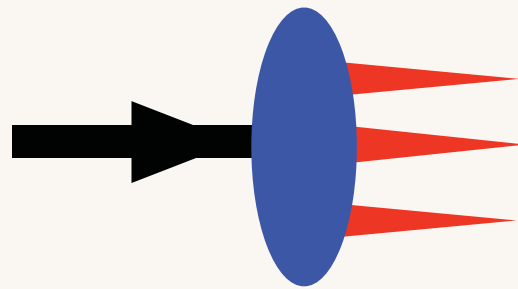


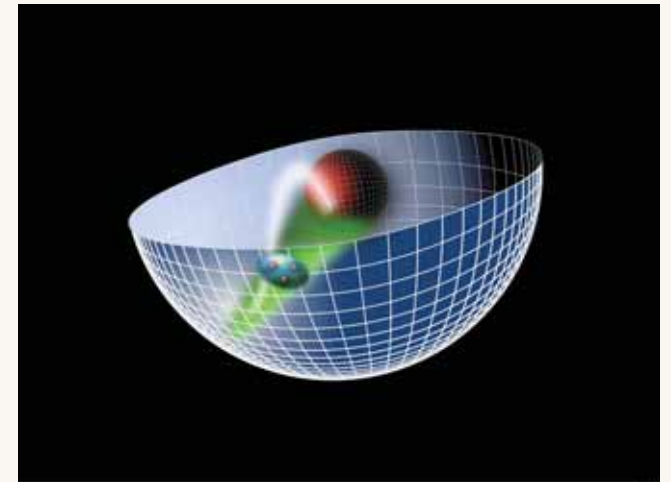
# *New Horizons in QCD*

*From Particles and Fields to Nuclei and Fields:  
An International Symposium in Celebration of  
Al Mueller's 70th Birthday*

*Columbia University October 23, 2009*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

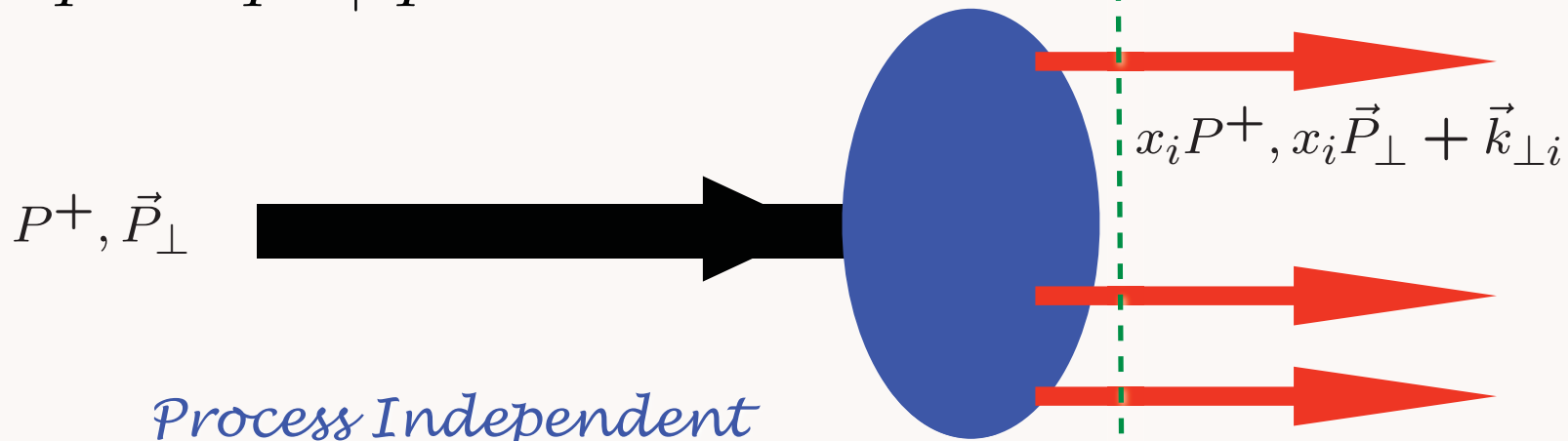


*Stan Brodsky, SLAC National Accelerator Laboratory*

# Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed  $\tau = t + z/c$



*Process Independent  
Direct Link to QCD Lagrangian!*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

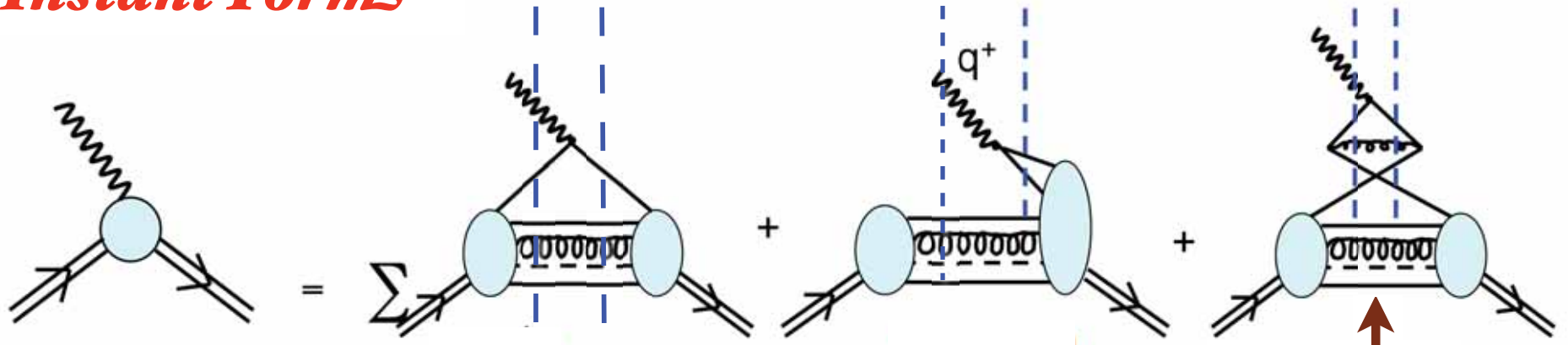
$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}$$

*Invariant under boosts! Independent of  $P^\mu$*

# Calculation of Form Factors in Equal-Time Theory

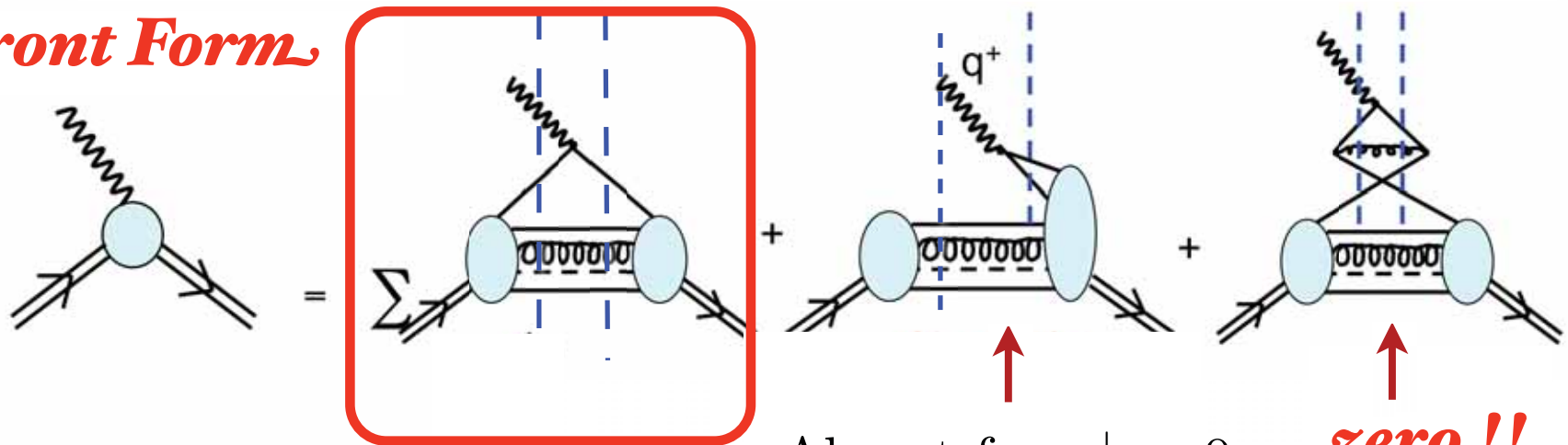
## Instant Form



Need vacuum-induced currents

# Calculation of Form Factors in Light-Front Theory

## Front Form



Complete Answer

Absent for  $q^+ = 0$

zero !!

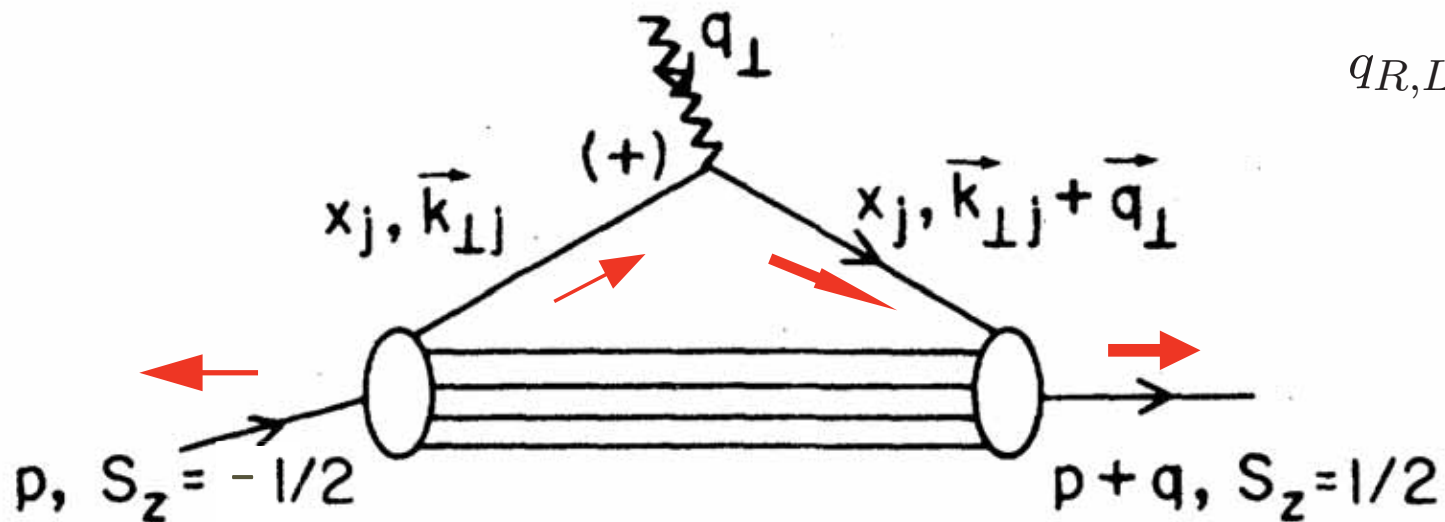
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

Drell, sjb

$$\left[ -\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

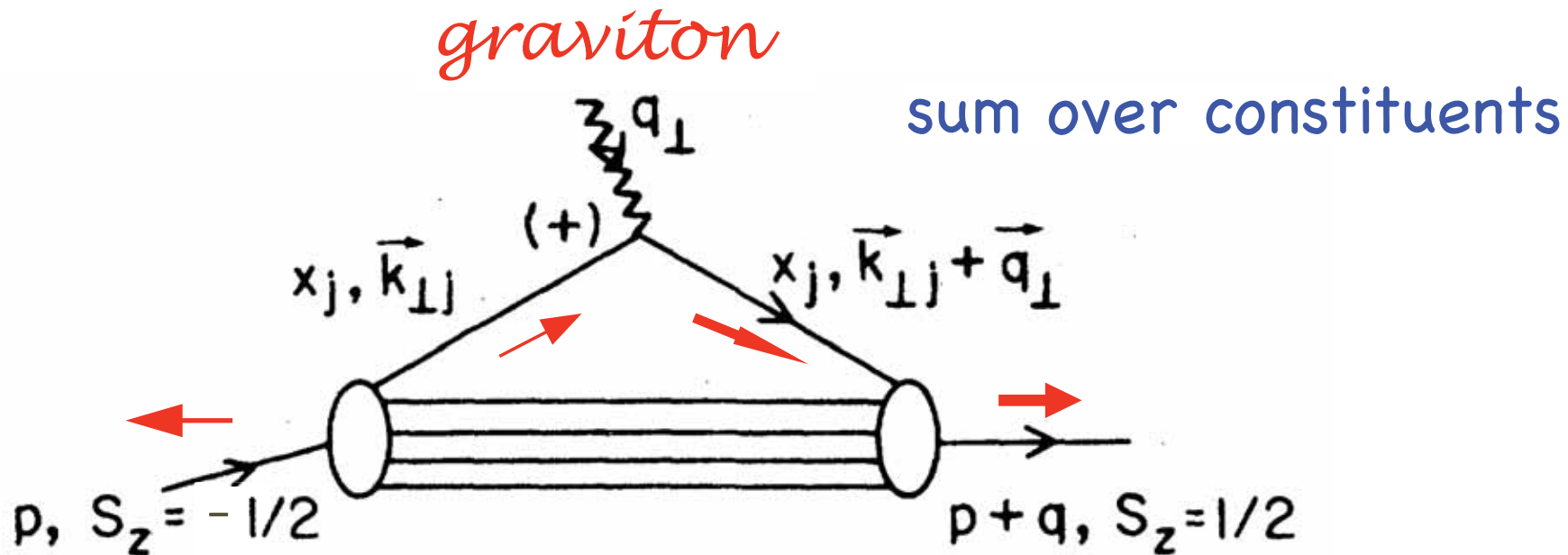


Must have  $\Delta l_z = \pm 1$  to have nonzero  $F_2(q^2)$

*Same matrix elements appear in Sivers effect  
-- connection to quark anomalous moments*

# Anomalous gravitomagnetic moment $B(0)$

**Terayev, Okun, et al:**  $B(0)$  Must vanish because of Equivalence Theorem



**Hwang, Schmidt, sjb;  
Holstein et al**

$B(0) = 0$

*Each Fock State*

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

*sum over states with  $n=3, 4, \dots$  constituents*

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum  $P^\mu$ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

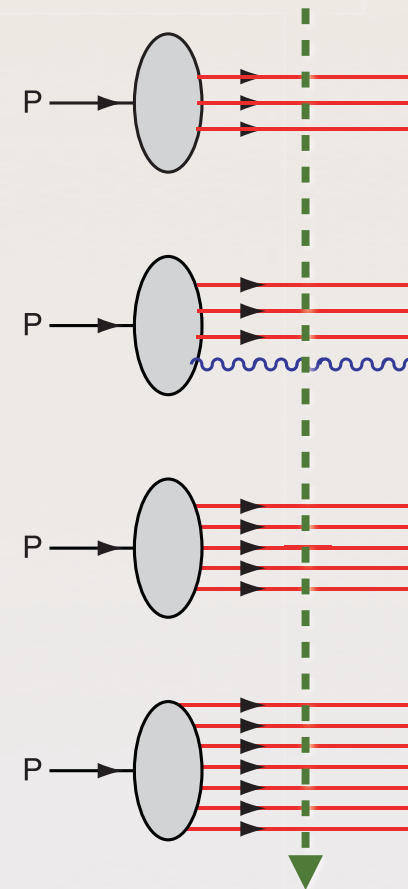
$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

**Intrinsic heavy quarks**

$c(x), b(x)$  at high  $x$

$$\bar{s}(x) \neq s(x)$$

$$\bar{u}(x) \neq \bar{d}(x)$$



*Fixed LF time*

# Soft gluons in the infinite-momentum wave function and the BFKL pomeron \*

A.H. Mueller

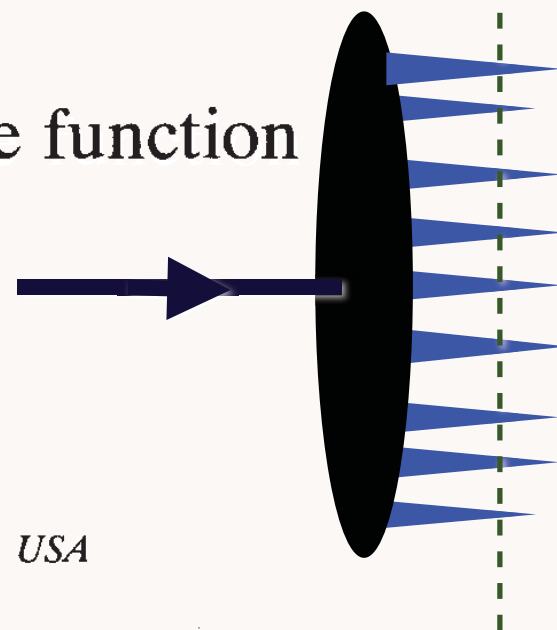
*Stanford Linear Accelerator Center, Stanford, CA 94309, USA*

and

*Department of Physics, Columbia University <sup>1</sup>, New York, NY 10027, USA*

Received 27 August 1993

Accepted for publication 8 November 1993



*Fixed LF time*

We construct the infinite-momentum wave function for arbitrary numbers of soft gluons in a heavy quark–antiquark, onium, state. The soft gluon part of the wave function is constructed exactly within the leading logarithmic and large- $N_c$  limits. The BFKL pomeron emerges when gluon number densities are evaluated.



## INTRINSIC CHEVROLETS AT THE SSC

Stanley J. Brodsky

Stanford Linear Accelerator Center, Stanford University, Stanford CA 94305

John C. Collins

Department of Physics, Illinois Institute of Technology, Chicago IL 60616  
and  
High Energy Physics Division, Argonne National Laboratory, Argonne IL 60439

Stephen D. Ellis

Department of Physics, FM-15, University of Washington, Seattle WA 98195

John F. Gunion

Department of Physics, University of California, Davis CA 95616

Alfred H. Mueller

Department of Physics, Columbia University, New York NY 10027

$$\mathcal{L}_{QCD}^{eff} = -\frac{1}{4}F_{\mu\nu a}F^{\mu\nu a} - \frac{g^2 N_C}{120\pi^2 M_Q^2} D_\alpha F_{\mu\nu a} D^\alpha F^{\mu\nu a} + C \frac{g^2 N_C}{120\pi^2 M_Q^2} F_\mu^{a\nu} F_\nu^{b\tau} F_\tau^{c\mu} f_{abc} + \mathcal{O}\left(\frac{1}{M_Q^4}\right)$$

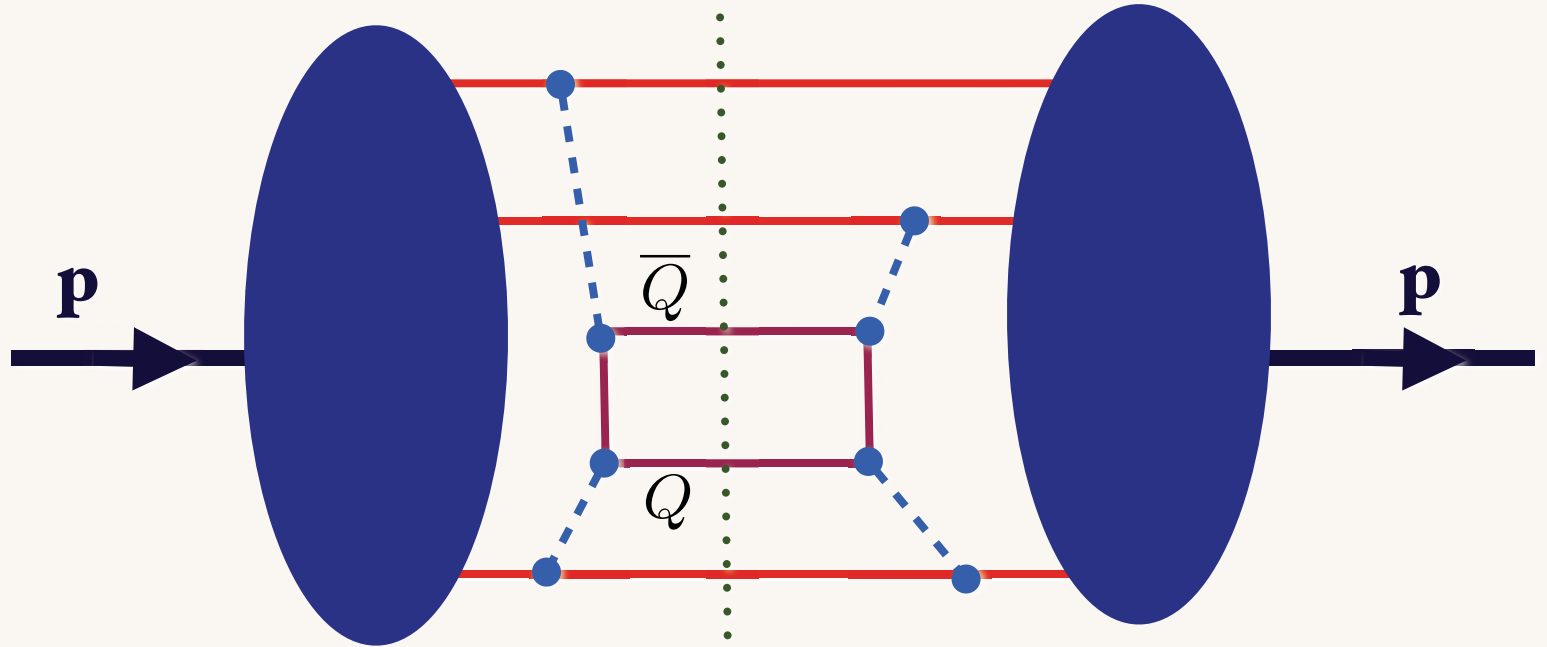
*Probability of Intrinsic Heavy Quarks  $\sim 1/M_Q^2$*



*Proton Self Energy  
Intrinsic Heavy Quarks*

*Fixed LF time*

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$



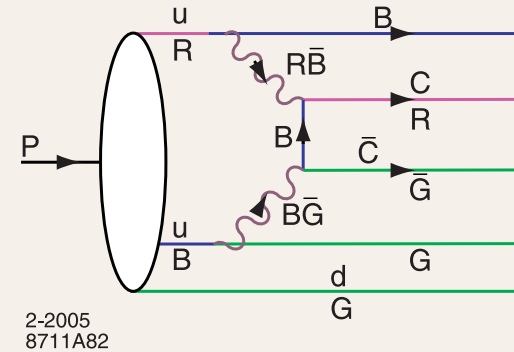
$$\text{Probability (QED)} \propto \frac{1}{M_{\ell}^4}$$

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

**Collins, Ellis, Gunion, Mueller, sjb**

# Intrinsic Heavy-Quark Fock States

- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!



- Probability  $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$   $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$   $P_{c\bar{c}/p} \simeq 1\%$

- Large Effect at high x

**Collins, Ellis, Gunion, Mueller, sjb**

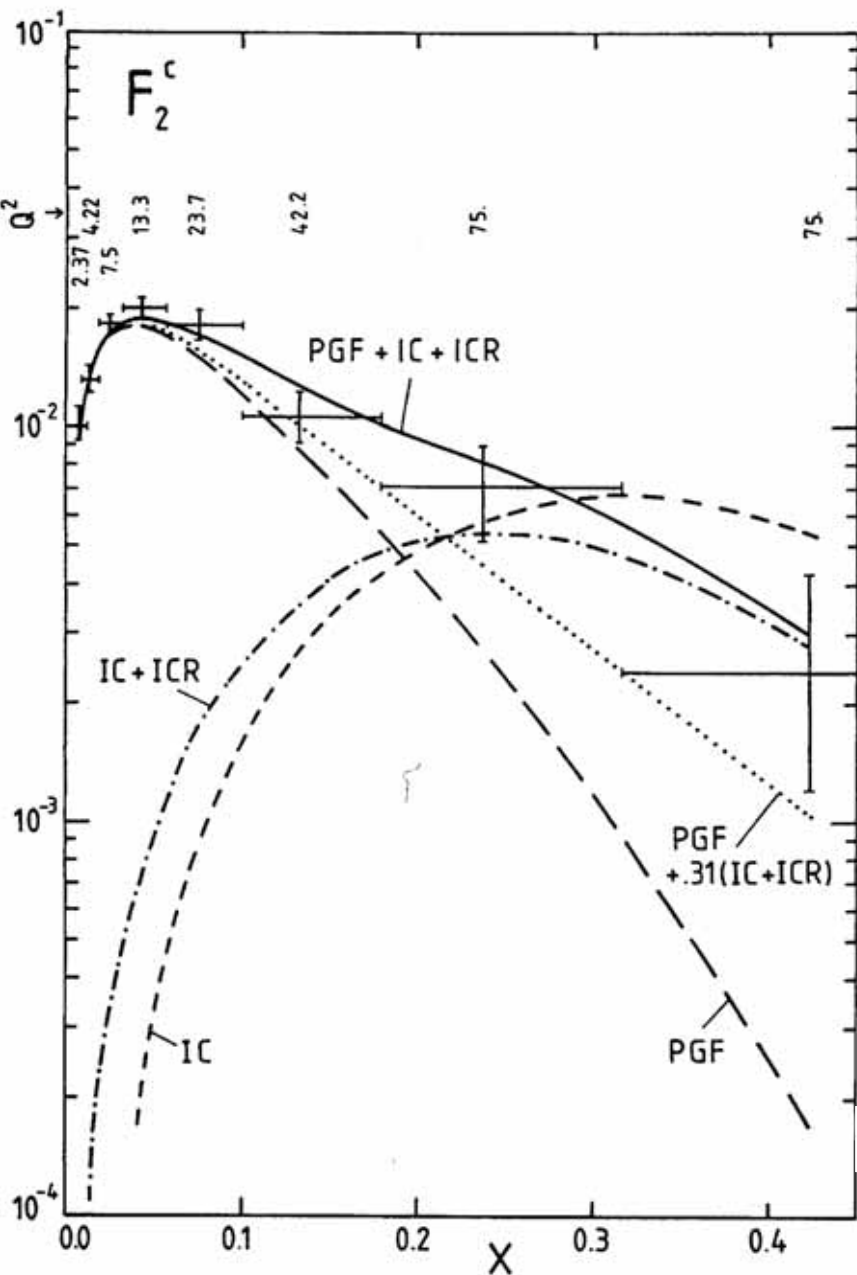
- Greatly increases kinematics of colliders such as Higgs production (**Kopeliovich, Schmidt, Soffer, sjb**)

- Severely underestimated in conventional parameterizations of heavy quark distributions (**Pumplin, Tung**)

- Many empirical tests

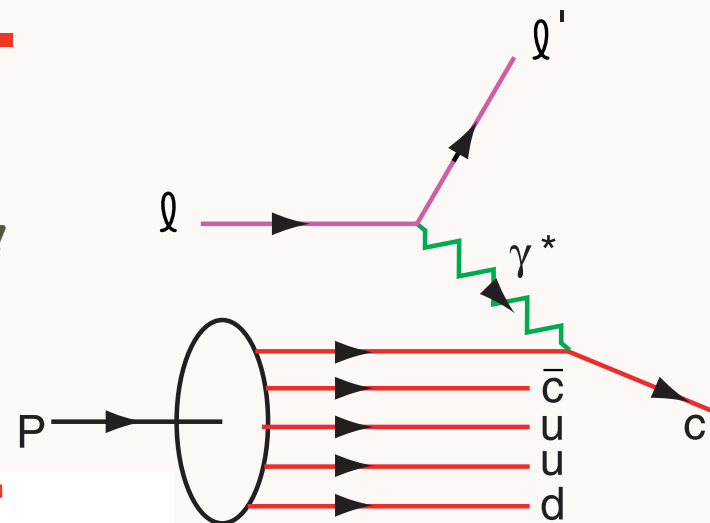
# Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).



## First Evidence for Intrinsic Charm

factor of 30!



**DGLAP / Photon-Gluon Fusion: factor of 30 too small**

- EMC data:  $c(x, Q^2) > 30 \times \text{DGLAP}$   
 $Q^2 = 75 \text{ GeV}^2, x = 0.42$
- High  $x_F$   $pp \rightarrow J/\psi X$
- High  $x_F$   $pp \rightarrow J/\psi J/\psi X$
- High  $x_F$   $pp \rightarrow \Lambda_c X$
- High  $x_F$   $pp \rightarrow \Lambda_b X$
- High  $x_F$   $pp \rightarrow \Xi(ccd) X$  (SELEX)

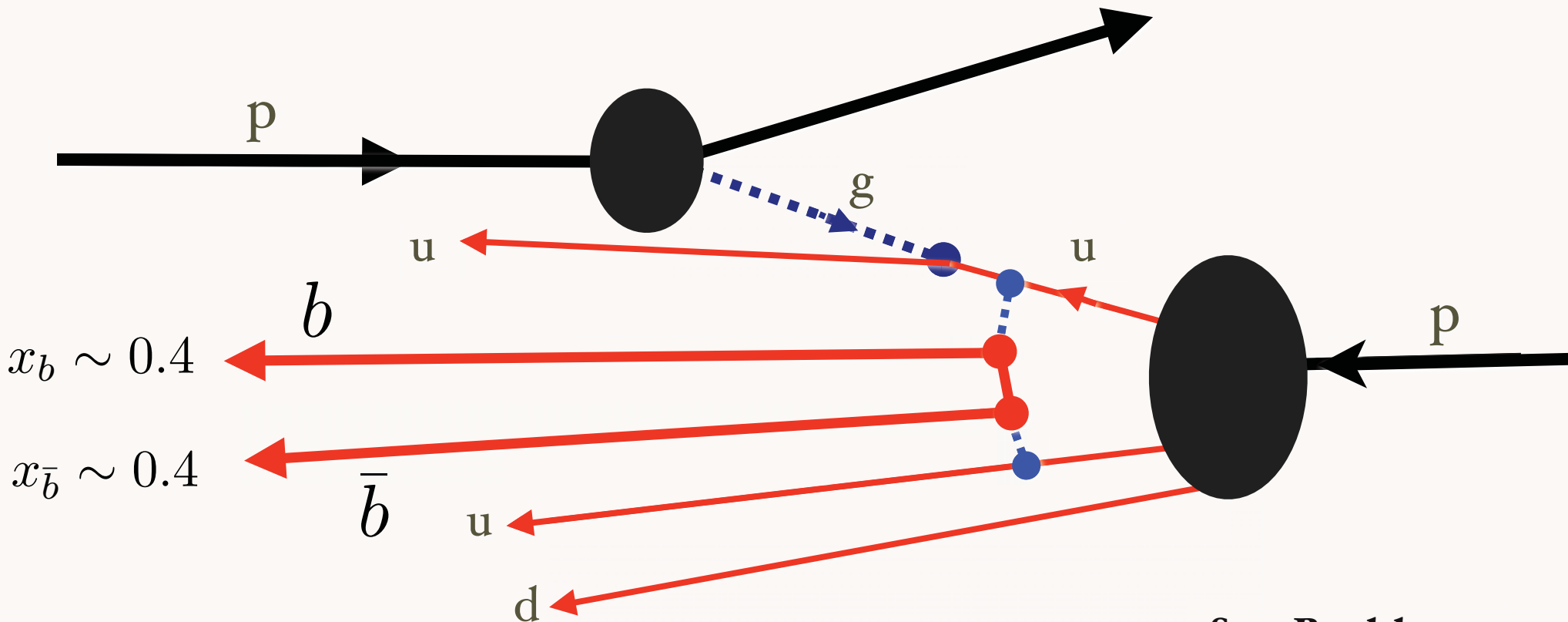
**C.H. Chang, J.P. Ma, C.F. Qiao and X.G. Wu,**  
**Hadronic production of the doubly charmed baryon  $\Xi_{cc}$  with intrinsic charm,"** arXiv:hep-ph/0610205.

# Excitation of Intrinsic Heavy Quarks in Proton

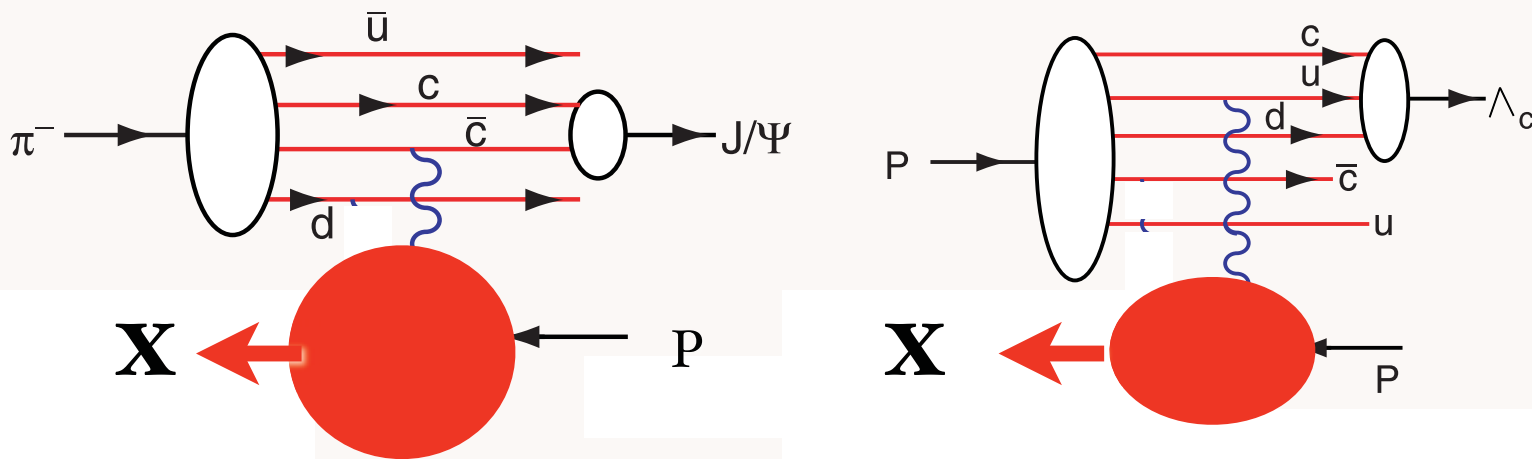
Amplitude maximal at small invariant mass, equal rapidity

$$x_i \sim \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

Produce forward, high  $x_F$   
 $\Upsilon(b\bar{b}), \Lambda_b(bud), B^+(\bar{b}u), B^0(\bar{b}d)$

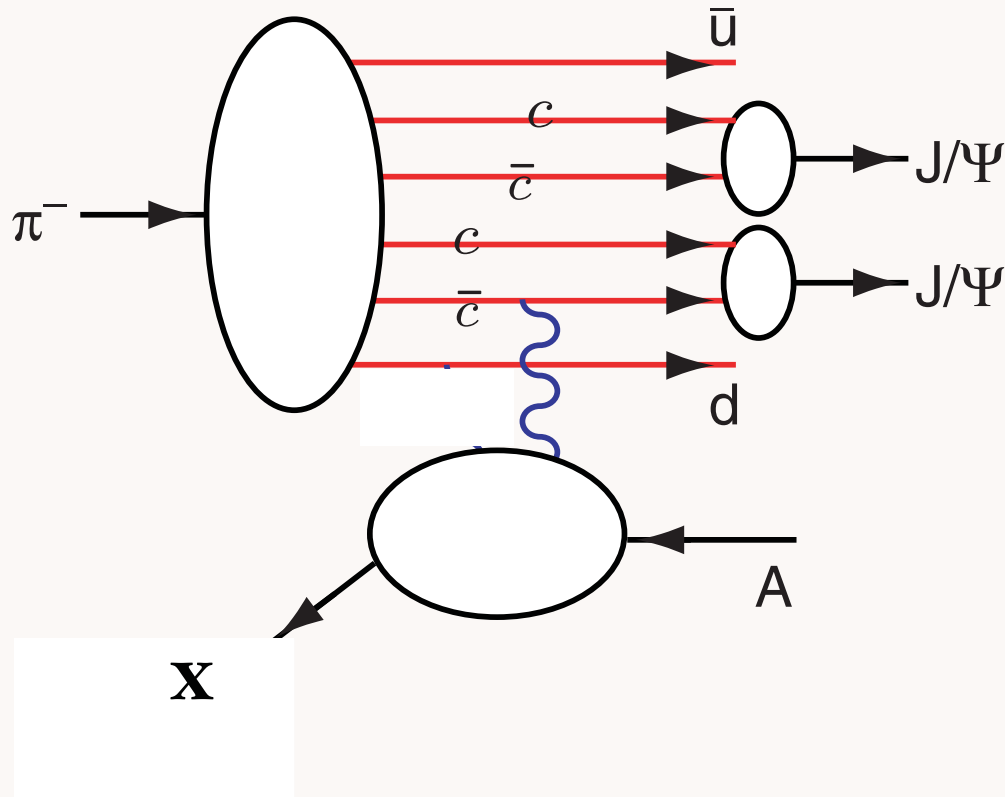


# Leading Hadron Production from Intrinsic Charm



Coalescence of Comoving Charm and Valence Quarks  
Produce  $J/\psi$ ,  $\Lambda_c$  and other Charm Hadrons at High  $x_F$

# Production of Two Charmonia at High $x_F$



All events have  $x_{\psi\psi}^F > 0.4$  !

## Excludes color drag model

$$\pi A \rightarrow J/\psi J/\psi X$$

Intrinsic charm contribution to double quarkonium hadroproduction <sup>\*</sup>

R. Vogt <sup>a</sup>, S.J. Brodsky <sup>b</sup>

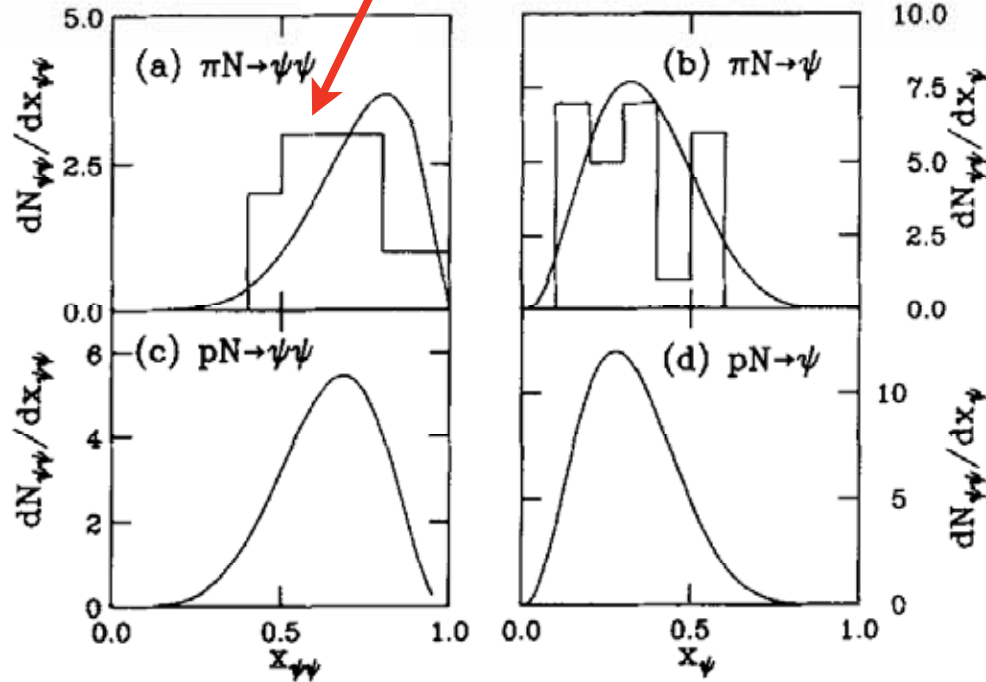


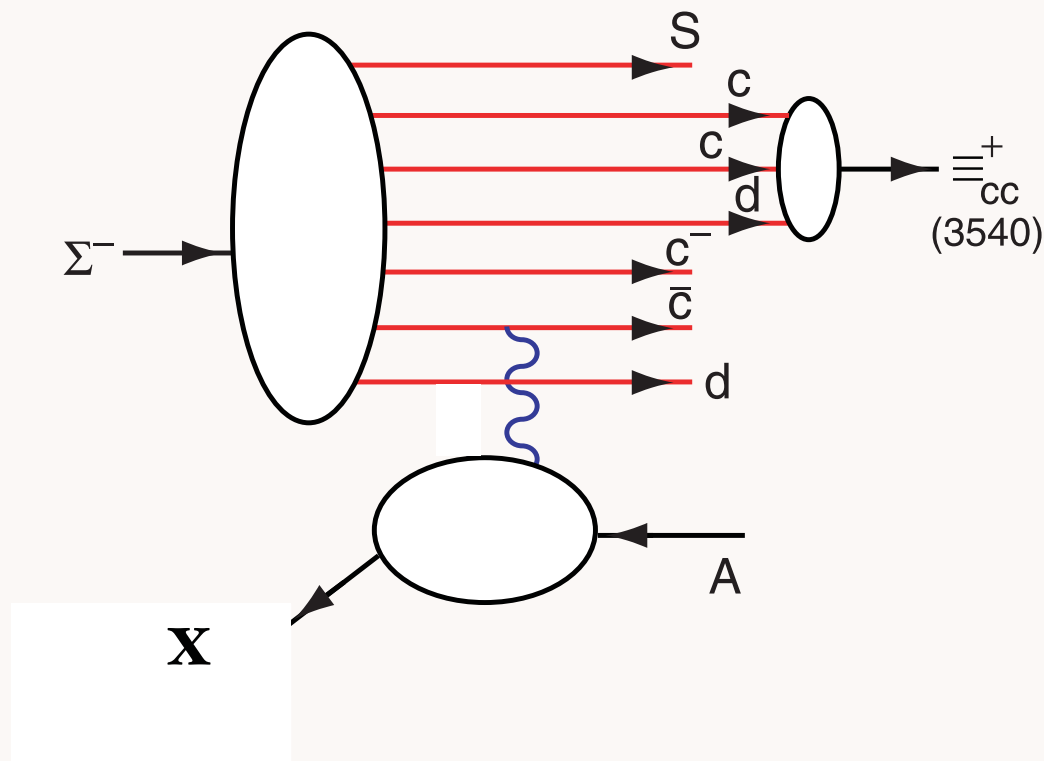
Fig. 3. The  $\psi\psi$  pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of  $J/\psi$ 's from the pairs are shown in (b) and (d). Our calculations are compared with the  $\pi^- N$  data at 150 and 280 GeV/c [1]. The  $x_{\psi\psi}$  distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single  $J/\psi$ 's is twice the number of pairs.

The probability distribution for a general  $n$ -parton intrinsic  $c\bar{c}$  Fock state as a function of  $x$  and  $k_T$  written as

$$\frac{dP_{ic}}{\prod_{i=1}^n dx_i d^2k_{T,i}} = N_n \alpha_s^4 (M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n k_{T,i}) \delta(1 - \sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2},$$

## NA3 Data

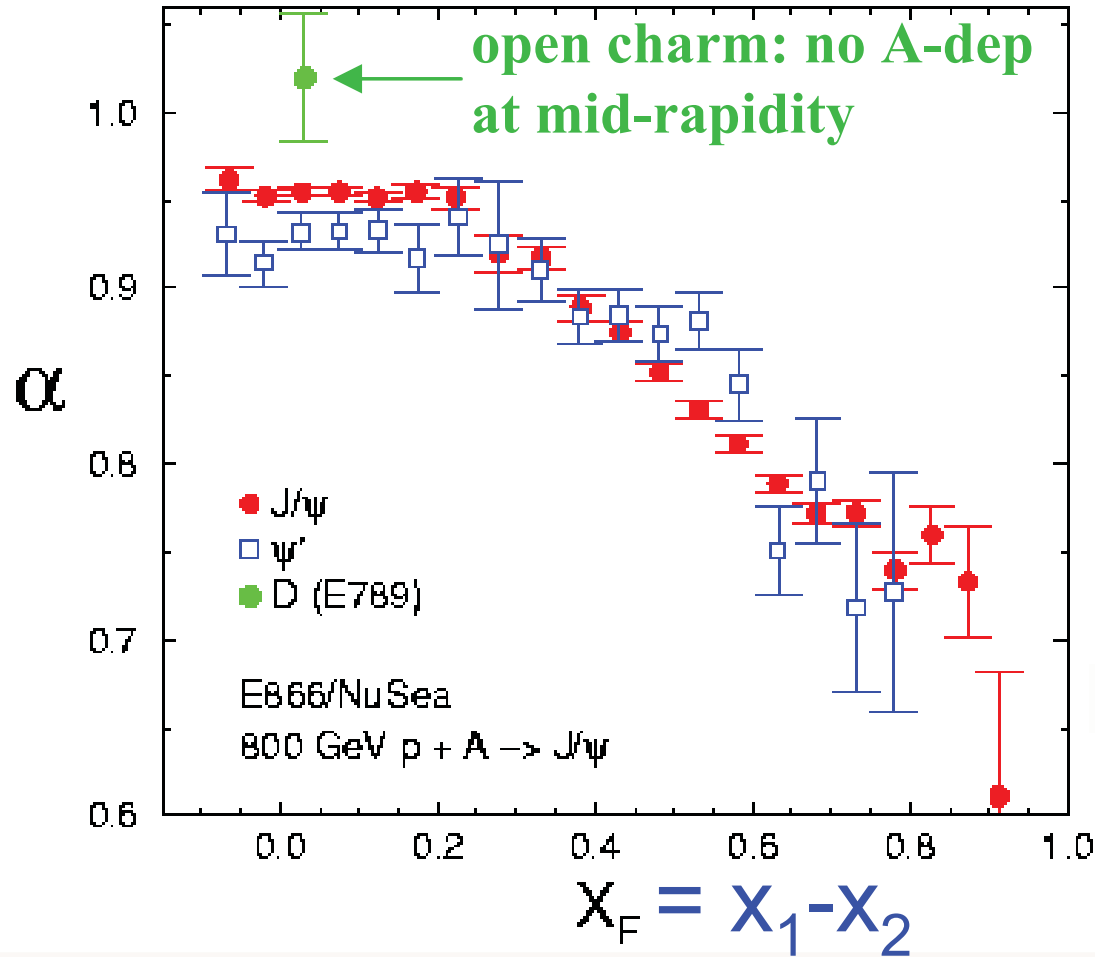




## *Production of a Double-Charm Baryon*

**SELEX high  $x_F$**        $\langle x_F \rangle = 0.33$

800 GeV p-A (FNAL)  $\sigma_A = \sigma_p * A^\alpha$   
*PRL 84, 3256 (2000); PRL 72, 2542 (1994)*



$$\frac{d\sigma}{dx_F} (pA \rightarrow J/\psi X)$$

*Remarkably Strong Nuclear Dependence for Fast Charmonium*

*Violation of PQCD Factorization!*

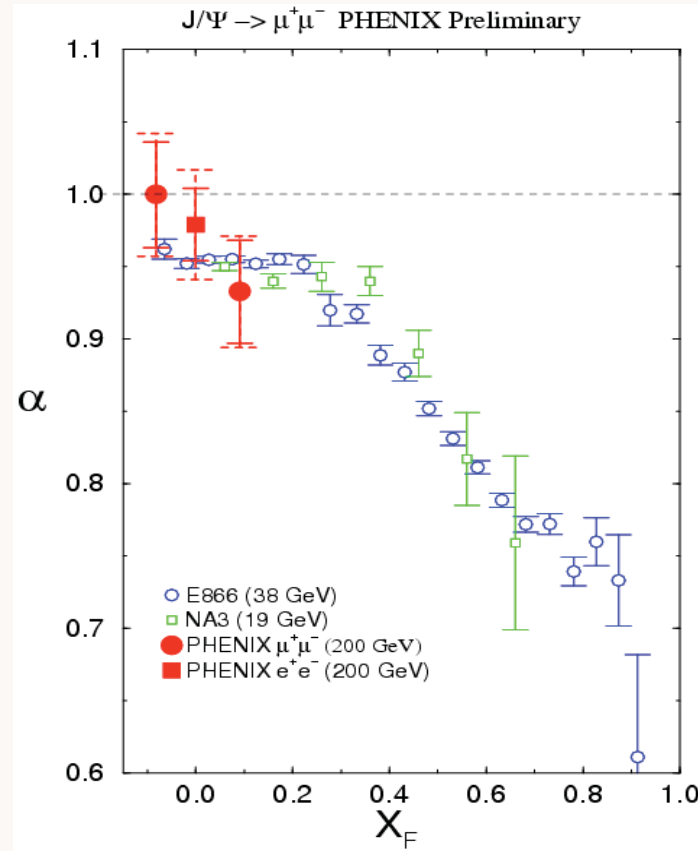
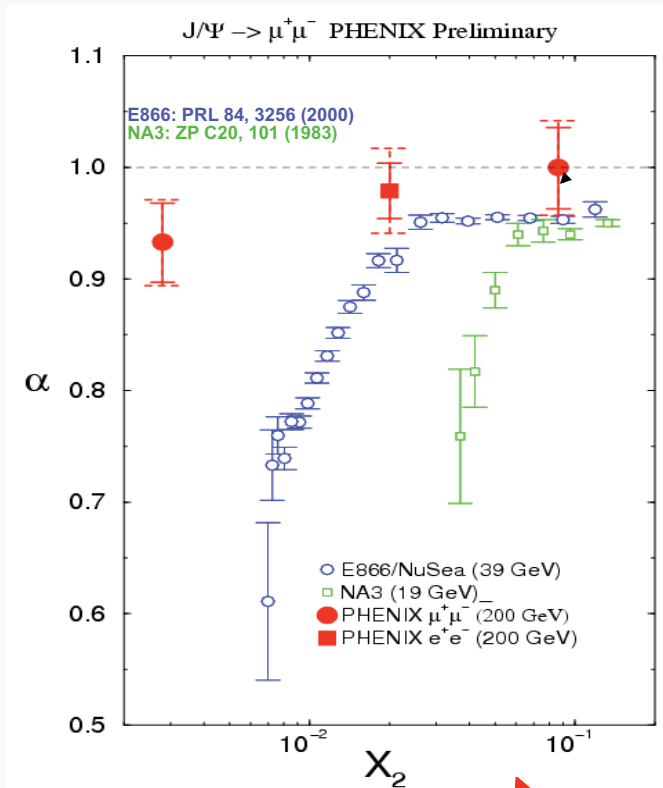
Violation of factorization in charm hadroproduction.

[P. Hoyer](#), [M. Vanttinen \(Helsinki U.\)](#), [U. Sukhatme \(Illinois U., Chicago\)](#). HU-TFT-90-14, May 1990. 7pp.  
 Published in Phys.Lett.B246:217-220,1990

# J/ψ nuclear dependence vrs rapidity, $x_{Au}$ , $x_F$

M. Leitch

## PHENIX compared to lower energy measurements



Huge  
“absorption”  
effect



Klein, Vogt, PRL 91:142301, 2003  
Kopeliovich, NP A696:669, 2001

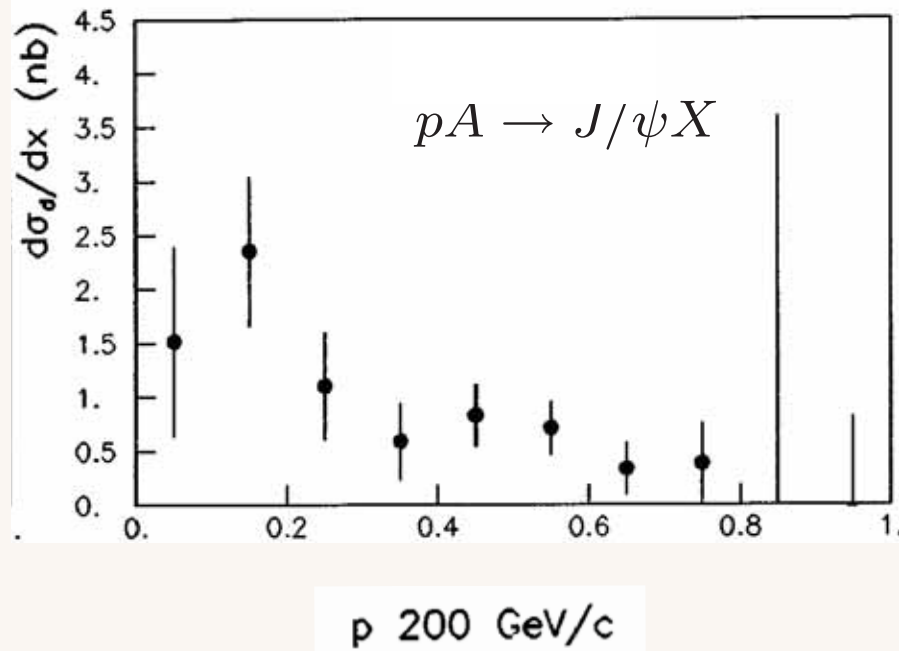
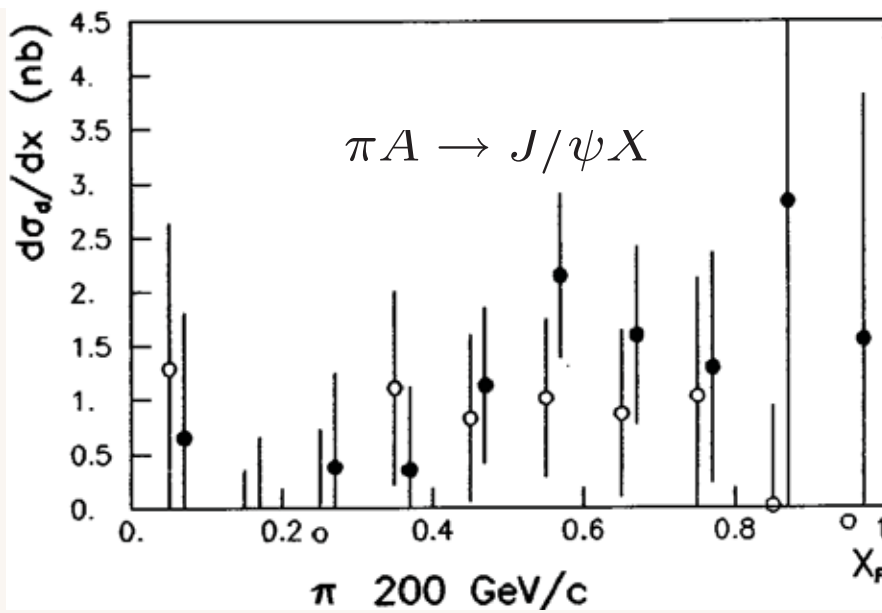
*Violates PQCD  
factorization!*

$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X)$$

Hoyer, Sukhatme, Vanttinen

J. Badier et al, NA3

$A^{2/3}$  component



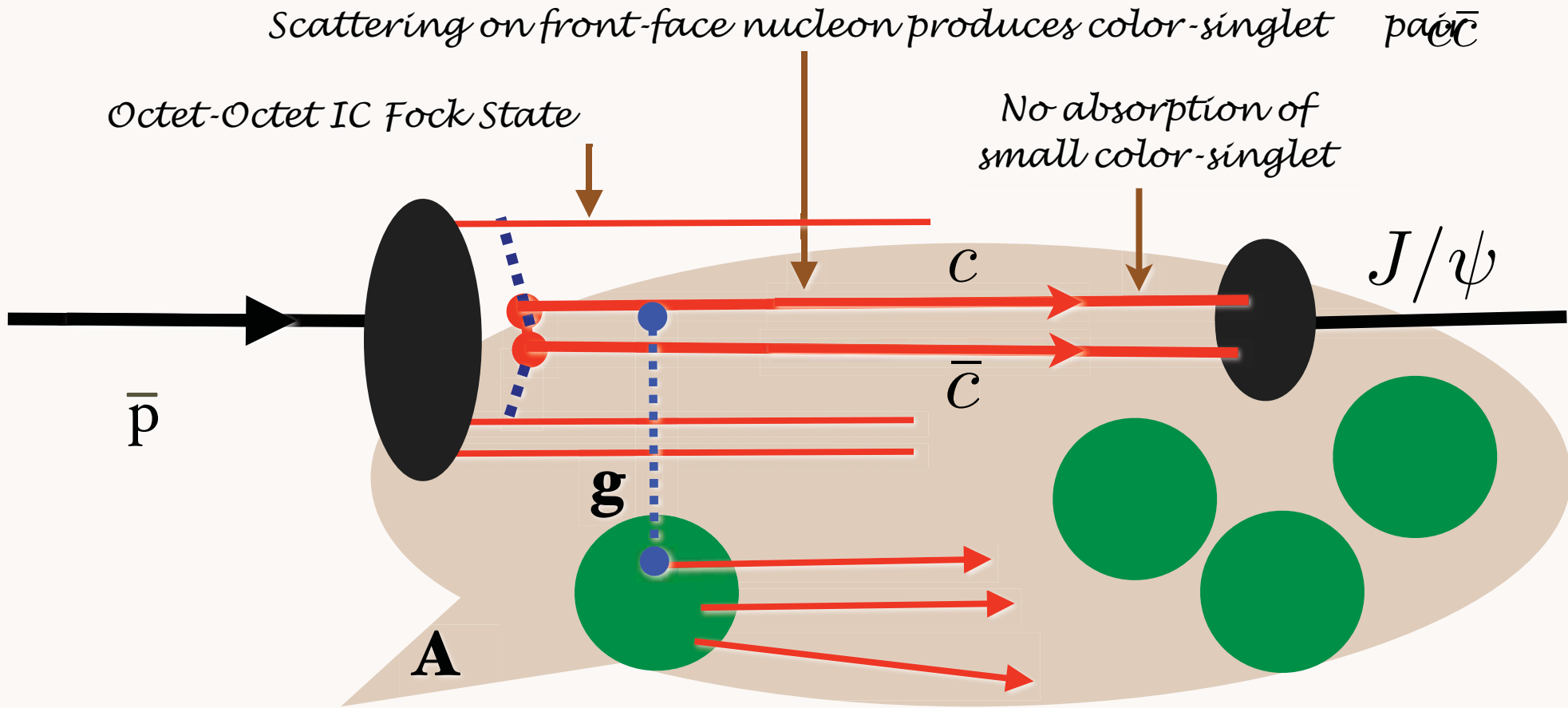
$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$

$\uparrow$   
*g g fusion*

$\uparrow$   
*Intrinsic charm*

**Excess beyond conventional PQCD subprocesses**

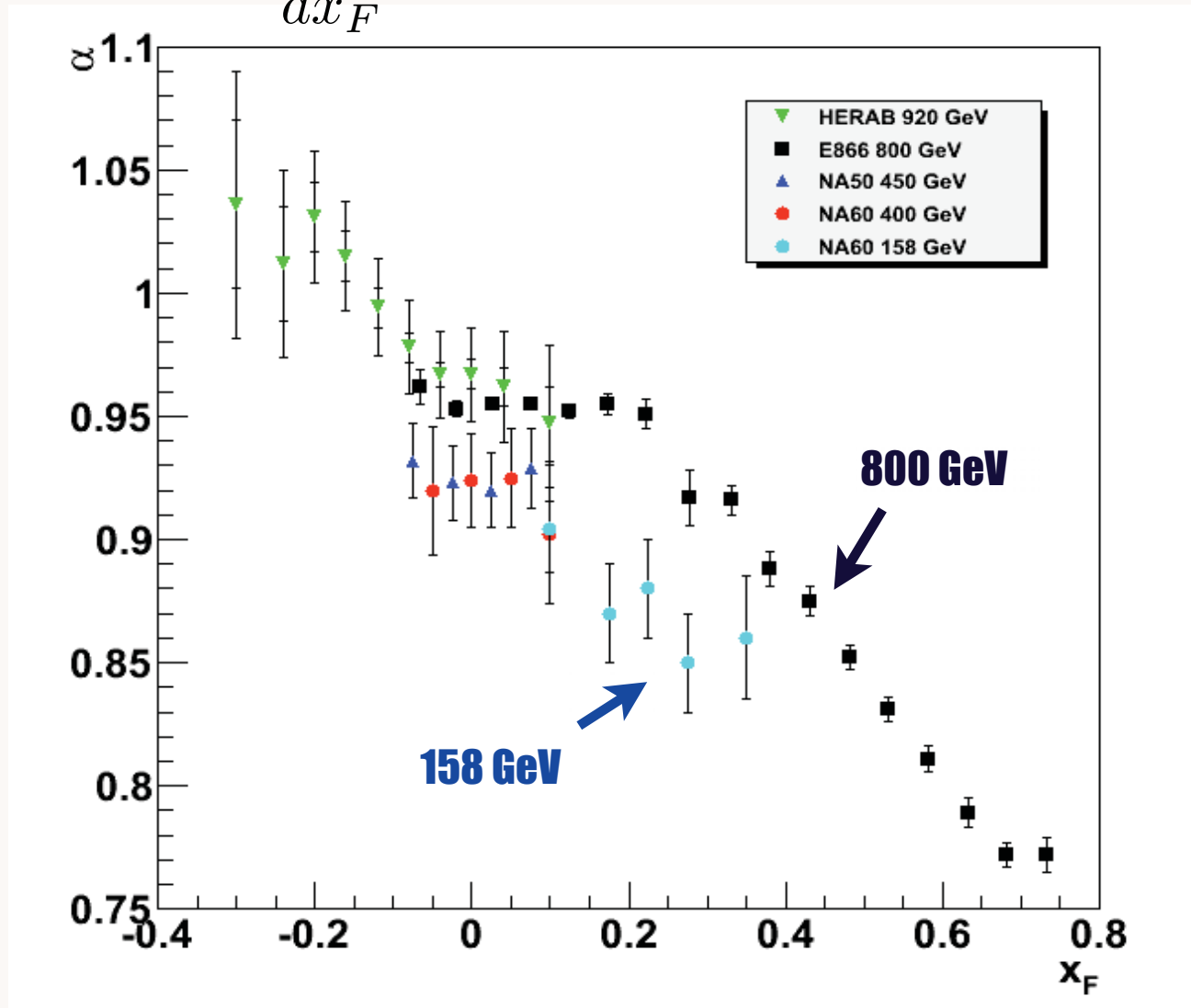
*Color-Opaque IC Fock state  
interacts on nuclear front surface*



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$

# NA60 pA data @ 158GeV

$$\frac{d\sigma}{dx_F} (pA \rightarrow J/\psi X) \propto A^\alpha$$

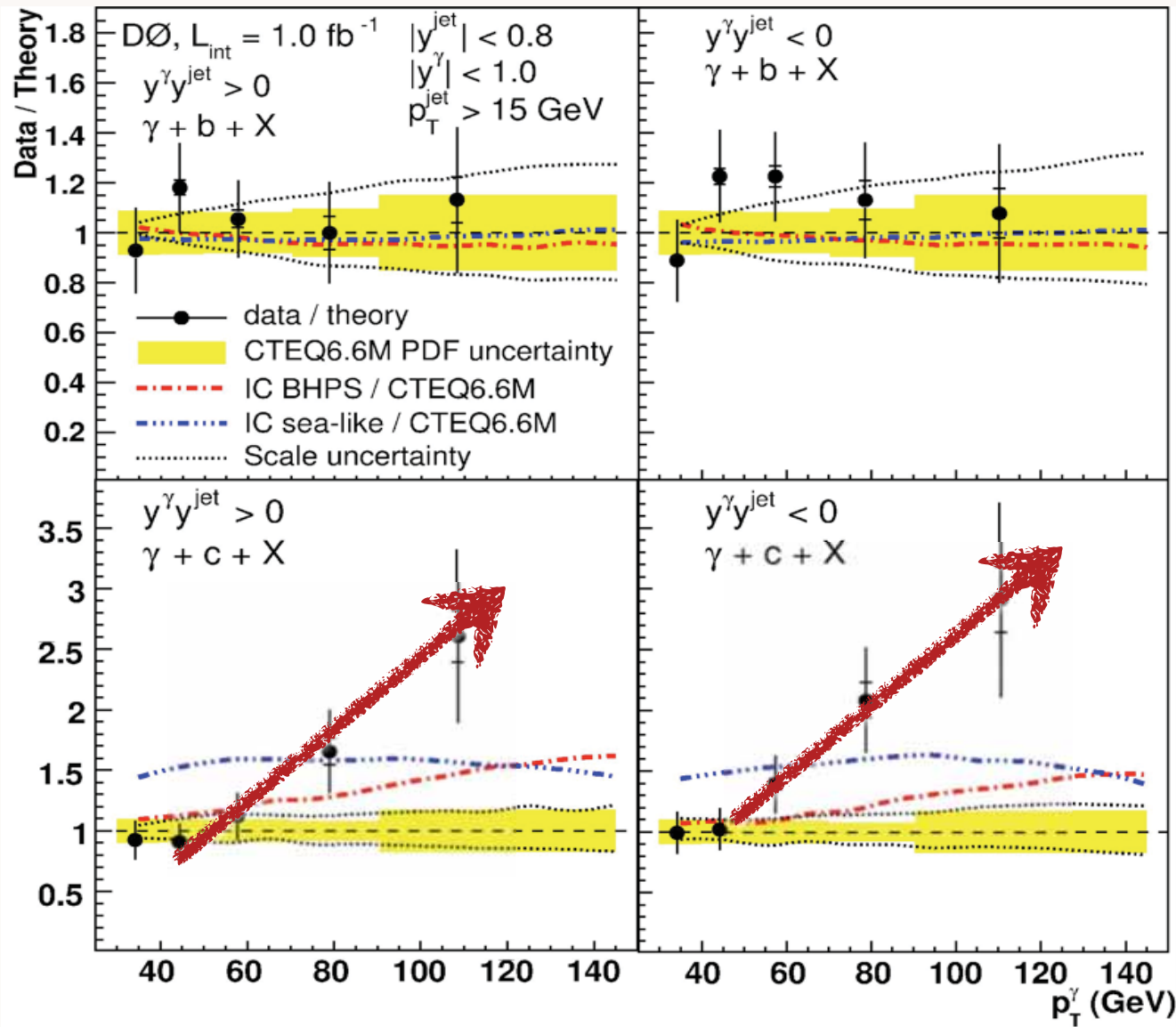


*Clear dependence  
on  $x_F$  and  
beam energy*

- IC Explains Anomalous  $\alpha(x_F)$  not  $\alpha(x_2)$  dependence of  $pA \rightarrow J/\psi X$   
(Mueller, Gunion, Tang, SJB)
- Color Octet IC Explains  $A^{2/3}$  behavior at high  $x_F$  (NA3, Fermilab) *Color Opacity*  
(Kopeliovitch, Schmidt, Soffer, SJB)
- IC Explains  $J/\psi \rightarrow \rho\pi$  puzzle  
(Karliner, SJB)
- IC leads to new effects in  $B$  decay  
(Gardner, SJB)

## Higgs production at $x_F = 0.8$

Measurement of  $\gamma + b + X$  and  $\gamma + c + X$  Production Cross Sections  
in  $p\bar{p}$  Collisions at  $\sqrt{s} = 1.96$  TeV



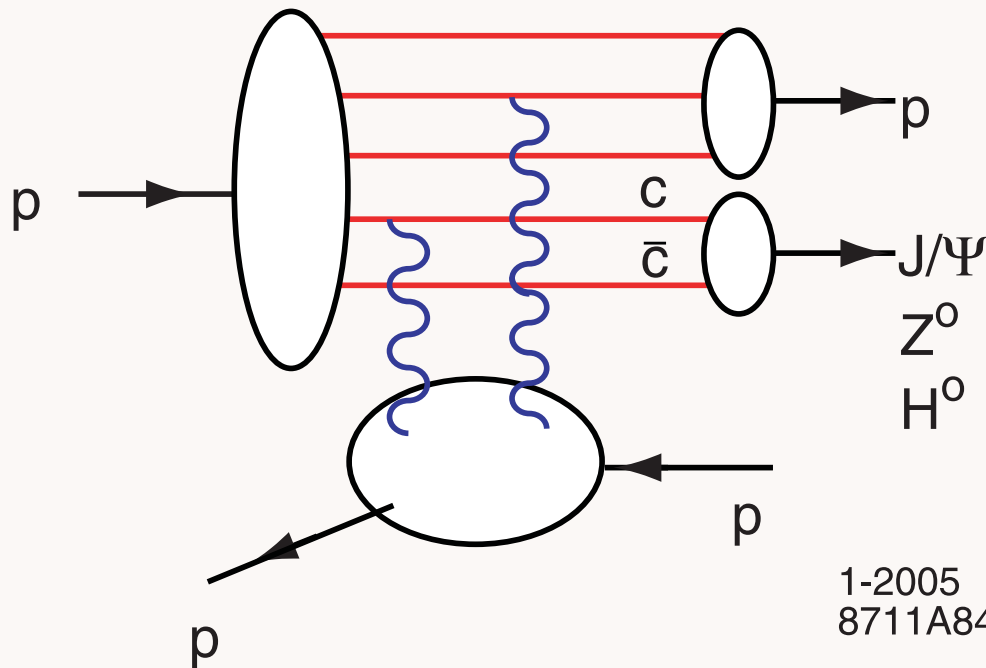
$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

**Ratio  
insensitive to  
gluon PDF,  
scales**

**Signal for  
significant IC  
at  $x > 0.1$ ?**



# Intrinsic Charm Mechanism for Exclusive Diffraction Production



$$p p \rightarrow J/\psi p p$$

$$x_{J/\psi} = x_c + x_{\bar{c}}$$

Exclusive Diffractive  
and Non-Diffractive  
High- $X_F$  Higgs Production

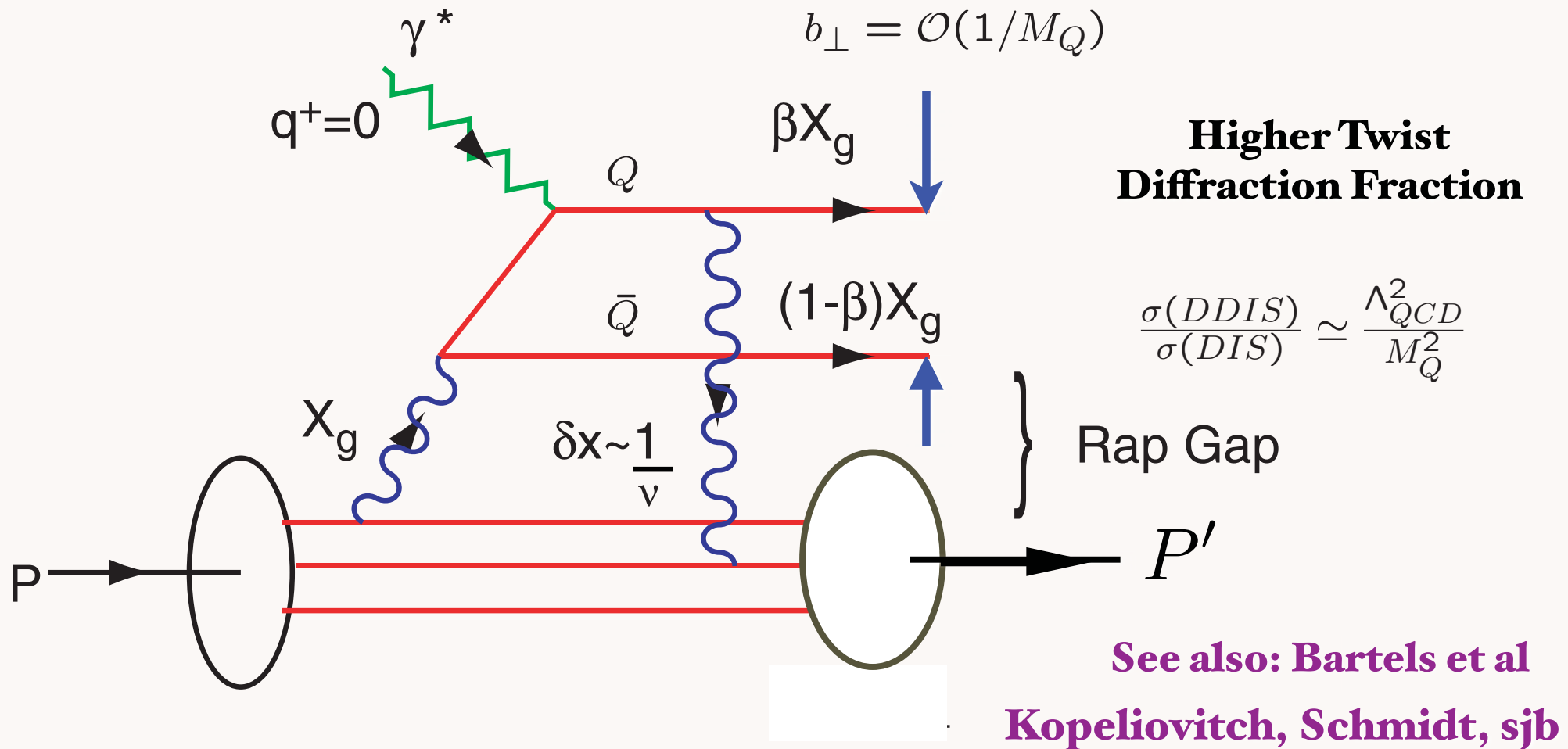
Goldhaber, Kopeliovitch,  
Schmidt, Soffer, sjb

Intrinsic  $c\bar{c}$  pair formed in color octet  $8_C$  in proton wavefunction Large Color Dipole

Collision produces color-singlet  $J/\psi$  through color exchange

RHIC Experiment

# Predict: Reduced DDIS/DIS for Heavy Quarks



**Reproduces lab-frame color dipole approach**

# New QCD production mechanisms for hard processes at large $x$ \*

Stanley J. Brodsky and Paul Hoyer \*\*

*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*

A.H. Mueller \*\*\* and Wai-Keung Tang \*\*\*

*Department of Physics, Columbia University, New York, NY 10027, USA*

Received 12 August 1991

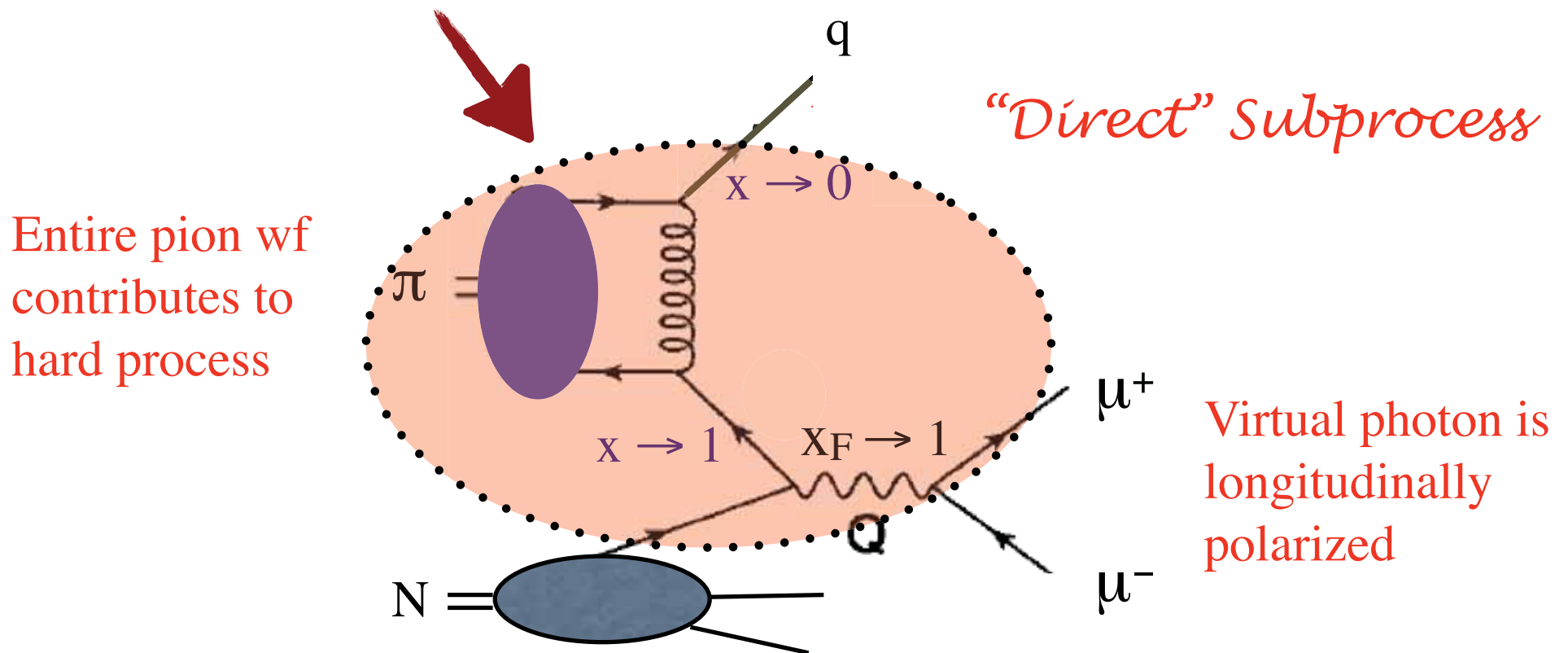
Accepted for publication 24 September 1991

We study the QCD production mechanisms for lepton and quark pairs of high mass  $M$  that carry a large fraction  $x$  of the projectile momentum. We show that the dominant contribution comes from peripheral processes in which low- $x$  spectator quarks interact with the target, with the hardness scale of the collision being given by  $Q^2 = M^2(1-x)$ . In the high- $M^2$  limit with fixed  $M^2(1-x)$  we identify new leading-order perturbative contributions from the hadron wave function which involve more than one constituent. These “intrinsic” contributions cannot be expressed in terms of the usual single-parton structure functions, implying a breakdown of QCD factorization. In a numerical study of a simple gauge theory model, we show that such contributions can dominate the standard single-parton factorizable terms. These results appear to explain several anomalies seen in the data: the excess production and the anomalous nuclear-number of dependence of open and bound charm at large  $x$ ; the “cumulative” effects ( $x > 1$ ) observed in hadron production from nuclei; and the large target-polarization asymmetry observed for hadron production at high  $x$ . The intrinsic multi-parton processes provide new mechanisms for the hadro-production of the  $J/\psi$  directly in a color-singlet state and also for the production of heavy flavor systems in lepto-production at high momentum.

$$\pi N \rightarrow \mu^+ \mu^- X \text{ at high } x_F$$

In the limit where  $(1-x_F)Q^2$  is fixed as  $Q^2 \rightarrow \infty$

*Light-Front Wavefunctions from AdS/CFT*



Berger, sjb

Hoyer, Mueller, Tang, sjb

Khoze, Brandenburg, Muller, sjb

$$\pi^- N \rightarrow \mu^+ \mu^- X \text{ at } 80 \text{ GeV}/c$$

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

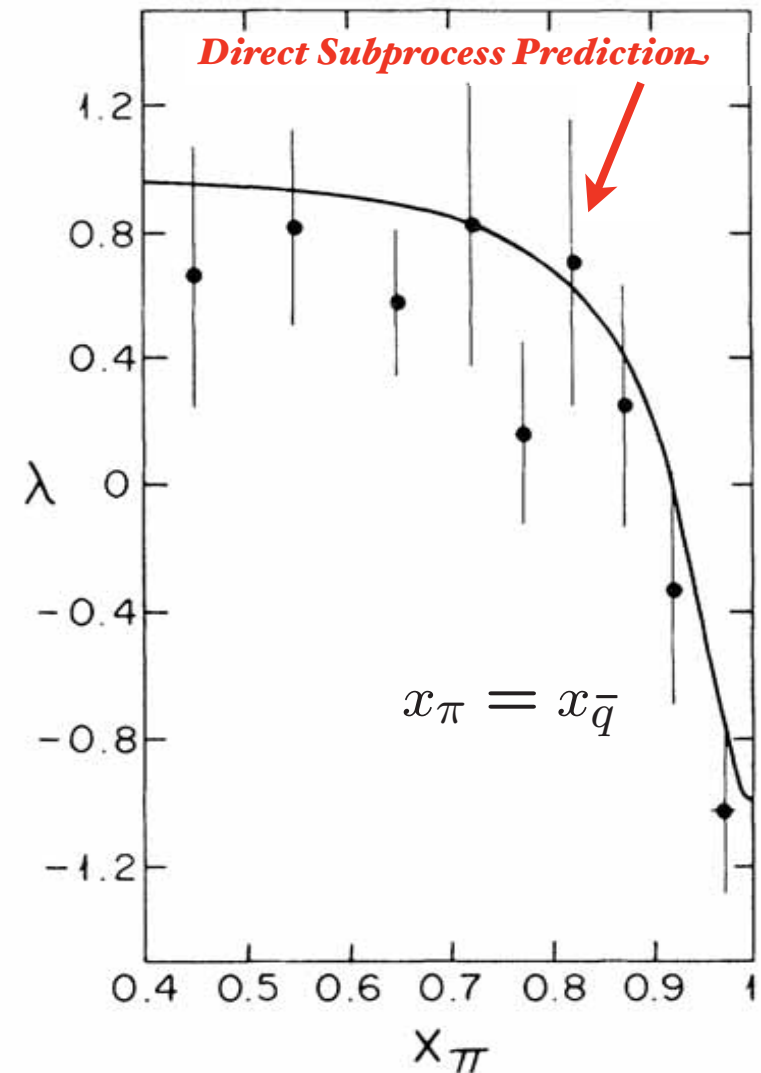
$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[ (1-x_\pi)^2 (1 + \cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \text{ GeV}^2/c^2$$

$$Q^2 = M^2$$

*Dramatic change in angular distribution at large  $x_f$*

**Example of a higher-twist direct subprocess**



Chicago-Princeton  
Collaboration

Phys.Rev.Lett.55:2649,1985

## USING NUCLEI TO PROBE HADRONIZATION IN QCD

Stanley J. BRODSKY<sup>1,a,b</sup> and A.H. MUELLER<sup>2,c,b</sup>

<sup>a</sup> *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305, USA*

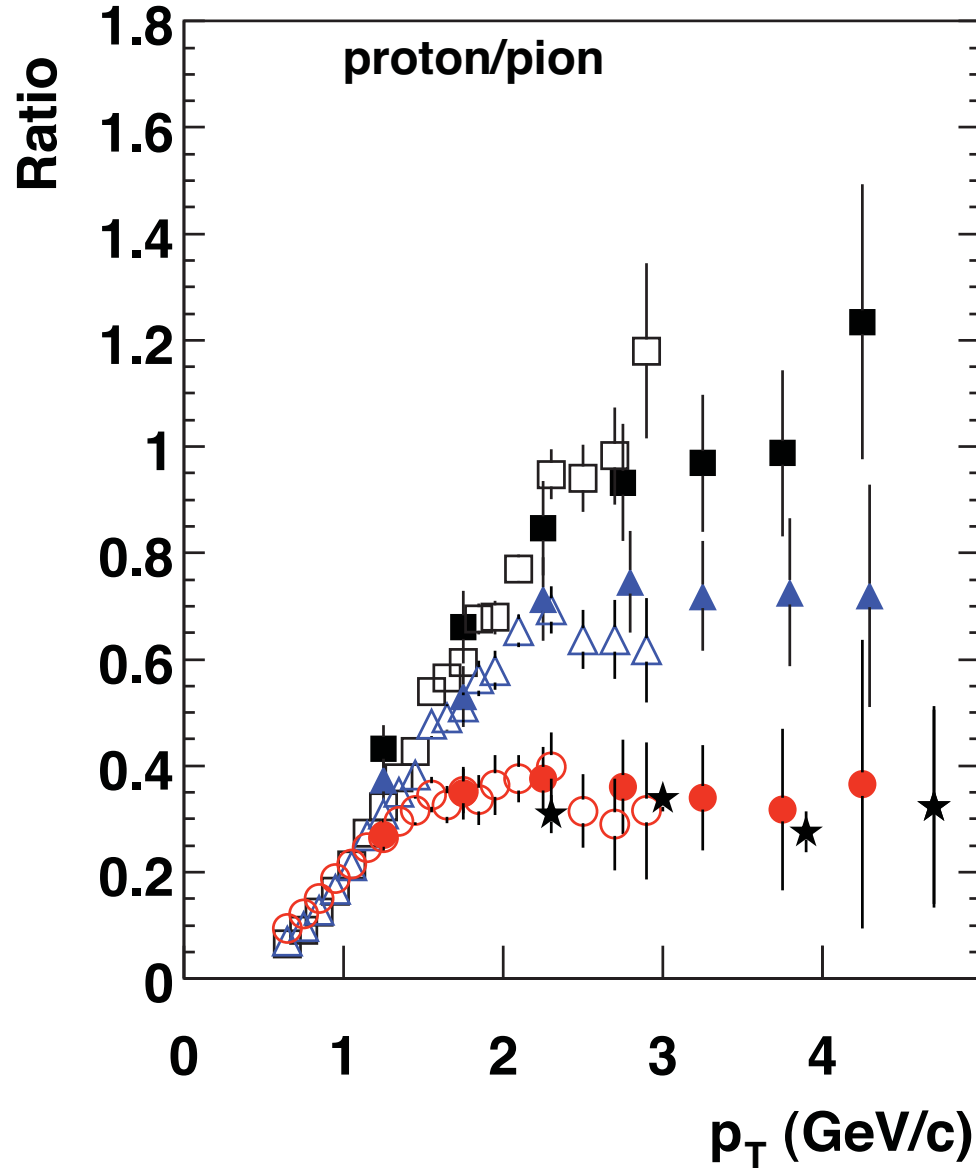
<sup>b</sup> *Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*

<sup>c</sup> *Physics Department, Columbia University, New York, NY 10027, USA*

Received 15 March 1988

The behavior of quasi-exclusive and inclusive  $\rho$  and  $J/\psi$  photoproduction, electroproduction and hadroproduction in nuclei are discussed for small and large  $p_{\perp}$ . In particular we argue that  $J/\psi$  production in ion-ion collisions is likely to be suppressed relative to the background lepton pair production, independent of whether or not a QCD plasma is formed. We point out that previous extractions of the  $J/\psi$  inelastic cross section do not actually measure the cross section for the interaction of physical  $J/\psi$ 's with nucleons.

*Baryon Anomaly: Particle ratio changes with centrality!*



*Protons less absorbed in nuclear collisions than pions*

← Central

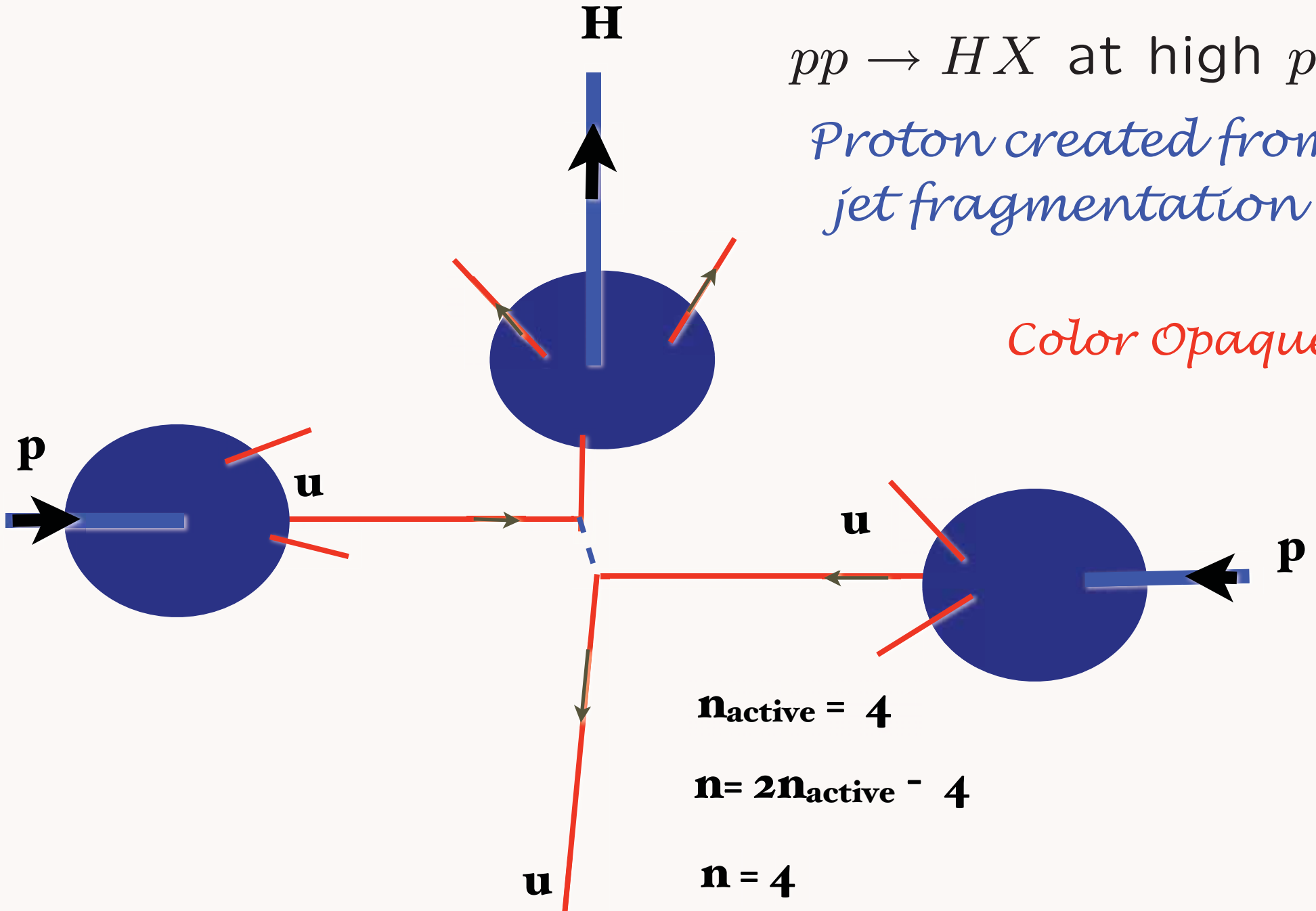
- ■ Au+Au 0-10%
- △ ▲ Au+Au 20-30%
- ● Au+Au 60-92%
- ★ p+p,  $\sqrt{s} = 53$  GeV, ISR
- e<sup>+</sup>e<sup>-</sup>, gluon jets, DELPHI
- ..... e<sup>+</sup>e<sup>-</sup>, quark jets, DELPHI

← Peripheral

Sickles, sjb

$pp \rightarrow HX$  at high  $p_T$   
*Proton created from  
jet fragmentation*

*Color Opaque*



$$n_{\text{active}} = 4$$

$$n = 2n_{\text{active}} - 4$$

$$n = 4$$

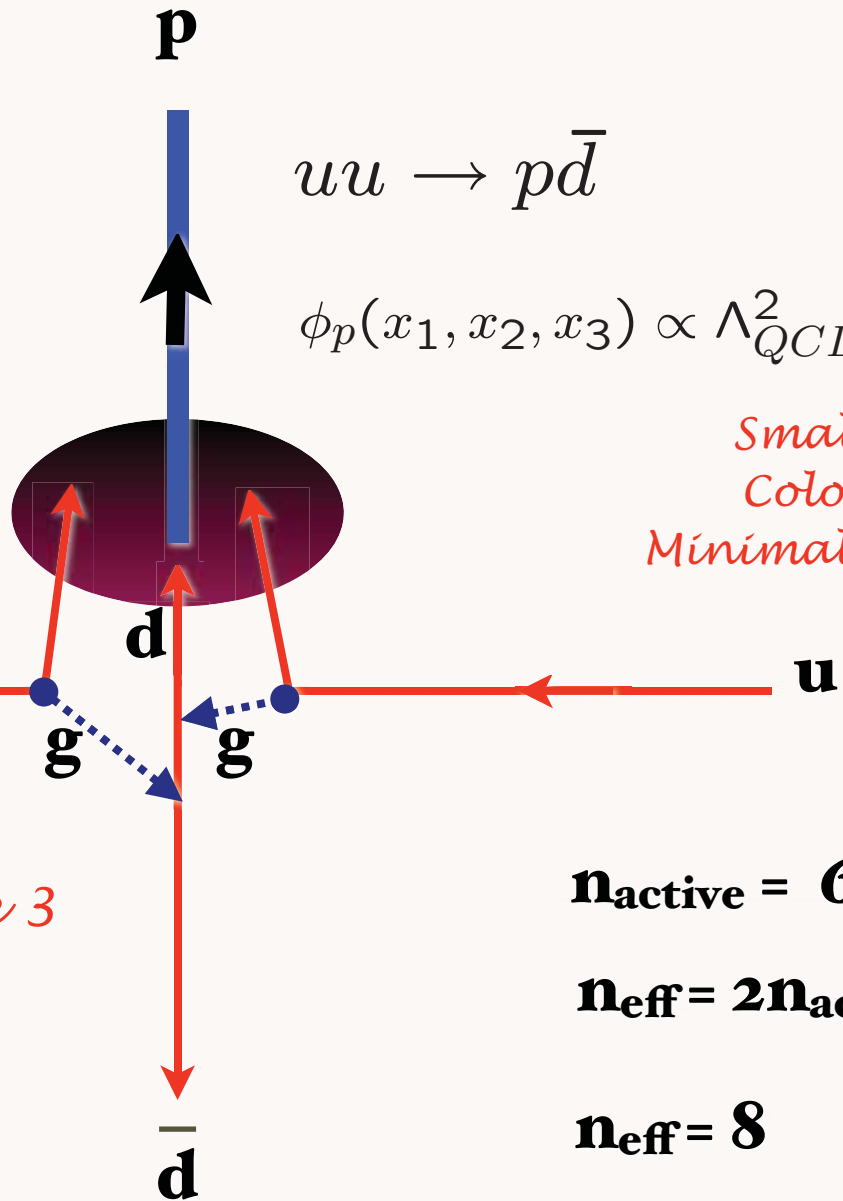


*Baryon can be made directly within hard subprocess*

**Coalescence  
within hard  
subprocess**

$$b_{\perp} \simeq 1/p_T$$

*Collision can produce 3  
collinear quarks*



$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$

Bjorken  
Blankenbecler, Gunion, sjb  
Berger, sjb  
Hoyer, et al: Semi-Exclusive

Sickles, sjb

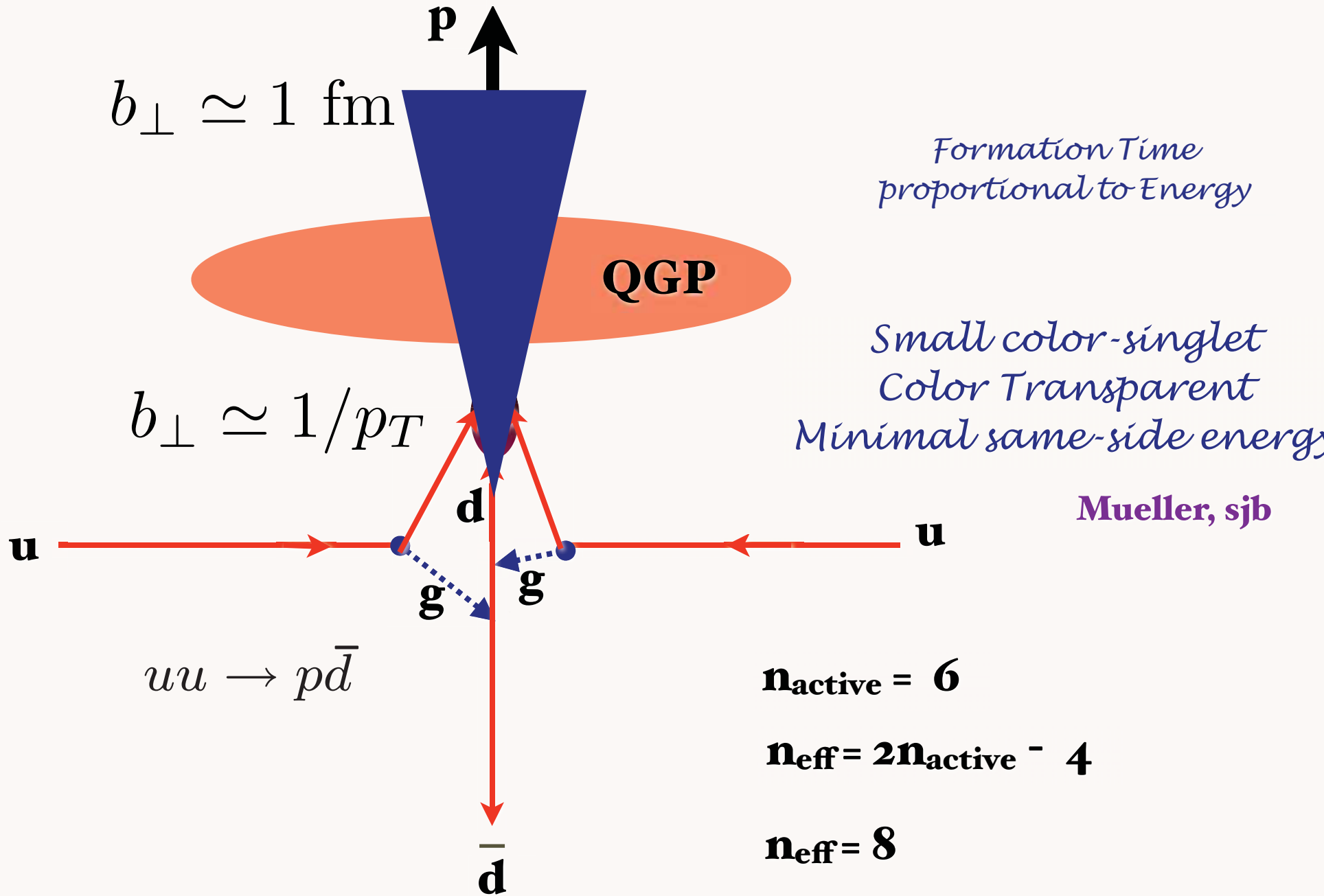
*Small color-singlet  
Color Transparent  
Minimal same-side energy*

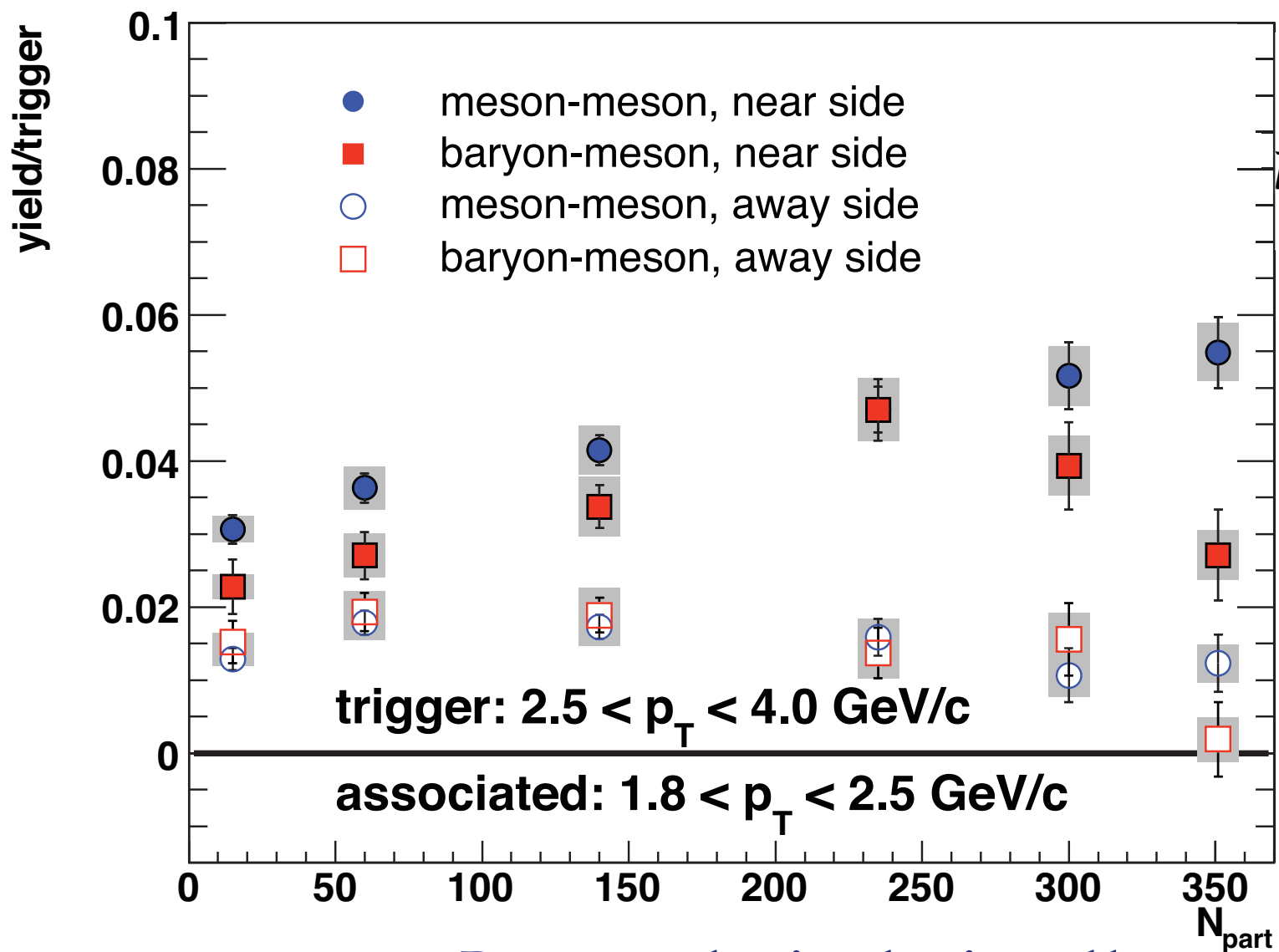
$$n_{\text{active}} = 6$$

$$n_{\text{eff}} = 2n_{\text{active}} - 4$$

$$n_{\text{eff}} = 8$$

*Baryon made directly within hard subprocess*





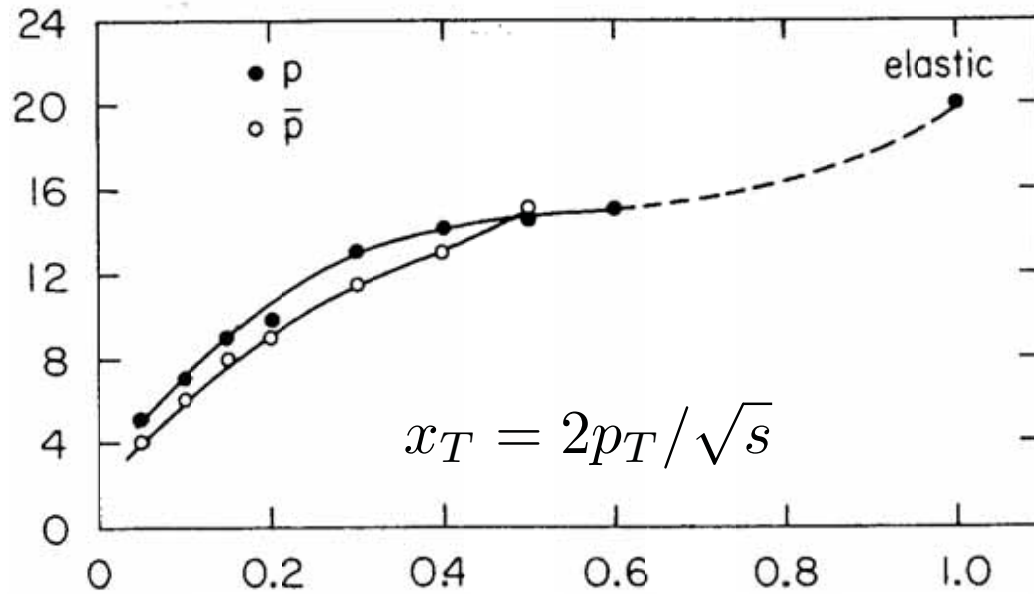
proton trigger:  
# same-side  
particles *decreases*  
with centrality



**Proton production dominated by  
color-transparent direct high- $n_{\text{eff}}$  subprocesses**

$$E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^{n_{eff}}}$$

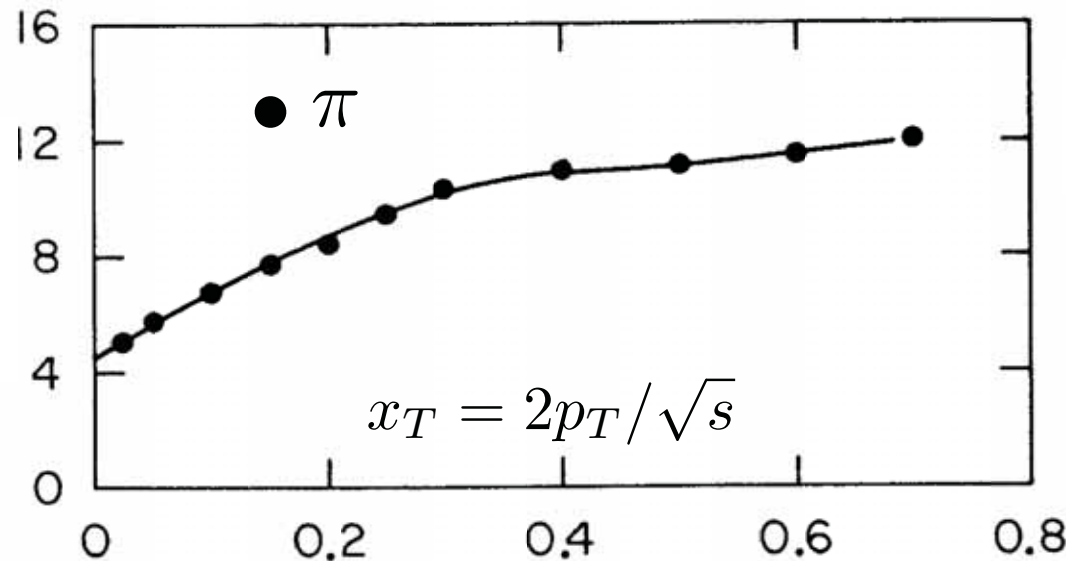
$n_{eff}$



*Clear evidence for higher-twist contributions*

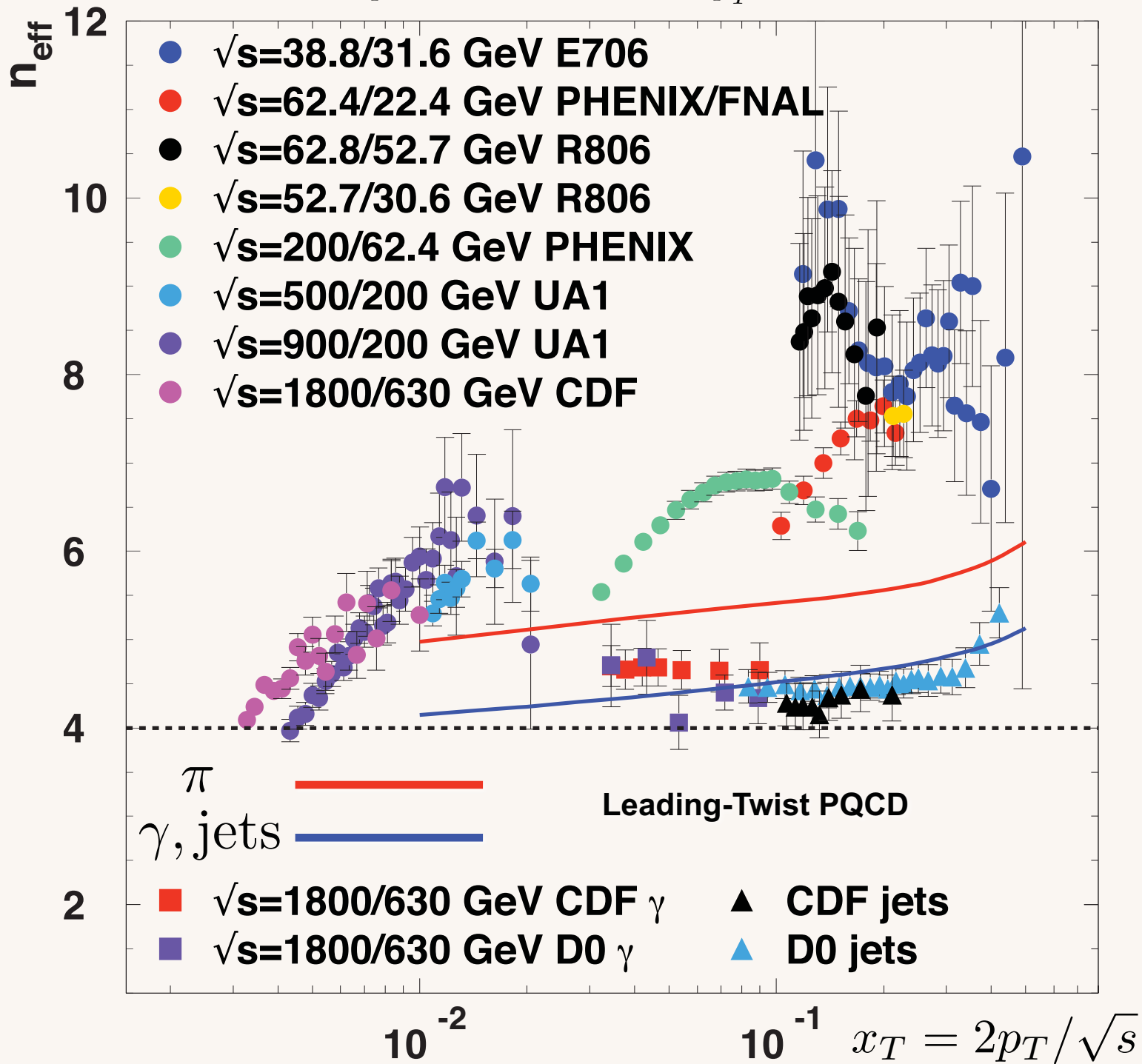
**Fermilab, ISR data**

$n_{eff}$

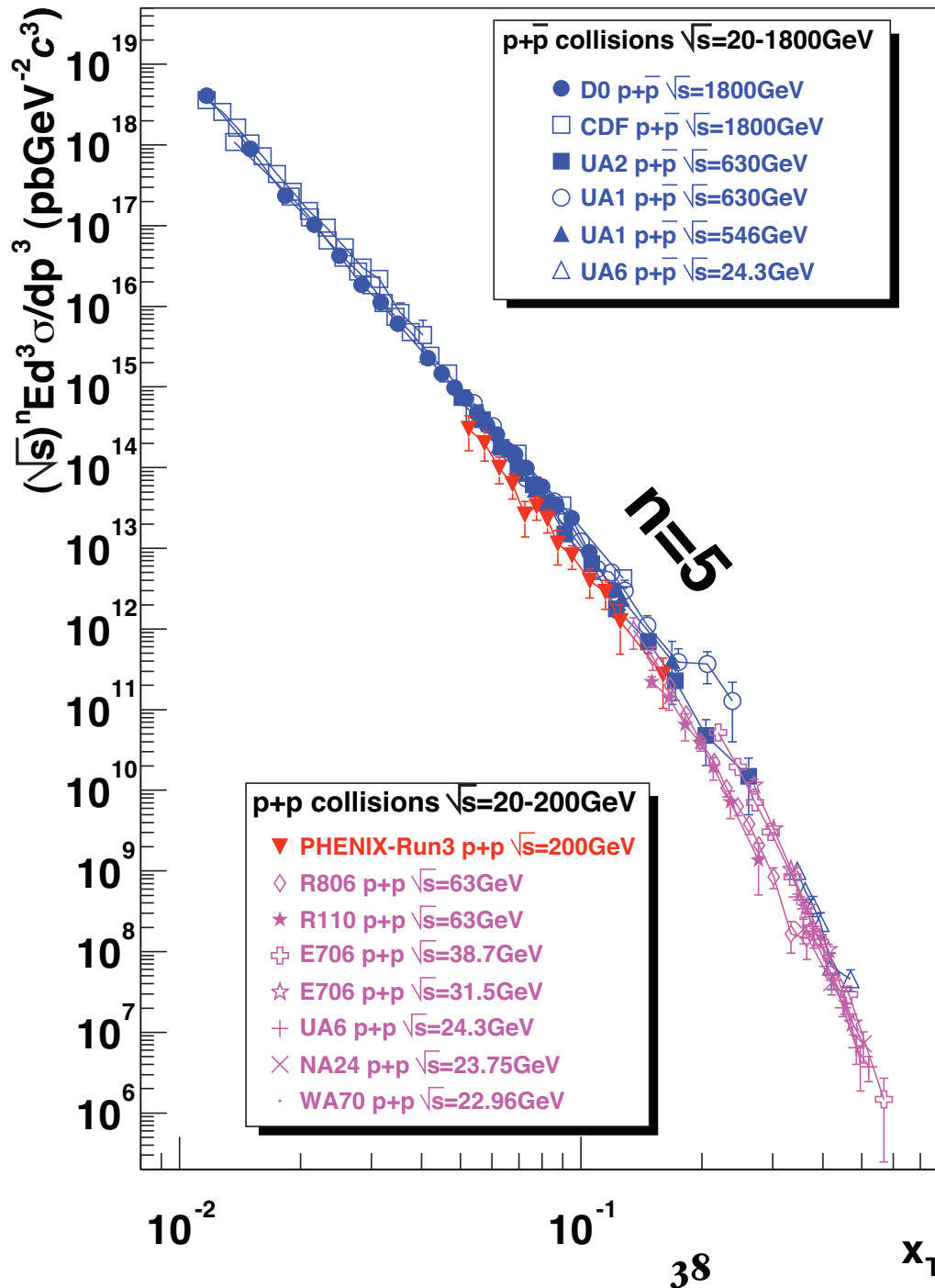


*Continuous Rise of  $n_{eff}$*

$$E \frac{d\sigma}{d^3p}(pp \rightarrow HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\text{eff}}}}$$



$$\sqrt{s}^n E \frac{d\sigma}{d^3p} (pp \rightarrow \gamma X) \text{ at fixed } x_T$$

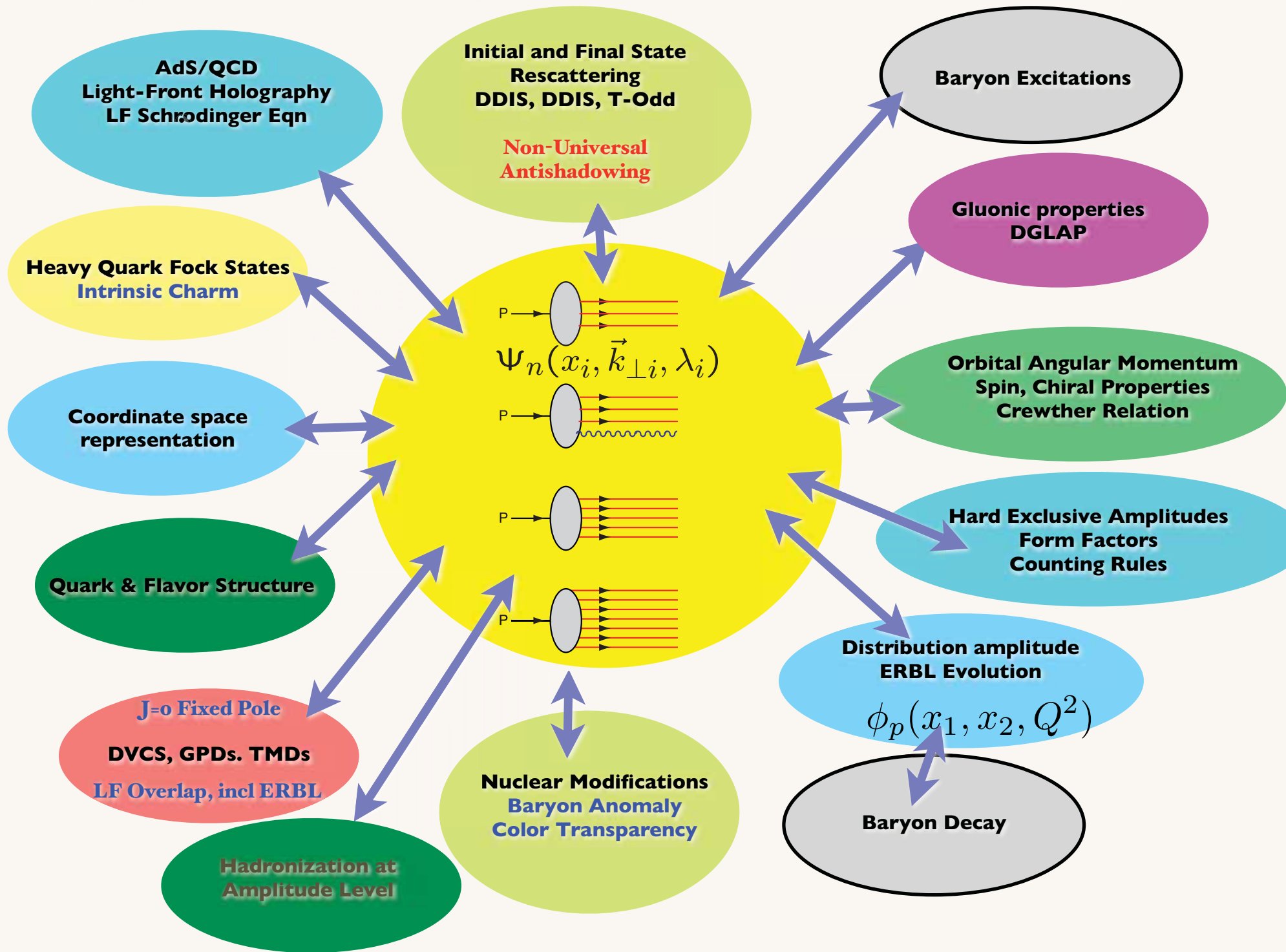


**Scaling of direct  
photon production  
consistent with  
PQCD**

# *Light-Front QCD Features and Phenomenology*

- **Hidden color, Intrinsic glue, sea, Color Transparency**
- **Physics of spin, orbital angular momentum**
- **Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints**
- **Vanishing anomalous gravitomagnetic moment**
- **Relation between edm and anomalous magnetic moment**
- **Cluster Decomposition Theorem for relativistic systems**
- **OPE: DGLAP, ERBL evolution; invariant mass scheme**

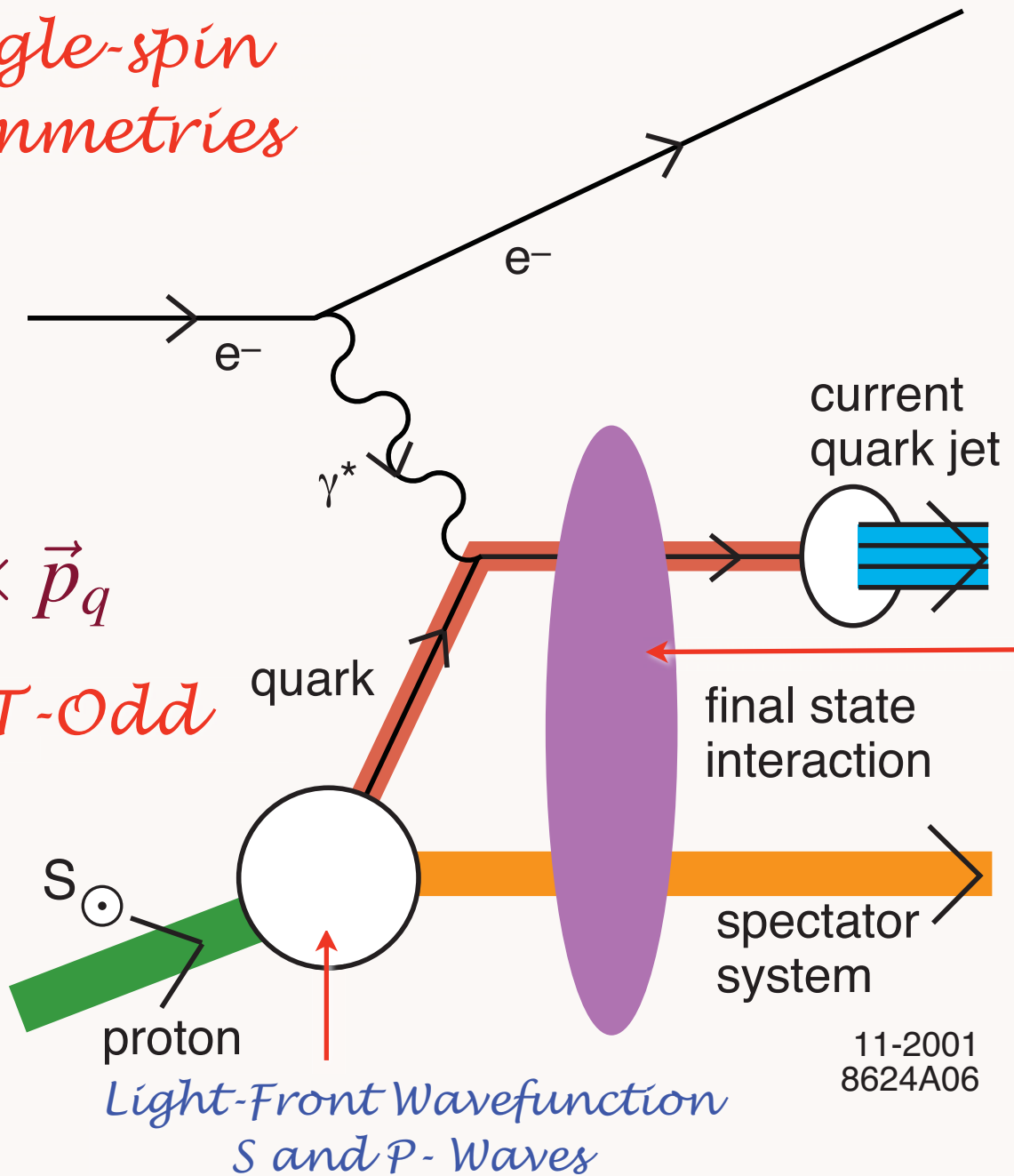
# QCD and the LF Hadron Wavefunctions





*Single-spin asymmetries*

# Leading Twist Sivers Effect



$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

*Pseudo-T-Odd*

*Light-Front Wavefunction  
S and P-Waves*

current quark jet

final state interaction

spectator system

11-2001  
8624A06

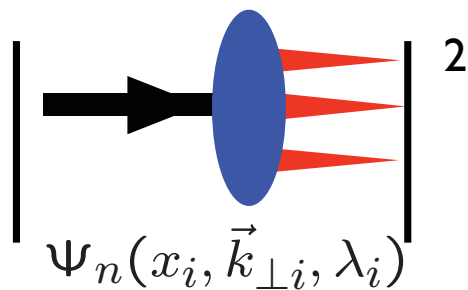
Hwang,  
Schmidt, sjb

Collins, Burkardt  
Ji, Yuan

*QCD S- and P-  
Coulomb Phases  
--Wilson Line*

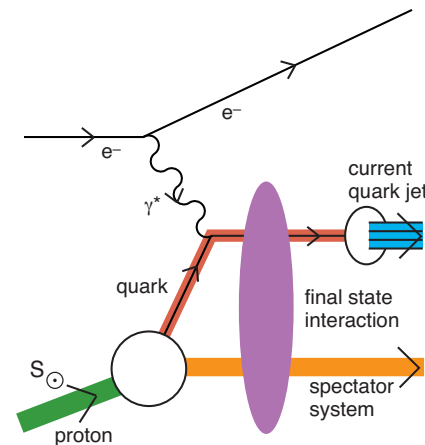
# Static

- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and  $J^z$
- DGLAP Evolution; mod. at large  $x$
- No Diffractive DIS



# Dynamic

- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
  - No Probabilistic Interpretation
  - Process-Dependent - From Collision
  - T-Odd (Sivers, Boer-Mulders, etc.)
  - Shadowing, Anti-Shadowing, Saturation
  - Sum Rules Not Proven
  - DGLAP Evolution
  - Hard Pomeron and Odderon Diffractive DIS



# Double Initial-State Interactions

generate anomalous  $\cos 2\phi$

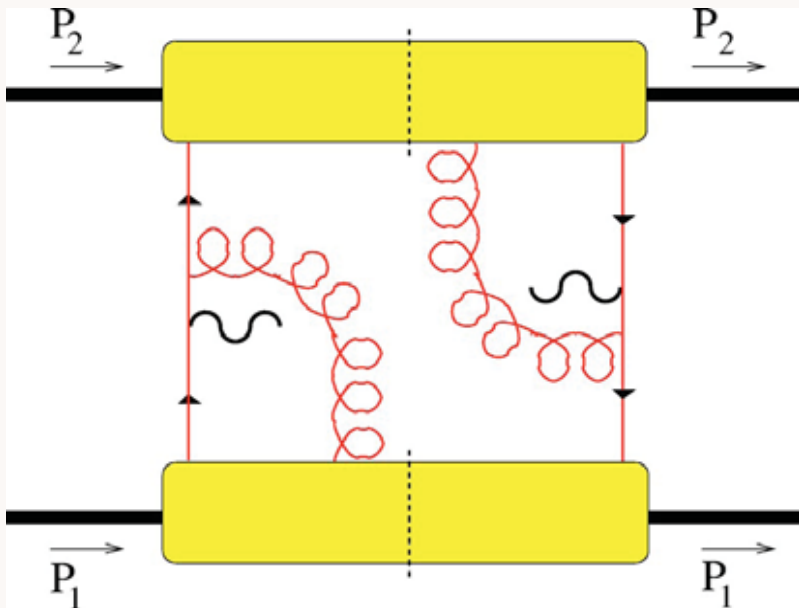
Boer, Hwang, sjb

## Drell-Yan planar correlations

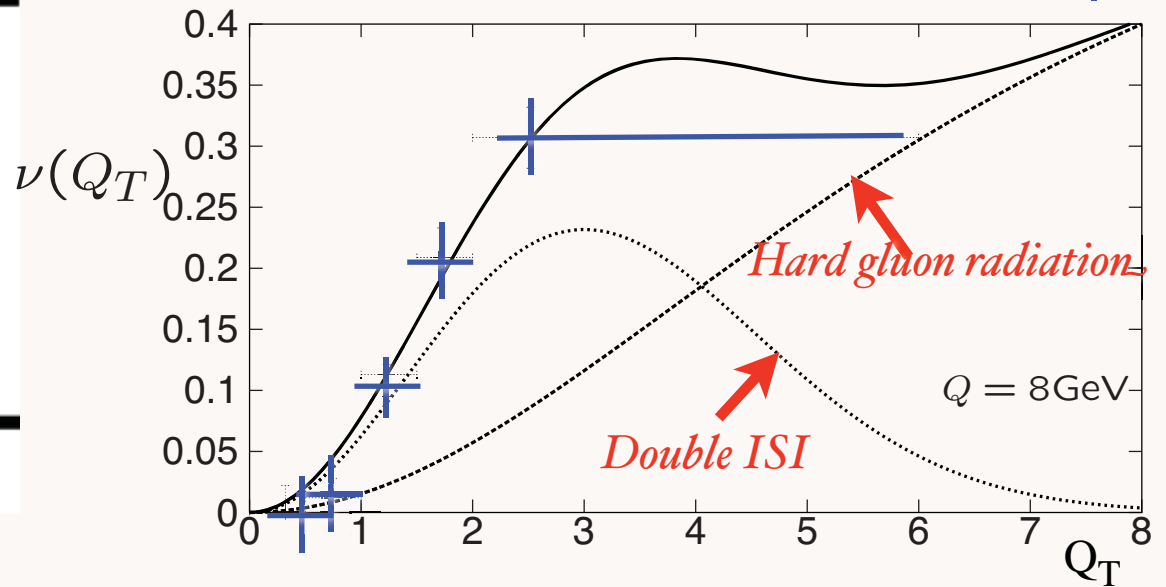
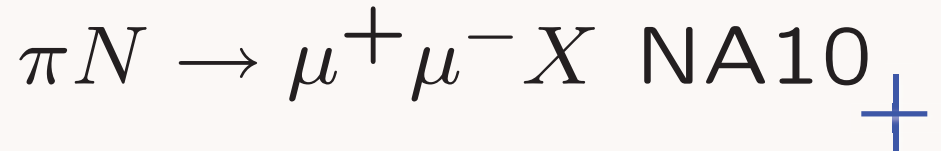
$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)$$

PQCD Factorization (Lam Tung):  $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^\perp(\pi) h_1^\perp(N)$$

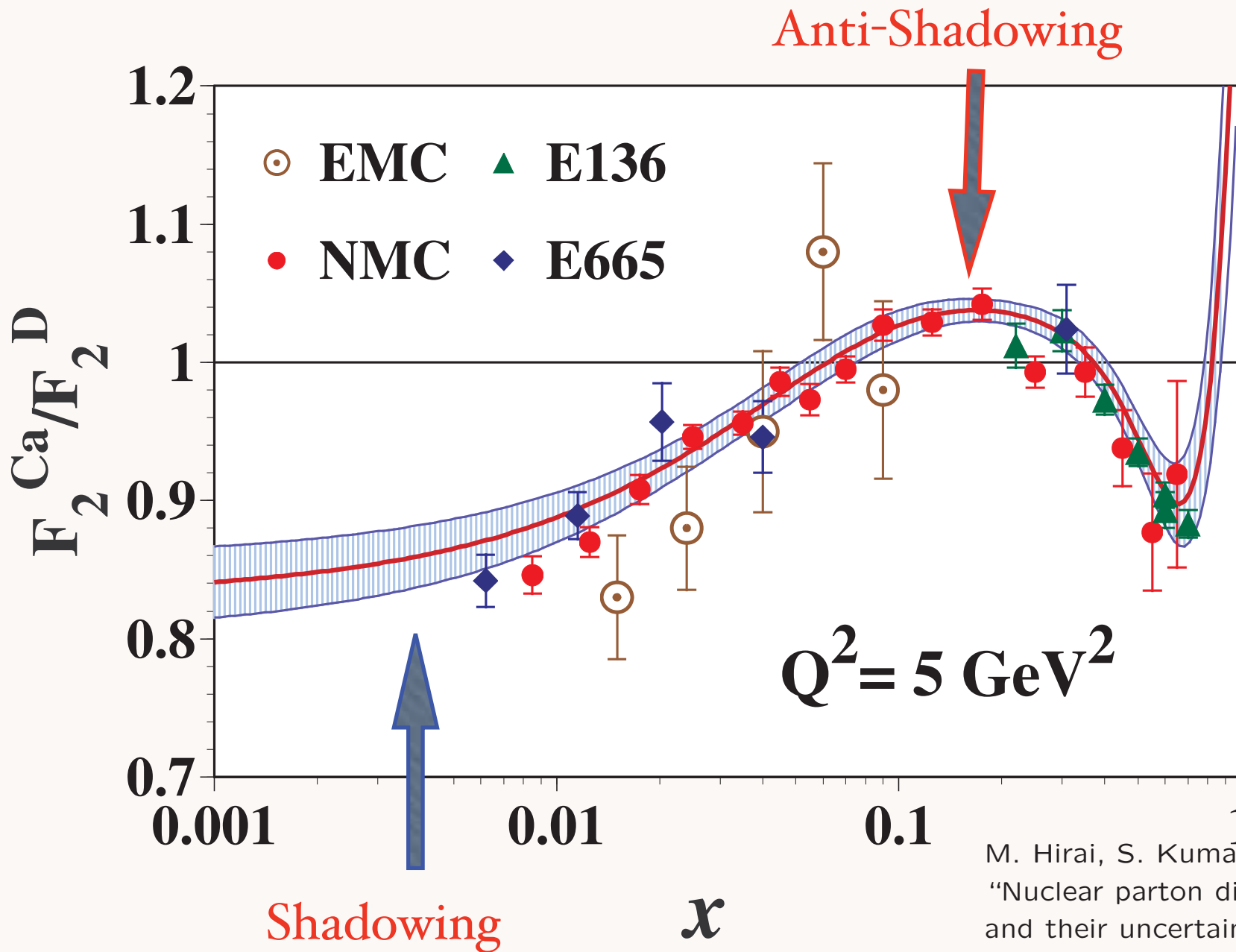


**Violates Lam-Tung relation!**



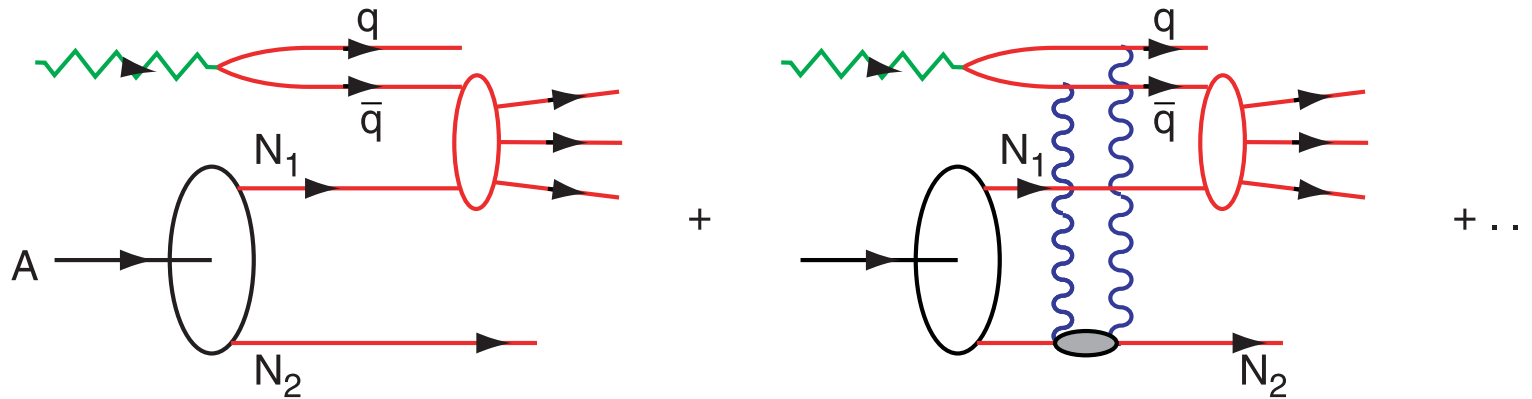
Model: Boer,

**Stan Brodsky**  
SLAC



M. Hirai, S. Kumano and T. H. Nagai,  
 "Nuclear parton distribution functions  
 and their uncertainties,"  
 Phys. Rev. C **70**, 044905 (2004)  
 [arXiv:hep-ph/0404093].

# Nuclear Shadowing in QCD



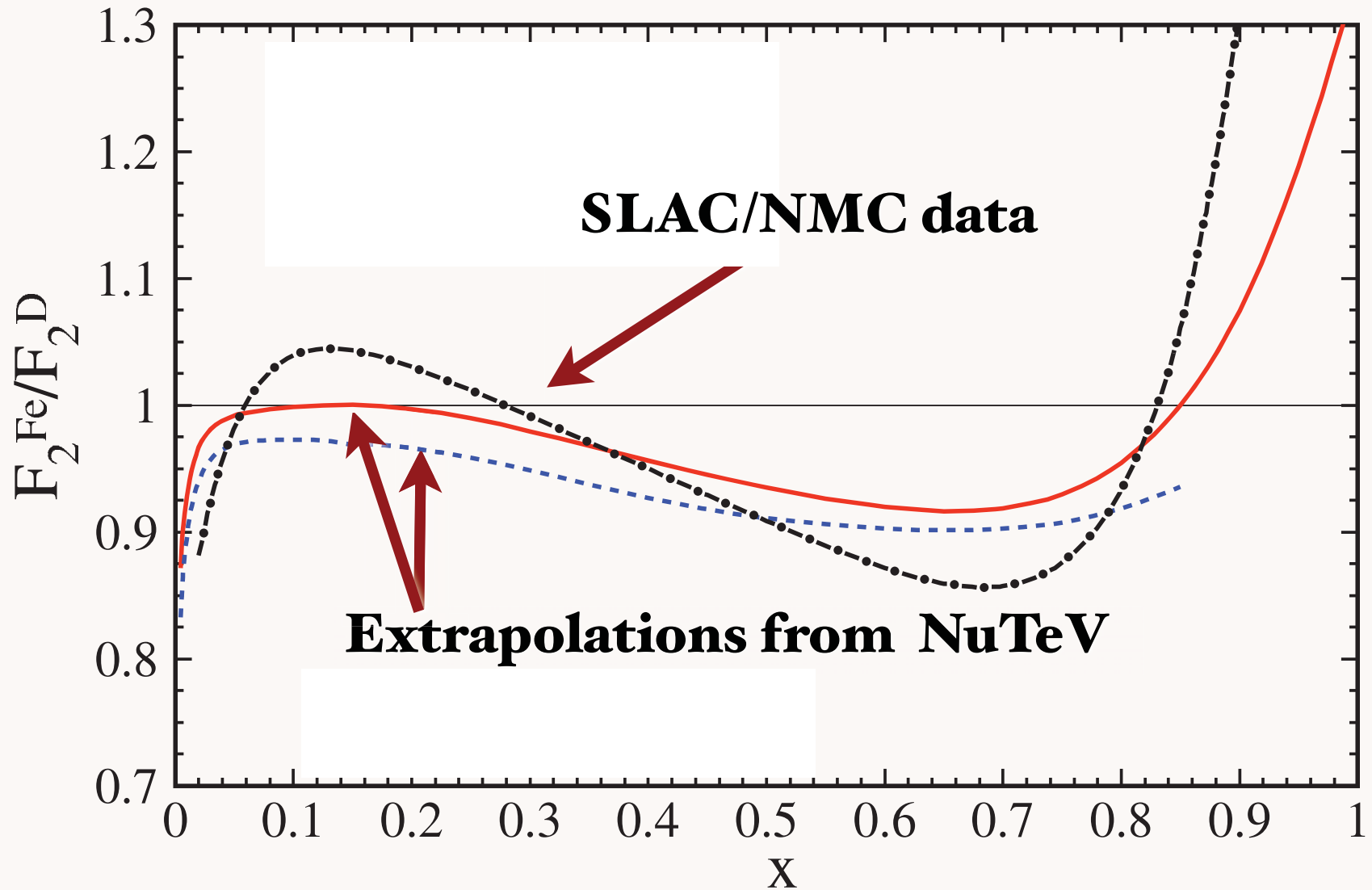
*Shadowing depends on understanding leading twist-diffraction in DIS*

**Nuclear Shadowing not included in nuclear LFWF !**

**Dynamical effect due to virtual photon interacting in nucleus**

*Antishadowing (Reggeon exchange) is not universal!*

$$Q^2 = 5 \text{ GeV}^2$$



*Scheinbein, Yu, Keppel, Morfin, Olness, Owens*

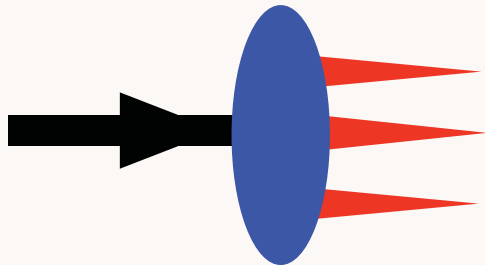
# Mueller: AdS/CFT and Deep Inelastic Scattering

## Light-Front Holography and Non-Perturbative QCD

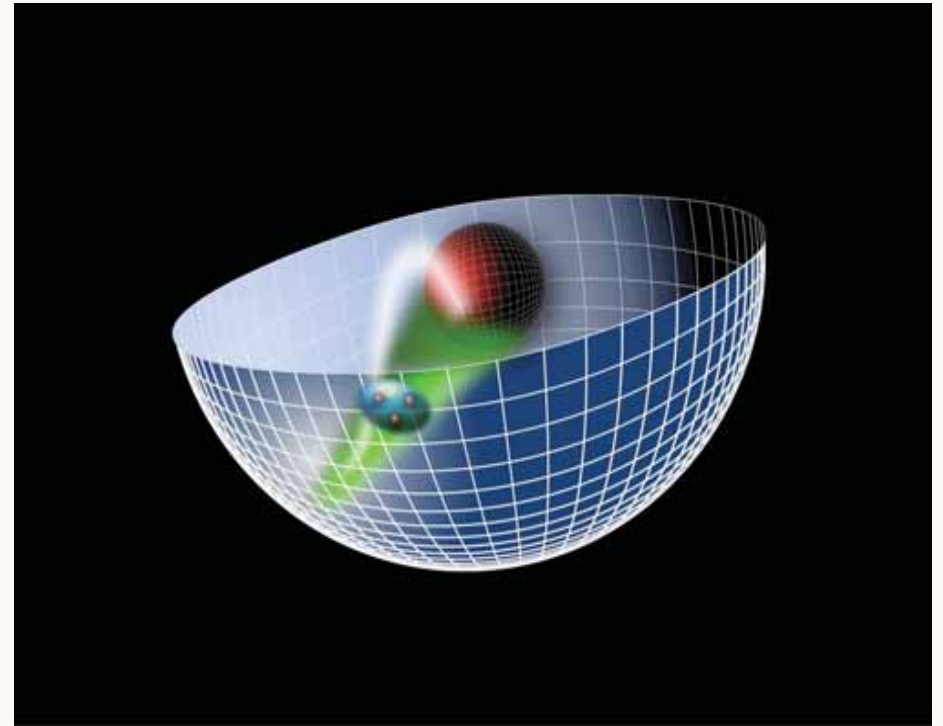
**Goal:**

**Use AdS/QCD duality to construct  
a first approximation to QCD**

*Hadron Spectrum  
Light-Front Wavefunctions,  
Form Factors, DVCS, etc*

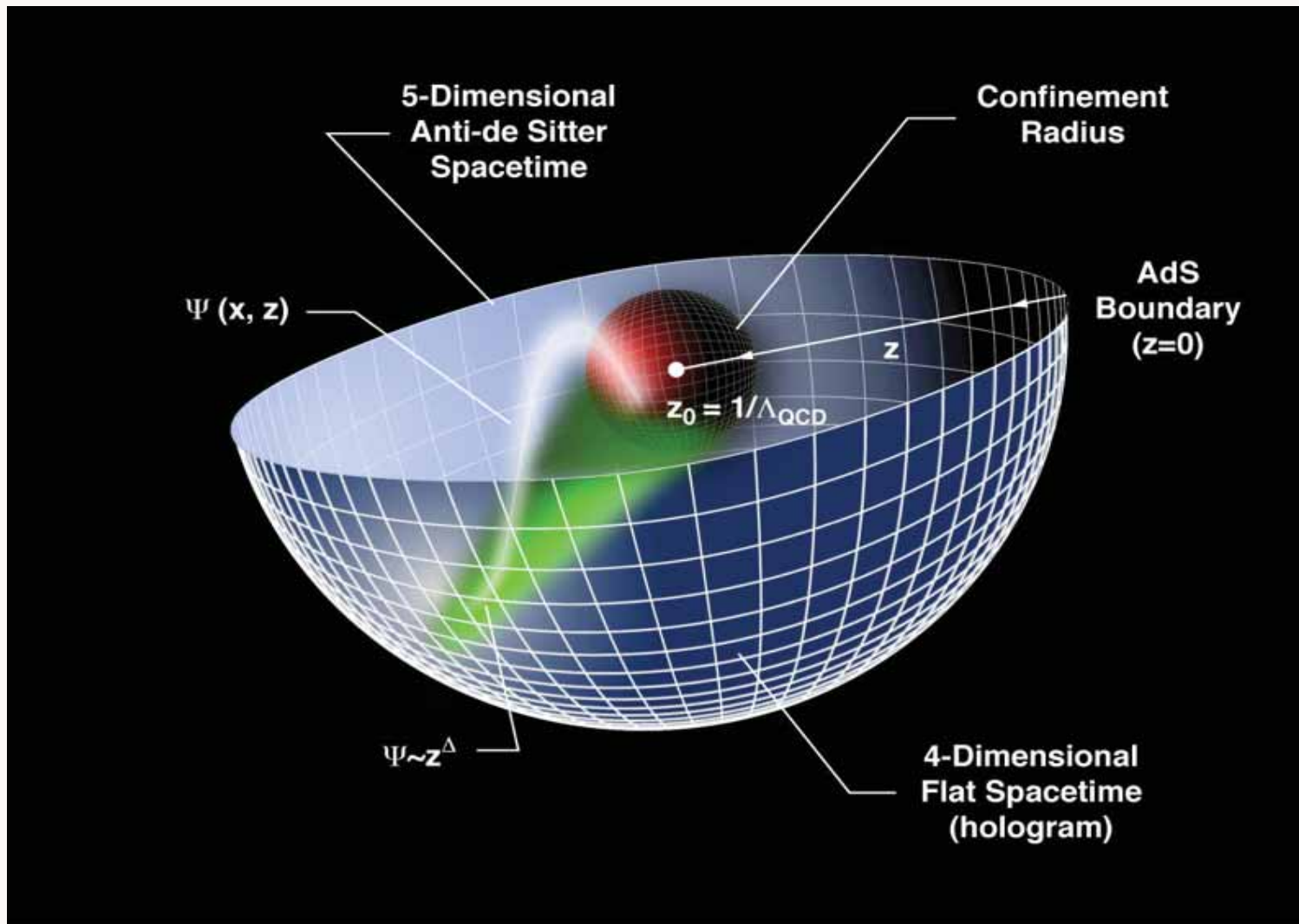


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



**in collaboration with  
Guy de Teramond**

# Applications of AdS/CFT to QCD



*Changes in physical length scale mapped to evolution in the 5th dimension  $z$*

**in collaboration with Guy de Teramond**

Columbia  
October 23, 2009

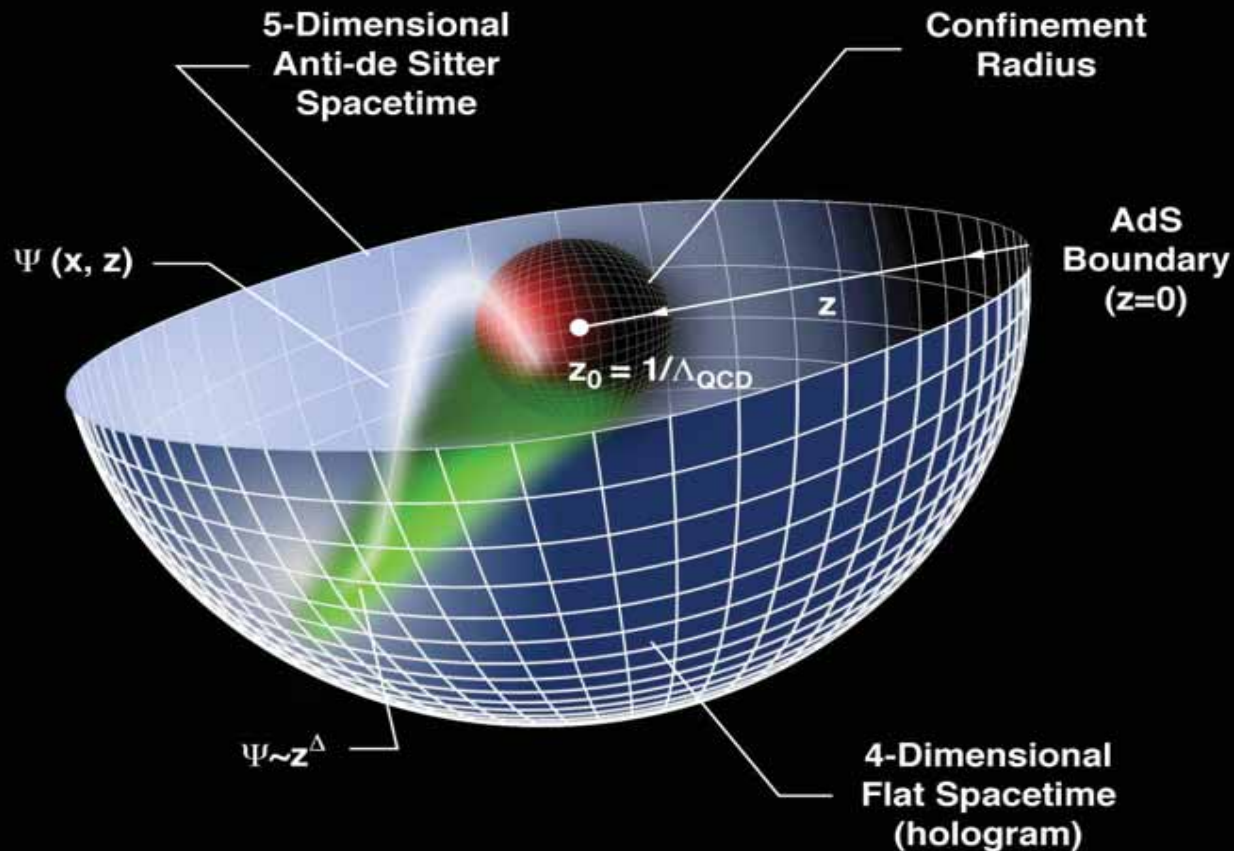
**New Horizons in QCD**

48

Stan Brodsky  
**SLAC**



# Applications of AdS/CFT to QCD



*Changes in physical length scale mapped to evolution in the 5th dimension  $z$*

**String Theory**



**Bottom-Up**



**Top-Down**

# Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

*Conformal Theories are invariant under the Poincare and conformal transformations with*

$$M^{\mu\nu}, P^\mu, D, K^\mu,$$


*the generators of  $SO(4,2)$*

**$SO(4,2)$  has a mathematical representation on  $AdS_5$**

## Scale Transformations

- Isomorphism of  $SO(4, 2)$  of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

*invariant measure* 

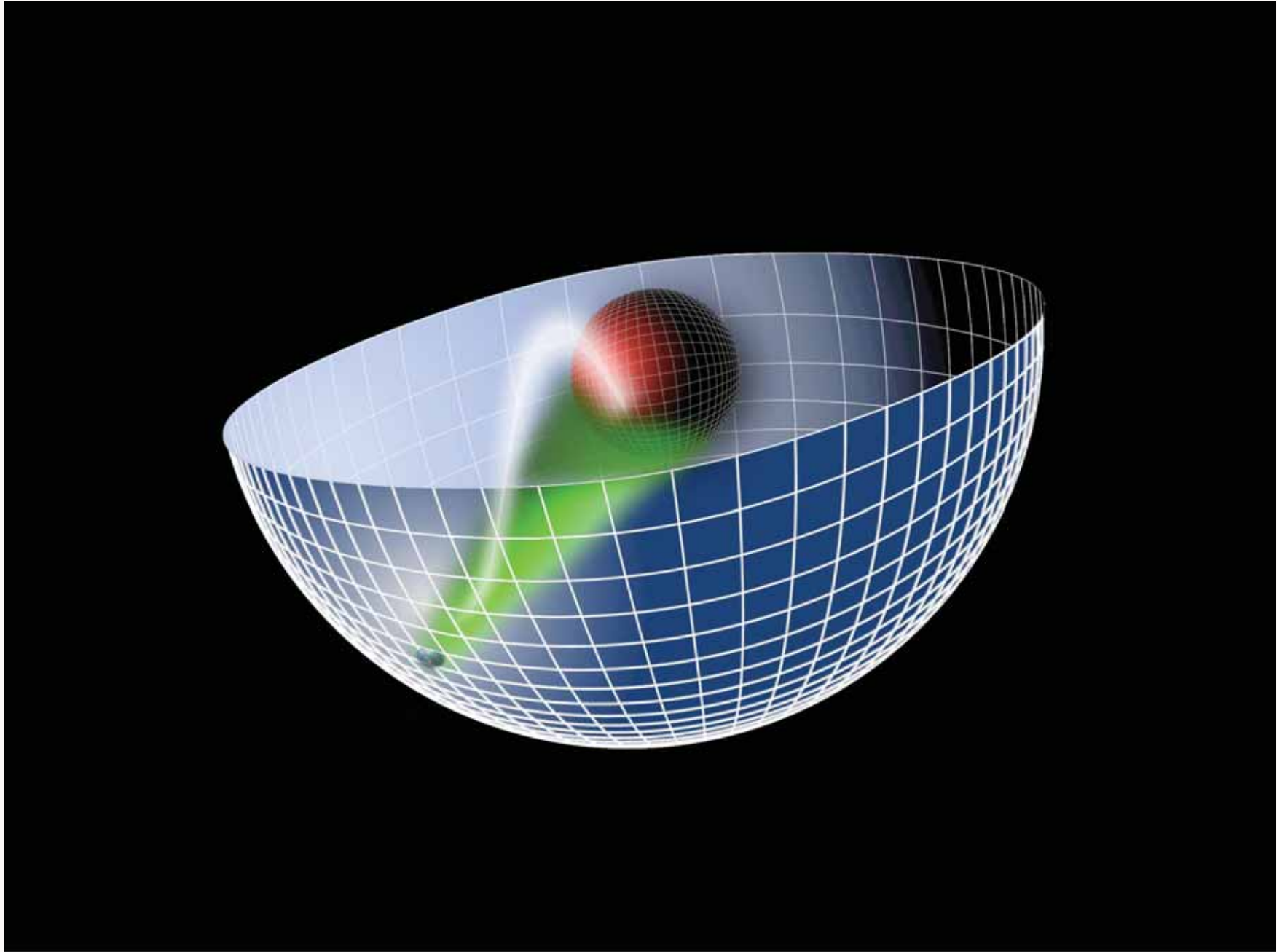
$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$ , maps scale transformations into the holographic coordinate  $z$ .

- AdS mode in  $z$  is the extension of the hadron wf into the fifth dimension.
- Different values of  $z$  correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$ : invariant separation between quarks

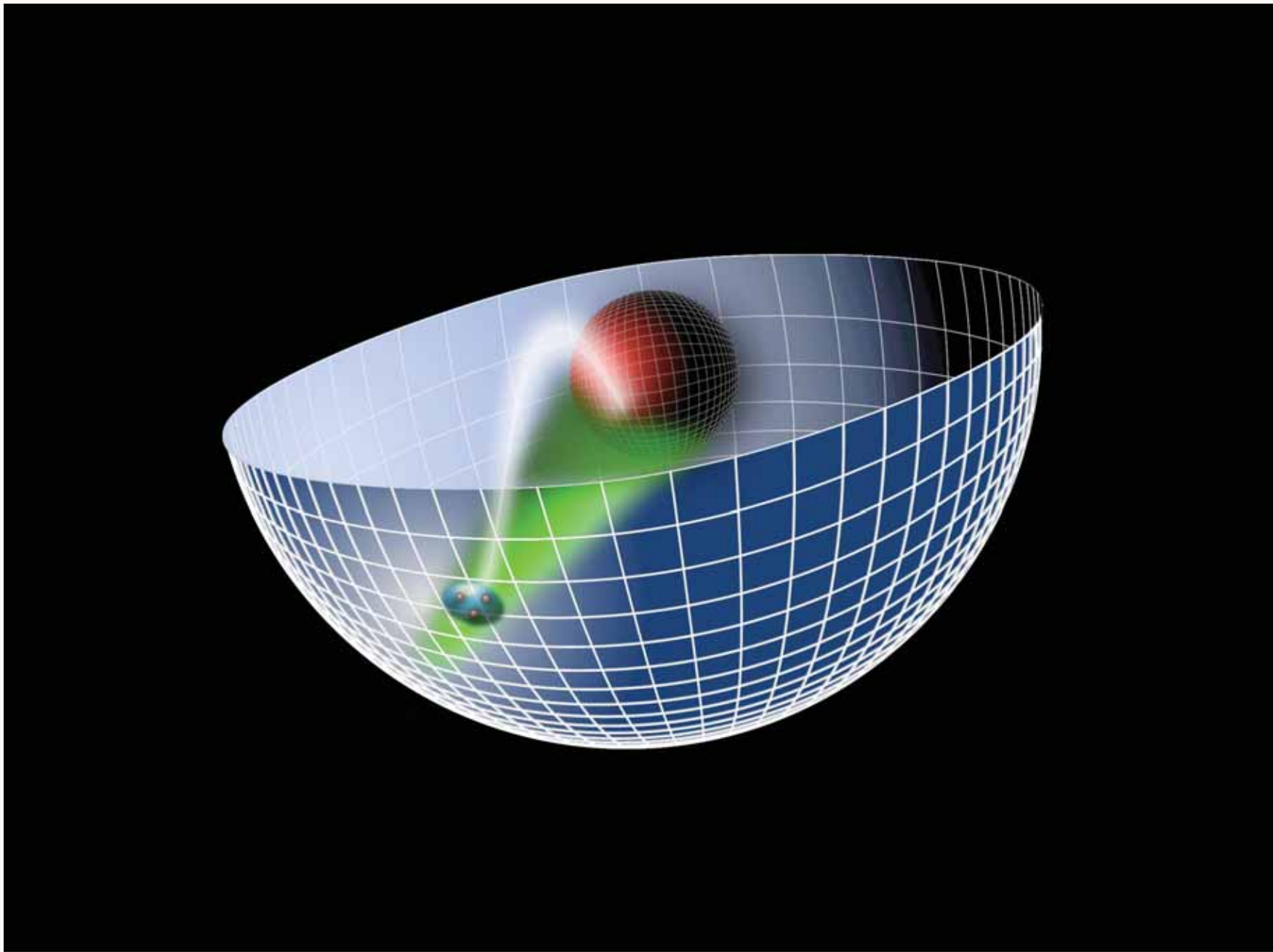
- The AdS boundary at  $z \rightarrow 0$  correspond to the  $Q \rightarrow \infty$ , UV zero separation limit.



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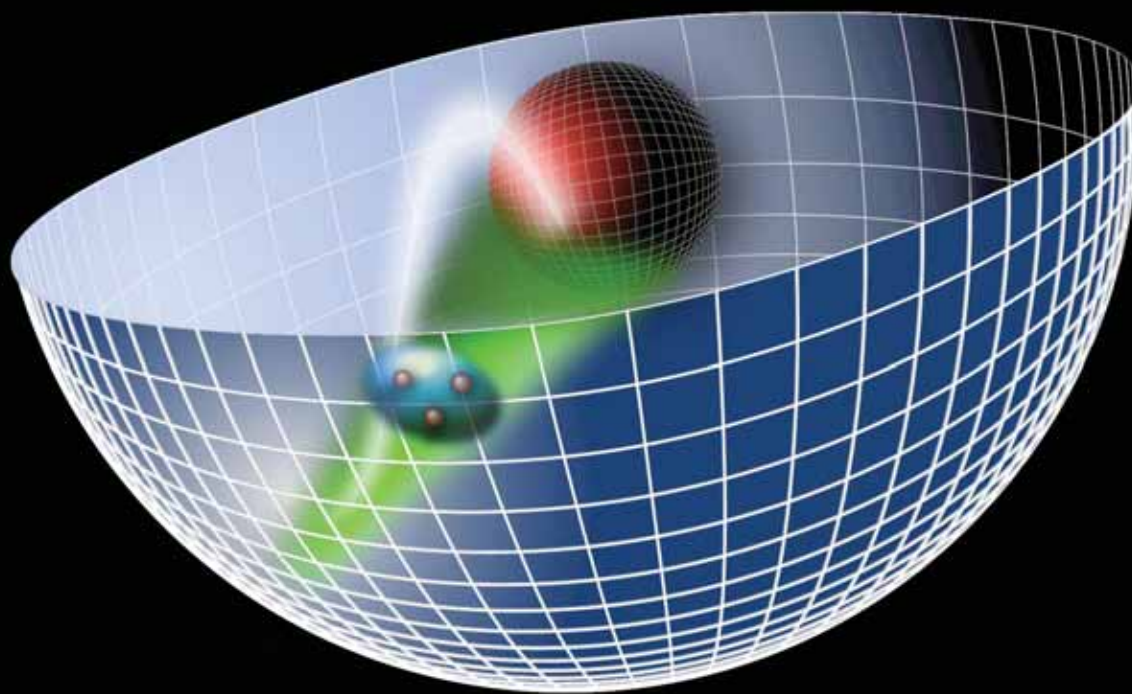


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**New Horizons in QCD**

**54**

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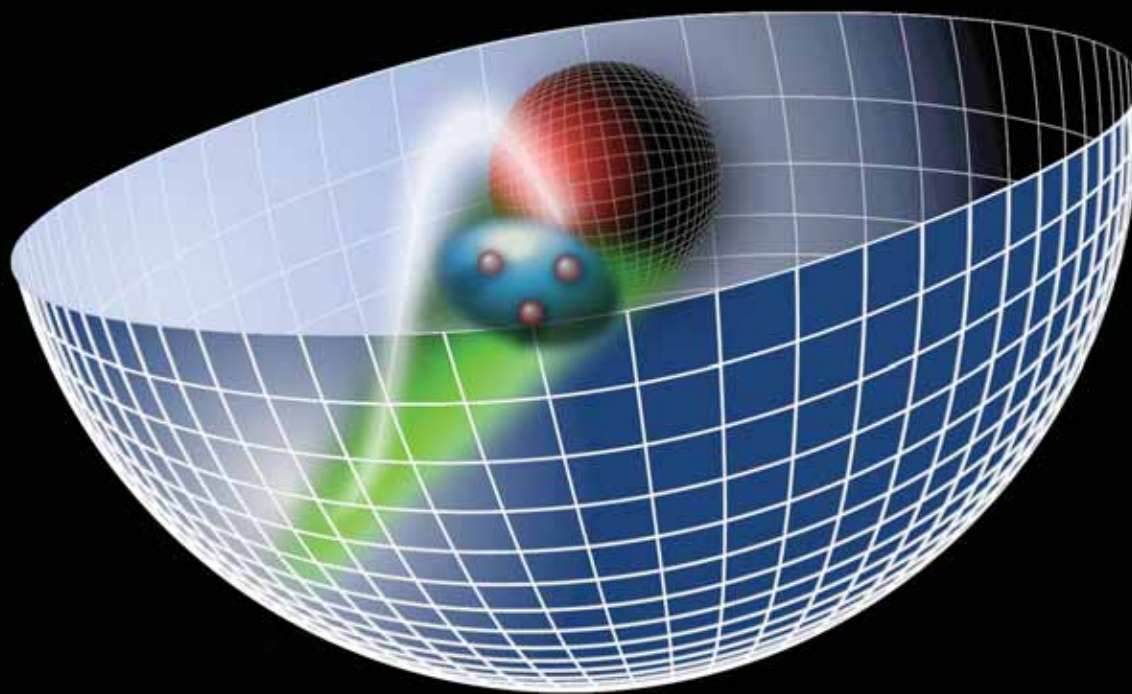


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**New Horizons in QCD**

**55**

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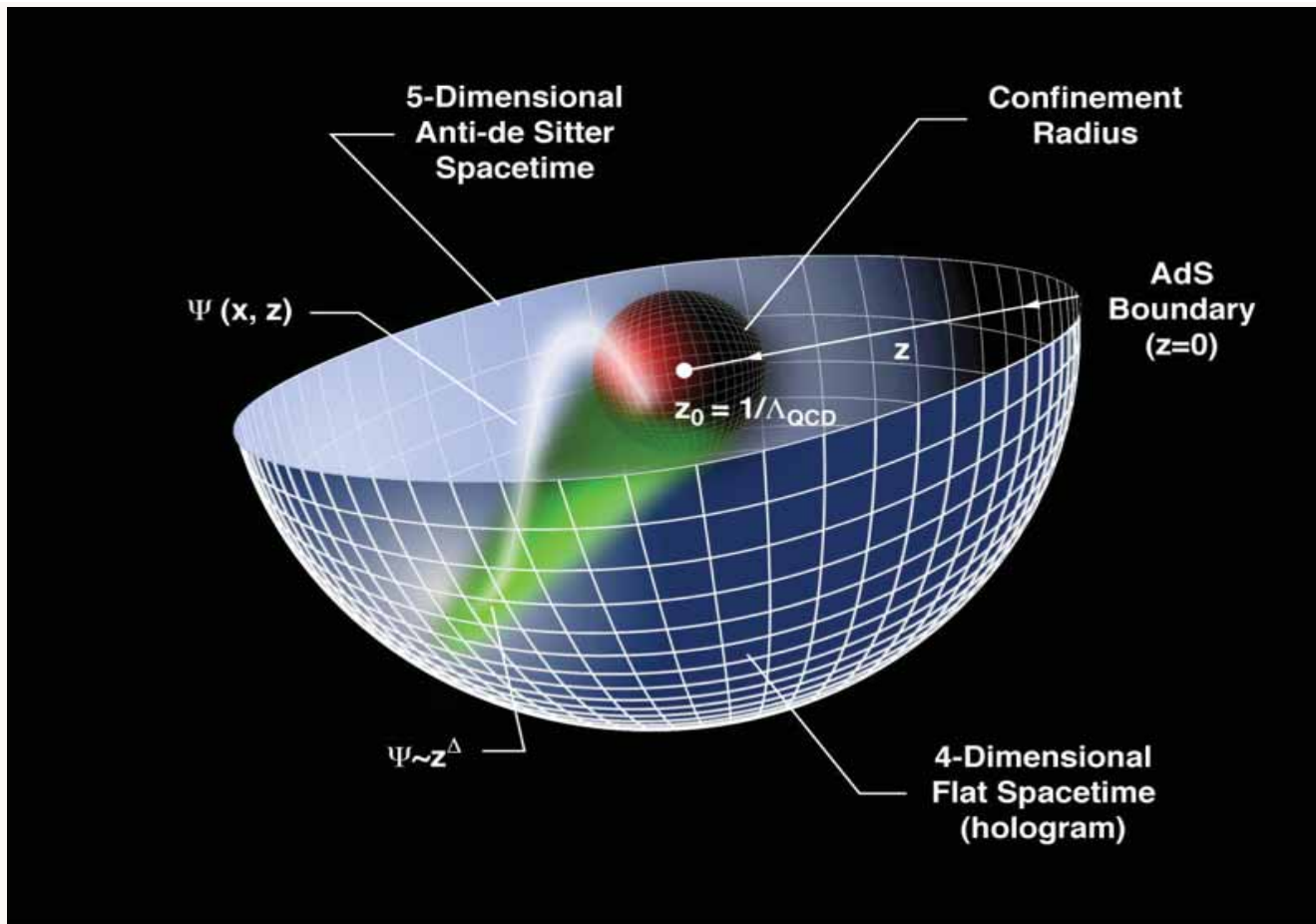
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**New Horizons in QCD**

**56**

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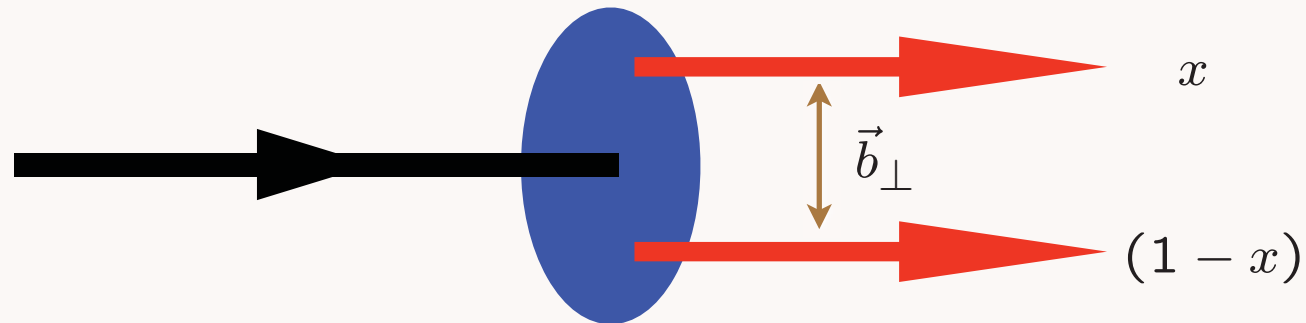
- Truncated AdS/CFT (Hard-Wall) model: cut-off at  $z_0 = 1/\Lambda_{\text{QCD}}$  breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) [Polchinski and Strassler \(2001\)](#).
- Smooth cutoff: introduction of a background dilaton field  $\varphi(z)$  – usual linear Regge dependence can be obtained (Soft-Wall Model) [Karch, Katz, Son and Stephanov \(2006\)](#).

$LF(3+1)$

$AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

*Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements*

# *AdS/CFT*: Anti-de Sitter Space / Conformal Field Theory

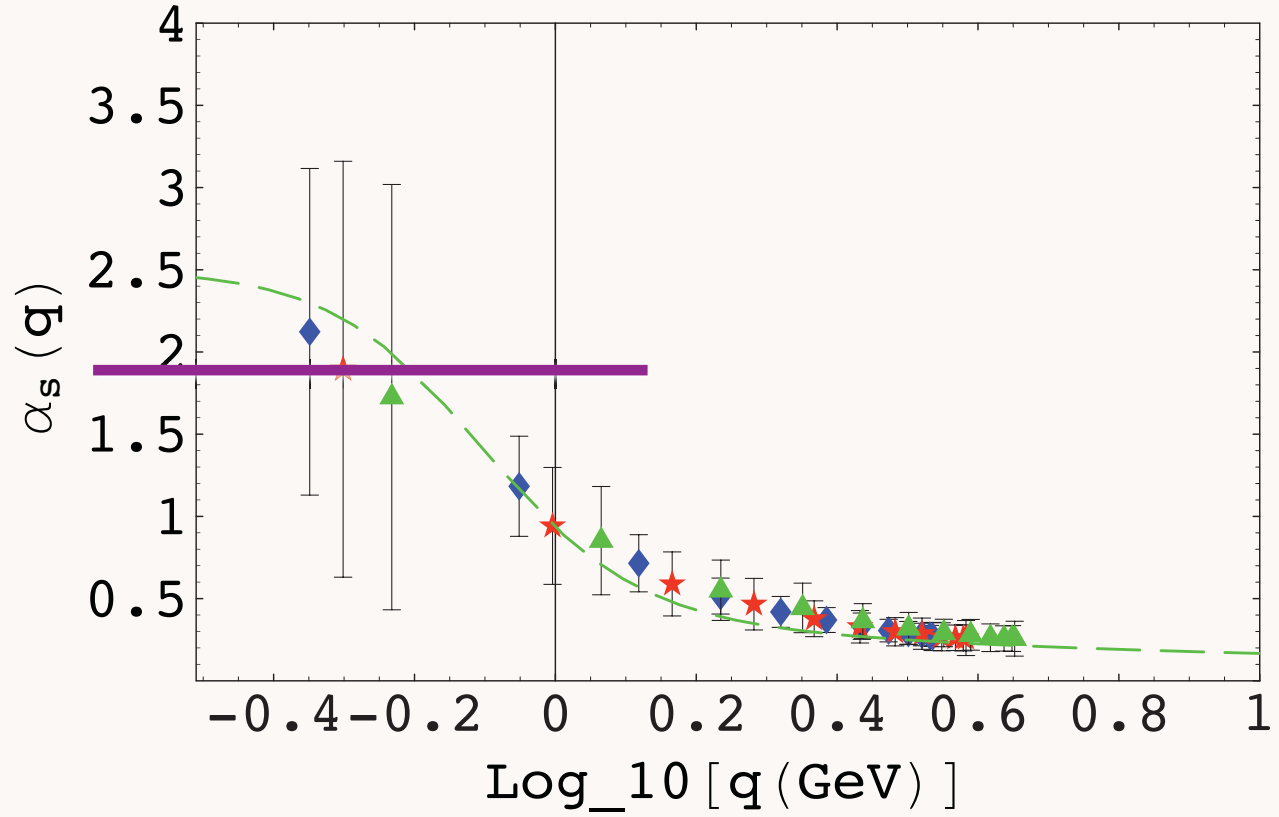
Maldacena:

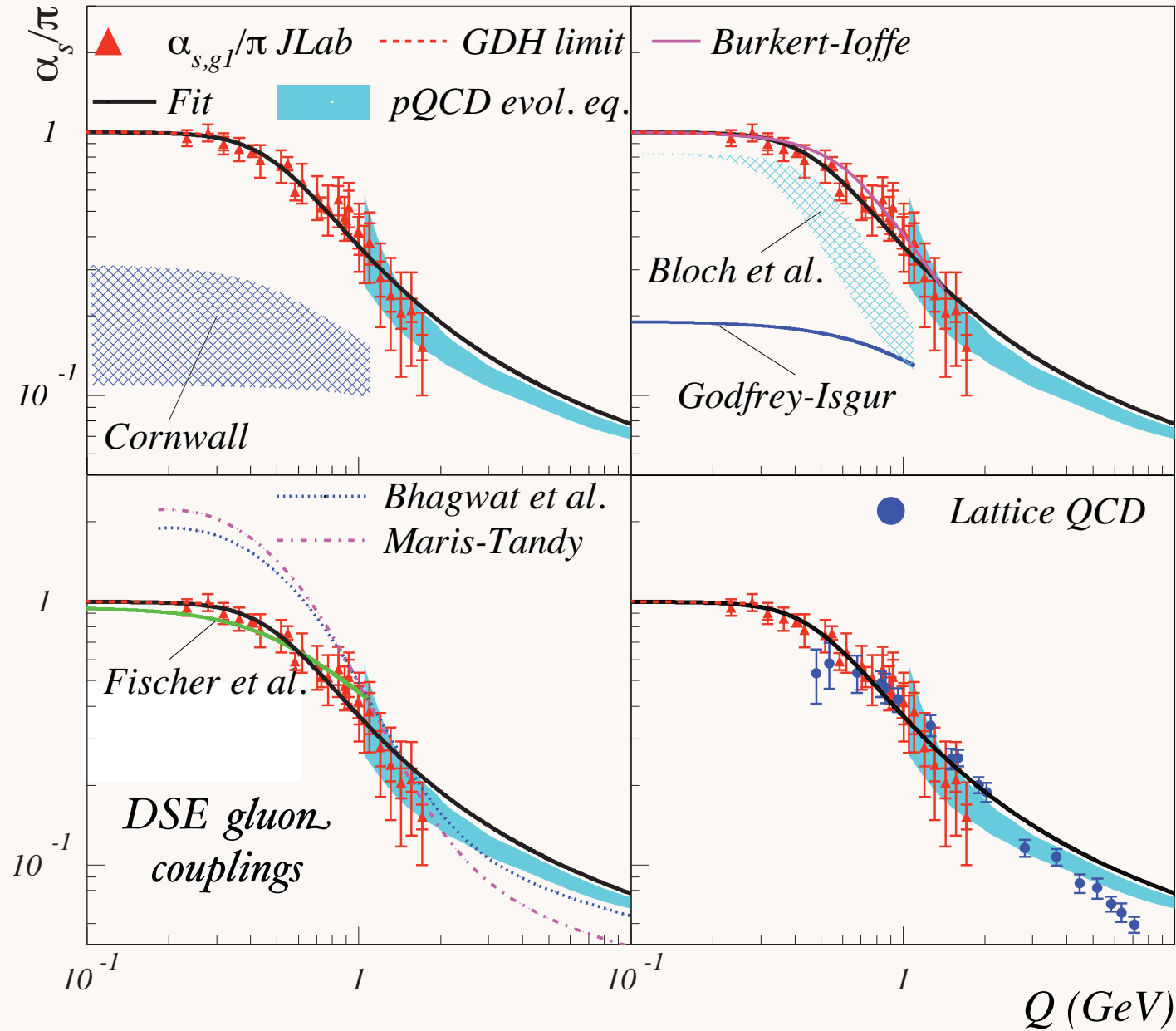
Map  $AdS_5 \times S^5$  to conformal  $N=4$  SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window**:  $\alpha_s(Q^2) \simeq \text{const}$  at small  $Q^2$
- **Use mathematical mapping of the conformal group  $SO(4,2)$  to  $AdS_5$  space**

# Conformal QCD Window in Exclusive Processes

- Does  $\alpha_s$  develop an IR fixed point? Dyson–Schwinger Equation [Alkofer, Fischer, LLanes-Estrada, Deur ...](#)
- Recent lattice simulations: evidence that  $\alpha_s$  becomes constant and is not small in the infrared [Furui and Nakajima, hep-lat/0612009](#) (Green dashed curve: DSE).





# Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain**  $(x - y)^2 < \Lambda_{QCD}^{-2}$
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

*Quark and Gluon vacuum polarization insertions  
decouple: IR fixed Point*

**Shrock, sjb**

J. D. Bjorken,  
SLAC-PUB 1053  
Cargese Lectures 1989

*A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).*

**Columbia**  
**October 23, 2009**

**New Horizons in QCD**

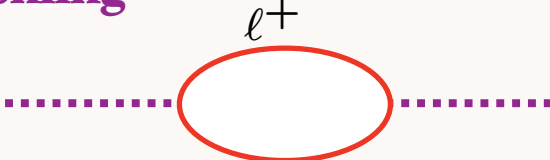
62

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**SLAC**

# IR Conformal Window for QCD

- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**
- *Evidence from Lattice Gauge Theory*
- *Stability of  $\Upsilon \rightarrow ggg$*  Shrock, sjb
- Define coupling from observable: **indications of IR fixed point for QCD effective charges** Deur, Chen, Burkert, Korsch,
- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small  $Q^2$**

Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2$$


- **Justifies application of AdS/CFT in strong-coupling conformal window**

# AdS/CFT

- Use mapping of conformal group  $SO(4,2)$  to  $AdS_5$
- Scale Transformations represented by wavefunction in 5th dimension  
$$x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z$$
- Match solutions at small  $z$  to conformal twist dimension of hadron wavefunction at short distances  $\psi(z) \sim z^\Delta$  at  $z \rightarrow 0$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates “bag” boundary conditions

$$0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$$



*AdS Schrodinger Equation for bound state  
of two scalar constituents:*

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

**L: light-front orbital angular  
momentum**

*Derived from variation of Action in AdS<sub>5</sub>*

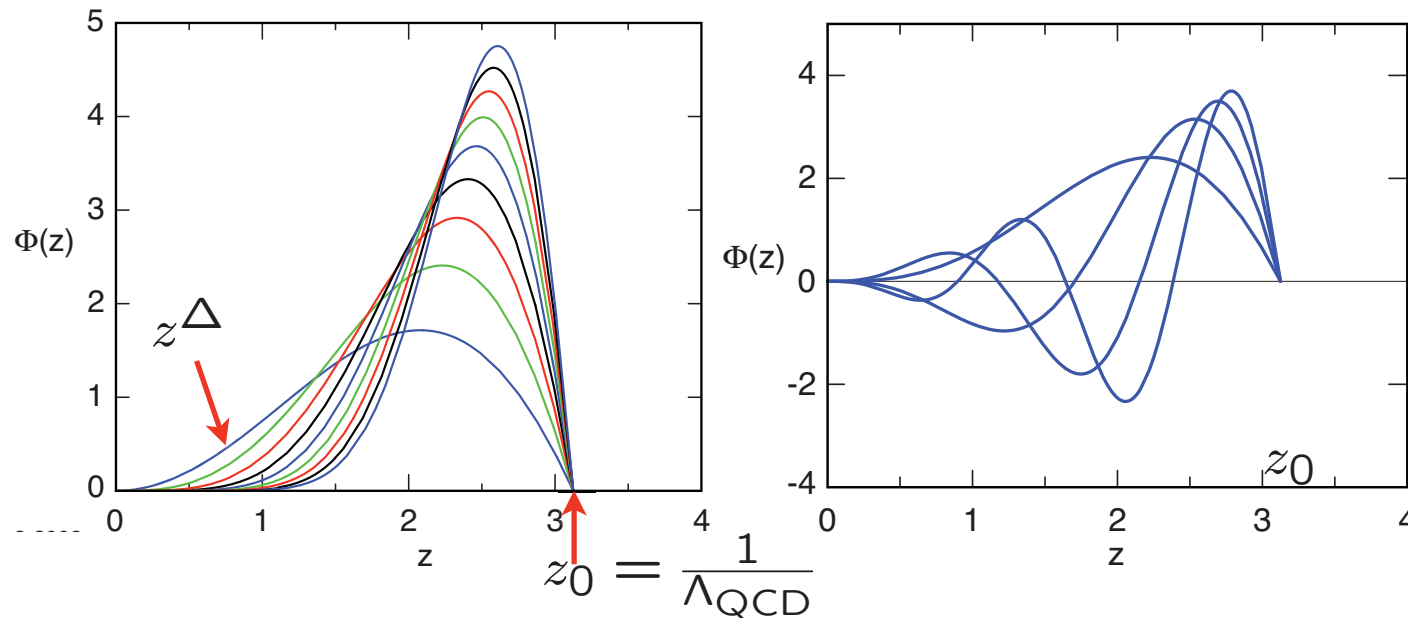
*Hard wall model: truncated space*

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

***Match fall-off at small  $z$  to conformal twist-dimension  
at short distances***

*twist*

- Pseudoscalar mesons:  $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\{\ell_1 \dots \ell_m\}} \psi$  ( $\Phi_\mu = 0$  gauge).  $\Delta = 2 + L$
- 4- $d$  mass spectrum from boundary conditions on the normalizable string modes at  $z = z_0$ ,  $\Phi(x, z_0) = 0$ , given by the zeros of Bessel functions  $\beta_{\alpha,k}$ :  $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$
- Normalizable AdS modes  $\Phi(z)$



$S = 0$  Meson orbital and radial AdS modes for  $\Lambda_{QCD} = 0.32$  GeV.

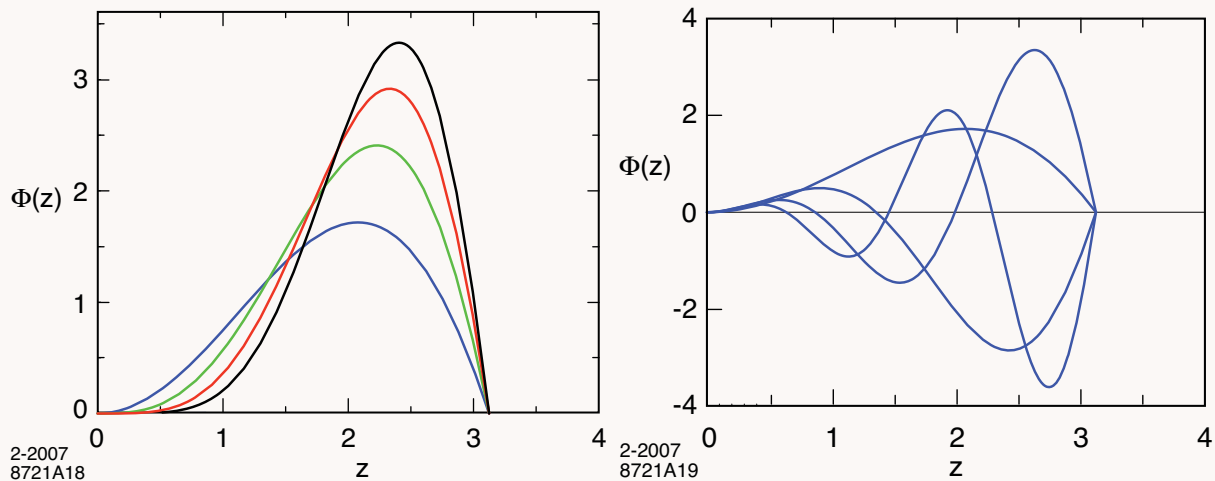


Fig: Orbital and radial AdS modes in the hard wall model for  $\Lambda_{QCD} = 0.32$  GeV .

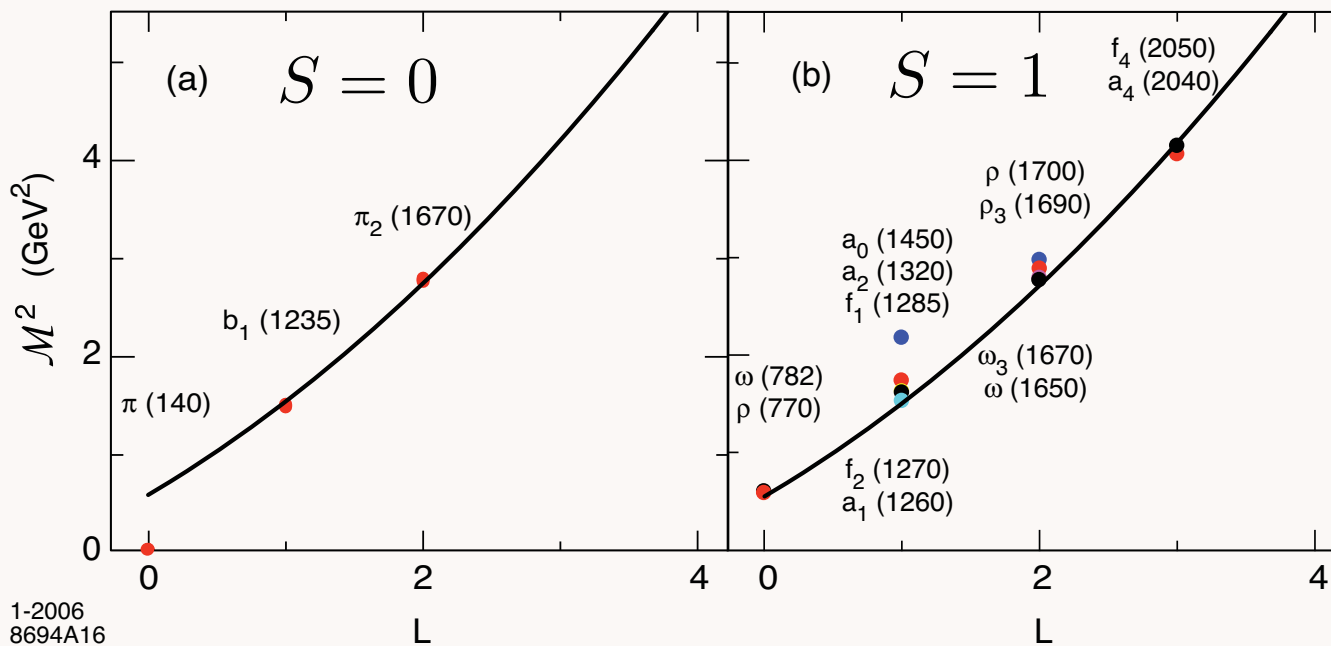
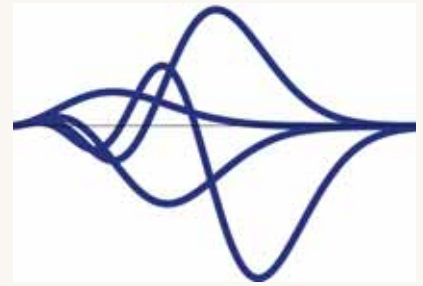


Fig: Light meson and vector meson orbital spectrum  $\Lambda_{QCD} = 0.32$  GeV



## Soft-Wall Model

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field  $\varphi(z) = \pm \kappa^2 z^2$

$$S = \int d^d x dz \sqrt{g} e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field  $\mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2)$

$$[z^2 \partial_z^2 - (d - 1 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0$$

with  $(\mu R)^2 \geq -4$ . See also [Metsaev (2002), Andreev (2006)] + sign: **Fen Zuo(2009)**

- LH holography requires 'plus dilaton'  $\varphi = +\kappa^2 z^2$ . Lowest possible state  $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 4\kappa^2 n, \quad \Phi_n(z) \sim z^2 e^{-\kappa^2 z^2} L_n(\kappa^2 z^2)$$

$\Phi_0(z)$  a chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

*Massless pion*