QCD at high energy and statistical physics

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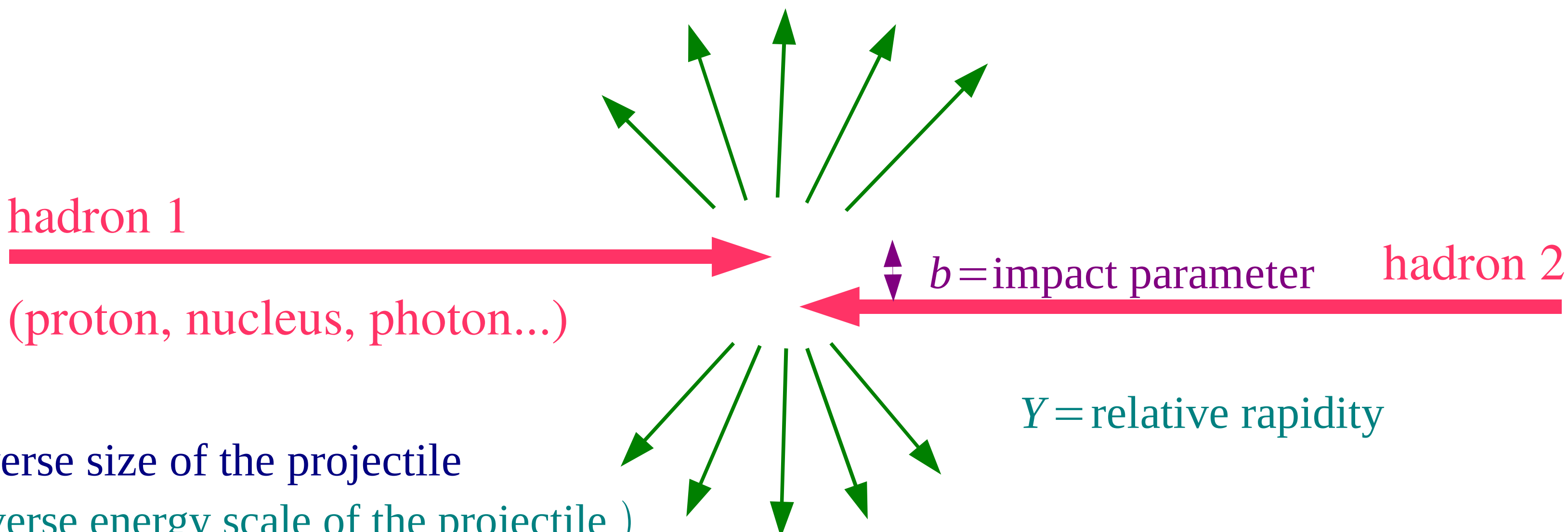
High-energy QCD



r : transverse size of the projectile

$(k$: transverse energy scale of the projectile $)$

$\rho = \log(k^2/\Lambda^2)$ or $\log(1/(r^2\Lambda^2))$

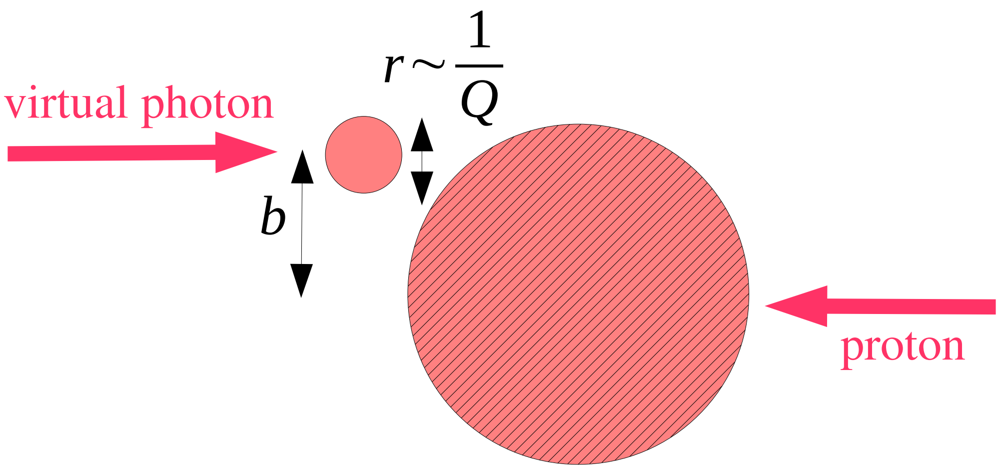
$$A(Y,r) = \int d^2b\, A(Y,b,r) = \text{elastic amplitude}$$

$$A(Y,b,r) = \text{fixed impact parameter amplitude}$$

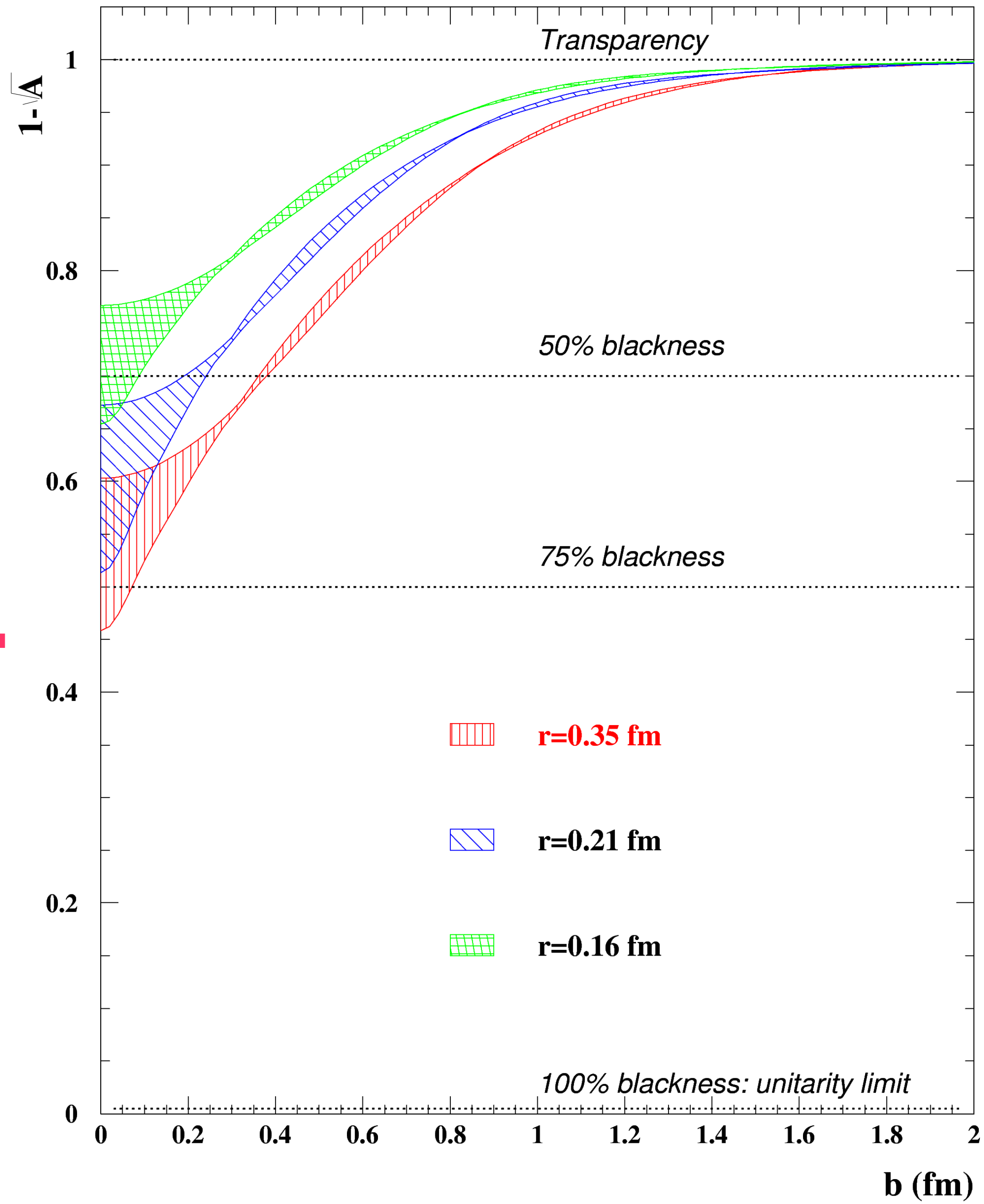
$$\sim \text{interaction probability} \leq 1$$

"Measurement" of A at HERA

Scatter a virtual photon elastically...



...and measure *A* as a function of *b*:



SM, Stasto, Mueller (2001)

Work done in after my first stay at Columbia (2000)

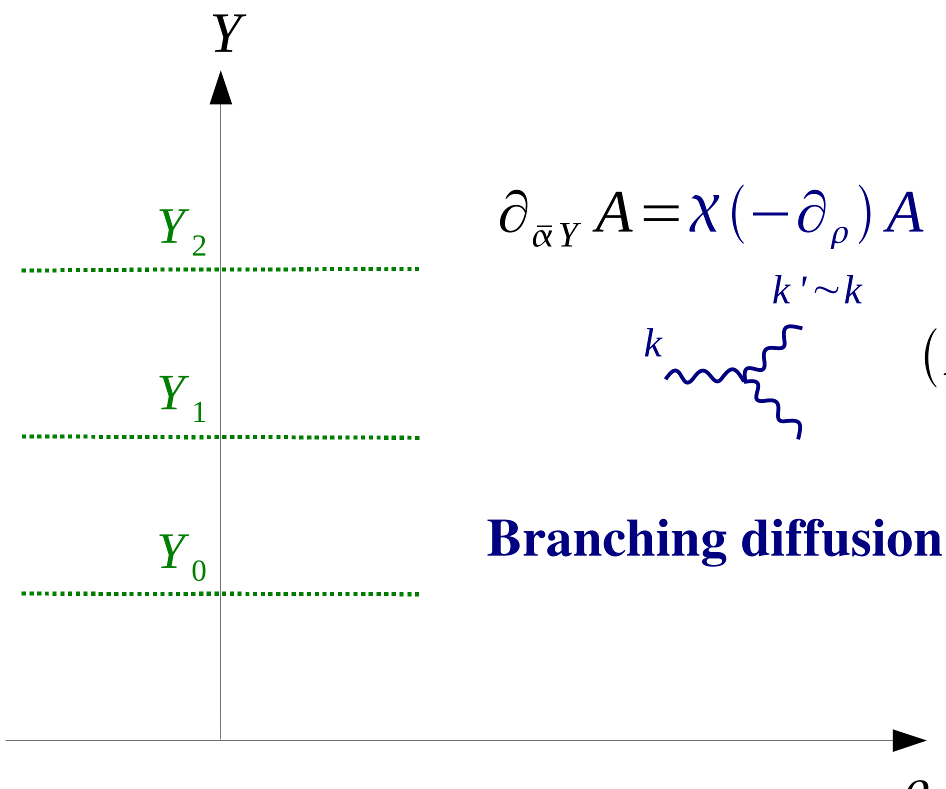
Outline

- ☆ Evolution equations for scattering amplitudes in high-energy QCD and their solutions
- ☆ Stochasticity/discreteness and its consequences: Example of a simple toy model
- ☆ What we have learnt so far for QCD

Evolution equations

BFKL equation

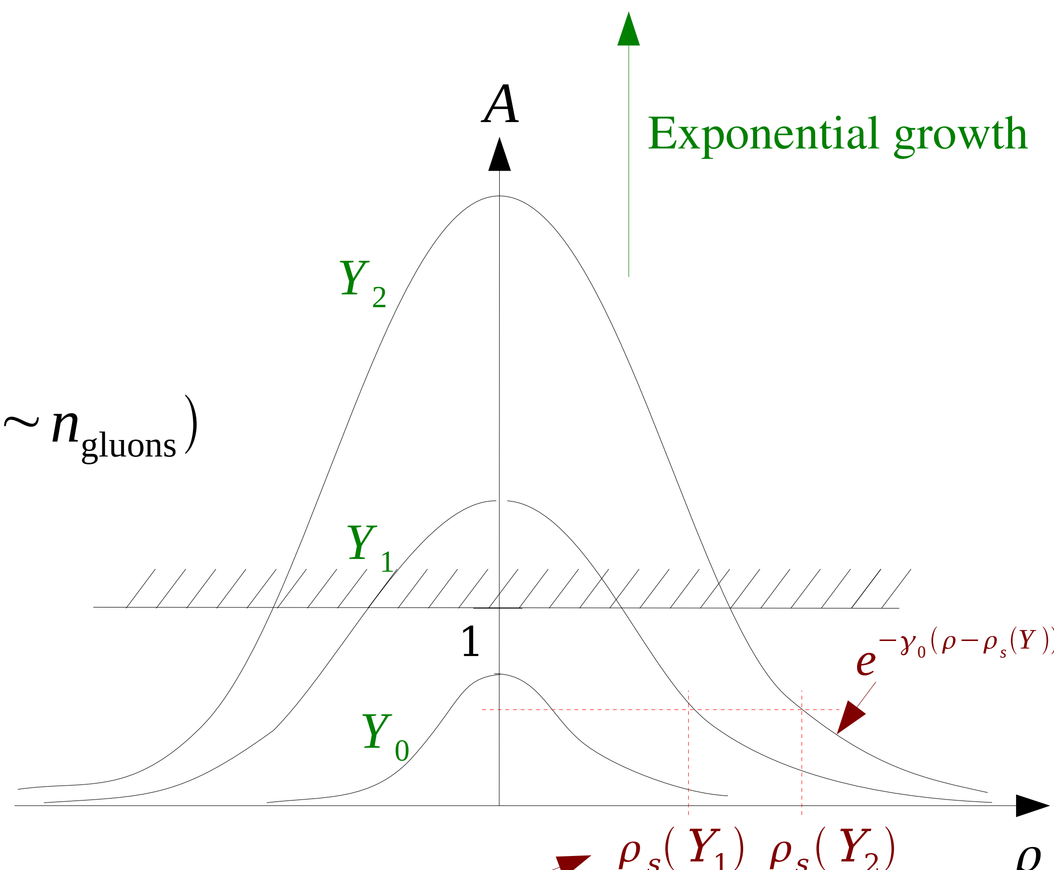
$$\partial_{\bar{\alpha} Y} A = \chi(-\partial_\rho) A$$



$$\partial_{\bar{\alpha} Y} A = \chi(-\partial_\rho) A$$

$$(A \sim n_{\text{gluons}})$$

Branching diffusion



Saturation scale; governs the rapidity-dependence

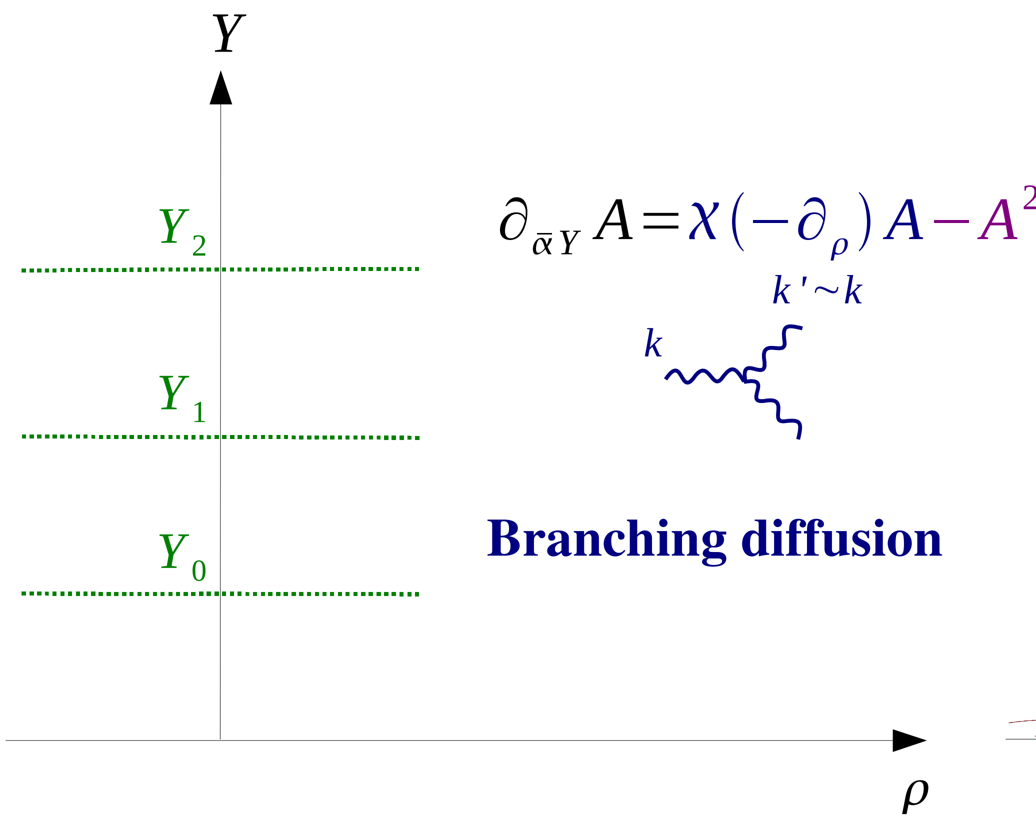
$$\frac{d\rho_s}{dY} = \bar{\alpha}\frac{\chi(\gamma_0)}{\gamma_0} = \bar{\alpha}\chi'(\gamma_0)$$

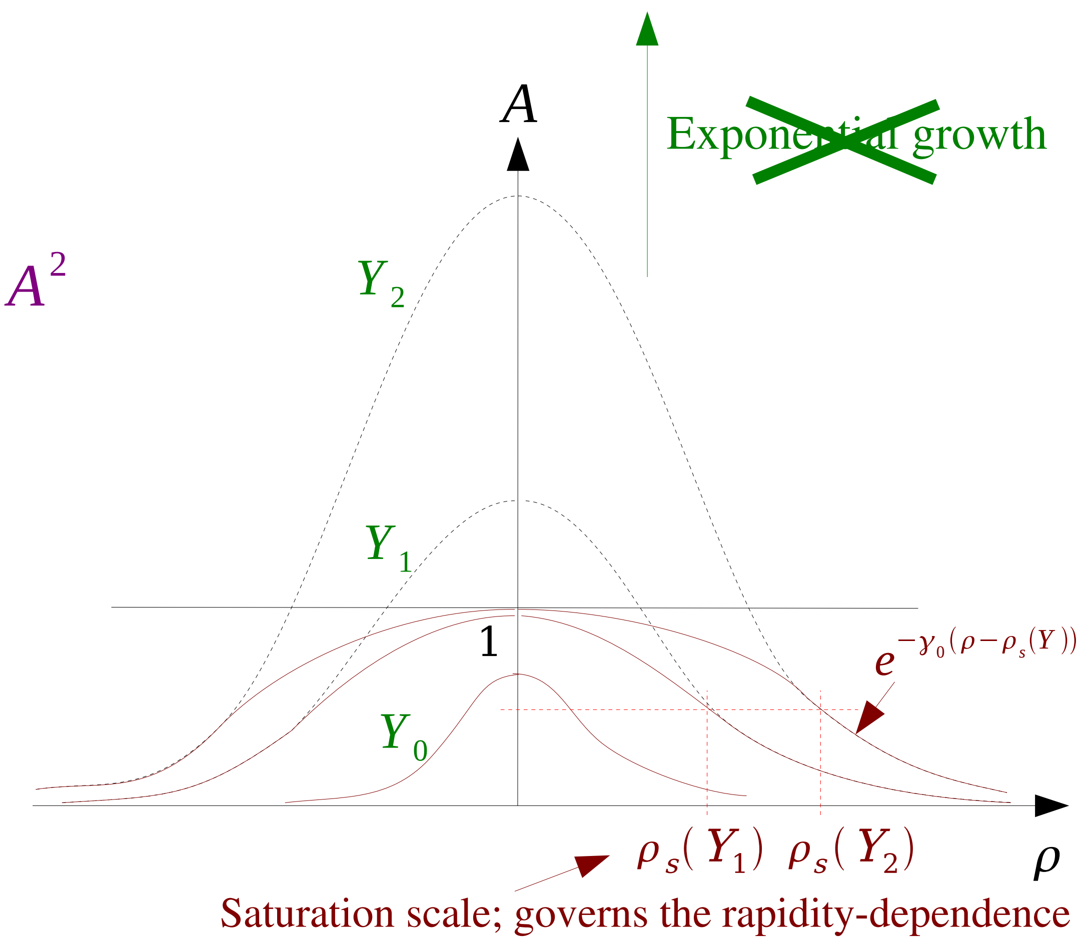
Gribov, Levin, Ryskin (1983)
Golec-Biernat, Motyka, Stasto (2001)
Iancu, Itakura, McLerran (2002)

Evolution equations

BK equation

$$\partial_{\bar{\alpha} Y} A = \chi(-\partial_\rho) A - A^2$$





Evolution equations

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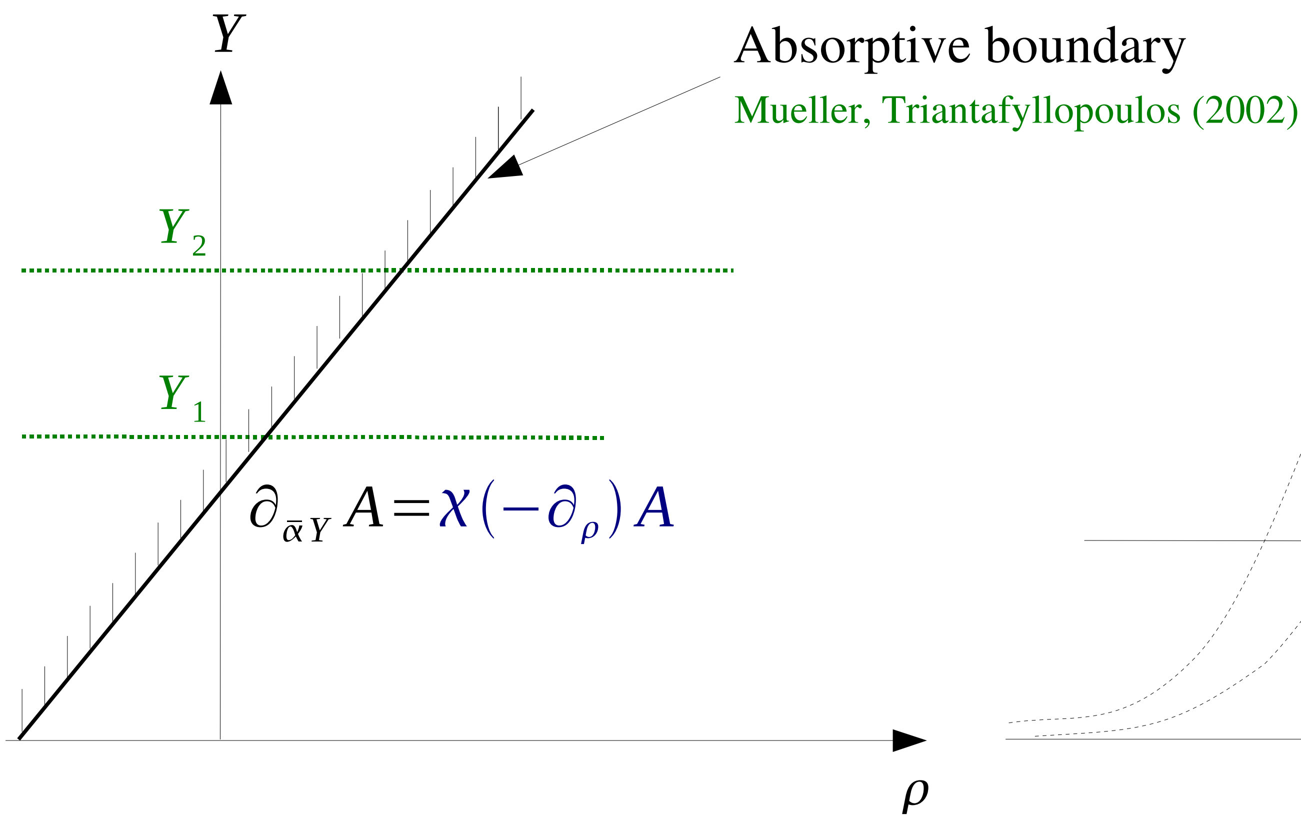
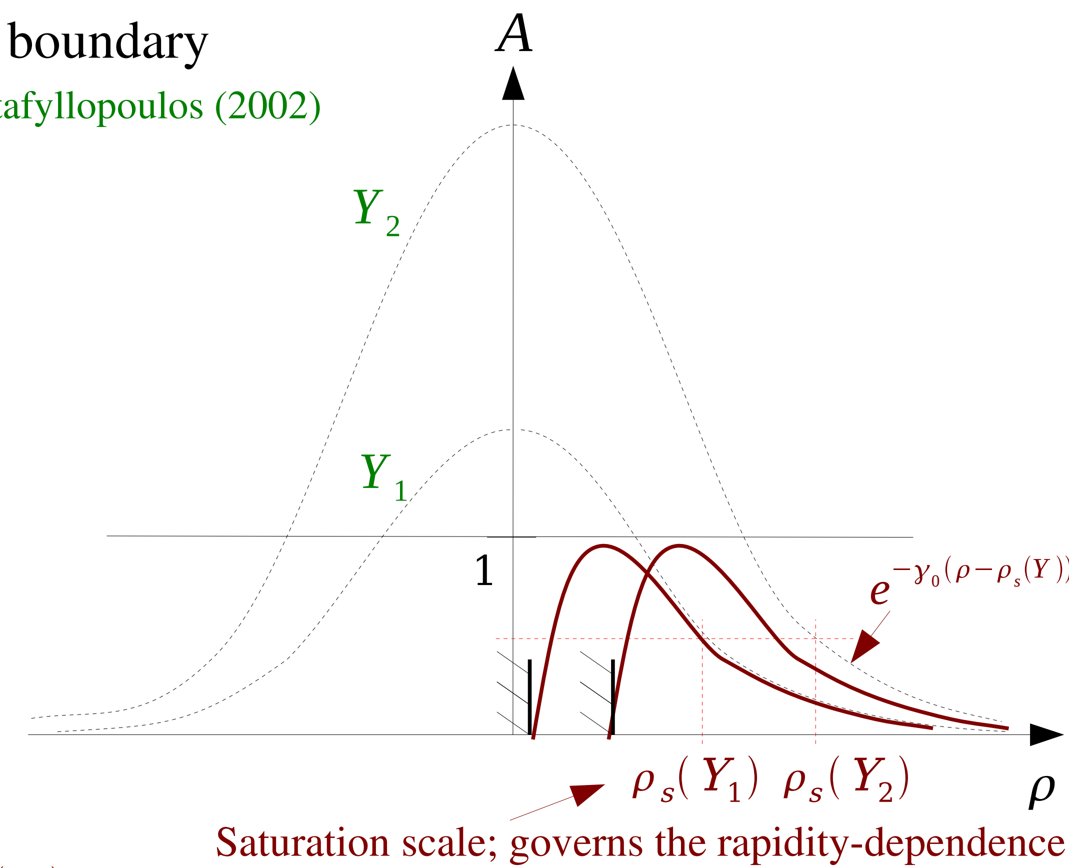




$$\frac{d\rho_s}{dY} = \bar\alpha \frac{\chi(\gamma_0)}{\gamma_0} - \frac{3}{2\gamma_0}\frac{1}{Y}$$

Mueller, Triantafyllopoulos (2002)

Evolution equations

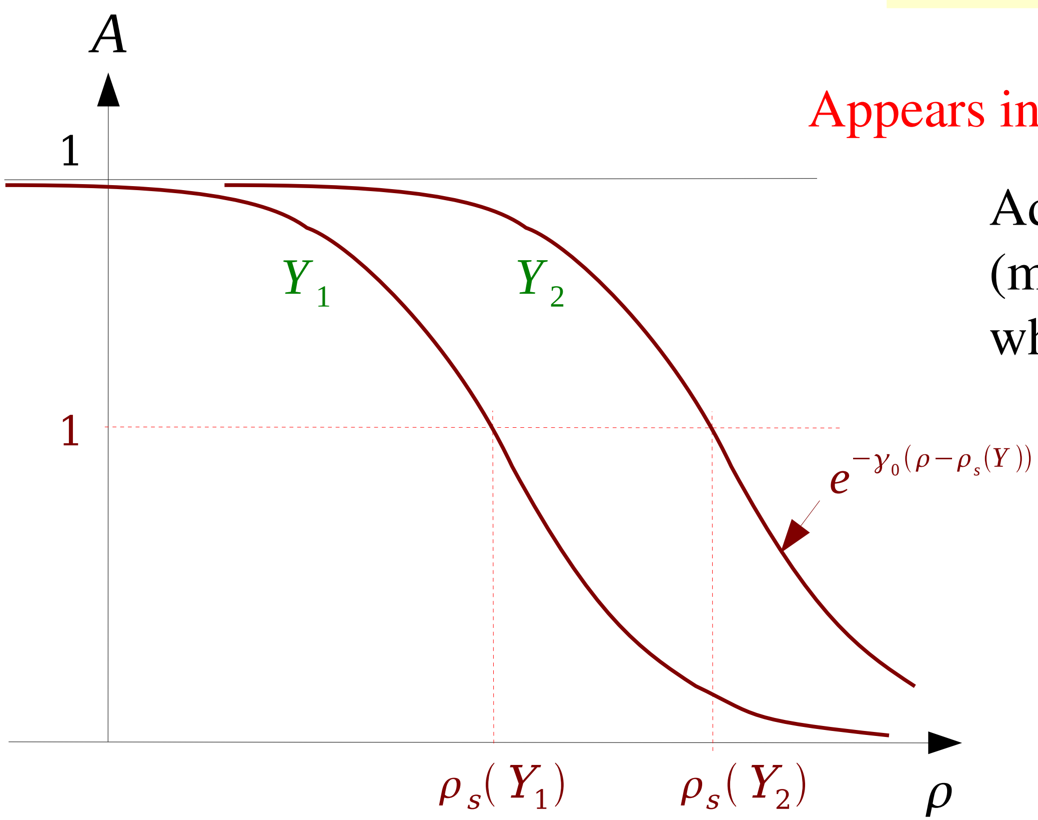
BK equation

$$\partial_{\bar{\alpha} Y} A = \chi(-\partial_\rho) A - A^2$$

branching diffusion of gluons: $\chi(-\partial_\rho) A \sim \partial_\rho^2 A + A$

BK equation $\sim$ $\partial_{\bar{\alpha} Y} A = \partial_\rho^2 A + A - A^2$ **FKPP equation**

Fisher; Kolmogorov, Petrovsky, Piscunov (1937)



Appears in a variety of problems: Reaction-diffusion etc...

Admits traveling wave solutions $A = A(\rho - \rho_s(Y))$
(mathematical result)
whose main characteristics are universal:

$$A \sim e^{-\gamma_0(\rho - \rho_s(Y))}$$

$$\frac{d\rho_s}{dY} = \bar{\alpha}\frac{\chi(\gamma_0)}{\gamma_0} - \frac{3}{2\gamma_0}\frac{1}{Y}$$

SM, Peschanski (2003)

Evolution equations

BK equation

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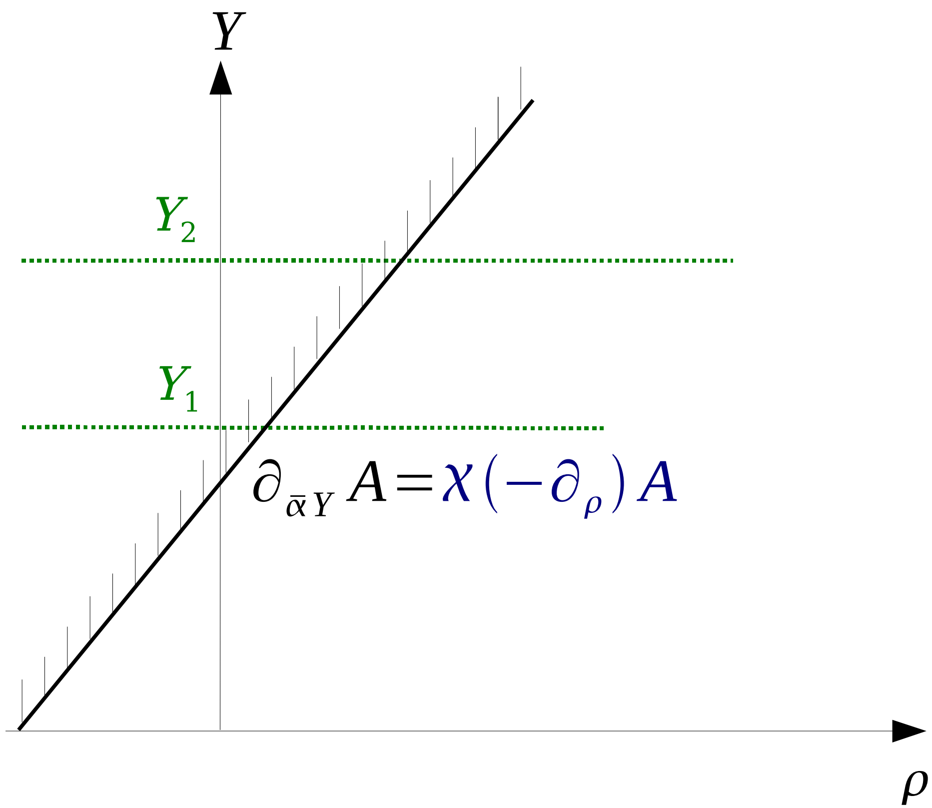
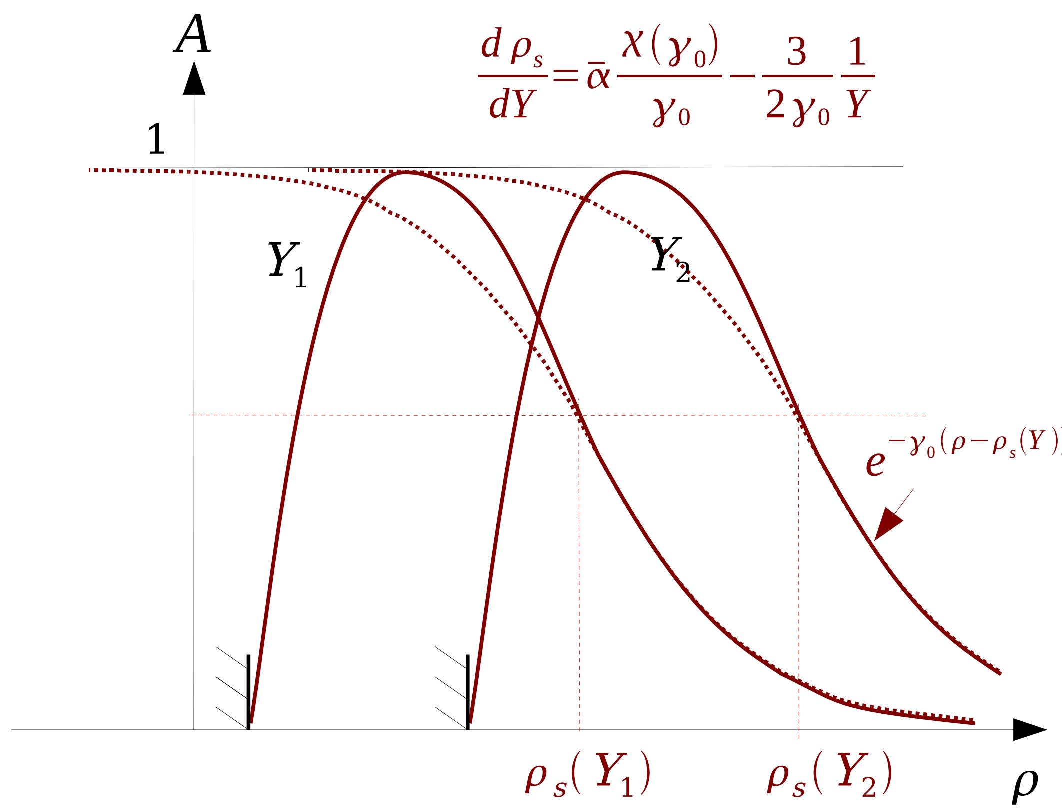




- The traveling wave property had been found in the data under the name « **geometric scaling** »

Stasto, Golec-Biernat, Kwiecinski (2000)

- The most sophisticated QCD equations known at that time **(B-JIMWLK)** were shown numerically to give results very close to BK...

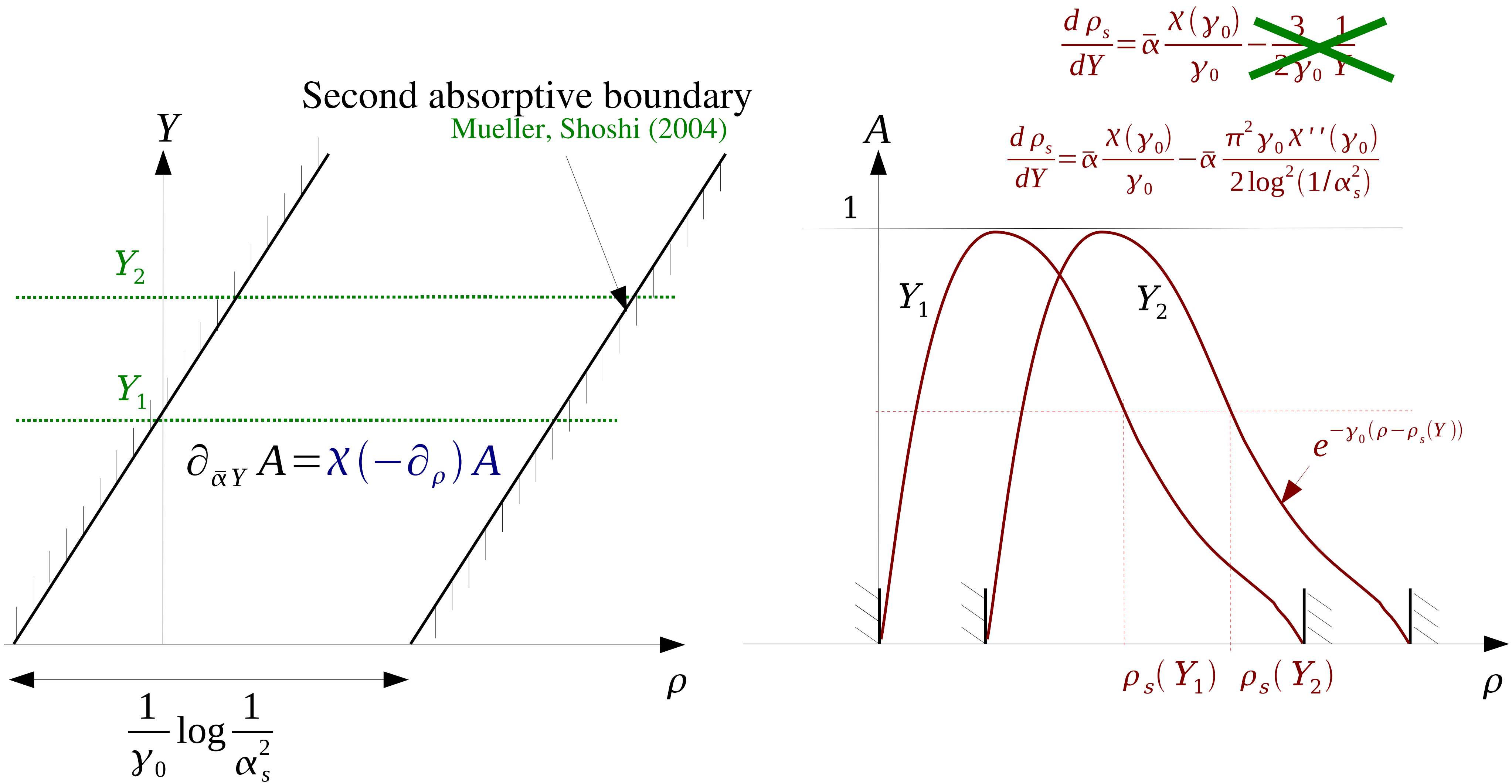
Balitsky (1995)

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1994-...)

Rummukainen, Weigert (2003)

The end of the story?

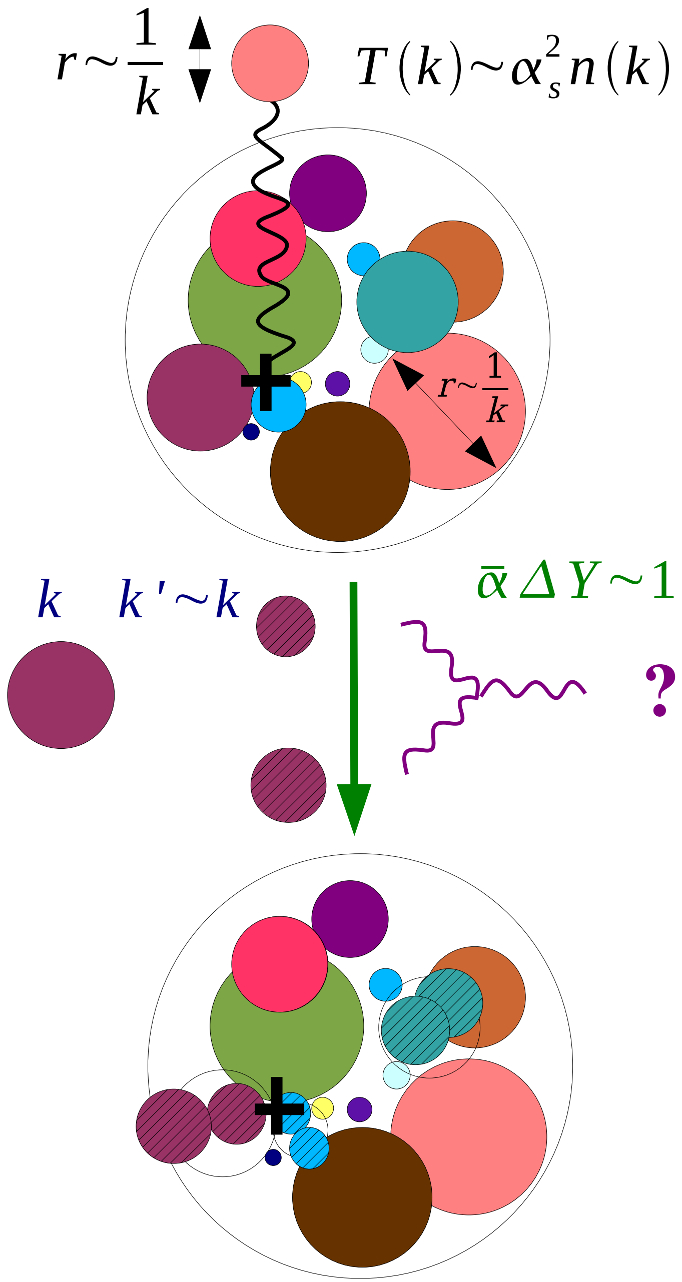
Beyond known equations

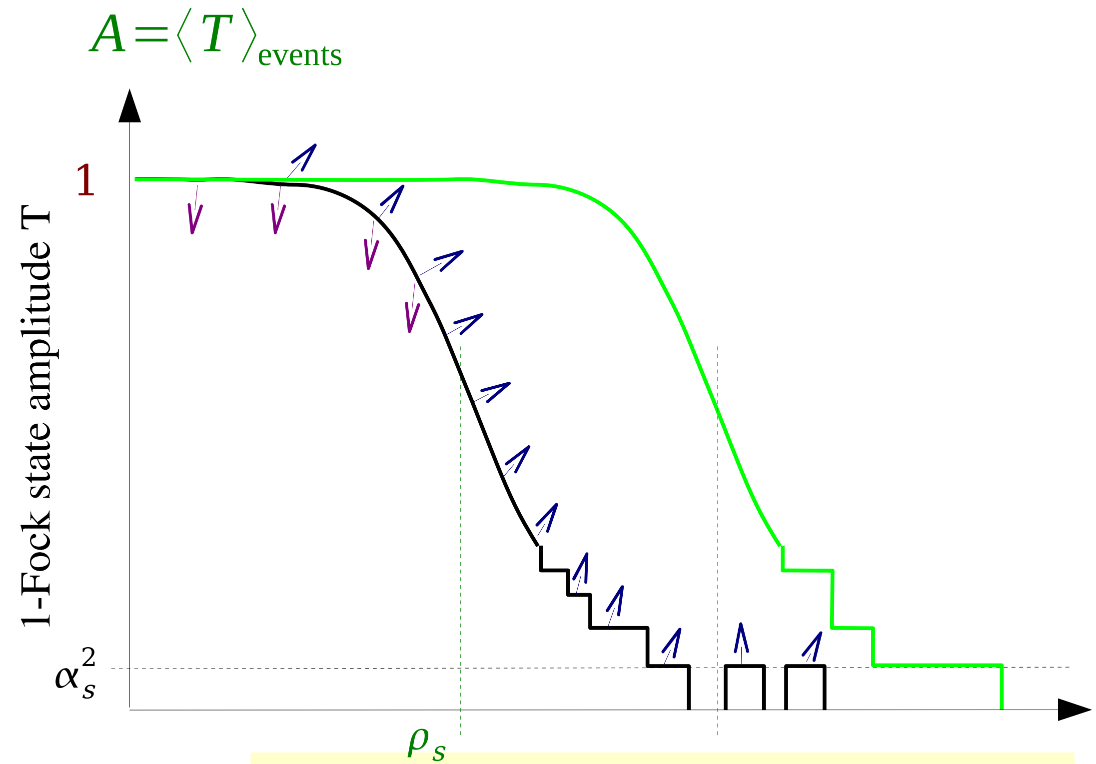


Even more weird: Predicted violations of geometric scaling!

Microscopic picture of the scattering process

Iancu, Mueller, Munier (2004)





$$\partial_{\bar{\alpha} Y} T = \chi(-\partial_\rho) T - T^2 + \alpha_s \sqrt{T}\, \nu$$

$\text{BFKL} \sim \partial_\rho^2 T + T$

Noise due to discreteness

Reggeon field theory equation

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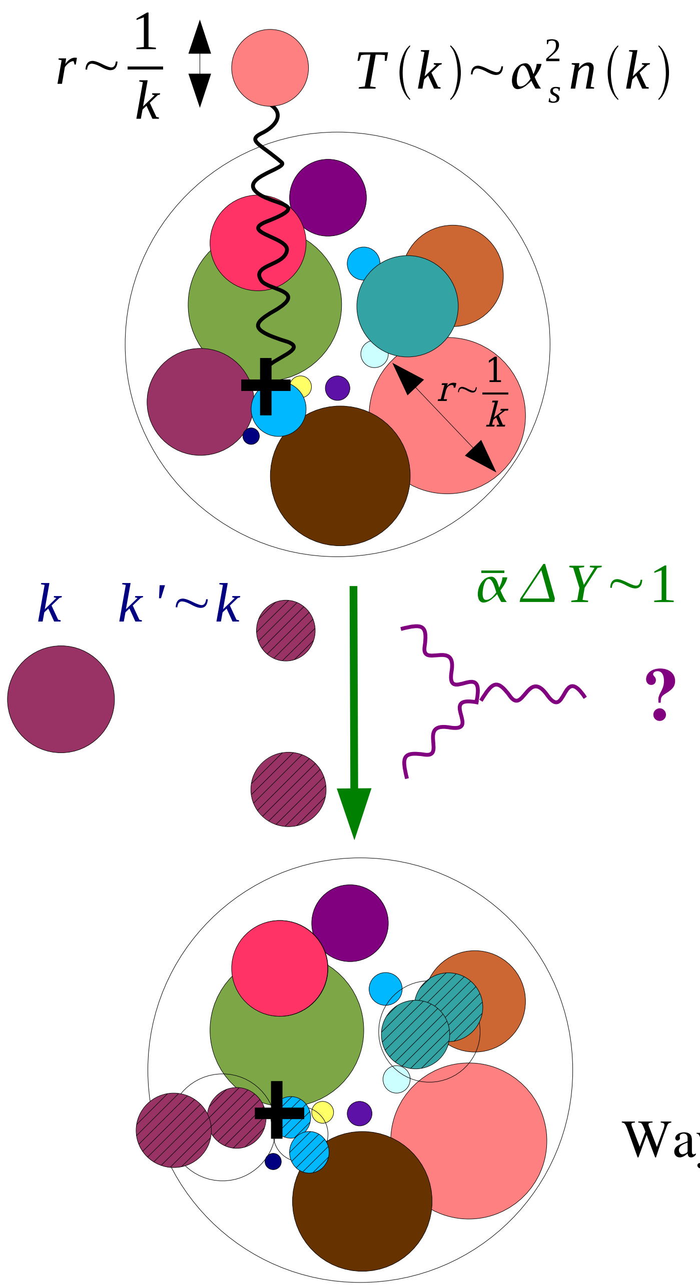

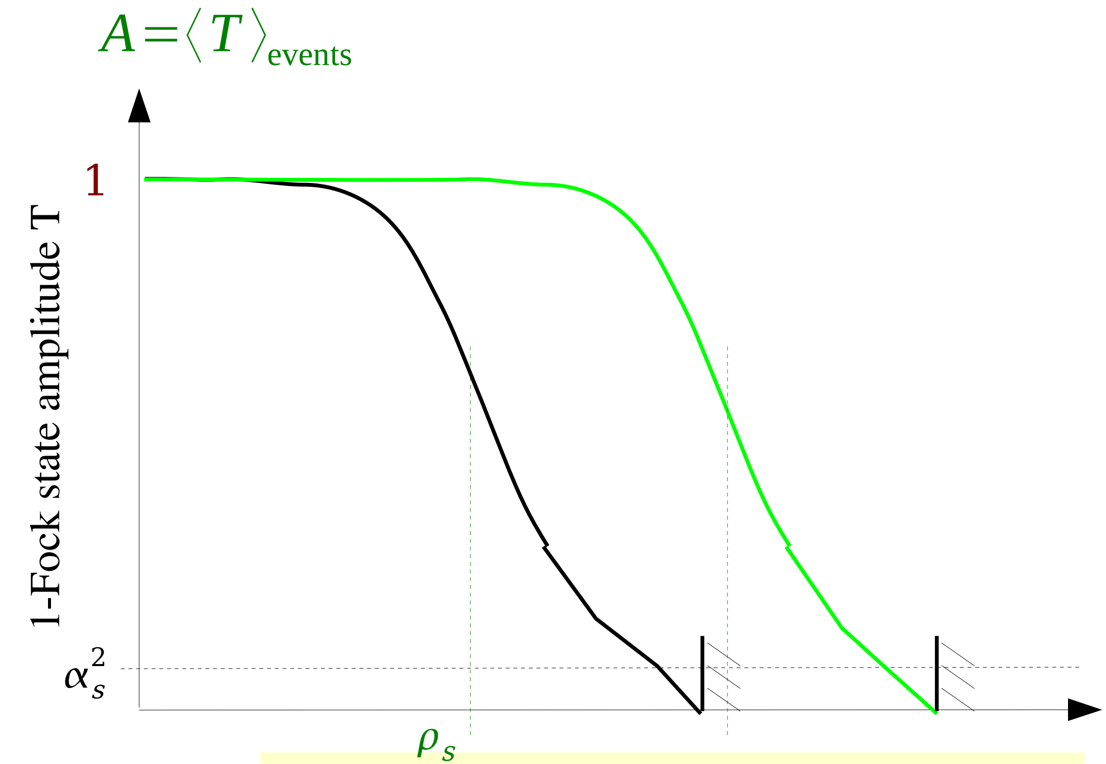

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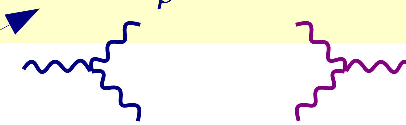

Noise due to discreteness

Reggeon field theory equation

Way to get the first-order effect of the noise: **put a cutoff!**

$$\frac{d\rho_s}{dY} = \bar{\alpha}\frac{\chi(\gamma_0)}{\gamma_0} - \bar{\alpha}\frac{\pi^2 \gamma_0 \chi''(\gamma_0)}{2\log^2(1/\alpha_s^2)}$$

Brunet, Derrida (1997)

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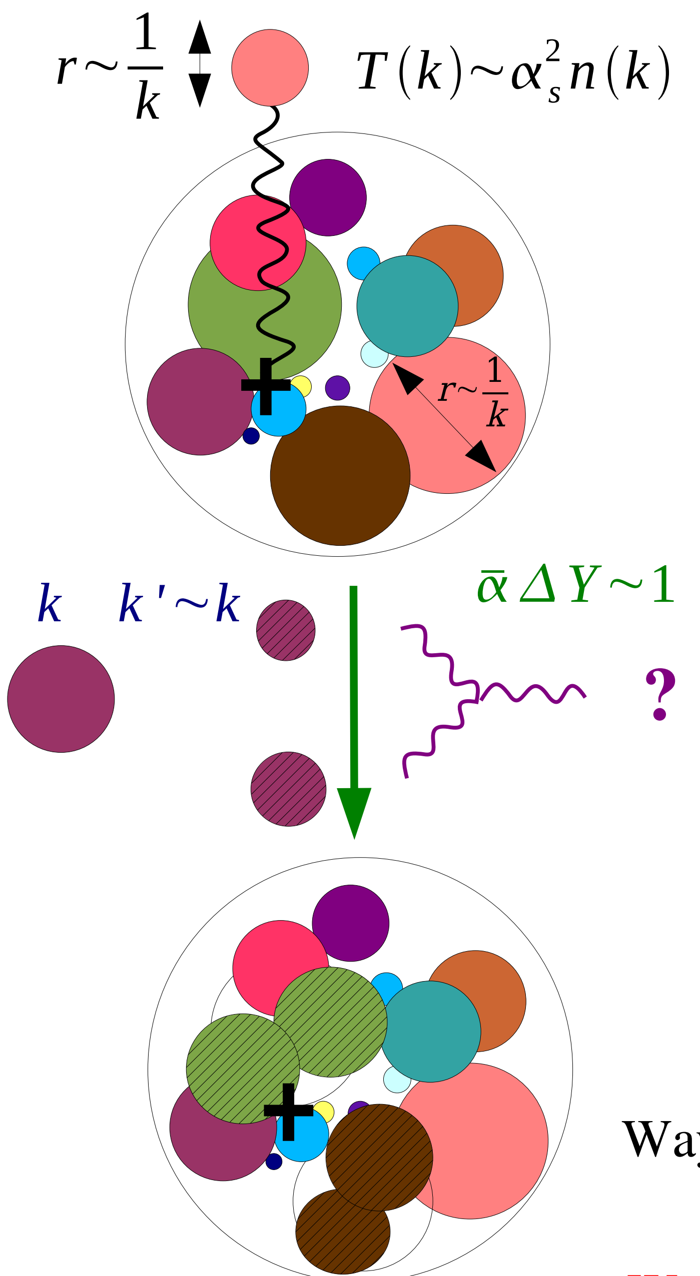
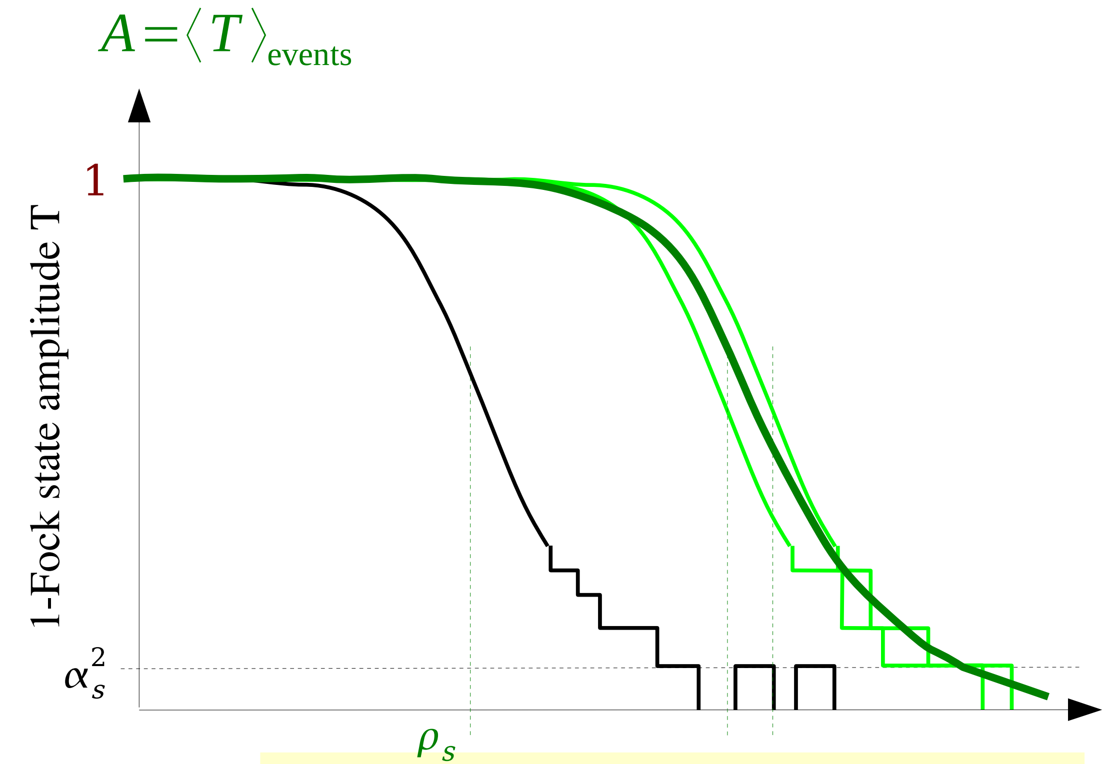
Iancu, Mueller, Munier (2004)





$$\partial_{\bar{\alpha}Y}T=\chi(-\partial_\rho)T-T^2+\alpha_s\sqrt{T}\,\nu$$

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Brunet, Derrida (1997)

We went beyond (in particular stochasticity of the saturation scale) more recently

Brunet, Derrida, Mueller, Munier (2005)

Summary of part I

We have seen that a nonlinear equation is needed to preserve the unitarity of scattering amplitudes: BK, B-JIMWLK equations

The nonlinearity is a priori difficult to treat mathematically, but it can be replaced by a **cutoff** in phase space. The found properties of the amplitude does not depend on the exact form of the nonlinearity.

Consistency with unitarity/relativity imposes to add another cutoff, that does not correspond to any known equation in QCD. Some progress in finding the corresponding equation is being done.

Iancu, Triantafyllopoulos (2005)
Levin et al.
Kovner, Lublinsky et al.

However, from physical argument, we know that the cutoff may represent the discreteness of the gluons, which results in a new noise term in the BK equation.

Outline

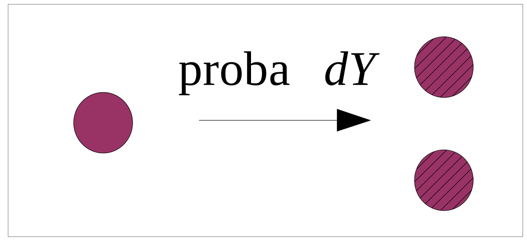
- Evolution equations for scattering amplitudes in high-energy QCD and their solutions
- Stochasticity/discreteness and its consequences: Example of a simple toy model
- What we have learnt so far for QCD

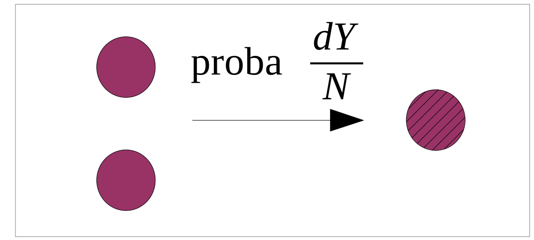
The simplest parton model

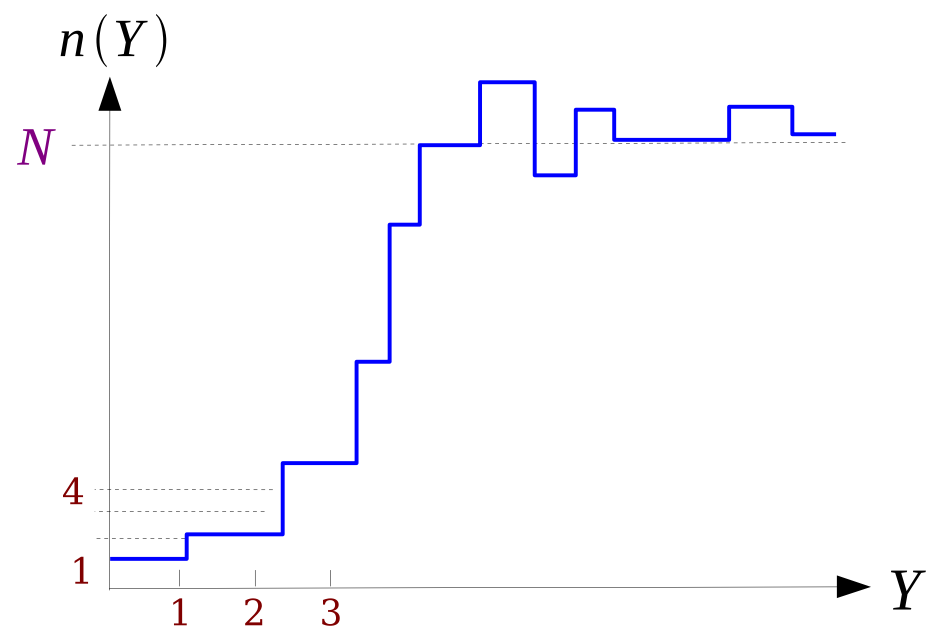
See Mueller, Salam (1995)

only one size

Evolution over the rapidity interval dY:







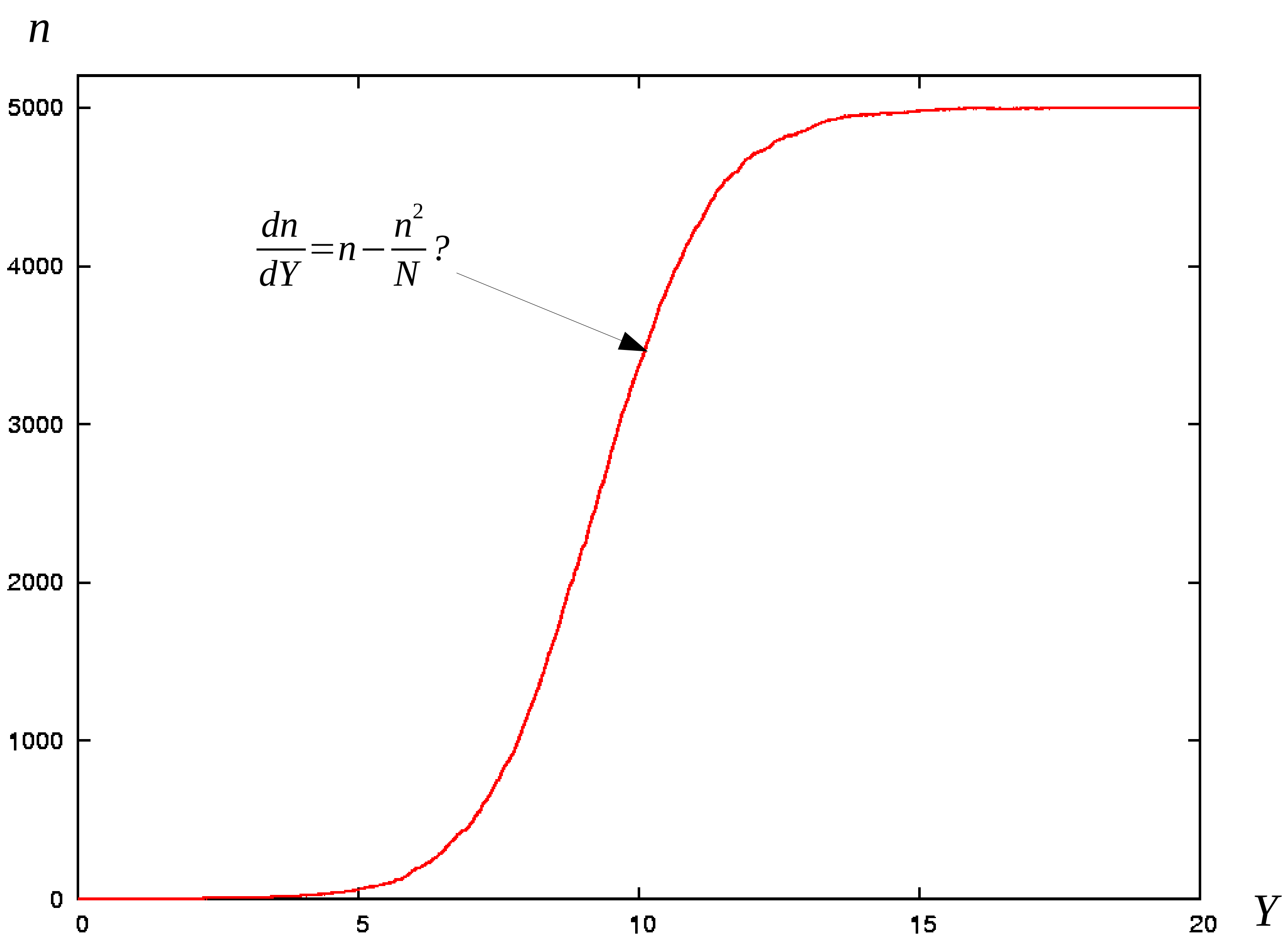
Simplest equation for the evolution of the number of particles with rapidity:

$$\frac{dn}{dY} = n - \frac{n^2}{N}$$

logistic equation $\sim$ BK equation

Numerical illustration

$N = 5000$



Numerical illustration

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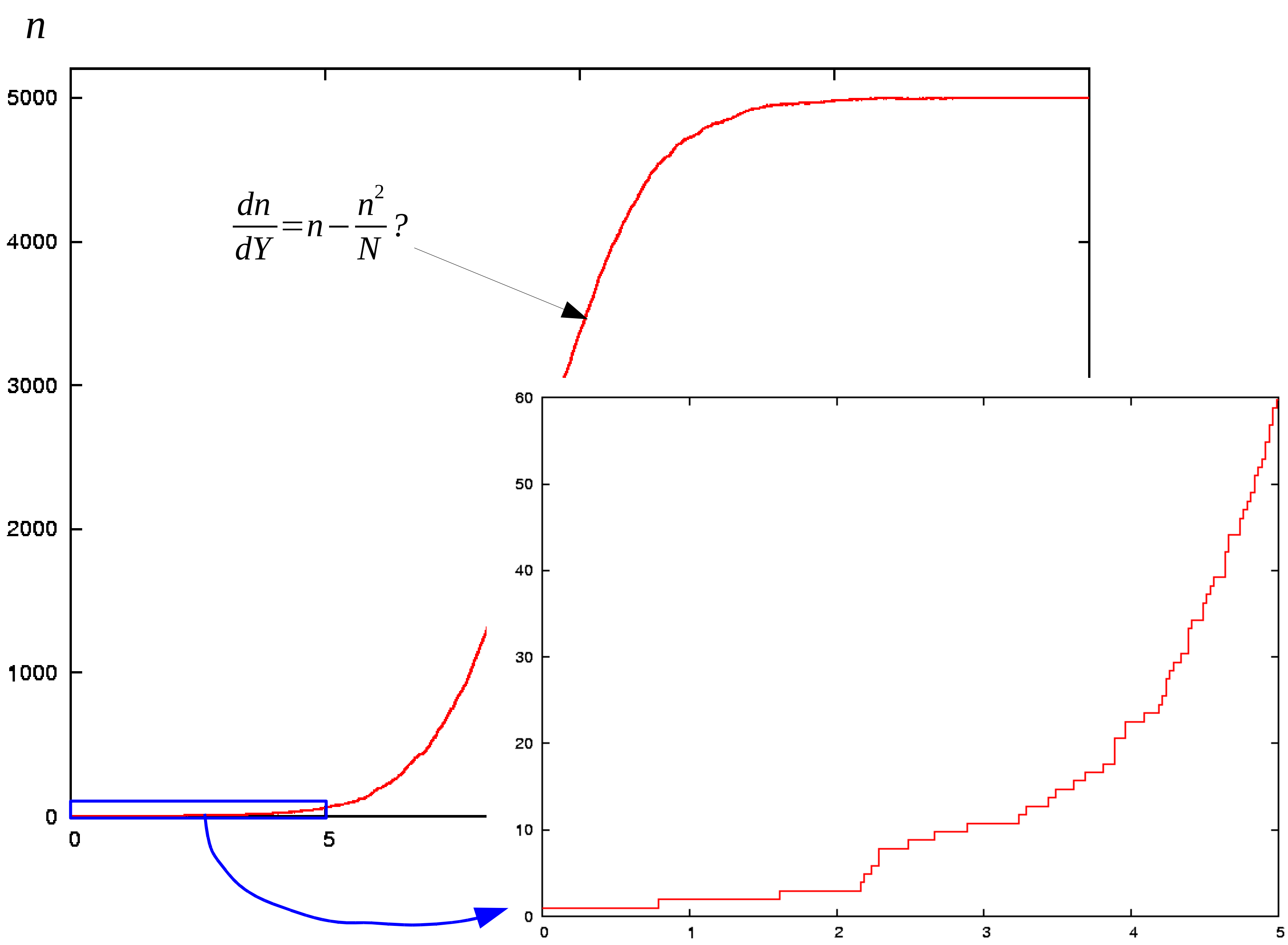

Numerical illustration

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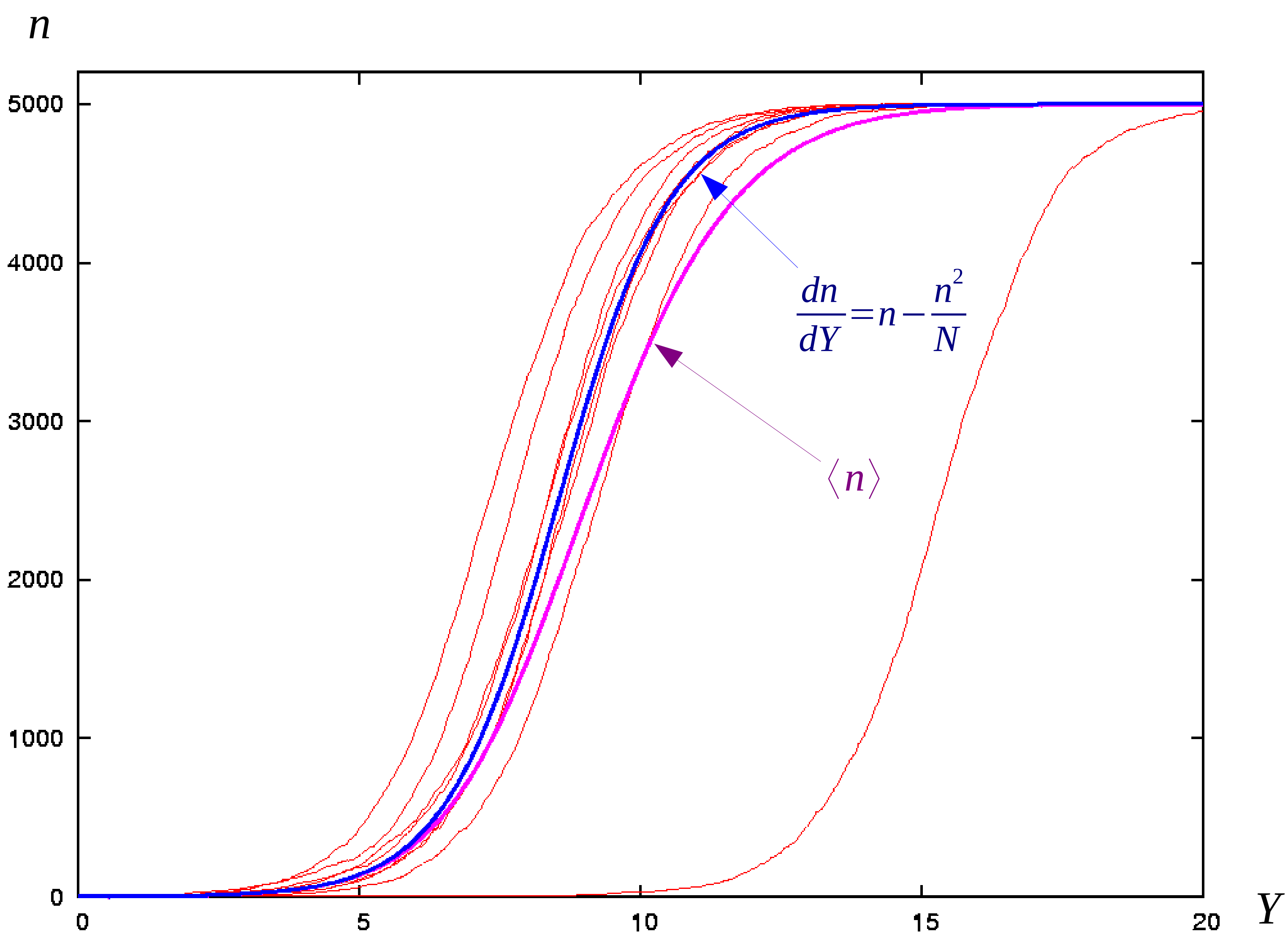


« Field theory » treatment

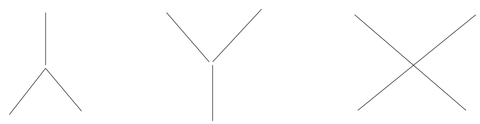
Shoshi, Xiao (2005)
Bondarenko, Motyka, Mueller, Shoshi, Xiao (2006)
Kozlov, Levin (2006)

Generating function for the factorial moments of n:

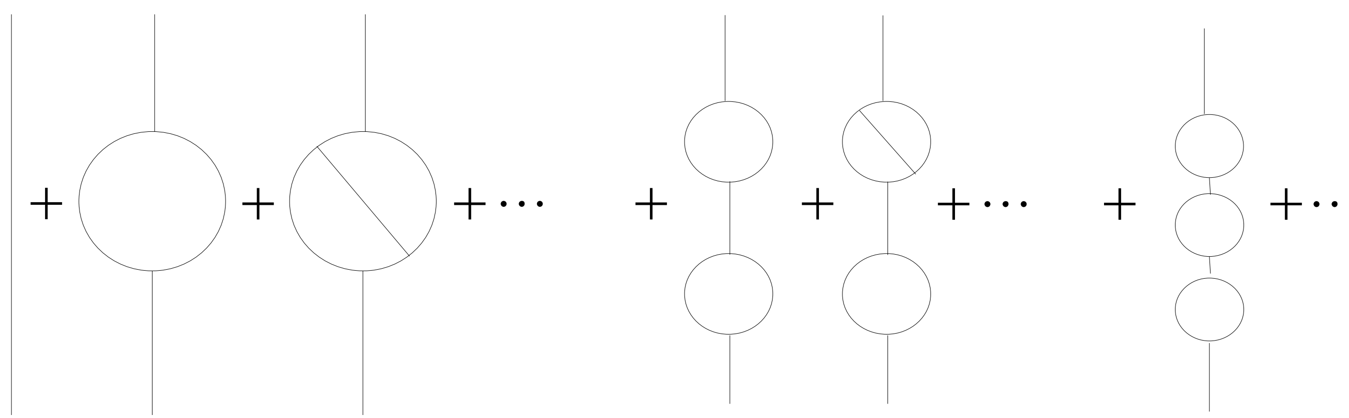
$$Z(Y,z)=\sum_{n=0}^{\infty}(1+z)^n P(Y,n)=\sum_k \frac{z^k}{k!}\left\langle\frac{n!}{(n-k)!}\right\rangle$$

obeys the equation $\quad \dfrac{\partial Z}{\partial Y}=z(1+z)\left(\dfrac{\partial Z}{\partial z}-\dfrac{1}{N}\dfrac{\partial^2 Z}{\partial z^2}\right)=-(H_0+H_1)Z$

Eigenstates of H_0 : k- "Pomeron" states, such that $|k\rangle_Y=e^{kY}|k\rangle_0$ and $\langle k|Z\rangle=\left\langle\dfrac{n!}{(n-k)!}\right\rangle$

$H_1=$ 

Shoshi, Xiao (2005)

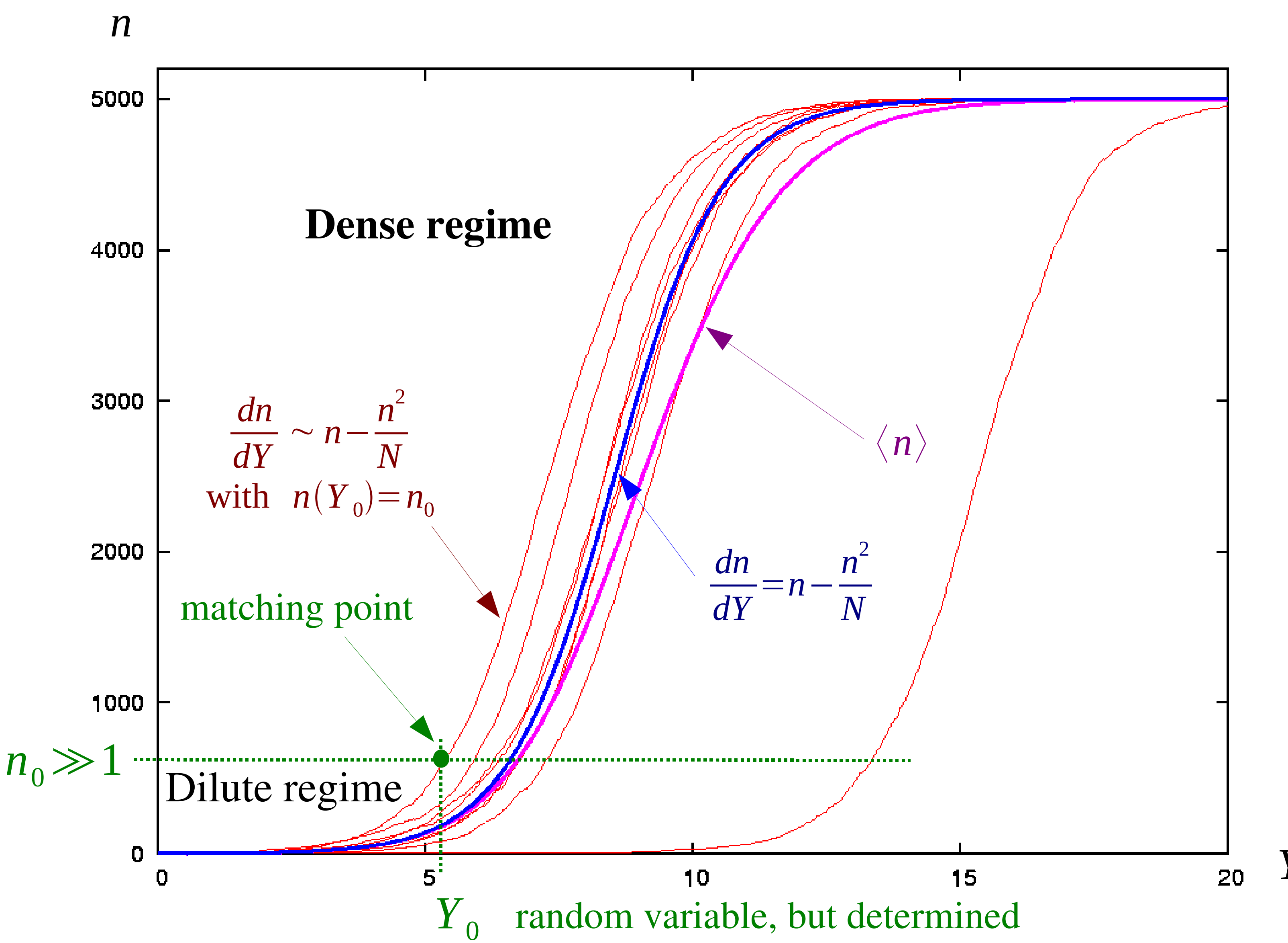
$\langle n\rangle=$ 

$e^Y \quad -2!\,\dfrac{e^{2Y}}{N} \quad +3!\,\dfrac{e^{3Y}}{N^2} \quad\quad +4Y\,\dfrac{e^{2Y}}{N^2}$

$$\frac{\langle n\rangle}{N}=\sum_{k=1}^{\infty}(-1)^{k-1}k!\,e^{kY}N^{k-2}\quad+\quad O\left(\frac{\log N}{N}\right)=\int_0^{\infty}du\ e^{-u}\frac{u\,e^Y}{u\,e^Y+N}+O\left(\frac{\log N}{N}\right)$$

"Event-by-event" treatment

SM (2006)



random variable, but determined by solving a linear equation

We have obtained an expression for <n>/N to order $O\left(\frac{1}{N^2}\right)$

Derrida, SM (in progress)

Summary of part II

The noise/discreteness has a crucial effect on physical observables, especially at the transition between the dilute and dense phase. In some situations (cf population dynamics, spread of illness etc...), **the noise changes the very nature of the problem.**

It is difficult to address in field theory. In particular, it seems to require the full resummation of asymptotic divergent series.

General problem to be addressed: *the matching of a mean-field and a discrete description of the same model.*
See corresponding works in statistical physics!

Outline

- ⋆ Evolution equations for scattering amplitudes in high-energy QCD and their solutions
- ⋆ Stochasticity/discreteness and its consequences: Example of a simple toy model
- ➡ ⋆ What we have learnt so far for QCD

Predictions for QCD amplitudes

$$\partial_{\bar\alpha Y} T = \chi(-\partial_\rho) T - T^2 + \alpha_s \sqrt{T}\,\nu$$

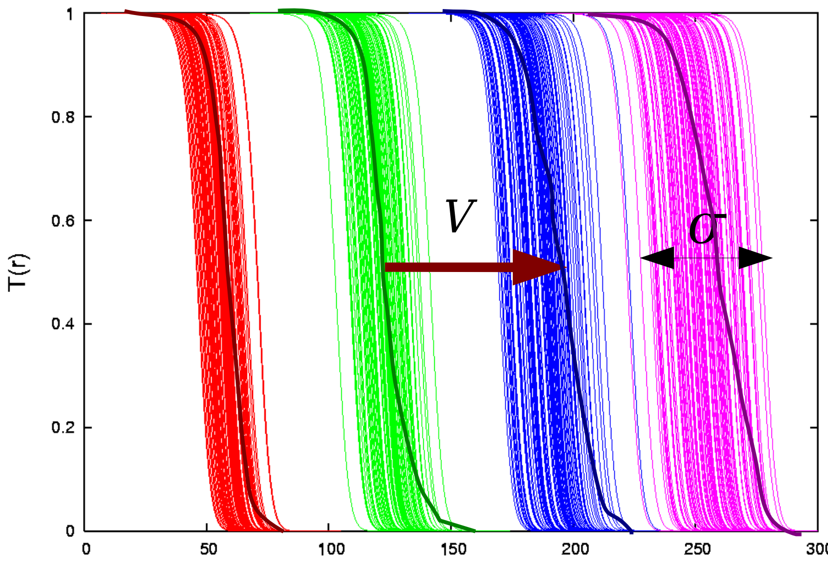
Rapidity dependence of the saturation scale:

$$\frac{d\langle\rho_s\rangle}{d(\bar\alpha Y)} = \frac{\chi(\gamma_0)}{\gamma_0} - \frac{\pi^2\gamma_0\chi''(\gamma_0)}{2\log^2(1/\alpha_s^2)} + \pi^2\gamma_0\chi''(\gamma_0)\frac{3\log\log(1/\alpha_s^2)}{\log^3(1/\alpha_s^2)}$$

$$\langle\rho_s^n\rangle_{cumulant} = \pi^2\chi''(\gamma_0)\frac{n!\zeta(n)}{\gamma_0^{n-2}}\left[\frac{\bar\alpha Y}{\log^3(1/\alpha_s^2)}\right]$$

New scaling that should replace geometric scaling:

$$A \sim A\left(\frac{\rho-\rho_s(Y)}{\sqrt{\bar\alpha Y/\log^3(1/\alpha_s^2)}}\right)$$



Traveling waves

Brunet, Derrida, Mueller, Munier (2004)

These formulas are independent of the precise form of the stochasticity and of the nonlinearity.

Summary

We have identified, from the physics, the universality class of high energy QCD as the one of reaction-diffusion processes, whose dynamics are governed by an equation of the form

$$\partial_{\bar{\alpha} Y} T = \chi(-\partial_\rho) T - T^2 + \alpha_s \sqrt{T}\, \nu$$

Discreteness of the partons, reflected by the noise term, plays parametrically an important role.

Such an equation has not been derived yet within QCD.
A lot of work is being done in this direction.

Nevertheless, this framework may enable one to get asymptotic properties of QCD.

So far, results at fixed (or uniform) impact parameter.
Work in progress: try and understand the spatial correlations.

SM, Salam, Soyez (2008)
Mueller, SM, work in progress