Jets at weak and strong coupling

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``From Particles and Partons to Nuclei and Fields”

An international workshop and symposium in celebration of Al’s 70th birthday.

Columbia University, October 23-25, 2009

In October 2006, I was starting my second postdoc at Saclay.

I was very lucky that at the same time Al had been visiting Saclay on sabbatical !

YH and Mueller, "Correlation of small-x gluons in impact parameter space" hep-ph/0702023 Young scientist award of JPS (nuclear theory division)

YH, Iancu, Mueller, "Deep inelastic scattering at strong coupling from gauge/string duality: the saturation line" 0710.2148[hep-th]

YH, Iancu, Mueller, "Deep inelastic scattering off a N=4 SYM plasma at strong coupling" 0710.5297[hep-th]

YH, Iancu, Mueller, "Jet evolution in the N=4 SYM plasma at strong coupling" 0803.2481[hep-th]

Contents

- Jets at weak coupling
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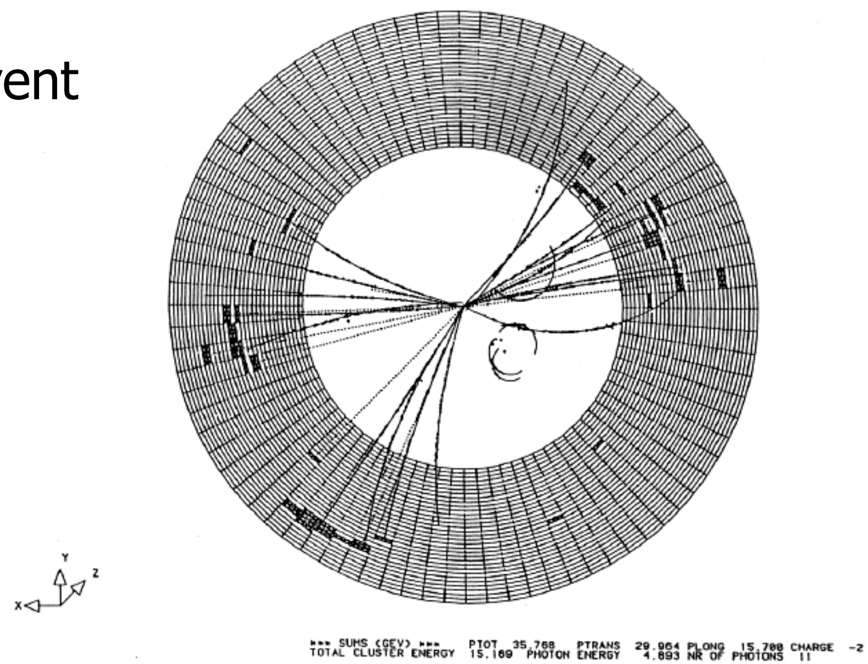
- Thermal hadron production from gauge/string duality
- Relation between e+e- annihilation and high energy scattering

Refs.
YH and Matsuo, PLB670 (2008) 150
YH and Matsuo, PRL102 (2009) 062001
YH, JHEP 0811 (2008) 057
Avsar, YH, Matsuo, JHEP 0906 (2009) 011
YH and Ueda, PRD80 (2009) 074018

Jets in QCD

Observation of jets in 1975 has provided one of the most striking confirmations of QCD

A three-jet event

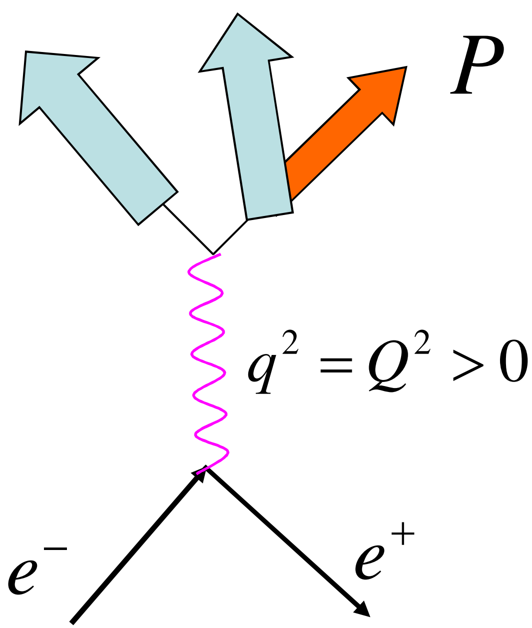


Average angular distribution of two jets $1+\cos^2\theta$
reflecting fermionic degrees of freedom (quarks)

The inclusive spectrum

Cross section to produce one hadron plus anything else

$$\frac{d\sigma}{dE\,d\cos\theta} = \frac{e^4 p}{16\pi^2 Q^6}\left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k\cdot k'\right)$$
$$\times \sum_X \int d^4y\, e^{iqy} \langle 0|J^\mu(y)|PX\rangle\langle PX|J^\nu(0)|0\rangle$$



$$\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx} = D_T(x, Q^2)$$

Fragmentation function

Count how many hadrons are there inside a quark.

Feynman-x $\quad x \equiv \frac{2P\cdot q}{Q^2} = \frac{2E}{Q}$

Evolution equation

The fragmentation function satisfies a DGLAP-type equation

$$\frac{\partial}{\partial \ln Q^2} D_T(x, Q^2) = \int_x^1 \frac{dz}{z} P_T(z) D_T\left(\frac{x}{z}, Q^2\right)$$

Mellin transform

$$D_T(j, Q^2) = \int_0^1 dx\ x^{j-1} D_T(x, Q^2)$$

$$\frac{\partial}{\partial \ln Q^2} D_T(j, Q^2) = \gamma_T(j) D_T(j, Q^2)$$

Timelike anomalous dimension

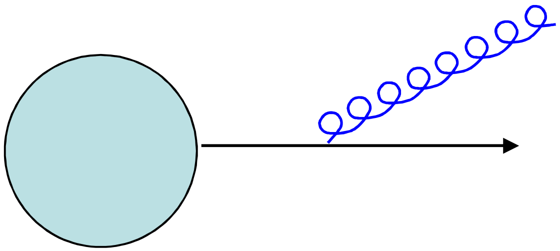
First moment gives the average multiplicity

$$\langle n \rangle = D_T(1, Q^2) \propto Q^{2\gamma_T(1)}$$

(assume $\beta = 0$)

Timelike anomalous dimension

Lowest order perturbation

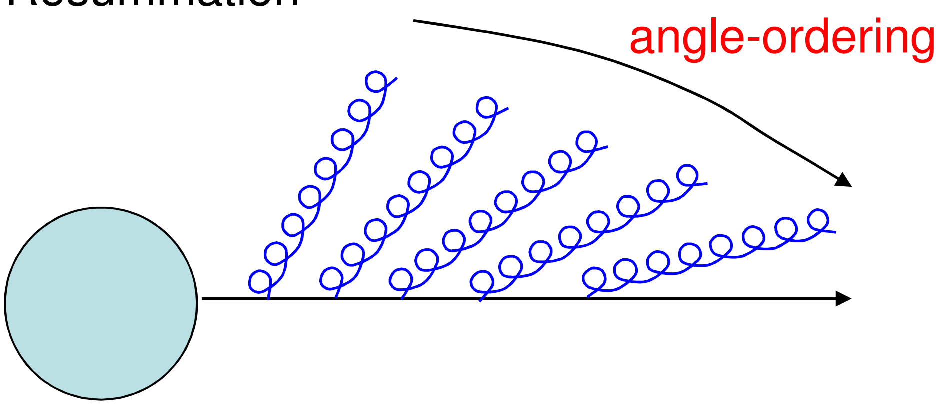


$$\sim \frac{1}{x} \quad \longleftrightarrow \quad \gamma_T(j) \sim \frac{\alpha_s}{j-1}$$

Soft singularity

$$\gamma_T(1) = \infty \quad !!$$ Nonsense !

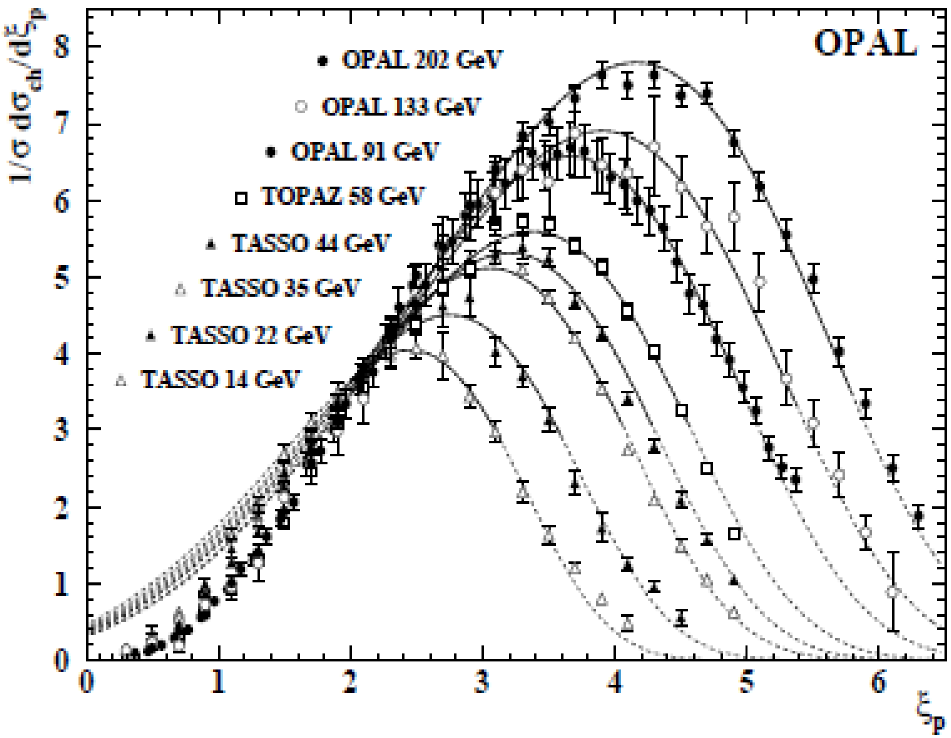
Resummation



$$\gamma_T(j) = \frac{1}{4}\left[\sqrt{(j-1)^2 + \frac{8N\alpha_s}{\pi}} - (j-1)\right]$$

$$\gamma_T(1) = \sqrt{\frac{N\alpha_s}{2\pi}}$$ Mueller (1981)

Inclusive spectrum



largel-x ↔ small-x

$$\frac{x}{\sigma}\frac{d\sigma}{dx} \propto x D_T(x, Q^2)$$

"hump-backed" distribution peaked at

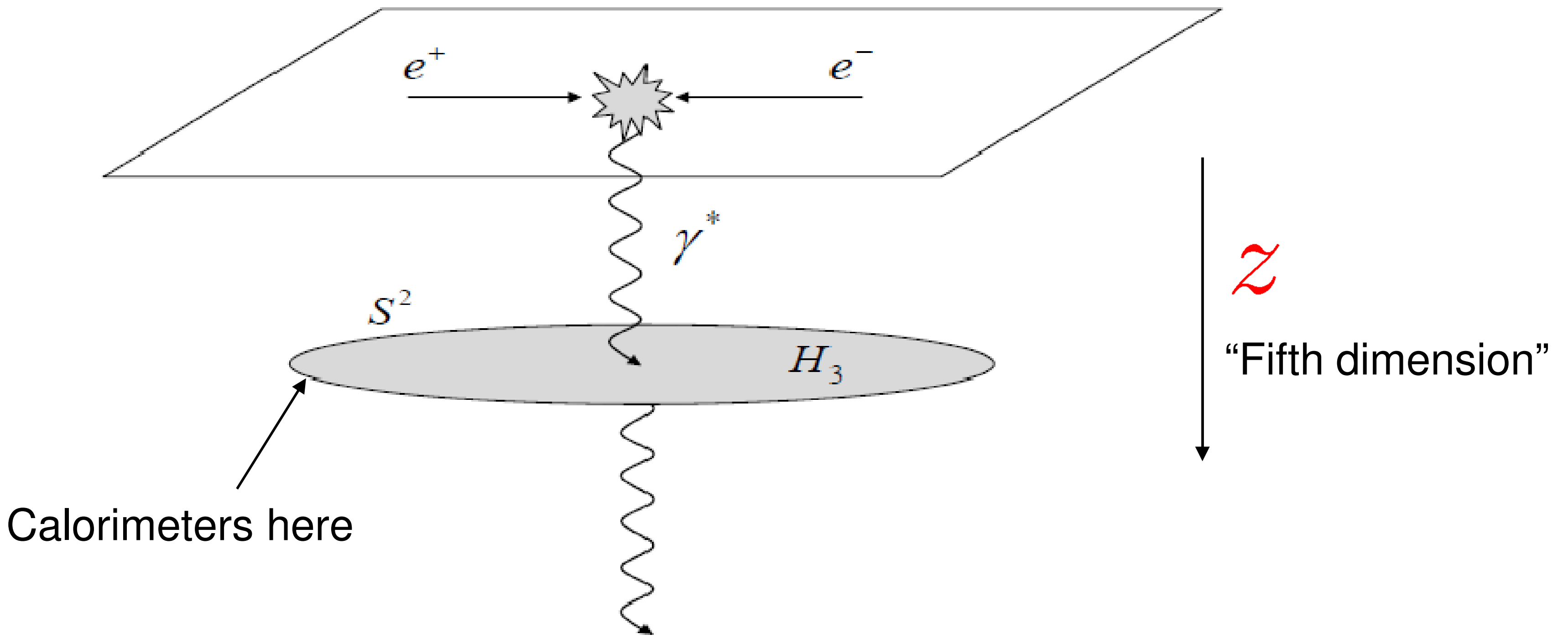
$$\frac{1}{x} \sim \sqrt{\frac{Q}{\Lambda}}$$

Mueller (1981)
Ermolaev, Fadin (1981)
Dokshitzer, Fadin, Khoze (1982)

Double logs + QCD coherence. Structure of jets well understood in pQCD.

e+e- annihilation in N=4 SYM at strong coupling

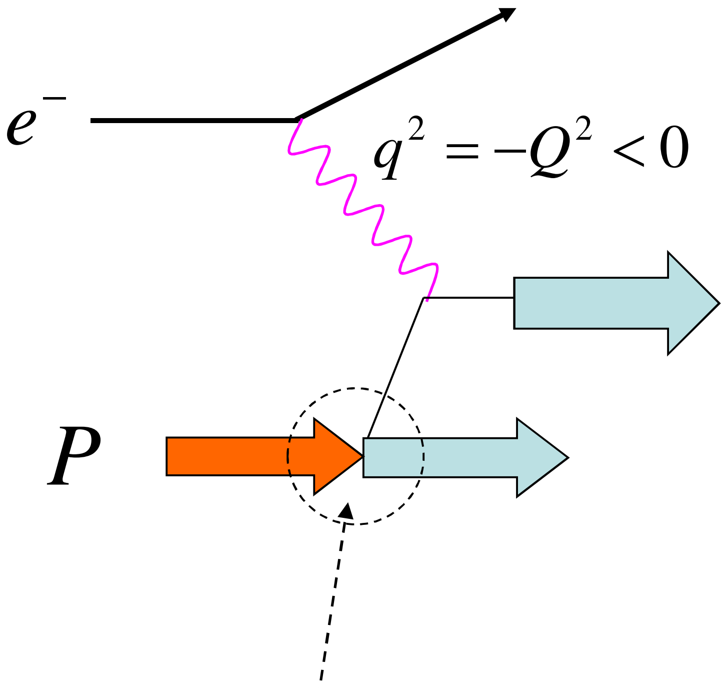
Hofman & Maldacena, 0803.1467; YH, Iancu & Mueller, 0803.2481;
YH & Matsuo, 0804.4733, 0807.0098; YH, 0810.0889.



Possible phenomenological application at the LHC ? Strassler (2008)

The AdS/CFT dictionary doesn't tell you how to compute the timelike anomalous dimension !

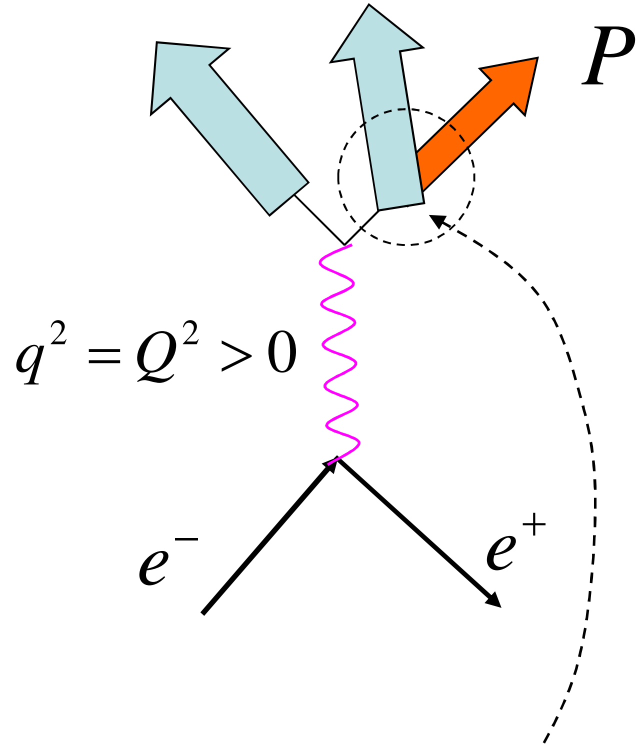
DIS vs. e+e- : crossing symmetry



Parton distribution function

$$D_S(x_B, Q^2)$$

Bjorken variable $x_B \equiv \frac{Q^2}{2P \cdot q}$



Fragmentation function

$$D_T(x_F, Q^2)$$

Feynman variable $x_F \equiv \frac{2P \cdot q}{Q^2}$

Gribov-Lipatov reciprocity

DGLAP equation $$\frac{\partial}{\partial \ln Q^2} D_{S/T}(j, Q^2) = \gamma_{S/T}(j) D_{S/T}(j, Q^2)$$

An intriguing relation in DLA $$\gamma_T(j) = \gamma_S(j + 2\gamma_T(j))$$ Mueller (1983)

The two anomalous dimensions derive from a single function

$$\gamma_S(j) = f(j - \gamma_S(j))$$
$$\gamma_T(j) = f(j + \gamma_T(j))$$

Dokshitzer, Marchesini, Salam (2005)

Nontrivial check up to three loops (!) in QCD Mitov, Moch, Vogt (2006)

Average multiplicity

$$\gamma_S(j) = \frac{j}{2} - \frac{1}{2}\sqrt{2\sqrt{\lambda}(j - j_0)} \quad \xleftrightarrow{\text{crossing}} \quad \gamma_T(j) = -\frac{1}{2}\left(j - j_0 - \frac{j^2}{2\sqrt{\lambda}}\right)$$

Lipatov et al. (2005)
Brower et al. (2006)

$\lambda = g^2 N_c$

$$n(Q) \propto (Q/\Lambda)^{2\gamma_T(1)} = (Q/\Lambda)^{1-3/2\sqrt{\lambda}}$$

YH, Matsuo (2008)

c.f. in perturbation theory, $n(Q) \propto Q^{\sqrt{\frac{\lambda}{2\pi^2}}}$ Mueller (1981)

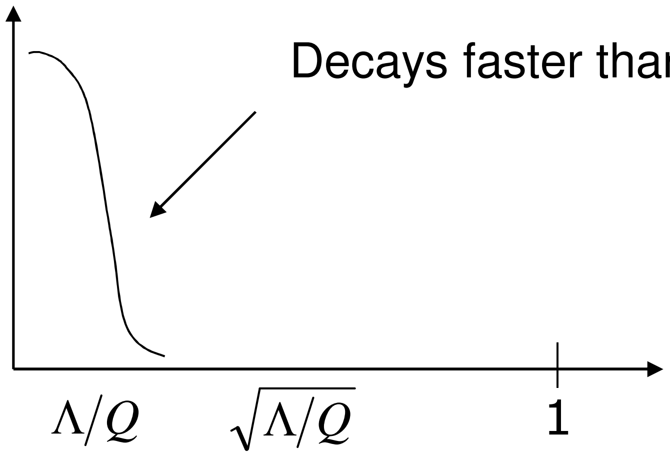
c.f. heuristic argument $n(Q) \propto Q$ YH, Iancu, Mueller (2008)

Jets at strong coupling?

$$\gamma_T(j) \approx 1 - \frac{j}{2}$$ in the supergravity limit $\lambda \to \infty$

The inclusive distribution is peaked at the kinematic lower limit

$$D_T(x, Q^2) = Q^2 F\left(\frac{Qx}{\Lambda}\right)$$



Decays faster than the power-law when $x > \Lambda/Q$

$x = 2E/Q$

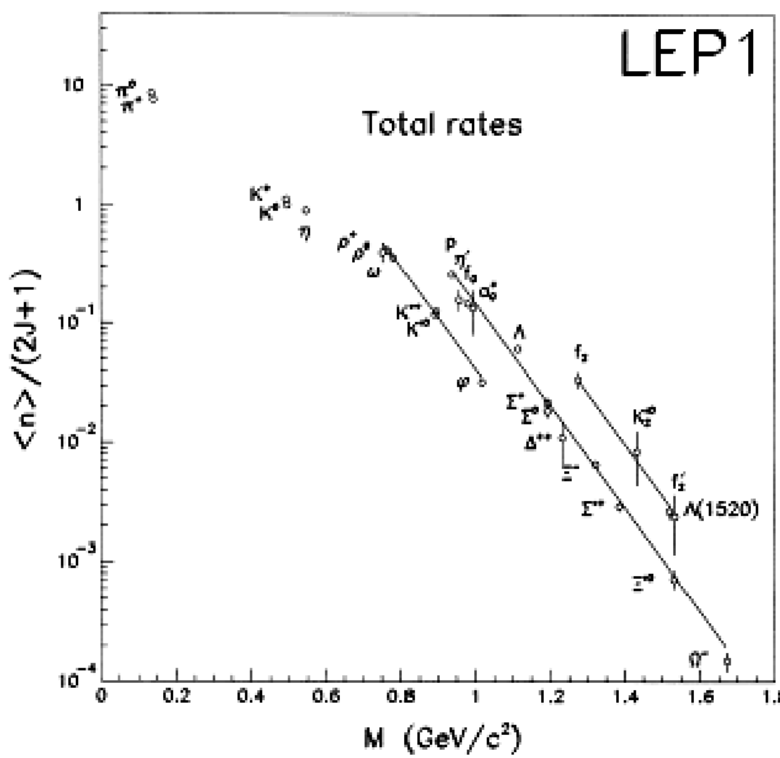
At strong coupling, branching is so fast and complete.
There are no “partons” at large-x.

Thermal hadron production

Identified particle yields are well described by a thermal model

$$\frac{N^*}{N} \propto \exp\left(-\frac{M^* - M}{T}\right)$$

$T \sim 170\ \mathrm{MeV}$

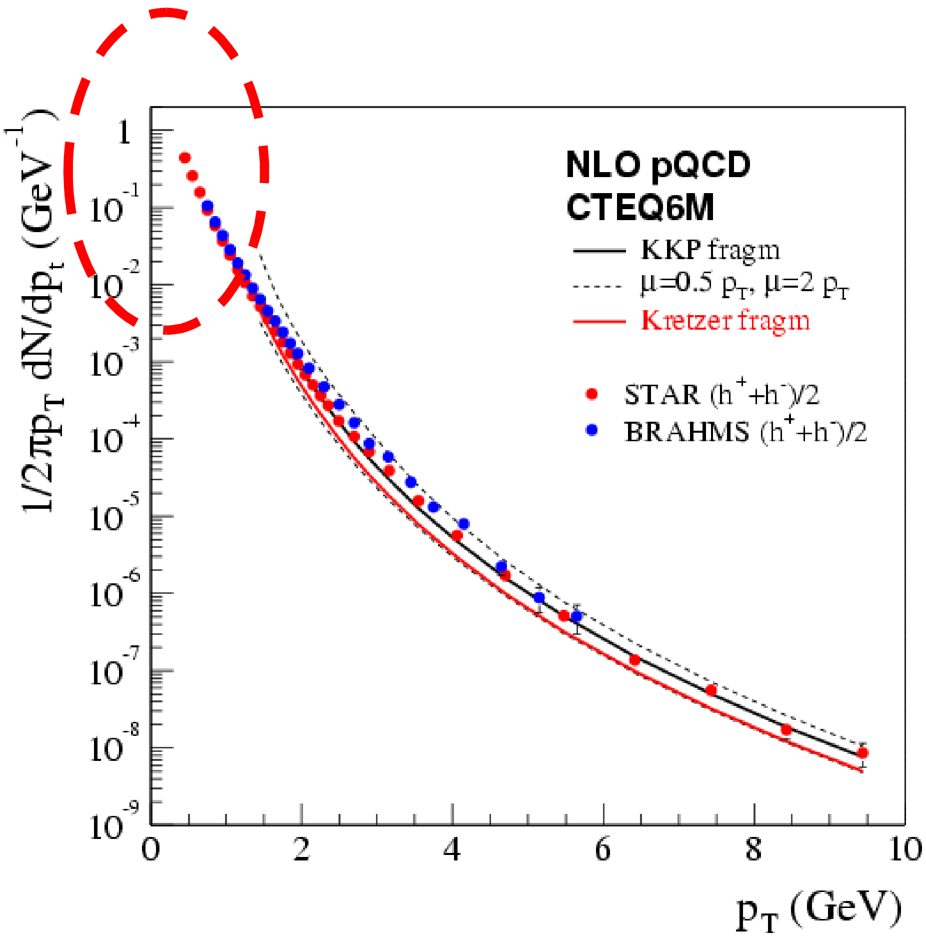


The model works in e+e- annihilation, hadron collisions, and heavy-ion collisions

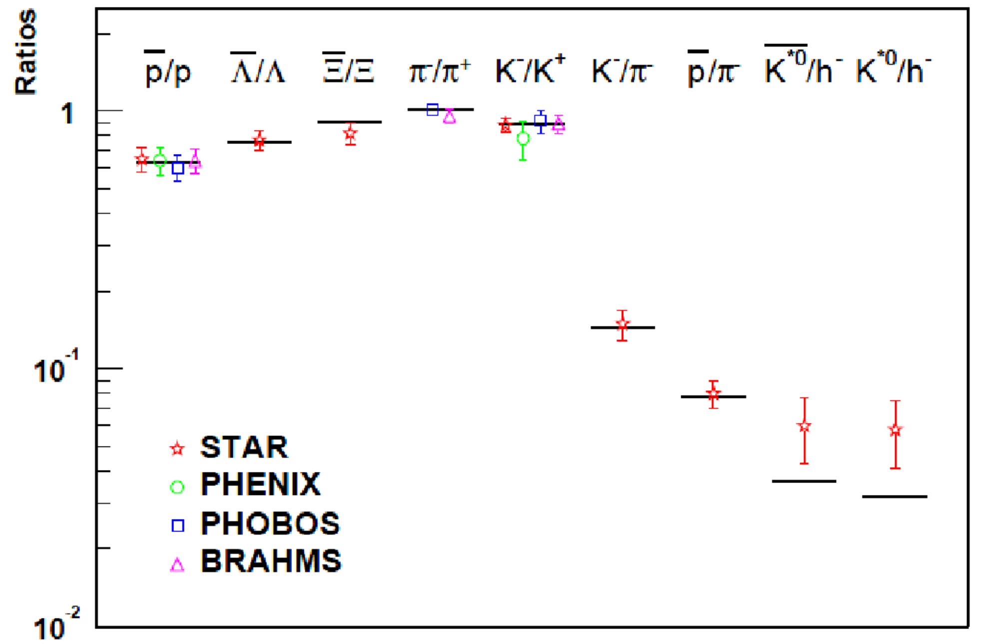
Becattini (1996)
Chliapnikov,
Braun-Munzinger et al.

Hadronization not understood in perturbation theory.
Need nonperturbative arguments, e.g., Kharzeev et al. (2007)

Thermal hadron production at RHIC



Charged hadrons in pp at 200 GeV



Multiplicity ratios in AA

A statistical model

Bjorken and Brodsky (1970)

Cross section to produce exactly $n \sim \mathcal{O}(Q/\Lambda)$ 'pions'

$$\sigma_n = \frac{e^4}{2Q^6}(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k') \prod_{i=1}^{n} \int \frac{d^3 p_i}{2E_i (2\pi)^3}$$

$$\times \langle 0 | j^\mu(0) | p_1, \ldots p_n \rangle \langle p_1, \ldots p_n | j^\nu(0) | 0 \rangle (2\pi)^4 \delta^{(4)}(q - \sum_i p_i)$$

Assume

$$\langle 0 | j^\mu(0) | p_1, \ldots p_n \rangle \langle p_1, \ldots p_n | j^\nu(0) | 0 \rangle$$

$$\to a_n (q^\mu q^\nu - g^{\mu\nu} q^2) e^{-\beta Q}$$

Then

$$2E \frac{dN}{d^3 p} \sim e^{-\beta E}$$

Thermal distribution from gauge/string duality

$$\langle 0|\epsilon\cdot j(0)|p_1,...p_n\rangle \sim \frac{g_c^{n+1}}{\alpha' g_c^2}\int dz d\Omega_5 \sqrt{-G} F(\alpha'\partial^2)(\Phi)^n A_\mu$$

string amplitude

5D hadron w.f.

5D photon

Polchinski, Strassler (2001)

When $n \sim \mathcal{O}(1)$ amplitude dominated by $z_s \sim \frac{1}{p}$

⟶ Dimensional counting rule by Brodsky, Farrar (1973)

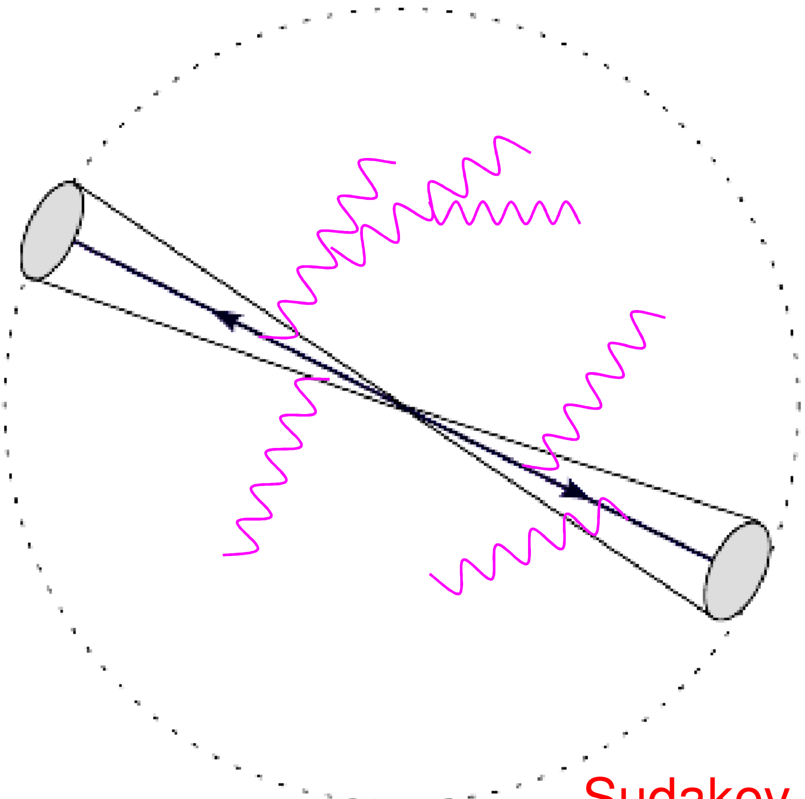
YH, Matsuo (2008)

When $n \sim \mathcal{O}(Q)$ and $A_\mu \propto H_1^{(1)}(Qz) \sim e^{iQz}$

Saddle point at $z_s \sim \frac{i}{\Lambda}$ $\quad e^{iQz_s} \sim e^{-Q/\Lambda}$

Thermal !

Away-from-jets region



Gluons emitted at large angle,
insensitive to the collinear singularity

Resum only the soft logarithms

$$\left(\alpha_s \ln 1/x\right)^n$$

There are two types of logarithms.

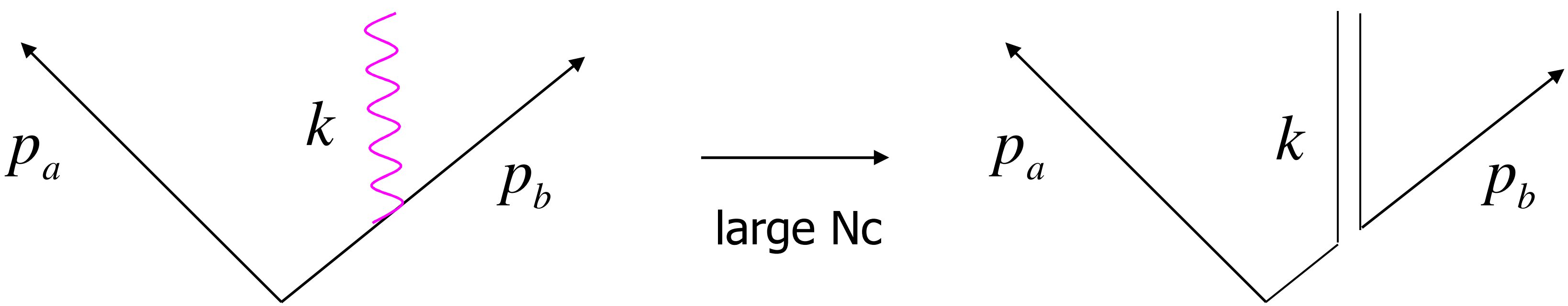
Sudakov logs. (emission from primary partons)
Kidonakis, Oderda, Sterman (1997)
Non-global logs. (emission from secondary gluons)
Dasgupta, Salam (2001)

Marchesini-Mueller equation

Marchesini, Mueller (2003)

Differential probability for the soft gluon emission

$$dP = \bar{\alpha}_s \omega d\omega \frac{d\Omega_k}{4\pi} \frac{p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} \approx \bar{\alpha}_s \frac{d\omega}{\omega} \frac{d\Omega_k}{4\pi} \frac{1 - \cos\theta_{ab}}{(1 - \cos\theta_{ak})(1 - \cos\theta_{bk})}$$



Evolution of the interjet gluon number. Non-global logs included.

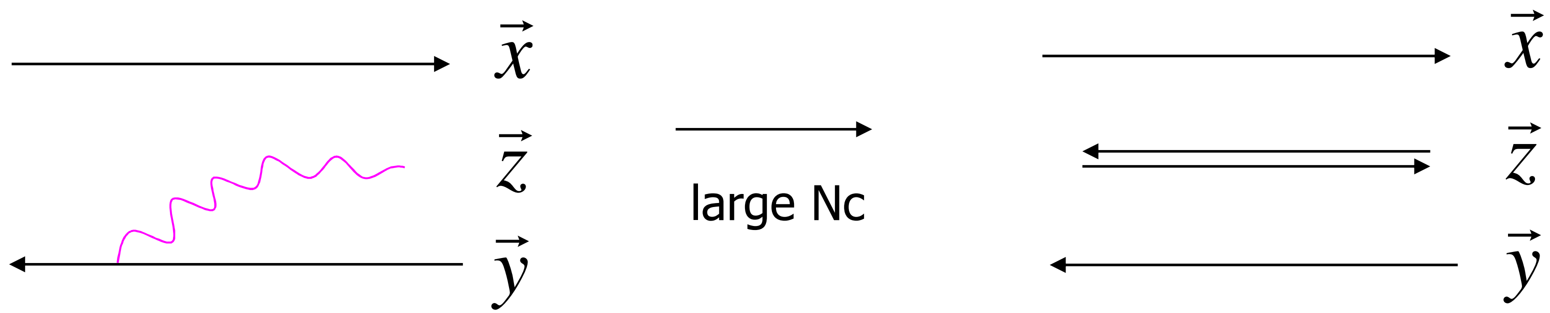
$$\partial_Y n(\theta_{ab}, \theta_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2\Omega_k}{4\pi} \frac{1 - \cos\theta_{ab}}{(1 - \cos\theta_{ak})(1 - \cos\theta_{bk})}$$
$$\times \left(n(\theta_{ak}, \theta_{cd}, Y) + n(\theta_{bk}, \theta_{cd}, Y) - n(\theta_{ab}, \theta_{cd}, Y) \right).$$

$Y = \ln 1/x$

BFKL equation

Differential probability for the dipole splitting

$$dP = \overline{\alpha}_s \frac{d\omega}{\omega} d^2\vec{z} \frac{(\vec{x}-\vec{y})^2}{(\vec{x}-\vec{z})^2(\vec{z}-\vec{y})^2}$$



Dipole version of the BFKL equation Mueller (1995)

$$\partial_Y n(x_{ab}, x_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2\vec{x}_k}{2\pi} \frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2}$$
$$\times \left(n(x_{ak}, x_{cd}, Y) + n(x_{bk}, x_{cd}, Y) - n(x_{ab}, x_{cd}, Y) \right)$$

BFKL dynamics in jets

The two equations become formally identical after the small angle approximation

$$1 - \cos\theta \approx \theta^2/2 \qquad d^2\Omega \approx d^2\vec{\theta}.$$

$$n(\theta_{ab}, \theta_{cd}, Y) \sim n(x_{ab}, x_{cd}, Y) \sim e^{4\bar{\alpha}_s \ln 2\, Y}$$

The interjet soft gluon number grows like the BFKL Pomeron !

Question : Is this just a coincidence, or is there any deep relationship between the two processes ?

⟶ Hint from AdS/CFT

Two Poincaré coordinates

Cornalba (2007)

AdS_5 as a hypersurface in 6D

$$W_{-1}^2 + W_0^2 - W_1^2 - W_2^2 - W_3^2 - W_4^2 = R^2$$

Introduce two Poincaré coordinate systems

Our universe

Poincaré 1 : $W_{-1} + W_4 = \frac{1}{z}, \quad W_\mu = \frac{x^\mu}{z}. \quad (\mu = 0,1,2,3)$

Poincaré 2 : $W_0 + W_3 = \frac{1}{y_5} \quad W_{-1} = -\frac{y^0}{y_5}, \quad W_4 = -\frac{y^3}{y_5}, \quad W_{1,2} = \frac{y^{1,2}}{y_5}$

Related via a conformal transformation on the boundary $\quad y^+ = -\frac{1}{2x^+}, \quad y^- = x^- - \frac{x_1^2 + x_2^2}{2x^+}, \quad \vec{y}_T = \frac{\vec{x}_T}{\sqrt{2}x^+}$

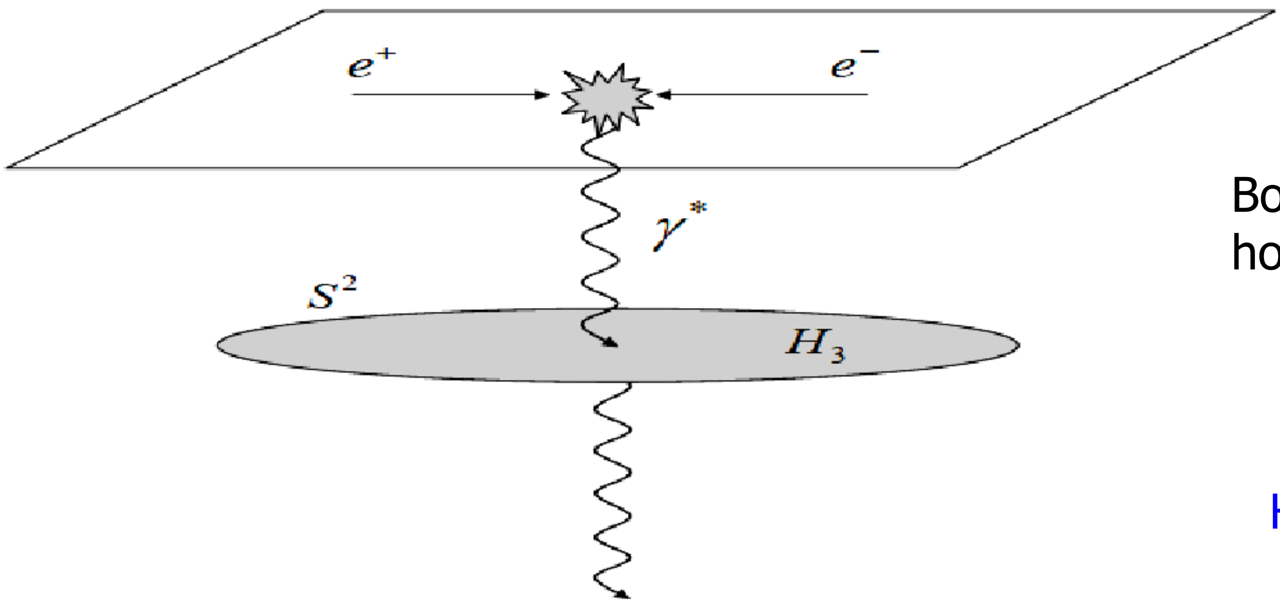
Shock wave picture of e+e- annihilation

YH (2008)

angular distribution of energy $\mathcal{E}(\Omega) \equiv \lim_{r\to\infty} r^2 \int_0^\infty dx^0\, n_i T^{0i}(x^0, r\vec{n})$

The sphere Ω can be mapped onto the transverse plane $\vec{y}_T$ of Poincare 2 via the stereographic projection

Treat the photon as a shock wave in Poincare 2 $T_{--} = q^+ \delta(y_5 - 1)\delta^{(2)}(\vec{y}_T)\delta(y^-)$



Boundary energy from the holographic renormalization

$$\langle \mathcal{E}(\Omega) \rangle = \frac{Q}{4\pi}$$

Hofman, Maldacena (2008)

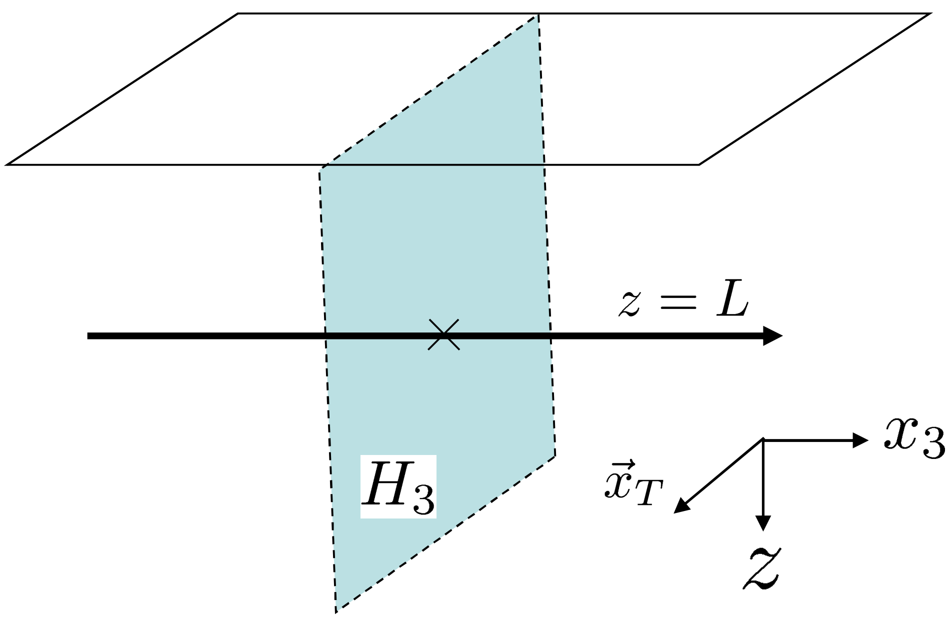
Shock wave picture of a high energy "hadron"

A color singlet state lives in the bulk.
At high energy, it is a shock wave in Poincare 1.

$$T^{++} = z^7 p^+ \delta(z - L)\delta^{(2)}(\vec{x}_T)\delta(x^-)$$

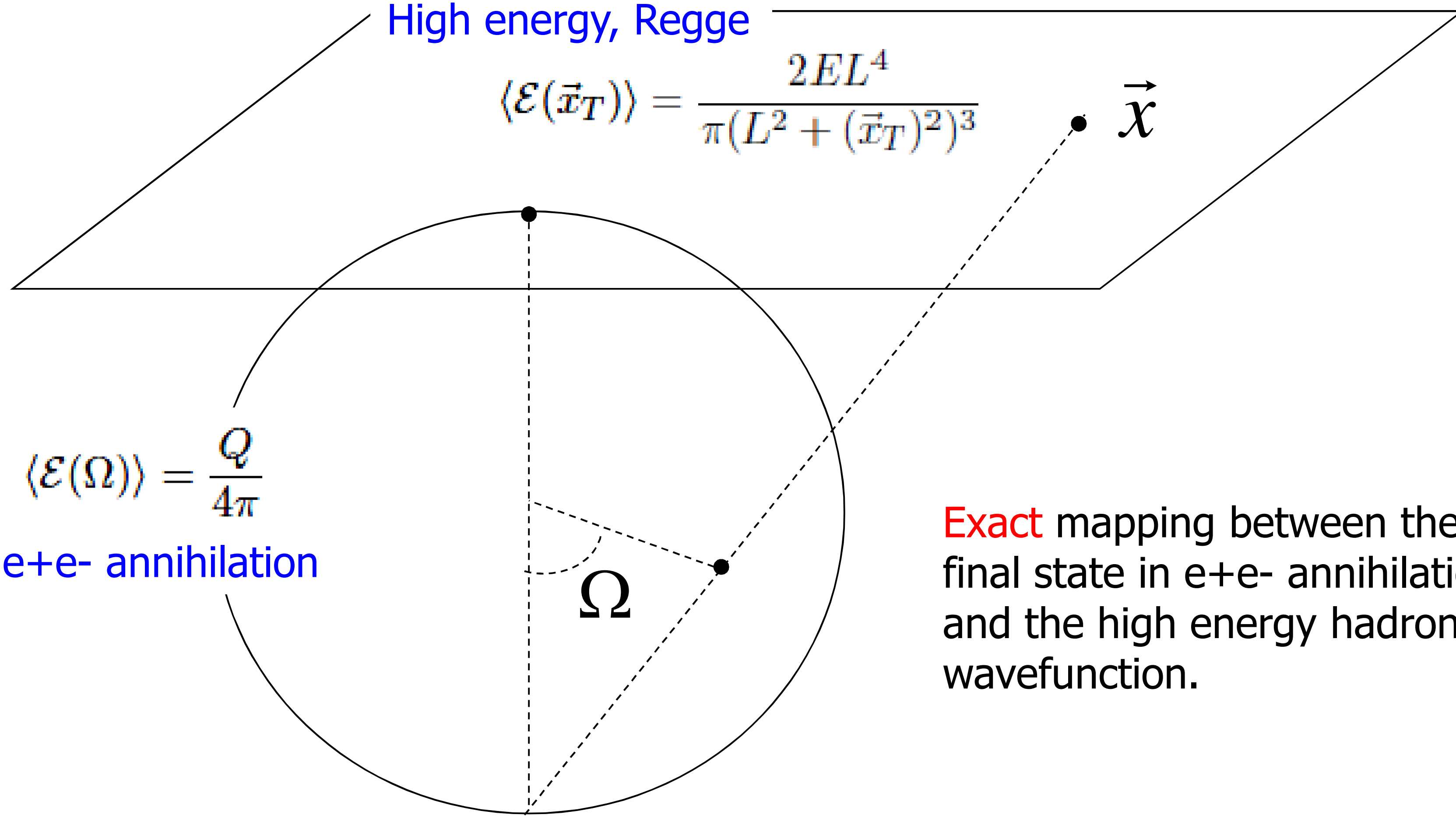
Energy distribution on the boundary transverse plane

$$\langle T^{++}(x^-, \vec{x}_T)\rangle \propto \lim_{z\to 0} \frac{1}{\kappa^2 z^2}\delta g_{\mu\nu} = \frac{2p^+ L^4}{\pi(L^2 + \vec{x}_T^2)^3}\delta(x^-)$$



Gubser, Pufu & Yarom (2008)

The stereographic map



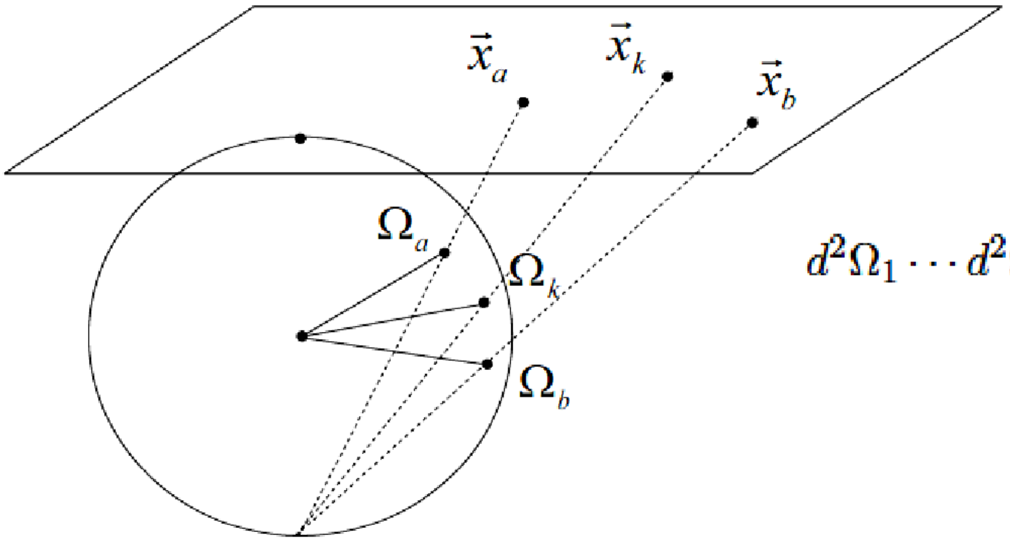
Exact mapping between the final state in e+e- annihilation and the high energy hadronic wavefunction.

The two processes are mathematically identical. The only difference is the choice of the coordinate system in which to express its physics content.

Exact map at weak coupling

The same stereographic map transforms BFKL into the Marchesini-Mueller equation

$$\frac{d^2\Omega_k}{4\pi}\frac{1-\cos\theta_{ab}}{(1-\cos\theta_{ak})(1-\cos\theta_{bk})} = \frac{d^2\vec{x}_k}{2\pi}\frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2}$$



k-gluon emission probability

$$d^2\Omega_1\cdots d^2\Omega_k\frac{1-\cos\theta_{ab}}{(1-\cos\theta_{a1})(1-\cos\theta_{12})\cdots(1-\cos\theta_{kb})}$$

$$= d^2x_1\cdots d^2x_k\frac{x_{ab}^2}{x_{a1}^2x_{12}^2\cdots x_{kb}^2}$$

Make the most of conformal symmetry SL(2,C) of the BFKL kernel.
Exact solution to the Marchesini—Mueller equation YH (2008)
and much more ! Avsar, YH, Matsuo (2009)

The issue of the evolution ‘time’

Timelike gluon cascade → ordered in the transverse momentum,
the angle is more or less constant.

The evolution time $$Y_t = \ln \frac{Q}{p_t}$$

Spacelike gluon cascade → ordered in angle, the transverse momentum
is more or less constant.

The evolution time $$Y_s = \ln \frac{Q}{1/\theta}$$

Obstacle to the equivalence? NO !
The stereographic projection is clever enough. Avsar, YH, Matsuo (2009)

$$p_t \leftrightarrow x_t \sim \frac{1}{\theta} \qquad\qquad Y_t \leftrightarrow Y_s$$

NLL timelike dipole evolution in N=4 SYM

Stereographic projection works both in the weak and strong coupling limits

$\lambda \to 0$ and $\lambda \to \infty$ Valid to all orders?

Apply the stereographic projection to the result by Balitsky & Chirilli (2008).

$$\partial_Y n_Y(\Omega_{ab}) = \bar{\alpha}_s \left(1 - \bar{\alpha}_s \frac{\pi^2}{12}\right) \int d^2\Omega_c K_{ab}(\Omega_c) \left[n_Y(\Omega_{ac}) + n_Y(\Omega_{cb}) - n_Y(\Omega_{ab})\right]$$
$$+\bar{\alpha}_s^2 \int d^2\Omega_c d^2\Omega_d K'_{ab}(\Omega_c, \Omega_d) n_Y(\Omega_{cd}) \,,$$

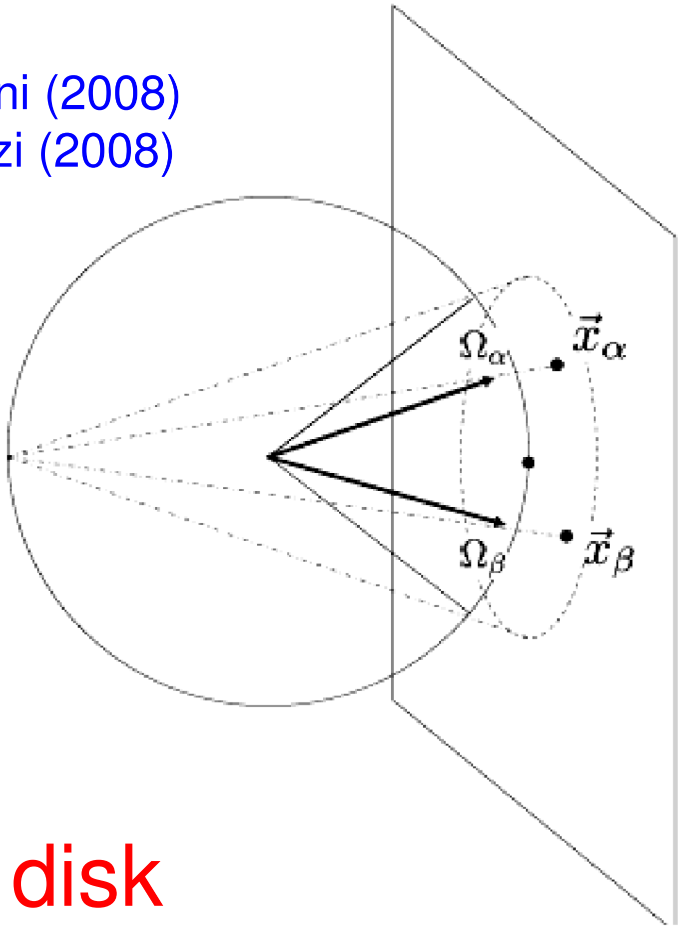
$$K'_{ab}(\Omega_c, \Omega_d) = \frac{1}{8\pi^2}\left\{ \frac{(1-\cos\theta_{ab})}{(1-\cos\theta_{ac})(1-\cos\theta_{cd})(1-\cos\theta_{db})} \right.$$
$$\times \left[\left(1 + \frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ac})(1-\cos\theta_{bd}) - (1-\cos\theta_{ad})(1-\cos\theta_{bc})} \right) \right.$$
$$\left. \times \ln\frac{(1-\cos\theta_{ac})(1-\cos\theta_{bd})}{(1-\cos\theta_{ad})(1-\cos\theta_{bc})} + 2\ln\frac{(1-\cos\theta_{ab})(1-\cos\theta_{cd})}{(1-\cos\theta_{ad})(1-\cos\theta_{bc})} \right]$$
$$\left. + 12\pi^2\zeta(3)\delta^{(2)}(\Omega_{ac})\delta^{(2)}(\Omega_{bd}) \right\}.$$

Energy flow as a jet identifier

YH & Ueda, 0909.0056

Want to discriminate highly boosted ($p_t \sim 1\ \mathrm{TeV}$) weak-boson jets from the QCD background.

Agashe, Gopalakrishna, Han, Huang, Soni (2008)
Almeida, Lee, Perez, Sterman, Sung, Virzi (2008)



Quantify the amount of energy radiated outside the jet cone in the two cases. Less energy in the weak-jet case due to the QCD coherence.

SL(2,C) conformal symmetry broken down to a subgroup SU(1,1).

Jet cone = Poincare disk

Summary

- Timelike anomalous dimension and multiplicity computed in strongly coupled N=4 SYM. Hadron spectrum thermal.

- Exact map between the final state in e+e- and hadron w.f. in the transverse plane. Works both at weak and strong coupling.