High Energy Scattering Amplitudes in AdS/CFT

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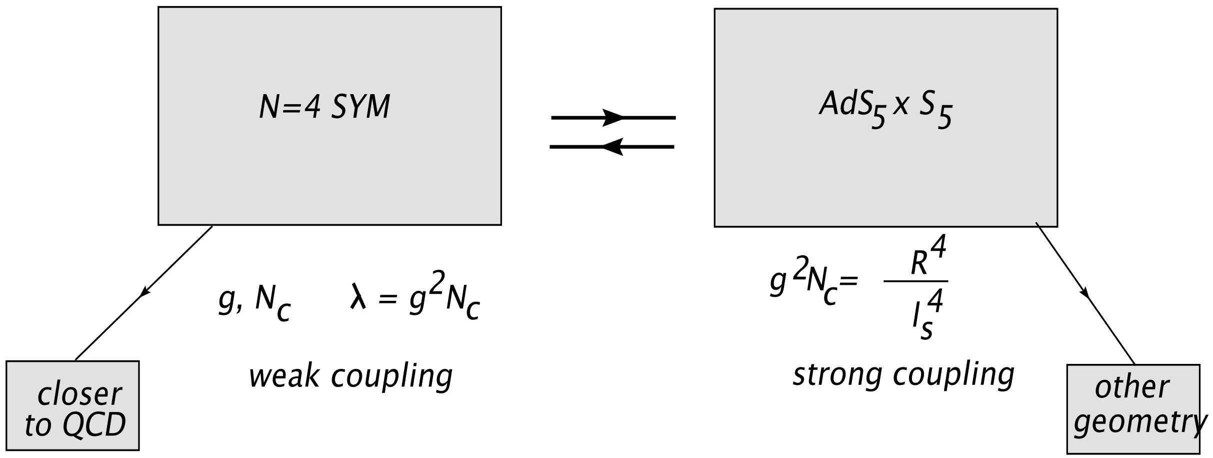
From Particles and Partons to Nuclei and Fields:
'Alfest', Columbia University October 23 - 25, 2009

- Introduction
- High energy scattering of planar amplitudes
- The Pomeron in AdS/CFT
- What is next: triple Pomeron vertex, integrability in high energy scattering amplitudes
- Conclusions

Introduction

Frame of this talk is the AdS/CFT correspondence hypothesis:



On both sides expansion in $1/N_c$ (expansion in toplogy).

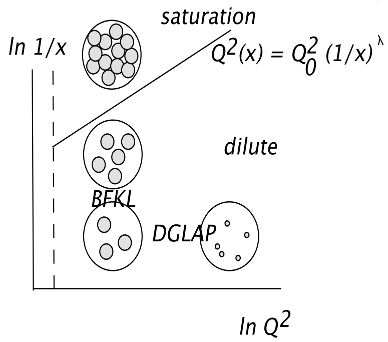
Is $N = 4$ SYM soluble: hope so far based (mainly) upon anomalous dimensions (integrability).
What about scattering amplitudes? Regge limit historically important.

This talk: two parts

(a) scattering amplitudes in the planar limit.
Main interest: n point amplitudes in $N=4$, guide for multiloop/multileg amplitudes in QCD, BDS formula.
Is $N=4$SYM soluble: integrability?

(b) Vacuum exchange (Pomeron, cylinder):
(Soft) Pomeron in hadron-hadron scattering is non-pertubative: need methods other the pQCD.
But: (Soft) Pomeron is also sensitive to low-energy features of QCD (slope α': chiral dynamics).

Hard Pomeron: in scattering of small-size projectiles (virtual photon)
Soft Pomeron: in hadron-hadron scattering
Transition in deep inelastic scattering (saturation, unitarization)

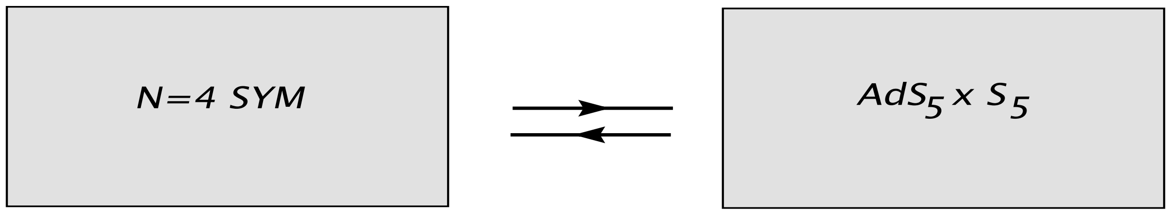


transition from hard to soft

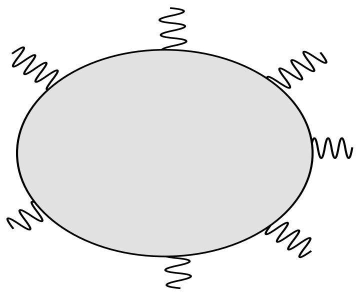
AdS/CFT correspondence: first the hard Pomeron, unitarization.
For soft Pomeron: need more sophisticated geometry on the string theory side (modelling).

Planar scattering amplitudes at high energies

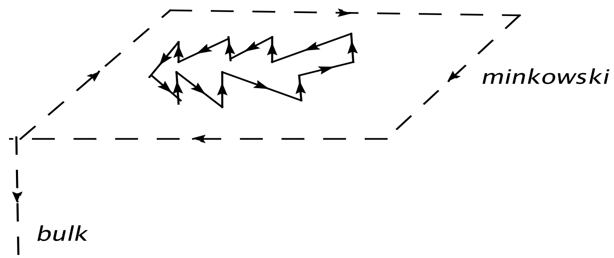
$N = 4$, MHV amplitudes. Duality:



$g, N_c \qquad \lambda = g^2 N_c \qquad\qquad g^2 N_c = \frac{R^4}{l_s^4}$



Compute planar amplitudes



Compute minimal surfaces
Amplitude has exponential form

Gauge theory side: enormous activity in two loop calculations, beyond MHV.
String theory side: mimimal surfaces are hard to compute, a few cases are known (Alday, Maldacena).

Most remarkable: Bern-Dixon-Smirnow (BDS) formula for planar n-gluon scattering amplitude:

Remove color factors, factor out tree amplitude, IR singular:

$$tr(T^{a1}...T^{an}) + noncycl.perm, \;\; A_n = A_n^{tree} \cdot M_n(\epsilon)$$

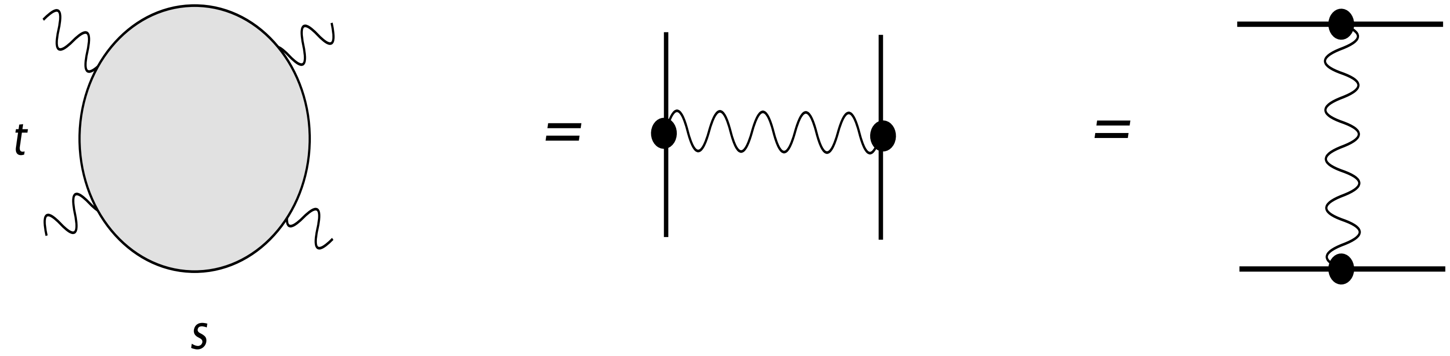
$$\ln M_n = \sum_l a^l \Big[\Big(f^{(l)}(\epsilon) I_n(l\epsilon) + F_n(0) \Big) + C^{(l)} + E_n^{(l)}[\epsilon] \Big]$$

$$a = \frac{N_c \alpha}{2\pi} (4\pi e^{-\gamma})^{\epsilon}, \;\; d = 4 - 2\epsilon$$

Based upon: universality of IR singularities (=poles in ϵ), and 1-loop calculation.

Several tests (Alday,Maldacena; Drummond,Korchemsky,Sokatchev; JB,Lipatov,Sabio-Vera):
partly successful ($n \leq 5$, partly disagreement $n \geq 6$).
This talk: high energy limit (Regge limit) of BDS formula (JB, Lipatov, Sabio Vera):

Four-point function:

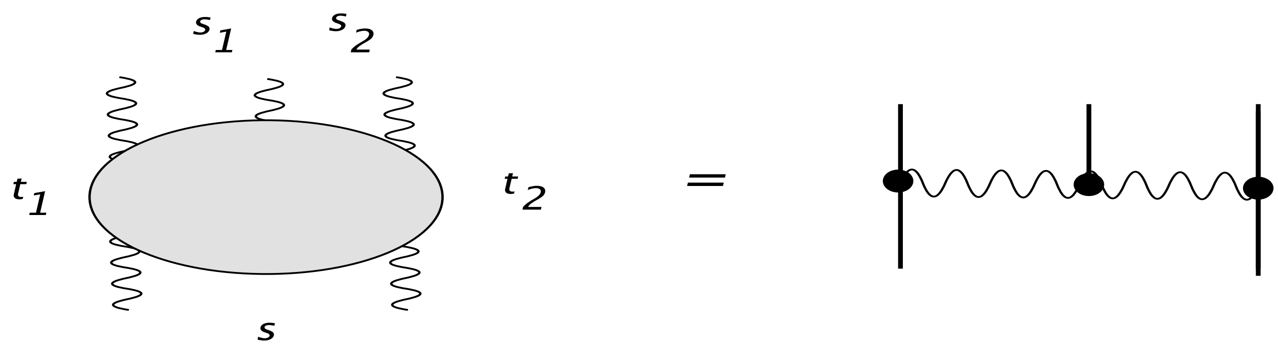


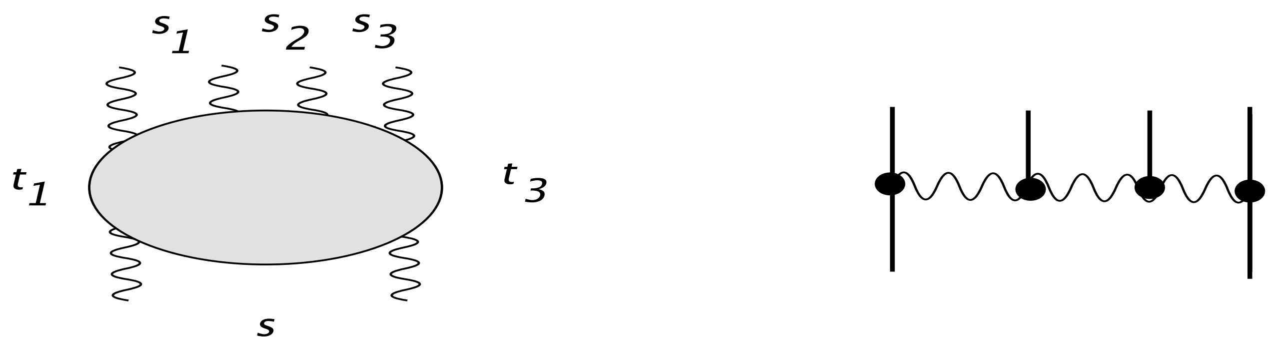
$$A_4(s,t) = \Gamma(t)\left(\frac{-s}{\mu^2}\right)^{\omega(t)}\Gamma(t) = \Gamma(s)\left(\frac{-t}{\mu^2}\right)^{\omega(s)}\Gamma(s)$$

All order gluon trajectory function, vertex function.

Comparison with Veneziano amplitude $B_4(s,t)$.

Five, six point functions:



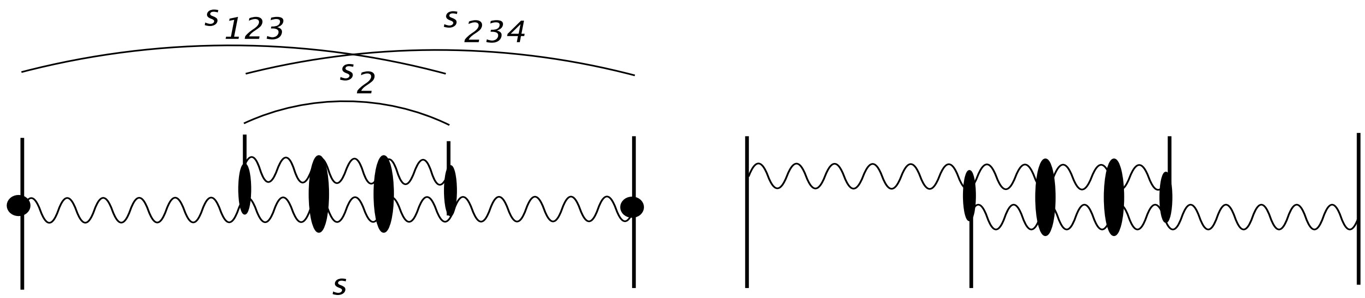


Same trajectory, vertex function, production vertex:

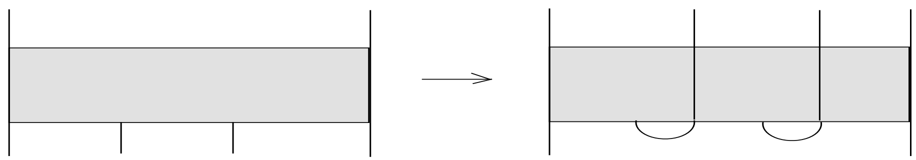
all seems to be consistent. But for $n \geq 6$:

Problem with the analytic structure:
scattering amplitudes = functions of several complex-valued variables: Steinmann relations

Comparison with leading-log calculations in QCD (JB, Lipatov,Sabio-Vera):
disagreement for $2 \longrightarrow 4$, $3 \longrightarrow 3$, ...:
piece is missing (beyond one loop): known since 1980



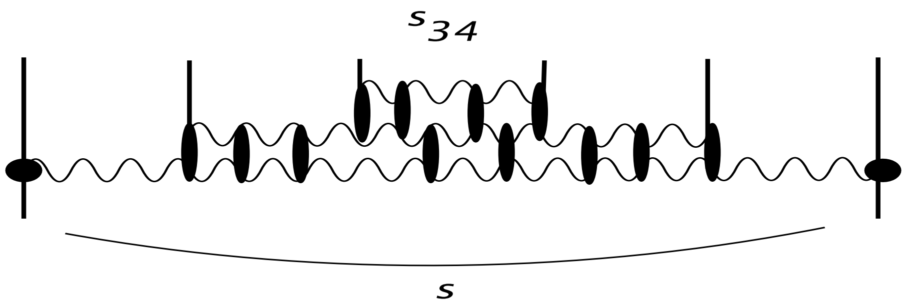
Visible in energy discontinuity or in another physical region:
$s, s_2 > 0$, $s_{123}, s_{234} < 0$:



Recent verfication through comparison with exact two-loop calculation (Schabinger)

Special feature of this extra piece: integrability.

Go to multi-leg amplitudes $n > 8$, e.g.

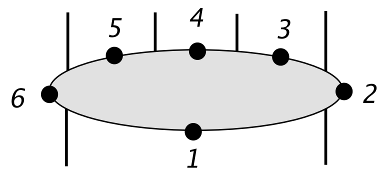


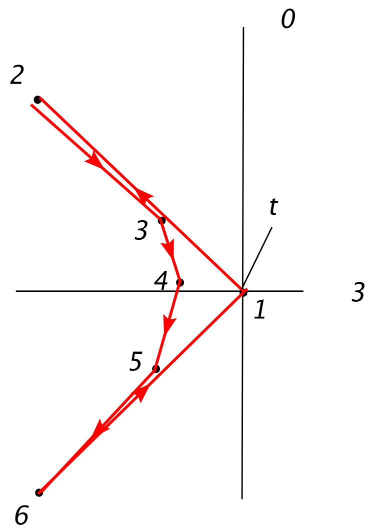
This Regge-cut piece, again, is visible in (double) energy discontinuities or in special physical regions. Dependence upon s_{34}:

$$A_8 \sim s_{34}^{-E_3}, \qquad \text{where} \qquad H_{3,open}\psi = E_3\psi$$

is the lowest energy of the BKP Hamiltonian describing the rapidity evolution in the t_3 channel. In the planar limit the t_3 channel is in a octet state: $H_{3,open}$ is integrable ($\longrightarrow$ Lipatov).

On the string side:
High energy limit contours on the string side have characteristic spike





Surfaces not known fo general n.
Analytic continuation of kinematic regions $\leftrightarrow$ relates different contours and minimal surfaces.
Study of these deformations might provide some guidance.

Task: correct the BDS formula.

The Pomeron in AdS/CFT

The 'Hard Pomeron': $\gamma^*\gamma^*$-scattering in QCD
In $N = 4$ SYM use R-currents (global $SU(4)$ symmetry) as substitute for the photon.
Elastic scattering:

$$< R_{\mu_1}(x_1) R_{\mu_2}(x_2) R_{\mu_3}(x_3) R_{\mu_4}(x_4) >$$

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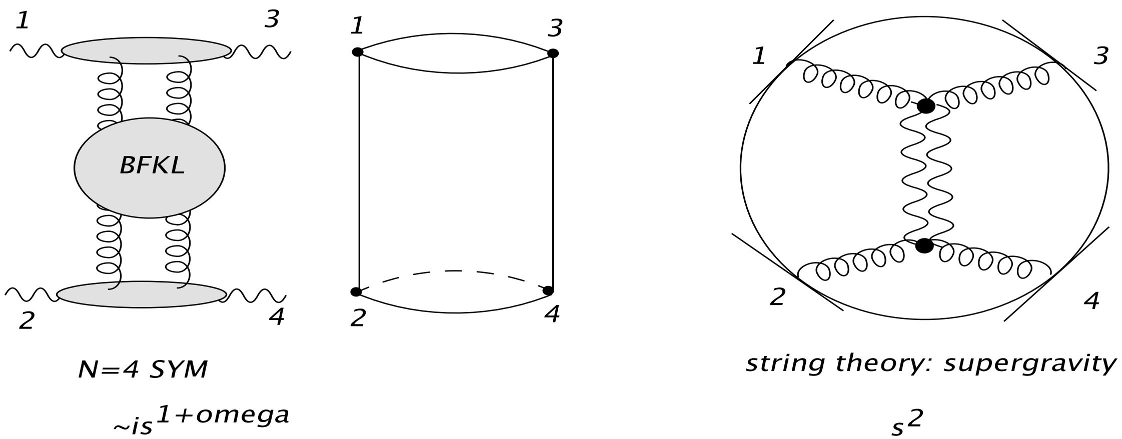


Basic message: BFKL in $N = 4$ SYM is dual to the graviton in AdS_5

In more detail:
on the weak coupling side the BFKL amplitude

$$A(s,t) = is \int \frac{d\omega}{2\pi i} \left(\frac{s}{kk'}\right)^{\omega} \Phi_1(Q_A^2, k, q-k) \otimes G_\omega(k, q-k; k', q-k') \otimes \Phi_2(Q_B^2, k', q-k'$$

LO impact factors for R-currents (JB,Mischler,Salvadore; Balitsky),
NLO characteristic BFKL function (Lipatov et al):

$$G_\omega(k, q-k; k', q-k') \sim \frac{1}{\omega - \chi(n,\nu)}$$

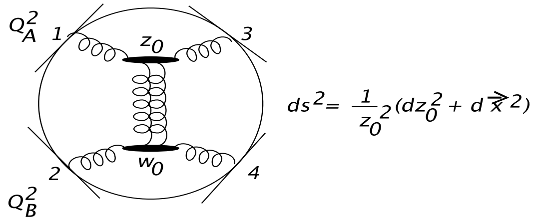
Connection between small x-limit and short distance limit (DIS):
leading twist anomalous dimension near $\omega = j - 1 \approx 0$

$$A(s, t=0) \sim \frac{is}{Q^2} \int \frac{d\omega}{2\pi i} \left(\frac{s}{Q_1^2}\right)^{\omega} \int \frac{d\nu}{2\pi i} \left(\frac{Q_1^2}{Q_2^2}\right)^{i\nu + \omega/2} \Phi_1(n,\nu) \frac{1}{\omega - \chi(\nu, 0)} \Phi_2(n,\nu)$$

The strong coupling side:

the leading term (in $1/\lambda$) is given by supergravity (Witten diagram): graviton exchange.
Calculation (Kotanski et al) gives:

$$I^{\mathrm{GR}} = \frac{1}{4} \int \frac{d^4 z dz_0}{z_0} \int \frac{d^4 w dw_0}{w_0} T_{(13)\mu\nu}(z) G_{\mu\nu;\mu'\nu'}(z,w) T_{(24)\mu'\nu'}(w) \, .$$



Fouriertransform, high energy limit, polarization vectors, helicity structure of the exchanged graviton:

$$\frac{2p_{2;\mu}p_{1;\mu'}}{s} \frac{2p_{2;\nu}p_{1;\nu'}}{s}$$

leads to

$$\mathcal{A}^{\mathrm{GR}}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t) = s^2 \int dz_0 dw_0 \Phi_{\lambda_1\lambda_3}(|\vec{p}_1|, |\vec{p}_3|; z_0) \, \Sigma(|\vec{p}_1 + \vec{p}_3|, z_0, w_0) \, \Phi_{\lambda_2\lambda_4}(|\vec{p}_2|, |\vec{p}_4|; w_0) \, .$$

Comparison with gauge theory side: 'Impact factors', integral over fifth coordinate analogous to transve
momentum.

Forward scattering:

$$\mathcal{A}^{\mathrm{GR}}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t=0) = s^2 \int_0^\infty dz_0 z_0^3 \Phi_{\lambda_1\lambda_3}(|\vec{p}_1|,|\vec{p}_3|;z_0) \times \int_0^\infty dw_0 w_0^3 \Phi_{\lambda_2\lambda_4}(|\vec{p}_2|,|\vec{p}_4|;w_0) \frac{1}{2} G_{\Delta=2,d=0}(\hat{u})$$

with

$$G_{\Delta=2,d=0}(\hat{u}) = \frac{1}{4w_0^2 z_0^2}(\theta(w_0-z_0)z_0^4 + \theta(z_0-w_0)w_0^4),$$

Limit of $Q_A^2 \gg Q_B^2$: dominant region close to the boundary (z_0 small, $r=1/z_0$ large): 'graviton $\leftrightarrow$ hard Pomeron' lives close to the boundary'.

Powers of $\ln Q_A^2/Q_B^2$, dependence upon polarization. beginning of OPE expansion?

Cannot see in Witten diagram: reggeization of the graviton. $j = 2 \to j = 2 - \frac{2}{\sqrt{\lambda}} + \mathcal{O}(\lambda)$.
More general (Lipatov et al, Polchinski et al): it exists function $j(\nu, \lambda)$

$$1 + \chi(\nu, \lambda) < j(\nu, \lambda) < 2 - \frac{4 + \nu^2}{2\sqrt{\lambda}} + \ldots$$

Diffusion in $\ln z$ (Polchinski et al.).

Result for $\gamma^* \gamma^*$ scattering

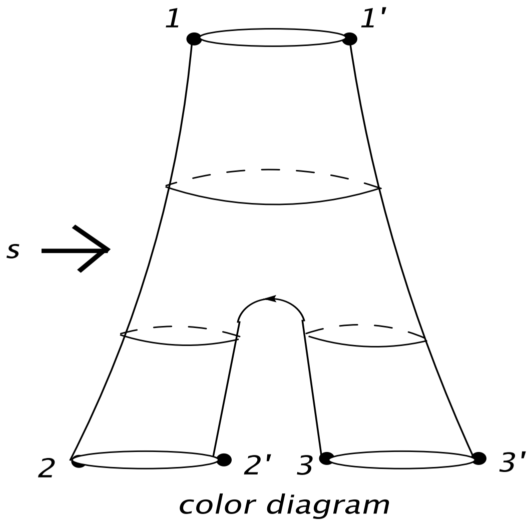
- intercept: function $j(\nu, \lambda)$ interpolates between weak and strong coupling: $1 < j(\nu, \lambda) < 2$.
  We know the first two corrections for $\lambda \to 0$, first correction at $\lambda \to \infty$.
  Connection with anomalous dimension.
- impact factor: we know the first term at $\lambda \to 0$, the first term at $\lambda \to \infty$.

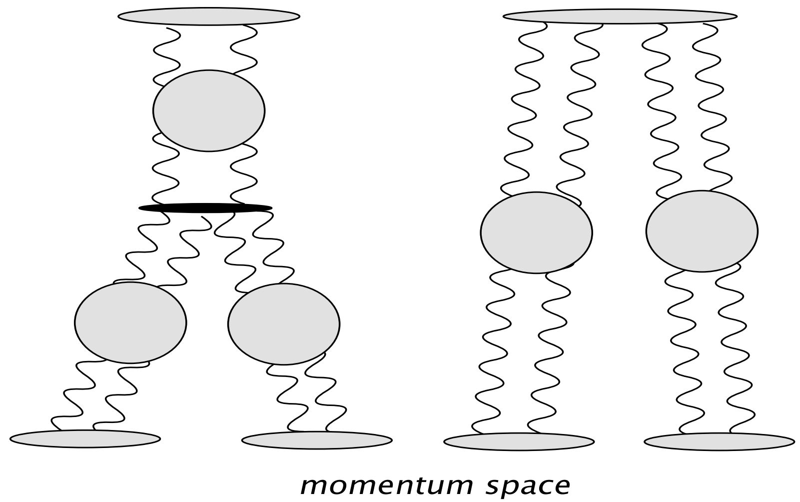
Wanted: represention which can be used for both weak and strong coupling
(Cornalba et al, Banks et al.).

What next: unitarization, integrability

A.Unitarization:
problem worse than BFKL: single graviton $\sim s^2$, double graviton $\sim s^3$,...
Need to go beyond planar (large-N_c limit): decided to study six-point function.

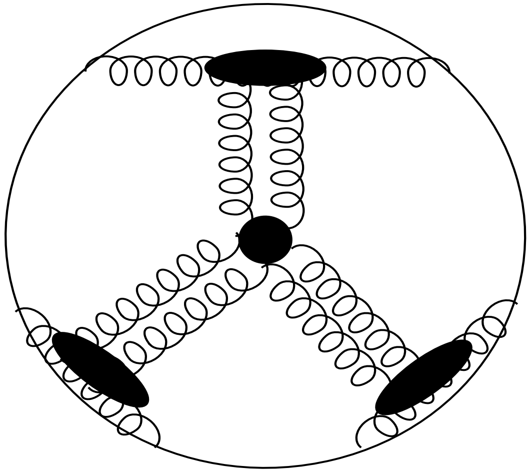
On the gauge theory side: pair-of pants topology:



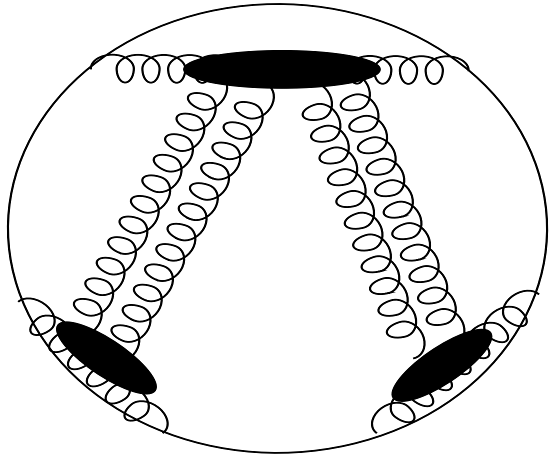


← new impact factor:
appears only in N=4 (not QCD)

On the string theory side:



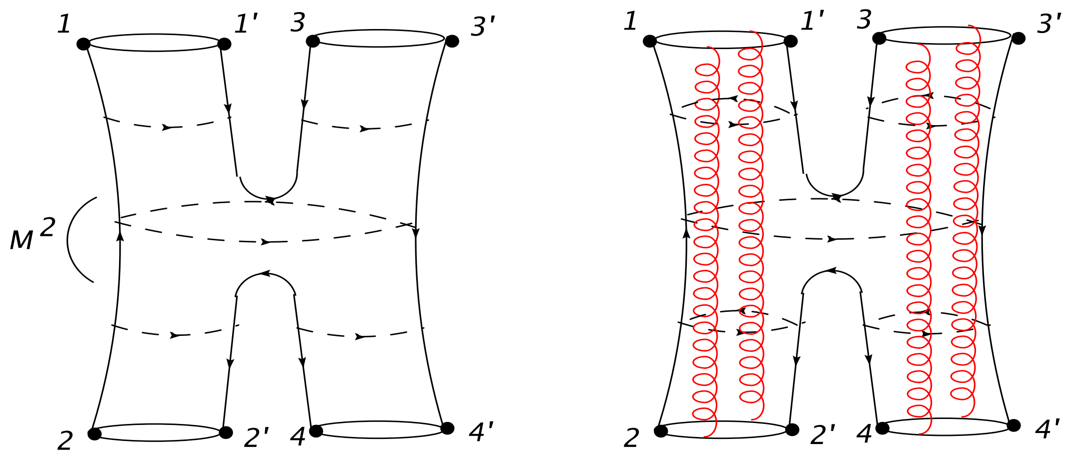
triple graviton vertex vanishes:
need string theory calculation



compute new impact factor
(leading order in $1/\lambda$)

B.Integrability:

Important feature of BFKL: generalize from 2 to $n > 2$ gluons,
(LO) Hamiltonian of BKP states is integrable for large N_c.
Where to find large-N_c BKP states: in multi-leg amplitudes, e.g. eight point correlator for $4 \longrightarrow 4$:



$$A_8 \sim {M^2}^{1-E}, \qquad \text{where} \qquad H_{4,closed}\psi = E\psi$$

E is the lowest eigenvalue of the energy spectrum of the 4 gluon BKP Hamiltonian (closed chain).

Conclusions

We are only at the beginning of exciting investigations.

- Planar amplitudes:
  'Islands' of integrability
  BDS formula: Regge limit should help to get correct the expression
- Pomeron: ‘Hard‘ Pomeron, interpolation from strong to weak coupling.

To work on:

- correct the BDS formula
- What is the role of integrability in high energy scattering on the string side?
- Unitarization? ‘Soft‘ Pomeron needs modelling, dual analogue of QCD.

Dear Al,

best wishes

and many more happy years!