QCD REGGEON FIELD THEORY AND EIKONAL SCATTERING

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IN HONOR OF AL MUELLER'S 70th BIRTHDAY

AL IMERICK

A LANKY MAGICIAN NAMED **AL**

OWNS NEITHER A MAC NOR A DELL.

SO PARTONS TO FIELDS

WITH HIS BARE HANDS HE WIELDS,

THIS UBIQUITOUS WIZARD NAMED **AL**

AL AT HIGH ENERGY

MUELLER - QIU DERIVATION OF GLR;

COMPREHENSIBLE INTERPRETATION OF BFKL;

MUELLER-NAVELET PROCESS;

GLAUBER-MUELLER MULTIPLE SCATTERING;

DIPOLE PICTURE OF HIGH ENERGY SCATTERIN;

IMPORTANCE OF POMERON LOOPS;

STATISTICAL MODELS OF QCD EVOLUTION...

AND THE LIST GOES ON

FIELD THEORY AT HIGH ENERGY

GRIBOV'S POMERON: BASIC SCATTERING AMPLITUDE AT FIXED IMPACT PARAMETER $\Phi_\eta(X)$

EFFECTIVE POMERON FIELD THEORY: $H[\Phi, \Pi]$

Φ IS A QUANTUM OPERATOR:

$$\frac{d}{d\eta}\Phi = -[H, \Phi]$$

QCD AT HIGH ENERGY

BASIC t-CHANNEL EXCHANGE (COLOR OCTET): REGGEIZED GLUON

$$G(k, \eta) \propto e^{-a\eta \ln k/\mu}$$

IR DIVERGENT (VANISHING) - COLOR IS NOT CARRIED OVER FINITE RAPIDITY INTERVAL

"‘BOUND STATES"’ OF REGGEIZED GLUONS - SINGLET EXCHANGE AMPLITUDES (IR FINITE!)

BFKL POMERON: $P(X, Y, \eta) \propto e^{\omega_0 \eta}$

n - REGGEON BKP STATES

$$P_N(X_1, ...X_n, \eta) \propto e^{(n \pm \epsilon)\omega_0 \eta}$$

MANY "‘SUPERCRITICAL"’ BKP STATES - SCATTERING MATRIX GROWS WITH ENERGY AS A POWER.

UNITARIZATION: PERHAPS MANY GROWING EXCHANGES SUM INTO A DECREASING FUNCTION

$$\Sigma_n a_n e^{(n\pm\epsilon)\omega_0\eta} \sim_{\eta\to\infty} const$$

ALSO REGGEONS INTERACT: THEY SPLIT AND THEY MERGE (WHEN THERE ARE MANY OF THEM). THIS MUST ALSO CONTRIBUTE TO UNITARIZATION.

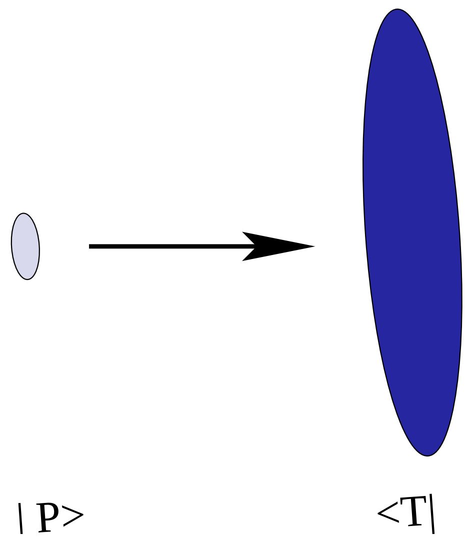
THE REGGEONS ARE ALSO "QUANTUM OBJECTS": MERGINGS AND SPLITTINGS HAVE TO BE "'VERTICES"' IN THE "'HAMILTONIAN"'.

WE KNOW SOME OF THE ELEMENTS THROUGH FEYNMAN DIAGRAMS, BUT WE STILL DON'T KNOW THE HAMILTONIAN.

WE'LL TRY A DIFFERENT APPROACH (TO THE SAME QCD).

EIKONAL QCD AT HIGH ENERGY

SCATTER A "‘PROJECTILE"’ HADRON $|P\rangle$ ON A "‘TARGET"’ HADRON $|T\rangle$ AT HIGH ENERGY



$|P\rangle$ - A DISTRIBUTION OF COLOR CHARGES DENSITY $j^a(X)$.

$|T\rangle$ - AN ENSEMBLE OF (STRONG) COLOR FIELDS $\alpha^a(X)$.

ENERGY IS HIGH - SCATTERING IS EIKONAL

THE S - MATRIX

THE EIKONAL S - MATRIX:

EVERY PROJECTILE GLUON KEEPS THE TRANSVERSE POSITION BUT ACQUIRES A PHASE

$$|X,a\rangle \to S^{ab}(X)|X,b\rangle$$

WITH

$$S^{ab}(X) = \mathcal{P}\,\exp\left\{i\int dX^-\,T^a\,\alpha^a(X,X^-)\right\}^{ab}\;.$$

THE FORWARD SCATTERING AMPLITUDE:

$$\mathcal{S} \; = \langle \mathrm{IN}|\mathrm{OUT}\rangle =< \langle P|\hat{S}|P\rangle >_T$$

$$=< \int dj \; W^P[j] \;\; \exp\left\{ i \int d^2X \, j^a(X) \; \alpha^a(X) \right\} >_T$$

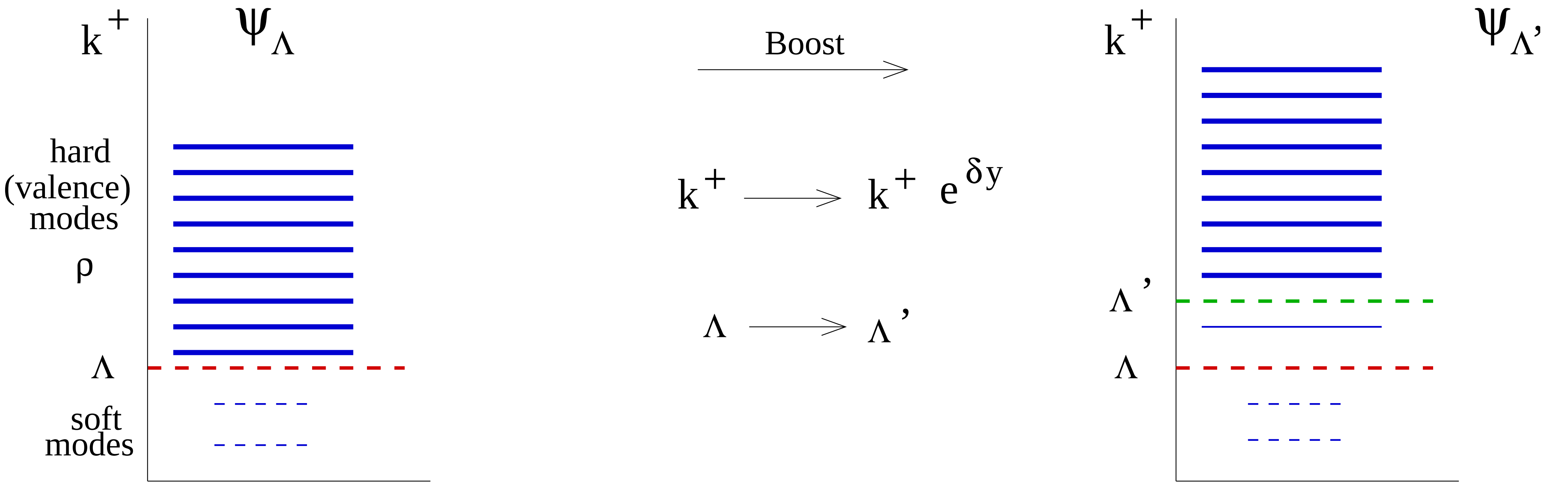
$W^P[j]$ - PROBABILITY DISTRIBUTION OF THE PROJECTILE COLOR CHARGE DENSITY

AT HIGHER RAPIDITY $|P\rangle$ EVOLVES, MEANING $W^P[j]$ EVOLVES: $W^P[j] \longrightarrow W^P_\eta[j]$

THE RFT HAMILTONIAN IS WHAT EVOLVES IT:

$$\frac{d}{d\eta} W^P[j] = -H_{RFT}[j, \delta/\delta j] \; W^P[j]$$

AS THE HADRON IS BOOSTED, ITS WAVE FUNCTION AND THE PROBABILITY DISTRIBUTION CHANGE



UNDER BOOST THE LONGITUDINAL MOMENTA SCALE .

NEW GLUONS RISE FROM THE "BOTTOMLESS PIT" WHICH IS THE ZERO MODE.

COLOR FIELD BECOMES STRONGER BECAUSE OF THESE EXTRA WEIZSACKER-WILIAMS GLUONS

NEED TO KNOW THE "'SOFT VACUUM"' WAVEFUNCTION

THEN:

$$\mathcal{S} = \langle \mathrm{IN} | \mathrm{OUT} \rangle = \langle P_{\mathrm{valence}} | \langle P_{\mathrm{soft}} | \hat{S} | P_{\mathrm{soft}} \rangle | P_{\mathrm{valence}} \rangle$$

$|P_{\mathrm{soft}}\rangle$ ALSO DEFINES THE RFT HAMILTONIAN

substitute

$$S^{ab}(X) = \exp\{i\,T^c\,\alpha^c(X)\}^{ab} \rightarrow R^{ab}(X) = \exp\left\{g\,T^c\,\frac{\delta}{\delta j^c(X)}\right\}^{ab}$$

$$\langle P_{\mathrm{soft}} | \hat{S} | P_{\mathrm{soft}} \rangle = 1 - H_{RFT}[j, \delta/\delta j]\ \Delta\eta + \ldots$$

H_{RFT} EVOLVES THE PROBABILITY DISTRIBUTION WITH RAPIDITY η:

$$\frac{d}{d\eta} W^P[j] = -H_{RFT}[j, \delta/\delta j]\ W^P[j]$$

THE "'SOFT VACUUM"' - DIAGONALIZE H_{QCD}

$H_{QCD}[a, a^\dagger, j]$ - QCD HAMILTONIAN ON THE SOFT HILBERT

SPACE WITH VALENCE BACKGROUND COLOR CHARGE

HERE:

$a, \ a^\dagger$ - SOFT GLUE CREATION AND ANNIHILATION OPERATORS

j - VALENCE COLOUR CHARGE DENSITY

FIND $\Omega[a, a^\dagger, j]$ SUCH THAT

$$\Omega^\dagger H_{QCD} \Omega \ = \ H_{\text{diagonal}}$$

Ω PERTURBATIVELY

$$\Omega = CB$$

$$C = \exp \quad i \quad d^2X b_i^a(X) \quad d\eta[a_i^a(X,\eta) + a_i^{a\dagger}(X,\eta)]$$

$$\partial_i b_i^a(X) = j^a(X), \qquad \partial_i b_j^a(X) - \partial_j b_i^a(X) - gf^{abc} b_i^b(X) b_j^c(X) = 0$$

C COHERENT OPERATOR - CREATES WEIZSACKER-WILLIAMS FIELD $b_i^a(X)$

B - BOGOLYUBOV OPERATOR: $B = \exp\{\Lambda[j]a^2 + a^{\dagger 2} + \ldots\}$

B - DEFINES GLUON QUASIPARTICLES ABOVE THE WEISZACKER-WILLIAMS BACKGROUND

Ω IS PERTURBATIVELY ACCURATE: E.G.

$$\langle VAC|a^\dagger a|VAC\rangle = b^2 \left(O(1/\alpha_s)\right) + \Lambda^2 \left(O(1)\right) + O(\alpha_s) + \ldots$$

FOR "PROTON" $b \sim O(g)$; $\Lambda \sim O(g^2)$

FOR "'NUCLEUS"' $b \sim O(1/g)$; $\Lambda \sim O(1)$

THIS IS GOOD FOR THE "BULK" OF THE WAVE FUNCTION - MOST OF THE PROBABILITY AND "TYPICAL" CONFIGURATIONS ARE DESCRIBED WELL

REMEMBER: WE NEED

$$\langle IN|OUT\rangle$$

WHERE, GIVEN OUR Ω

$$\langle A|IN\rangle \sim \exp \quad ib[j]A + \Lambda[j]A^2$$

$$\langle A|OUT\rangle \sim \exp \quad ib[Rj]RA + \Lambda[Rj](RA)^2$$

WITH

$$R(X) = \exp\{gT^a \frac{\delta}{\delta j^a(X)}\}$$

SO OVERLAP OF TWO GAUSSIAN WAVE FUNCTIONS

$$\langle IN|OUT\rangle \sim \quad DAe^{-ib(j,X)A(X)+\Lambda^*[j]A^2} e^{ib(Rj,X)RA(X)+\Lambda[Rj](RA)^2}$$

WOULD LIKE TO KEEP IT GENERAL:

$g \leq b \leq 1/g$ - KEEP NONLINEARITIES IN THE WAVE FUNCTION

$R \sim O(1)$, BUT NOT NECESSARILY $1 - O(\alpha_s)$ - KEEP ALL MULTIPLE SCATTERINGS

WE CANNOT QUITE MANAGE THAT YET IN FULL GENERALITY

CAN MANAGE THE SITUATION WHEN THE $|IN\rangle$ AND $|OUT\rangle$ STATES ARE PERTURBATIVELY CLOSE

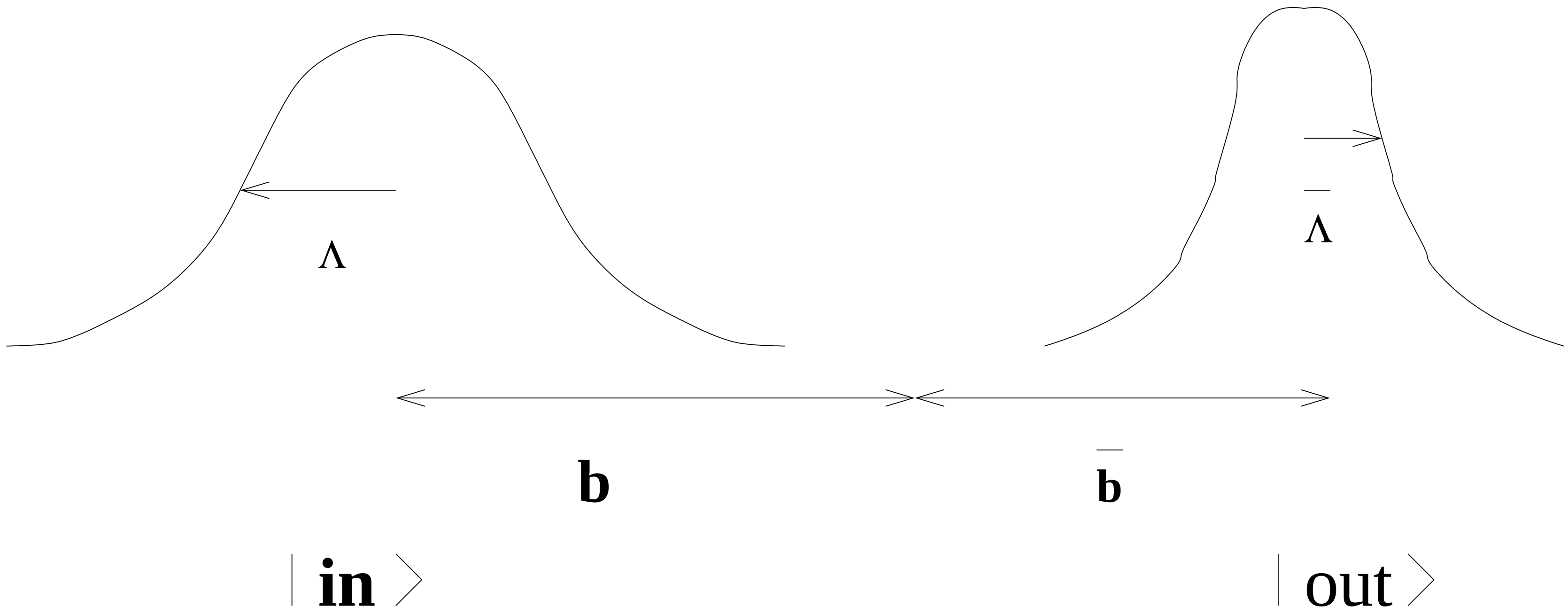


Figure 1: THE TAIL OF THE TWO GAUSSIANS.

WHEN THE OVERLAP COMES FROM THE REGION OF MOST PROBABILITY WE ARE OK

IF IT'S THE TAILS - WE DON'T KNOW THE TAILS

WHEN ARE WE OK?

PROJECTILE IS DENSE BUT TARGET IS DILUTE (JIMWLK LIMIT):

$\Lambda \sim O(1); \quad b \sim O(1/g); \quad \bar{b} \sim O(1/g); \quad \mathrm{BUT}\ R = 1 - O(\alpha_s)\ \mathrm{AND}\ b - \bar{b} \sim O(g)$

PROJECTILE IS DILUTE (PERTURBATIVE), BUT TARGET IS DENSE (KLWMIJ LIMIT):

$b \sim O(g); \quad \bar{b} \sim O(g); \quad \Lambda \sim O(1)$

POMERON LOOPS ("'DIPOLE-DIPOLE"' SCATTERING): TARGET IS ALWAYS PERTURBATIVE

NUCLEUS-NUCLEUS - NOT SO GOOD...

$R \sim 1; \quad b \sim O(1/g) \quad \rightarrow \quad b - \bar{b} \sim O(1/g); \quad \Lambda \sim O(1)$

OVERLAP IS DOMINATED BY THE TAILS OF THE TWO WAVEFUNCTIONS - WE NEED TO KNOW THE TAILS

WKB APPROXIMATION WOULD BE APPROPRIATE ($A \sim O(1/g)$) - BUT NOT SUPER EASY

E.G. JIMWLK

THE DILUTE TARGET LIMIT (SMALL $\delta/\delta j$ LIMIT) - JIMWLK EQUATION

$$H^{JIMWLK} = \frac{\alpha_s}{2\pi^2} \quad d^2Z\, Q_i^a(Z)\, Q_i^a(Z)$$

THE HERMITIAN AMPLITUDES $Q_i^a(Z)$

$$Q_i^a(Z) = \quad d^2X \frac{(X-Z)_i}{(X-Z)^2}[S^{ab}(Z) - S^{ab}(X)]J_R^b(X)\,.$$

THE GENERATORS OF RIGHT COLOR ROTATIONS J_R

$$J_R^a(X) = -\mathrm{tr} \quad S(X)T^a \frac{\delta}{\delta S^\dagger(X)}$$

ITS A HAMILTONIAN QUANTUM FIELD THEORY - RFT

IN THE WEAK FIELD LIMIT $\alpha \to 0$ [EXPAND IN $\alpha \propto j^T$]

$$Q \to Q_i^{\mathrm{BFKL}\,a}(Z) = \int d^2X \frac{(X-Z)_i}{(X-Z)^2} f^{abc} \left[\frac{\delta}{\delta j^b(X)} - \frac{\delta}{\delta j^b(Z)} \right] j^c(X)$$

AND

$$H_{JIMWLK} \to H_{BFKL} = \frac{\alpha_s}{2\pi^2} \int d^2Z\, Q_i^{\mathrm{BFKL}\,a}(Z)\, Q_i^{\mathrm{BFKL}\,a}(Z)$$

THIS IS "EASY" TO "SOLVE". SPECTRUM SCHEMATICALLY

$|\Psi_1\rangle = f_1^a(X) j^a(X)$ - REGGEIZED GLUON

$|\Psi_2\rangle = f_2^{ab}(X,Y) j^a(X) j^b(Y)$ - BFKL POMERON

$|\Psi_n\rangle = f_n^{ab\ldots c}(X,Y\ldots Z) j^a(X) j^b(Y)\ldots j^c(Z)$ - n-GLUON BKP STATE

AT LEADING ORDER IN j, H_{JIMWLK} CONTAINS ALL THE BKP STATES - "'QCD REGGEONS"' WITH THE BKP EIGENVALUES

EXPANDING H_{JIMWLK} IN j - GENERATES REGGEON INTERACTIONS $2 \longrightarrow n$ VERTICES.

EIGENFUNCTIONS WILL CONTAIN ADMIXTURE OF STATES WITH MORE REGGEIZED GLUONS

$$|\Psi_2\rangle = |\mathrm{BFKL\ \ POMERON}\rangle + |4\ \ \mathrm{REGGEIZED\ \ GLUON\ \ BKP}\rangle + \dots$$

ETC...

H_{JIMWLK} IS THE HAMILTONIAN OF THE QCD REGGEON FIELD THEORY:

IT CONTAINS "POMERONS" IN THE LEADING ORDER IN j; AND POMERON INTERACTIONS (TRIPLE POMERON VERTEX ETC.) IN HIGHER ORDERS

RFT VS BFKL

H_{JIMWLK} HAS NONNEGATIVE SPECTRUM - SO THE AMPLITUDE SATURATES.

$$S_\eta \propto \sum_i a_i e^{-\omega_i \eta}; \qquad \text{All} \;\; \omega_i \geq 0$$

BFKL HAS NEGATIVE EIGENVALUES - UNITARY IS VIOLATED

BUT H_{BFKL} IS THE LIMIT OF H_{JIMWLK} AS $j \to 0$.

SO HOW COME?

IT'S LIKE UNSTABLE EQUILIBRIUM: $H = \frac{1}{2}\pi^2 - \phi^2 + \phi^4$

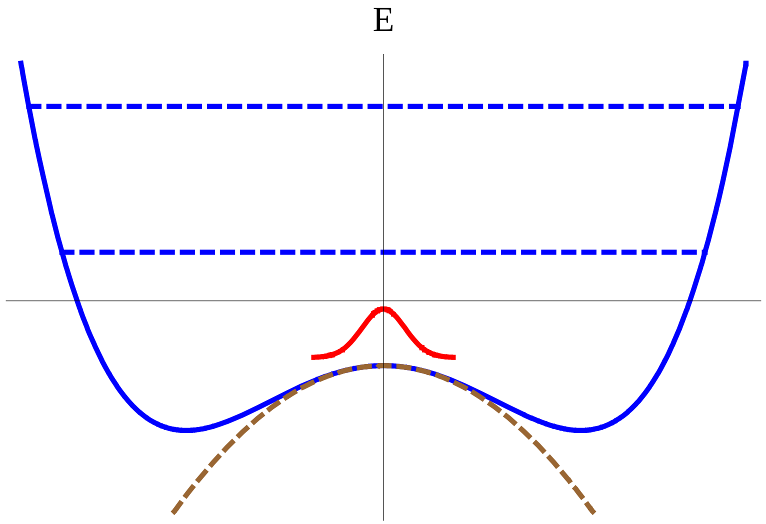


Figure 2: JIMWLK vs BFKL.

INITIAL WAVE PACKET LOCALIZED AT THE MAXIMUM AROUND THE ORIGIN $j \sim 0$

Ψ_n - EXACT EIGENFUNCTIONS

Φ_n - EIGENFUNCTIONS (NONORMALIZABLE) OF AN UPSIDE DOWN HARMONIC OSCILLATOR.

ALWAYS TRUE:

$$\Psi(t) = \sum_n \alpha_n e^{-\epsilon_n t} \Psi_n, \qquad \text{all } \epsilon_n \text{ positive}$$

BUT FOR SMALL TIMES ALSO

$$\Psi(t) = \sum_n \beta_n e^{-\bar{\epsilon}_n t}\Phi_n, \qquad \text{some } \bar{\epsilon}_\mathrm{n} \text{ negative}$$

IF FOR LARGE n THE OVERLAPS β_n ARE SMALL - THE ENERGIES CLOSE TO THE MAXIMUM DOMINATE EVOLUTION FOR A WHILE

UNITARIZATION OF BFKL - TURN THE SECOND SERIES INTO FIRST!

RFT PROVIDES THE ”‘MINIMUM”’ OF THE ”‘POTENIAL”’ AWAY FROM THE ORIGIN

JIMWLK TO RFT

JIMWLK IS POMERON TREES - ONLY "'POMERON SPLITTINGS"'

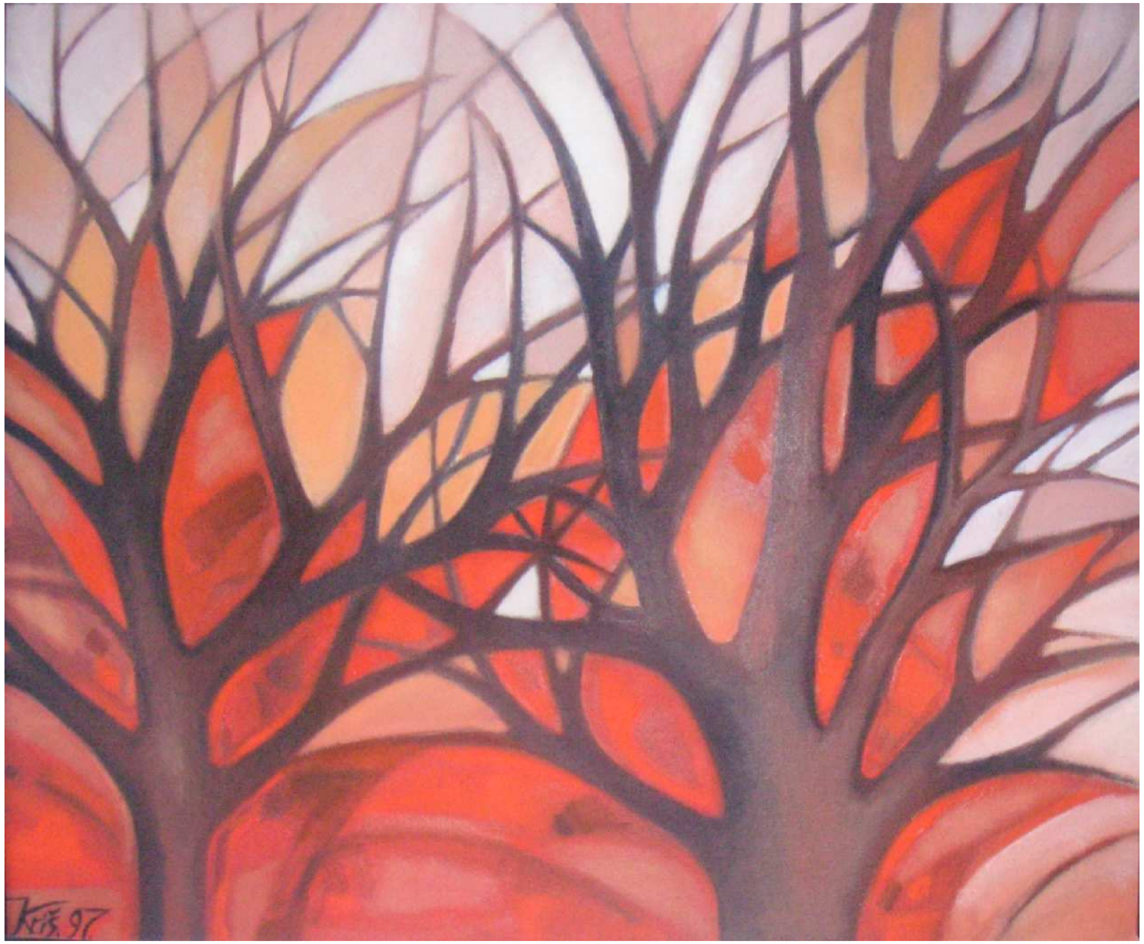


Figure 3: Red Pomeron Trees by Natalija Krisciuniene.