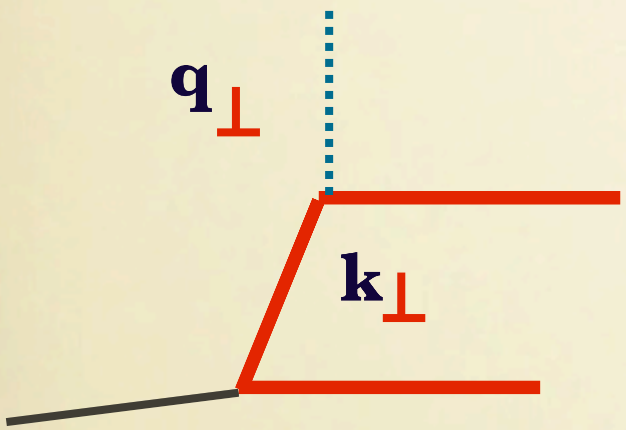
"Lipatov vertex"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Inelastic dissociation.

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

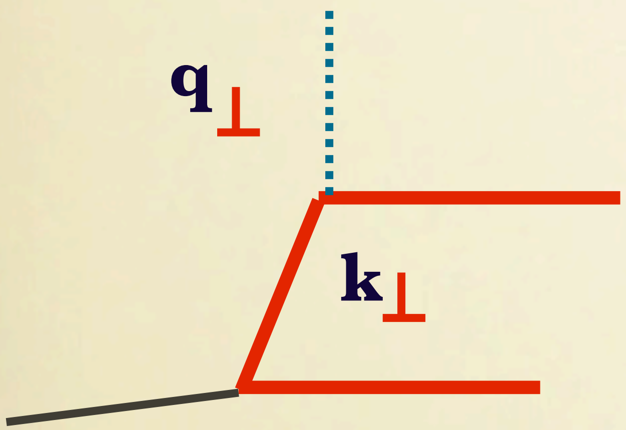
"*Lipatov vertex*"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Inelastic dissociation. Into a ***compact*** state : $\Delta\rho_\perp \sim k_\perp^{-1}$

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

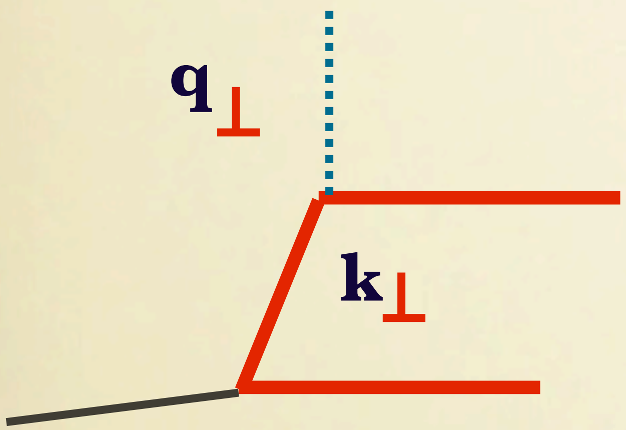
"*Lipatov vertex*"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Inelastic dissociation. Into a ***compact*** state : $\Delta\rho_\perp \sim k_\perp^{-1}$

In a ***long-range*** potential : $\lambda_\perp \sim q_\perp^{-1} \gg \Delta\rho_\perp$

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

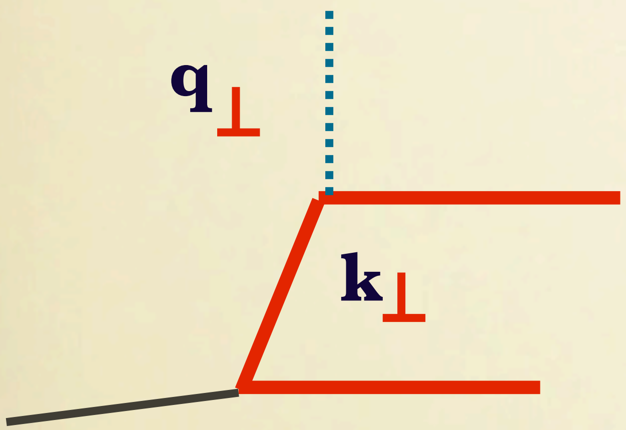
"*Lipatov vertex*"

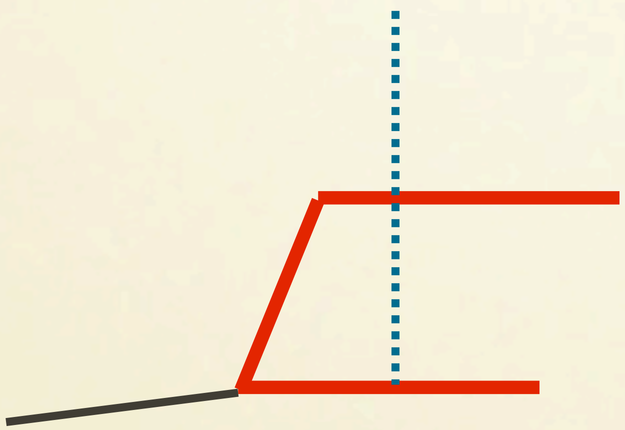
$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.





Inelastic dissociation. Into a ***compact*** state : $\Delta\rho_\perp \sim k_\perp^{-1}$

In a ***long-range*** potential : $\lambda_\perp \sim q_\perp^{-1} \gg \Delta\rho_\perp$

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

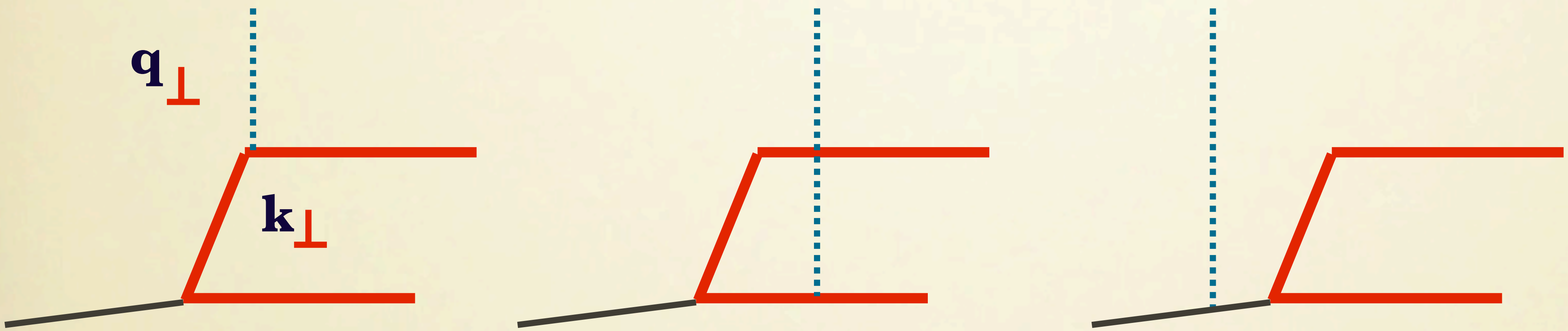
"*Lipatov vertex*"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Inelastic dissociation. Into a ***compact*** state : $\Delta\rho_\perp \sim k_\perp^{-1}$

In a ***long-range*** potential : $\lambda_\perp \sim q_\perp^{-1} \gg \Delta\rho_\perp$

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(q,k) \, \phi(y,k)$$

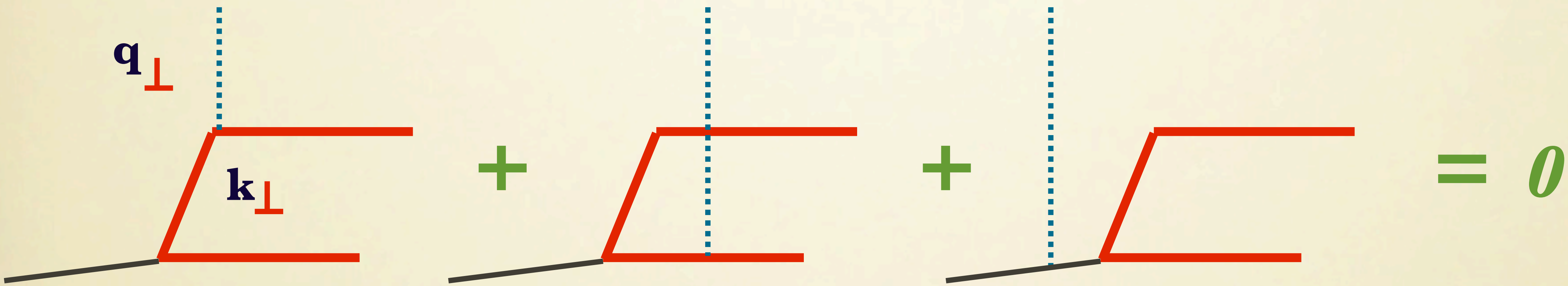
"Lipatov vertex"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Inelastic dissociation. Into a ***compact*** state : $\Delta\rho_\perp \sim k_\perp^{-1}$

In a ***long-range*** potential : $\lambda_\perp \sim q_\perp^{-1} \gg \Delta\rho_\perp$

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \, K(q,k) \, \phi(y,k)$$

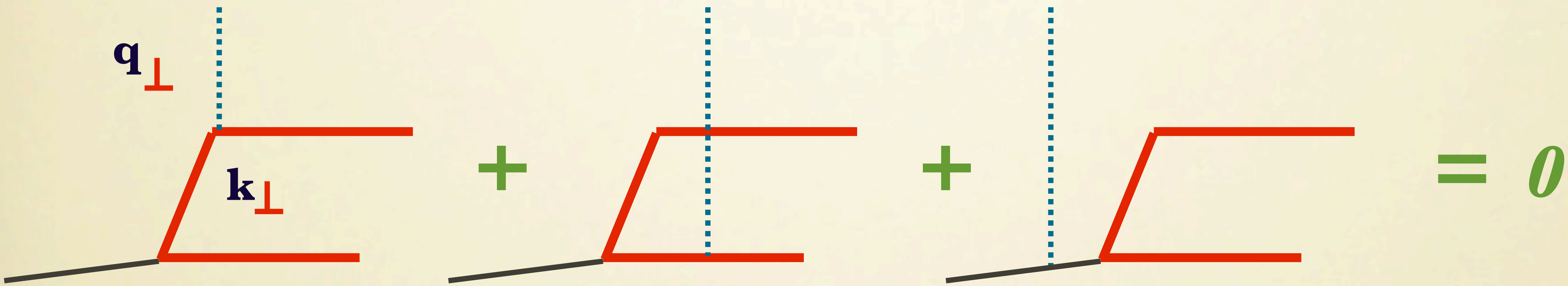
"Lipatov vertex"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Inelastic dissociation. Into a ***compact*** state : $\Delta\rho_\perp \sim k_\perp^{-1}$

In a ***long-range*** potential : $\lambda_\perp \sim q_\perp^{-1} \gg \Delta\rho_\perp$

In QCD/QED vanishing of forward inelastic processes follows from gauge invariance.

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

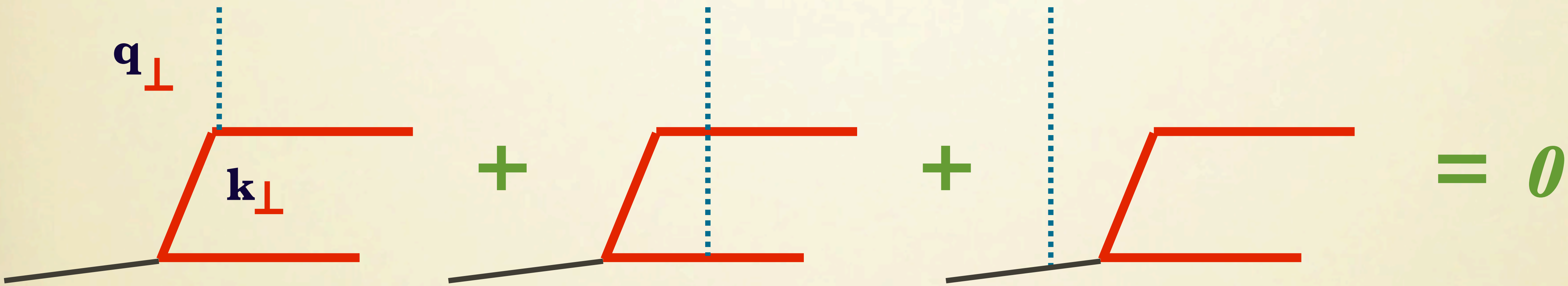
"Lipatov vertex"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}\right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Inelastic dissociation. Into a ***compact*** state : $\Delta\rho_\perp \sim k_\perp^{-1}$

In a ***long-range*** potential : $\lambda_\perp \sim q_\perp^{-1} \gg \Delta\rho_\perp$

In QCD/QED vanishing of forward inelastic processes follows from gauge invariance.

In a more general context, it is due to orthogonality of the initial and final state wave functions, *provided* the initial and final systems *interact identically* with "the probe".

Rapidity evolution of ***unintegrated parton density*** ***(BFKL)***

$$\frac{\partial \phi(y,q)}{\partial y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(q,k)\, \phi(y,k)$$

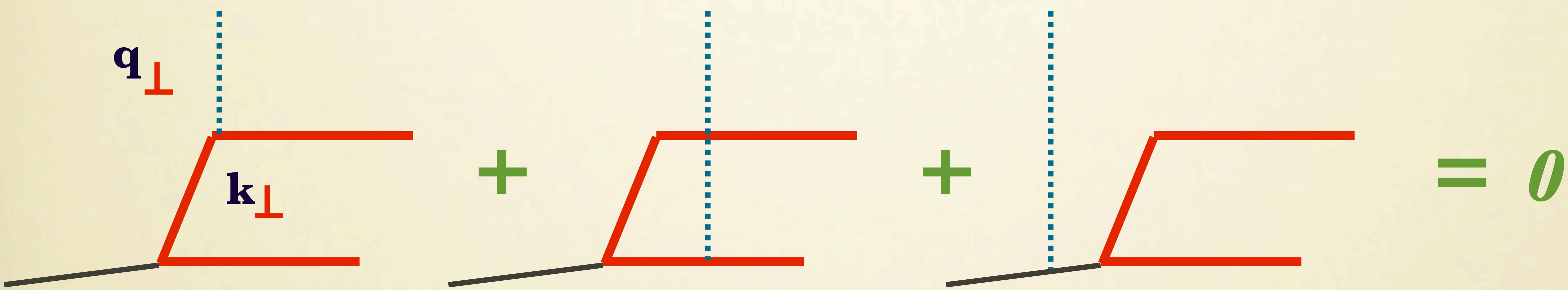
"Lipatov vertex"

$$K(k,q) = \left[\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2} \right]^2$$

rapidity-independent kernel **!**

$$\frac{k_\perp}{k_\perp^2} - \frac{(k-q)_\perp}{(k-q)_\perp^2}$$

How has it happened? Once again, quantum-mechanical coherence at work.



Inelastic dissociation. Into a ***compact*** state : $\Delta\rho_\perp \sim k_\perp^{-1}$

In a ***long-range*** potential : $\lambda_\perp \sim q_\perp^{-1} \gg \Delta\rho_\perp$

In QCD/QED vanishing of forward inelastic processes follows from gauge invariance.

In a more general context, it is due to orthogonality of the initial and final state wave functions, *provided* the initial and final systems *interact identically* with "the probe".

By hook or by crook, the "kinematical" *fluctuation time ordering* seems to be of little relevance as it misses essential physics.

$$t_{[q]} \simeq \frac{\beta_q}{q_\perp^2} \ll \frac{\beta_k}{k_\perp^2} \simeq t_{[k]}$$

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

space-like parton evolution; DIS structure functions

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

space-like parton evolution; DIS structure functions

Angular ordering (Mueller & Fadin)

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)
space-like parton evolution; DIS structure functions

$$d\xi = d \ln \frac{k_\perp^2}{1}$$

Angular ordering (Mueller & Fadin)
time-like parton multiplication; jet fragmentation functions

$$d\xi = d \ln \frac{k_\perp^2}{z^2}$$

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)

space-like parton evolution; DIS structure functions

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

Angular ordering (Mueller & Fadin)

time-like parton multiplication; jet fragmentation functions

$$d\xi = d\ln\frac{k_\perp^2}{z^2}$$

Each is a consequence of taking into full consideration soft gluon coherence to prevent explosively large terms $(\alpha_s \ln^2 x)^n$ from appearing in higher loop anomalous dimensions.

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)

space-like parton evolution; DIS structure functions

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

Angular ordering (Mueller & Fadin)

time-like parton multiplication; jet fragmentation functions

$$d\xi = d\ln\frac{k_\perp^2}{z^2}$$

Each is a consequence of taking into full consideration soft gluon coherence to prevent explosively large terms $(\alpha_s \ln^2 x)^n$ from appearing in higher loop anomalous dimensions.

However, having abandoned ***fluctuation time ordering***, we've lost quite a bit of wisdom along with it ...

$$d\xi = d\ln\frac{k_\perp^2}{z}$$

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)

space-like parton evolution; DIS structure functions

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

Angular ordering (Mueller & Fadin)

time-like parton multiplication; jet fragmentation functions

$$d\xi = d\ln\frac{k_\perp^2}{z^2}$$

Each is a consequence of taking into full consideration soft gluon coherence to prevent explosively large terms $(\alpha_s \ln^2 x)^n$ from appearing in higher loop anomalous dimensions.

However, having abandoned ***fluctuation time ordering***, we've lost quite a bit of wisdom along with it ...

$$d\xi = d\ln\frac{k_\perp^2}{z}$$

The fluctuation lifetime variable was not just a "wrong" one, for either e+e- or DIS

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)

space-like parton evolution; DIS structure functions

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

Angular ordering (Mueller & Fadin)

time-like parton multiplication; jet fragmentation functions

$$d\xi = d\ln\frac{k_\perp^2}{z^2}$$

Each is a consequence of taking into full consideration soft gluon coherence to prevent explosively large terms $(\alpha_s \ln^2 x)^n$ from appearing in higher loop anomalous dimensions.

However, having abandoned ***fluctuation time ordering***, we've lost quite a bit of wisdom along with it ...

$$d\xi = d\ln\frac{k_\perp^2}{z}$$

The fluctuation lifetime variable was not just a "wrong" one, for either e+e- or DIS

It was equally, ***symmetrically wrong***, lying right in between the two "***clever ones***".

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)

space-like parton evolution; DIS structure functions

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

Angular ordering (Mueller & Fadin)

time-like parton multiplication; jet fragmentation functions

$$d\xi = d\ln\frac{k_\perp^2}{z^2}$$

Each is a consequence of taking into full consideration soft gluon coherence to prevent explosively large terms $(\alpha_s \ln^2 x)^n$ from appearing in higher loop anomalous dimensions.

However, having abandoned ***fluctuation time ordering***, we've lost quite a bit of wisdom along with it ...

$$d\xi = d\ln\frac{k_\perp^2}{z}$$

The fluctuation lifetime variable was not just a "wrong" one, for either e+e- or DIS

It was equally, ***symmetrically wrong***, lying right in between the two "***clever ones***".

So, it preserves the ***symmetry*** between annihilation and scattering channels !

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)
space-like parton evolution; DIS structure functions

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

Angular ordering (Mueller & Fadin)
time-like parton multiplication; jet fragmentation functions

$$d\xi = d\ln\frac{k_\perp^2}{z^2}$$

Each is a consequence of taking into full consideration soft gluon coherence to prevent explosively large terms $(\alpha_s \ln^2 x)^n$ from appearing in higher loop anomalous dimensions.

However, having abandoned ***fluctuation time ordering***, we've lost quite a bit of wisdom along with it ...

$$d\xi = d\ln\frac{k_\perp^2}{z}$$

The fluctuation lifetime variable was not just a "wrong" one, for either e+e- or DIS

It was equally, ***symmetrically wrong***, lying right in between the two "***clever ones***".

So, it preserves the ***symmetry*** between annihilation and scattering channels !

Gribov-Lipatov reciprocity relation :

$$P_{BA}^{(T)}(x_{\,\text{Feynman}}) \;=\; P_{BA}^{(S)}(x_{\,\text{Bjorken}})$$

Beyond the 1st loop, it starts to matter how to order successive parton splittings that is, which variable to choose as a "*parton evolution time*".

The "clever choices" had been established quite some time ago:

Transverse momentum ordering (Gribov & Lipatov)
space-like parton evolution; DIS structure functions

$$d\xi = d\ln\frac{k_\perp^2}{1}$$

Angular ordering (Mueller & Fadin)
time-like parton multiplication; jet fragmentation functions

$$d\xi = d\ln\frac{k_\perp^2}{z^2}$$

Each is a consequence of taking into full consideration soft gluon coherence to prevent explosively large terms $(\alpha_s \ln^2 x)^n$ from appearing in higher loop anomalous dimensions.

However, having abandoned ***fluctuation time ordering***, we've lost quite a bit of wisdom along with it ...

$$d\xi = d\ln\frac{k_\perp^2}{z}$$

The fluctuation lifetime variable was not just a "wrong" one, for either e+e- or DIS

It was equally, ***symmetrically wrong***, lying right in between the two "***clever ones***".

So, it preserves the ***symmetry*** between annihilation and scattering channels !

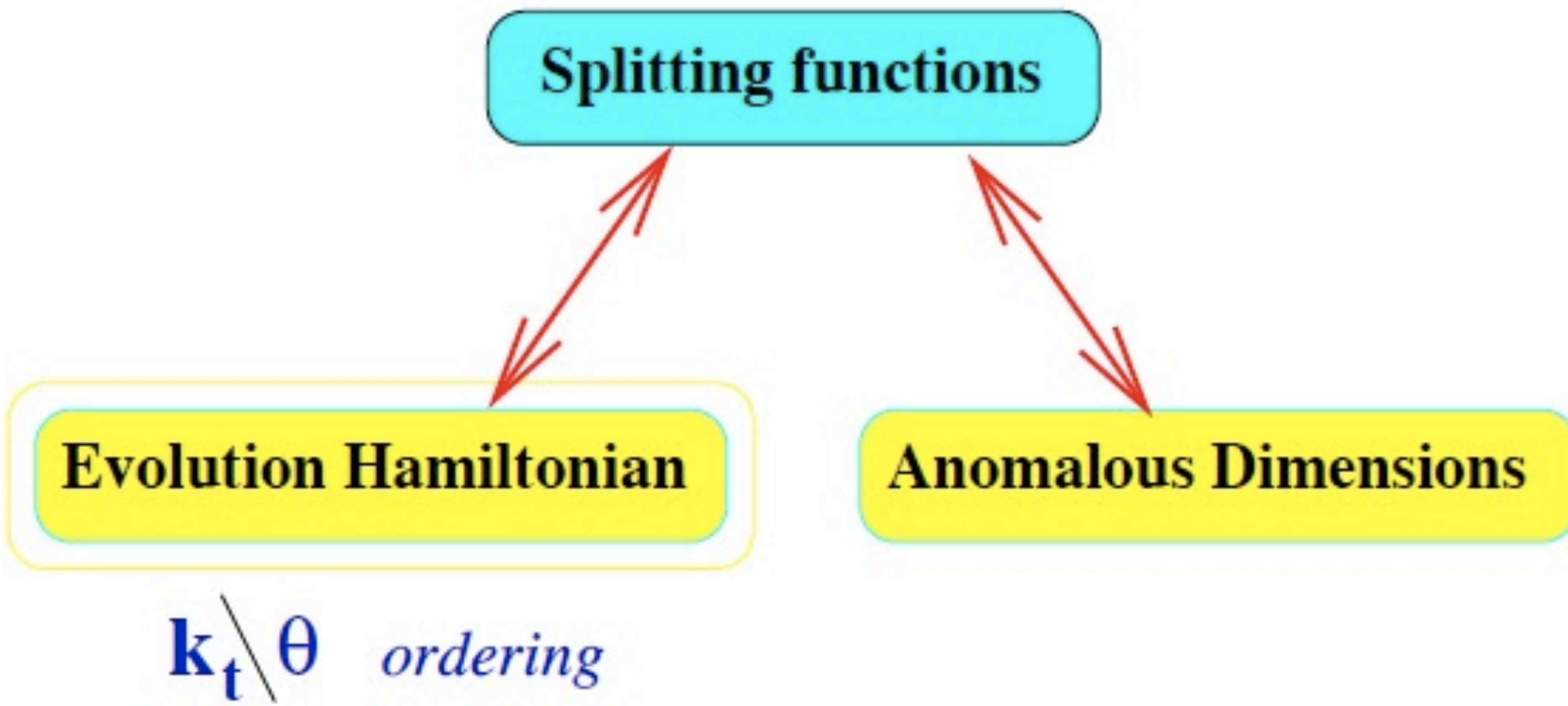
Gribov-Lipatov reciprocity relation :

$$P_{BA}^{(T)}(x_{\text{Feynman}}) = P_{BA}^{(S)}(x_{\text{Bjorken}})$$

$$x_B = \frac{-q^2}{2pq}, \quad x_F = \frac{2pq}{q^2}$$

Have a look at "key names" of the game of partons :

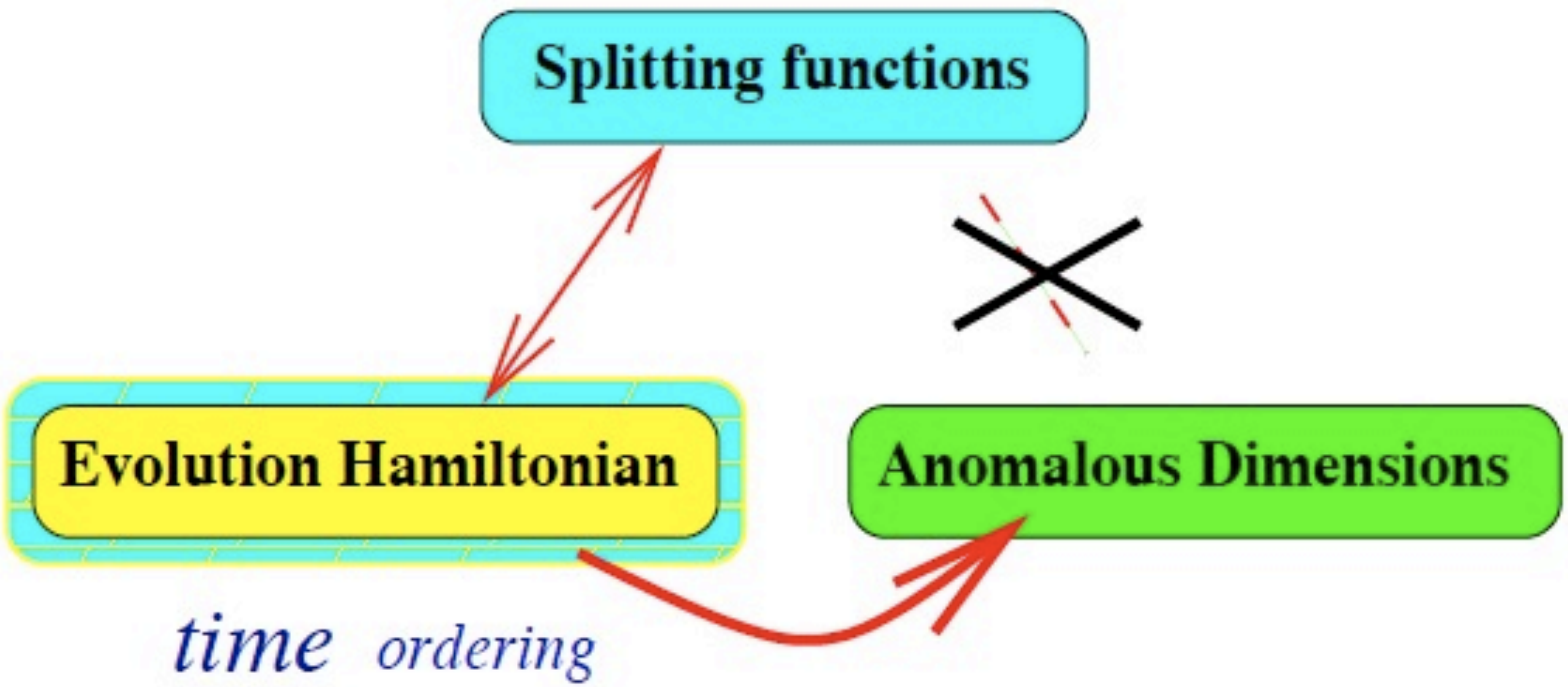
In the standard approach,



- parton splitting functions are equated with anomalous dimensions;
- they are different for DIS and e^+e^- evolution;
- "clever evolution variables" are different too

Have a look at "key names" of the game of partons :

In the new approach,



- splitting functions are disconnected from the anomalous dimensions;
- the evolution kernel is identical for space- and time-like cascades (Gribov–Lipatov reciprocity relation true in all orders);
- unique evolution variable — parton fluctuation time

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\; z^\sigma\,Q^2\right)$$

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\; z^\sigma\, Q^2\right) \qquad \sigma = \begin{cases} +1, & (\mathrm{T}) \\ -1, & (\mathrm{S}) \end{cases}$$

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\; z^\sigma\,Q^2\right) \qquad \sigma = \begin{cases} +1, & (\mathrm{T}) \\ -1, & (\mathrm{S}) \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}} D(Q^2)$

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\ z^\sigma\,Q^2\right) \qquad \sigma = \begin{cases} +1, & (\mathrm{T}) \\ -1, & (\mathrm{S}) \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}} D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\ z^\sigma\,Q^2\right) \qquad \sigma = \begin{cases} +1, & \text{(T)} \\ -1, & \text{(S)} \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}}\, D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\hat{d}\,D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\ z^\sigma\,Q^2\right) \qquad \sigma = \begin{cases} +1, & (\mathrm{T}) \\ -1, & (\mathrm{S}) \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}} D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\hat{d}\,D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$, expanding the operator argument of the kernel $\mathcal{P}$ we get a non-linear equation for the anomalous dimension γ_N :

$$\gamma[\alpha] = \mathcal{P} + \dot{\mathcal{P}}\cdot(\sigma\gamma+\beta/\alpha)+\tfrac{1}{2}\ddot{\mathcal{P}}\cdot\left[\gamma^2+\sigma(2\beta/\alpha\,\gamma+\beta\partial_\alpha\gamma)+\beta/\alpha\,\partial_\alpha\beta\right]+\mathcal{O}(\alpha^4).$$

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\, \mathcal{P}_B^A(z;\alpha_s)\, D^B\left(\frac{x}{z},\; z^\sigma Q^2\right) \qquad \sigma = \begin{cases} +1, & \text{(T)} \\ -1, & \text{(S)} \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}} D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\hat{d}\, D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$, expanding the operator argument of the kernel $\mathcal{P}$ we get a non-linear equation for the anomalous dimension γ_N :

$$\gamma[\alpha] = \mathcal{P} + \dot{\mathcal{P}} \cdot (\sigma\gamma + \beta/\alpha) + \tfrac{1}{2}\ddot{\mathcal{P}} \cdot \left[\gamma^2 + \sigma(2\beta/\alpha\,\gamma + \beta\partial_\alpha\gamma) + \beta/\alpha\,\partial_\alpha\beta\right] + \mathcal{O}(\alpha^4).$$

Hypothesis of ***universality*** of the "evolution kernel" $\mathcal{P}$ *(D-r, Marchesini & Salam, 05)*

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\; z^\sigma Q^2\right) \qquad \sigma = \begin{cases} +1, & (\mathrm{T}) \\ -1, & (\mathrm{S}) \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}} D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\hat{d}\,D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$, expanding the operator argument of the kernel $\mathcal{P}$ we get a non-linear equation for the anomalous dimension γ_N :

$$\gamma[\alpha] = \mathcal{P} + \dot{\mathcal{P}}\cdot\left(\sigma\gamma + \beta/\alpha\right) + \tfrac{1}{2}\ddot{\mathcal{P}}\cdot\left[\gamma^2 + \sigma(2\beta/\alpha\,\gamma + \beta\partial_\alpha\gamma) + \beta/\alpha\,\partial_\alpha\beta\right] + \mathcal{O}(\alpha^4).$$

Hypothesis of ***universality*** of the "evolution kernel" $\mathcal{P}$ *(D-r, Marchesini & Salam, 05)*
was verified by the calculation of the 3-loop time-like an. dim. *(Mitov, Moch & Vogt, 06)*

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\ z^\sigma Q^2\right) \qquad \sigma = \begin{cases} +1, & \text{(T)} \\ -1, & \text{(S)} \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}} D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\hat{d}\, D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$, expanding the operator argument of the kernel $\mathcal{P}$
we get a non-linear equation for the anomalous dimension γ_N :

$$\gamma[\alpha] = \mathcal{P} + \dot{\mathcal{P}} \cdot (\sigma\gamma + \beta/\alpha) + \tfrac{1}{2}\ddot{\mathcal{P}} \cdot \left[\gamma^2 + \sigma(2\beta/\alpha\,\gamma + \beta\partial_\alpha\gamma) + \beta/\alpha\,\partial_\alpha\beta\right] + \mathcal{O}(\alpha^4).$$

Hypothesis of ***universality*** of the "evolution kernel" $\mathcal{P}$ *(D-r, Marchesini & Salam, 05)*
was verified by the calculation of the 3-loop time-like an. dim. *(Mitov, Moch & Vogt, 06)*
and checked against all known higher loop QFT calculations (in the large-N limit) :

- 3loop singlet unpolarized
- 2loop quark transversity
- 2loop linearly polarized gluon
- 2loop singlet polarized
- Also true for SUSYs,
- in 4 loops in $\lambda\phi^4$,
- in QCD $\beta_0 \to \infty$, all loops,
- AdS/CFT ($\mathcal{N}=4$ SYM $\alpha \gg 1$)

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\; z^\sigma\,Q^2\right) \qquad \sigma = \begin{cases} +1, & \text{(T)} \\ -1, & \text{(S)} \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma\frac{d}{d\ln Q^2}}\,D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\hat{d}\,D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$, expanding the operator argument of the kernel $\mathcal{P}$

we get a non-linear equation for the anomalous dimension γ_N :

$$\gamma[\alpha] = \mathcal{P} + \dot{\mathcal{P}}\cdot\left(\sigma\gamma + \beta/\alpha\right) + \tfrac{1}{2}\ddot{\mathcal{P}}\cdot\left[\gamma^2 + \sigma(2\beta/\alpha\,\gamma + \beta\partial_\alpha\gamma) + \beta/\alpha\,\partial_\alpha\beta\right] + \mathcal{O}(\alpha^4).$$

Hypothesis of ***universality*** of the "evolution kernel" $\mathcal{P}$ *(D-r, Marchesini & Salam, 05)*

was verified by the calculation of the 3-loop time-like an. dim. *(Mitov, Moch & Vogt, 06)*

and checked against all known higher loop QFT calculations (in the large-N limit) :

(Basso & Korchemsky, 06)

- 3loop singlet unpolarized
- 2loop quark transversity
- 2loop linearly polarized gluon
- 2loop singlet polarized
- Also true for SUSYs,
- in 4 loops in $\lambda\phi^4$,
- in QCD $\beta_0 \to \infty$, all loops,
- AdS/CFT ($\mathcal{N}=4$ SYM $\alpha \gg 1$)

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\, D^B\left(\frac{x}{z},\ z^\sigma Q^2\right) \qquad \sigma = \begin{cases} +1, & \text{(T)} \\ -1, & \text{(S)} \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}} D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\hat{d}\, D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$, expanding the operator argument of the kernel $\mathcal{P}$ we get a non-linear equation for the anomalous dimension γ_N :

$$\gamma[\alpha] = \mathcal{P} + \dot{\mathcal{P}}\cdot(\sigma\gamma+\beta/\alpha)+\tfrac{1}{2}\ddot{\mathcal{P}}\cdot\left[\gamma^2+\sigma(2\beta/\alpha\,\gamma+\beta\partial_\alpha\gamma)+\beta/\alpha\,\partial_\alpha\beta\right]+\mathcal{O}(\alpha^4).$$

Hypothesis of ***universality*** of the "evolution kernel" $\mathcal{P}$ *(D-r, Marchesini & Salam, 05)*
was verified by the calculation of the 3-loop time-like an. dim. *(Mitov, Moch & Vogt, 06)*
and checked against all known higher loop QFT calculations (in the large-N limit) :

- 3loop singlet unpolarized
- 2loop quark transversity
- 2loop linearly polarized gluon
- 2loop singlet polarized
- Also true for SUSYs,
- in 4 loops in $\lambda\phi^4$,
- in QCD $\beta_0 \to \infty$, all loops,
- AdS/CFT ($\mathcal{N}=4$ SYM $\alpha \gg 1$)

(Basso & Korchemsky, 06)
conformal symmetry

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\,\mathcal{P}_B^A(z;\alpha_s)\,D^B\left(\frac{x}{z},\ z^\sigma\,Q^2\right) \qquad \sigma = \begin{cases} +1, & (\mathrm{T}) \\ -1, & (\mathrm{S}) \end{cases}$$

Using the Taylor expansion trick $\quad D\left(z^\sigma Q^2\right) = z^{\sigma\frac{d}{d\ln Q^2}}\,D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\quad \hat{d}\,D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$, expanding the operator argument of the kernel $\mathcal{P}$ we get a non-linear equation for the anomalous dimension γ_N :

$$\gamma[\alpha] = \mathcal{P} + \dot{\mathcal{P}}\cdot\left(\sigma\gamma + \beta/\alpha\right) + \tfrac{1}{2}\ddot{\mathcal{P}}\cdot\left[\gamma^2 + \sigma\left(2\beta/\alpha\,\gamma + \beta\partial_\alpha\gamma\right) + \beta/\alpha\,\partial_\alpha\beta\right] + \mathcal{O}(\alpha^4).$$

Hypothesis of ***universality*** of the "evolution kernel" $\mathcal{P}$ *(D-r, Marchesini & Salam, 05)*
was verified by the calculation of the 3-loop time-like an. dim. *(Mitov, Moch & Vogt, 06)*
and checked against all known higher loop QFT calculations (in the large-N limit) :

- 3loop singlet unpolarized
- 2loop quark transversity
- 2loop linearly polarized gluon
- 2loop singlet polarized
- Also true for SUSYs,
- in 4 loops in $\lambda\phi^4$,
- in QCD $\beta_0 \to \infty$, all loops,
- AdS/CFT ($\mathcal{N}=4$ SYM $\alpha \gg 1$)

(Basso & Korchemsky, 06)

conformal symmetry

"parity preserving series" in $[N(N+1)]^{-k}$

The transverse and longitudinal variable mix, and we end up with ***non-local*** evolution

$$\frac{dD^A(x,Q^2)}{d\ln Q^2} = \int_0^1 \frac{dz}{z}\, \mathcal{P}_B^A(z;\alpha_s)\, D^B\left(\frac{x}{z},\, z^\sigma Q^2\right) \qquad \sigma = \begin{cases} +1, & (T) \\ -1, & (S) \end{cases}$$

Using the Taylor expansion trick $D\left(z^\sigma Q^2\right) = z^{\sigma \frac{d}{d\ln Q^2}} D(Q^2)$

in the Mellin moment space we get the operator relation

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

Since $\hat{d}\, D_N(Q^2) \equiv \gamma_N \cdot D_N(Q^2)$, expanding the operator argument of the kernel $\mathcal{P}$ we get a non-linear equation for the anomalous dimension γ_N :

$$\gamma[\alpha] = \mathcal{P} + \dot{\mathcal{P}}\cdot(\sigma\gamma + \beta/\alpha) + \tfrac{1}{2}\ddot{\mathcal{P}}\cdot\left[\gamma^2 + \sigma(2\beta/\alpha\,\gamma + \beta\partial_\alpha\gamma) + \beta/\alpha\,\partial_\alpha\beta\right] + \mathcal{O}(\alpha^4).$$

Hypothesis of ***universality*** of the "evolution kernel" $\mathcal{P}$ *(D-r, Marchesini & Salam, 05)*
was verified by the calculation of the 3-loop time-like an. dim. *(Mitov, Moch & Vogt, 06)*
and checked against all known higher loop QFT calculations (in the large-N limit) :

- 3loop singlet unpolarized
- 2loop quark transversity
- 2loop linearly polarized gluon
- 2loop singlet polarized
- Also true for SUSYs,
- in 4 loops in $\lambda\phi^4$,
- in QCD $\beta_0 \to \infty$, all loops,
- AdS/CFT ($\mathcal{N}=4$ SYM $\alpha \gg 1$)

(Basso & Korchemsky, 06)
conformal symmetry
"parity preserving series" in $[N(N+1)]^{-k}$

The equation relating two channels

has already delivered some marvels :

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

The equation relating two channels

has already delivered some marvels :

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

- gave an all-order prediction for first sub-leading singularities in the limit $x \to 1$

The equation relating two channels

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

has already delivered some marvels :

- gave an all-order prediction for first sub-leading singularities in the limit $x \to 1$
- significantly simplified the structure of higher-loop anomalous dimensions

The equation relating two channels has already delivered some marvels :

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

- gave an all-order prediction for first sub-leading singularities in the limit $x \longrightarrow 1$
- significantly simplified the structure of higher-loop anomalous dimensions
- hinted at a common nature of two long-standing ***puzzles*** ...

The equation relating two channels

has already delivered some marvels :

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

- gave an all-order prediction for first sub-leading singularities in the limit $x \to 1$
- significantly simplified the structure of higher-loop anomalous dimensions
- hinted at a common nature of two long-standing ***puzzles*** ...

Here the introduction stops.

The equation relating two channels

has already delivered some marvels :

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

- gave an all-order prediction for first sub-leading singularities in the limit $x \to 1$
- significantly simplified the structure of higher-loop anomalous dimensions
- hinted at a common nature of two long-standing ***puzzles*** ...

Here the introduction stops.

"D'GLAP *is the correct name, 'cause it sounds* ***Dutch***" (c) AHM

The equation relating two channels

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

has already delivered some marvels :

- gave an all-order prediction for first sub-leading singularities in the limit $x \to 1$
- significantly simplified the structure of higher-loop anomalous dimensions
- hinted at a common nature of two long-standing ***puzzles*** ...

Here the introduction stops.

“D’GLAP *is the correct name, ‘cause it sounds* ***Dutch*****”** (c) AHM

M’BKDMS *should be the correct name for the new Reciprocity-Respecting Evolution.*

The equation relating two channels

has already delivered some marvels :

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

- gave an all-order prediction for first sub-leading singularities in the limit $x \to 1$
- significantly simplified the structure of higher-loop anomalous dimensions
- hinted at a common nature of two long-standing ***puzzles*** ...

Here the introduction stops.

"D'GLAP *is the correct name, 'cause it sounds* ***Dutch***" (c) AHM

M'BKDMS *should be the correct name for the new Reciprocity-Respecting Evolution.*

Not only "*'cause it sounds* ***delicious***".

The equation relating two channels

has already delivered some marvels :

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

- gave an all-order prediction for first sub-leading singularities in the limit $x \to 1$
- significantly simplified the structure of higher-loop anomalous dimensions
- hinted at a common nature of two long-standing ***puzzles*** ...

Here the introduction stops.

"D'GLAP *is the correct name, 'cause it sounds **Dutch**"* (c) AHM

M'BKDMS *should be the correct name for the new Reciprocity-Respecting Evolution.*

Not only "*'cause it sounds **delicious***". Al was discussing this relation *in early 1980's.*

The equation relating two channels has already delivered some marvels :

$$\gamma_N \cdot D_N(Q^2) = \mathcal{P}_{N+\sigma\hat{d}} \cdot D_N(Q^2)$$

- gave an all-order prediction for first sub-leading singularities in the limit $x \to 1$
- significantly simplified the structure of higher-loop anomalous dimensions
- hinted at a common nature of two long-standing ***puzzles*** ...

Here the introduction stops.

"D'GLAP *is the correct name, 'cause it sounds* ***Dutch*****"** (c) AHM

M'BKDMS *should be the correct name for the new Reciprocity-Respecting Evolution.*

Not only **"***'cause it sounds* ***delicious*****"**. Al was discussing this relation *in early 1980's.*

The two underexplored puzzles I wanted to bring your attention to today, are :

the "***Malaza puzzle***" ***vs.*** the "***BFKL puzzle***"

The “*fifth form factor*” emerges, depending on the color charge of the *t-channel exchange*

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right)$$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi} \int^{\sqrt{-t}} \frac{dk_\perp}{k_\perp} \int_{\Theta_s^2}^{1} \frac{d\Theta^2}{\Theta^2} \right)$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

The “*fifth form factor*” emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi} \int^{\sqrt{-t}} \frac{dk_\perp}{k_\perp} \int_{\Theta_s^2}^{1} \frac{d\Theta^2}{\Theta^2} \right) \propto \left(\frac{s}{-t} \right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi} \int_0^{\sqrt{-t}} \frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung.

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung.
Logarithms survive.

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung. ***Logarithms*** survive.   Real particle production - an addition source of ***s***-dependence.

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung. ***Logarithms*** survive. Real particle production - an addition source of ***s***-dependence.

BFKL $\alpha_s \int \frac{d\beta}{\beta}$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi} \int^{\sqrt{-t}} \frac{dk_\perp}{k_\perp} \int_{\Theta_s^2}^{1} \frac{d\Theta^2}{\Theta^2} \right) \propto \left(\frac{s}{-t} \right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi} \int_0^{\sqrt{-t}} \frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung. ***Logarithms*** survive. Real particle production - an addition source of ***s***-dependence.

BFKL $\alpha_s \int \frac{d\beta}{\beta} \longrightarrow \alpha_s \ln s$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung. ***Logarithms*** survive. Real particle production - an addition source of ***s***-dependence.

BFKL $\alpha_s \int \frac{d\beta}{\beta} \longrightarrow \alpha_s \ln s$

How the "BFKL ladder" is organized?

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi} \int^{\sqrt{-t}} \frac{dk_\perp}{k_\perp} \int_{\Theta_s^2}^{1} \frac{d\Theta^2}{\Theta^2} \right) \propto \left(\frac{s}{-t} \right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

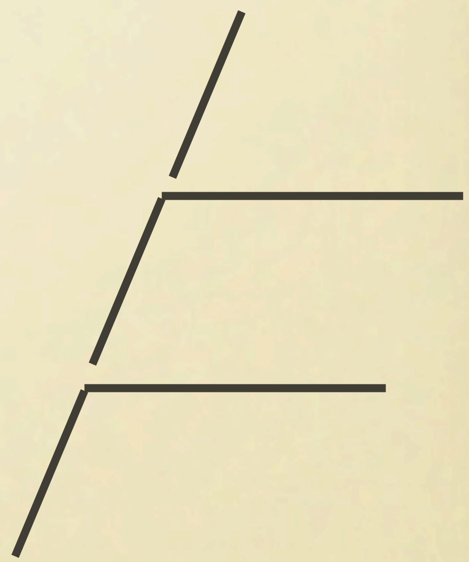
$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi} \int_0^{\sqrt{-t}} \frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung. ***Logarithms*** survive. Real particle production - an addition source of ***s***-dependence.

BFKL $\alpha_s \int \frac{d\beta}{\beta} \longrightarrow \alpha_s \ln s$

How the "BFKL ladder" is organized?



The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

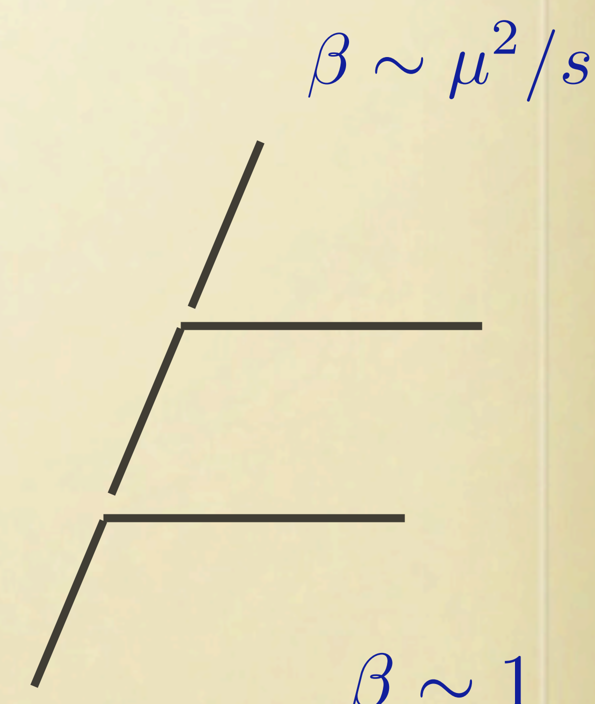
$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung.
Logarithms survive. Real particle production - an addition source of ***s***-dependence.

BFKL $\alpha_s \int \frac{d\beta}{\beta} \Longrightarrow \alpha_s \ln s$

How the "BFKL ladder" is organized?



The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

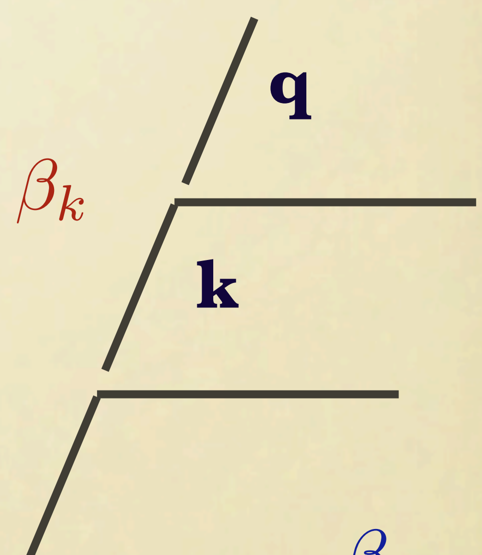
In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung.
Logarithms survive. Real particle production - an addition source of ***s***-dependence.

BFKL $\quad \alpha_s \int \frac{d\beta}{\beta} \longrightarrow \alpha_s \ln s$

How the "BFKL ladder" is organized?

$\beta \sim \mu^2/s$

$\beta_q \ll \beta_k$



$\beta \sim 1$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung.
Logarithms survive. Real particle production - an addition source of ***s***-dependence.

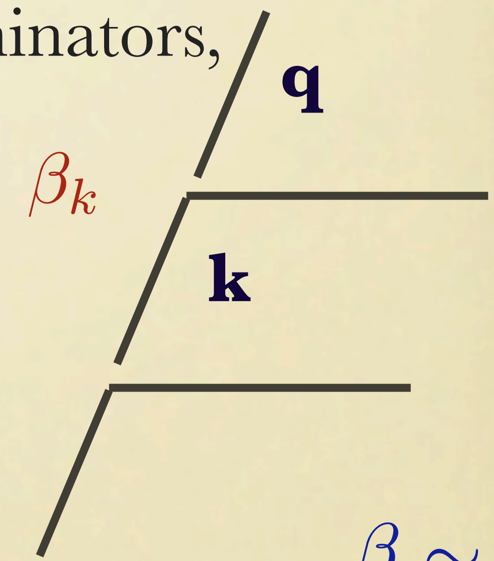
BFKL $\alpha_s \int \frac{d\beta}{\beta} \Longrightarrow \alpha_s \ln s$

$\beta \sim \mu^2/s$

How the "BFKL ladder" is organized? Analyzing Feynman denominators,

$$\frac{\beta_q}{q_\perp^2} \ll \frac{\beta_k}{k_\perp^2}$$

$\beta_q \ll \beta_k$



$\beta \sim 1$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung.
Logarithms survive. Real particle production - an addition source of ***s***-dependence.

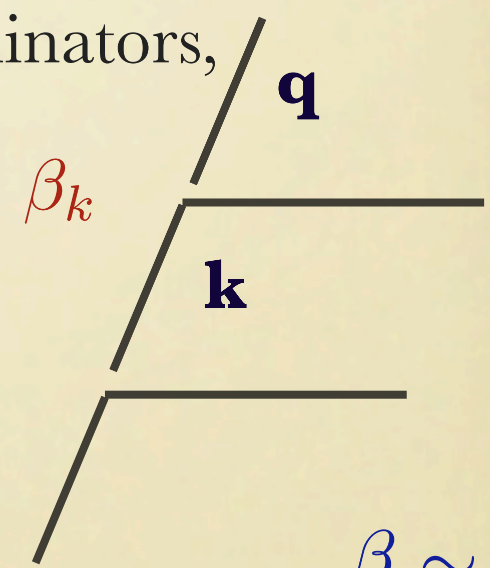
BFKL $\alpha_s \int \frac{d\beta}{\beta} \Longrightarrow \alpha_s \ln s$

$\beta \sim \mu^2/s$

How the "BFKL ladder" is organized? Analyzing Feynman denominators,

$$t_{[q]} \simeq \frac{\beta_q}{q_\perp^2} \ll \frac{\beta_k}{k_\perp^2} \simeq t_{[k]}$$

$\beta_q \ll \beta_k$



$\beta \sim 1$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi} \int^{\sqrt{-t}} \frac{dk_\perp}{k_\perp} \int_{\Theta_s^2}^{1} \frac{d\Theta^2}{\Theta^2} \right) \propto \left(\frac{s}{-t} \right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi} \int_0^{\sqrt{-t}} \frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung.
Logarithms survive. Real particle production - an addition source of ***s***-dependence.

BFKL $\quad \alpha_s \int \frac{d\beta}{\beta} \;\longrightarrow\; \alpha_s \ln s$

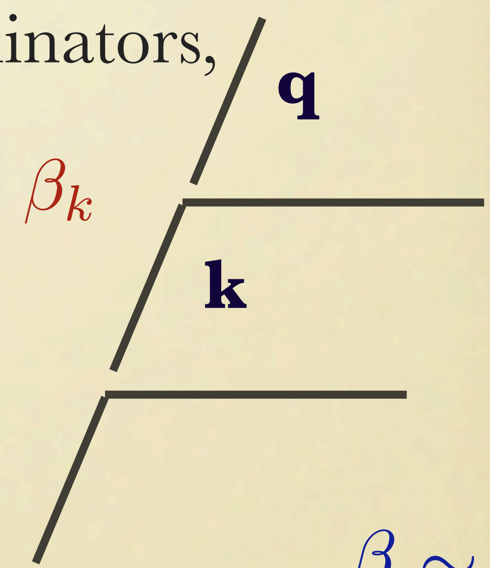
$\beta \sim \mu^2/s$

How the "BFKL ladder" is organized? Analyzing Feynman denominators,

$$t_{[q]} \simeq \frac{\beta_q}{q_\perp^2} \ll \frac{\beta_k}{k_\perp^2} \simeq t_{[k]}$$

$\beta_q \ll \beta_k$

$k_\perp$ integration phase space swells when $\beta_q/\beta_k \longrightarrow 0$



$\beta \sim 1$

The "*fifth form factor*" emerges, depending on the color charge of the *t-channel exchange*

$$\sigma \propto \mathcal{F}^2(s,t) \simeq \exp\left(-\frac{\alpha_s N_c}{\pi}\int^{\sqrt{-t}}\frac{dk_\perp}{k_\perp}\int_{\Theta_s^2}^{1}\frac{d\Theta^2}{\Theta^2}\right) \propto \left(\frac{s}{-t}\right)^{2(\alpha_G(t)-1)}$$

$$\Theta_s \simeq \sqrt{\frac{-t}{s}}$$

gluon Regge trajectory :

$$\alpha_G(t) \simeq 1 - \frac{\alpha_s N}{2\pi}\int_0^{\sqrt{-t}}\frac{dk}{k}$$

Reggeized gluon exchange in elastic amplitude = the t-channel elastic form factor.

In physical observables, infrared ***infinities*** are cancelled by real bremsstrahlung.
Logarithms survive. Real particle production - an addition source of ***s***-dependence.

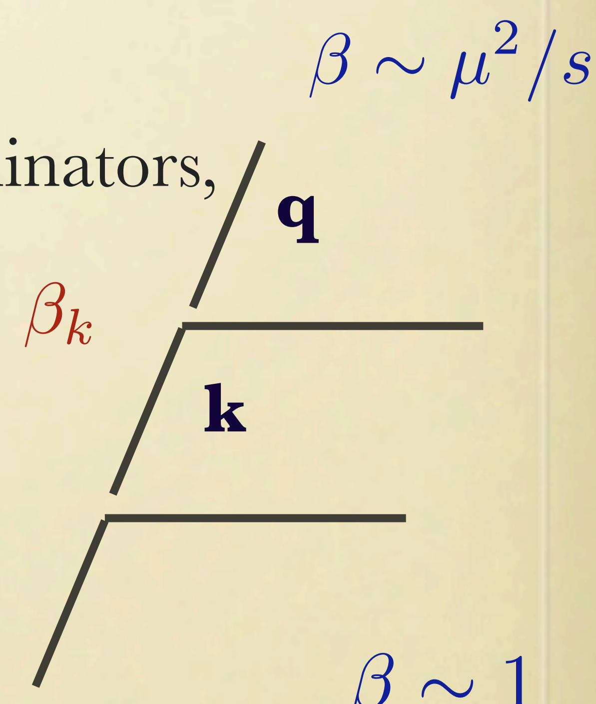
BFKL $\quad \alpha_s \int \frac{d\beta}{\beta} \longrightarrow \alpha_s \ln s$

How the "BFKL ladder" is organized? Analyzing Feynman denominators,

$$t_{[q]} \simeq \frac{\beta_q}{q_\perp^2} \ll \frac{\beta_k}{k_\perp^2} \simeq t_{[k]}$$

$\beta_q \ll \beta_k$

$k_\perp$ integration phase space swells when $\beta_q / \beta_k \longrightarrow 0$



If this were the whole story, the $k_\perp$ and y dependencies would mix and the "evolution" of the system with ***rapidity*** would be non-local...

PT series for anomalous dimension
in the small-x region :

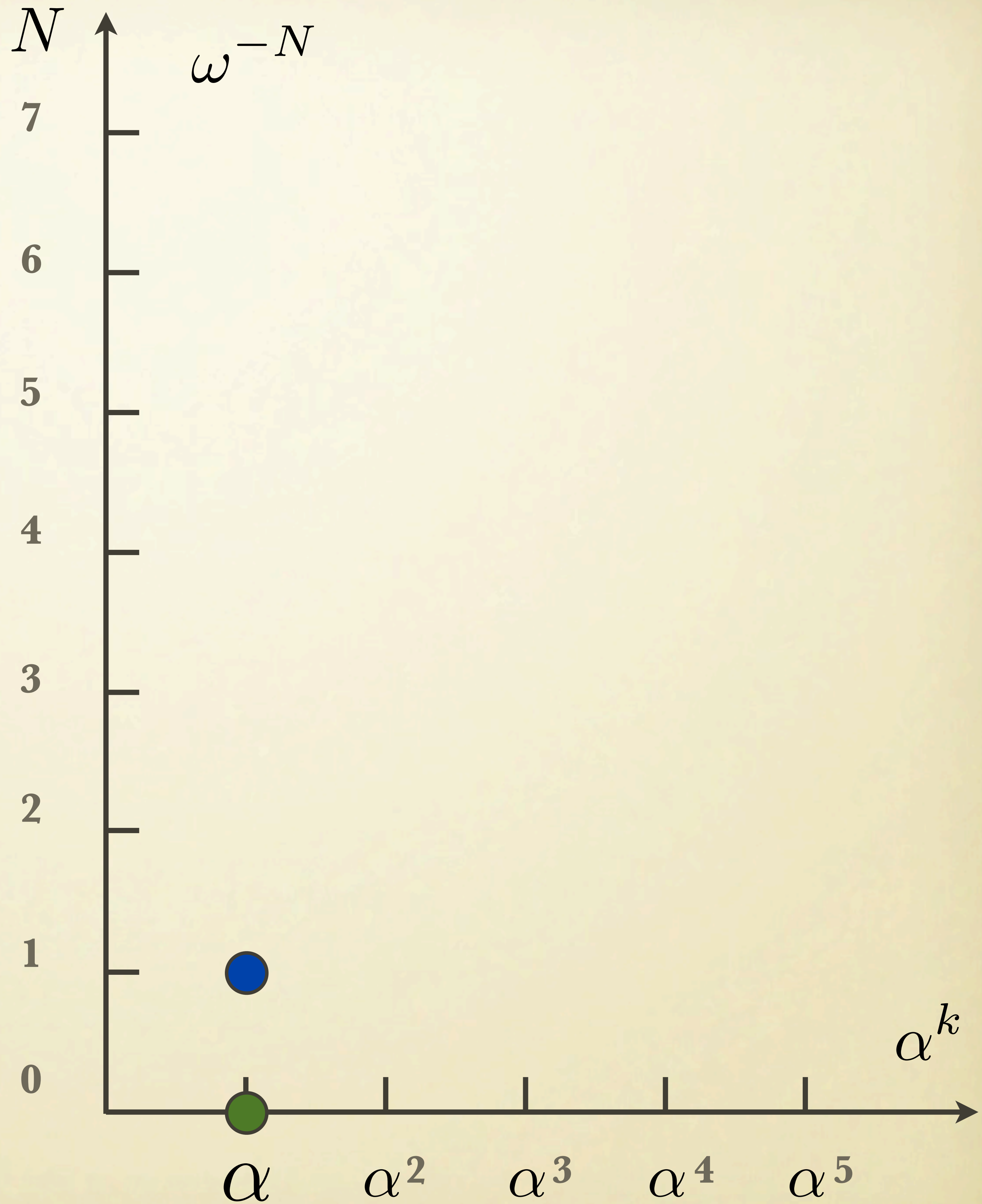
$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

PT series for anomalous dimension in the small-x region :

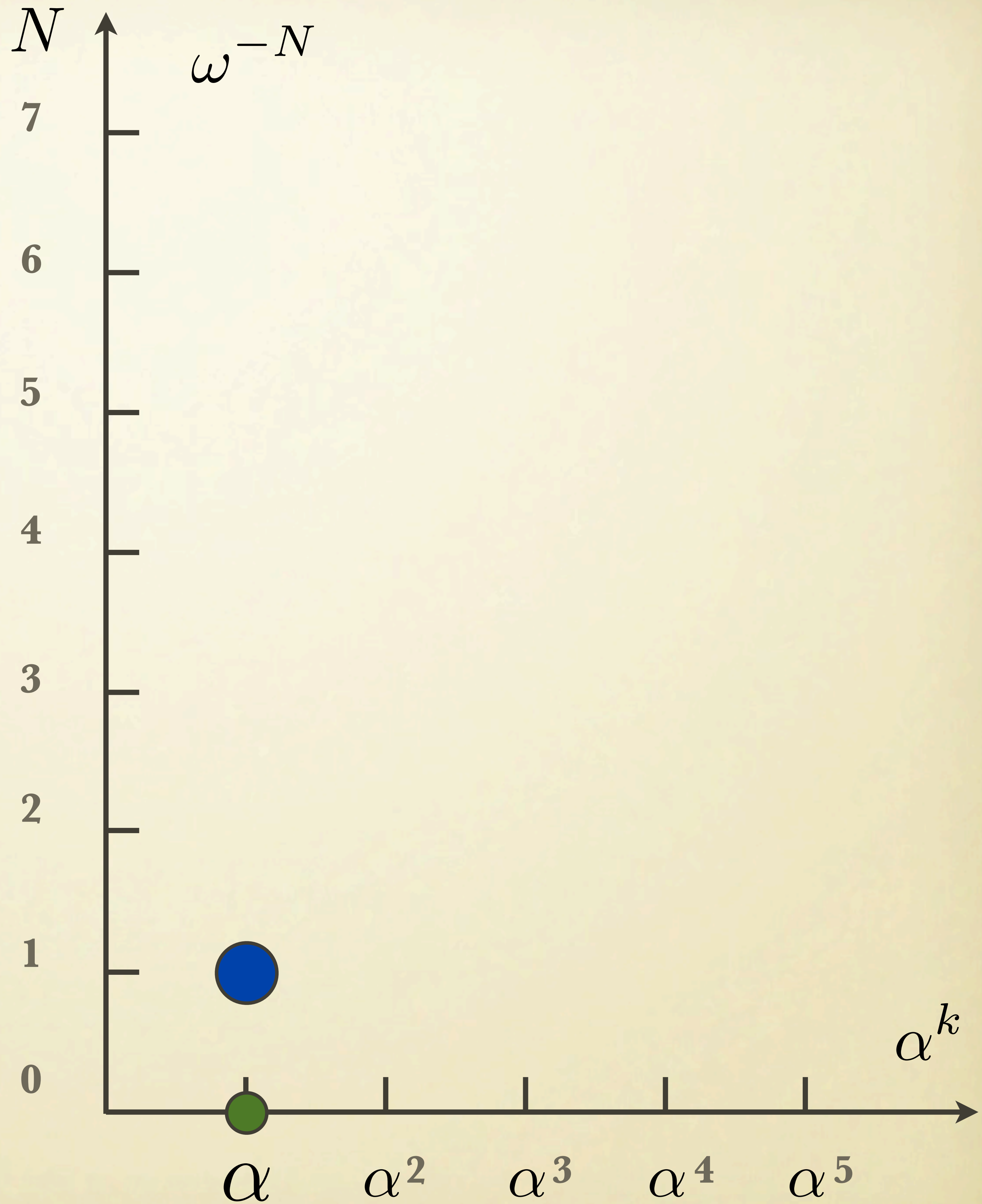
$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$



PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

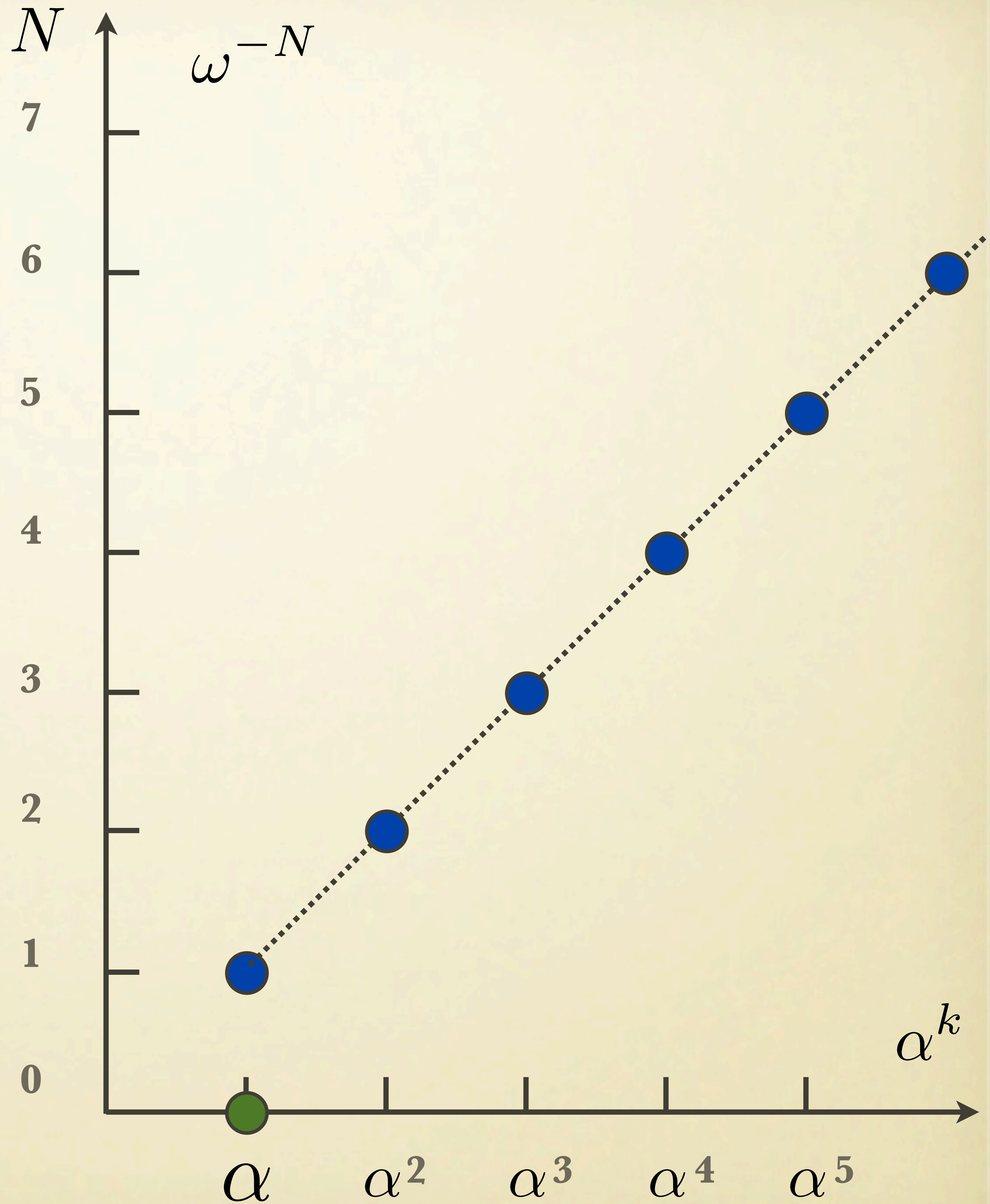
The leading singularity generates, via the BFKL eq., the ***N=k*** series.



PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

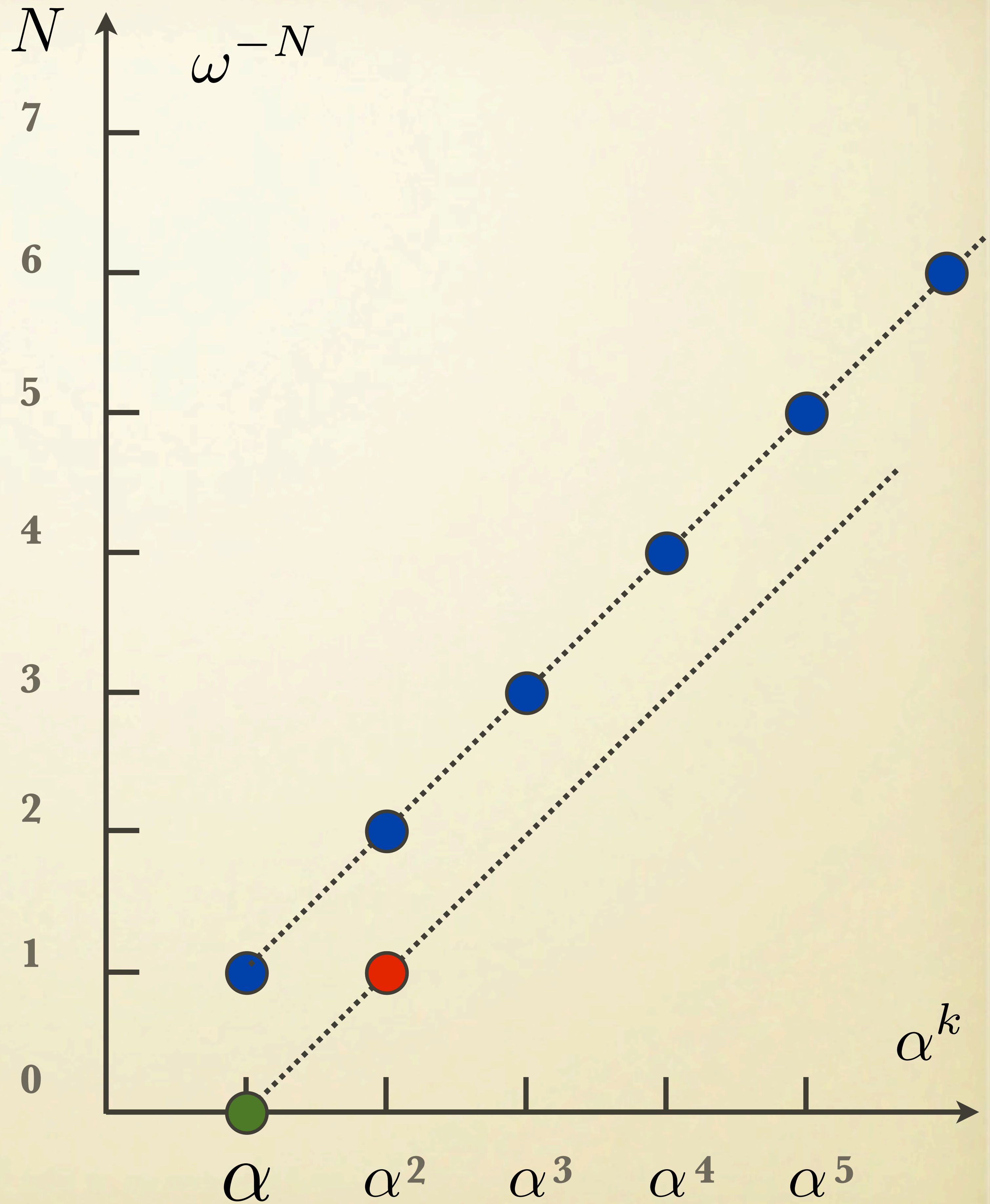


PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

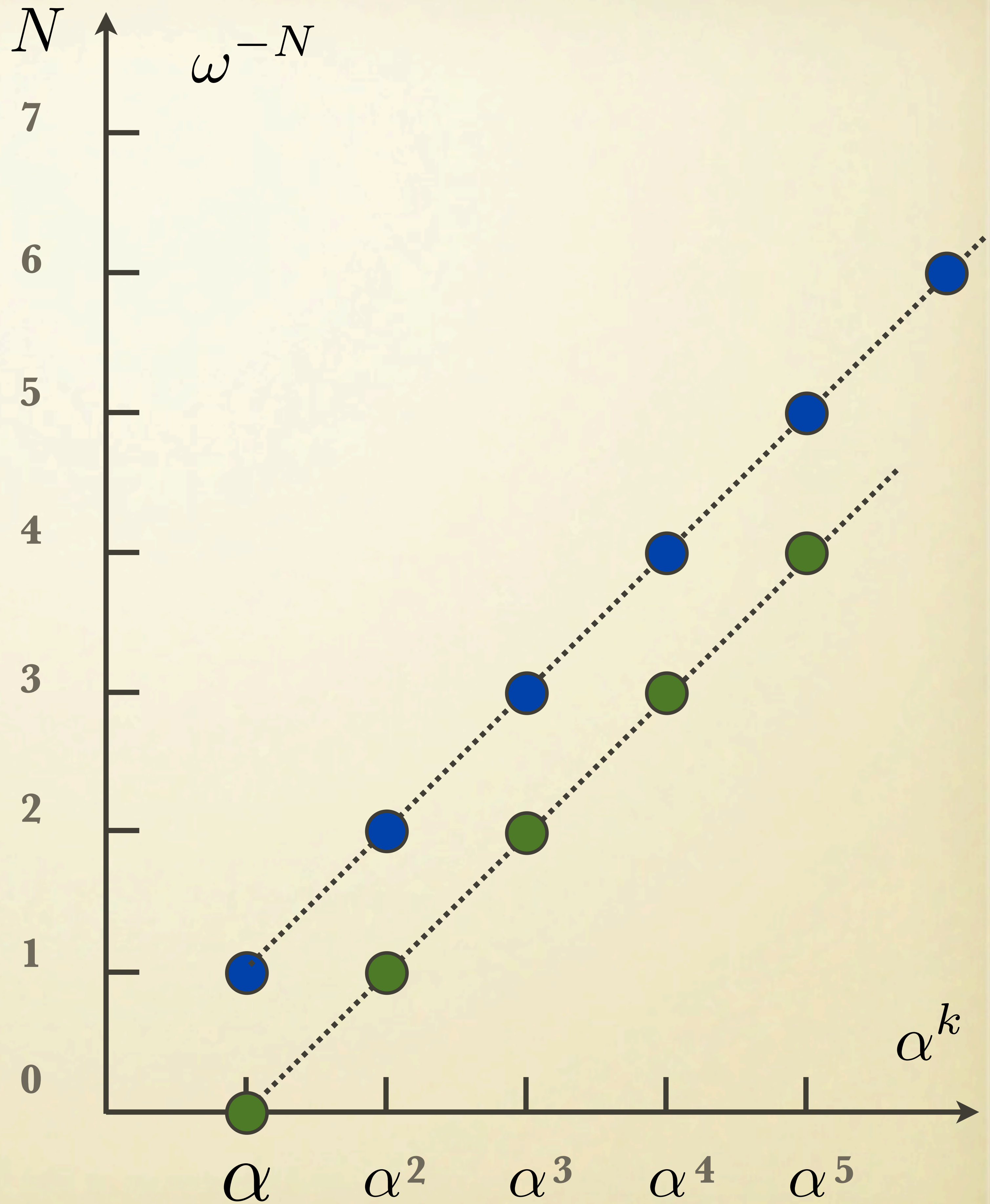


PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.



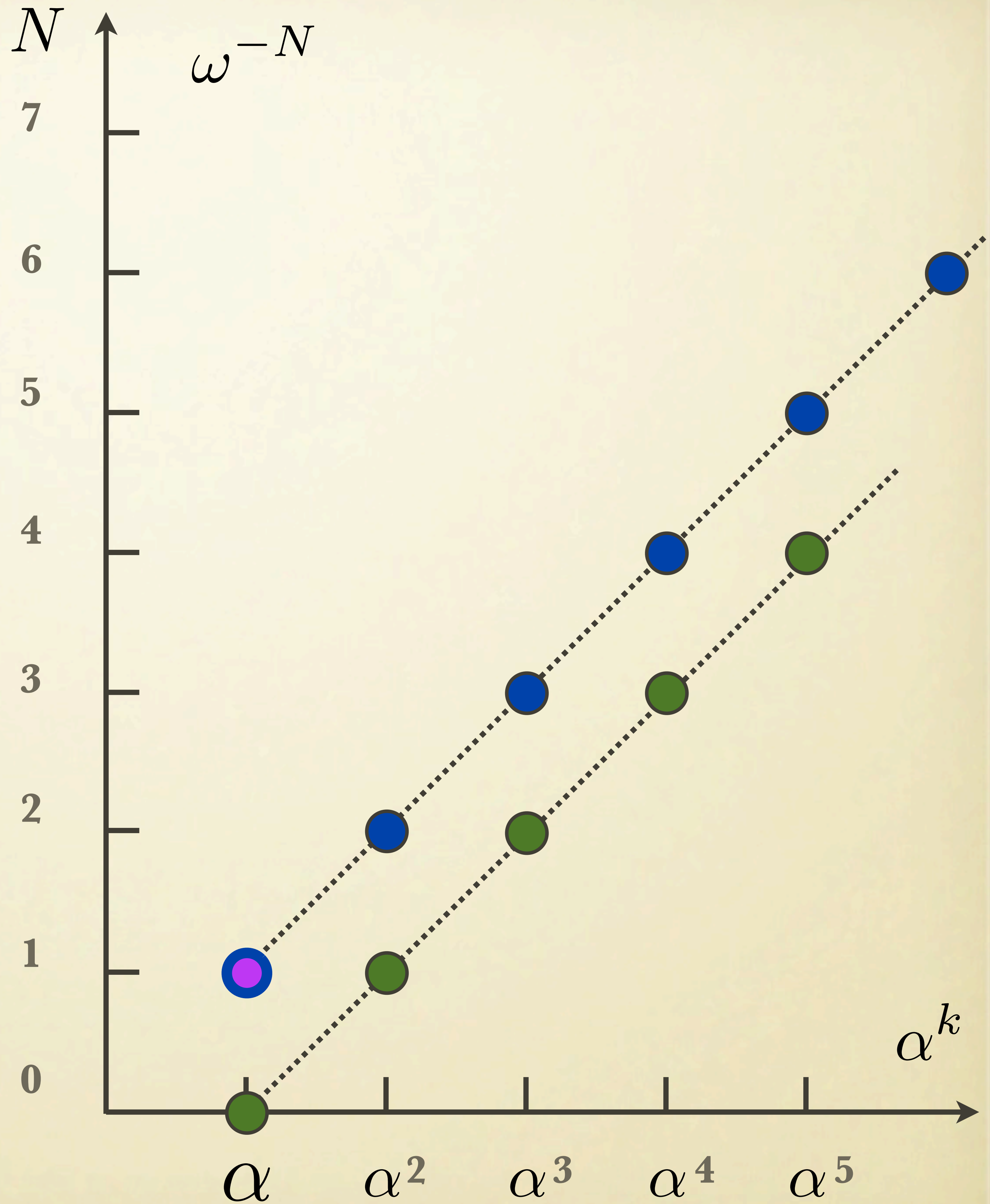
PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***,



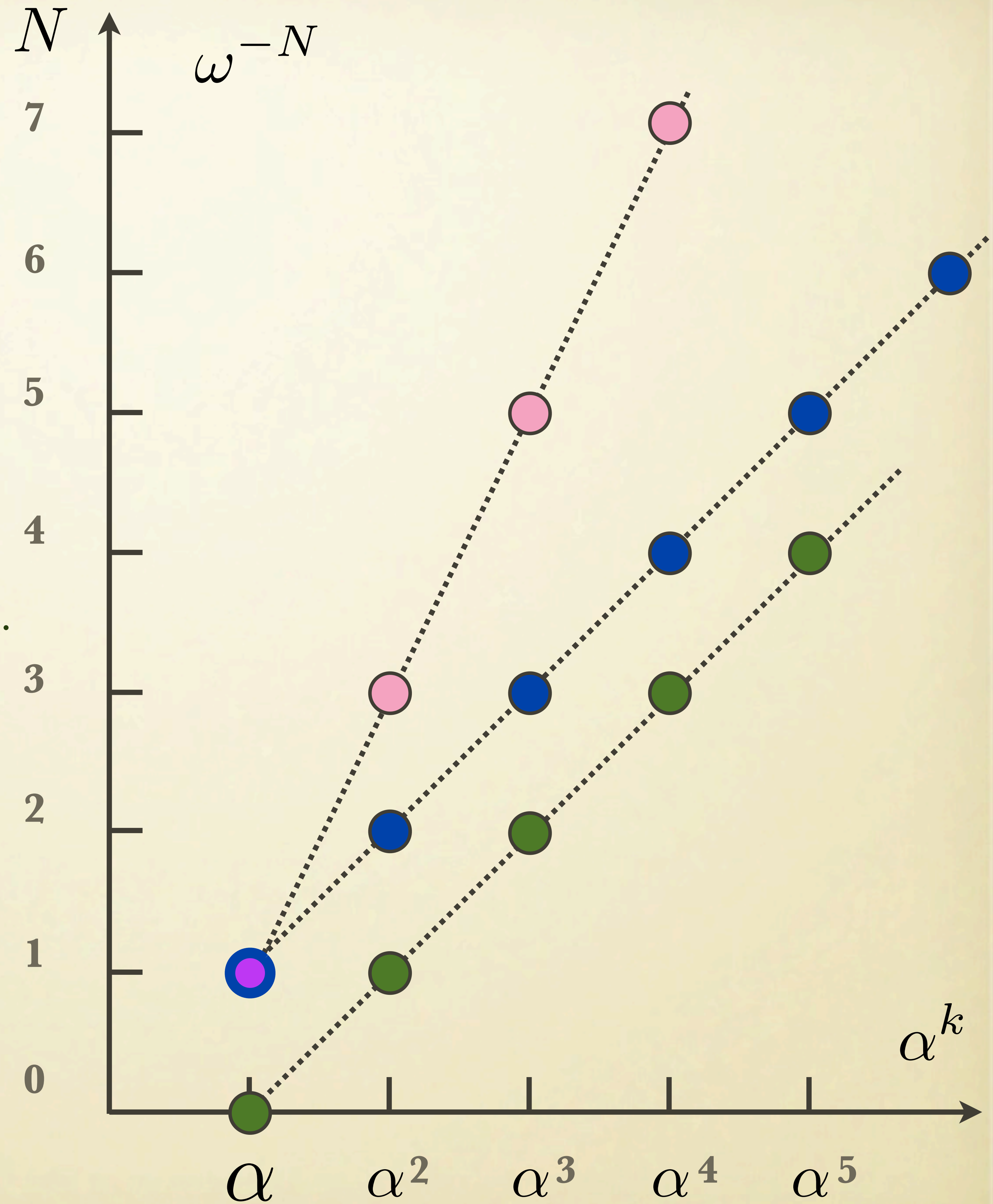
PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.



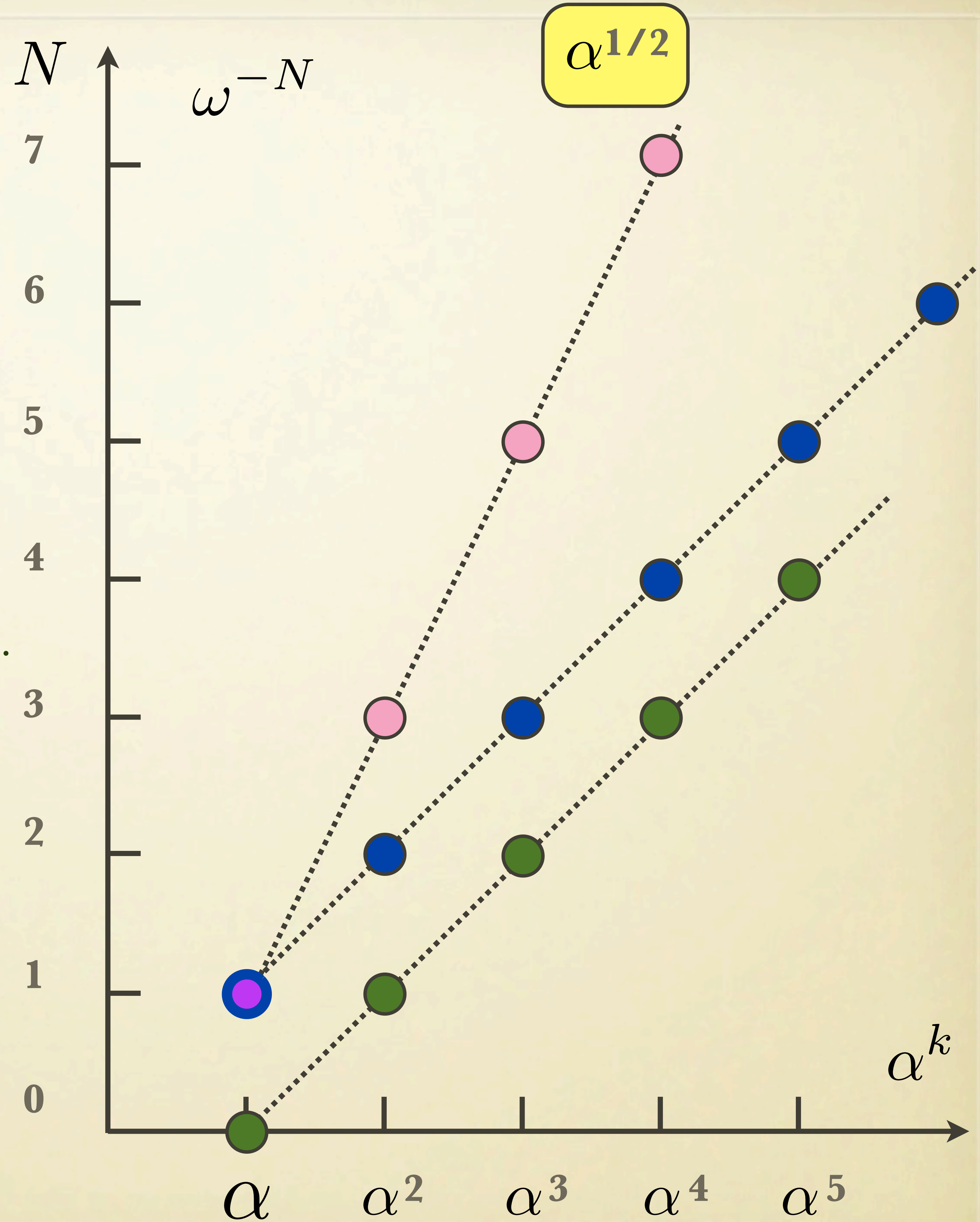
PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.



PT series for anomalous dimension in the small-x region :

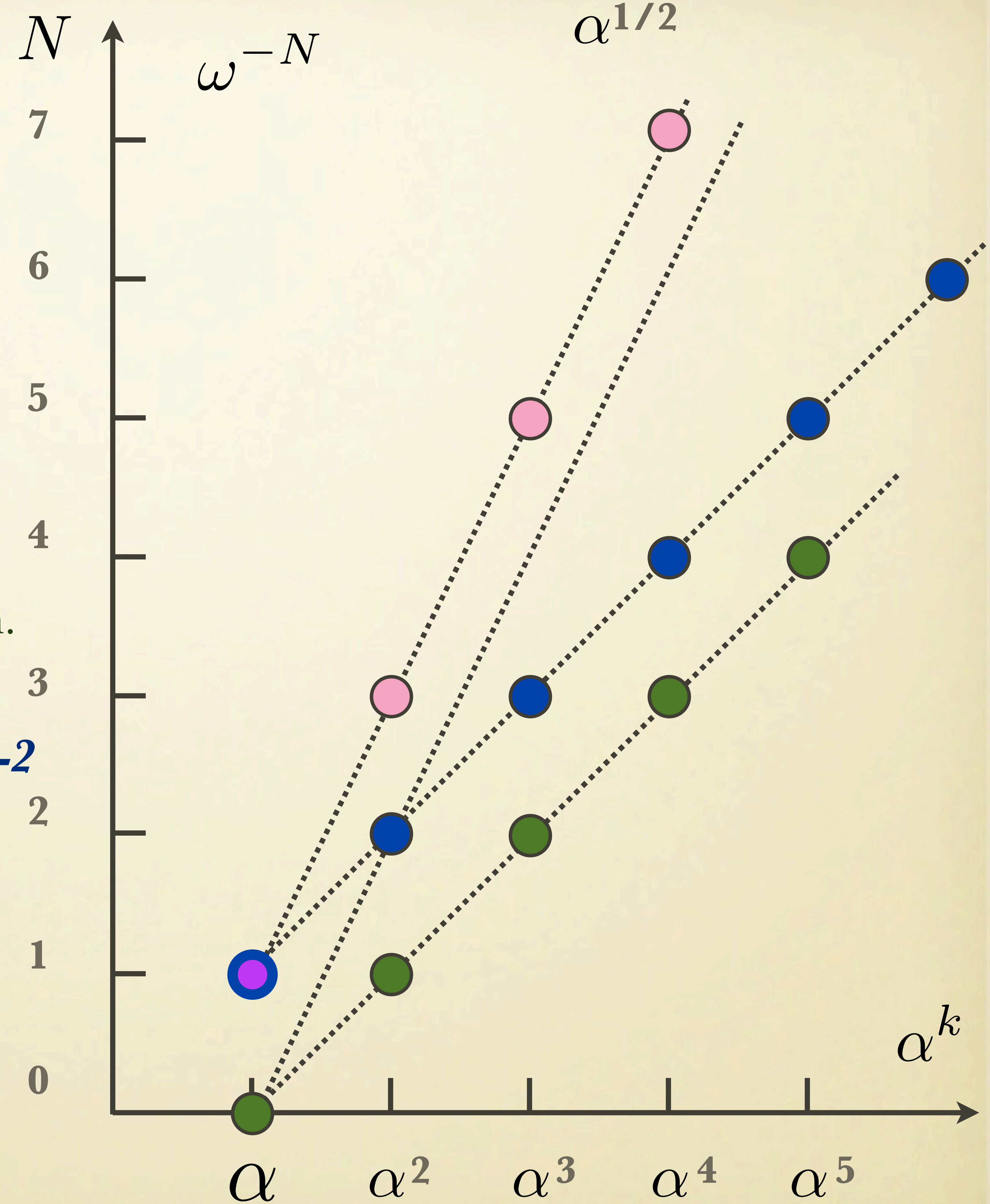
$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.

Next-to-leading correction to $\gamma^{(T)}$ -the double slope, step below: ***N=2k-2***



PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

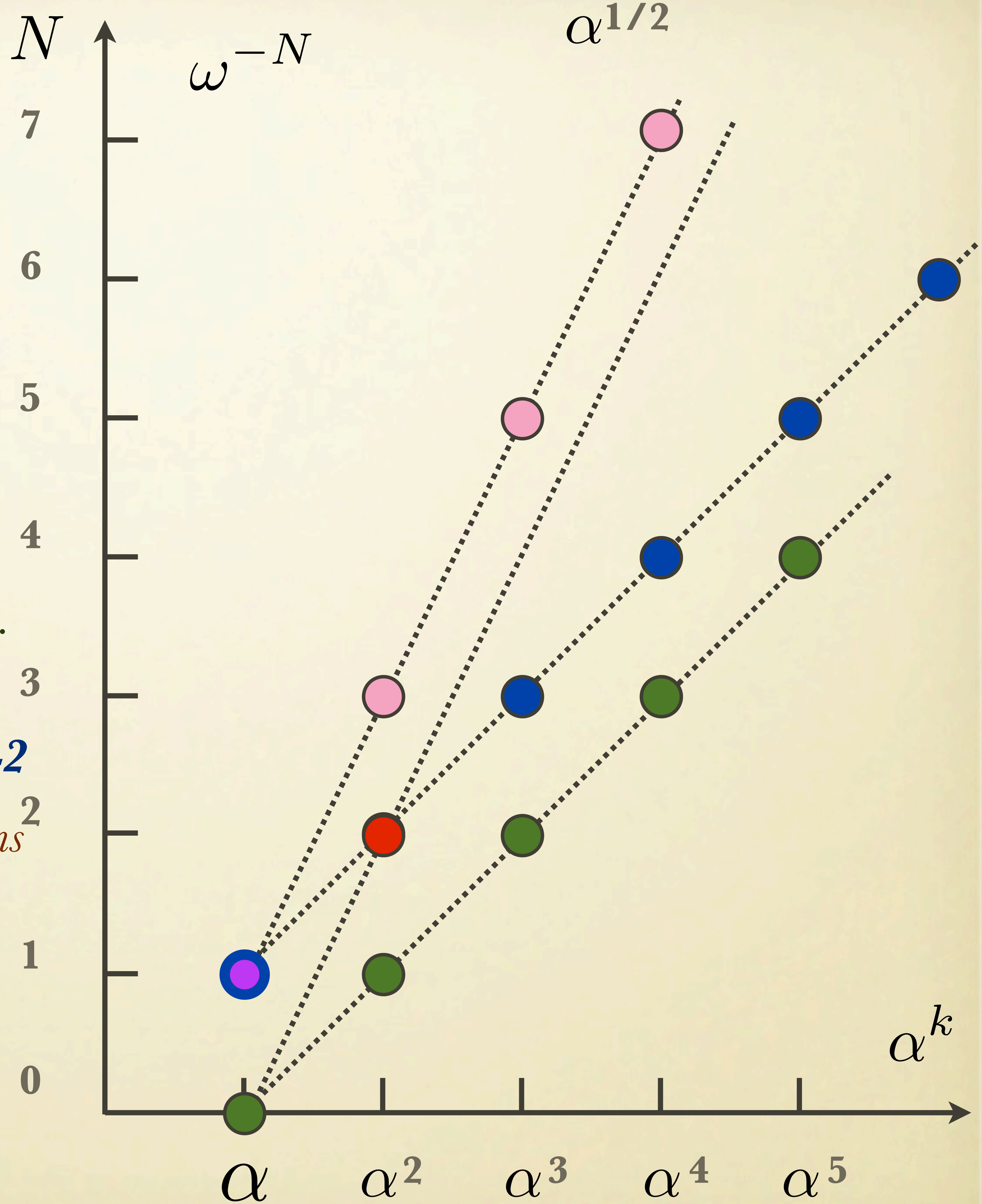
The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.

Next-to-leading correction to $\gamma^{(T)}$ -the double slope, step below: ***N=2k-2***

"*simultaneous*" emission of *two soft gluons* could have modified the answer ...



PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

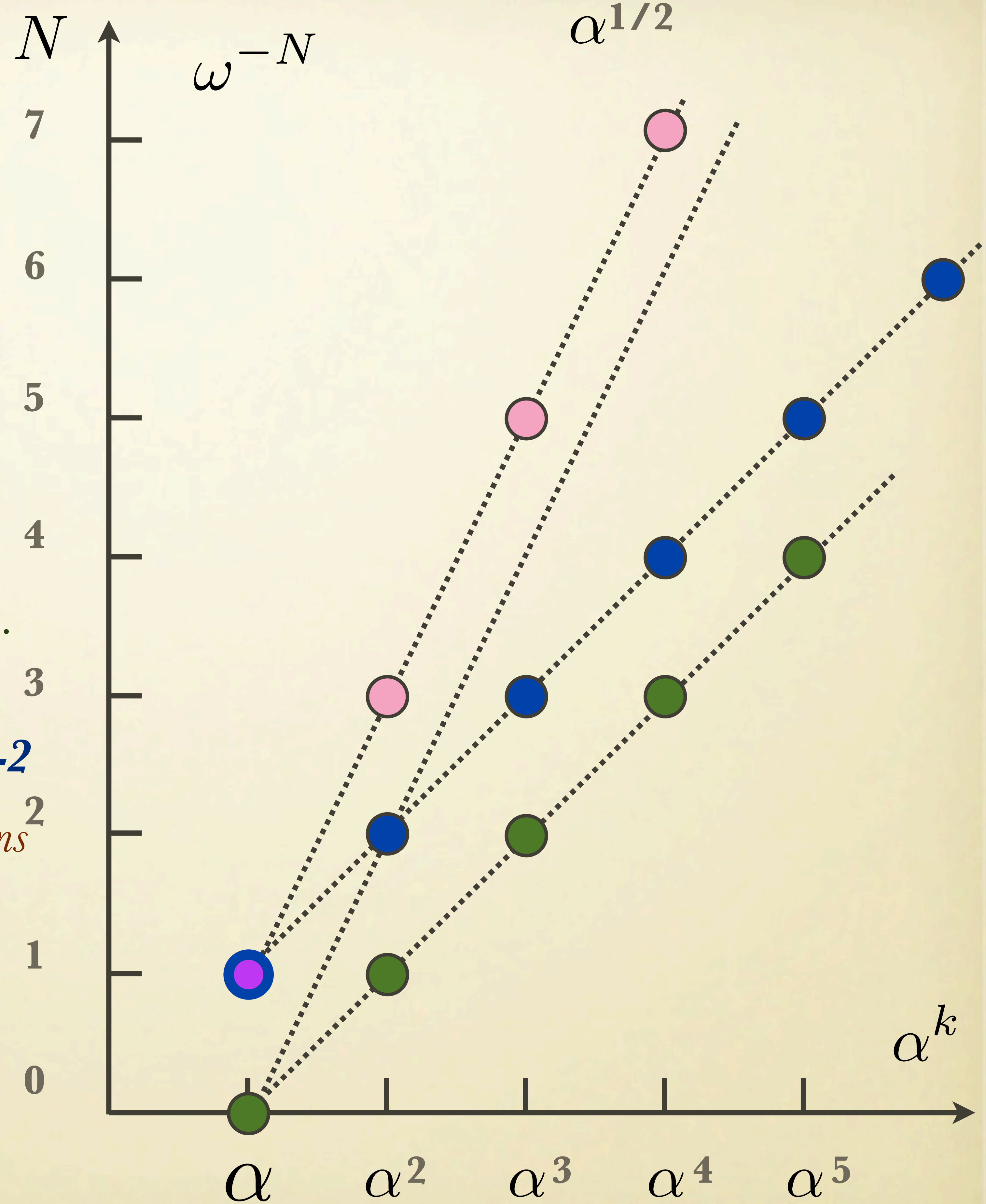
The next line - N-BFKL : ***N=k-1***.

DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.

Next-to-leading correction to $\gamma^{(T)}$ -the double slope, step below: ***N=2k-2***

"*simultaneous*" emission of *two soft gluons* could have modified the answer ...

However, it was shown to ***factorize*** :



PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

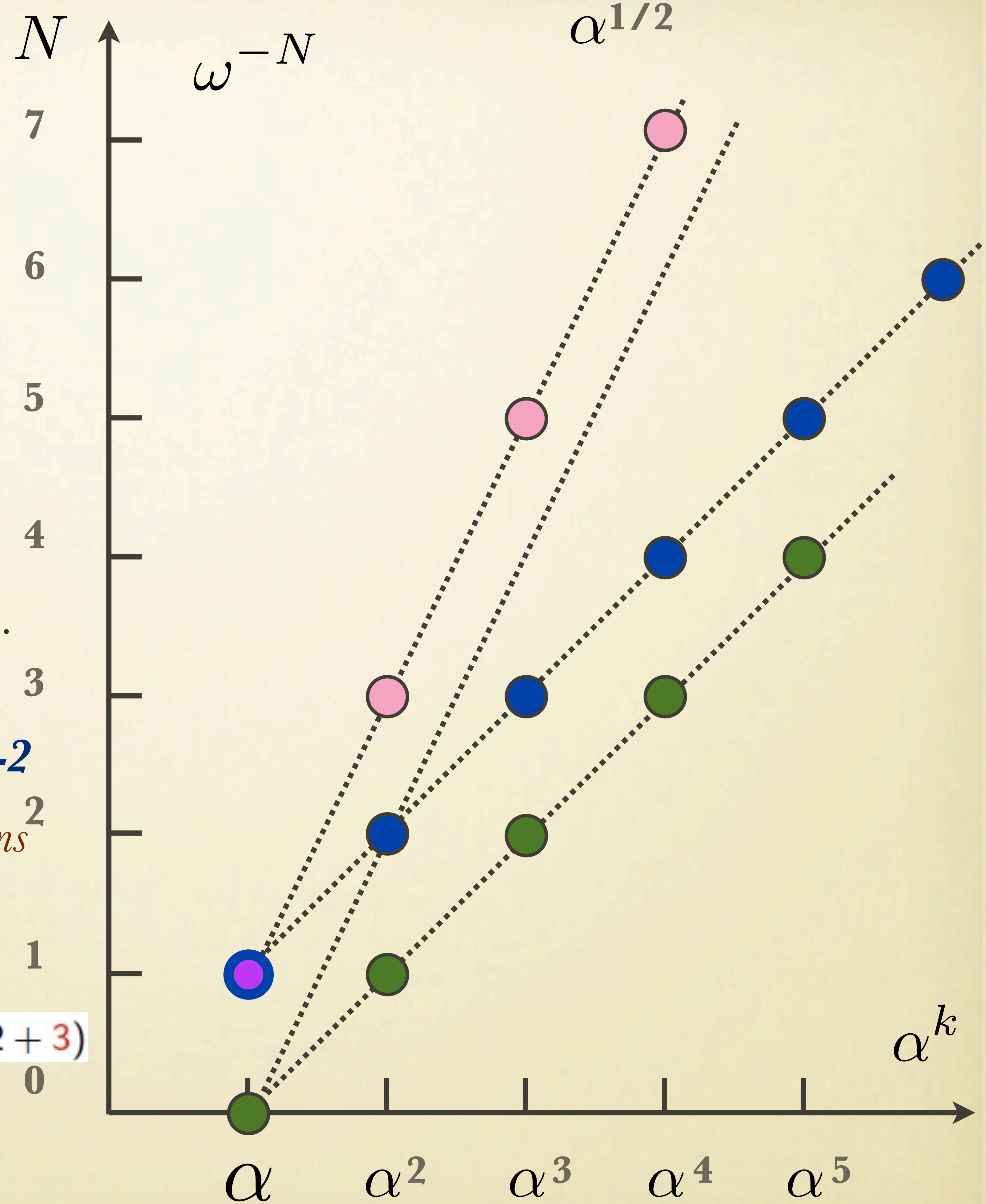
DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.

Next-to-leading correction to $\gamma^{(T)}$ -the double slope, step below: ***N=2k-2***

"*simultaneous*" emission of *two soft gluons* could have modified the answer ...

However, it was shown to ***factorize*** :

$1 \to 1 + 2 + 3$ ⟹ $(1 \to 1 + 2) \otimes (2 \to 2 + 3)$



PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

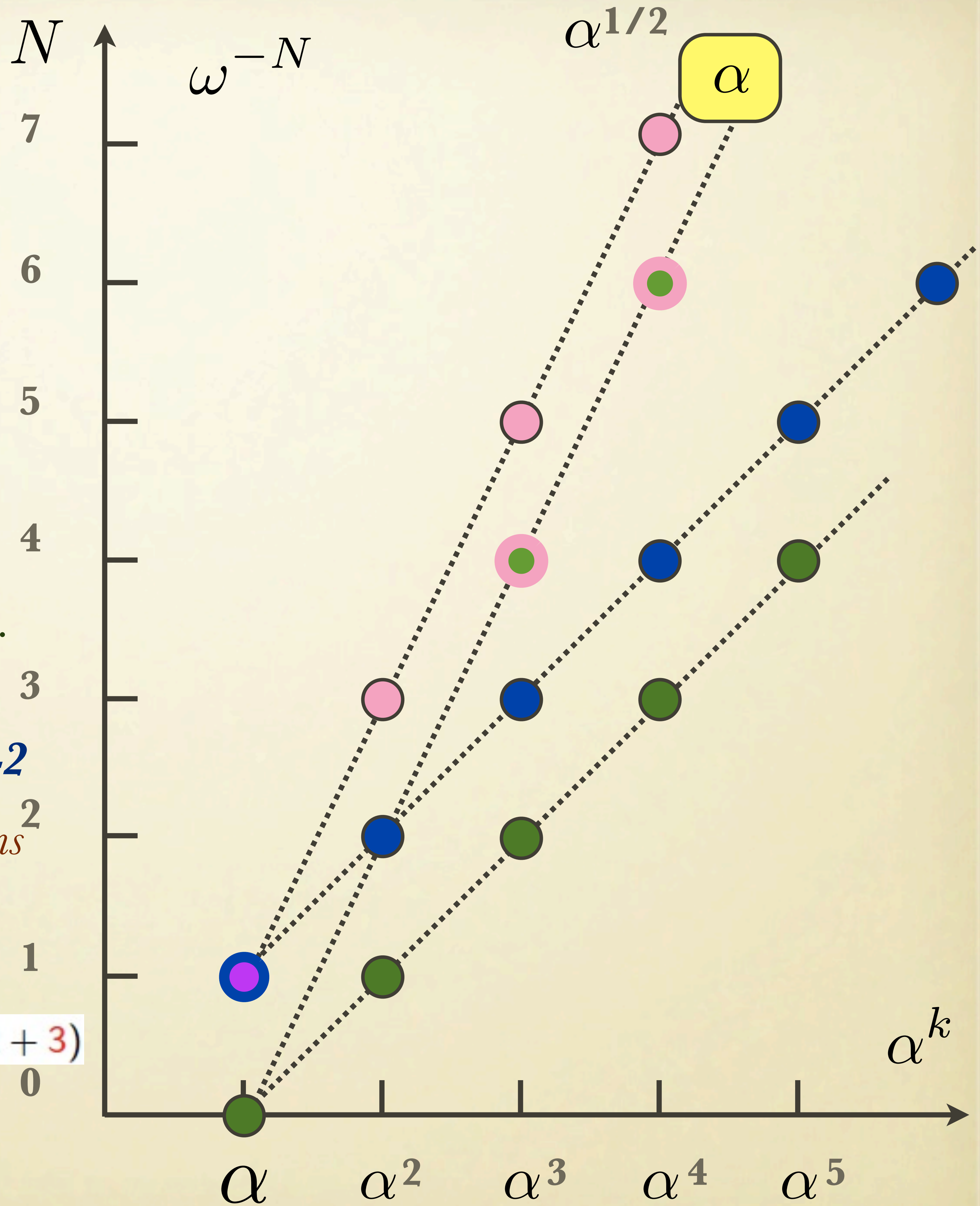
DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.

Next-to-leading correction to $\gamma^{(T)}$ -the double slope, step below: ***N=2k-2***

"*simultaneous*" emission of *two soft gluons* could have modified the answer ...

However, it was shown to ***factorize*** :

$1 \to 1+2+3$ $\Rightarrow$ $(1 \to 1+2) \otimes (2 \to 2+3)$



PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.

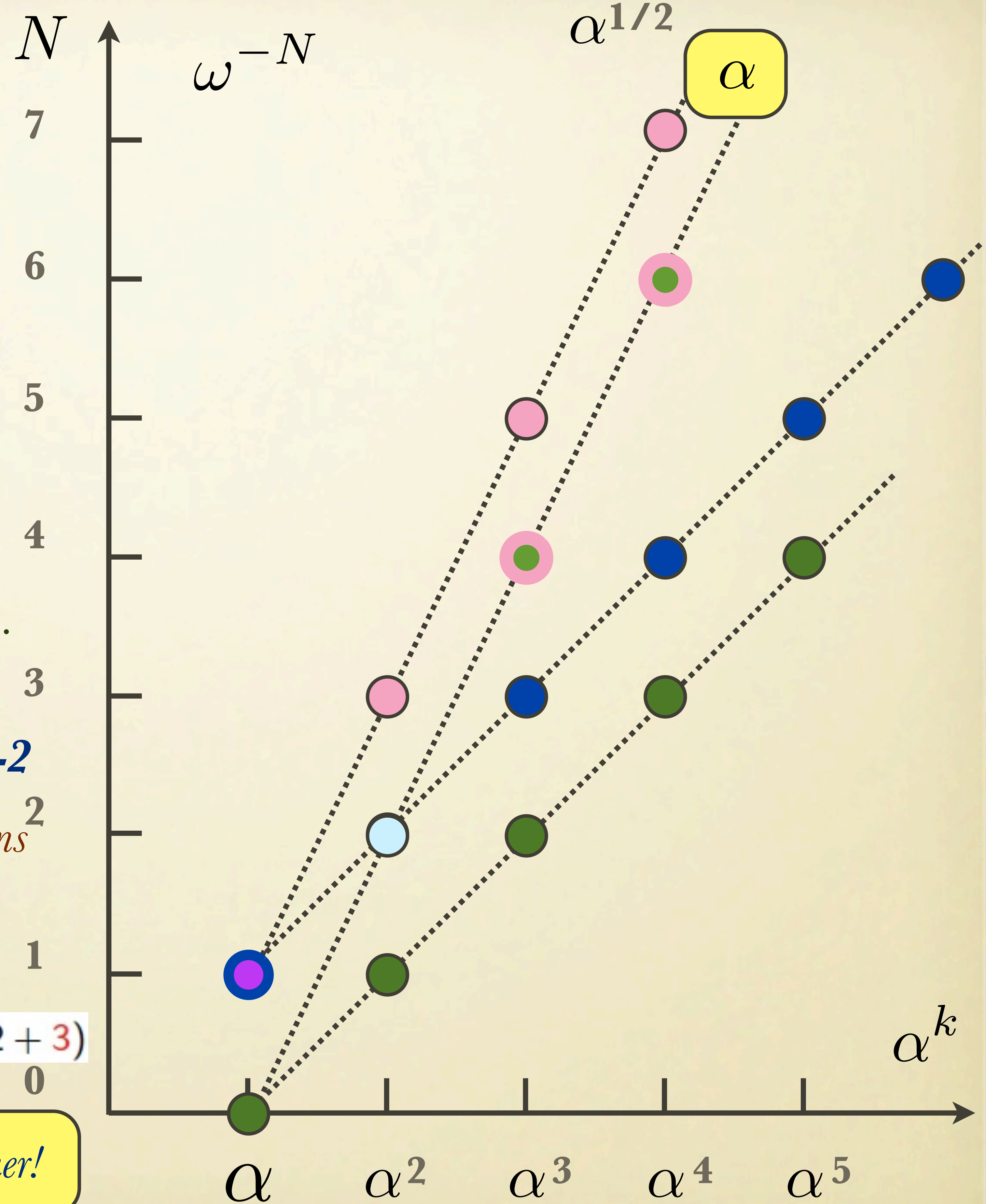
Next-to-leading correction to $\gamma^{(T)}$ -the double slope, step below: ***N=2k-2***

"*simultaneous*" emission of *two soft gluons* could have modified the answer ...

However, it was shown to ***factorize*** :

$1 \to 1 + 2 + 3$ ➡ $(1 \to 1 + 2) \otimes (2 \to 2 + 3)$

Lo and behold, $\gamma^{(S)}$ *has no this piece either!*



PT series for anomalous dimension in the small-x region :

$$\gamma(\omega) = \sum \frac{\alpha^k}{\omega^N}$$

The leading singularity generates, via the BFKL eq., the ***N=k*** series.

The next line - N-BFKL : ***N=k-1***.

DL series for the ***time-like*** an.dim. start off the same point, ***N=2k-1***, generated by the Evolution Equation.

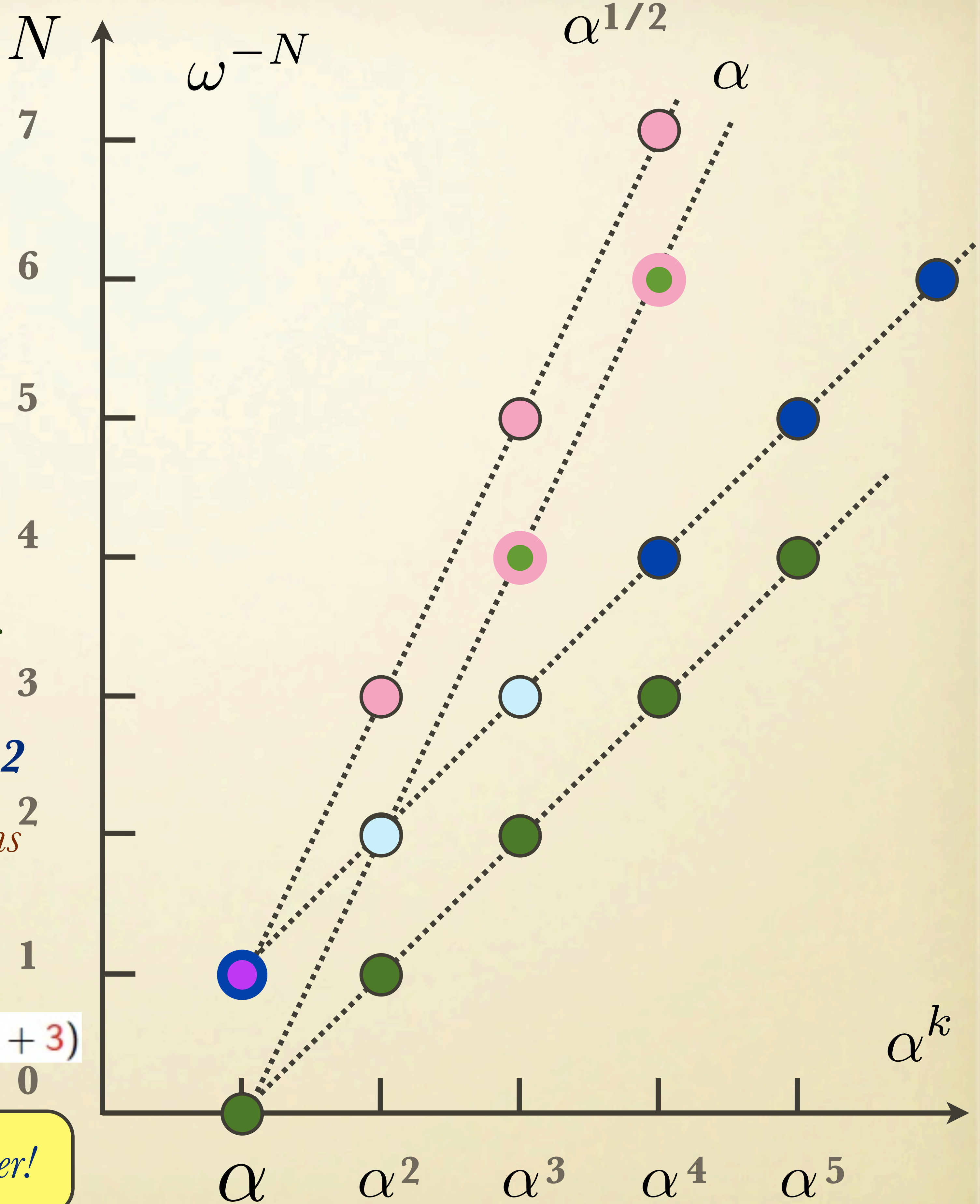
Next-to-leading correction to $\gamma^{(T)}$ -the double slope, step below: ***N=2k-2***

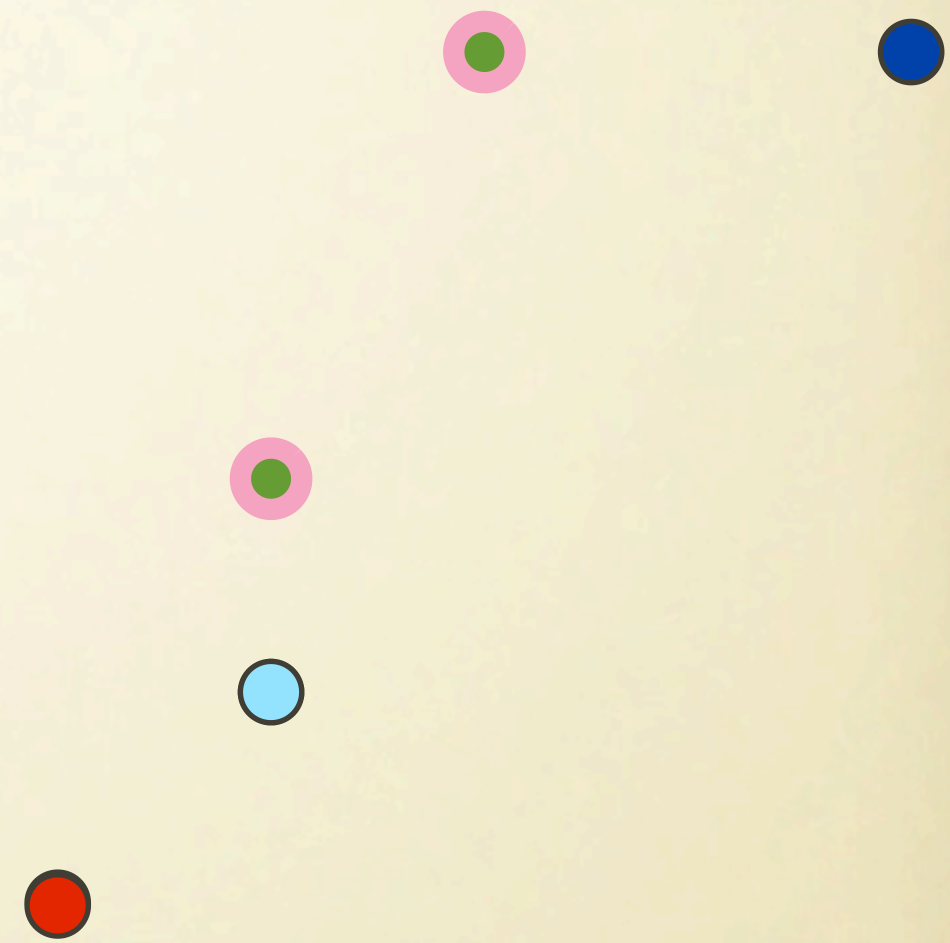
"*simultaneous*" emission of *two soft gluons* could have modified the answer ...

However, it was shown to ***factorize*** :

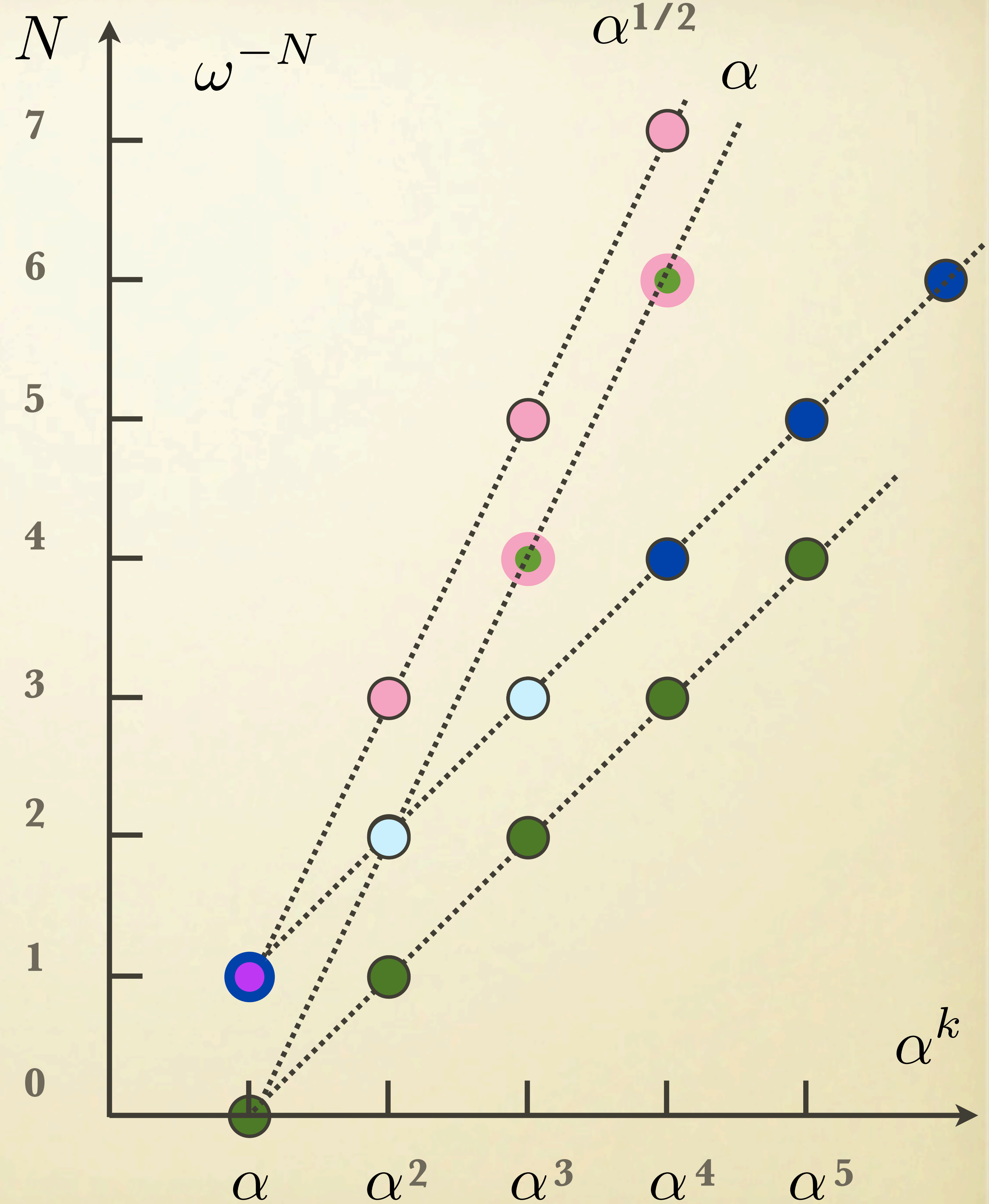
$1 \to 1 + 2 + 3$ ➡ $(1 \to 1 + 2) \otimes (2 \to 2 + 3)$

Lo and behold, $\gamma^{(S)}$ has no this piece either!





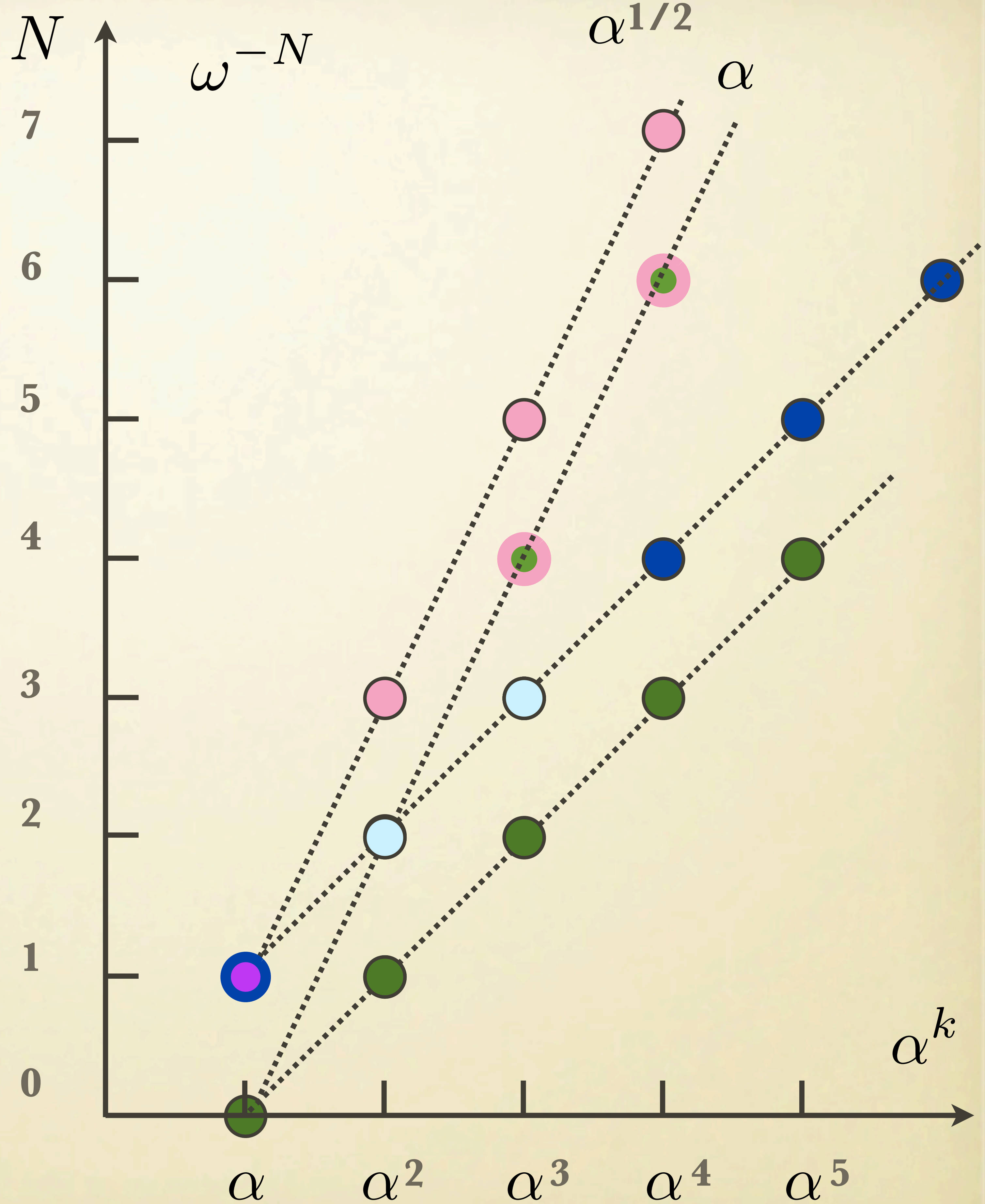
The “***second BFKL zero***” also has its “*time-like image*”.



The "***second BFKL zero***"
also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$1 \to 1 + 2 + 3 + 4$



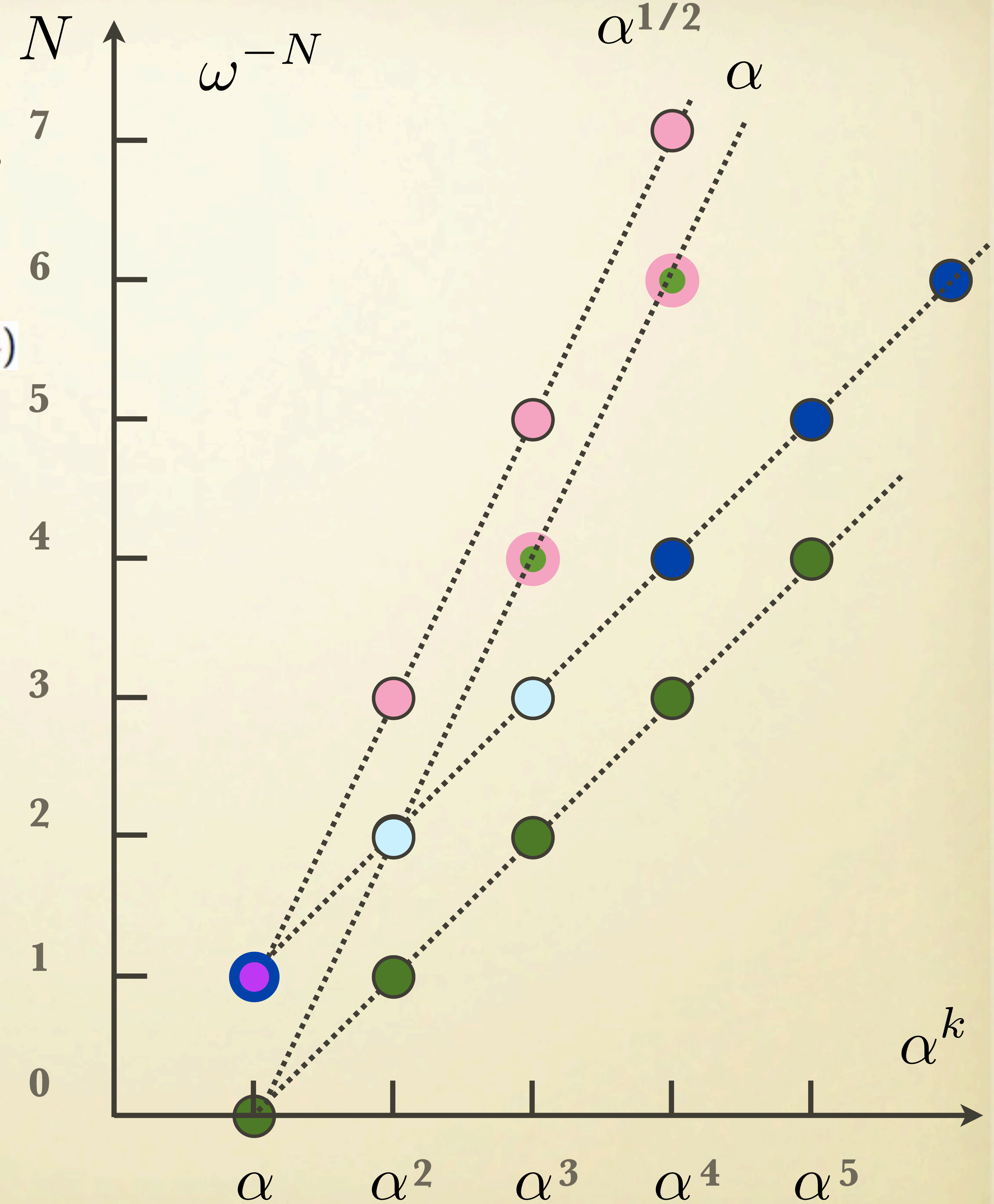
The "***second BFKL zero***"
also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$$1 \to 1 + 2 + 3 + 4$$

factorizes too : *(D-r & Troyan, 84)*

$$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$$



The "***second BFKL zero***" also has its "*time-like image*".

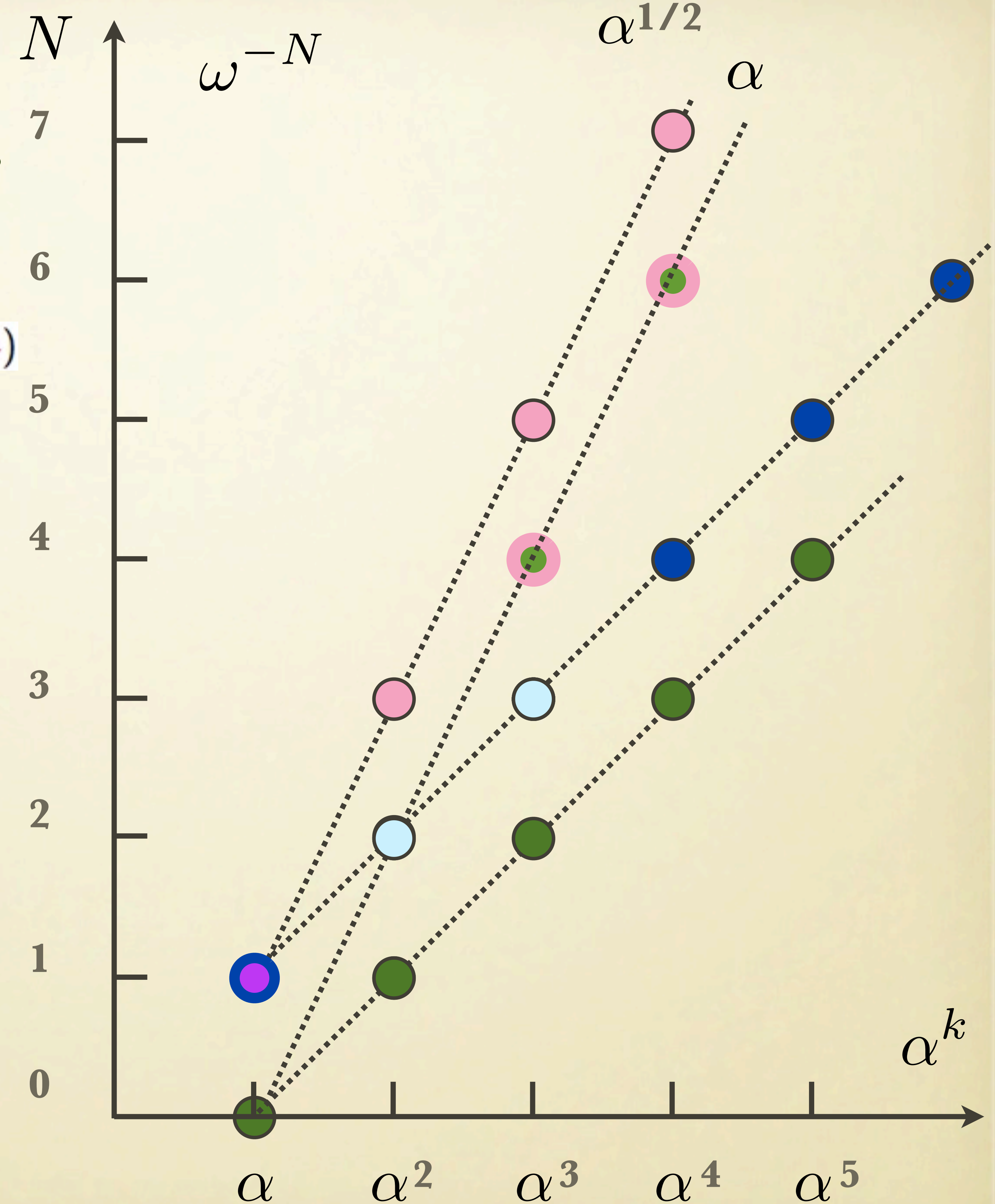
Namely, radiation of *three* soft gluons

$1 \to 1 + 2 + 3 + 4$

factorizes too : *(D-r & Troyan, 84)*

$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.



The "***second BFKL zero***"
also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$1 \to 1+2+3+4$

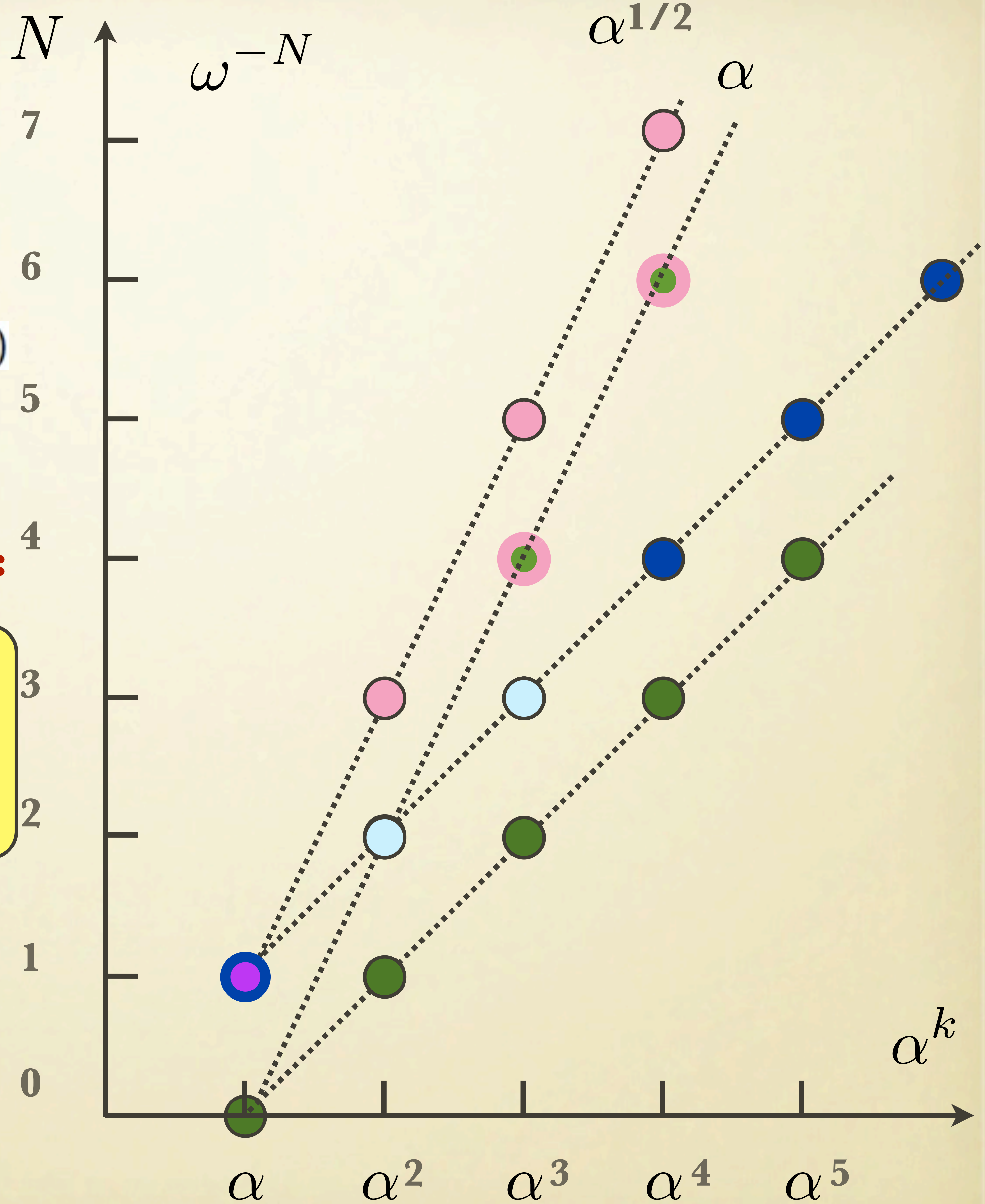
factorizes too : *(D-r & Troyan, 84)*

$(1 \to 1+2) \otimes (2 \to 2+3) \otimes (3 \to 3+4)$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.



The "***second BFKL zero***" also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$1 \to 1 + 2 + 3 + 4$

factorizes too : *(D-r & Troyan, 84)*

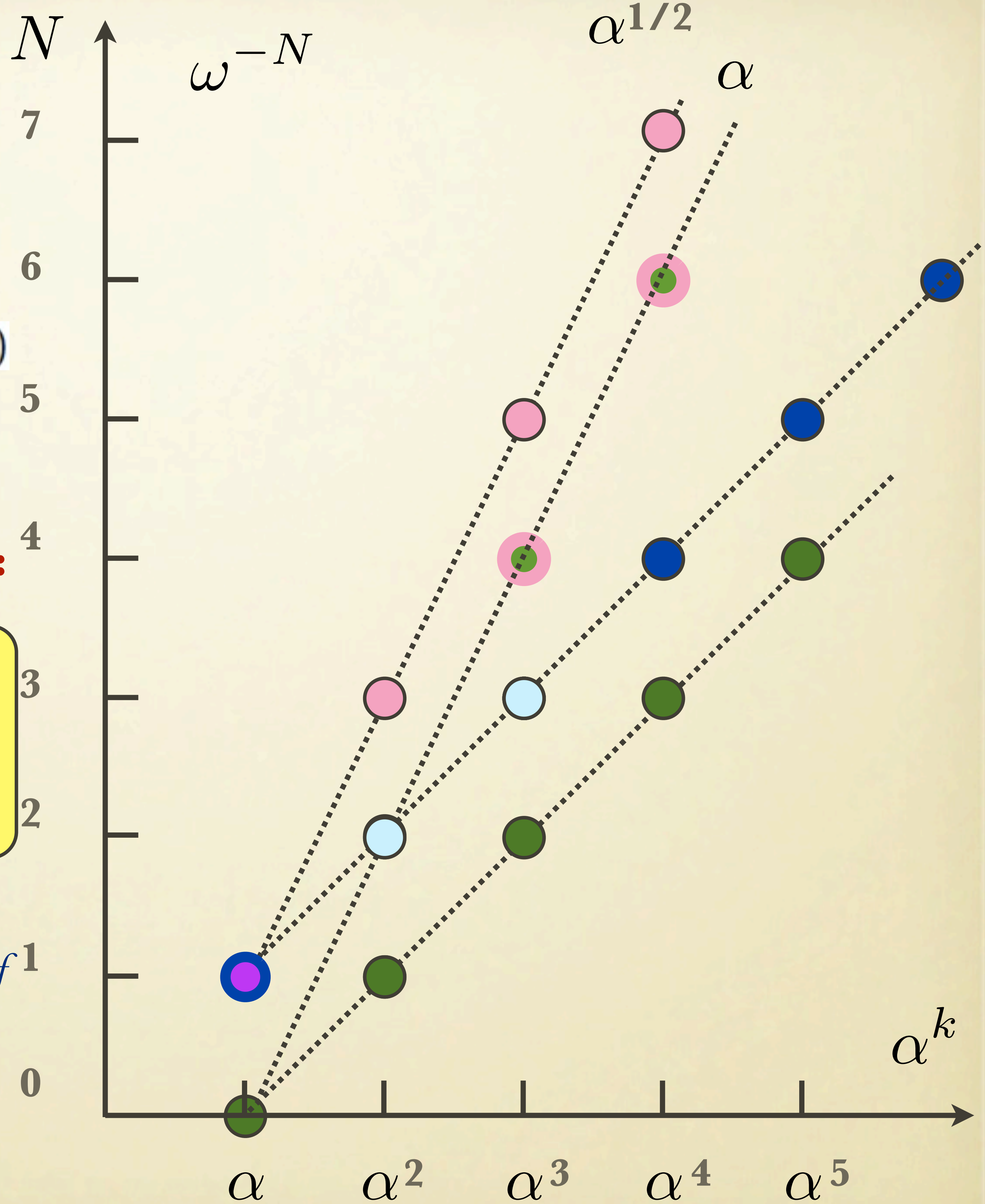
$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of



The "***second BFKL zero***" also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$$1 \to 1 + 2 + 3 + 4$$

factorizes too : *(D-r & Troyan, 84)*

$$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

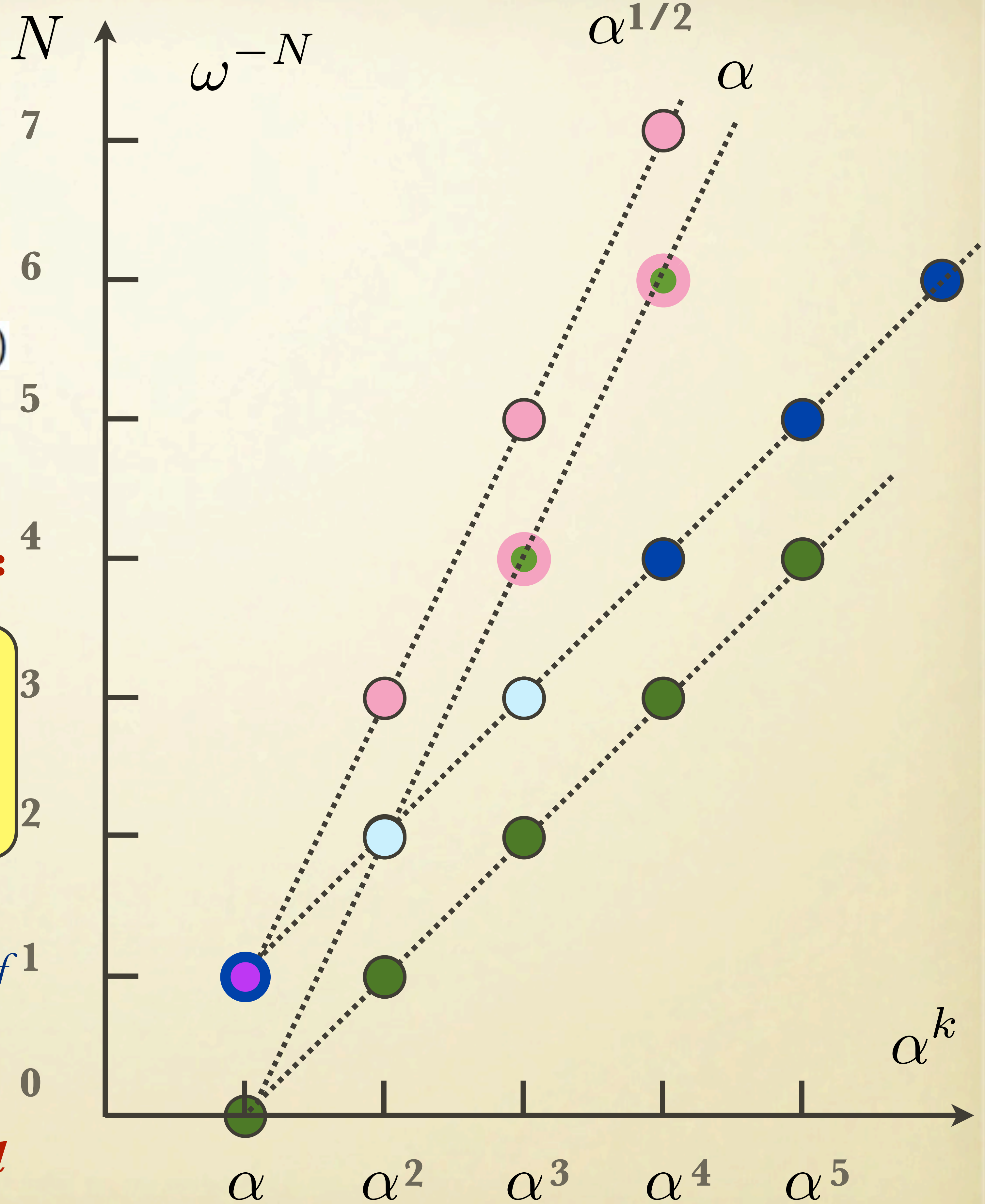
Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of

NNLL

time-like *an. dim. as a* ***free special***



The "***second BFKL zero***"
also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$$1 \to 1 + 2 + 3 + 4$$

factorizes too : *(D-r & Troyan, 84)*

$$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

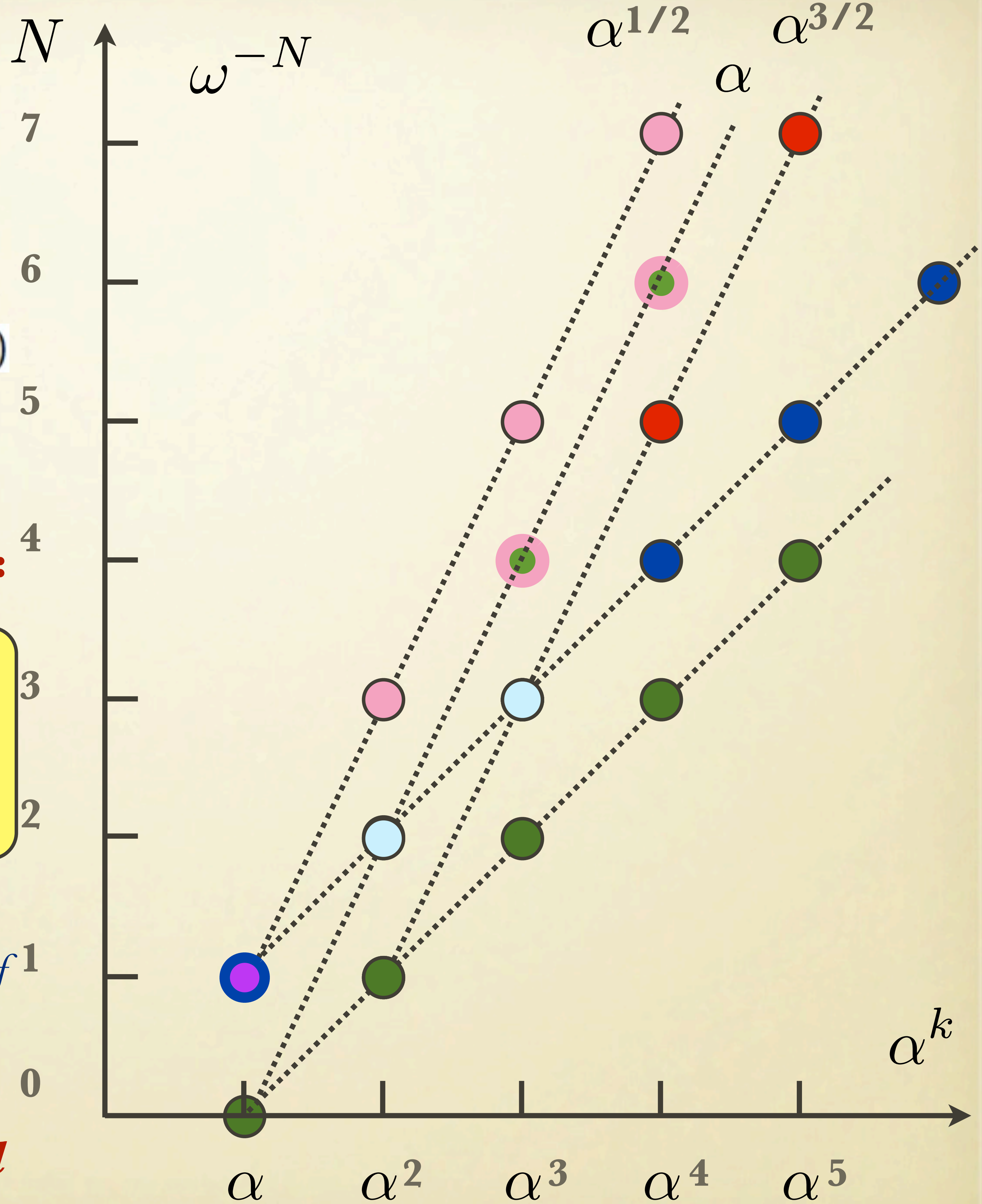
Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of

NNLL

time-like *an. dim. as a* ***free special***



The "***second BFKL zero***" also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$$1 \to 1 + 2 + 3 + 4$$

factorizes too : *(D-r & Troyan, 84)*

$$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

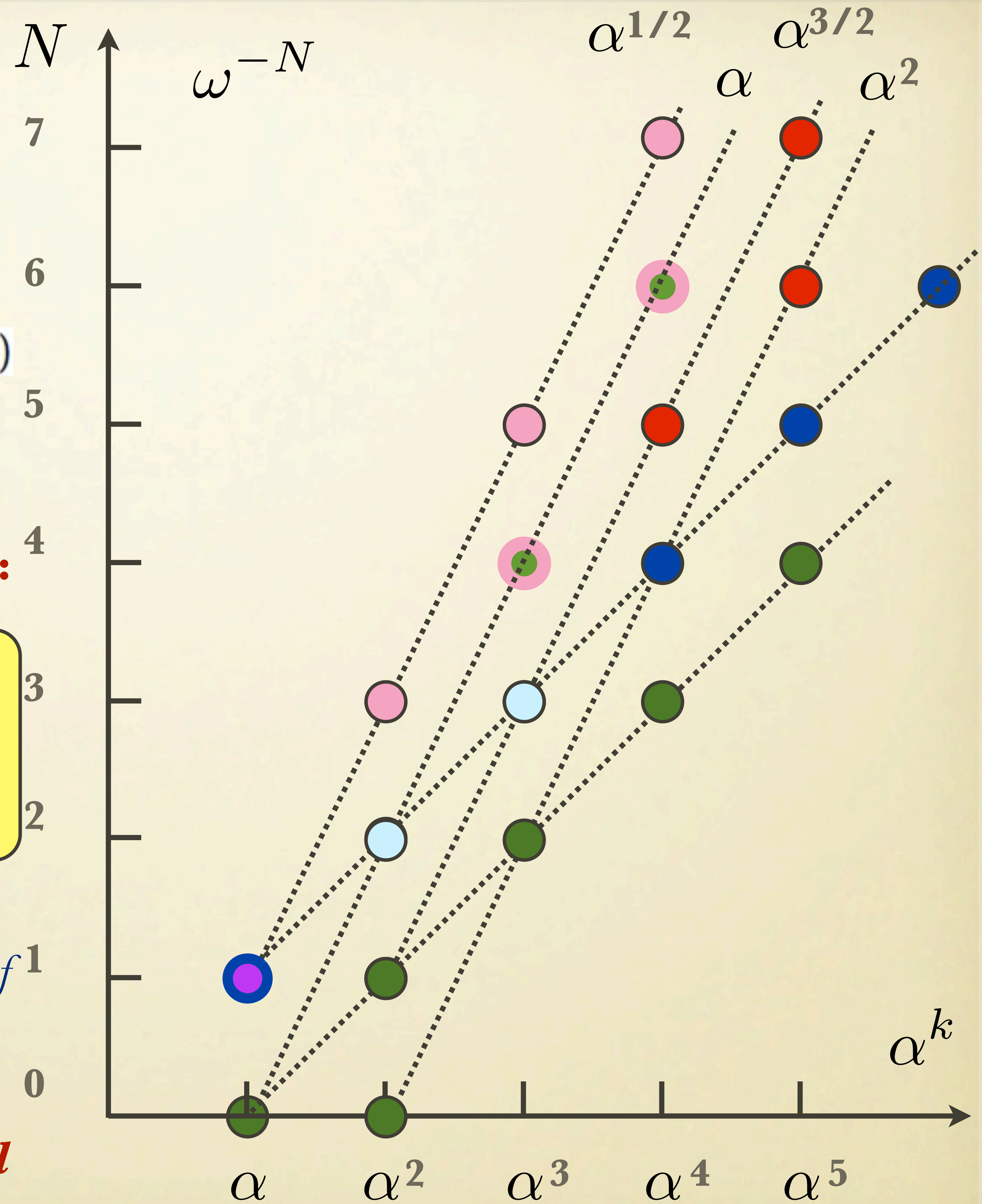
Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of

NNLL, *NNNLL*

time-like *an. dim. as a* ***free special***



The "***second BFKL zero***"
also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$$1 \to 1 + 2 + 3 + 4$$

factorizes too : *(D-r & Troyan, 84)*

$$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

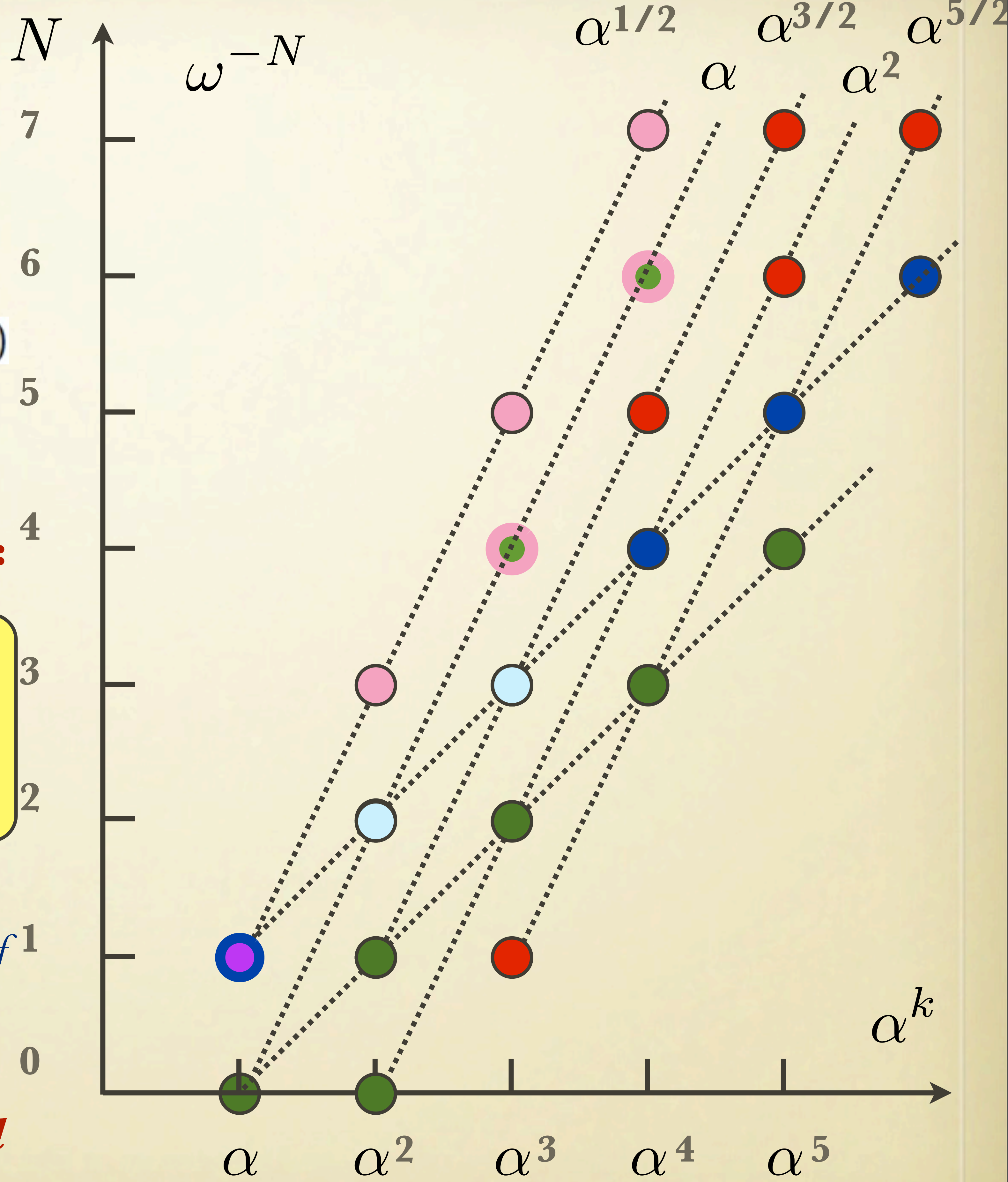
Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of

NNLL, NNNLL, N^4LL

time-like *an. dim. as a* ***free special***



The "***second BFKL zero***" also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$1 \to 1 + 2 + 3 + 4$

factorizes too :   *(D-r & Troyan, 84)*

$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

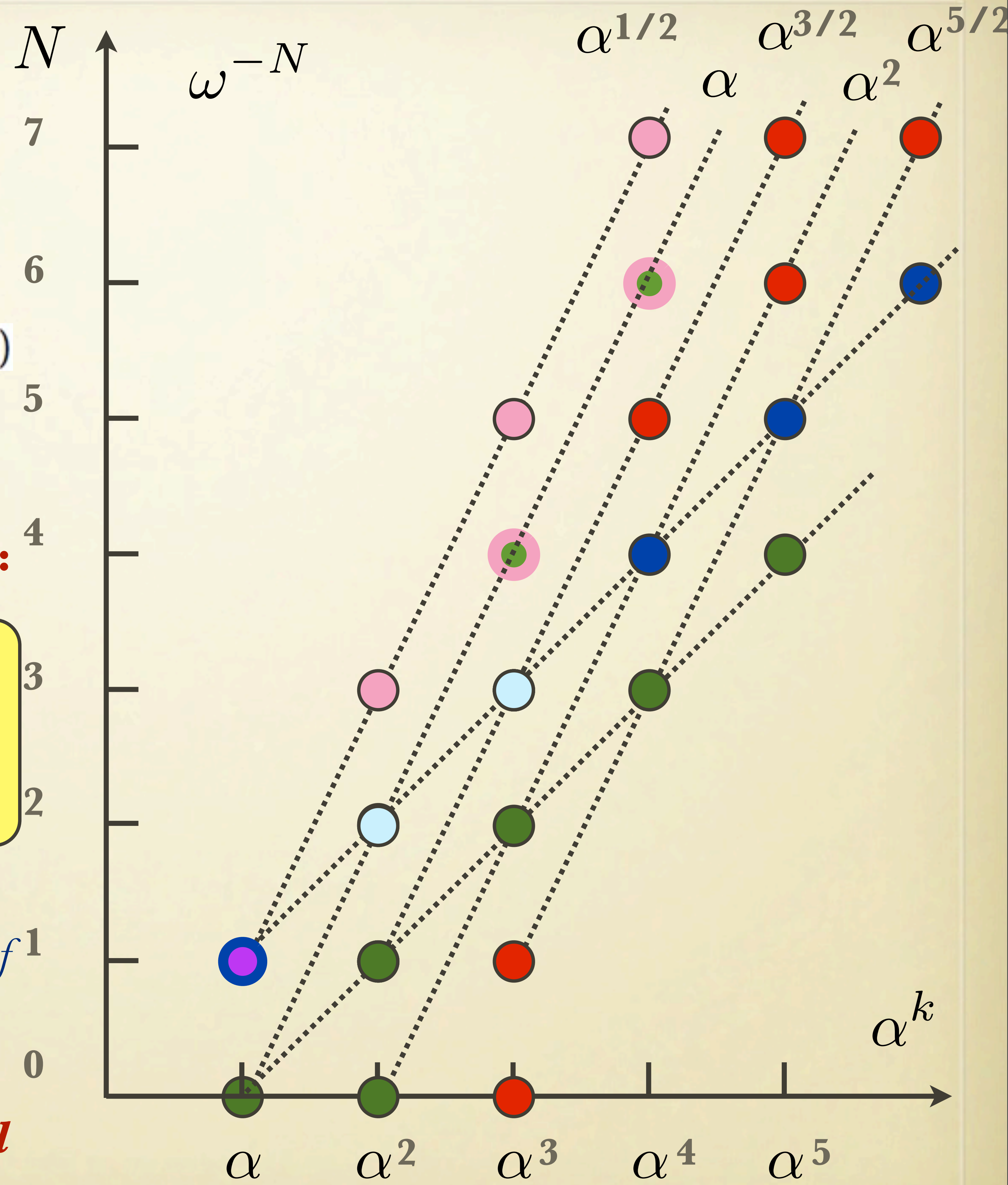
Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of

NNLL, NNNLL, N⁴LL

time-like *an. dim. as a* ***free special***



The “***second BFKL zero***” also has its “*time-like image*”.

Namely, radiation of *three* soft gluons

$$1 \to 1 + 2 + 3 + 4$$

factorizes too : *(D-r & Troyan, 84)*

$$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

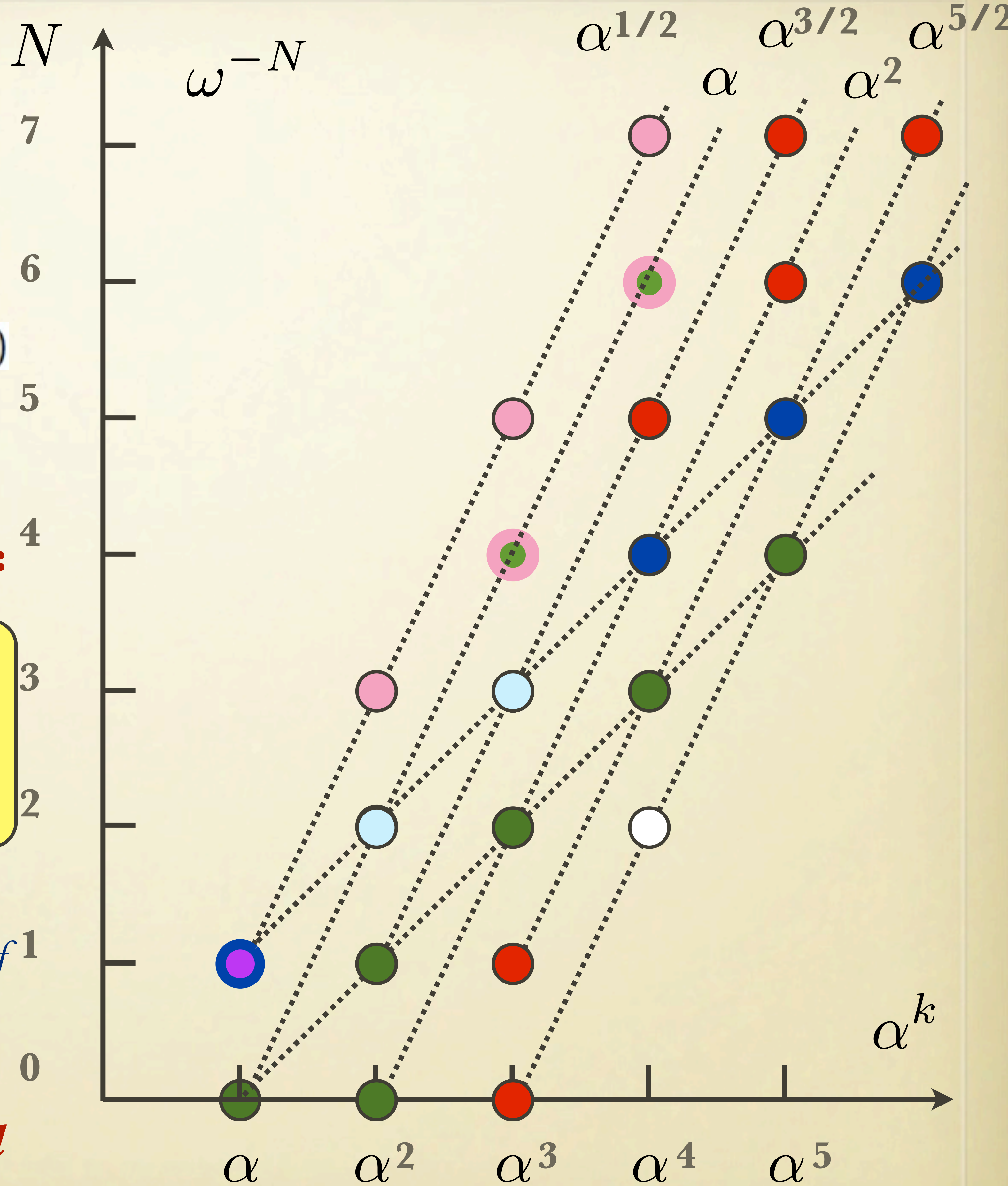
Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of

NNLL, NNNLL, N^4LL, N^5LL

time-like *an. dim. as a* ***free special***



The "***second BFKL zero***" also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$1 \to 1 + 2 + 3 + 4$

factorizes too : *(D-r & Troyan, 84)*

$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

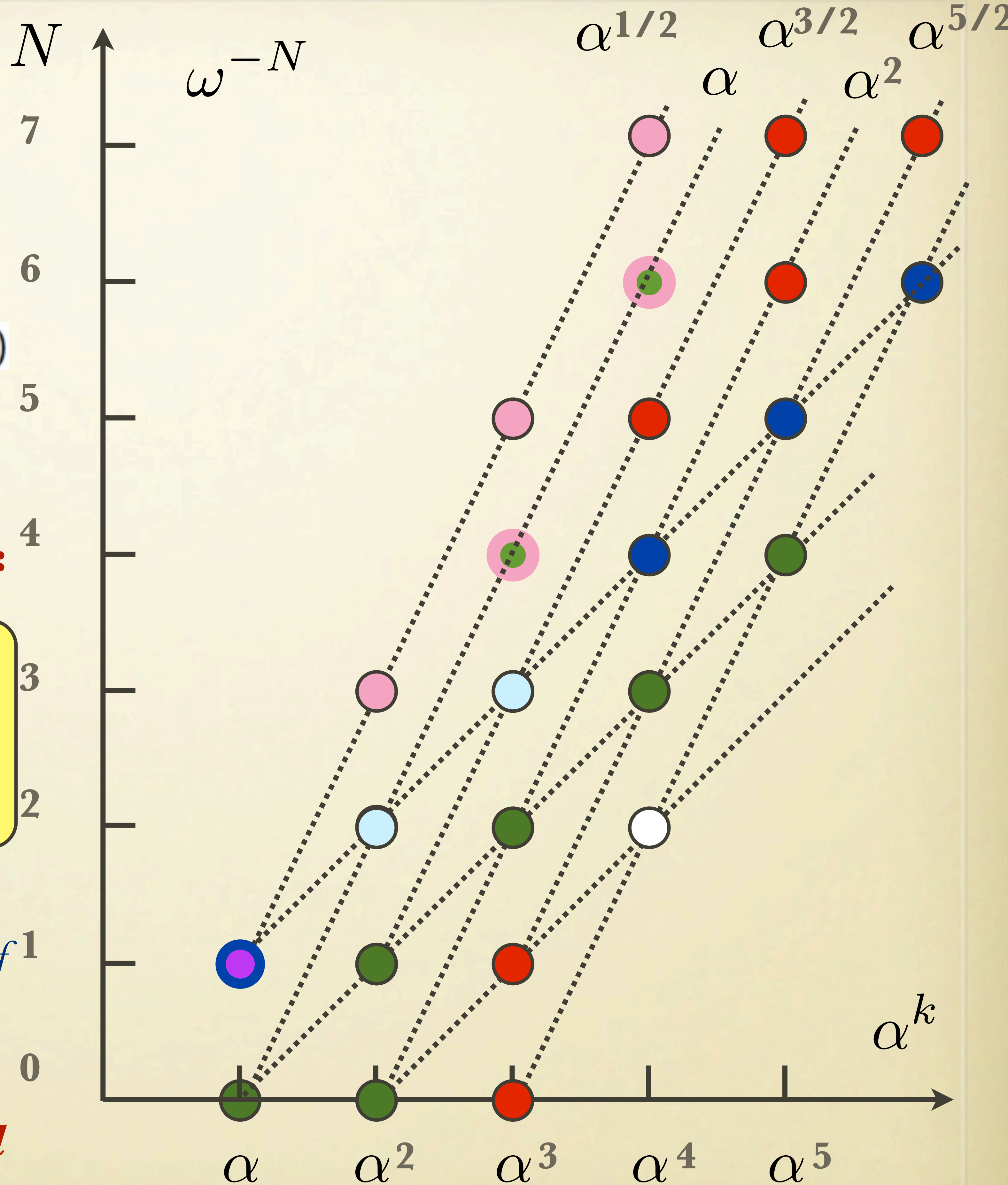
Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of

NNLL, NNNLL, N^4LL, N^5LL

time-like *an. dim. as a* ***free special***



The "***second BFKL zero***"
also has its "*time-like image*".

Namely, radiation of *three* soft gluons

$$1 \to 1 + 2 + 3 + 4$$

factorizes too : *(D-r & Troyan, 84)*

$$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes (3 \to 3 + 4)$$

Exact angular ordering takes good care of emission of **1**, **2**, **3** soft gluons.

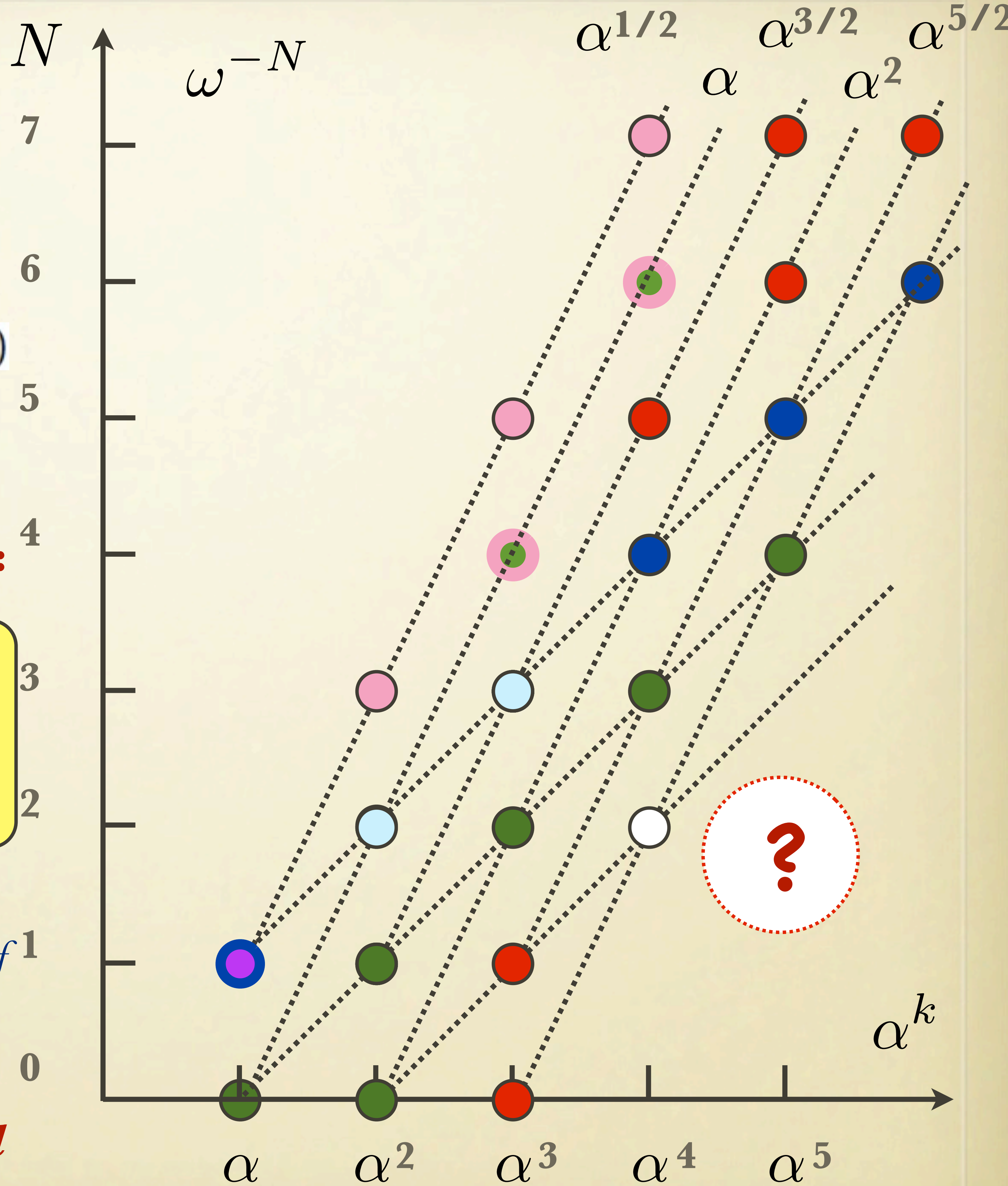
Known consequence - *Malaza puzzle* **:**

derivation of the *N-N-LL* correction to the ratio of quark and gluon jet multiplicities *(Gaffney & AHM, 85)* from the ***1-loop*** AO evolution eq.

Employ M'BKDMS RREE to get hold of

NNLL, NNNLL, N^4LL, N^5LL

time-like *an. dim. as a **free special***



The "***second BFKL zero***"
also has its "*time-like image*".

Namely, radiation of *three* s

$1 \to 1 + 2 + 3 + 4$

factorizes too : *(D-r & T*

$(1 \to 1 + 2) \otimes (2 \to 2 + 3) \otimes$

Exact angular ordering takes g
of emission of **1**, **2**, **3** sof

Known consequence - *Mal*

derivation of the *N-N-LL* c
to the ratio of quark and
multiplicities *(Gaffney & A*
from the ***1-loop*** AO evol

Employ M'BKDMS RREE t

NNLL, NNNLL, N^4LL

time-like *an. dim. as a* ***free special***